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Retraction

Retracted: A Family of Bayesian Estimators for the Two-Parametric Burr Type II Distribution

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This article has been retracted by Hindawi following an investigation undertaken by the publisher [1]. This investigation has uncovered evidence of one or more of the following indicators of systematic manipulation of the publication process:

(1) Discrepancies in scope

(2) Discrepancies in the description of the research reported

(3) Discrepancies between the availability of data and the research described

(4) Inappropriate citations

(5) Incoherent, meaningless and/or irrelevant content included in the article

(6) Peer-review manipulation

The presence of these indicators undermines our confidence in the integrity of the article's content and we cannot, therefore, vouch for its reliability. Please note that this notice is intended solely to alert readers that the content of this article is unreliable. We have not investigated whether authors were aware of or involved in the systematic manipulation of the publication process.

Wiley and Hindawi regrets that the usual quality checks did not identify these issues before publication and have since put additional measures in place to safeguard research integrity.

We wish to credit our own Research Integrity and Research Publishing teams and anonymous and named external researchers and research integrity experts for contributing to this investigation.

The corresponding author, as the representative of all authors, has been given the opportunity to register their agreement or disagreement to this retraction. We have kept a record of any response received.

References

[1] R. Alshenawy, N. Feroze, A. Al-Alwan, M. Saleem, and S. Islam, "A Family of Bayesian Estimators for the Two-Parametric Burr Type II Distribution," *Journal of Function Spaces*, vol. 2022, Article ID 6347192, 12 pages, 2022.

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Research Article

A Family of Bayesian Estimators for the Two-Parametric Burr Type II Distribution

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This study discusses the posterior estimation for the parameters of the Burr type II distribution (BIID). The informative and noninformative priors along with different loss functions have also been assumed for the posterior estimation. The applicability of the proposed distribution has also been discussed. The modeling capability of the proposed model has been compared with seven classes of the lifetime distributions using real data. The generalizations of Weibull, exponential, Rayleigh, gamma, log normal, Pareto, Maxwell, Levy, Laplace, inverse gamma, Gompertz, chi-square, inverse chi-square, half normal, and log-logistic distributions have been considered for the comparison. The comparison has been made based on different goodness-of-fit criteria, such as Akaike information criteria (AIC), Bayesian information criteria (BIC), and Kolmogorov-Smirnov (KS) test. Based on the results from the study, it can be suggested that the BIID can efficiently replace commonly used lifetime distributions and their modifications. The results under this model were comparable with different conventional/modified distributions having up to six parameters.

1. Introduction

Lifetime distributions are very useful in reliability analysis. There are many conventional models available in literature to model life data. A class of life distribution including twelve models was introduced by Burr [1]. From this family of distribution, the Burr type III (BIIID), Burr type X (BXD), and Burr type XII (BXIID) have been frequently used for lifetime analysis. However, other members of the family have not been considered much for estimating the lifetimes. Similarly, BIID has not received much attention in modeling life datasets. The analysis of such deprived distribution is always of interest for research for exploring their hidden properties and applications.

The contributions regarding the Burr family of distributions can be seen from the following works. Abd-Elfattah and Alharbey [2] used the trimmed samples to estimate the parameters of Burr type III distribution (BIIID). The comparison between Bayesian approach and maximum likelihood (ML) approach was considered. Azimi and Yaghmaei [3] estimated the reliability function of the BIIID using doubly censored samples. The Bayes estimates were obtained using various loss functions and priors. Pant and Hedrick [4] derived the BIIID family of distribution in the context of univariate L correlations and the L moments. Altindag et al. [5] used ML estimation via EM algorithm to analyze the parameters of BIIID using type II censored samples.

Recently, Feroze and Aslam [6] obtained the ML estimates for the parameters of the Burr type V distribution (BVD) under left censored samples. Feroze and Aslam [7] considered the Bayes point and interval estimators for the parameters of the BVD. The contributions regarding BXD can be seen from the following. Surles and Padgett [8] studied the applications and analysis the BXD. Raqab and Kundu [9] suggested that the two parametric BXD can be used to model the skewed datasets efficiently as compared to the Weibull and gamma distributions. Moussa [10] considered the point and interval estimation of the BXD using the

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Table 1: BEs and PRs under SELF and noninformative prior using $\lambda = 0.1, 0.5$ and $\gamma = 0.1, 0.5$.

n	$\lambda = 0.1$	$\gamma = 0.1$	$\lambda = 0.1$	$\gamma = 0.5$	$\lambda = 0.5$	$\gamma = 0.1$
20	0.157254	0.185589	0.137233	0.650650	0.866755	0.253996
	0.010308	**0.009465**	**0.001353**	**0.016990**	**0.147738**	**0.013761**
30	0.131436	0.157923	0.112264	0.476765	0.849495	0.184519
	0.004495	**0.004911**	**0.000710**	**0.015689**	**0.142248**	**0.004290**
50	0.103888	0.131996	0.092479	0.514952	0.804073	0.158216
	0.001258	**0.002408**	**0.000294**	**0.010094**	**0.084603**	**0.001585**
70	0.115840	0.110648	0.095085	0.509778	0.594757	0.114315
	0.001237	**0.001478**	**0.000224**	**0.007072**	**0.036282**	**0.001253**
100	0.099165	0.108867	0.099649	0.522362	0.448178	0.088012
	0.000700	**0.000894**	**0.000128**	**0.004845**	**0.017950**	**0.000862**

Table 2: BEs and PRs under PLF and noninformative prior using $\lambda = 0.1, 0.5$ and $\gamma = 0.1, 0.5$.

n	$\lambda = 0.1$	$\gamma = 0.1$	$\lambda = 0.1$	$\gamma = 0.5$	$\lambda = 0.5$	$\gamma = 0.1$
20	0.197366	0.182009	0.132471	0.760875	1.228240	0.266986
	0.157923	**0.059902**	**0.019544**	**0.079208**	**0.383388**	**0.079540**
30	0.181923	0.171854	0.117223	0.670827	0.909363	0.197881
	0.067085	**0.036632**	**0.006547**	**0.100008**	**0.138713**	**0.030437**
50	0.168380	0.167398	0.112032	0.576744	0.706192	0.174474
	0.027564	**0.030268**	**0.003448**	**0.025514**	**0.121010**	**0.026254**
70	0.125518	0.123353	0.111416	0.464453	0.616973	0.154574
	0.020934	**0.014598**	**0.003678**	**0.047047**	**0.065171**	**0.011682**
100	0.117152	0.118159	0.107443	0.479754	0.513104	0.118903
	0.011649	**0.014547**	**0.001431**	**0.010331**	**0.062256**	**0.007771**

Table 3: BEs and PRs under QLF and noninformative prior using $\lambda = 0.1, 0.5$ and $\gamma = 0.1, 0.5$.

