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Research Article

Fractional Analysis of Coupled Burgers Equations within Yang Caputo-Fabrizio Operator

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This work applies a novel analytical technique to the fractional view analysis of coupled Burgers equations. The proposed problems have been fractionally analyzed in the Caputo-Fabrizio sense. The Yang transformation was initially applied to the specified problem in the current approach. The series form solution is then obtained using the Adomian decomposition technique. The desired analytical solution is obtained after performing the inverse transform. Specific examples of fractional Burgers couple systems are used to validate the proposed technique. The current strategy has been found to be a useful methodology with a close match to actual solutions. The proposed method offers a lower computing cost and a faster convergence rate. As a result, the suggested technique can be applied to a variety of fractional order problems.

1. Introduction

The branch of mathematics, which deals with the study of derivatives and integrals of non-integer orders, is known as fractional calculus (FC). It was born in 1695 on September 30 due to an important question asked by L'Hospital in a letter to Leibniz. The answer of Leibniz [1] gives motivation to a series of interesting results during the last 325 years [2–4]. In the last decades, FC has been used as a powerful tool by many researchers in various fields of science and engineering, for example, the fractional control theory [2, 5], anomalous diffusion, fractional neutron point kinetic model, fractional filters, soft matter mechanics, non-Fourier heat conduction, notably control theory, Levy statistics, nonlocal phenomena, fractional signal and image processing, porous media, fractional Brownian motion, relaxation, groundwater problems, rheology, acoustic dissipation, creep, fractional phase-locked loops, and fluid dynamics [6–10].

In recent years, fractional partial differential equations (FPDEs) have gained considerable interest because of their applications in various fields such as finance, biological processes and systems, fluid flow [11, 12], chaotic dynamics, electrochemistry, diffusion processes, material science, electromagnetic, turbulent flow [13–18], elastoplastic indentation problems [19], dynamics of van der Pol equation [20], and statistical mechanics model [21].

To find the solution of FPDEs is a hard task, however, many mathematicians devoted their sincere work and developed numerical and analytical techniques to solve FPDEs. Some of these techniques include homotopy analysis method (HAM) [22], operational matrix [23], Adomian decomposition method (ADM) [24], homotopy perturbation method (HPM) [25], meshless method [26], variational iteration method (VIM) [27], tau method [28], Bernstein polynomials [29], the Haar wavelet method [30], the Laplace transform method [31], the Legendre base method [32], Laplace variational iteration method [33], G'/G-expansion method [34], Jacobi spectral collocation method [35], Yang-Laplace transform [36], new spectral algorithm [37], fractional complex transform method [38], cylindrical-coordinate method [39], and spectral Legendre-Gauss-Lobatto collocation method [40].

The Burgers equation was initially introduced by Harry Bateman in the year 1915 [41]. They have many applications in various fields, especially in equations having nonlinear form. This equation describes many phenomena such as acoustic waves, heat conduction, dispersive water, shock waves [42], continuous stochastic processes [43], and modeling of dynamics [44–46]. The one-dimensional Burgers equations have many applications in plasma physics, gas dynamics, etc. [47]. Various techniques were developed by mathematicians to find the numerical and analytical solutions of Burgers equations. Some of these methods are a direct variational iteration method by Ozis and Ozdes [48]. Jaiswal [49] solved the equations numerically by finite difference method. Group explicit method was used by Evans and Abdullah [50]. Singhal and Mittal applied the Galerkin method [51] to solve these equations numerically. A weighted residue method was applied by Caldwell et al. [52]. Fractional Riccati expansion method was applied by Kurt et al. [53], and variational iteration method was applied by Inc [54] to solve space-time fractional Burgers equation. Esen et al. [55] used HAM to solve time-fractional Burgers equation. The cubic B-spline finite elements method was applied by Esen and Tasbozan to solve these equations [56].

Yang decomposition method (YDM) is one of the straightforward and effective techniques to solve nonlinear FPDEs. YDM possesses the combined behavior of Yang transformation and Adomian decomposition method (ADM). It is observed that the suggested method require no predefined declaration size like RK4. Laplace Adomian decomposition method required less number of parameters, no discritization, and linerization as compared to other analytical technique. Laplace Adomian decomposition method is also compared with ADM to analyze the solution of FPDEs given in [57]. The solution of Kundu-Eckhaus equation is discussed in [58], via Laplace Adomian decomposition method. Multistep Laplace Adomian decomposition method is implemented to solve FPDEs in [59]. Laplace Adomian decomposition method is also used for the solution of fractional Navier-Stokes and smoke models [60–62].

In the current study, we implemented YDM for the solution of coupled Burgers equations. The desired degree of accuracy is achieved. The procedure of the suggested technique is very simple and straightforward. The accuracy is calculated in terms of absolute error. The results have shown the present method has the desired accuracy as compared to other analytical techniques.

2. Preliminary Concepts

We provide the fundamental definitions that will be used throughout the article. For the purpose of simplification, we write the exponential decay kernel as, $K(\Psi, \varrho) = e^{[-\wp(\Psi-\varrho/1-\wp)]}$.

Definition 1. If the Caputo-Fabrizio derivative is given as follows [63]:

$$ {}^{CF}D^{\wp}_{\Psi}[\mathbb{P}(\Psi)] = \frac{N(\wp)}{1-\wp}\int_0^{\Psi} \mathbb{P}'(\varrho)K(\Psi,\varrho)d\varrho, \quad n-1<\wp\leq n. \tag{1}$$

$N(\wp)$ is the normalization function with $N(0)=N(1)=1$.

$$ {}^{CF}D^{\wp}_{\Psi}[\mathbb{P}(\Psi)] = \frac{N(\wp)}{1-\wp}\int_0^{\Psi} [\mathbb{P}(\Psi)-\mathbb{P}(\varrho)]K(\Psi,\varrho)d\varrho. \tag{2}$$

Definition 2. The fractional integral Caputo-Fabrizio is given as [63]

$$ {}^{CF}I^{\wp}_{\Psi}[\mathbb{P}(\Psi)] = \frac{1-\wp}{N(\wp)}\mathbb{P}(\Psi) + \frac{\wp}{N(\wp)}\int_0^{\Psi}\mathbb{P}(\varrho)d\varrho, \quad \Psi\geq 0, \wp\in(0,1]. \tag{3}$$

Definition 3. For $N(\wp)=1$, the following result shows the Caputo-Fabrizio derivative of Laplace transformation [63]:

$$ L\left[{}^{CF}D^{\wp}_{\Psi}[\mathbb{P}(\Psi)]\right] = \frac{\nu L[\mathbb{P}(\Psi)-\mathbb{P}(0)]}{\nu+\wp(1-\nu)}. \tag{4}$$

Definition 4. The Yang transformation of $\mathbb{P}(\Psi)$ is expressed as [64].

$$ \mathbb{Y}[\mathbb{P}(\Psi)] = \chi(\nu) = \int_0^{\infty}\mathbb{P}(\Psi)e^{-\frac{\Psi}{\nu}}d\Psi. \quad \Psi>0. \tag{5}$$

Remarks 5. Yang transformation of few useful functions is defined as below.

$$ \begin{aligned} \mathbb{Y}[1] &= \nu, \\ \mathbb{Y}[\Psi] &= \nu^2, \\ \mathbb{Y}\left[\Psi^i\right] &= \Gamma(i+1)\nu^{i+1}. \end{aligned} \tag{6}$$

Lemma 6 (Laplace-Yang duality). *Let the Laplace transformation of $\mathbb{P}(\Psi)$ is $F(\nu)$, then $\chi(\nu)=F(1/\nu)$ [65].*

Proof. From equation (5), we can achieve another type of the Yang transformation by putting $\Psi/\nu=\zeta$ as

$$ L[\mathbb{P}(\Psi)] = \chi(\nu) = \nu\int_0^{\infty}\mathbb{P}(\nu\zeta)e^{\zeta}d\zeta. \quad \zeta>0, \tag{7}$$

Since $L[\mathbb{P}(\Psi)]=F(\nu)$, this implies that

$$ F(\nu) = L[\mathbb{P}(\Psi)] = \int_0^{\infty}\mathbb{P}(\Psi)e^{-\nu\Psi}d\Psi. \tag{8}$$

Put $\Psi=\zeta/\nu$ in (8), we have

$$ F(\nu) = \frac{1}{\nu}\int_0^{\infty}\mathbb{P}\left(\frac{\zeta}{\nu}\right)e^{\zeta}d\zeta. \tag{9}$$

Thus, from equation (7), we achieve

$$F(\nu) = \chi\left(\frac{1}{\nu}\right). \tag{10}$$

Also from equations. (5) and (8), we achieve

$$F\left(\frac{1}{\nu}\right) = \chi(\nu). \tag{11}$$

The connections (10) and (11) represent the duality link between the Laplace and Yang transformation. □

Lemma 7. *Let $\mathbb{P}(\Psi)$ be a continuous function; then, the Caputo-Fabrizio derivative Yang transformation of $\mathbb{P}(\Psi)$ is define by [65].*

$$\mathbb{Y}[\mathbb{P}(\Psi)] = \frac{\mathbb{Y}[\mathbb{P}(\Psi) - \nu\mathbb{P}(0)]}{1+\wp(\nu-1)}. \tag{12}$$

