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Research Article

Novel Evaluation of Fuzzy Fractional Cauchy Reaction-Diffusion Equation

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The present research correlates with a fuzzy hybrid approach merged with a new iterative transform method known as the fuzzy new iterative transform method. With the help of Atangana-Baleanu under generalized Hukuhara differentiability, we show that this system works well by getting fractional fuzzy Cauchy reaction-diffusion equations with the initial fuzzy condition. Fractional Cauchy reaction-diffusion equations play a significant role in diffusion, and instabilities can lead to formation and stabilization. The suggested technique looks at fuzzy set theory to figure out how to solve the fuzzy Cauchy reaction-diffusion equations. In this way, the components can be quickly defined and a couple of numerical solutions with the uncertainty parameter can be found. Several numerical instances are looked at to show how effective and valuable the proposed technique is to see if the given problem will come to a solution. The simulation results show that the fuzzy new iterative transform method is an excellent way to study a proposed model's behaviour precisely and accurately.

1. Introduction

Fractional calculus is often used in fields where data is inaccurate, such as natural, biological, and physical engineering and science [1, 2]. Before getting into such challenges, let us first familiarise ourselves with the fuzzy set. Zadeh invented the notion of fuzzy set in 1965 [3], which shows how to measure uncertainty in particular phenomena. As a result, the fuzzy set theory is extended to various other fields of mathematics and science, including algebra, topology, fuzzy logic, analysis, and automata. They develop the basic concept by describing fuzzy function and control. Based on these findings, the scholars expanded on the concept by introducing essential fuzzy calculus. The fractional integral and differential equations have become famous because they can be used to describe real-world phenomena [4, 5]. In [6], Babolian et al. evaluated a few vital mathematical analytical results. Fractional fuzzy integral equations (FFIEs) and fractional fuzzy differential equations (FFDEs) can be employed to represent these types of problems. Several more authors and scientists believe that this technique can be utilized to evaluate, respectively, fractional and integer fuzzy differential equations. Due to the multiple applications of the fuzzy differential equation to simulate unknown processes in a range of fields, including business, physical sciences, and biology, see references [7–11].

In recent decades, a wide variety of applied sciences and engineering fields, including porous media, signal processing, electrical circuits, thermal systems, acoustics, robotics, and optimal control, have turned to fuzzy differential equations to describe physical processes and uncertain parameters. By using fractional operators to fuzzy equations, one can further faithfully represent physical occurrences and gain a better understanding of their underlying causes [12]. As a result, the uniqueness and existence of solutions to fuzzy fractional differential equations have been demonstrated in [13], building on the concept proposed in [14]. In this regard, Agarwal et al. [14] developed the concept of fuzzy fractional differential equations. Fuzzy fractional differential equations with Riemann-Liouville H-differentiability have been analyzed using Laplace fuzzy transformations [15]. Allahviranloo et al. used Mittag-Leffler functions to find explicit solutions to fuzzy fractional differential equations via Riemann-Liouville generalized H-differentiability [16]. The fractional derivative generalized Atangana-Baleanu method has been developed in [17] for the purpose of solving fuzzy fractional differential equations. Additionally, Allahviranloo and Ghanbari [18] studied the ABC fractional derivative as a method for dealing with fuzzy fractional differential equations given in parametric interval form. Hoa [19] proposed a new technique for solving fuzzy fractional differential equations analytically utilizing the Caputo Katugampola fractional derivative.

Modelling real-world and industrial issues using partial differential equations (PDEs) is difficult for scientists and researchers. Modelling nonlinear schemes of the differential equation give rise to a slew of issues [20–22]. Scholars have sought to analyze these issues numerically or analytically using various approaches and formulae to achieve better precision [23–26]. In numerical analysis, the implicit and explicit Finite Difference Method (FDM), spectral collocation method, subdomain least squares method, Galerkin and modified Galerkin methods, shooting method, and decomposition method are all often employed [27–30]. Despite its limitations, the FDM has been frequently utilized; we may acquire solutions at specific grid points using this approach, but it cannot come up with a solution at every single point between two points on the grid. The expense of computing to obtain more precision is the next disadvantage. For a while, the Galerkin finite element method played an essential role in solving industrial and engineering challenges, including intricate geometries and material characteristics, to overcome these difficulties. Even in the problem's complicated domain, this widely applied Galerkin weighted residual approach gives the numerical outcomes among any grid two-point. The Galerkin finite element technique is commonly used to solve ordinary, partial equations [31, 32], and fractional-order differential equations, both linear and nonlinear [33, 34].