n	$\lambda = 0.1$	$\gamma = 0.1$	$\lambda = 0.1$	$\gamma = 0.5$	$\lambda = 0.5$	$\gamma = 0.1$
20	0.265642	0.233581	0.154377	0.344405	0.051382	0.019315
	0.068621	**0.058633**	**0.228267**	**0.215957**	**0.784434**	**0.784260**
30	0.172735	0.193308	0.060130	0.575012	0.092165	0.033785
	0.061714	**0.040254**	**0.099529**	**0.058981**	**0.522815**	**0.556817**
50	0.126520	0.164437	0.117663	0.523112	0.383178	0.134679
	0.023375	**0.035005**	**0.039567**	**0.055168**	**0.121901**	**0.120160**
70	0.104709	0.108598	0.108708	0.519747	0.430361	0.117992
	0.018765	**0.017472**	**0.029487**	**0.032670**	**0.103622**	**0.094063**
100	0.102616	0.105773	0.101436	0.510275	0.462315	0.111016
	0.017547	**0.016516**	**0.015426**	**0.038728**	**0.066965**	**0.092619**

classical and Bayesian methods. Aludaat et al. [11] obtained Bayesian and classical estimators under grouped data for the parameter of the BXD. Feroze and Aslam [12] analyzed the BXD using Bayesian methods. Ahmad et al. [13] obtained Bayes estimates and maximum likelihood estimates for the BXD based on doubly type II censored.

The analysis of BXIID in different situations has been quite frequent. Following contributions present the glimpses regarding the use of BXIID. Moussa and Jaheen [14] obtained the Bayes estimation for reliability function of BXIID. Soliman [15] suggested the ML and Bayes estimators for estimation of reliability from BXIID. Shao et al. [16] used BXIID to analyze the flood frequency. Yarmohammadi and Pazira [17] worked on the classical estimators for the BXIID such as Minimum Mean Squared Error (min MSE), the ML and the minimal estimators. Rastogi and Tripathi [18] considered the problem of estimating reliability function of BXIID on the basis of a censored type II sample. The comparison between Bayes estimation and ML estimates was

Table 4: BEs and PRs under ELF and noninformative prior using $\lambda = 0.1, 0.5$ and $\gamma = 0.1, 0.5, 0.1$.

n	$\lambda = 0.1$	$\gamma = 0.1$	$\lambda = 0.1$	$\gamma = 0.5$	$\lambda = 0.5$	$\gamma = 0.1$
20	0.189698	0.208199	0.114647	0.414412	0.663113	0.074151
	0.106172	**0.218758**	**0.023635**	**0.074339**	**0.338068**	**0.415012**
30	0.141398	0.144890	0.110247	0.543223	0.631751	0.112822
	0.087738	**0.125240**	**0.020013**	**0.034407**	**0.095608**	**0.135670**
50	0.107190	0.105097	0.107017	0.517596	0.554093	0.103459
	0.080897	**0.093947**	**0.015419**	**0.018677**	**0.058840**	**0.080647**
70	0.093787	0.098241	0.096126	0.497539	0.546376	0.086534
	0.069283	**0.073107**	**0.009544**	**0.026246**	**0.050055**	**0.051566**
100	0.104310	0.100240	0.102307	0.415012	0.491680	0.108994
	0.028607	**0.041435**	**0.007323**	**0.040970**	**0.046790**	**0.030830**

Table 5: BEs and PRs under SELF and informative prior using $\lambda = 0.1, 0.5$ and $\gamma = 0.1, 0.5$.

n	$\lambda = 0.1$	$\gamma = 0.1$	$\lambda = 0.1$	$\gamma = 0.5$	$\lambda = 0.5$	$\gamma = 0.1$
20	0.169785	0.176204	0.121090	0.612655	0.567322	0.152033
	0.008308	**0.008726**	**0.001064**	**0.030319**	**0.124357**	**0.014980**
30	0.149047	0.134234	0.109111	0.449949	0.533977	0.079138
	0.004014	**0.002839**	**0.000581**	**0.014065**	**0.075869**	**0.005891**
50	0.114144	0.117050	0.094292	0.469182	0.486764	0.091117
	0.001539	**0.001841**	**0.000297**	**0.008764**	**0.035180**	**0.001895**
70	0.088691	0.095301	0.128427	0.516571	0.507531	0.092319
	0.000953	**0.000951**	**0.000257**	**0.007015**	**0.031436**	**0.001313**
100	0.104642	0.102893	0.096613	0.484467	0.502793	0.501757
	0.000842	**0.000776**	**0.000141**	**0.004595**	**0.016138**	**0.001067**

Table 6: BEs and PRs under PLF and informative prior using $\lambda = 0.1, 0.5$ and $\gamma = 0.1, 0.5$.

n	$\lambda = 0.1$	$\gamma = 0.1$	$\lambda = 0.1$	$\gamma = 0.5$	$\lambda = 0.5$	$\gamma = 0.1$
20	0.203755	0.253304	0.116120	0.662923	0.928086	0.174503
	0.006686	**0.010915**	**0.010723**	**0.065767**	**0.205545**	**0.036420**
30	0.143397	0.157053	0.088055	0.523088	0.715546	0.151350
	0.005376	**0.004545**	**0.007910**	**0.032785**	**0.126093**	**0.035820**
50	0.114974	0.131342	0.093188	0.519436	0.602093	0.111399
	0.001459	**0.002137**	**0.004949**	**0.019371**	**0.063768**	**0.015857**
70	0.095253	0.086459	0.101497	0.504966	0.419085	0.097850
	0.001363	**0.000928**	**0.004472**	**0.018876**	**0.048392**	**0.013454**
100	0.104621	0.103657	0.098566	0.498382	0.564189	0.099489
	0.000713	**0.000851**	**0.002358**	**0.018232**	**0.032803**	**0.007902**

made. Tahir et al. [19] discussed the estimation of mixture of BXIID under type I censored dataset. Amein and Sayed-Ahmed [20] introduced extended BXIID and estimated its parameters using type I hybrid censoring. Xin et al. [21] considered Bayes estimators and ML estimators of the parameters of three parametric exponentiated BXIID. Rabies and Li [22] investigated the behavior of reliability function from the BXIID using empirical Bayes methods.