Proof. The Caputo-Fabrizio fractional Laplace transformation is given by

$$L[\mathbb{P}(\Psi)] = \frac{L[\nu\mathbb{P}(\Psi) - \mathbb{P}(0)]}{\nu+\wp(1-\nu)}. \tag{13}$$

Also, we have that the connection among Laplace and Yang property, i.e., $\chi(\nu) = F(1/\nu)$. To achieve the necessary result, we substitute ν by $1/\nu$ in equation (13), and we get

$$\begin{aligned} \mathbb{Y}[\mathbb{P}(\Psi)] &= \frac{(1/\nu)\mathbb{Y}[\mathbb{P}(\Psi) - \mathbb{P}(0)]}{(1/\nu)+\wp(1-1/\nu)}, \\ \mathbb{Y}[\mathbb{P}(\Psi)] &= \frac{\mathbb{Y}[\mathbb{P}(\Psi) - \nu\mathbb{P}(0)]}{1+\wp(\nu-1)}. \end{aligned} \tag{14}$$

The proof is completed. □

3. Implementation of YDM with Caputo-Fabrizio

To explain the fundamental concept of this technique, we consider a particular fractional-order nonlinear partial differential equation:

$$^{CF}D^{\delta}u(\xi,\Psi) + Lu(\xi,\Psi) + Nu(\xi,\Psi) = q(\xi,\Psi), \quad \xi,\Psi \geq 0, \quad m-1 < \delta < m, \tag{15}$$

where the fractional derivative in equation (15) is defined in Caputo-Fabrizio. The operator $\mathscr{R}$ and $\mathscr{N}$ describe the linear and nonlinear operators, respectively, and $g(\zeta,\Psi)$ is the source term.

The initial condition is

$$u(\xi,0) = k(\xi), \tag{16}$$

Using Yang transformation to equation (15), we get

$$\mathscr{Y}\left[D^{\delta}u(\xi,\Psi)\right] + \mathscr{Y}[Lu(\xi,\Psi) + Nu(\xi,\Psi)] = \mathscr{Y}[q(\xi,\Psi)], \tag{17}$$

with the help of fractional derivative Yang property, we have

$$\begin{aligned} &\frac{1}{(1+\delta(s-1))}\mathscr{Y}\{u(\xi,0)\} - su(\xi,0) \\ &\quad = \mathscr{Y}[q(\xi,\Psi)] - \mathscr{Y}[Lu(\xi,\Psi) + Nu(\xi,\Psi)], \end{aligned} \tag{18}$$

$$\begin{aligned} \mathscr{Y}[u(\xi,\Psi)] = {} & sk(\xi) + (1+\delta(s-1))\mathscr{Y}[q(\xi,\Psi)] \\ & - (1+\delta(s-1))\mathscr{Y}[Lu(\xi,\Psi) + Nu(\xi,\Psi)]. \end{aligned} \tag{19}$$

Using YDM procedure, the solution is expressed as

$$u(\xi,\Psi) = \sum_{j=0}^{\infty} u_j(\xi,\Psi), \tag{20}$$

The nonlinear term can be decomposed as

$$Nu(\xi,\Psi) = \sum_{j=0}^{\infty} A_j, \tag{21}$$

$$A_j = \frac{1}{j!}\left[\frac{d^j}{d\lambda^j}\left[N\sum_{j=0}^{\infty}\left(\lambda^j u_j\right)\right]\right]_{\lambda=0}, \quad j = 0, 1, 2\cdots, \tag{22}$$

substitution (20) and (21) in equation (18), we get

$$\begin{aligned} \mathscr{Y}\left[\sum_{j=0}^{\infty} u(\xi,\Psi)\right] = {} & sk(\xi) + (1+\delta(s-1))\mathscr{Y}[q(\xi,\Psi)] \\ & - (1+\delta(s-1))\mathscr{Y}\left[L\sum_{j=0}^{\infty} u_j(\xi,\Psi) + \sum_{j=0}^{\infty} A_j\right]. \end{aligned} \tag{23}$$

$$\mathscr{Y}[u_0(\xi,\Psi)] = su(\xi,0) + (1+\delta(s-1))\mathscr{Y}[q(\xi,\Psi)], \tag{24}$$

$$\mathscr{Y}[u_1(\xi,\Psi)] = -(1+\delta(s-1))\mathscr{Y}[Lu_0(\xi,\Psi) + A_0]. \tag{25}$$

Generally, we can write

$$\mathscr{Y}\left[u_{j+1}(\xi,\Psi)\right] = -(1+\delta(s-1))\mathscr{Y}\left[Lu_j(\xi,\Psi) + A_j\right], \quad j \geq 1. \tag{26}$$

Taking the inverse Yang transformation of Eq. (26), we get

$$u_0(\xi,\Psi) = k(\xi,\Psi) + \mathscr{Y}^{-1}[(1+\delta(s-1))\mathscr{Y}[q(\xi,\Psi)]], \tag{27}$$

$$u_{j+1}(\xi,\Psi)=-\mathscr{Y}^{-1}\left[(1+\delta(s-1))\mathscr{Y}\left[Lu_j(\xi,\Psi)+A_j\right]\right]. \tag{28}$$

4. Example

Consider the following fractional-order coupled Burgers equations:

$$\begin{aligned}\frac{{}^{CF}\partial^\delta\mu}{\partial\Psi^\delta}+\frac{\partial^2\mu}{\partial\zeta^2}-2\mu\frac{\partial\mu}{\partial\zeta}-\frac{\partial(\mu\nu)}{\partial\zeta}&=0,\\ \frac{{}^{CF}\partial^\delta\nu}{\partial\Psi^\delta}+\frac{\partial^2\nu}{\partial\zeta^2}-2\nu\frac{\partial\nu}{\partial\zeta}-\frac{\partial(\mu\nu)}{\partial\zeta}&=0,\quad 0<\delta\le 1,\end{aligned} \tag{29}$$

with initial conditions

$$\mu(\zeta,0)=\sin(\zeta),\quad \nu(\zeta,0)=-\sin(\zeta). \tag{30}$$

Taking Yang transform of (29),

$$\mathscr{Y}\left[\frac{\partial^\delta\mu}{\partial\Psi^\delta}\right]=-\mathscr{Y}\left[\frac{\partial^2\mu}{\partial\zeta^2}-2\mu\frac{\partial\mu}{\partial\zeta}-\frac{\partial(\mu\nu)}{\partial\zeta}\right], \tag{31}$$

$$\mathscr{Y}\left[\frac{\partial^\delta\nu}{\partial\Psi^\delta}\right]=-\mathscr{Y}\left[\frac{\partial^2\nu}{\partial\zeta^2}-2\nu\frac{\partial\nu}{\partial\zeta}-\frac{\partial(\mu\nu)}{\partial\zeta}\right], \tag{32}$$

$$\frac{1}{(1+\delta(s-1))}\mathscr{Y}\{\mu(\zeta,0)\}-s\mu(\zeta,0)=-\mathscr{Y}\left[\frac{\partial^2\mu}{\partial\zeta^2}-2\mu\frac{\partial\mu}{\partial\zeta}-\frac{\partial(\mu\nu)}{\partial\zeta}\right], \tag{33}$$

$$\frac{1}{(1+\delta(s-1))}\mathscr{Y}\{\nu(\zeta,0)\}-s\nu(\zeta,0)=-\mathscr{Y}\left[\frac{\partial^2\nu}{\partial\zeta^2}-2\nu\frac{\partial\nu}{\partial\zeta}-\frac{\partial(\mu\nu)}{\partial\zeta}\right]. \tag{34}$$

Applying inverse Yang transform

$$\mu(\zeta,\Psi)=\mathscr{Y}^{-1}\left[s\mu(\zeta,0)-(1+\delta(s-1))\mathscr{Y}\left\{\frac{\partial^2\mu}{\partial\zeta^2}-2\mu\frac{\partial\mu}{\partial\zeta}-\frac{\partial(\mu\nu)}{\partial\zeta}\right\}\right], \tag{35}$$

$$\nu(\zeta,\Psi)=\mathscr{Y}^{-1}\left[s\mu(\zeta,0)-(1+\delta(s-1))\mathscr{Y}\left\{\frac{\partial^2\nu}{\partial\zeta^2}-2\nu\frac{\partial\nu}{\partial\zeta}-\frac{\partial(\mu\nu)}{\partial\zeta}\right\}\right], \tag{36}$$

$$\mu(\zeta,\Psi)=\sin(\zeta)-\mathscr{Y}^{-1}\left[(1+\delta(s-1))\mathscr{Y}\left\{\frac{\partial^2\mu}{\partial\zeta^2}-2\mu\frac{\partial\mu}{\partial\zeta}-\frac{\partial(\mu\nu)}{\partial\zeta}\right\}\right], \tag{37}$$

$$\nu(\zeta,\Psi)=-\sin(\zeta)-\mathscr{Y}^{-1}\left[(1+\delta(s-1))\mathscr{Y}\left\{\frac{\partial^2\nu}{\partial\zeta^2}-2\nu\frac{\partial\nu}{\partial\zeta}-\frac{\partial(\mu\nu)}{\partial\zeta}\right\}\right]. \tag{38}$$