The convection-diffusion-reaction (CDR) equation, for example, may illustrate actual situations, and because of its relevance, numerous academics have devised numerical systems. Its prospective applications have sparked a lot of interest. The Finite Element Approach (FEM) is the most accurate method for solving linear and nonlinear CDR models among the methods discussed above. Diffusion, convection, and reaction are essential because they may be used to solve a variety of physical issues involving how the concentration of one or more substances dispersed in a medium changes as a result of the three processes [35, 36]. Convection describes the movement of imports caused by the transport medium in a circle. Diffusion, conversely, preserves the uniform distribution of the material by verifying the substance's migration from a higher concentration to a lower one and vice versa, depending on the applications. In a nutshell, the CDR model is a mathematical representation of how a substance's concentration is spread [37, 38].

2. Fundamental Definitions

Definition 1. Let a fuzzy continuous term $\tilde{\Psi}(\eta)$ on $[0,\sigma] \subset R$ in the presence of Atangana-Baleanu-Caputo (ABC) operator with respect to η as [39].

The ABC derivative of $\tilde{\Psi}(\eta) \in \mathscr{H}^1(0,\eta)$ is expressed as

$$D_\eta^\varrho \tilde{\Psi}(\eta) = \frac{\mathrm{ABC}(\varrho)}{1-\varrho}\int_0^\eta \frac{d}{d\varepsilon}\tilde{\Psi}(\varepsilon) M_\varrho\left[\frac{-\varrho}{1-\varrho}(\eta-\varepsilon)^\varrho\right] d\varepsilon. \quad (1)$$

Replacing $E_\rho[(-\varrho/1-\varrho)(\eta-\varepsilon)^\varrho]$ by $E_1[(-\varrho/1-\varrho)(\eta-\varepsilon)]$, we get "differential operator Caputo Fabrizio." Moreover, if $\tilde{\Psi}(\eta) \in C^F[0,\sigma] \cap L^F[0,\sigma]$, such that $\tilde{\Psi}(\eta) = [\underline{\Psi}_\varrho, \bar{\Psi}_\varrho(\eta)]$, $\varrho \in [0,1]$ and $\eta_0 \in (0,\sigma)$, then the fractional fuzzy ABC derivative is given as

$$\left[D_\eta^\varrho \tilde{\Psi}(\eta)\right]_\delta = \left[D_\eta^\varrho \underline{\Psi}_\varrho(\eta), D_\eta^\varrho \bar{\Psi}_\delta(\eta)\right], \quad 0 \le \delta \le 1, \quad (2)$$

such that

$$\begin{aligned} D_\eta^\varrho \underline{\Psi}_\varrho(\eta) &= \frac{\mathrm{ABC}(\varrho)}{1-\varrho}\int_0^\eta \frac{d}{d\varepsilon}\underline{\Psi}(\varepsilon) E_\varrho\left[\frac{-\varrho}{1-\varrho}(\eta-\varepsilon)^\varrho\right] d\varepsilon, \\ D_\eta^\varrho \tilde{\Psi}_\varrho(\eta) &= \frac{\mathrm{ABC}(\varrho)}{1-\varrho}\int_0^\eta \frac{d}{d\varepsilon}\tilde{\Psi}(\varepsilon) E_\varrho\left[\frac{-\varrho}{1-\varrho}(\eta-\varepsilon)^\varrho\right] d\varepsilon, \\ D_\eta^\varrho&[\mathrm{constant}] = 0. \end{aligned} \quad (3)$$

Here, $\mathrm{ABC}(\varrho)$ represent "function of normalization" and defined by $\kappa(0)=\kappa(1)=1$, and E_ϱ is named as "Mittag-Leffler" function.