However, the BIID has not yet received the desired attention of the researchers. Feroze et al. [23] addressed the problem of estimating parameters of the BIID on the basis of the ML estimates when the samples were left censored. Sindhu et al. [24] addressed the problem of estimating the Burr type II distribution using trimmed samples. However, the said contributions have considered the Bayesian analysis of a single parameter from the BIID. We have attempted to analyze the two-parametric BIID using Bayesian methods. Different priors and loss functions have been assumed for the estimation of the model parameters. Since the Bayes estimators in closed form were unavailable,

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Table 7: BEs and PRs under QLF and informative prior using $\lambda = 0.1, 0.5$ and $\gamma = 0.1, 0.5$.

n	$\lambda = 0.1$	$\gamma = 0.1$	$\lambda = 0.1$	$\gamma = 0.5$	$\lambda = 0.5$	$\gamma = 0.1$
20	0.031917	0.021145	0.069926	0.250091	0.317637	0.050336
	0.438336	**0.515107**	**0.156016**	**0.299545**	**0.232090**	**0.459284**
30	0.034400	0.022518	0.072537	0.540022	0.411084	0.066457
	0.309041	**0.291528**	**0.125372**	**0.058406**	**0.221973**	**0.330387**
50	0.070767	0.066201	0.089179	0.523281	0.431181	0.072356
	0.162426	**0.214098**	**0.026938**	**0.037279**	**0.107217**	**0.270354**
70	0.077508	0.155997	0.109425	0.495028	0.558997	0.073810
	0.078948	**0.166401**	**0.026090**	**0.018845**	**0.095172**	**0.113494**
100	0.112070	0.089452	0.097947	0.495452	0.509506	0.106796
	0.074227	**0.155439**	**0.000145**	**0.006604**	**0.073167**	**0.079320**

Table 8: BEs and PRs under ELF and informative prior using $\lambda = 0.1, 0.5$ and $\gamma = 0.1, 0.5$.

n	$\lambda = 0.1$	$\gamma = 0.1$	$\lambda = 0.1$	$\gamma = 0.5$	$\lambda = 0.5$	$\gamma = 0.1$
20	0.117625	0.173324	0.152185	0.555533	0.091634	0.017041
	0.039466	**0.038557**	**0.030947**	**0.051604**	**0.202605**	**0.305823**
30	0.135209	0.148623	0.083186	0.531343	0.391556	0.072322
	0.090961	**0.119785**	**0.030084**	**0.034599**	**0.185538**	**0.210423**
50	0.074773	0.054819	0.114484	0.470447	0.409648	0.082887
	0.065321	**0.113163**	**0.012113**	**0.020801**	**0.083286**	**0.071606**
70	0.119968	0.112488	0.093079	0.475603	0.534828	0.107432
	0.039046	**0.078766**	**0.014788**	**0.013970**	**0.045444**	**0.059365**
100	0.101468	0.098225	0.094819	0.511975	0.442827	0.096037
	0.002085	**0.057727**	**0.016771**	**0.010364**	**0.028694**	**0.037519**

Table 9: BEs and PEs under ELF and informative prior $\lambda = 0.5, 1$ and $\gamma = 0.5, 1, 2$.

n	$\lambda = 0.5$	$\gamma = 0.5$	$\lambda = 1$	$\gamma = 1$	$\lambda = 1$	$\gamma = 2$
20	0.597987	0.608535	0.726261	0.682525	1.076590	1.633710
	0.040999	**0.058660**	**0.043068**	**0.036224**	**0.023058**	**0.051033**
30	0.572712	0.428040	1.145180	1.128690	0.947092	1.816010
	0.031513	**0.055039**	**0.016838**	**0.018957**	**0.018969**	**0.043565**
50	0.554941	0.440153	0.912421	0.916108	0.965428	1.934090
	0.021730	**0.038584**	**0.013168**	**0.013222**	**0.010207**	**0.010319**
70	0.464667	0.463372	0.929114	0.931033	0.974551	1.957400
	0.011865	**0.017363**	**0.010720**	**0.013375**	**0.007204**	**0.011405**
100	0.529465	0.512466	0.960924	0.954053	1.010080	2.016630
	0.007549	**0.010875**	**0.008035**	**0.005809**	**0.000410**	**0.005855**

the numerical integrations were employed to obtain the said estimates numerically.

2. Materials and Methods

The probability density function for BIID is

$$f(x \mid \lambda, \gamma) = \frac{\gamma}{\lambda} e^{-x/\lambda} \left(1 + e^{-x/\lambda}\right)^{-(\gamma+1)}, \quad -\infty < x < \infty, \lambda, \gamma > 0, \tag{1}$$

where the shape parameter is $\gamma \geq 0.5$ and scale parameter is $\lambda > 0$.

Cumulative distribution functions (CDF) for BIID are

$$F(x, \lambda, \gamma) = \left(1 + e^{-x/\lambda}\right)^{\gamma}. \tag{2}$$

In this study, the Bayesian analysis of the BIID has been considered. The numerical integration has been used to obtain the Bayes estimates. Further, different loss functions have been used for the posterior estimation. A real dataset

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Table 10: BEs and PRs under ELF and informative prior $\lambda = 1.5, 2$ and $\gamma = 1, 2$.

n	$\lambda = 1.5$	$\gamma = 1$	$\lambda = 2$	$\gamma = 1$	$\lambda = 2$	$\gamma = 2$
20	1.399280	0.742399	1.694110	1.372880	1.428260	1.777290
	0.053497	**0.044837**	**0.025475**	**0.025766**	**0.040175**	**0.030803**
30	1.438440	1.093850	2.263420	0.716414	1.794970	2.169420
	0.031082	**0.042397**	**0.012449**	**0.025564**	**0.016407**	**0.016641**
50	1.449630	0.901676	1.929650	1.061130	1.861960	1.915240
	0.020512	**0.019625**	**0.012337**	**0.020647**	**0.010285**	**0.014451**
70	1.457860	0.924060	2.044770	0.959895	1.949720	1.927030
	0.015360	**0.009956**	**0.008384**	**0.012976**	**0.004061**	**0.009009**
100	1.478970	0.962216	1.987570	0.987196	2.026100	1.941500
	0.004428	**0.007936**	**0.005000**	**0.009314**	**0.002972**	**0.001045**

Table 11: BEs and PRs under ELF and informative prior $\lambda = 10, 1$ and $\gamma = 1, 10$.

n	$\lambda = 10$	$\gamma = 1$	$\lambda = 1$	$\gamma = 10$	$\lambda = 10$	$\gamma = 10$
20	5.422610	0.667664	1.391310	3.511930	9.407280	5.474190
	0.028824	**0.054879**	**0.028180**	**0.027550**	**0.073539**	**0.024276**
30	6.330470	0.777774	1.225670	4.554940	9.713680	5.661200
	0.019923	**0.032914**	**0.020072**	**0.021528**	**0.039601**	**0.023192**
50	8.292970	0.837974	1.160800	5.343760	10.066200	6.814900
	0.005905	**0.030191**	**0.009809**	**0.014363**	**0.023219**	**0.015703**
70	9.443730	0.915281	1.084830	6.855400	10.032100	7.077170
	0.005154	**0.005283**	**0.004968**	**0.012120**	**0.013149**	**0.013293**
100	9.744990	0.932157	1.064740	6.838310	9.983510	7.944030
	0.003741	**0.001581**	**0.008591**	**0.008731**	**0.017631**	**0.012076**

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has been used to illustrate the application of the proposed model. The suitably proposed model has been investigated by comparing it with different classes of lifetime distributions such as exponentiated distribution (ED), Beta Extended Generalized Distribution (BEGD), Weibull Extended Generalized Distribution (WEGD), Exponentiated Generalized G Distribution (EGGD), Gamma G-I Distribution (GGID), Gamma G-II Distribution (GGIID), and Life Time Distribution (LTD).