Using ADM procedure, we get

$$\sum_{j=0}^{\infty}\mu_j(\zeta,\Psi)=\sin(\zeta)-\mathscr{Y}^{-1}\left[(1+\delta(s-1))\mathscr{Y}\left\{\sum_{j=0}^{\infty}(\mu_{\zeta\zeta})_j-2\sum_{j=0}^{\infty}A_j(\mu\mu_\zeta)-\sum_{j=0}^{\infty}B_j(\mu\nu)_\zeta\right\}\right], \tag{39}$$

$$\sum_{j=0}^{\infty}\nu_j(\zeta,\Psi)=-\sin(\zeta)-\mathscr{Y}^{-1}\left[(1+\delta(s-1))\mathscr{Y}\left\{\sum_{j=0}^{\infty}(\nu_{\zeta\zeta})_j-2\sum_{j=0}^{\infty}C_j(\nu\nu_\zeta)-\sum_{j=0}^{\infty}D_j(\mu\nu)_\zeta\right\}\right], \tag{40}$$

where $A_j(\mu\mu_\zeta)$, $B_j(\mu\nu)_\zeta$, $C_j(\nu\nu_\zeta)$, and $D_j(\mu\nu)_\zeta$ are Adomian polynomials are given below,

$$\begin{aligned}A_0(\mu\mu_\zeta)&=\mu_0\frac{\partial\mu_0}{\partial\zeta}, & B_0(\mu\nu)_\zeta&=\frac{\partial\mu_0}{\partial\zeta}\frac{\partial\nu_0}{\partial\zeta},\\ A_1(\mu\mu_\zeta)&=\mu_0\frac{\partial\mu_1}{\partial\zeta}+\mu_1\frac{\partial\mu_0}{\partial\zeta}, & B_1(\mu\nu)_\zeta&=\frac{\partial\mu_0}{\partial\zeta}\frac{\partial\nu_1}{\partial\zeta}+\frac{\partial\mu_1}{\partial\zeta}\frac{\partial\nu_0}{\partial\zeta},\\ A_2(\mu\mu_\zeta)&=\mu_0\frac{\partial\mu_2}{\partial\zeta}+\mu_1\frac{\partial\mu_1}{\partial\zeta}+\mu_2\frac{\partial\mu_0}{\partial\zeta}. & B_2(\mu\nu)_\zeta&=\frac{\partial\mu_0}{\partial\zeta}\frac{\partial\nu_2}{\partial\zeta}+\frac{\partial\mu_1}{\partial\zeta}\frac{\partial\nu_1}{\partial\zeta}+\frac{\partial\mu_2}{\partial\zeta}\frac{\partial\nu_0}{\partial\zeta}.\end{aligned} \tag{41}$$

$$\begin{aligned}C_0(\nu\nu_\zeta)&=\nu_0\frac{\partial\nu_0}{\partial\zeta}, & D_0(\mu\nu)_\zeta&=\frac{\partial\mu_0}{\partial\zeta}\frac{\partial\nu_0}{\partial\zeta},\\ C_1(\nu\nu_\zeta)&=\nu_0\frac{\partial\nu_1}{\partial\zeta}+\nu_1\frac{\partial\nu_0}{\partial\zeta}, & D_1(\mu\nu)_\zeta&=\frac{\partial\mu_0}{\partial\zeta}\frac{\partial\nu_1}{\partial\zeta}+\frac{\partial\mu_1}{\partial\zeta}\frac{\partial\nu_0}{\partial\zeta},\\ C_2(\nu\nu_\zeta)&=\nu_0\frac{\partial\nu_2}{\partial\zeta}+\nu_1\frac{\partial\nu_1}{\partial\zeta}+\nu_2\frac{\partial\nu_0}{\partial\zeta}. & D_2(\mu\nu)_\zeta&=\frac{\partial\mu_0}{\partial\zeta}\frac{\partial\nu_2}{\partial\zeta}+\frac{\partial\mu_1}{\partial\zeta}\frac{\partial\nu_1}{\partial\zeta}+\frac{\partial\mu_2}{\partial\zeta}\frac{\partial\nu_0}{\partial\zeta}.\end{aligned} \tag{42}$$

$$\begin{aligned}\mu_0(\zeta,\Psi)&=\sin\zeta,\\ \nu_0(\zeta,\Psi)&=-\sin(\zeta),\end{aligned} \tag{43}$$

$$\mu_{j+1}(\zeta,\Psi)=-\mathscr{Y}^{-1}\left[(1+\delta(s-1))\mathscr{Y}\left\{\sum_{j=0}^{\infty}(\mu_{\zeta\zeta})_j-2\sum_{j=0}^{\infty}A_j(\mu\mu_\zeta)-\sum_{j=0}^{\infty}B_j(\mu\nu)_\zeta\right\}\right], \tag{44}$$

$$\nu_{j+1}(\zeta,\Psi)=-\mathscr{Y}^{-1}\left[(1+\delta(s-1))\mathscr{Y}\left\{\sum_{j=0}^{\infty}(\nu_{\zeta\zeta})_j-2\sum_{j=0}^{\infty}C_j(\nu\nu_\zeta)-\sum_{j=0}^{\infty}D_j(\mu\nu)_\zeta\right\}\right], \tag{45}$$

for $j=0,1,2\cdots$

$$\begin{aligned}\mu_1(\zeta,\Psi)&=-\mathscr{Y}^{-1}\left[(1+\delta(s-1))\mathscr{Y}\left\{\frac{\partial^2\mu_0}{\partial\zeta^2}-2\mu_0\frac{\partial\mu_0}{\partial\zeta}-\frac{\partial\mu_0}{\partial\zeta}\frac{\partial\nu_0}{\partial\zeta}\right\}\right],\\ \mu_1(\zeta,\Psi)&=-\mathscr{Y}^{-1}\left[(1+\delta(s-1))\times\frac{-\sin\zeta}{s}\right]=\sin(\zeta)\{\delta\Psi+(1-\delta)\},\\ \nu_1(\zeta,\Psi)&=-\mathscr{Y}^{-1}\left[(1+\delta(s-1))\mathscr{Y}\left\{\frac{\partial^2\nu_0}{\partial\zeta^2}-2\nu_0\frac{\partial\nu_0}{\partial\zeta}-\frac{\partial\mu_0}{\partial\zeta}\frac{\partial\nu_0}{\partial\zeta}\right\}\right]\\ \nu_1(\zeta,\Psi)&=-\mathscr{Y}^{-1}\left[(1+\delta(s-1))\times\frac{\sin(\zeta)}{s}\right]=-\sin(\zeta)\{\delta\Psi+(1-\delta)\},\end{aligned} \tag{46}$$

The subsequent terms are

$$\mu_2(\zeta,\Psi)=-\mathscr{Y}^{-1}\left[(1+\delta(s-1))\mathscr{Y}\left\{\frac{\partial^2\mu_1}{\partial\zeta^2}-2\mu_0\frac{\partial\mu_1}{\partial\zeta}-2\mu_1\frac{\partial\mu_0}{\partial\zeta}-\frac{\partial\mu_0}{\partial\zeta}\frac{\partial\nu_1}{\partial\zeta}-\frac{\partial\mu_1}{\partial\zeta}\frac{\partial\nu_0}{\partial\zeta}\right\}\right],$$
$$\mu_2(\zeta,\Psi)=\sin(\zeta)\left\{(1-\delta)^2+2\delta(1-\delta)\Psi+\frac{\delta^2\Psi^2}{2}\right\},$$
$$\nu_2(\zeta,\Psi)=-\mathscr{Y}^{-1}\left[(1+\delta(s-1))\mathscr{Y}\left\{\frac{\partial^2\nu_1}{\partial\zeta^2}-2\nu_0\frac{\partial\nu_1}{\partial\zeta}-2\nu_1\frac{\partial\nu_0}{\partial\zeta}-\frac{\partial\mu_0}{\partial\zeta}\frac{\partial\nu_1}{\partial\zeta}-\frac{\partial\mu_1}{\partial\zeta}\frac{\partial\nu_0}{\partial\zeta}\right\}\right]$$
$$\nu_2(\zeta,\Psi)=-\sin(\zeta)\left\{(1-\delta)^2+2\delta(1-\delta)\Psi+\frac{\delta^2\Psi^2}{2}\right\},\tag{47}$$