Definition 2. Then, the ABC integral is defined as [39]

$$\mathbb{I}_\eta^\varrho \tilde{\Psi}(\eta) = \frac{(1-\varrho)\tilde{\Psi}(\eta)}{\mathrm{ABC}(\varrho)} + \frac{\varrho}{\mathrm{ABC}(\varrho)}\int_0^\eta \frac{(\eta-\varepsilon)^{\varrho-1}}{\Gamma(\varrho)}\tilde{\Psi}(\varepsilon) d\varepsilon. \quad (4)$$

Then, fuzzy fractional ABC integral is defined as

$$\left[\mathbb{I}_0^\varrho \tilde{\Psi}(\eta)\right]_\delta = \left[\mathbb{I}_0^\delta \underline{\Psi}_\delta(\eta), \mathbb{I}_0^\delta \bar{\Psi}_\delta(\eta)\right], \quad 0 \le \delta \le 1, \quad (5)$$

such that

$$\mathbb{I}_0^\delta \underline{\Psi}_\delta(\eta) = \frac{1-\varrho}{\mathrm{ABC}(\varrho)}\underline{\Psi}(\eta) + \frac{\varrho}{\mathrm{ABC}(\varrho)\Gamma(\varrho)}\int_0^\eta (\eta-\varepsilon)^{\varrho-1}\underline{\Psi}(\varepsilon)d\varepsilon,$$
$$\mathbb{I}_0^\delta \bar{\Psi}_\delta(\eta) = \frac{1-\varrho}{\mathrm{ABC}(\varrho)}\bar{\Psi}(\eta) + \frac{\varrho}{\mathrm{ABC}(\varrho)\Gamma(\varrho)}\int_0^\eta (\eta-\varepsilon)^{\varrho-1}\bar{\Psi}(\varepsilon)d\varepsilon. \tag{6}$$

Definition 3. The "fuzzy Laplace transform" of ABC derivative of $\tilde{\Psi}(\eta)$ is given as [39]

$$\mathscr{L}\left[D_0^\varrho \tilde{\Psi}(\eta)\right] = \frac{\mathrm{ABC}(\varrho)}{[s^\varrho(1-\varrho)+\varrho]}\left[s^\varrho \mathscr{L}\left[\tilde{\Psi}(\eta) - s^{\varrho-1}\tilde{\Psi}(0)\right]\right]. \tag{7}$$

Definition 4. The "Mittag-Leffler" function $E_\beta(\eta)$ is given by [39]

$$E_\beta(\eta) = \sum_{n=0}^{\infty}\frac{\eta^n}{\Gamma(n\beta+1)}, \quad \beta>0. \tag{8}$$

Definition 5. A mapping $\kappa : R \longrightarrow [0,1]$. If it holds, it is considered to be number of fuzzy [39].

(i) κ is upper semicontinuous

$$\kappa\{\mu(\varepsilon_1)+\mu(\varepsilon_2)\} \geq \min\{\kappa(\varepsilon_1), \kappa(\varepsilon_2)\}. \tag{9}$$

(ii) $\exists \varepsilon_0 \in R$ such that $\kappa(\varepsilon_0)=1$

(iii) $\mathrm{cl}\{r \in R, \kappa(r)>0\}$ is compact

Definition 6. The fuzzy number of parametric form is $(\underline{\kappa}(\delta), \bar{\kappa}(\delta))$ such that $0 \leq \delta \leq 1$; and conditions [39]

(i) $\underline{k}(\delta)$ increasing, left continuous over $(0,1]$ and right continuous at 0

(ii) $\bar{k}(\delta)$ decreasing, left continuous over $(0,1]$ and right continuous at 0

$$k(\delta) \leq \bar{k}(\delta). \tag{10}$$

3. Methodology

In this section, we apply Laplace transform to analysis general solution of fuzzy fractional PDE. On both sides using Laplace transform, we have

$$\mathscr{L}\left[D_\eta^\varrho\left(\tilde{\Psi}(\vartheta,\eta)\right)\right] = \mathscr{L}\left[A\frac{\partial^2}{\partial\vartheta^2}\left(\tilde{\Psi}(\vartheta,\eta)\right) + \frac{\partial}{\partial x}\left(h(\vartheta)\tilde{\Psi}(\vartheta,\eta)\right)\right]. \tag{11}$$