2.1. Loss Function. The following four loss functions have been used for obtaining Bayes estimates of the model parameters. The loss functions are used to estimate the model parameters and associated posterior risks. The loss functions can be either symmetric or asymmetric. The symmetric loss function gives equal weights to over- or underestimation. On the other hand, the asymmetric loss function considers different weights for over- and underestimation.

2.1.1. Squared Error Loss Function (SELF). SELF is defined as:

$$L(\psi, \psi_{\mathrm{SELF}}) = (\psi - \psi_{\mathrm{SELF}})^2, \tag{3}$$

where $\psi = (\lambda, \gamma)$.

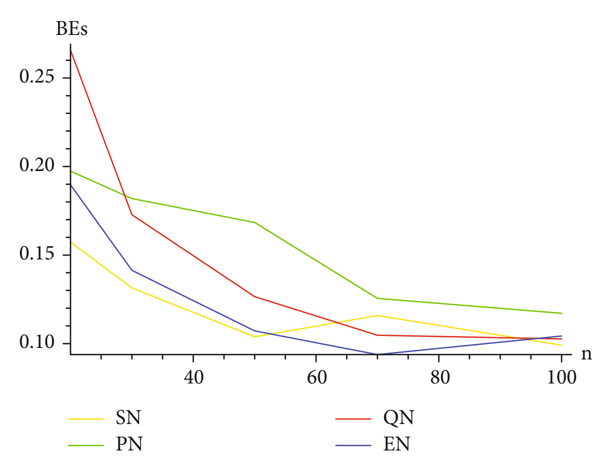

Figure 1: Graph of BEs under different LFs using NIP for $\lambda = 0.1$, using $\gamma = 0.1$.

The SELF under the Bayes estimator is

$$\psi_{\mathrm{SELF}} = E(\psi). \tag{4}$$

The posterior risk under SELF is

$$R_{\mathrm{SELF}} = E(\psi - \psi_{\mathrm{SELF}})^2 = E(\psi)^2 - [E(\psi)]^2. \tag{5}$$

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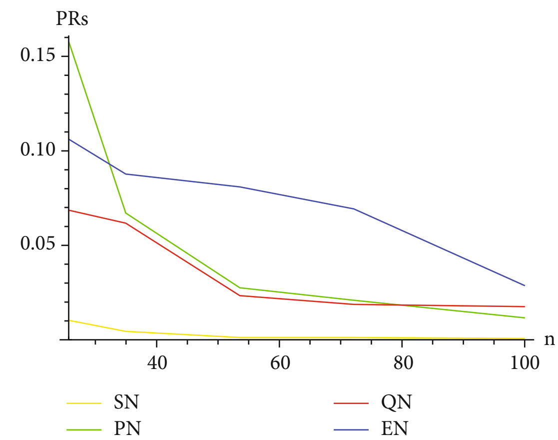

Figure 2: Graph of PRs under different LFs using NIP for $\lambda = 0.1$, using $\gamma = 0.1$.

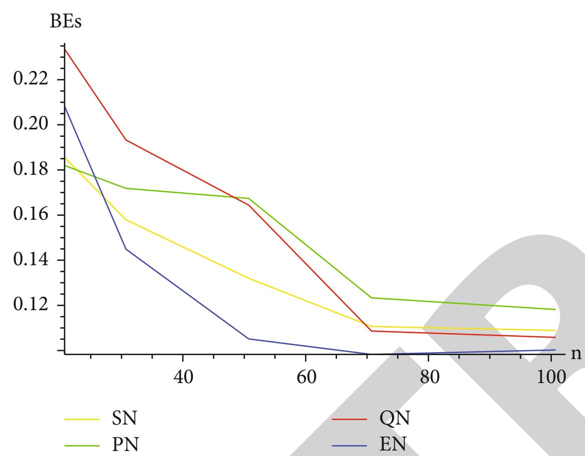

Figure 3: Graph of BEs under different LFs using NIP for $\lambda = 0.1$, using $\gamma = 0.5$.

2.1.2. Quadratic Loss Function (QLF). The QLF is define as

$$L\left(\psi, \psi_{\mathrm{QLF}}\right) = \left(\frac{\psi - \psi_{\mathrm{QLF}}}{\psi}\right)^2. \quad (6)$$

These are the estimators related to the Bayes under QLF which is given as

$$B_{\mathrm{QLF}} = E\left(\psi^{-1}\right)\left\{E\left(\psi^{-2}\right)\right\}^{-1}. \quad (7)$$

The risk under QLF using is

$$R_{\mathrm{QLF}} = E\left[\frac{\psi - \psi_{\mathrm{QLF}}}{\psi}\right]^{-2} = 1 - \frac{\left\{E\left(\psi^{-1}\right)\right\}^2}{E\left(\psi^{-2}\right)}. \quad (8)$$

2.1.3. Precautionary Loss Function (PLF). The PLF can be presented as

$$L(\psi_{\mathrm{PLF}}, \psi) = \frac{(\psi_{\mathrm{PLF}} - \psi)^2}{\psi_{\mathrm{PLF}}}. \quad (9)$$

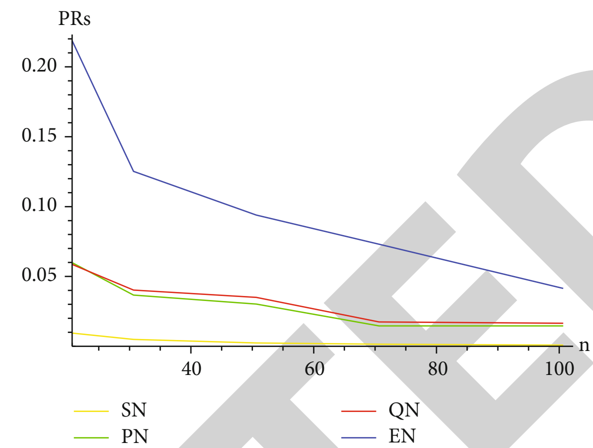

Figure 4: Graph of PRs under different LFs using NIP for $\lambda = 0.1$, using $\gamma = 0.5$.

