Retraction

Retracted: Computing First and Second Fuzzy Zagreb Indices of Linear and Multiacyclic Hydrocarbons

Journal of Function Spaces

This article has been retracted by Hindawi following an investigation undertaken by the publisher [1]. This investigation has uncovered evidence of one or more of the following indicators of systematic manipulation of the publication process:

(1) Discrepancies in scope

(2) Discrepancies in the description of the research reported

(3) Discrepancies between the availability of data and the research described

(4) Inappropriate citations

(5) Incoherent, meaningless and/or irrelevant content included in the article

(6) Peer-review manipulation

The presence of these indicators undermines our confidence in the integrity of the article's content and we cannot, therefore, vouch for its reliability. Please note that this notice is intended solely to alert readers that the content of this article is unreliable. We have not investigated whether authors were aware of or involved in the systematic manipulation of the publication process.

Wiley and Hindawi regrets that the usual quality checks did not identify these issues before publication and have since put additional measures in place to safeguard research integrity.

We wish to credit our own Research Integrity and Research Publishing teams and anonymous and named external researchers and research integrity experts for contributing to this investigation.

The corresponding author, as the representative of all authors, has been given the opportunity to register their agreement or disagreement to this retraction. We have kept a record of any response received.

References

[1] Z. S. Mufti, E. Fatima, R. Anjum, F. Tchier, Q. Xin, and M. M. Hossain, "Computing First and Second Fuzzy Zagreb Indices of Linear and Multiacyclic Hydrocarbons," *Journal of Function Spaces*, vol. 2022, Article ID 6281592, 8 pages, 2022.

Hindawi

Research Article

Computing First and Second Fuzzy Zagreb Indices of Linear and Multiacyclic Hydrocarbons

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Fuzzy graph theory was invented by Rosenfeld. It is the extension of the work of L.A. Zadeh on fuzzy sets. Rosenfeld extracted the fuzzy-related concepts using the graph theoretic concepts. Topological indices for crisp theory have already been discussed in the literature but these days, topological index-related fuzzy graphs are much popular. Fuzzy graphs are being used as an application in different fields of sciences such as broadcast, communications, producing, social network, man-made reasoning, data hypothesis, and neural systems. In this paper, we have computed some fuzzy topological indices such as first and second Zagreb indices, Randic index, and harmonic index of fuzzy chemical graph named phenylene.

1. Introduction

Graph theory is a very ancient branch of discrete mathematics but miraculously applications of graph theory hit the modern world quite nicely. Nowadays, graph theory has roots in almost every field of science. The mathematical structures which are broadly used to represent relations between objects are based upon graphs. Graphs can conveniently express the structure for providing us information to employ and understand the behavior over the assumption tested on it [1–3, 9, 11].

The graph is basically a well-defined set $G = (V, E)$ of vertices V and edges E, where vertices are the points or dots and edges are the links connecting those points or nodes. The sets of vertices and edges are crisp sets in graph theory. But in fuzzy graph theory, the super set of a crisp set, namely, fuzzy set, deals with the notion of partial truth between absolute true and absolute false. Graph plays an important role in our life as it has widen its range to real-life applications. Most of the important fields like, networking, communication, data mining, clustering, image processing, image segmentation, planning, and scheduling rely on graphs. Graph theory represents real-life phenomena, but most of the times, graphs are not capable of properly representing many phenomena because vagueness of different imputes of the systems exists naturally. Many real-world facts are motivated to define the fuzzy graphs [6–8, 13, 26].

Bhattacharya [4] elaborated the notion of the fuzzy group with fuzzy graphs. Bhutani [5] has focused on automorphism in fuzzy graphs. Gani and Latha [14] explained the concept of irregularity of fuzzy graphs. Gani and Ahamed [15] examined the degree and size of fuzzy graphs. Mathew and Sunitha [20] introduced the join, cartesian product, union, and composition of fuzzy subgraphs of graphs. Morderson and Peng [21] explained the basic applications of fuzzy graphs. Most popular applications regarding topological indices of fuzzy graphs are human trafficking and internet routing. The theory which usually defines the membership of an element in a set is called a set theory, which is appraise in binary terms in accordance with an

associated condition if an element relates or does not relate to the set. By contrast, fuzzy set allows the progressive estimation of the membership of elements in a set; it can be evoked with the assistance of a membership function (a membership function (MF) is a curve which defines that how every point in the given space is mapped on a membership value (or degree of membership) between 0 and 1) valued in the real unit interval [0,1]. Fuzzy sets or uncertain sets are sets whose elements have a degree of membership. Simply, it is the generalization of the classical set. L.A. Zadeh in 1965 proposed one of the earliest uncertainty based models that assigns categorized memberships to elements rather than binary membership given by crisp sets. Through this soft computing technique, the ambiguity, vagueness, and uncertainty in real-world knowledge bases can be fixed [10, 12, 17, 19].

