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Research Article

Fractional-Stochastic Solutions for the Generalized $(2+1)$-Dimensional Nonlinear Conformable Fractional Schrödinger System Forced by Multiplicative Brownian Motion

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In this paper, the $(2+1)$-dimensional nonlinear conformable fractional stochastic Schrödinger system (NCFSSS) generated by the multiplicative Brownian motion is treated. To get new rational, trigonometric, hyperbolic, and elliptic stochastic solutions, we use two different methods: the sine-cosine and the Jacobi elliptic function methods. Moreover, we use the MATLAB tools to plot our figures to introduce a variety of 2D and 3D graphs to highlight the effect of the multiplicative noise on the exact solutions of the NCFSSS. Finally, we illustrate that the multiplicative Brownian motion stabilizes the solutions of NCFSSS a round zero.

1. Introduction

Stochastic partial differential equations (SPDEs) can be used to represent a wide range of complicated nonlinear physical processes. These kinds of equations appear in a variety of areas including physics, finance, climate dynamics, chemistry, biology, geophysical, engineering, and other fields [1–3].

On the other side, fractional partial differential equations (FPDEs) have gotten a lot of interest because they may illustrate the fundamental components underlying real-world issues. They have been seen in a number of physical phenomena, such as viscoelastic materials with relaxation and creeping functions, the motion of a heavy meager surface in a Newtonian fluid, and relapse subordinate dissipative occupancy of components. As a result, FPDEs are employed in a range of fields, including predicting, describing, and modeling the mechanisms engaged in finance, polymeric materials, a kinematic model of neutron points, engineering, electrical circuits, solid-state physics, optical fibers, chemical kinematics, biogenetics, plasma physics, physics of condensed matter, meteorology, electromagnetic, elasticity, and oceanic spectacles [4–9].

The exact solutions of PDEs are important in nonlinear science. As a result, various analytical techniques, such as tanh-sech [10, 11], Darboux transformation [12], sine-cosine [13, 14], extended simple equation [15], extended sinh-Gordon equation expansion [16], F-expansion [17], Kudryashov technique [18], generalized Kudryashov [19–21], $\exp(-\phi(\varsigma))$-expansion [22], (G'/G)-expansion [23–25], Hirota's function [26], perturbation [5, 27], the Jacobi elliptic function [28, 29], and Riccati-Bernoulli sub-ODE [30], have been created to deal with these types of equations.

To reach a better level of qualitative agreement, the following (2 + 1)-dimensional nonlinear conformable fractional stochastic Schrödinger system (NCFSSS) is addressed:

$$idu + \left[\gamma_1 \mathbb{D}^{\alpha}_{xy} u + \gamma_2 uv\right] dt + i\sigma u d\mathbb{B} = 0, \quad (1)$$

$$\gamma_3 \mathbb{D}^{\alpha}_x v + \gamma_4 \mathbb{D}^{\alpha}_y \left(|u|^2\right) = 0, \quad (2)$$

where $v \in \mathbb{R}$ while $u \in \mathbb{C}$. $\mathbb{D}^{\alpha}$ is the conformable derivative (CD) [31], and γ_i are arbitrary constants for $i = 1, .., 4$. $\mathbb{B}(t)$ is a Brownian motion (BM), and $ud\mathbb{B}$ is multiplicative noise in the Itô sense.

The NCFSSS ((1) and (2)) is crucial in atomic physics, and the functions v and u have diverse physical meanings in various disciplines of physics such as plasma physics [32] and fluid dynamics [33]. In the hydrodynamic context, v is the induced mean flow, and u is the envelope of the wave packet [33], while, in the context of water waves, v is the velocity potential of the mean flow interacting with the surface waves and u is the amplitude of a surface wave packet [34]. The multiplicative noise $i\sigma u d\mathbb{B}$ plays an important role in the theory of measurements continuous in time in open quantum systems. For more physical interpretations, we refer to [35, 36] and the references therein.

Recently, many authors have established exact solutions of NCFSSS ((1) and (2)) with $\sigma = 0$ and $\alpha = 1$ by employing various techniques, such as applied Kudryashov approach [37], direct approach [38], and the extended modified auxiliary equation [39]. Moreover, Bilal and Ahmad [40] applied three methods such as generalized Kudryashov, modified direct algebraic, and (G'/G^2)-expansion function to attain diverse forms of optical solutions of the NCFSSS ((1) and (2)) with $\sigma = 0$, while the exact solutions of the NCFSSS ((1) and (2)) has not yet been studied.

The motivations of this work are to obtain the exact fractional stochastic solutions of NCFSSS ((1) and (2)). This is the first investigation to acquire the exact solutions of NCFSSS ((1) and (2)) in the presence of stochastic term and fractional-space derivatives. To accomplish a wide variety of solutions, such as trigonometric, hyperbolic, elliptic, and rational functions, we apply two different methods such as the Jacobi elliptic function and the sine-cosine methods. Also, we study the effect of BM on the obtained solutions of NCFSSS ((1) and (2)) by using MATLAB to create 3D and 2D diagrams for some of the obtained solutions here.

The document is laid out as follows: we define and state some features of the CD and BM in Section 2. We employ an appropriate wave transformation in Section 3 to derive the wave equation of NCFSSS ((1) and (2)). While in Section 4, we utilize two methods to create the analytic solutions of the NCFSSS ((1) and (2)). In Section 5, the influence of the BM on the obtained solutions is investigated. The conclusion of the document is displayed last.

2. Preliminaries

Here, we define and state some features of the CD and BM.