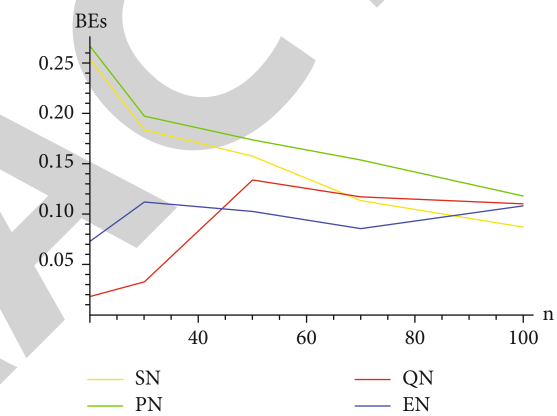

Figure 5: Graph of BEs under different LFs using NIP for $\gamma = 0.1$, using $\lambda = 0.5$.

The Bayes estimator for PLF is

$$\psi_{\mathrm{PLF}} = \left[E\left(\psi^2\right)\right]^{1/2}. \quad (10)$$

Under PLF, the Bayes risk is

$$R_{\mathrm{PLF}} = 2(\psi_{\mathrm{PLF}} - E(\psi)). \quad (11)$$

2.1.4. Entropy Loss Function (ELF). The ELF is defined as

$$L(\psi, \psi_{\mathrm{ELF}}) = \left(\frac{\psi_{\mathrm{ELF}}}{\psi}\right) - \ln\left(\frac{\psi_{\mathrm{ELF}}}{\psi}\right). \quad (12)$$

Under the Bayes estimator is written as

$$\psi_{\mathrm{ELF}} = E\left(\psi^{-1}\right)^{-1}. \quad (13)$$

The risk under ELF can be written as

$$R_{\mathrm{ELF}} = E\{L(\psi_{\mathrm{ELF}}, \psi)\} = E\{\ln(\psi)\} - \ln(\psi_{\mathrm{ELF}}). \quad (14)$$

2.2. Bayesian Estimation of the Burr Type II Distribution.

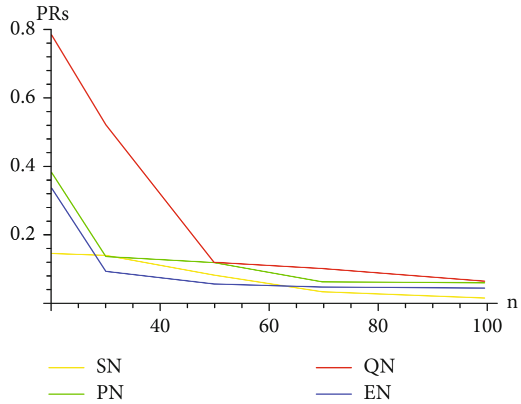

Figure 6: Graph of PRs under different LFs using NIP for $\gamma = 0.1$, using $\lambda = 0.5$.

Table 12: Comparison of BIID with class of EGDs using real dataset-1.

Model	AIC	BIC	LL	KS-statistic	KS *P* value
BIID	72.78	74.77	-34.39	0.09	0.98
E exponential	70.22	72.21	-33.11	0.15	0.65
E Rayleigh	67.61	69.60	-31.80	0.11	0.90
E Weibull	68.95	71.94	-31.47	0.11	0.92
E gamma	71.10	74.09	-32.55	0.14	0.75
E log normal	80.89	83.87	-37.44	0.25	0.12
E Burr XII	79.41	82.39	-36.70	0.22	0.20
E chi-square	70.34	72.34	-33.17	0.15	0.69
E Frechet	89.43	92.42	-41.71	0.23	0.17
E Gompertz	68.86	71.85	-31.43	0.11	0.92
E log Frechet	69.31	72.30	-31.65	0.09	0.98
E Lomax	73.09	76.08	-33.54	0.17	0.50
E log-logistic	69.30	72.29	-31.65	0.09	0.99

AIC: Akaike information criteria; BIC: Bayesian information criteria; LL: log-likelihood; KS: Kolmogorov-Smirnov.

The main advantage of the Bayesian methods to the classical methods is that these methods allow us to incorporate the prior information to analyze the model parameters. In addition, the Bayesian methods also provide more efficient results as compared to classical methods, especially in the case of the small samples. The Bayesian estimation of the BIID has been given in the following.

For a sample of size "n," the likelihood function for the BIID is

$$L(x;\lambda,\gamma) \propto \lambda^{-n}\gamma^{n}\prod_{i=1}^{n}\exp\left(-\frac{x_i}{\lambda}\right)\left\{1+\exp\left(-\frac{x_i}{\lambda}\right)\right\}^{-\gamma-1}. \tag{15}$$

The noninformative prior for $\psi = (\lambda, \gamma)$ is given as

$$g_1(\psi) \propto 1. \tag{16}$$

Table 13: Comparison of BIID with class of BEGDs using real dataset-1.

Model	AIC	BIC	LL	KS-statistic	KS *P* value
BIID	72.78	74.77	-34.39	0.09	0.98
BE exponential	74.57	78.55	-33.28	0.15	0.64
BE Rayleigh	72.46	76.45	-32.23	0.13	0.81
BE Weibull	72.97	77.95	-31.48	0.10	0.96
BE gamma	73.47	78.45	-31.73	0.12	0.86
BE log normal	75.01	79.99	-32.50	0.15	0.67
BE Burr XII	83.77	88.75	-36.88	0.21	0.25
BE chi-square	71.15	75.13	-31.57	0.10	0.95
BE Frechet	86.90	91.87	-38.45	0.23	0.19
BE Gompertz	73.16	78.14	-31.58	0.08	0.99
BE log Frechet	74.69	79.67	-32.34	0.09	0.98
BE Lomax	77.81	82.79	-33.90	0.16	0.61
BE log-logistic	73.04	78.02	-31.52	0.10	0.96

Table 14: Comparison of BIID with class of WEGDs using real dataset-1.