Chemical graph theory is a topology branch of mathematical chemistry which deals with the combination of chemistry and graph theory in which graph theory is applied to mathematically model of chemical phenomena. Alexandru Balaban, Ante Graovac, Iván Gutman, Haruo Hosoya, Milan Randic, and Nenad Trinajsti´ c (also Harry´ Wiener and others) are the pioneers of chemical graph theory. In 1988, it was reported for producing about 500 articles annually and hundreds of researchers worked in chemical graph theory. By using tools of graph theory, this comprehensive science takes problems like isomer enumeration from chemistry, thus influencing both chemistry and mathematics. The chemical graph theory is a profound field for solving chemical problems by combining methods and theorems from mathematics and chemistry where molecular structures are usually represented as mathematical graphs. In such molecular graphs, vertices and edges represent atoms and bonds, respectively. To predict the chemical, physical, and biological activities, one may use some indices such as Wiener index which is one of the most famous examples of a graph-based molecular index. The applications of graph theory in chemistry include isomer enumeration and molecular structure generation [16, 22, 24]

The numerical parameters of a graph which are usually graph invariants are called topological indices as they characterize its topology. Topological indices are used for example in the progress of (QSARs) Quantitative Structure-Activity Relationships. It describes the biological activity or other properties of molecules which are correlated with their chemical structure. Particular topological indices include the Zagreb index index, Randic index, and harmonic topological index. To correlate and predict several physical and chemical properties of chemical compounds especially carbon containing compounds, topological indices are used. A wide range of applications of the topological index of graph are in theoretical chemistry, network design, data transmission, etc. These topological indices have complete connotations in fuzzy graph. These topological indices are new in fuzzy graphs so we name them as modified topological indices. Here are some topological indices of fuzzy graphs which are first and second Zagreb indices, Randic index, and harmonic index.

Fuzzy graphs are being used these days as an application tool in the field of mathematics because of its fuzziness which is very friendly in nature. Some basics of fuzzy graphs and notion related to these graphs are discussed here. What is actually a fuzzy graph? How it relates with topological indices? How can we find different topological indices of fuzzy graphs? A fuzzy graph $G = (V, \sigma, \mu)$ has a pair of functions σ and μ. σ maps the vertex set to the closed interval [0,1], and μ is mapping from the cartesian product of the vertex set with itself to the closed interval [0,1] such that it satisfies the following condition $\mu(p, q) \leq \min(\sigma(p), \sigma(q))$ for all $p, q \in V$, and E is the set of collection of edges in G. The degree of vertex in fuzzy graphs can easily be calculated by taking the overall sum of all weights of edges, i.e., $d(p) = \Sigma\mu(p, q)$. In crisp graph, the weight of all the edges is one by default but in fuzzy graph, the weight of every edge may vary from zero to one. The edges which contain fuzzy weights are called fuzzy edges. The order of the fuzzy graphs can be represented by the sum of the weights of all the vertices of a graph G, and the size of the fuzzy graphs can be found as the sum of all the weighs of its edges. By the definition of fuzzy graphs, the order of a fuzzy graphs is always greater than or equal to size of the fuzzy graphs [23, 25, 27]. Phenylene belongs to the special class of conjugated hydrocarbons which plays an important role in the field of chemistry. The graphical form of phenylene is a combination of two geometrical figures such as hexagon and square. These geometrical figures are joined in such a way that a square is adjusted between two hexagons.

The motivation of this work came from [18]. We have generalized this work on chemical structure and compute the fuzzy topological indices of chemical structure mentioned in [18].

2. Topological Indices of Fuzzy Graph of Linear Phenylene

Topological index is a numeric value which describes the physicochemical properties of molecular structures.

2.1. Zagreb Index of First in Fuzzy Graphs. Kalathian et al. [18] introduced the fuzzy Zagreb index of first kind.

$$M(G) = \sum_{i=1}^{n} \sigma(u_i)[d(u_i)]^2. \tag{1}$$

2.2. Zagreb Index of Second Kind in Fuzzy Graphs. Kalathian et al. [18] introduced the fuzzy Zagreb index of second kind.

$$M^*(G) = \frac{1}{2}\left[\sum_{ij\in E(G)} \sigma(u_i)d(u_i)\sigma(v_j)d(v_j)\right]. \tag{2}$$

2.3. Randic Index in Fuzzy Graphs. Kalathian et al. [18] introduced the fuzzy Randic index.

$$R(G) = \frac{1}{2}\left[\sum_{i=1}^{n} \sigma(u_i)d(u_i)\sigma(v_j)d(u_j)\right]^{-1/2}. \tag{3}$$

2.4. Harmonic Index in Fuzzy Graphs. Kalathian et al. [18] introduced the fuzzy harmonic index.

$$H(G) = \frac{1}{2}\left[\sum_{ij \in E(G)}\left[\frac{1}{\sigma(u_i)d(u_i) + \sigma(v_j)d(v_j)}\right]\right]. \quad (4)$$