The YDM solution for example (4) is

$$\mu(\zeta,\Psi)=\mu_0(\zeta,\Psi)+\mu_1(\zeta,\Psi)+\mu_2(\zeta,\Psi)+\mu_3(\zeta,\Psi)+\cdots,\tag{48}$$

$$\nu(\zeta,\Psi)=\nu_0(\zeta,\Psi)+\nu_1(\zeta,\Psi)+\nu_2(\zeta,\Psi)+\nu_3(\zeta,\Psi)+\cdots,\tag{49}$$

$$\mu(\zeta,\Psi)=\sin(\zeta)+\sin(\zeta)\{\delta\Psi+(1-\delta)\}+\sin(\zeta)\left\{(1-\delta)^2+2\delta(1-\delta)\Psi+\frac{\delta^2\Psi^2}{2}\right\}+\cdots,\tag{50}$$

$$\nu(\zeta,\Psi)=-\sin(\zeta)-\sin(\zeta)\{\delta\Psi+(1-\delta)\}-\sin(\zeta)\left\{(1-\delta)^2+2\delta(1-\delta)\Psi+\frac{\delta^2\Psi^2}{2}\right\}-\cdots,\tag{51}$$

when $\delta=1$, then YDM solution is

$$\mu(\zeta,\Psi)=\sin(\zeta)+\sin(\zeta)\Psi+\sin(\zeta)\frac{\Psi^2}{2}+\sin(\zeta)\frac{\Psi^3}{6}+\sin(\zeta)\frac{\Psi^4}{24}+\cdots,\tag{52}$$

$$\nu(\zeta,\Psi)=-\sin(\zeta)-\sin(\zeta)\Psi-\sin(\zeta)\frac{\Psi^2}{2}-\sin(\zeta)\frac{\Psi^3}{6}-\sin(\zeta)\frac{\Psi^4}{24}-\cdots.\tag{53}$$

The exact solutions are

$$\mu(\zeta,\Psi)=e^{\Psi}\sin(\zeta),\quad \nu(\zeta,\Psi)=-e^{\Psi}\sin(\zeta).\tag{54}$$

5. Example

Consider the following fractional-order couple Burgers equations [17]:

$$\frac{{}^{CF}\partial^\delta\mu}{\partial\Psi^\delta}+\mu\frac{\partial\mu}{\partial\zeta}+\nu\frac{\partial\mu}{\partial\xi}-\frac{\partial^2\mu}{\partial\zeta^2}-\frac{\partial^2\mu}{\partial\xi^2}=0,$$
$$\frac{{}^{CF}\partial^\delta\nu}{\partial\Psi^\delta}+\mu\frac{\partial\nu}{\partial\zeta}+\nu\frac{\partial\nu}{\partial\xi}-\frac{\partial^2\nu}{\partial\zeta^2}-\frac{\partial^2\nu}{\partial\xi^2}=0,\quad 0<\delta\le 1,\tag{55}$$

with initial condition

$$\mu(\zeta,\xi,0)=\zeta+\xi,\quad \nu(\zeta,\xi,0)=\zeta-\xi.\tag{56}$$

Taking Yang transform of (55),

$$\mathscr{Y}\left[\frac{\partial^\delta\mu}{\partial\Psi^\delta}\right]=-\mathscr{Y}\left[\mu\frac{\partial\mu}{\partial\zeta}+\nu\frac{\partial\mu}{\partial\xi}-\frac{\partial^2\mu}{\partial\zeta^2}-\frac{\partial^2\mu}{\partial\xi^2}\right],\tag{57}$$

$$\mathscr{Y}\left[\frac{\partial^\delta\nu}{\partial\Psi^\delta}\right]=-\mathscr{Y}\left[\mu\frac{\partial\nu}{\partial\zeta}+\nu\frac{\partial\nu}{\partial\xi}-\frac{\partial^2\nu}{\partial\zeta^2}-\frac{\partial^2\nu}{\partial\xi^2}\right],\tag{58}$$

$$\frac{1}{(1+\delta(s-1))}\mathscr{Y}\{\mu(\zeta,\xi,0)\}-s\mu(\zeta,\xi,0)=-\mathscr{Y}\left[\mu\frac{\partial\mu}{\partial\zeta}+\nu\frac{\partial\mu}{\partial\xi}-\frac{\partial^2\mu}{\partial\zeta^2}-\frac{\partial^2\mu}{\partial\xi^2}\right],\tag{59}$$

$$\frac{1}{(1+\delta(s-1))}\mathscr{Y}\{\nu(\zeta,\xi,0)\}-s\nu(\zeta,\xi,0)=-\mathscr{Y}\left[\mu\frac{\partial\nu}{\partial\zeta}+\nu\frac{\partial\nu}{\partial\xi}-\frac{\partial^2\nu}{\partial\zeta^2}-\frac{\partial^2\nu}{\partial\xi^2}\right].\tag{60}$$

Applying inverse Yang transform

$$\mu(\zeta,\xi,\Psi)=\mathscr{Y}^{-1}\left[s\mu(\zeta,\xi,0)-(1+\delta(s-1))\mathscr{Y}\left\{\mu\frac{\partial\mu}{\partial\zeta}+\nu\frac{\partial\mu}{\partial\xi}-\frac{\partial^2\mu}{\partial\zeta^2}-\frac{\partial^2\mu}{\partial\xi^2}\right\}\right],\tag{61}$$

$$\nu(\zeta,\xi,\Psi)=\mathscr{Y}^{-1}\left[s\nu(\zeta,\xi,0)-(1+\delta(s-1))\mathscr{Y}\left\{\mu\frac{\partial\nu}{\partial\zeta}+\nu\frac{\partial\nu}{\partial\xi}-\frac{\partial^2\nu}{\partial\zeta^2}-\frac{\partial^2\nu}{\partial\xi^2}\right\}\right],\tag{62}$$

$$\mu(\zeta,\xi,\Psi)=\zeta+\xi-\mathscr{Y}^{-1}\left[(1+\delta(s-1))\mathscr{Y}\left\{\mu\frac{\partial\mu}{\partial\zeta}+\nu\frac{\partial\mu}{\partial\xi}-\frac{\partial^2\mu}{\partial\zeta^2}-\frac{\partial^2\mu}{\partial\xi^2}\right\}\right],\tag{63}$$

$$\nu(\zeta,\xi,\Psi)=\zeta-\xi-\mathscr{Y}^{-1}\left[(1+\delta(s-1))\mathscr{Y}\left\{\mu\frac{\partial\nu}{\partial\zeta}+\nu\frac{\partial\nu}{\partial\xi}-\frac{\partial^2\nu}{\partial\zeta^2}-\frac{\partial^2\nu}{\partial\xi^2}\right\}\right].\tag{64}$$

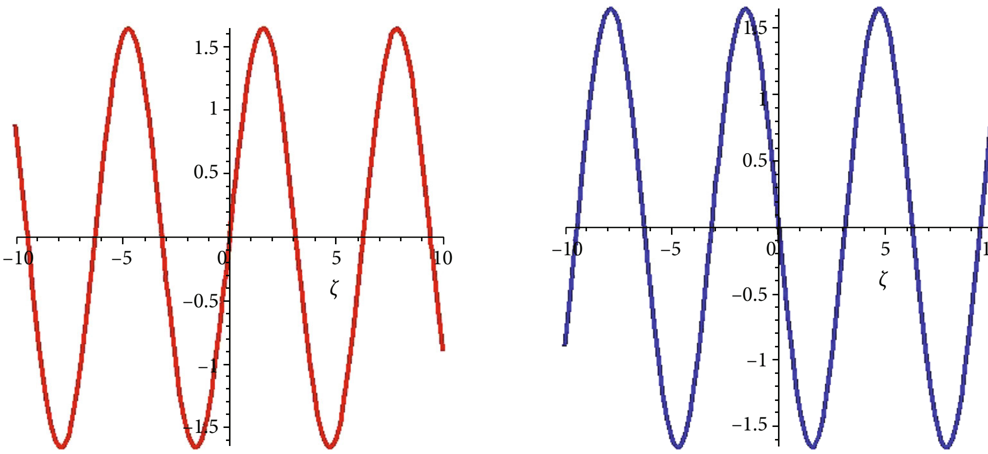

Figure 1: YDM solutions of $\mu(\zeta, \Psi)$ and $\nu(\zeta, \Psi)$ for example 1 at $\delta = 1$.

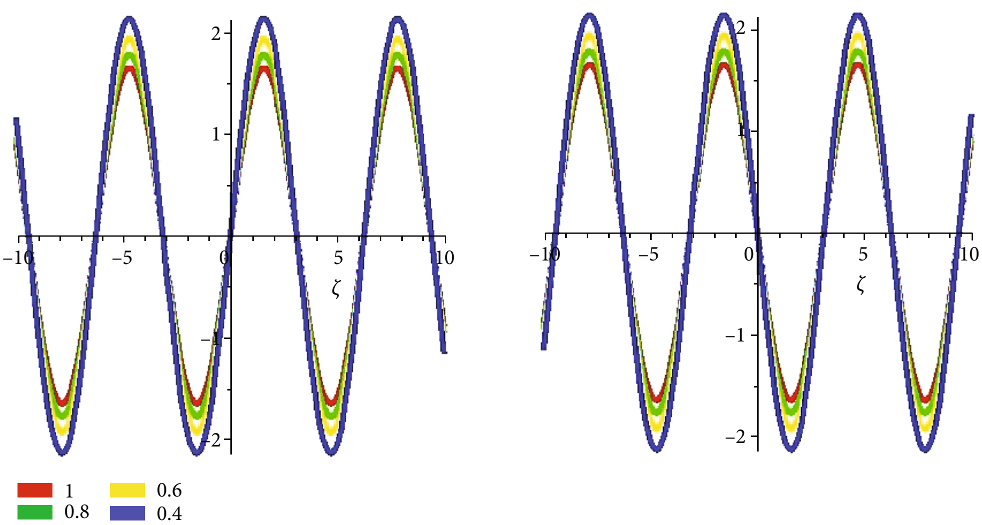

Figure 2: YDM solutions of $\mu(\zeta, \Psi)$ and $\nu(\zeta, \Psi)$ for example 1 at different value of δ.