Model	AIC	BIC	LL	KS-statistic	KS *P* value
BIID	72.78	74.77	-34.39	0.09	0.98
WE exponential	73.80	77.78	-32.90	0.12	0.87
WE Rayleigh	77.40	82.55	-33.30	0.10	0.93
WE Weibull	74.47	79.45	-32.23	0.12	0.86
WE gamma	77.36	82.34	-33.68	0.11	0.92
WE log normal	74.25	79.23	-32.12	0.12	0.90
WE Burr XII	77.70	82.68	-33.85	0.14	0.75
WE chi-square	75.08	79.06	-33.54	0.13	0.87
WE Frechet	77.53	82.51	-33.76	0.11	0.93
WE Gompertz	76.22	81.19	-33.11	0.12	0.84
WE log Frechet	74.25	79.23	-32.12	0.11	0.92
WE Lomax	76.55	81.53	-33.27	0.11	0.92
WE log-logistic	76.92	81.89	-33.46	0.11	0.91

The conjugate gamma prior for $\psi = (\lambda, \gamma)$ is

$$g_2(\psi) \propto \psi^{a-1}\exp(-b\psi). \tag{17}$$

Using (15) and (16), the posterior distribution under noninformative prior is

$$\kappa_1(\lambda,\gamma|\mathrm{x}) \propto \lambda^{-n}\gamma^{n}\prod_{i=1}^{n}\exp\left(-\frac{x_i}{\lambda}\right)\left\{1+\exp\left(-\frac{x_i}{\lambda}\right)\right\}^{-\gamma-1}. \tag{18}$$

Similarly, using (15) and (17), the posterior distribution under the informative prior is

$$\kappa_2(\lambda,\gamma|\mathrm{x}) \propto \lambda^{-n+a_1-1}\gamma^{n+a_2-1}\exp(-b_1\lambda)\exp(-b_2\gamma)\prod_{i=1}^{n}\exp\left(-\frac{x_i}{\lambda}\right) \cdot \left\{1+\exp\left(-\frac{x_i}{\lambda}\right)\right\}^{-\gamma-1}. \tag{19}$$

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Table 15: Comparison of BIID with class of EGGDs using real dataset-1.

Model	AIC	BIC	LL	KS-statistic	KS P value
BIID	72.78	74.77	-34.39	0.09	0.98
EG exponential	72.22	75.20	-33.11	0.15	0.63
EG Rayleigh	69.50	72.48	-31.75	0.11	0.92
EG Weibull	70.93	74.92	-31.46	0.13	0.80
EG gamma	71.67	75.65	-31.83	0.12	0.85
EG log normal	74.39	78.37	-33.19	0.17	0.51
EG BIID	81.41	85.39	-36.70	0.23	0.19
EG chi-square	69.83	72.82	-31.91	0.10	0.96
EG Frechet	83.05	87.03	-37.52	0.25	0.13
EG Gompertz	70.87	74.85	-31.43	0.10	0.95
EG log Frechet	71.27	75.25	-31.63	0.11	0.915
EG Burr XII	72.78	74.77	-34.39	0.09	0.98
EG exponential	72.22	75.20	-33.11	0.15	0.63

Table 16: Comparison of BIID with class of GGIDs using real dataset-1.

Model	AIC	BIC	LL	KS-statistic	KS P value
BIID	72.78	74.77	-34.39	0.09	0.98
GI exponential	70.14	72.13	-33.07	0.15	0.67
GI Rayleigh	67.76	69.75	-31.88	0.11	0.93
GI Weibull	68.95	71.94	-31.47	0.10	0.94
GI gamma	69.89	72.88	-31.94	0.11	0.92
GI log normal	78.63	81.61	-36.31	0.24	0.16
GI Burr XII	78.97	81.96	-36.48	0.21	0.25
GI chi-square	70.39	72.38	-33.19	0.14	0.73
GI Frechet	87.09	90.08	-40.65	0.28	0.06
GI Gompertz	68.79	71.78	-31.39	0.98	0.01
GI log Frechet	69.38	72.37	-31.69	0.09	0.98
GI Lomax	75.30	79.28	-33.65	0.16	0.60
GI log-logistic	69.07	71.77	-31.39	0.09	0.97

From (18) and (19), it can be assessed that the Bayes estimators under proposed loss functions cannot be obtained explicitly. Hence, the numerical integrations have been used for approximate solutions of the proposed estimates.

3. Results and Discussions

This section starts with the simulation study for the proposed Bayes estimates using different sample sizes, different parametric values, different priors, and different loss functions. The following true parametric values have been used for generation of the simulated samples: $(\lambda, \gamma) = \{(0.1, 0.1)$ $, (0.1, 0.5), \{(0.5, 0.1)\}$. After verifying the necessary characteristics (such as convergence and consistency) of the Bayes estimates from the proposed model, the modeling capability of the proposed model has been compared with different classes of the life distributions. These classes include class of LTD, BEGD, WEGD, ED, EKGD, GGID, GGIID, and EGD. The Mathematica and R software have been used for the numerical calculations. Both the parameters of the proposed distribution have been estimated jointly. In the tables, the values given in standard fonts represent the Bayes estimates, whereas the values given in bold fonts represent the amounts of associated posterior risks.

The results from the simulation study have been presented in Tables 1–11. In the tables, the amounts of posterior

Table 17: Comparison of BIID with class of GGIIDs using real dataset-1.

Model	AIC	BIC	LL	KS-statistic	KS P value
BIID	72.78	74.77	-34.39	0.09	0.98
GII exponential	70.16	72.15	-33.08	0.98	0.01
GII Rayleigh	70.24	72.23	-33.12	0.99	0.01
GII Weibull	71.46	74.45	-32.73	0.99	0.01
GII gamma	72.13	75.11	-33.06	0.99	0.01
GII log normal	78.47	81.46	-36.23	0.99	0.01
GII Burr XII	79.00	81.99	-36.50	0.21	0.26
GII chi-square	70.39	72.38	-33.19	0.14	0.72
GII Frechet	84.89	87.87	-39.44	0.99	0.01
GII Gompertz	68.85	71.84	-31.42	0.10	0.96
GII log Frechet	69.66	72.65	-31.83	0.98	0.01
GII Lomax	73.96	76.94	-33.98	0.98	0.01
GII log-logistic	72.72	75.71	-33.36	0.99	0.01

Table 18: Comparison of BIID with class of LTDs using real dataset-1.

Model	AIC	BIC	LL	KS-statistic	KS P value
BIID	72.78	74.77	-34.39	0.09	0.98
LT Weibull	69.57	71.56	-32.78	0.12	0.53
LT exponential	70.41	72.40	-33.20	0.17	0.28
LT Rayleigh	79.03	81.03	-37.51	0.19	0.16
LT gamma	70.14	72.13	-33.07	0.15	0.25
LT inverse Gaussian	77.50	79.49	-36.75	0.21	0.01
LT Pareto	72.99	84.98	-39.49	0.38	0.01
LT Maxwell	97.81	99.80	-36.90	0.25	0.01
LT Levy	87.00	88.99	-31.50	0.29	0.01
LT Laplace	76.06	78.05	-36.03	0.10	0.81
LT inverse gamma	92.40	94.39	-34.20	0.30	0.01
LT gamble	79.97	81.96	-37.98	0.15	0.19
LT chi-square	70.39	72.38	-33.19	0.14	0.40
LT inverse chi-square	82.64	84.63	-39.322	0.29	0.01
LT half normal	67.86	69.85	-31.93	0.09	0.95
LT inverse Gaussian	88.49	90.48	-42.24	0.38	0.01

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Table 19: Comparison of BT-IID with class of EGDs using dataset-2.