Theorem 1. *Let G be a fuzzy graph of linear phenylene; then, the first fuzzy Zagreb index of linear phenylene is $M(G) = 0.472n - 0.116$.*
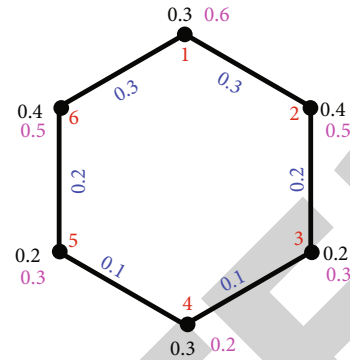
Proof. In linear phenylene structures (see Figures 1 and 2), the total number of vertices is $6n$ and the total number of edges is $8n - 2$, respectively. The partition of the total vertex set with respect to weight has the following form by using Table 1: the vertex set $V_{0.2}$ (0.2 is the weight of the vertex) has a total vertex count $2n$ out of which 2 vertices are of degree 0.3 and $2n - 2$ vertices are of degree 0.4, vertex set $V_{0.3}$ (0.3 is the weight of the vertex) has a total vertex count $2n$ out of which n vertices are of degree 0.2 and n vertices are of degree 0.6, and vertex set $V_{0.4}$ (0.4 is the weight of the vertex) has a total vertex count $2n$ out of which 2 vertices are of degree 0.5 and $2n - 2$ vertices are of degree 0.6.

$$
\begin{aligned}
M(G) &= \sum_{i=1}^{n}\sigma(u_i)[d(u_i)]^2 = (0.2)\left[2(0.3)^2 + (2n-2)(0.4)^2\right] \\
&\quad + (0.3)\left[n(0.2)^2 + n(0.6)^2\right] + (0.4)\left[2(0.5)^2 + (2n-2)(0.6)^2\right] \\
&= 0.036 + 0.064n - 0.064 + 0.012n + 0.108n + 0.2 \\
&\quad + 0.288n - 0.288 = 0.472n - 0.116.
\end{aligned}
\quad (5)
$$

□

Theorem 2. *Let G be a fuzzy graph of linear phenylene; then, the second fuzzy Zagreb index of linear phenylene is $M*(G) = 0.0672n - 0.0156$.*

Proof. The linear phenylene structures have a total number of vertices $6n$ and total number of edges $8n - 2$, respectively. The partition of the total edge sets has the following form by using Table 2: the edge set $E_1 = (u_{0.3}, v_{0.4})$ (where 0.3 and 0.4 are the weights of the vertices u and v, respectively) has three type of partitions. The total count of the vertices of the type $u_{0.3}$ is $2n$ out of which two vertices are of degree 0.6 and $2n - 2$ vertices are of degree 0.6, and the total count of the vertices of the type $v_{0.4}$ is $2n$ out of which two vertices are of degree 0.5 and $2n - 2$ vertices are of degree 0.6. The edge set $E_2 = (u_{0.2}, v_{0.3})$ (where 0.2 and 0.3 are the weights of the vertices u and v, respectively) has two types of partitions. The total count of the vertices of the type $u_{0.2}$ is $2n$ out of which two vertices are of degree 0.3 and $2n - 2$ vertices of degree 0.4, and the total count of the vertices of the type $v_{0.3}$ is $2n$ out of which two vertices are of degree 0.2 and $2n - 2$ vertices are of degree 0.2. The edge set $E_3 = (u_{0.2}, v_{0.4})$ (where 0.2 and 0.4 are the weights of the vertices u and v, respectively) has two types of partitions. The total count of the vertices of the type $u_{0.2}$ is $2n$ out of which two vertices are of degree 0.3 and $2n - 2$ vertices of degree 0.4, and the total count of the vertices of the type $v_{0.4}$ is $2n$ out of which

Final Degrees

	d(1)=0.6
	d(2)=0.5
0.6 = 1	d(3)=0.3
0.5 = 2	d(4)=0.2
0.3 = 2	d(5)=0.3
0.2 = 1	d(6)=0.5

FIGURE 1: (1, 1) unit of fuzzy graph of phenylene.

two vertices are of degree 0.5 and $2n - 2$ vertices are of degree 0.6.

$$
\begin{aligned}
M^*(G) &= \frac{1}{2}\left[\sum_{ij \in E(G)}\sigma(u_i)d(u_i)\sigma(v_j)d(v_j)\right] \\
&= \frac{1}{2}[2(0.3)(0.6)(0.4)(0.5) + (2n-2)(0.3)(0.6)(0.4)(0.6)] \\
&\quad + \frac{1}{2}[2(0.2)(0.3)(0.3)(0.2) + (2n-2)(0.3)(0.2)(0.2)(0.4)] \\
&\quad + \frac{1}{2}[2(0.2)(0.3)(0.4)(0.5) + (2n-2)(0.2)(0.4)(0.4)(0.6)] \\
&= \frac{1}{2}[0.072 + 0.0864n - 0.0864 + 0.0072 + 0.0096n - 0.0096 \\
&\quad + 0.024 + 0.0384n - 0.0384] = \frac{1}{2}[0.1344n - 0.0312] \\
&= 0.0672n - 0.0156.
\end{aligned}
\quad (6)
$$

□

Theorem 3. *Let G be a fuzzy graph of linear phenylene; then, the Randic index of linear phenylene is $R(G) = [4.6039 + 26.462n]$.*