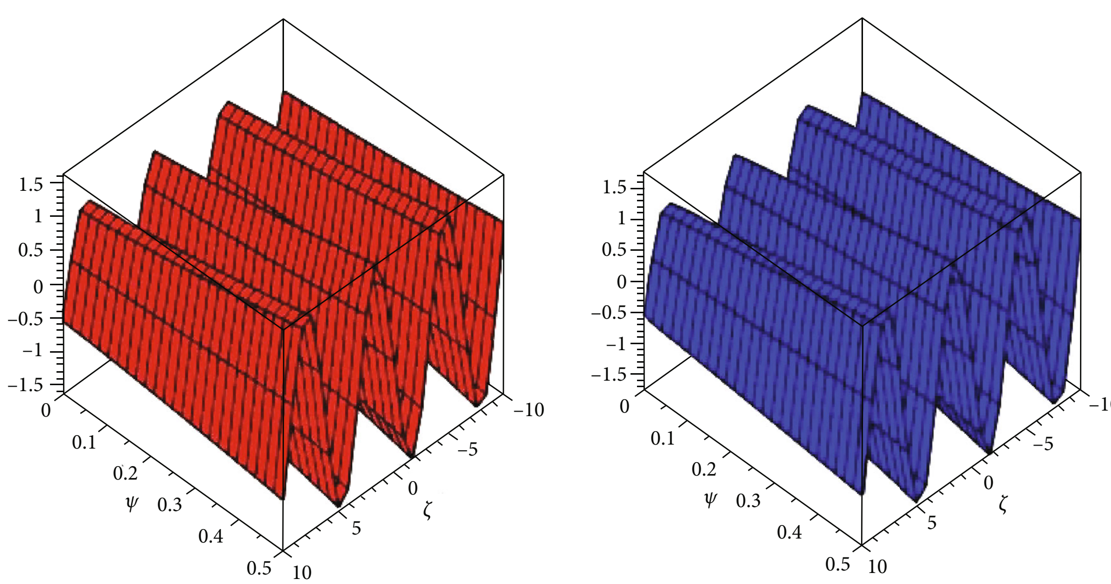

Figure 3: The YDM solution of example 1 of $\mu(\zeta, \Psi)$ at $\delta = 1$, and 0.8.

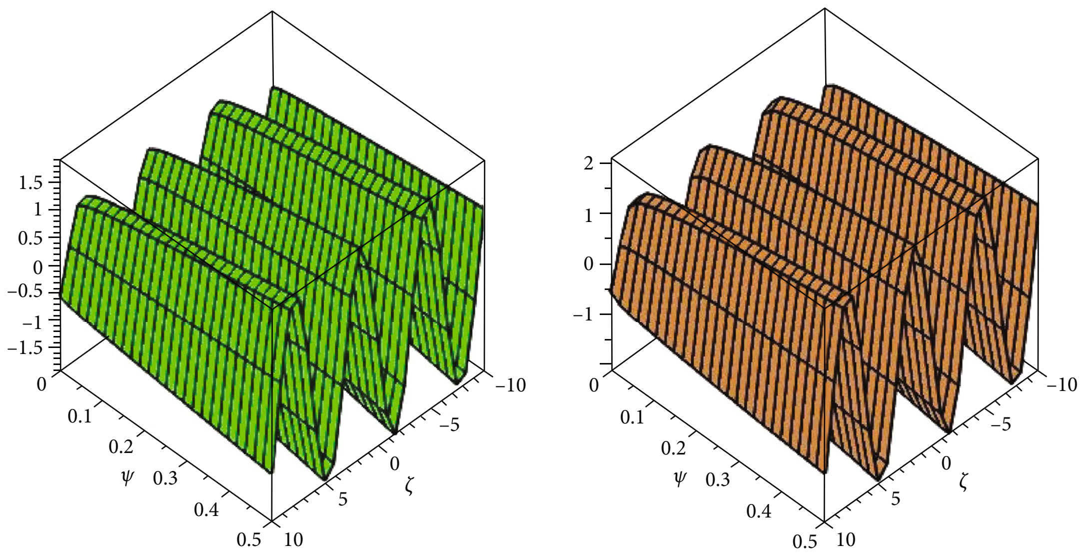

FIGURE 4: The YDM solution of example 1 of $\mu(\zeta, \Psi)$ at $\delta = 0.6$, and 0.4.

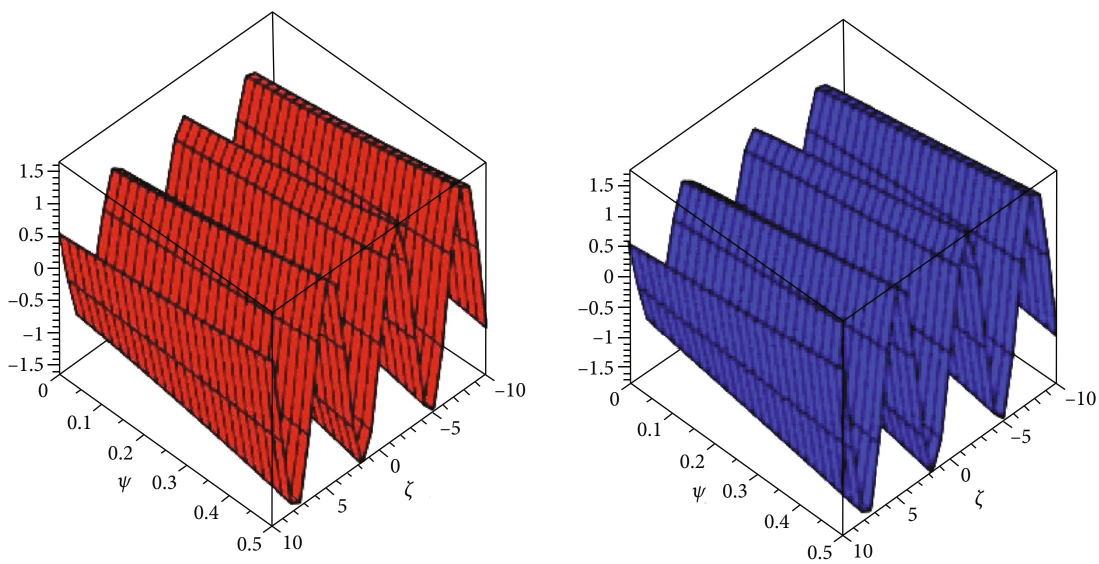

FIGURE 5: The YDM solution of example 1 of $\mu(\zeta, \Psi)$ at $\delta = 1$, and 0.8.

Using ADM procedure, we get

$$\sum_{j=0}^{\infty}\mu_j(\zeta,\xi,\Psi)=\zeta+\xi-\mathscr{Y}^{-1}\left[(1+\delta(s-1))\mathscr{Y}\left\{\sum_{j=0}^{\infty}A_j(\mu\mu_\zeta)+\sum_{j=0}^{\infty}B_j(\nu\mu_\xi)-\sum_{j=0}^{\infty}\mu_{\zeta\zeta}-\sum_{j=0}^{\infty}\mu_{\xi\xi}\right\}\right], \tag{65}$$

$$\sum_{j=0}^{\infty}\nu_j(\zeta,\xi,\Psi)=\zeta-\xi-\mathscr{Y}^{-1}\left[(1+\delta(s-1))\mathscr{Y}\left\{\sum_{j=0}^{\infty}C_j(\mu\nu_\zeta)+\sum_{j=0}^{\infty}D_j(\nu\nu_\xi)-\sum_{j=0}^{\infty}\nu_{\zeta\zeta}-\sum_{j=0}^{\infty}\nu_{\xi\xi}\right\}\right], \tag{66}$$

where $A_j(\mu\mu_\zeta)$, $B_j(\nu\mu_\xi)$, $C_j(\mu\nu_\zeta)$, and $D_j(\nu\nu_\xi)$, the Adomian polynomials are given below,

$$\begin{aligned}
A_0(\mu\mu_\zeta)&=\mu_0\frac{\partial\mu_0}{\partial\zeta}, & B_0(\nu\mu_\xi)&=\nu_0\frac{\partial\mu_0}{\partial\xi},\\
A_1(\mu\mu_\zeta)&=\mu_0\frac{\partial\mu_1}{\partial\zeta}+\mu_1\frac{\partial\mu_0}{\partial\zeta}, & B_1(\nu\mu_\xi)&=\nu_0\frac{\partial\mu_1}{\partial\xi}+\nu_1\frac{\partial\mu_0}{\partial\xi},\\
A_2(\mu\mu_\zeta)&=\mu_0\frac{\partial\mu_2}{\partial\zeta}+\mu_1\frac{\partial\mu_1}{\partial\zeta}+\mu_2\frac{\partial\mu_0}{\partial\zeta}. & B_2(\nu\mu_\xi)&=\nu_0\frac{\partial\mu_2}{\partial\xi}+\nu_1\frac{\partial\mu_1}{\partial\xi}+\nu_2\frac{\partial\mu_0}{\partial\xi}.
\end{aligned} \tag{67}$$

$$\begin{aligned}
C_0(\mu\nu_\zeta)&=\mu_0\frac{\partial\nu_0}{\partial\zeta}, & D_0(\nu\nu_\xi)&=\nu_0\frac{\partial\nu_0}{\partial\xi},\\
C_1(\mu\nu_\zeta)&=\mu_0\frac{\partial\nu_1}{\partial\zeta}+\mu_1\frac{\partial\nu_0}{\partial\zeta}, & D_1(\nu\nu_\xi)&=\nu_0\frac{\partial\nu_1}{\partial\xi}+\nu_1\frac{\partial\nu_0}{\partial\xi},\\
C_2(\mu\nu_\zeta)&=\mu_0\frac{\partial\nu_2}{\partial\zeta}+\mu_1\frac{\partial\nu_1}{\partial\zeta}+\mu_2\frac{\partial\nu_0}{\partial\zeta}. & D_2(\nu\nu_\xi)&=\nu_0\frac{\partial\nu_2}{\partial\xi}+\nu_1\frac{\partial\nu_1}{\partial\xi}+\nu_2\frac{\partial\nu_0}{\partial\xi}.
\end{aligned} \tag{68}$$