Name of distribution	Dataset-2 AIC	BIC	LL	KS-statistic	KS P value
Burr II	-60.61	-58.62	-32.30	0.16	0.68
E exponential	-61.90	-59.78	-32.88	0.17	0.60
E Rayleigh	-45.46	-43.47	-24.73	0.34	0.01
E Weibull	-59.36	-56.37	-32.68	0.16	0.62
E gamma	-59.90	-56.92	-32.95	0.17	0.56
E log normal	-66.64	-63.65	-36.32	0.14	0.79
E Burr XII	-59.75	-56.76	-32.87	0.24	0.19
E chi-square	-60.93	-40.94	-35.46	0.15	0.01
E Frechet	-71.25	-68.27	-38.62	0.11	0.95
E Gompertz	-64.46	-61.48	-35.23	0.19	0.41
E log Frechet	-60.17	-57.19	-33.08	0.19	0.40
E Lomax	-64.72	-61.73	-35.36	0.14	0.77
E log-logistic	-70.60	-67.61	-38.30	0.11	0.96

Table 20: Comparison of BT-IID with class of BEGDs using dataset-2.

Name of distribution	Dataset-2 AIC	BIC	LL	KS-statistic	KS P value
Burr II	-60.61	-58.62	-32.30	0.16	0.68
BE exponential	-66.06	-62.07	-37.03	0.10	0.98
BE Rayleigh	-59.60	-55.61	-33.80	0.15	0.69
BE Weibull	-61.37	-56.39	-35.68	0.14	0.78
BE gamma	-65.26	-60.28	-37.63	0.13	0.83
BE log normal	-67.14	-62.16	-38.57	0.12	0.93
BE Burr XII	-61.25	-56.27	-35.62	0.12	0.93
BE chi-square	-59.12	-55.14	-33.56	0.19	0.42
BE Frechet	-67.45	-62.47	-38.72	0.13	0.84
BE Gompertz	-63.16	-58.18	-36.58	0.12	0.91
BE log Frechet	-64.45	-59.47	-37.22	0.12	0.91
BE Lomax	-56.96	-51.98	-33.48	0.16	0.67
BE log-logistic	-67.33	-62.35	-38.66	0.12	0.93

risks (PRs) have been presented in bold fonts. From the results, it can be assessed that the estimated values of the model parameters are converging to the true values by increasing the sample size. In addition, the amounts of PRs are decreasing with increase in sample size, which shows that the proposed estimators are consistent in nature. Further, the larger values of λ result in improved estimation for γ. On the other hand, the smaller values of γ result in improved estimation for λ. In comparison of the prior, it can be observed that the estimation under the informative prior is better than that under the noninformative prior. As far as the comparison of loss functions is concerned, ELF is better for the estimation of both of the model parameters. Figures 1–6 also confirm these findings.

3.1. Real Data Analysis. In this subsection, the performance of the BIID has been compared with different classes of the lifetime distributions in modeling the real dataset. The real dataset contains lifetimes of 20 electronic components reported by Murthy et al. [25]. The values of the dataset are 0.03, 0.12, 0.22, 0.35, 0.73, 0.79, 1.25, 1.41, 1.52, 1.79, 1.80, 1.94, 2.38, 2.40, 2.87, 2.99, 3.14, 3.17, 4.72, and 5.09. We have named this data as dataset-1. The analysis using the real dataset has been presented in Tables 12–18. Another dataset containing failure times (in years) of electrical products has also been used for the analysis. These data contain the following observations: 0.0003, 0.0298, 0.1648, 0.3529, 0.4044, 0.5712, 0.5808, 0.7607, 0.8188, 1.1296, 1.2228, 1.2773, 1.9115, 2.2333, 2.3791, 3.0916, 3.4999, 3.7744, 7.4339, and 13.6866. These data have also been reported by Murthy et al. [25] and have been named as dataset-2. The results regarding analysis of dataset-2 have been placed in Tables 19–25.

From Tables 12–18, it can be assessed that performance of BIID is

Table 21: Comparison of BT-IID with class of WEGDs using dataset-2.

Name of distribution	Dataset-2 AIC	BIC	LL	KS-statistic	KS P value
Burr II	-60.61	-58.62	32.30	0.16	0.68
WE Weibull	-64.41	-59.43	37.20	0.23	0.20
WEB Rayleigh	-66.20	-62.10	36.05	1.00	0.01
WE Weibull	-65.46	-59.49	38.73	0.15	0.72
WE gamma	-62.85	-56.87	37.42	0.21	0.33
WE log normal	-67.17	-58.88	35.29	0.12	0.66
WE Burr XII	-64.10	-58.12	38.05	0.15	0.74
WE chi-square	-56.24	-51.26	33.12	0.28	0.06
WE Frechet	-66.17	-60.20	39.08	0.11	0.93
WE Gompertz	-61.26	-59.55	38.48	0.13	0.87
WE log Frechet	-59.58	-53.60	35.79	0.22	0.24
WE Lomax	-65.26	-59.29	38.63	0.16	0.68
WE log-logistic	-64.73	-58.76	38.36	0.13	0.86

(i) Comparable with the class of exponentiated distributions

(ii) Comparable with the class of beta extended distributions

(iii) Slightly better than the Weibull extended generalized family of distributions

(iv) Comparable with the exponentiated generalized G distributions

(v) Comparable with the gamma G I distributions

(vi) Comparable with the gamma G II distributions

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Table 22: Comparison of BT-IID with class of EGGDs using dataset-2.

Name of distribution	Dataset-2 AIC	BIC	LL	KS-statistic	KS *P* value
Burr II	-60.61	-58.62	-32.30	0.16	0.68
EG exponential	-59.95	-56.96	-32.97	0.15	0.69
EG Rayleigh	-44.55	-41.56	-25.27	0.31	0.03
EG Weibull	-59.05	-55.06	-33.52	0.18	0.47
EG gamma	-58.17	-54.18	-33.08	0.17	0.57
EG log normal	-63.99	-60.01	-35.99	0.12	0.92
EG Burr XII	-60.28	-56.29	-34.14	0.17	0.56
EG chi-square	-61.82	-58.83	-33.91	0.17	0.58
EG Frechet	-69.72	-65.74	-38.86	0.13	0.85
EG Gompertz	-64.52	-60.53	-36.26	0.14	0.77
EG log Frechet	-62.28	-58.30	-35.14	0.16	0.66
EG Lomax	-63.92	-59.94	-35.96	0.14	0.81
EG log-logistic	-68.15	-64.16	-38.07	0.12	0.89

Table 23: Comparison of BT-IID with class of GGIDs using dataset-2.