Evaluating the Laplace transform, Equation (11) implies that

$$\begin{aligned}\frac{\mathrm{ABC}(\varrho)}{[s^\varrho(1-\varrho)+\varrho]}&\left[s^\varrho\mathscr{L}\left[\tilde{\Psi}(\vartheta,\eta)\right] - s^{\varrho-1}\tilde{\Psi}(\vartheta,0)\right]\\ &= \mathscr{L}\left[A\frac{\partial^2}{\partial\vartheta^2}\left(\tilde{\Psi}(\vartheta,\eta)\right) + \frac{\partial}{\partial x}\left(h(\vartheta)\tilde{\Psi}(\vartheta,\eta)\right)\right].\end{aligned} \tag{12}$$

By using initial condition, we get

$$\begin{aligned}s^\varrho\mathscr{L}\left[\tilde{\Psi}(\vartheta,\eta)\right] &= s^{\varrho-1}\tilde{g}(\vartheta,\eta) + \frac{[s^\varrho(1-\varrho)+\varrho]}{\mathrm{ABC}(\varrho)}\mathscr{L}\\ &\cdot\left[A\frac{\partial^2}{\partial\vartheta^2}\left(\tilde{\Psi}(\vartheta,\eta)\right) + \frac{\partial}{\partial x}\left(h(\vartheta)\tilde{\Psi}(\vartheta,\eta)\right)\right],\end{aligned} \tag{13}$$

or

$$\begin{aligned}\mathscr{L}\left[\tilde{\Psi}(\vartheta,\eta)\right] &= \frac{1}{s}\tilde{g}(\vartheta,\eta) + \frac{[s^\varrho(1-\varrho)+\varrho]}{s^\varrho\mathrm{ABC}(\varrho)}\mathscr{L}\\ &\cdot\left[A\frac{\partial^2}{\partial\vartheta^2}\left(\tilde{\Psi}(\vartheta,\eta)\right) + \frac{\partial}{\partial x}\left(h(\vartheta)\tilde{\Psi}(\vartheta,\eta)\right)\right].\end{aligned} \tag{14}$$

The analysis of series form solution, we can write as $\tilde{\Psi}(\vartheta,\eta) = \sum_{n=0}^{\infty}\tilde{\Psi}_n(\vartheta,\eta)$. In these form representations, Equation (11) becomes

$$\begin{aligned}\mathscr{L}\left[\sum_{n=0}^{\infty}\tilde{\Psi}_n(\vartheta,\eta)\right] &= \frac{1}{s}\tilde{g}(\vartheta,\eta) + \frac{[s^\varrho(1-\varrho)+\varrho]}{s^\varrho\mathrm{ABC}(\varrho)}\mathscr{L}\\ &\cdot\left[A\frac{\partial^2}{\partial\vartheta^2}\left(\sum_{n=0}^{\infty}\tilde{\Psi}_n(\vartheta,\eta)\right) + \frac{\partial}{\partial x}\left(h(\vartheta)\sum_{n=0}^{\infty}\tilde{\Psi}_n(\vartheta,\eta)\right)\right].\end{aligned} \tag{15}$$

Comparisons terms by terms of Equation (15), we have

$$\begin{aligned}\mathscr{L}\left[\tilde{\Psi}_0(\vartheta,\eta)\right] &= \frac{1}{s}\tilde{g}(\vartheta,\eta),\\ \mathscr{L}\left[\tilde{\Psi}_1(\vartheta,\eta)\right] &= \frac{[s^\varrho(1-\varrho)+\varrho]}{s^\varrho\mathrm{ABC}(\varrho)}\mathscr{L}\left[A\frac{\partial^2}{\partial\vartheta^2}\left(\tilde{\Psi}_0(\vartheta,\eta)\right) + \frac{\partial}{\partial x}\left(h(\vartheta)\tilde{\Psi}_0(\vartheta,\eta)\right)\right],\\ \mathscr{L}\left[\tilde{\Psi}_2(\vartheta,\eta)\right] &= \frac{[s^\varrho(1-\varrho)+\varrho]}{s^\varrho\mathrm{ABC}(\varrho)}\mathscr{L}\left[A\frac{\partial^2}{\partial\vartheta^2}\left(\tilde{\Psi}_1(\vartheta,\eta)\right) + \frac{\partial}{\partial x}\left(h(\vartheta)\tilde{\Psi}_1(\vartheta,\eta)\right)\right],\\ &\vdots\\ \mathscr{L}\left[\tilde{\Psi}_{n+1}(\vartheta,\eta)\right] &= \frac{[s^\varrho(1-\varrho)+\varrho]}{s^\varrho\mathrm{ABC}(\varrho)}\mathscr{L}\\ &\cdot\left[A\frac{\partial^2}{\partial\vartheta^2}\left(\tilde{\Psi}_n(\vartheta,\eta)\right) + \frac{\partial}{\partial x}\left(h(\vartheta)\tilde{\Psi}_n(\vartheta,\eta)\right)\right], \quad n\geq 0.\end{aligned} \tag{16}$$