Definition 1 (see [31]). Let $\phi : (0,\infty) \longrightarrow \mathbb{R}$, then the CD of ϕ of order $\alpha \in (0, 1]$ is defined as

$$\mathbb{D}^{\alpha}_x \phi(x) = \lim_{\kappa \longrightarrow 0} \frac{\phi\left(x + \kappa x^{1-\alpha}\right) - \phi(x)}{\kappa}. \quad (3)$$

Theorem 2. *Let $\phi, H : (0,\infty) \longrightarrow \mathbb{R}$ be differentiable and also α be differentiable functions, then*

$$\mathbb{D}^{\alpha}_x (\phi \circ H)(x) = x^{1-\alpha} H'(x) \phi'(H(x)). \quad (4)$$

Let us state some properties of the CD. If a and b are constant, then

(1) $\mathbb{D}^{\alpha}_x [a\phi(x) + bH(x)] = a\mathbb{D}^{\alpha}_x \phi(x) + b\mathbb{D}^{\alpha}_x H(x)$

(2) $\mathbb{D}^{\alpha}_x [a] = 0$

(3) $\mathbb{D}^{\alpha}_x [x^b] = bx^{b-\alpha}$

(4) $\mathbb{D}^{\alpha}_x H(x) = x^{1-\alpha}(dH/dx)$

In next definition, we define Brownian motion $\mathbb{B}(t)$.

Definition 3. Stochastic process $\{\mathbb{B}(t)\}_{t\geq 0}$ is said a Brownian motion if it satisfies:

(1) $\mathbb{B}(0) = 0$

(2) $\mathbb{B}(t)$ is continuous function of $t \geq 0$

(3) $\mathbb{B}(t_2) - \mathbb{B}(t_1)$ is independent for $t_1 < t_2$

(4) $\mathbb{B}(t_2) - \mathbb{B}(t_1)$ has a normal distribution $\aleph(0, t_2 - t_1)$

3. Wave Equation for NCFSSS

The next wave transformation is used to get the wave equation of the NCFSSS ((1) and (2)):

$$u(x, y, t) = \varphi(\zeta) e^{\left(i\hbar - \sigma\mathbb{B}(t) - (1/2)\sigma^2 t\right)}, \quad v(x, y, t) = \psi(\zeta) e^{\left(-\sigma\mathbb{B}(t) - (1/2)\sigma^2 t\right)}, \quad (5)$$

with

$$\zeta = \frac{\zeta_1}{\alpha} x^{\alpha} + \frac{\zeta_2}{\alpha} y^{\alpha} - \zeta_3 t, \quad h = \frac{h_1}{\alpha} x^{\alpha} + \frac{h_2}{\alpha} y^{\alpha} - h_3 t, \quad (6)$$

where φ and ψ are deterministic functions and ζ_k and $\hbar_k$ for $k = 1, 2, 3$, are nonzero constants. Plugging Equation (5) into

Equations (1) and (2) and using

$$du = \left[\left(-\zeta_3\varphi' + i\hbar_3\varphi\right)dt - \sigma\varphi d\mathbb{B}\right]e^{\left(i\hbar - \sigma\mathbb{B}(t) - (1/2)\sigma^2 t\right)},$$
$$\mathbb{D}^{\alpha}_{xy}u = \left[\zeta_1\zeta_2\varphi'' + i(\hbar_2\zeta_1 + \hbar_1\zeta_2)\varphi' - \hbar_1\hbar_2\varphi\right]e^{\left(i\hbar - \sigma\mathbb{B}(t) - (1/2)\sigma^2 t\right)},$$
$$\mathbb{D}^{\alpha}_{y}\left(|u|^2\right) = 2\zeta_2\varphi\varphi' e^{\left(-\sigma\mathbb{B}(t) - (1/2)\sigma^2 t\right)}, \mathbb{D}^{\alpha}_{x}v = \zeta_1\psi' e^{\left(-\sigma\mathbb{B}(t) - (1/2)\sigma^2 t\right)}, \tag{7}$$

we get for imaginary part

$$\zeta_3 = \hbar_2\zeta_1 + \hbar_1\zeta_2, \tag{8}$$

and for real part

$$\zeta_1\zeta_2\varphi'' - (\hbar_3 + \gamma_1\hbar_1\hbar_2)\varphi + \gamma_2\varphi\psi e^{\left(-\sigma\mathbb{B}(t) - (1/2)\sigma^2 t\right)} = 0, \tag{9}$$

$$\gamma_3\zeta_1\psi' + 2\gamma_4\zeta_2\varphi\varphi' e^{\left(-\sigma\mathbb{B}(t) - (1/2)\sigma^2 t\right)} = 0. \tag{10}$$

Taking expectation $\mathbb{E}(\cdot)$ on both sides for Equations (9) and (10), we have

$$\zeta_1\zeta_2\varphi'' - (\hbar_3 + \gamma_1\hbar_1\hbar_2)\varphi + \gamma_2\varphi\psi e^{-(1/2)\sigma^2 t}\mathbb{E}\left(e^{-\sigma\mathbb{B}(t)}\right) = 0, \tag{11}$$

$$\gamma_3\zeta_1\psi' + 2\gamma_4\zeta_2\varphi\varphi' e^{-(1/2)\sigma^2 t}\mathbb{E}\left(e^{-\sigma\mathbb{B}(t)}\right) = 0. \tag{12}$$