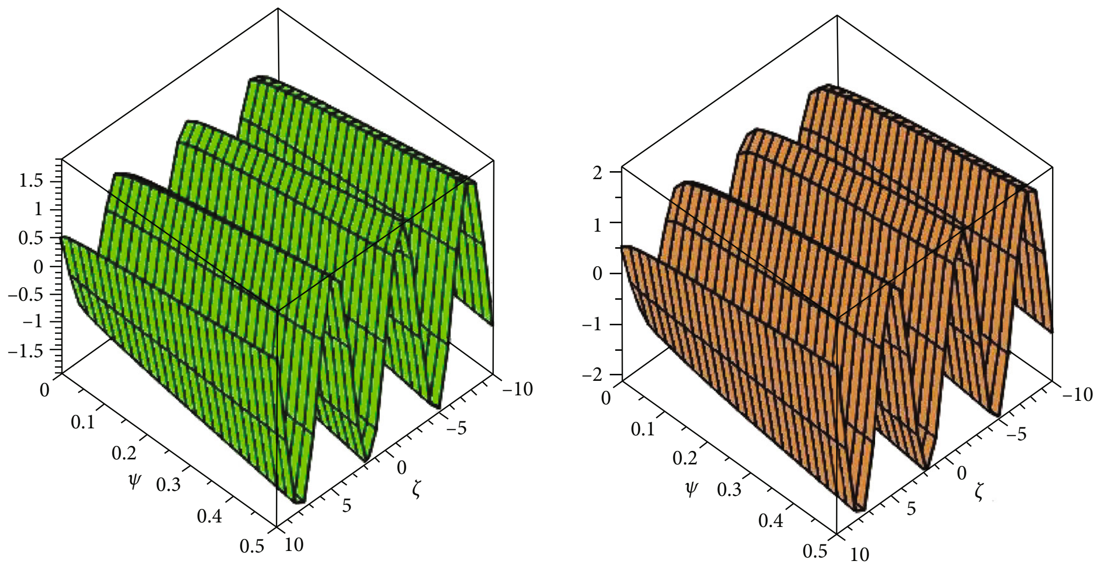

Figure 6: The YDM solution of example 1 of $\nu(\zeta, \Psi)$ at $\delta = 1$, and 0.8.

Table 1: YDM-solutions of example 1 $\mu(\zeta, \Psi)$ and $\nu(\zeta, \Psi)$ different fractional-order of δ.

Ψ	ζ	Absolute error ($\delta = 0.4$)	Absolute error ($\delta = 0.6$)	Absolute error ($\delta = 0.8$)	Absolute error ($\delta = 1$)
	1	$1.6795833810 \times 10^{-02}$	$5.4134672620 \times 10^{-03}$	$7.0667992140 \times 10^{-04}$	$7.0683562720 \times 10^{-09}$
	2	$1.8149655480 \times 10^{-02}$	$5.8498176890 \times 10^{-03}$	$7.6364158210 \times 10^{-04}$	$7.6380983850 \times 10^{-09}$
0.1	3	$2.8167675970 \times 10^{-03}$	$9.0787271070 \times 10^{-04}$	$1.1851469400 \times 10^{-04}$	$1.1854080680 \times 10^{-09}$
	4	$1.5105843420 \times 10^{-02}$	$4.8687662520 \times 10^{-03}$	$6.3557405730 \times 10^{-04}$	$6.3571409610 \times 10^{-09}$
	5	$1.9140211660 \times 10^{-02}$	$6.1690839760 \times 10^{-03}$	$8.0531895140 \times 10^{-04}$	$8.0549639070 \times 10^{-09}$
	1	$2.2895909420 \times 10^{-02}$	$8.0513685670 \times 10^{-03}$	$2.3663474260 \times 10^{-04}$	$2.3207769760 \times 10^{-07}$
	2	$2.4741425310 \times 10^{-02}$	$8.7003460040 \times 10^{-03}$	$2.5570859420 \times 10^{-04}$	$2.5078423030 \times 10^{-07}$
0.2	3	$3.8397888720 \times 10^{-03}$	$1.3502654490 \times 10^{-03}$	$3.9685143520 \times 10^{-05}$	$3.8920898230 \times 10^{-08}$
	4	$2.0592131750 \times 10^{-02}$	$7.2412429330 \times 10^{-03}$	$2.1282464510 \times 10^{-04}$	$2.0872612820 \times 10^{-07}$
	5	$2.6091741410 \times 10^{-02}$	$9.1751859580 \times 10^{-03}$	$2.6966443650 \times 10^{-04}$	$2.6447131500 \times 10^{-07}$
	1	$2.7579918610 \times 10^{-02}$	$1.0158028710 \times 10^{-02}$	$3.1243332140 \times 10^{-04}$	$1.7930063740 \times 10^{-06}$
	2	$2.9802987250 \times 10^{-02}$	$1.0976812670 \times 10^{-02}$	$3.3761688790 \times 10^{-04}$	$1.9375309570 \times 10^{-06}$
0.3	3	$4.6253268490 \times 10^{-02}$	$1.7035656840 \times 10^{-03}$	$5.2397044750 \times 10^{-05}$	$3.0069851330 \times 10^{-07}$
	4	$2.4804837720 \times 10^{-02}$	$9.1359317340 \times 10^{-02}$	$2.8099639980 \times 10^{-04}$	$1.6125947570 \times 10^{-06}$
	5	$3.1429548890 \times 10^{-02}$	$1.1575895650 \times 10^{-02}$	$3.5604305020 \times 10^{-04}$	$2.0432758450 \times 10^{-06}$
	1	$3.1561849440 \times 10^{-02}$	$1.1987254260 \times 10^{-02}$	$3.7995256610 \times 10^{-04}$	$7.6880155060 \times 10^{-06}$
	2	$3.4105880060 \times 10^{-02}$	$1.2953482240 \times 10^{-02}$	$4.1057849520 \times 10^{-04}$	$8.3077050100 \times 10^{-06}$
0.4	3	$5.2931218410 \times 10^{-02}$	$2.0103383830 \times 10^{-03}$	$6.3720449280 \times 10^{-05}$	$1.2893288420 \times 10^{-06}$
	4	$2.8386108190 \times 10^{-02}$	$1.0781101310 \times 10^{-02}$	$3.4172188380 \times 10^{-04}$	$6.9144503180 \times 10^{-06}$
	5	$3.5967281260 \times 10^{-02}$	$1.3660446180 \times 10^{-02}$	$4.3298669290 \times 10^{-04}$	$8.7611157430 \times 10^{-06}$
	1	$3.5108679510 \times 10^{-02}$	$1.3639899070 \times 10^{-02}$	$4.4195033070 \times 10^{-04}$	$2.3878506280 \times 10^{-06}$
	2	$3.7938600990 \times 10^{-02}$	$1.4739337840 \times 10^{-02}$	$4.7757356550 \times 10^{-04}$	$2.5803224010 \times 10^{-06}$
0.5	3	$5.8879476850 \times 10^{-02}$	$2.2874973730 \times 10^{-03}$	$7.4117866660 \times 10^{-05}$	$4.0045765820 \times 10^{-06}$
	4	$3.1576057570 \times 10^{-02}$	$1.2267457630 \times 10^{-02}$	$3.9748145700 \times 10^{-04}$	$2.1475860090 \times 10^{-05}$
	5	$4.0009181120 \times 10^{-02}$	$1.5543768660 \times 10^{-02}$	$5.0363816220 \times 10^{-04}$	$2.7211490040 \times 10^{-05}$

Table 2: YDM-solutions of example 1 at $\mu(\zeta, \Psi)$ different fractional-order of δ.