Proof. The linear phenylene structures have a total number of vertices $6n$ and total number of edges $8n - 2$, respectively. The partition of the total edge sets has the following form by using Table 2: the edge set $E_1 = (u_{0.3}, v_{0.4})$ (where 0.3 and 0.4 are the weights of the vertices u and v, respectively) has two types of partitions. The total count of the vertices of the type $u_{0.3}$ is $2n$ out of which two vertices are of degree 0.6 and $2n - 2$ vertices are of degree 0.6, and the total count of the vertices of the type $v_{0.4}$ is $2n$ out of which two vertices are of degree 0.5 and $2n - 2$ vertices are of degree 0.6. The edge set $E_2 = (u_{0.2}, v_{0.3})$ (where 0.2 and 0.3 are the weights of the vertices u and v, respectively) has two types of partitions. The total count of the vertices of the type $u_{0.2}$ is $2n$ out of
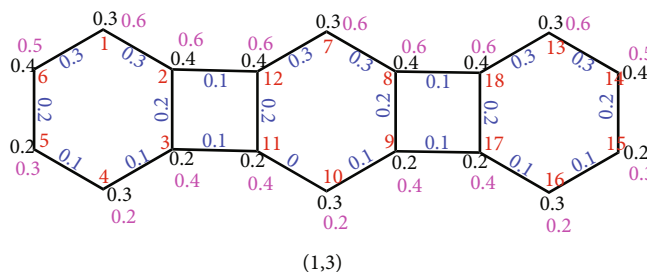
(1,3)

Figure 2: (1, 3) unit of fuzzy graph of phenylene.

Table 1: Vertex count for fuzzy graph of linear phenylene.

Vertex w.r.t. vertex weight	Count
0.2	$2n$
0.3	$2n$
0.4	$2n$

Table 2: Edge type according to the degrees for linear fuzzy graph.

Edge w.r.t vertex weight	Degree type	Count
(0.3,0.4)	(0.6,0.5)	2
(0.3,0.4)	(0.6,0.6)	(2n-2)
(0.2,0.3)	(0.3,0.2)	(2)
(0.3,0.2)	(0.2,0.4)	(2n-2)
(0.2,0.4)	(0.3,0.5)	(2)
(0.2,0.4)	(0.4,0.6)	(2n-2)

which two vertices are of degree 0.3 and $2n - 2$ vertices of degree 0.4, and the total count of the vertices of the type $v_{0.3}$ is $2n$ out of which two vertices of degree 0.2 and $2n - 2$ vertices are of degree 0.2. The edge set $E_3 = (u_{0.2}, v_{0.4})$ (where 0.2 and 0.4 are the weights of the vertices u and v, respectively) has two types of partitions. The total count of the vertices of the type $u_{0.2}$ is $2n$ out of which two vertices are of degree 0.3 and $2n - 2$ vertices are of degree 0.4, and the total count of the vertices of the type $v_{0.4}$ is $2n$ out of which two vertices are of degree 0.5 and $2n - 2$ vertices are of degree 0.6.

$$
\begin{aligned}
R(G) &= \frac{1}{2} \left[\sum_{i=1}^{n} \sigma(u_i) d(u_i) \sigma(v_j) d(u_j) \right]^{-1/2} \\
&= \frac{1}{2} [2][(0.3)(0.6)(0.4)(0.5)]^{-1/2} \\
&\quad + \frac{1}{2} [2n-2][(0.3)(0.6)(0.4)(0.6)]^{-1/2} \\
&\quad + \frac{1}{2} [2][(0.2)(0.3)(0.3)(0.2)]^{-1/2} \\
&\quad + \frac{1}{2} [2n-2][(0.3)(0.2)(0.2)(0.4)]^{-1/2} \\
&\quad + \frac{1}{2} [2][(0.2)(0.3)(0.4)(0.5)]^{-1/2} \\
&\quad + \frac{1}{2} [2n-2][(0.2)(0.4)(0.4)(0.6)]^{-1/2} \\
&= [5.2705 + (4.8113n - 4.8113) + 16.6667 \\
&\quad + (14.4338n - 14.4338) + 9.1287 + (7.2169n - 7.2169)] \\
&= [4.6039 + 26.462n].
\end{aligned}
\tag{7}
$$

□

Theorem 4. *Let G be a fuzzy graph of linear phenylene; then, the harmonic index for fuzzy graph of linear phenylene is H* $(G) = [12.6488n + 2.1624]$.