Name of distribution	Dataset-2 AIC	BIC	LL	KS-statistic	KS *P* value
Burr II	-60.61	-58.62	-32.30	0.16	0.68
GI exponential	-55.78	-53.79	-29.89	0.22	0.26
GI Rayleigh	-44.59	-42.60	-24.29	0.34	0.01
GI Weibull	-53.14	-50.15	-29.57	0.25	0.15
GI gamma	-56.81	-52.82	-32.40	0.19	0.44
GI log normal	-62.54	-59.56	-34.27	0.15	0.71
GI Burr XII	-53.31	-50.32	-29.65	0.28	0.08
GI chi-square	-55.63	-53.64	-34.81	0.48	0.01
GI Frechet	-71.25	-68.27	-38.62	0.11	0.94
GI Gompertz	-51.37	-48.38	-28.68	0.26	0.11
GI log Frechet	-50.73	-47.74	-28.36	0.26	0.11
GI Lomax	-62.76	-59.77	-34.38	0.14	0.81
GI log-logistic	-69.85	-66.86	-37.92	0.10	0.97

(vii) Comparable with the conventional distributions

The comparison of BIID with class of lifetime distribution has been made using three criteria, i.e., AIC, BIC, and KS tests. Whenever the BIID was better in modeling the respective real dataset using all three criteria, the BIID was declared better to the corresponding class of life models. On the other hand, whenever the BIID was better to some but not all the modes in the respective class, the BIID distribution was considered comparable to the respective class of lifetime models. Keeping in view that the different classes of distribution contain higher number of parameters (from two to six), the BIID can provide lots of ease in modeling lifetimes regarding the real data containing lifetimes of 20 electronic components. Similar trends can be seen from the analysis of the dataset-2, reported in Tables 19–25.

Table 24: Comparison of BT-IID with class of GGIDs using dataset-2.

Name of distribution	Dataset-2 AIC	BIC	LL	KS-statistic	KS *P* value
Burr II	-60.61	-58.62	-32.30	0.16	0.68
GII exponential	-42.61	-40.62	-23.30	0.97	0.01
GII Rayleigh	-47.58	-45.59	-25.79	0.99	0.12
GII Weibull	-54.91	-51.92	-30.45	0.99	0.01
GII gamma	-53.27	-50.28	-29.63	0.25	0.13
GII log normal	-59.33	-56.35	-32.66	0.99	0.01
GII Burr XII	-52.30	-49.32	-29.15	0.24	0.17
GII chi-square	-52.95	-50.96	-28.47	0.98	0.01
GII Frechet	-71.70	-68.71	-38.85	0.99	0.01
GII Gompertz	-49.99	-47.00	-27.99	0.28	0.07
GII log Frechet	-54.20	-51.21	-30.10	0.25	0.15
GII Lomax	-39.52	-36.54	-22.76	0.96	0.01
GII log-logistic	-70.20	-67.21	-38.10	0.99	0.01

Table 25: Comparison of BT-IID with class of LTDs using dataset-2.

Name of distribution	Dataset-2 AIC	BIC	LL	KS-statistic	KS *P* value
Burr II	-60.61	-58.62	32.30	0.16	0.68
LT Weibull	-48.84	-46.85	26.42	0.11	0.62
LT exponential	-40.29	-38.30	22.14	0.12	0.61
LT Rayleigh	-46.55	-44.56	25.27	0.30	0.58
LT gamma	-55.78	-53.79	29.89	0.13	0.54
LT inverse Gaussian	-63.12	-61.13	33.56	0.12	0.54
LT Pareto	-75.12	-73.12	39.56	0.14	0.56
LT Maxwell	42.53	40.54	23.26	0.15	0.66
LT Levy	-89.23	-91.22	42.61	0.15	0.59
LT Laplace	-57.15	-55.15	30.57	0.11	0.61
LT inverse gamma	-67.96	-65.97	35.98	0.10	0.60
LT gamble	-18.11	-16.12	11.05	0.10	0.64
LT chi-square	-4.46	-2.47	4.23	0.14	0.62
LT inverse chi-square	-66.49	-64.50	35.24	0.13	0.52
LT half normal	-42.98	-40.98	23.49	0.11	0.55
LT inverse Gaussian	-62.15	-60.15	33.07	0.10	0.61

4. Conclusion

Unlike other distributions from the family of Burr distributions, the lack of literature regarding the BIID motivated us to conduct an in-depth study regarding this distribution.

We have proposed the Bayesian analysis for the parameters of the BIID assuming different priors (informative and noninformative) and loss functions (SELF, PLF, QLF, and ELF) to estimate the parameters of the proposed distribution. As the Bayes estimates for the said parameters cannot be obtained in the closed form, the numerical integration was used for the computations. The performance of the proposed estimators has been discussed in terms of associated posterior risks. The applications of the proposed model have been explored using real data. Finally, the performance of the distribution, in modeling real data, has been compared with different classes of conventional and modified distributions. These classes include LTD, BEGD, WEGD, ED, EKGD, GGID, GGIID, and EGD having ninety-nine (two to six parametric) distributions. The following distributions along with their modifications were used for comparison: Weibull, exponential, and Rayleigh, gamma, log normal, Pareto, Maxwell, Levy, Laplace, inverse gamma, Gompertz, chi-square, inverse chi-square, half normal, and log-logistic distribution. The comparison has been carried out using a real dataset. The AIC, BIC, and KS tests were used as goodness-of-fit criteria, for comparison.

The highlights of the contribution have been presented in the following.

(i) It was encouraging to observe that the estimated values of the model parameters converged to the true figures, especially in the large samples. Likewise, the magnitudes of the posterior tend to become smaller in the larger samples. Therefore, the Bayes estimates from the BIID were consistent in nature under all priors and for all loss functions

(ii) The results under noninformative and informative priors were quite comparable, with slight advantages in case of informative priors. So, the Bayes estimators from the proposed distribution were insensitive with respect to change in prior parameters

(iii) The larger values for the parameter λ provided the improved estimation for parameter γ. On the other hand, smaller values of the parameter γ provided more efficient estimation for the parameter λ

(iv) The Bayesian estimates under ELF were better than their counterparts under all priors, all true parametric values, and for both parameters. Hence, the employment of ELF provided substantial gain in efficiencies

(v) The applications of the BIID have been demonstrated using ten real datasets. The modeling ability of this model has been compared with ninety-nine life distributions containing two to six parameters. From the said comparison, it can be assessed that the results under the BIID were almost as efficient as under the competing models having up to six parameters. Therefore, the BIID has been explored as an appealing alternative to the competing distributions with significant reduction in estimation complexities

The study can be extended for analyzing censored data while comparing BIID with other life distributions. The applicability of this distribution can also be explored in modeling the censored heterogeneous datasets.

Data Availability

The data used in the paper is available in the paper.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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