Ψ	ζ	Absolute error ($\delta = 0.4$)	Absolute error ($\delta = 0.6$)	Absolute error ($\delta = 0.8$)	Absolute error ($\delta = 1$)
	1	$5.1388035000 \times 10^{-03}$	$8.5868730000 \times 10^{-04}$	$1.2436850000 \times 10^{-05}$	$1.4585000000 \times 10^{-10}$
	2	$1.0422343800 \times 10^{-02}$	$1.7352281000 \times 10^{-03}$	$2.2053740000 \times 10^{-05}$	$2.1331000000 \times 10^{-09}$
0.1	3	$1.4825705100 \times 10^{-02}$	$2.4804600000 \times 10^{-03}$	$3.3501640000 \times 10^{-05}$	$2.6843000000 \times 10^{-09}$
	4	$2.1340157300 \times 10^{-02}$	$3.3450207000 \times 10^{-03}$	$4.5130530000 \times 10^{-05}$	$3.3374000000 \times 10^{-09}$
	5	$2.5732507500 \times 10^{-02}$	$4.2223515000 \times 10^{-03}$	$5.6568430000 \times 10^{-05}$	$4.1714000000 \times 10^{-09}$
	1	$6.5526269000 \times 10^{-03}$	$1.2845443000 \times 10^{-04}$	$1.9524030000 \times 10^{-05}$	$8.9043500000 \times 10^{-08}$
	2	$1.3585147000 \times 10^{-02}$	$2.5823037000 \times 10^{-03}$	$3.9081030000 \times 10^{-05}$	$1.2243480000 \times 10^{-08}$
0.2	3	$2.0617667100 \times 10^{-03}$	$3.8800631000 \times 10^{-03}$	$5.8638020000 \times 10^{-05}$	$1.5582620000 \times 10^{-08}$
	4	$2.7650187200 \times 10^{-02}$	$5.1778225000 \times 10^{-03}$	$7.8195020000 \times 10^{-05}$	$1.8921750000 \times 10^{-08}$
	5	$3.4682707200 \times 10^{-02}$	$6.4755819000 \times 10^{-03}$	$9.7752020000 \times 10^{-05}$	$2.2260880000 \times 10^{-08}$
	1	$7.5239217000 \times 10^{-03}$	$1.6203247000 \times 10^{-04}$	$2.6503270000 \times 10^{-05}$	$9.9570730000 \times 10^{-08}$
	2	$1.5715901300 \times 10^{-02}$	$3.2621458000 \times 10^{-03}$	$5.3069340000 \times 10^{-05}$	$1.2801951000 \times 10^{-08}$
0.3	3	$2.3907880800 \times 10^{-02}$	$4.9039669000 \times 10^{-03}$	$7.9635420000 \times 10^{-05}$	$1.5646829000 \times 10^{-08}$
	4	$3.2099860300 \times 10^{-02}$	$6.5457880000 \times 10^{-03}$	$1.0620149000 \times 10^{-05}$	$1.8491707000 \times 10^{-08}$
	5	$4.0291839800 \times 10^{-02}$	$8.1876092000 \times 10^{-03}$	$1.3276756000 \times 10^{-05}$	$2.1336585000 \times 10^{-08}$
	1	$8.2762123000 \times 10^{-03}$	$1.9075950000 \times 10^{-03}$	$3.2874570000 \times 10^{-05}$	$5.7825882000 \times 10^{-09}$
	2	$1.7398405800 \times 10^{-02}$	$3.8455493000 \times 10^{-03}$	$6.5848290000 \times 10^{-05}$	$6.7463529000 \times 10^{-08}$
0.4	3	$2.6520599300 \times 10^{-02}$	$5.7835036000 \times 10^{-03}$	$9.8822010000 \times 10^{-05}$	$7.7101176000 \times 10^{-08}$
	4	$3.5642792800 \times 10^{-02}$	$7.7214580000 \times 10^{-03}$	$1.3179574000 \times 10^{-05}$	$8.6738823000 \times 10^{-08}$
	5	$4.4764986200 \times 10^{-02}$	$9.6594122000 \times 10^{-03}$	$1.6476946000 \times 10^{-05}$	$9.6376470000 \times 10^{-08}$
	1	$8.8947364000 \times 10^{-03}$	$2.1627817000 \times 10^{-03}$	$3.8817930000 \times 10^{-04}$	$2.3900000000 \times 10^{-08}$
	2	$1.8806520800 \times 10^{-02}$	$4.3652454000 \times 10^{-03}$	$7.7777130000 \times 10^{-04}$	$3.5300000000 \times 10^{-08}$
0.5	3	$2.8718305200 \times 10^{-02}$	$6.5677091000 \times 10^{-03}$	$1.1673633000 \times 10^{-05}$	$4.6600000000 \times 10^{-08}$
	4	$3.8630089800 \times 10^{-02}$	$8.7701729000 \times 10^{-03}$	$1.5569554000 \times 10^{-04}$	$5.8000000000 \times 10^{-08}$
	5	$4.8541874400 \times 10^{-02}$	$1.0972636700 \times 10^{-03}$	$1.9465475000 \times 10^{-04}$	$6.9300000000 \times 10^{-08}$

$$\mu_0(\zeta, \xi, \Psi) = \zeta + \xi,$$
$$\nu_0(\zeta, \xi, \Psi) = \zeta - \xi, \tag{69}$$

$$\mu_{j+1}(\zeta, \xi, \Psi) = -\mathscr{Y}^{-1}\left[(1 + \delta(s-1))\mathscr{Y}\left\{\sum_{j=0}^{\infty} A_j(\mu\mu_\zeta) + \sum_{j=0}^{\infty} B_j(\nu\mu_\xi) - \sum_{j=0}^{\infty} \mu_{\zeta\zeta} - \sum_{j=0}^{\infty} \mu_{\xi\xi}\right\}\right], \tag{70}$$

$$\nu_{j+1}(\zeta, \xi, \Psi) = -\mathscr{Y}^{-1}\left[(1 + \delta(s-1))\mathscr{Y}\left\{\sum_{j=0}^{\infty} C_j(\mu\nu_\zeta) + \sum_{j=0}^{\infty} D_j(\nu\nu_\xi) - \sum_{j=0}^{\infty} \nu_{\zeta\zeta} - \sum_{j=0}^{\infty} \nu_{\xi\xi}\right\}\right], \tag{71}$$

for $j = 0, 1, 2 \cdots$

$$\mu_1(\zeta, \xi, \Psi) = -\mathscr{Y}^{-1}\left[(1 + \delta(s-1))\mathscr{Y}\left[\mu_0 \frac{\partial \mu_0}{\partial \zeta} + \nu_0 \frac{\partial \mu_0}{\partial \xi} - \frac{\partial^2 \mu_0}{\partial \zeta^2} - \frac{\partial^2 \mu_0}{\partial \xi^2}\right]\right] = -2\zeta\{\delta\Psi + (1-\delta)\},$$

$$\nu_1(\zeta, \xi, \Psi) = -\mathscr{Y}^{-1}\left[(1 + \delta(s-1))\mathscr{Y}\left[\mu_0 \frac{\partial \nu_0}{\partial \zeta} + \nu_0 \frac{\partial \nu_0}{\partial \xi} - \frac{\partial^2 \nu_0}{\partial \zeta^2} - \frac{\partial^2 \nu_0}{\partial \xi^2}\right]\right] = -2\xi\{\delta\Psi + (1-\delta)\}. \tag{72}$$

The subsequent terms are

$$\mu_2(\zeta, \xi, \Psi) = -\mathscr{Y}^{-1}\left[(1 + \delta(s-1))\mathscr{Y}\left[\mu_0 \frac{\partial \mu_1}{\partial \zeta} + \mu_1 \frac{\partial \mu_0}{\partial \zeta} + \nu_0 \frac{\partial \mu_1}{\partial \xi} + \nu_1 \frac{\partial \mu_0}{\partial \xi} - \frac{\partial^2 \mu_1}{\partial \zeta^2} - \frac{\partial^2 \mu_1}{\partial \xi^2}\right]\right],$$

$$\mu_2(\zeta, \xi, \Psi) = 2(\zeta + \xi)\left\{(1-\delta)^2 + 2\delta(1-\delta)\Psi + \frac{\delta^2\Psi^2}{2}\right\},$$

Table 3: YDM-solutions of example 2 at $\nu(\zeta, \Psi)$ different fractional-order of δ.