Since $\mathbb{B}(t)$ is standard Gaussian process, hence $\mathbb{E}(e^{-\sigma\mathbb{B}(t)}) = e^{(\sigma^2/2)t}$. Now Equations (11) and (12) have the form

$$\zeta_1\zeta_2\varphi'' - (\hbar_3 + \gamma_1\hbar_1\hbar_2)\varphi + \gamma_2\varphi\psi = 0, \tag{13}$$

$$\gamma_3\zeta_1\psi' + 2\gamma_4\zeta_2\varphi\varphi' = 0. \tag{14}$$

Integrating Equation (14) once and setting the integral constant equal zero yields

$$\psi = -\frac{\gamma_4\zeta_2}{\gamma_3\zeta_1}\varphi^2. \tag{15}$$

Plugging Equation (15) into Equation (13), we get the following wave equation

$$\varphi'' - \Lambda_1\varphi^3 - \Lambda_2\varphi = 0, \tag{16}$$

where

$$\Lambda_1 = \frac{\gamma_2\gamma_4}{\gamma_3\zeta_1^2},$$
$$\Lambda_2 = \frac{(\hbar_3 + \gamma_1\hbar_1\hbar_2)}{\zeta_1\zeta_2}. \tag{17}$$

4. The Exact Solutions of the NCFSSS

To find the exact solutions of Equation (16), we use two different methods such as sine-cosine [13, 14] and the Jacobi elliptic function [29] methods. As a result, we are able to obtain the exact solutions of the NCFSSS ((1) and (2)).

4.1. Sine-Cosine Method. Assume the solution φ of Equation (16) has the form

$$\varphi(\zeta) = A\mathbb{Y}^n, \tag{18}$$

where

$$\mathbb{Y} = \cos(B\zeta) \text{ or } \mathbb{Y} = \sin(B\zeta). \tag{19}$$

Setting Equation (18) into Equation (16) we get

$$-AB^2\left[-n^2\mathbb{Y}^n + n(n-1)\mathbb{Y}^{n-2}\right] - \Lambda_1A^3\mathbb{Y}^{3n} - \Lambda_2A\mathbb{Y}^n = 0, \tag{20}$$

rewriting the above equation

$$\left(\Lambda_2A - AB^2n^2\right)\mathbb{Y}^n + n(n-1)AB^2\mathbb{Y}^{n-2} + \Lambda_1A^3\mathbb{Y}^{3n} = 0. \tag{21}$$

Equalizing the term of $\mathbb{Y}$ in Equation (21), we attain

$$n - 2 = 3n \Rightarrow n = -1. \tag{22}$$

Substituting Equation (22) into Equation (21)

$$\left(\Lambda_2A - AB^2\right)\mathbb{Y}^{-1} + \left(\Lambda_1A^3 + 2AB^2\right)\mathbb{Y}^{-3} = 0. \tag{23}$$

Equating each coefficient of $\mathbb{Y}^{-3}$ and $\mathbb{Y}^{-1}$ to zero, we have

$$\Lambda_2A - AB^2 = 0,$$
$$\Lambda_1A^3 + 2AB^2 = 0. \tag{24}$$

By solving these equations, we get

$$B = \sqrt{\Lambda_2},$$
$$A = \sqrt{\frac{-2\Lambda_2}{\Lambda_1}}. \tag{25}$$

Hence, the solution of Equation (16) is

$$\varphi(\zeta) = A\sec(B\zeta) \text{ or } \varphi(\zeta) = A\csc(B\zeta). \tag{26}$$

There are many cases depending on the sign of Λ_1 and Λ_2.

Case 1. If $\Lambda_2 > 0$ and $\Lambda_1 < 0$, then the exact solutions of the NCFSSS ((1) and (2)) are

$$u(x, y, t) = \sqrt{\frac{-2\Lambda_2}{\Lambda_1}} \sec\left[\sqrt{\Lambda_2}\zeta\right] e^{\left(i\hbar - \sigma\mathbb{B}(t) - (1/2)\sigma^2 t\right)},$$
$$v(x, y, t) = \frac{2\Lambda_2\gamma_4\zeta_2}{\Lambda_1\gamma_3\zeta_1} \sec^2\left[\sqrt{\Lambda_2}\zeta\right] e^{\left(-\sigma\mathbb{B}(t) - (1/2)\sigma^2 t\right)}, \tag{27}$$

or

$$u(x, y, t) = \sqrt{\frac{-2\Lambda_2}{\Lambda_1}} \csc\left[\sqrt{\Lambda_2}\zeta\right] e^{\left(i\hbar - \sigma\mathbb{B}(t) - (1/2)\sigma^2 t\right)},$$
$$v(x, y, t) = \frac{2\Lambda_2\gamma_4\zeta_2}{\Lambda_1\gamma_3\zeta_1} \csc^2\left[\sqrt{\Lambda_2}\zeta\right] e^{\left(-\sigma\mathbb{B}(t) - (1/2)\sigma^2 t\right)}. \tag{28}$$

Case 2. If $\Lambda_2 < 0$ and $\Lambda_1 < 0$, then the exact solutions of the NCFSSS ((1) and (2)) are

$$u(x, y, t) = i\sqrt{\frac{2\Lambda_2}{\Lambda_1}} \operatorname{sech}\left[\sqrt{-\Lambda_2}\zeta\right] e^{\left(i\hbar - \sigma\mathbb{B}(t) - (1/2)\sigma^2 t\right)},$$
$$v(x, y, t) = \frac{2\Lambda_2\gamma_4\zeta_2}{\Lambda_1\gamma_3\zeta_1} \sec\mathrm{h}^2\left[\sqrt{-\Lambda_2}\zeta\right] e^{\left(-\sigma\mathbb{B}(t) - (1/2)\sigma^2 t\right)}, \tag{29}$$

or

$$u(x, y, t) = \sqrt{\frac{2\Lambda_2}{\Lambda_1}} \operatorname{csch}\left[\sqrt{-\Lambda_2}\zeta\right] e^{\left(i\hbar - \sigma\mathbb{B}(t) - (1/2)\sigma^2 t\right)},$$
$$v(x, y, t) = \frac{-2\Lambda_2\gamma_4\zeta_2}{\Lambda_1\gamma_3\zeta_1} \csc\mathrm{h}^2\left[\sqrt{-\Lambda_2}\zeta\right] e^{\left(-\sigma\mathbb{B}(t) - (1/2)\sigma^2 t\right)}. \tag{30}$$