Using inverse Laplace transformation in Equation (6), we get

$$\tilde{\Psi}_0(\vartheta,\eta)=\mathscr{L}^{-1}\left[\frac{1}{s}\tilde{g}(\vartheta,\eta)\right],$$
$$\tilde{\Psi}_1(\vartheta,\eta)=\mathscr{L}^{-1}\left[\frac{[s^{\varrho}(1-\varrho)+\varrho]}{s^{\varrho}\mathrm{ABC}(\varrho)}\mathscr{L}\left[A\frac{\partial^2}{\partial\vartheta^2}\left(\tilde{\Psi}_0(\vartheta,\eta)\right)+\frac{\partial}{\partial x}\left(h(\vartheta)\tilde{\Psi}_0(\vartheta,\eta)\right)\right]\right],$$
$$\tilde{\Psi}_2(\vartheta,\eta)=\mathscr{L}^{-1}\left[\frac{[s^{\varrho}(1-\varrho)+\varrho]}{s^{\varrho}\mathrm{ABC}(\varrho)}\mathscr{L}\left[A\frac{\partial^2}{\partial\vartheta^2}\left(\tilde{\Psi}_1(\vartheta,\eta)\right)+\frac{\partial}{\partial x}\left(h(\vartheta)\tilde{\Psi}_1(\vartheta,\eta)\right)\right]\right],$$
$$\vdots$$
$$\tilde{\Psi}_{n+1}(\vartheta,\eta)=\mathscr{L}^{-1}\left[\frac{[s^{\varrho}(1-\varrho)+\varrho]}{s^{\varrho}\mathrm{ABC}(\varrho)}\mathscr{L}\left[A\frac{\partial^2}{\partial\vartheta^2}\left(\tilde{\Psi}_n(\vartheta,\eta)\right)+\frac{\partial}{\partial x}\left(h(\vartheta)\tilde{\Psi}_n(\vartheta,\eta)\right)\right]\right],\quad n\geq 0. \tag{17}$$

Thus, the fuzzy solution is obtained as

$$\underline{\Psi}(\vartheta,\eta)=\sum_{n=0}^{\infty}\underline{\Psi}_n(\vartheta,\eta),\quad \bar{\Psi}(\vartheta,\eta)=\sum_{n=0}^{\infty}\bar{\Psi}_n(\vartheta,\eta). \tag{18}$$

4. Numerical Implementation

In this section, we analyze the following fuzzy fractional CRD equations by new iterative transform method.

4.1. Case I. Consider the fractional-order fuzzy CRDE is defined as

$$^{\mathrm{ABC}}D_{\eta}^{\varrho}\tilde{\Psi}(\vartheta,\eta)=D_{\eta}^{2}\tilde{\Psi}(\vartheta,\eta)-\tilde{\Psi}(\vartheta,\eta),\quad 0<\varrho\leq 1, \tag{19}$$

with the boundaries and initial conditions

$$\tilde{\Psi}(\vartheta,0)=\tilde{\kappa}\left(e^{-\vartheta}+\vartheta\right)=\tilde{\kappa}g(\vartheta),\quad \tilde{\Psi}(0,\eta)=\tilde{\kappa}1=\tilde{\kappa}f_0(\eta),$$
$$\frac{\partial\tilde{\Psi}(0,\eta)}{\partial\eta}=\tilde{\kappa}(e^{-\eta}-1)=\tilde{\kappa}f_1(\eta),\quad \tilde{\kappa}=(\underline{\kappa}(\delta)\bar{\kappa}(\delta))=(\delta-1,1-\delta). \tag{20}$$