Ψ	ζ	Absolute error ($\delta = 0.4$)	Absolute error ($\delta = 0.6$)	Absolute error ($\delta = 0.8$)	Absolute error ($\delta = 1$)
	1	$5.6770988000 \times 10^{-04}$	$8.7119340000 \times 10^{-05}$	$1.1548840000 \times 10^{-07}$	$1.6326000000 \times 10^{-10}$
	2	$5.3834512000 \times 10^{-03}$	$8.4544080000 \times 10^{-04}$	$9.5379000000 \times 10^{-07}$	$2.0407510200 \times 10^{-09}$
0.1	3	$5.0898036000 \times 10^{-03}$	$8.1968820000 \times 10^{-04}$	$7.5269600000 \times 10^{-07}$	$4.0814857200 \times 10^{-09}$
	4	$4.7961560000 \times 10^{-03}$	$7.9393560000 \times 10^{-04}$	$5.5160200000 \times 10^{-07}$	$6.1222204100 \times 10^{-09}$
	5	$4.5025084000 \times 10^{-03}$	$7.6818300000 \times 10^{-04}$	$3.5050800000 \times 10^{-07}$	$8.1629551100 \times 10^{-09}$
	1	$7.5124132000 \times 10^{-04}$	$1.3109744000 \times 10^{-05}$	$1.9589970000 \times 10^{-07}$	$2.2260880000 \times 10^{-08}$
	2	$6.9925200000 \times 10^{-03}$	$1.2577593000 \times 10^{-04}$	$1.5557000000 \times 10^{-07}$	$4.3444869600 \times 10^{-08}$
0.2	3	$6.4726268000 \times 10^{-03}$	$1.2045442000 \times 10^{-04}$	$1.1524030000 \times 10^{-07}$	$8.6867478200 \times 10^{-08}$
	4	$5.9527336000 \times 10^{-03}$	$1.1513291000 \times 10^{-04}$	$7.4910600000 \times 10^{-06}$	$1.3029008690 \times 10^{-08}$
	5	$5.4328404000 \times 10^{-03}$	$1.0981140000 \times 10^{-04}$	$3.4580900000 \times 10^{-06}$	$1.7371269560 \times 10^{-08}$
	1	$8.8600372900 \times 10^{-04}$	$1.6633175900 \times 10^{-05}$	$2.6628869000 \times 10^{-06}$	$4.2673170000 \times 10^{-08}$
	2	$8.1319795000 \times 10^{-03}$	$1.5818212000 \times 10^{-04}$	$2.0566070000 \times 10^{-06}$	$7.2886243900 \times 10^{-08}$
0.3	3	$7.4039217000 \times 10^{-03}$	$1.5003248000 \times 10^{-04}$	$1.4503270000 \times 10^{-06}$	$1.4534575610 \times 10^{-08}$
	4	$6.6758639000 \times 10^{-03}$	$1.4188284000 \times 10^{-04}$	$8.4404700000 \times 10^{-06}$	$2.1780526830 \times 10^{-08}$
	5	$5.9478061000 \times 10^{-03}$	$1.3373320000 \times 10^{-04}$	$2.3776700000 \times 10^{-06}$	$2.9026478050 \times 10^{-08}$
	1	$9.9681746700 \times 10^{-04}$	$1.9683136700 \times 10^{-04}$	$3.3072887000 \times 10^{-06}$	$3.8550588000 \times 10^{-09}$
	2	$9.0421934000 \times 10^{-03}$	$1.8579543000 \times 10^{-04}$	$2.4973720000 \times 10^{-06}$	$1.1668329410 \times 10^{-08}$
0.4	3	$8.1162122000 \times 10^{-03}$	$1.7475950000 \times 10^{-04}$	$1.6874560000 \times 10^{-06}$	$2.2951152940 \times 10^{-08}$
	4	$7.1902310000 \times 10^{-03}$	$1.6372357000 \times 10^{-04}$	$8.7754000000 \times 10^{-06}$	$3.4233976470 \times 10^{-08}$
	5	$6.2642497000 \times 10^{-03}$	$1.5268763000 \times 10^{-04}$	$6.7623000000 \times 10^{-06}$	$4.5516800000 \times 10^{-08}$
	1	$1.0928832650 \times 10^{-04}$	$2.2421458500 \times 10^{-04}$	$3.9100485000 \times 10^{-06}$	$1.2600000000 \times 10^{-08}$
	2	$9.8117844000 \times 10^{-03}$	$2.1024637000 \times 10^{-04}$	$2.8959200000 \times 10^{-06}$	$1.0050239900 \times 10^{-08}$
0.5	3	$8.6947361000 \times 10^{-03}$	$1.9627815000 \times 10^{-04}$	$1.8817910000 \times 10^{-06}$	$2.0100478600 \times 10^{-08}$
	4	$7.5776879000 \times 10^{-03}$	$1.8230994000 \times 10^{-04}$	$8.6766300000 \times 10^{-06}$	$3.0150717200 \times 10^{-08}$
	5	$6.4606397000 \times 10^{-03}$	$1.6834173000 \times 10^{-04}$	$1.4646500000 \times 10^{-06}$	$4.0200955900 \times 10^{-08}$

$$\nu_2(\zeta,\xi,\Psi) = -\mathscr{Y}^{-1}\left[(1+\delta(s-1))\mathscr{Y}\left[\mu_0\frac{\partial \nu_1}{\partial\zeta} + \mu_1\frac{\partial \nu_0}{\partial\zeta} + \nu_0\frac{\partial \nu_1}{\partial\xi} + \nu_1\frac{\partial \nu_0}{\partial\xi} - \frac{\partial^2\nu_0}{\partial\zeta^2} - \frac{\partial^2\nu_0}{\partial\xi^2}\right]\right],$$

$$\nu_2(\zeta,\xi,\Psi) = 2(\zeta-\xi)\left\{(1-\delta)^2 + 2\delta(1-\delta)\Psi + \frac{\delta^2\Psi^2}{2}\right\}. \tag{73}$$

The YDM solution for example (5) is

$$\mu(\zeta,\xi,\Psi) = \mu_0(\zeta,\xi,\Psi) + \mu_1(\zeta,\xi,\Psi) + \mu_2(\zeta,\xi,\Psi) + \mu_3(\zeta,\xi,\Psi) + \cdots, \tag{74}$$

$$\nu(\zeta,\xi,\Psi) = \nu_0(\zeta,\xi,\Psi) + \nu_1(\zeta,\xi,\Psi) + \nu_2(\zeta,\xi,\Psi) + \nu_3(\zeta,\xi,\Psi) + \cdots, \tag{75}$$

$$\mu(\zeta,\xi,\Psi) = \zeta + \xi - 2\zeta\{\delta\Psi + (1-\delta)\} + 2(\zeta+\xi) \cdot \left\{(1-\delta)^2 + 2\delta(1-\delta)\Psi + \frac{\delta^2\Psi^2}{2}\right\} + \cdots, \tag{76}$$

$$\nu(\zeta,\xi,\Psi) = \zeta - \xi - 2\xi\{\delta\Psi + (1-\delta)\} + 2(\zeta-\xi) \cdot \left\{(1-\delta)^2 + 2\delta(1-\delta)\Psi + \frac{\delta^2\Psi^2}{2}\right\} + \cdots, \tag{77}$$

when $\delta = 1$, then YDM solution is

$$\mu(\zeta,\xi,\Psi) = \zeta + \xi - 2\zeta\Psi + 2(\zeta+\xi)\Psi^2 - 4\Psi^3\zeta + 4(\zeta+\xi)\Psi^4 + \cdots, \tag{78}$$

$$\nu(\zeta,\xi,\Psi) = \zeta - \xi - 2\xi\Psi + 2(\zeta-\xi)\Psi^2 - 4\Psi^3\xi + 4(\zeta-\xi)\Psi^4 + \cdots. \tag{79}$$

The exact solutions are

$$\mu(\zeta, \xi, \Psi) = \frac{\zeta - 2\zeta\Psi + \xi}{1 - 2\Psi^2},$$
$$\nu(\zeta, \xi, \Psi) = \frac{\zeta - 2\xi\Psi - \xi}{1 - 2\Psi^2}. \quad (80)$$

6. Results and Discussion

In this section, we analyze the solution-figures of problem which have been investigated by applying Yang decomposition method in the sense of Caputo-Fabrizio operator. Figure 1 represents the two-dimensional solution-figures for variables $\mu(\zeta, \Psi)$ and $\nu(\zeta, \Psi)$ of example 1 at fractional order $\delta = 1$, respectively, in Figure 2 at different fractional-order of $\wp$. It is observed that Yang method solution-figures are identical and close contact with each other. In a similar way in Figures 3 and 4 represent the three-dimensional solution-figures for variables $\mu(\zeta, \Psi)$ of example 1 at fractional order $\delta = 1$, 0.8, 0.6, and 0.4. Figure 5 shows that the three dimensional figure of $\mu(\zeta, \Psi)$ of fractional order $\delta = 1$ and 0.8 of example 2 and Figure 6, approximate solution graphs of example 2 with respect to $\nu(\zeta, \Psi)$ at $\delta = 1$ and 0.8. Tables 1–3 show the absolute error of different fractional order of δ with respect to $\mu(\zeta, \Psi)$ and $\nu(\zeta, \Psi)$ of examples 1 and 2. The same graphs of the suggested methods attained and confirmed the applicability of the present technique. The convergence phenomenon of the fractional-solutions towards integer-solution is observed. The same accuracy is achieved by using the present techniques.

7. Conclusion

In this paper, Yang Adomian decomposition method is implemented for the solution of dynamic systems of fractional Burger equations. The derived results have been graphed and tables. The analytical solutions for some numerical problems represent the validity of the suggested technique. It is also analyzed that the fractional-order solution is convergence to the actual result for the problem as fractional-order approach integer-order. The higher accuracy of the suggested procedure is clearly demonstrated by this representation of the acquired results. The results for fractional systems that are closely akin to their actual solutions are obtained. It has been demonstrated that fractional solutions converge to integer-order solutions. The present method's valuable themes include fewer calculations and improved precision. The researchers modified it to solve fractional partial differential equations in various systems.

Data Availability

The numerical data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

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