Case 3. If $\Lambda_2 < 0$ and $\Lambda_1 > 0$, then the exact solutions of the NCFSSS ((1) and (2)) are

$$u(x, y, t) = \sqrt{\frac{-2\Lambda_2}{\Lambda_1}} \operatorname{sech}\left[\sqrt{-\Lambda_2}\zeta\right] e^{\left(i\hbar - \sigma\mathbb{B}(t) - (1/2)\sigma^2 t\right)},$$
$$v(x, y, t) = \frac{2\Lambda_2\gamma_4\zeta_2}{\Lambda_1\gamma_3\zeta_1} \sec\mathrm{h}^2\left[\sqrt{-\Lambda_2}\zeta\right] e^{\left(-\sigma\mathbb{B}(t) - (1/2)\sigma^2 t\right)}, \tag{31}$$

or

$$u(x, y, t) = -i\sqrt{\frac{-2\Lambda_2}{\Lambda_1}} \operatorname{csch}\left[\sqrt{-\Lambda_2}\zeta\right] e^{\left(i\hbar - \sigma\mathbb{B}(t) - (1/2)\sigma^2 t\right)},$$
$$v(x, y, t) = \frac{-2\Lambda_2\gamma_4\zeta_2}{\Lambda_1\gamma_3\zeta_1} \csc\mathrm{h}^2\left[\sqrt{-\Lambda_2}\zeta\right] e^{\left(-\sigma\mathbb{B}(t) - (1/2)\sigma^2 t\right)}. \tag{32}$$

Case 4. If $\Lambda_2 > 0$ and $\Lambda_1 > 0$, then the exact solution of the NCFSSS ((1) and (2)) are

$$u(x, y, t) = i\sqrt{\frac{2\Lambda_2}{\Lambda_1}} \sec\left[\sqrt{\Lambda_2}\zeta\right] e^{\left(i\hbar - \sigma\mathbb{B}(t) - (1/2)\sigma^2 t\right)},$$
$$v(x, y, t) = \frac{2\Lambda_2\gamma_4\zeta_2}{\Lambda_1\gamma_3\zeta_1} \sec^2\left[\sqrt{\Lambda_2}\zeta\right] e^{\left(-\sigma\mathbb{B}(t) - (1/2)\sigma^2 t\right)}, \tag{33}$$

or

$$u(x, y, t) = i\sqrt{\frac{2\Lambda_2}{\Lambda_1}} \csc\left[\sqrt{\Lambda_2}\zeta\right] e^{\left(i\hbar - \sigma\mathbb{B}(t) - (1/2)\sigma^2 t\right)},$$
$$v(x, y, t) = \frac{2\Lambda_2\gamma_4\zeta_2}{\Lambda_1\gamma_3\zeta_1} \csc^2\left[\sqrt{\Lambda_2}\zeta\right] e^{\left(-\sigma\mathbb{B}(t) - (1/2)\sigma^2 t\right)}, \tag{34}$$

where Λ_1 and Λ_2 are defined in (17) and $\zeta = (\zeta_1/\alpha)x^\alpha + (\zeta_2/\alpha)y^\alpha - \zeta_3 t$ and $h = (h_1/\alpha)x^\alpha + (h_2/\alpha)y^\alpha + h_3 t$.

4.2. The Jacobi Elliptic Function Method. We suppose that the solution to Equation (16) has the type

$$\varphi(\zeta) = a + bsn(\rho\zeta), \tag{35}$$

where a, b, and ρ are undefined constants and $sn(\rho\zeta) = sn(\rho\zeta, m)$ is the Jacobi elliptic sine function (Latin: sinus amplitudinis) for $0 < m < 1$. Differentiate Equation (35) two times, we have

$$\varphi''(\zeta) = -\left(m^2 + 1\right)b\rho^2 sn(\rho\zeta) + 2m^2 b\rho^2 sn^3(\rho\zeta). \tag{36}$$

Plugging Equations (35) and (36) into Equation (16), we attain

$$\begin{aligned} &\left(2m^2 b\rho^2 - \Lambda_1 b^3\right)sn^3(\rho\zeta) - 3\Lambda_1 ab^2 sn^2(\rho\zeta) \\ &- \left[\left(m^2 + 1\right)b\rho^2 + 3\Lambda_1 a^2 b + \Lambda_2 b\right]sn(\rho\zeta) \\ &- \left(\Lambda_1 a^3 + a\Lambda_2\right) = 0. \end{aligned} \tag{37}$$

Putting each coefficient of $[sn(\rho\zeta)]^n$ to be zero for $n = 0, 1, 2, 3$, we have

$$\begin{aligned} \Lambda_1 a^3 + a\Lambda_2 &= 0, \\ \left(m^2 + 1\right)b\rho^2 + 3\Lambda_1 a^2 b + \Lambda_2 b &= 0, \\ 3\Lambda_1 ab^2 sn^2 &= 0, \\ 2m^2 b\rho^2 - \Lambda_1 b^3 &= 0. \end{aligned} \tag{38}$$

When we solve the previous equations, we get

$$a = 0,\quad b = \pm\sqrt{\frac{-2m^2\Lambda_2}{(m^2+1)\Lambda_1}},\quad \rho^2 = \frac{-\Lambda_2}{(m^2+1)}. \tag{39}$$