Proof. The linear phenylene structures have a total number of vertices $6n$ and total number of edges $8n - 2$, respectively. The partition of the total edge sets has the following form by using Table 2: the edge set $E_1 = (u_{0.3}, v_{0.4})$ (where 0.3 and 0.4 are the weights of the vertices u and v, respectively) has two types of partitions. The total count of the vertices of the type $u_{0.3}$ is $2n$ out of which two vertices are of degree 0.6 and $2n - 2$ vertices are of degree 0.6, and the total count of the vertices of the type $v_{0.4}$ is $2n$ out of which two vertices are of degree 0.5 and $2n - 2$ vertices are of degree 0.6. The edge set $E_2 = (u_{0.2}, v_{0.3})$ (where 0.2 and 0.3 are the weights of the vertices u and v, respectively) has two types of partitions. The total count of the vertices of the type $u_{0.2}$ is $2n$ out of which two vertices are of degree 0.3 and $2n - 2$ vertices are of degree 0.4, and the total count of the vertices of the type $v_{0.4}$ is $2n$ out of which two vertices are of degree 0.5 and $2n - 2$ vertices are of degree 0.6. The edge set $E_3 = (u_{0.2}, v_{0.4})$ (where 0.2 and 0.4 are the weights of the vertices u and v, respectively) has two types of partitions. The total count of the vertices of the type $u_{0.2}$ is $2n$ out of which two vertices are of degree 0.3 and $2n - 2$ vertices are of degree 0.4, and the total count of the vertices of the type $v_{0.3}$ is $2n$ out of which two vertices are of degree 0.2 and $2n - 2$ vertices are of degree 0.2.

$$
\begin{aligned}
H(G) &= \frac{1}{2} \left[\sum_{ij \in E(G)} \left[\frac{1}{\sigma(u_i) d(u_i) + \sigma(v_j) d(v_j)} \right] \right] \\
&= \frac{1}{2} \left[\frac{2}{(0.3) + (0.6) + (0.4)(0.5)} \right] + \frac{1}{2} \left[\frac{2n-2}{(0.3) + (0.6) + (0.4)(0.6)} \right] \\
&\quad + \frac{1}{2} \left[\frac{2}{(0.2) + (0.3) + (0.3)(0.2)} \right] + \frac{1}{2} \left[\frac{2n-2}{(0.2) + (0.3) + (0.4)(0.2)} \right] \\
&\quad + \frac{1}{2} \left[\frac{2}{(0.2) + (0.3) + (0.4)(0.5)} \right] + \frac{1}{2} \left[\frac{2n-2}{(0.2) + (0.4) + (0.4)(0.6)} \right] \\
&= \frac{1}{2} \left[\frac{2}{0.38} + \frac{2n-2}{0.42} + \frac{2}{0.12} + \frac{2n-2}{0.14} + \frac{2}{0.26} + \frac{2n-2}{0.32} \right] \\
&= \frac{1}{2} [5.2632 + 4.7619n - 4.7619 + 16.6667 + 14.2857n \\
&\quad + 14.2857 + 7.6923 + 6.25n - 6.25] \\
&= \frac{1}{2} [25.2976n + 4.3246] = [12.6488n + 2.1623]
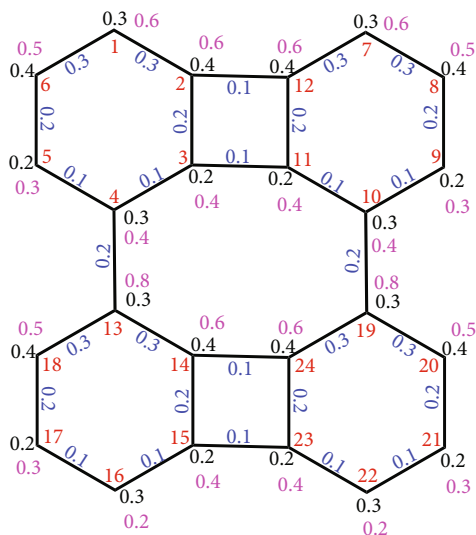\end{aligned}
\tag{8}
$$

□

FIGURE 3: (2, 2) unit of fuzzy graph of phenylene.

3. Topological Indices of Fuzzy Graph of Multiphenylene

Theorem 5. *Let G be a fuzzy graph of multiphenylene; then, the first fuzzy Zagreb index of multiphenylene is $M(G) = 0.592mn - 0.116\,m - 0.12n$.*

Proof. In multiphenylene structures (see Figure 3), the total number of vertices is $6mn$ and the total number of edges is $8n - 2$, respectively. The partition of the total vertex set with respect to weight has the following form by using Table 3: the vertex set $V_{0.2}$ (0.2 is the weight of the vertex) has a total vertex count $2mn$ out of which $2m$ vertex is of degree 0.3 and $2mn - 2m$ vertices are of degree 0.4, vertex set $V_{0.3}$ (0.3 is the weight of the vertex) has a total vertex count $2mn$ out of which n vertices are of degree 0.2, n vertices are of degree 0.6, $mn - n$ vertices of are degree 0.4, and $mn - n$ vertices are of degree 0.8, and vertex set $V_{0.4}$ (0.4 is the weight of the vertex) has a total vertex count $2mn$ out of which $2m$ vertex is of degree 0.5 and $2mn - 2\,m$ vertices are of degree 0.6.

$$\begin{aligned}
M(G) = \sum_{i=1}^{n} \sigma(u_i)[d(u_i)]^2 &= (0.2)\left[(2m)(0.3)^2 + (2mn - 2m)(0.4)^2\right] \\
&+ (0.3)\left[n(0.2)^2 + (mn - n)(0.4)^2\right] \\
&+ (0.3)\left[n(0.6)^2 + (mn - n)(0.8)^2\right] \\
&+ (0.4)\left[(2m)(0.5)^2 + (2mn - 2m)(0.6)^2\right] \\
&= 0.036m + 0.064mn - 0.064m + 0.012n \\
&+ 0.048mn - 0.048n + 0.108n + 0.192mn - 0.192n \\
&+ 0.2m + 0.288mm - 0.288m \\
&= 0.592mn - 0.116m - 0.12n.
\end{aligned}$$

(9)

□

TABLE 3: Vertex count for general fuzzy graph.