Applying the suggested technique, we get

$$\underline{\Psi}_0(\vartheta,\eta)=\underline{\kappa}(\delta)\left\{e^{-\vartheta}+\vartheta\right\},$$
$$\bar{\Psi}_0(\vartheta,\eta)=\bar{\kappa}(\delta)\left\{e^{-\vartheta}+\vartheta\right\},$$
$$\underline{\Psi}_1(\vartheta,\eta)=\underline{\kappa}(\delta)\frac{-\vartheta}{\mathrm{ABC}(\varrho)}\left[1-\varrho+\frac{\varrho\eta^{\varrho}}{\Gamma(\varrho+1)}\right],$$
$$\bar{\Psi}_1(\vartheta,\eta)=\bar{\kappa}(\delta)\frac{-\vartheta}{\mathrm{ABC}(\varrho)}\left[1-\varrho+\frac{\varrho\eta^{\varrho}}{\Gamma(\varrho+1)}\right],$$
$$\underline{\Psi}_2(\vartheta,\eta)=\underline{\kappa}(\delta)\frac{\vartheta}{(\mathrm{ABC}(\varrho))^2}\left((1-\varrho)^2+\frac{2\varrho(1-\varrho)\eta^{\varrho}}{\Gamma(\varrho+1)}+\frac{\varrho^2\eta^{2\varrho}}{\Gamma(2\varrho+1)}\right),$$
$$\bar{\Psi}_2(\vartheta,\eta)=\bar{\kappa}(\delta)\frac{\vartheta}{(\mathrm{ABC}(\varrho))^2}\left((1-\varrho)^2+\frac{2\varrho(1-\varrho)\eta^{\varrho}}{\Gamma(\varrho+1)}+\frac{\varrho^2\eta^{2\varrho}}{\Gamma(2\varrho+1)}\right). \tag{21}$$

In the same way, we can get the higher functions. Equation (19) is used to solve the series, so we write it down

$$\tilde{\Psi}(\vartheta,\eta)=\tilde{\Psi}_0(\vartheta,\eta)+\tilde{\Psi}_1(\vartheta,\eta)+\tilde{\Psi}_2(\vartheta,\eta)+\tilde{\Psi}_3(\vartheta,\eta)+\tilde{\Psi}_4(\vartheta,\eta)+\cdots. \tag{22}$$

In the upper and lower branches, it is

$$\underline{\Psi}(\vartheta,\eta)=\underline{\Psi}_0(\vartheta,\eta)+\underline{\Psi}_1(\vartheta,\eta)+\underline{\Psi}_2(\vartheta,\eta)+\underline{\Psi}_3(\vartheta,\eta)+\underline{\Psi}_4(\vartheta,\eta)+\cdots,$$
$$\bar{\Psi}(\vartheta,\eta)=\bar{\Psi}_0(\vartheta,\eta)+\bar{\Psi}_1(\vartheta,\eta)+\bar{\Psi}_2(\vartheta,\eta)+\bar{\Psi}_3(\vartheta,\eta)+\bar{\Psi}_4(\vartheta,\eta)+\cdots,$$
$$\underline{\Psi}(\vartheta,\eta)=\underline{\kappa}(\delta)\left\{e^{-\vartheta}+\vartheta\right\}+\underline{\kappa}(\delta)\frac{-\vartheta}{\mathrm{ABC}(\varrho)}\left[1-\rho+\frac{\varrho\eta^{\varrho}}{\Gamma(\varrho+1)}\right]+\underline{\kappa}(\delta)\frac{\vartheta}{(\mathrm{ABC}(\varrho))^2}\left((1-\varrho)^2+\frac{2\varrho(1-\varrho)\eta^{\varrho}}{\Gamma(\varrho+1)}+\frac{\varrho^2\eta^{2\varrho}}{\Gamma(2\varrho+1)}\right)+\cdots,$$
$$\bar{\Psi}(\vartheta,\eta)=\bar{\kappa}(\delta)\left\{e^{-\vartheta}+\vartheta\right\}+\bar{\kappa}(\delta)\frac{-\vartheta}{\mathrm{ABC}(\varrho)}\left[1-\varrho+\frac{\varrho\eta^{\varrho}}{\Gamma(\varrho+1)}\right]+\bar{\kappa}(\delta)\frac{\vartheta}{(\mathrm{ABC}(\varrho))^2}\left((1-\varrho)^2+\frac{2\varrho(1-\varrho)\eta^{\varrho}}{\Gamma(\varrho+1)}+\frac{\varrho^2\eta^{2\varrho}}{\Gamma(2\varrho+1)}\right)+\cdots. \tag{23}$$