As a result, using (35), the solution of Equation (16) is

$$\varphi(\zeta) = \pm\sqrt{\frac{-2m^2\Lambda_2}{(m^2+1)\Lambda_1}}\, sn\left(\sqrt{\frac{-\Lambda_2}{(m^2+1)}}\zeta\right). \tag{40}$$

Therefore, the exact solution of the NCFSSS ((1) and (2)) is

$$u(x,y,t) = \pm\sqrt{\frac{-2m^2\Lambda_2}{(m^2+1)\Lambda_1}}\, sn\left(\sqrt{\frac{-\Lambda_2}{(m^2+1)}}\zeta\right) e^{\left(i\hbar-\sigma\mathbb{B}(t)-(1/2)\sigma^2 t\right)}, \tag{41}$$

$$v(x,y,t) = \frac{\gamma_4\zeta_2 m^2\Lambda_2}{(m^2+1)\gamma_3\zeta_1\Lambda_1}\, sn^2\left(\sqrt{\frac{-\Lambda_2}{(m^2+1)}}\zeta\right) e^{\left(-\sigma\mathbb{B}(t)-(1/2)\sigma^2 t\right)}, \tag{42}$$

for $\Lambda_2 < 0$ and $\Lambda_1 > 0$. If $m \longrightarrow 1$, then the solutions (41) and (42) turn to:

$$\begin{aligned} u(x,y,t) &= \pm\sqrt{\frac{-\Lambda_2}{\Lambda_1}}\tanh\left(\sqrt{\frac{-\Lambda_2}{2}}\zeta\right) e^{\left(i\hbar-\sigma\mathbb{B}(t)-(1/2)\sigma^2 t\right)},\\ v(x,y,t) &= \frac{\gamma_4\zeta_2\Lambda_2}{2\gamma_3\zeta_1\Lambda_1}\tanh^2\left(\sqrt{\frac{-\Lambda_2}{2}}\zeta\right) e^{\left(-\sigma\mathbb{B}(t)-(1/2)\sigma^2 t\right)}. \end{aligned} \tag{43}$$

In the same way, we can substitute sn in (35) with $cn(\xi) = cn(\xi, m)$ (where cn is the elliptic cosine (Latin: cosinus amplitudinis)) and $dn(\xi, m) = dn(\xi, m)$ (where dn is the delta amplitude (Latin: delta amplitudinis)) to derive the solutions of Equation (16) as follows:

$$\begin{aligned} \varphi(\zeta) &= \pm\sqrt{\frac{-2m^2\Lambda_2}{(2m^2-1)\Lambda_1}}\, cn\left(\sqrt{\frac{-\Lambda_2}{(2m^2-1)}}\zeta\right),\\ \varphi(\zeta) &= \pm\sqrt{\frac{2m^2\Lambda_2}{(2-m^2)\Lambda_1}}\, dn\left(\sqrt{\frac{-\Lambda_2}{(2-m^2)}}\zeta\right). \end{aligned} \tag{44}$$

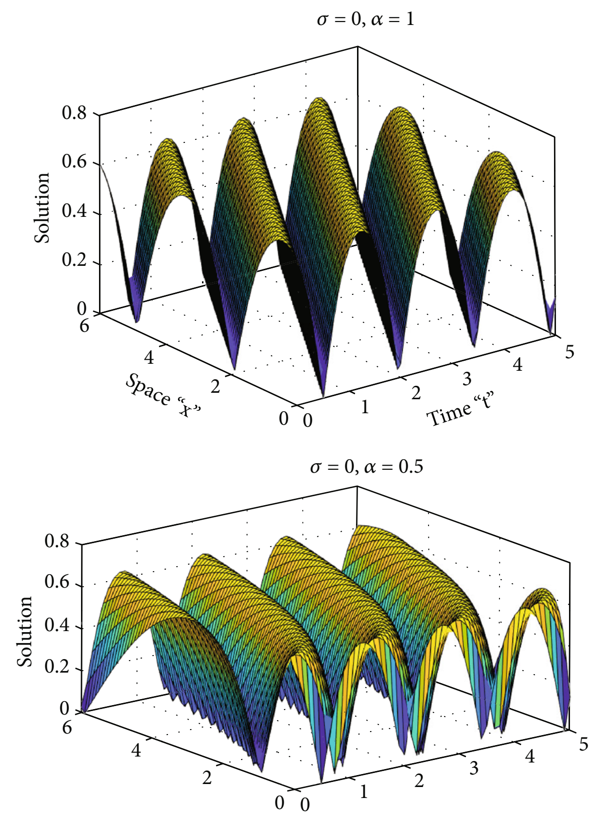

Figure 1: 3D diagrams of Equation (41).

Therefore, the solutions of the NCFSSS ((1) and (2)) are as follows:

$$u(x,y,t) = \pm\sqrt{\frac{-2m^2\Lambda_2}{(2m^2-1)\Lambda_1}}\, cn\left(\sqrt{\frac{-\Lambda_2}{(2m^2-1)}}\zeta\right) e^{\left(i\hbar-\sigma\mathbb{B}(t)-(1/2)\sigma^2 t\right)}, \tag{45}$$

$$v(x,y,t) = \frac{2\gamma_4\zeta_2 m^2\Lambda_2}{\gamma_3\zeta_1\Lambda_1(2m^2-1)}\, cn^2\left(\sqrt{\frac{-\Lambda_2}{(2m^2-1)}}\zeta\right) e^{\left(-\sigma\mathbb{B}(t)-(1/2)\sigma^2 t\right)}, \tag{46}$$