Vertex	Type	Count
0.2	0.3, 0.4	$2m$, $2mn\text{-}2m$
0.3	0.2, 0.4, 0.6, 0.8	n, $mn\text{-}n$, n, mnn
0.4	0.5, 0.6	$2m$, $2mn\text{-}2m$

TABLE 4: Edge type according to the degrees for general fuzzy graph.

Edge	Degree type	Count
(0.3,0.4)	(0.6,0.5)	2
(0.3,0.4)	(0.6,0.6)	$(2n\text{-}2)$
(0.3,0.4)	(0.8,0.5)	$(2m\text{-}2)$
(0.3,0.4)	(0.8,0.6)	$(2mn\text{-}2m\text{-}2n + 2)$
(0.2,0.2)	(0.4,0.4)	$(mn\text{-}m)$
(0.4,0.4)	(0.6,0.6)	$(mn\text{-}m)$
(0.2,0.4)	(0.3,0.5)	$(2m)$
(0.2,0.4)	(0.4,0.6)	$(2mn\text{-}2m)$
(0.2,0.3)	(0.3,0.2)	(2)
(0.2,0.3)	(0.4,0.2)	$(2n\text{-}2)$
(0.2,0.3)	(0.3,0.4)	$(2mn\text{-}2n)$
(0.2,0.3)	(0.4,0.4)	$(2mn\text{-}2m\text{-}2n + 2)$
(0.3,0.3)	(0.4,0.8)	$(mn\text{-}n)$

Theorem 6. *Let G be a fuzzy graph of multiphenylene; then, the second fuzzy Zagreb index of multiphenylene is $M * (G) = 0.14mn - 0.0584\,m - 0.0408n + 0.0108$.*

Proof. The multiphenylene structures have a total number of vertices $6mn$ and total number of edges $8n - 2$, respectively. The partition of the total edge sets has the following form by using Table 4: the edge set $E_1 = (u_{0.3}, v_{0.4})$ (where 0.3 and 0.4 are the weights of the vertices u and v, respectively) has four types of partitions. The total count of the vertices of the type $u_{0.3}$ is $2mn$ out of which 2 vertices are of degree 0.6, $2n - 2$ vertices are of degree 0.6, $2\,m - 2$ vertices are of degree 0.8, and $2mn - 2\,m - 2n + 2$ vertices are of degree 0.8, and similarly, the total count of the vertices of the type $v_{0.4}$ is $2mn$ out of which 2 vertices are of degree 0.5, $2n - 2$ vertices are of degree 0.6, $2\,m - 2$ vertices are of degree 0.5, and $2mn - 2\,m - 2n + 2$ vertices are of degree 0.6. The edge set $E_2 = (u_{0.2}, v_{0.2})$ (where 0.2 and 0.2 are the weights of the vertices u and v, respectively) has one type of partition. The total count of the vertices of the type $u_{0.2}$ is $2mn$ out of which $mn - m$ vertices are of degree 0.4 and $mn - m$ vertices are of degree 0.4. The edge set $E_3 = (u_{0.4}, v_{0.4})$ (where 0.4 and 0.4 are the weights of the vertices u and v, respectively) has one type of partition. The total count of the vertices of the type $u_{0.4}$ is $2mn$ out of which $mn - m$ vertices are of degree 0.6 and $mn - m$ vertices are of degree 0.6.

The edge set $E_4 = (u_{0.2}, v_{0.4})$ (where 0.2 and 0.4 are the weights of the vertices u and v, respectively) has two types of partitions. The total count of the vertices of the type $u_{0.2}$ is $2mn$ out of which $2m$ vertices are of degree 0.3 and $2mn$

$-2m$ vertices are of degree 0.4, and the total count of the vertices of the type $v_{0.4}$ is $2mn$ out of which $2m$ vertices are of degree 0.5 and $2mn - 2m$ vertices are of degree 0.6. The edge set $E_5 = (u_{0.2}, v_{0.3})$ (where 0.2 and 0.3 are the weights of the vertices u and v, respectively) has four types of partitions. The total count of the vertices of the type $u_{0.2}$ is $2mn$ out of which 2 vertices are of degree 0.3, $2n - 2$ vertices are of degree 0.4, $2mn - 2n$ vertices are of degree 0.3, and $2mn - 2m - 2n + 2$ vertices are of degree 0.4, and similarly, the total count of the vertices of the type $v_{0.3}$ is $2mn$ out of which 2 vertices are of degree 0.2, $2n - 2$ vertices are of degree 0.2, $2mn - 2n$ vertices are of degree 0.4, and $2mn - 2m - 2n + 2$ vertices are of degree 0.4. The edge set $E_6 = (u_{0.3}, v_{0.3})$ (where 0.3 and 0.3 are the weights of the vertices u and v, respectively) has one type of partition. The total count of the vertices of the type $u_{0.3}$ is $2mn$ out of which $mn - n$ vertices are of degree 0.4 and $mn - n$ vertices are of degree 0.8.