The exact result is

$$\tilde{\Psi}(\vartheta,\eta)=\tilde{\kappa}\left(e^{-\vartheta}+\vartheta e^{-\eta}\right). \tag{24}$$

In Figure 1, the first figure shows the 3D fuzzy upper and lower branches of analytical solution at $\rho=1$ and the 2nd figure shows the 2D fuzzy figure at $\varrho=1$. In Figure 2 are the various fractional order ρ figures of upper and lower branches of analytical solution.

4.2. Case II.

$$^{\mathrm{ABC}}D_{\eta}^{\varrho}\tilde{\Psi}(\vartheta,\eta)=D_{\eta}^{2}\tilde{\Psi}(\vartheta,\eta)-\left(1+4\vartheta^2\right)\tilde{\Psi}(\vartheta,\eta),\quad 0<\rho\leq 1, \tag{25}$$

with initial condition

$$\tilde{\Psi}(\vartheta,0)=\tilde{\kappa}e^{\vartheta^2},\quad \tilde{\kappa}=(\underline{\kappa}(\delta)\bar{\kappa}(\delta))=(\delta-1,1-\delta). \tag{26}$$

Using the proposed method, we have

$$\underline{\Psi}_0(\vartheta,\eta)=\underline{\kappa}(\delta)e^{\vartheta^2},$$
$$\bar{\Psi}_0(\vartheta,\eta)=\bar{\kappa}(\delta)e^{\vartheta^2},$$
$$\underline{\Psi}_1(\vartheta,\eta)=\underline{\kappa}(\delta)\frac{e^{\vartheta^2}}{\mathrm{ABC}(\varrho)}\left[1-\varrho+\frac{\varrho\eta^{\varrho}}{\Gamma(\varrho+1)}\right],$$
$$\bar{\Psi}_1(\vartheta,\eta)=\bar{\kappa}(\delta)\frac{e^{\vartheta^2}}{\mathrm{ABC}(\varrho)}\left[1-\varrho+\frac{\varrho\eta^{\varrho}}{\Gamma(\varrho+1)}\right],$$