for $(\Lambda_2/(2m^2-1)) < 0$, $\Lambda_1 > 0$, and

$$u(x,y,t) = \pm\sqrt{\frac{-2m^2\Lambda_2}{(2m^2-1)\Lambda_1}}\, dn\left(\sqrt{\frac{-\Lambda_2}{(2m^2-1)}}\zeta\right) e^{\left(i\hbar-\sigma\mathbb{B}(t)-(1/2)\sigma^2 t\right)}, \tag{47}$$

$$v(x,y,t) = \frac{2\gamma_4\zeta_2 m^2\Lambda_2}{\gamma_3\zeta_1\Lambda_1(2-m^2)}\, dn^2\left(\sqrt{\frac{-\Lambda_2}{(2-m^2)}}\zeta\right) e^{\left(-\sigma\mathbb{B}(t)-(1/2)\sigma^2 t\right)}, \tag{48}$$

for $\Lambda_2 < 0$ and $\Lambda_1 > 0$, respectively. If $m \longrightarrow 1$, then the

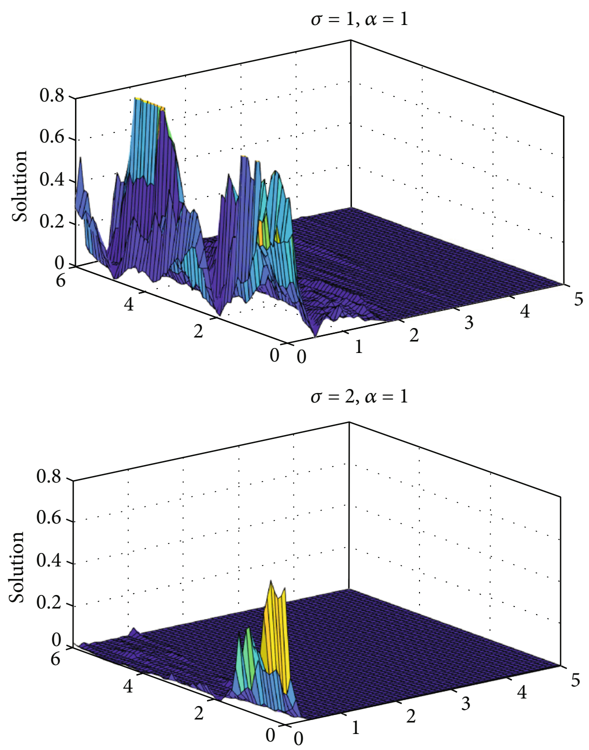

FIGURE 2: 3D diagrams of Equation (41) with $\alpha = 1$.

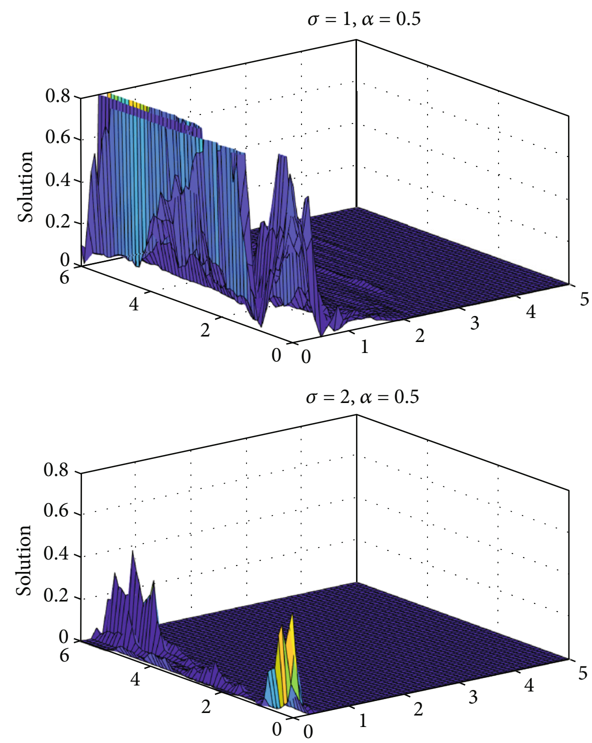

FIGURE 3: 3D diagrams of Equation (41) with $\alpha = 0.5$.

solutions (45) and (46) and (47) and (48) turn to:

$$u(x, y, t) = \pm\sqrt{\frac{-2\Lambda_2}{\Lambda_1}} \operatorname{sech}\left(\sqrt{-\Lambda_2}\zeta\right) e^{\left(i\hbar - \sigma\mathbb{B}(t) - (1/2)\sigma^2 t\right)},$$
$$v(x, y, t) = k^2 m^2 \Lambda_2 \sec\mathrm{h}^2\left(\sqrt{-\Lambda_2}\zeta\right) e^{\left(-\sigma\mathbb{B}(t) - (1/2)\sigma^2 t\right)}, \tag{49}$$

for $\Lambda_2 < 0$ and $\Lambda_1 > 0$.

5. The Effect of BM on NCFSSS Solutions

The effect of BM on the exact solutions of the NCFSSS ((1) and (2)) is discussed here. Fix the parameters $\gamma_1 = \gamma_2 = -1$, $\gamma_3 = 1, \gamma_4 = -2, \hbar_1 = \hbar_2 = \zeta_1 = \zeta_2 = 1, \hbar_3 = -1$, and $m = 0.5$. Hence, $\zeta_3 = -2, \Lambda_1 = 2$, and $\Lambda_2 = -2$. Now, for various values of α (the fractional derivative order) and σ (noise intensity), we provide a number of graphs for $t \in [0, 5]$ and $x \in [0, 6]$. To draw these graphs, we use the MATLAB tools. In the following Figure 1, if $\sigma = 0$, we can observe how the surface fluctuates as the value of α changes:

While in Figures 2 and 3, we can observe that after small transit patterns, the surface smooths significantly when noise is incorporated, and its intensity increases $\sigma = 1, 2$ for different value of α.