$$
\begin{aligned}
M^*(G) &= \frac{1}{2}\left[\sum_{ij\in E(G)} \sigma(u_i)d(u_i)\sigma(v_j)d(v_j)\right] \\
&= \frac{1}{2}[[2(0.3)(0.6)(0.4)(0.5) + (2n-2)(0.3)(0.6)(0.4)(0.6)]] \\
&\quad + \frac{1}{2}[[(mn-m)(0.4)(0.2)(0.2)(0.4) \\
&\quad + (2mn-2m-2n+2)(0.4)(0.3)(0.8)(0.6)]] \\
&\quad + \frac{1}{2}[[(mn-m)(0.4)(0.6)(0.4)(0.6) \\
&\quad + (2m-2)(0.4)(0.3)(0.8)(0.5)]] \\
&\quad + \frac{1}{2}[[(2m)(0.2)(0.3)(0.4)(0.5) \\
&\quad + (2mn-2m)(0.2)(0.4)(0.4)(0.6)]] \\
&\quad + \frac{1}{2}[[(2)(0.2)(0.3)(0.3)(0.2) \\
&\quad + (2n-2)(0.2)(0.4)(0.3)(0.2)]] \\
&\quad + \frac{1}{2}[[(2mn-2n)(0.2)(0.3)(0.3)(0.4) \\
&\quad + (2mn-2m-2n+2)(0.2)(0.4)(0.3)(0.4)]] \\
&\quad + \frac{1}{2}[[(mn-n)(0.3)(0.4)(0.3)(0.8))]] \\
&= \frac{1}{2}(0.072 + 0.0864n - 0.0864 + 0.096m - 0.096 \\
&\quad + 0.1152mn - 0.1152m - 0.1152n) + 0.1152 \\
&\quad + 0.0064mn - 0.0064m + 0.0576mn - 0.0576m \\
&\quad + 0.024m + 0.0384mn - 0.384m + 0.0072 \\
&\quad + 0.0096n - 0.0096 + 0.0144mn - 0.0144n \\
&\quad + 0.0192mn - 0.0192m - 0.0192n + 0.0192 \\
&\quad + 0.0288mn - 0.0288n = \frac{1}{2}(0.28mn - 0.1168m \\
&\quad - 0.0816n + 0.0216) = 0.14mn - 0.0584m \\
&\quad - 0.0408n + 0.0108.
\end{aligned}
$$

(10)

□

Theorem 7. *Let G be a fuzzy graph of multiphenylene; then, the Randic index of multiphenylene is $R(G) = 44.6542mn - 16.2297m - 9.8592n + 9.5543$.*

Proof.

$$
\begin{aligned}
R(G) &= \frac{1}{2}\left[\sum_{i=1}^{n} \sigma(u_i)d(u_i)\sigma(v_j)d(u_j)\right]^{-1/2} \\
&= \frac{1}{2}[(0.3)(0.6)(0.4)(0.5)]^{-1/2} \\
&\quad + \frac{1}{2}[2n-2][(0.3)(0.6)(0.4)(0.6)]^{-1/2} \\
&\quad + \frac{1}{2}[mn-m][(0.4)(0.2)(0.2)(0.4)]^{-1/2} \\
&\quad + \frac{1}{2}[2mn-2m-2n+2][(0.4)(0.3)(0.8)(0.6)]^{-1/2} \\
&\quad + \frac{1}{2}[mn-m][(0.4)(0.6)(0.4)(0.6)]^{-1/2} \\
&\quad + \frac{1}{2}[2m-2][(0.4)(0.3)(0.8)(0.5)]^{-1/2} \\
&\quad + \frac{1}{2}[2m][(0.2)(0.3)(0.4)(0.5)]^{-1/2} \\
&\quad + \frac{1}{2}[2mn-2m][(0.2)(0.4)(0.4)(0.6)]^{-1/2} \\
&\quad + \frac{1}{2}[2][(0.2)(0.3)(0.3)(0.2)]^{-1/2} \\
&\quad + \frac{1}{2}[2n-2][(0.2)(0.4)(0.3)(0.2)]^{-1/2} \\
&\quad + \frac{1}{2}[2mn-2n][(0.2)(0.3)(0.3)(0.4)]^{-1/2} \\
&\quad + \frac{1}{2}[2mn-2m-2n+2][(0.2)(0.4)(0.3)(0.4)]^{-1/2} \\
&\quad + \frac{1}{2}[mn-n][(0.3)(0.4)(0.3)(0.8)]^{-1/2} \\
&= \frac{1}{2}[5.2705 + 4.8113n - 48113 + 6.25mn - 6.25m \\
&\quad + 4.1667mn - 4.1667m - 4.1667n + 4.1667] \\
&\quad + [2.083mn - 2.083m + 4.5644m - 4.5644 \\
&\quad + 9.1287m + 7.2169mn - 7.2169m + 16.6667] \\
&\quad + [14.4338n - 14.4338 + 11.7851mn - 11.7851n \\
&\quad + 10.2062mn - 10.2062m - 10.2062n + 10.2062] \\
&\quad + [2.9463mn - 2.9463n] = 44.6542mn - 16.2297m \\
&\quad - 9.8592n + 9.5543.
\end{aligned}
$$