Figure 4 shows 2D graphs of Equation (41) with $\sigma = 0, 0.5, 1, 2$ and with $\alpha = 1$, which highlight the above results.

We may infer from Figures 1–4 the following:

(1) As fractional-order α decreases, the surface shrinks

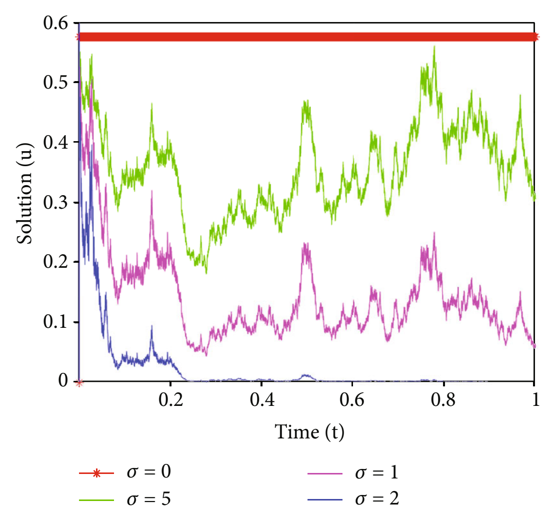

FIGURE 4: 2D diagrams of Equation (41).

(2) The solutions of NCFSSS are stabilized by BM around zero

6. Conclusions

In this paper, we considered the (2 + 1)-dimensional nonlinear conformable fractional stochastic Schrödinger system ((1) and (2)) which has never been examined before with stochastic term and fractional space at the same time. We employed two different methods such as the sine-cosine and the Jacobi elliptic function methods to get elliptic, trigonometric, rational, and hyperbolic fractional stochastic solutions. These obtained solutions are useful in describing some of interesting physical phenomena due to the importance of the NCFSSS in plasma physics and fluid dynamics. Finally, the effect of BM on the exact solution of the NCFSSS ((1) and (2)) is demonstrated by introducing 3D and 2D graphs for some analytical fractional stochastic solutions. In future study, we can address the NCFSSS ((1) and (2)) with multidimensional multiplicative noise.

Data Availability

All data are available in this paper.

Conflicts of Interest

The authors declare that they have no competing interests.

Authors' Contributions

All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

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References

[1] L. Arnold, *Random Dynamical Systems*, Springer-Verlag, 1998.

[2] P. Imkeller and A. H. Monahan, "Conceptual stochastic climate models," *Stochastics and Dynamics*, vol. 2, no. 3, pp. 311–326, 2002.

[3] W. W. Mohammed, "Fast-diffusion limit for reaction–diffusion equations with degenerate multiplicative and additive noise," *Journal of Dynamics and Differential Equations*, vol. 33, no. 1, pp. 577–592, 2021.

[4] S. B. Yuste, L. Acedo, and K. Lindenberg, "Reaction front in an $A + \vec{B}C$ reaction-subdiffusion process," *Physical Review E*, vol. 69, no. 3, article 036126, 2004.

[5] W. W. Mohammed and N. Iqbal, "Impact of the same degenerate additive noise on a coupled system of fractional space diffusion equations," *Fractals*, vol. 30, no. 1, article 2240033, 2022.

[6] D. A. Benson, S. W. Wheatcraft, and M. M. Meerschaert, "The fractional-order governing equation of Lévy motion," *Water Resources Research*, vol. 36, no. 6, pp. 1413–1423, 2000.

[7] S. B. Yuste and K. Lindenberg, "Subdiffusion-limited $A + A$ reactions," *Physical Review Letters*, vol. 87, no. 11, article 118301, 2001.

[8] F. M. Al-Askar, W. W. Mohammed, and M. Alshammari, "Impact of Brownian motion on the analytical solutions of the space-fractional stochastic approximate long water wave equation," *Symmetry*, vol. 14, no. 4, p. 740, 2022.

[9] E. Barkai, R. Metzler, and J. Klafter, "From continuous time random walks to the fractional Fokker-Planck equation," *Physical Review*, vol. 61, no. 1, pp. 132–138, 2000.

[10] A. M. Wazwaz, "The tanh method: exact solutions of the sine-Gordon and the sinh-Gordon equations," *Applied Mathematics and Computation*, vol. 167, no. 2, pp. 1196–1210, 2005.

[11] W. Malfliet and W. Hereman, "The tanh method. I. Exact solutions of nonlinear evolution and wave equations," *Physica Scripta*, vol. 54, no. 6, pp. 563–568, 1996.

[12] M. Wen-Xiu and B. Sumayah, "A binary Darboux transformation for multicomponent NLS equations and their reductions," *Analysis and Mathematical Physics*, vol. 11, no. 2, p. 44, 2021.

[13] A. M. Wazwaz, "A sine-cosine method for handlingnonlinear wave equations," *Mathematical and Computer Modelling*, vol. 40, no. 5-6, pp. 499–508, 2004.

[14] C. Yan, "A simple transformation for nonlinear waves," *Physics Letters A*, vol. 224, no. 1-2, pp. 77–84, 1996.

[15] A. R. Seadawy, A. Ali, and W. A. Albarakati, "Analytical wave solutions of the (2 + 1)-dimensional first integro- differential Kadomtsev-Petviashivili hierarchy equation by using modified mathematical methods," *Results in Physics*, vol. 15, article 102775, 2019.

[16] A. R. Seadawy, D. Kumar, and A. K. Chakrabarty, "Dispersive optical soliton solutions for the hyperbolic and cubic-quintic nonlinear Schrödinger equations via the extended sinh-Gordon equation expansion method," *The European Physical Journal Plus*, vol. 133, no. 5, p. 182, 2018.

[17] N. Celika, A. R. Seadawy, Y. S. Özkan, and E. Yasar, "A model of solitary waves in a nonlinear elastic circular rod: abundant different type exact solutions and conservation laws," *Chaos, Solitons & Fractals*, vol. 143, article 110486, 2021.

[18] A. R. Alharbi, M. B. Almatrafi, and A. R. Seadawy, "Construction of the numerical and analytical wave solutions of the Joseph–Egri dynamical equation for the long waves in nonlinear dispersive systems," *International Journal of Modern Physics B*, vol. 34, no. 30, article 2050289, 2020.

[19] S. T. Demiray and U. Bayrakci, "Soliton solutions for space-time fractional Heisenberg ferromagnetic spin chain equation by generalized Kudryashov method and modified exp -expansion function method," *Revista mexicana de física*, vol. 67, no. 3 May-Jun, pp. 393–402, 2021.

[20] S. T. Demiray, "New soliton solutions of optical pulse envelope $E(Z, \tau)$ with beta time derivative," *Optik*, vol. 223, article 165453, 2020.

[21] S. T. Demiray, "New solutions of Biswas-Arshed equation with beta time derivative," *Optik*, vol. 222, article 165405, 2020.

[22] K. Khan and M. A. Akbar, "The exp $(-\varphi(\varsigma))$-expansion method for finding travelling wave solutions of Vakhnenko-Parkes equation," *International Journal of Dynamical Systems and Differential Equations*, vol. 5, pp. 72–83, 2014.

[23] M. L. Wang, X. Z. Li, and J. L. Zhang, "The (G'/G)-expansion method and travelling wave solutions of nonlinear evolution equations in mathematical physics," *Physics Letters A*, vol. 372, pp. 417–423, 2008.

[24] H. Zhang, "New application of the (G'/G)-expansion method," *Communications in Nonlinear Science and Numerical Simulation*, vol. 14, pp. 3220–3225, 2009.

[25] W. W. Mohammed, M. Alesemi, S. Albosaily, N. Iqbal, and M. El-Morshedy, "The exact solutions of stochastic fractional-space Kuramoto-Sivashinsky equation by using (G'/G)-expansion method," *Mathematics*, vol. 9, p. 2712, 2021.

[26] R. Hirota, "Exact solution of the Korteweg-de Vries equation for multiple collisions of solitons," *Physical Review Letters*, vol. 27, no. 18, pp. 1192–1194, 1971.

[27] W. W. Mohammed, N. Iqbal, and T. Botmart, "Additive noise effects on the stabilization of fractional-space diffusion equation solutions," *Mathematics*, vol. 10, no. 1, p. 130, 2022.

[28] Z. L. Yan, "Abundant families of Jacobi elliptic function solutions of the (2+1)-dimensional integrable Davey-Stewartson-type equation via a new method," *Chaos, Solitons and Fractals*, vol. 18, no. 2, pp. 299–309, 2003.

[29] E. Fan and J. Zhang, "Applications of the Jacobi elliptic function method to special-type nonlinear equations," *Physics Letters A*, vol. 305, no. 6, pp. 383–392, 2002.

[30] X. F. Yang, Z. C. Deng, and Y. Wei, "A Riccati-Bernoulli sub-ODE method for nonlinear partial differential equations and its application," *Advances in Difference equations*, vol. 2015, no. 1, p. 133, 2015.

[31] R. Khalil, M. Al Horani, A. Yousef, and M. Sababheh, "A new definition of fractional derivative," *Journal of Computational and Applied Mathematics*, vol. 264, pp. 65–70, 2014.

[32] K. Nishinari, K. Abe, and J. Satsuma, "A new-type of soliton behavior in a two dimensional plasma system," *Journal of the Physical Society of Japan*, vol. 62, no. 6, pp. 2021–2029, 1993.

[33] M. J. Ablowitz and H. Segur, *Solitons and the Inverse Scattering Transform*, SIAM, Philadelphia, 1981.

[34] A. Davey and K. Stewartson, "On three-dimensional packets of surface waves," *Proceedings of the Royal Society of London. A. Mathematical and Physical Sciences*, vol. 338, pp. 101–110, 1974.

[35] V. Barbu, M. Röckner, and D. Zhang, "Stochastic nonlinear Schrödinger equations with linear multiplicative noise: rescaling approach," *Journal of Nonlinear Science*, vol. 24, no. 3, pp. 383–409, 2014.

[36] V. Barbu, M. Röckner, and D. Zhang, "Stochastic nonlinear Schrodinger equations," *Nonlinear Analysis*, vol. 136, pp. 168–194, 2016.

[37] K. Hosseini, K. Sadri, M. Mirzazadeh, and S. Salahshour, "An integrable (2+1)-dimensional nonlinear Schrodinger system and its optical soliton solutions," *Optik*, vol. 229, article 166247, 2021.

[38] R. Radha and M. Lakshmanan, "Singularity structure analysis and bilinear form of a (2+1) dimensional non-linear Schrodinger (NLS) equation," *Inverse Problems*, vol. 10, no. 4, pp. L29–L33, 1994.

[39] A. R. Seadawy, N. Cheemaa, and A. Biswas, "Optical dromions and domain walls in (2+1)-dimensional coupled system," *Optik*, vol. 227, article 165669, 2021.

[40] M. Bilal and J. Ahmad, "A variety of exact optical soliton solutions to the generalized (2+1)-dimensional dynamical conformable fractional Schrodinger model," *Results in Physics*, vol. 33, article 105198, 2022.