(11)

□

Theorem 8. *Let G be a fuzzy graph of multiphenylene; then, the harmonic index for fuzzy graph of multiphenylene is $H(G) = [21.3195mn - 5.1311m - 7.629n + 6.2518]$.*

Proof.

$$
\begin{aligned}
H(G) &= \frac{1}{2}\left[\sum_{ij\in E(G)}\left[\frac{1}{\sigma(u_i)d(u_i)+\sigma(v_j)d(v_j)}\right]\right] \\
&= \frac{1}{2}\left[\frac{2}{(0.3)(0.6)+(0.4)(0.5)}\right]+\frac{1}{2}\left[\frac{2n-2}{(0.3)(0.6)+(0.4)(0.6)}\right] \\
&\quad +\frac{1}{2}\left[\frac{2m-2}{(0.8)(0.3)+(0.4)(0.5)}\right]+\frac{1}{2}\left[\frac{2mn-2m-2n+2}{(0.8)(0.3)+(0.4)(0.6)}\right] \\
&\quad +\frac{1}{2}\left[\frac{mn-m}{(0.2)(0.4)+(0.4)(0.2)}\right]+\frac{1}{2}\left[\frac{mn-m}{(0.6)(0.4)+(0.4)(0.6)}\right] \\
&\quad +\frac{1}{2}\left[\frac{2m}{(0.2)(0.3)+(0.4)(0.5)}\right]+\frac{1}{2}\left[\frac{2mn-2m}{(0.2)(0.4)+(0.4)(0.6)}\right] \\
&\quad +\frac{1}{2}\left[\frac{2}{(0.2)(0.3)+(0.2)(0.3)}\right]+\frac{1}{2}\left[\frac{2n-2}{(0.2)(0.4)+(0.2)(0.3)}\right] \\
&\quad +\frac{1}{2}\left[\frac{2mn-2n}{(0.2)(0.3)+(0.4)(0.3)}\right]+\frac{1}{2}\left[\frac{2mn-2m-2n+2}{(0.2)(0.4)+(0.4)(0.3)}\right] \\
&\quad +\frac{1}{2}\left[\frac{mn-n}{(0.3)(0.4)+(0.8)(0.3)}\right] \\
&= \frac{1}{2}\left[\frac{2}{0.38}+\frac{2n-2}{0.42}+\frac{2m-2}{0.44}+\frac{2mn-2m-2n+2}{0.48}\right. \\
&\quad \left.+\frac{mn-m}{0.16}+\frac{mn-m}{0.48}\right]+\frac{1}{2}\left[\frac{2m}{0.26}+\frac{2mn-2m}{0.32}+\frac{2}{0.12}\right. \\
&\quad \left.+\frac{2n-2}{0.14}+\frac{2mn-2n}{0.18}+\frac{2mn-2m-2n+2}{0.2}+\frac{mn-n}{0.36}\right] \\
&= \frac{1}{2}\left[5.2632+4.7619n-4.7619+4.5455m-4.5455\right. \\
&\quad +4.1667mn-4.1667m\left]+\frac{1}{2}\right[-4.1667n+4.1667 \\
&\quad +6.25mn-6.25m+2.0833mn-2.0833m+7.6923m \\
&\quad +6.25mn-6.25m+16\left]+\frac{1}{2}\right[11.1111mn-11.1111n \\
&\quad +10mn-10m-10n+10+2.7778mn-2.7778n] \\
&= \frac{1}{2}\left[42.6389mn-10.2622m-15.258n+12.5035\right].
\end{aligned}
\tag{12}
$$

$\square$

4. Conclusion

In this paper, we find some topological indices of fuzzy chemical graph based on vertex-degree such as Zagreb first and second kind, Randic, and harmonic indices. The applications of fuzzy graphs are getting attraction due to its implementation in various fields of sciences such as medication, determination and treatment of sickness, and in telecommunication framework. Similarly, most popular applications regarding topological indices of fuzzy graphs are human trafficking and internet routing. We studied fuzzy graph of phenylene structure theocratically, not experimentally. Our results on fuzzy graph of phenylene can be very beneficial and helpful for the mankind to understand the physical properties, chemical reactivity, and biological activity. In future, we are focusing on the line graph of abovementioned structures.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

Acknowledgments

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