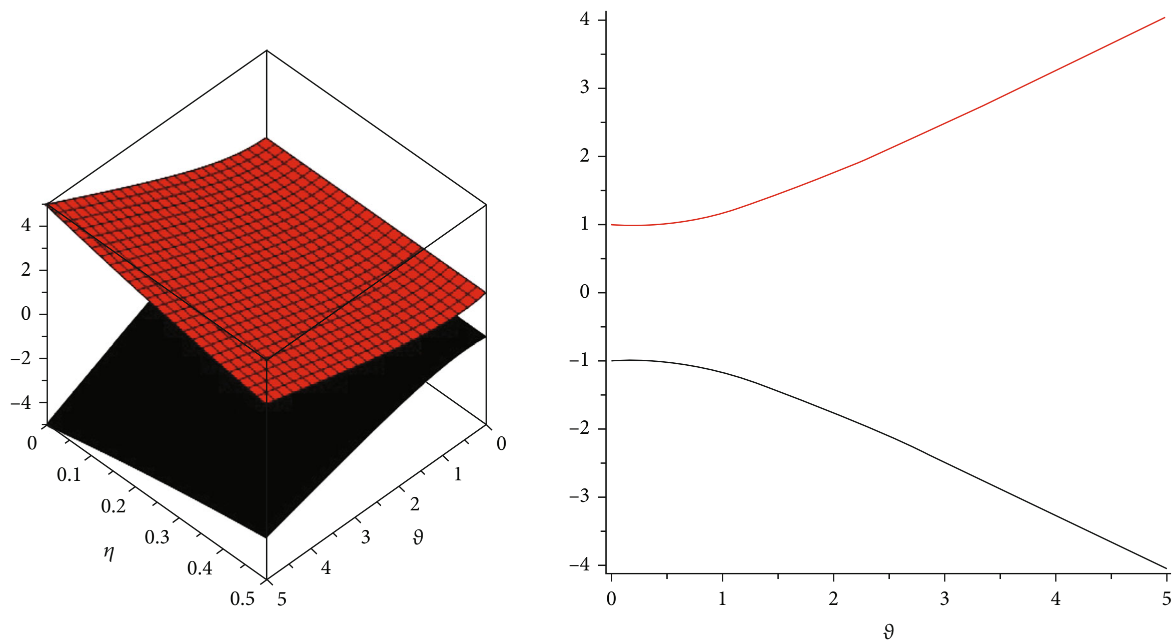

Figure 1: The first figure shows the 3D fuzzy upper and lower branches of analytical solution at $\rho = 1$ and the 2nd figure shows the 2D fuzzy figure at $\varrho = 1$.

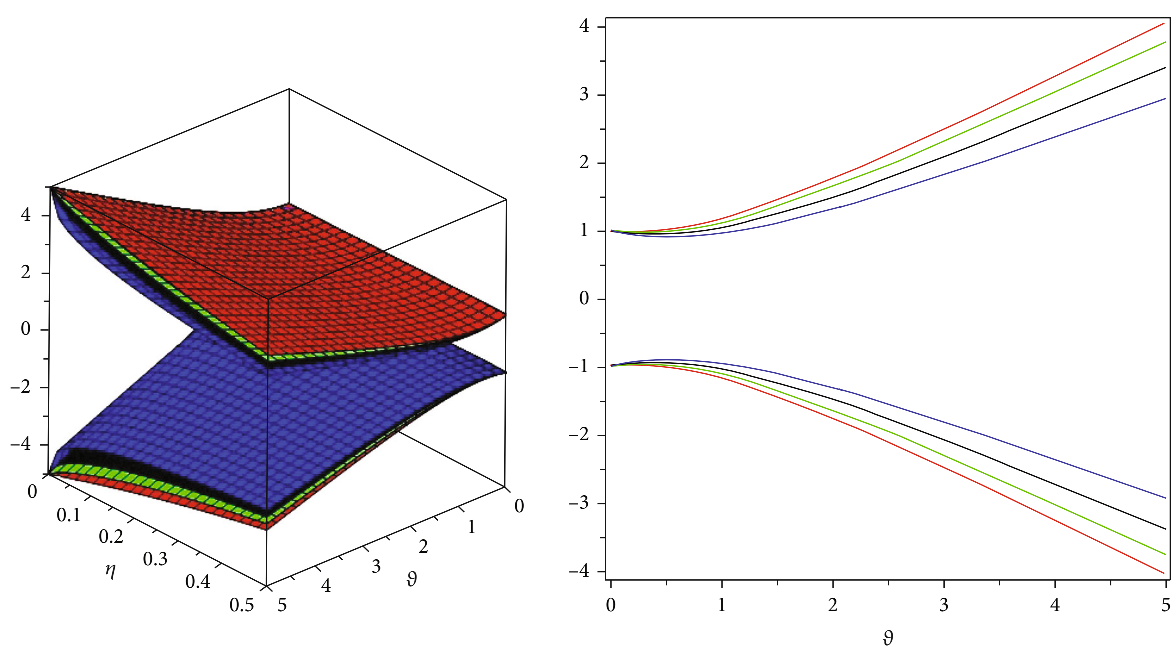

Figure 2: The various fractional order ϱ figures of upper and lower branches of analytical solution.

$$\underline{\Psi}_2(\vartheta, \eta) = \underline{\kappa}(\delta) \frac{e^{\vartheta^2}}{(\mathrm{ABC}(\varrho))^2} \left((1-\varrho)^2 + \frac{2\varrho(1-\varrho)\eta^{\varrho}}{\Gamma(\varrho+1)} + \frac{\varrho^2 \eta^{2\varrho}}{\Gamma(2\varrho+1)} \right),$$

$$\bar{\Psi}_2(\vartheta, \eta) = \bar{\kappa}(\delta) \frac{e^{\vartheta^2}}{(\mathrm{ABC}(\varrho))^2} \left((1-\varrho)^2 + \frac{2\varrho(1-\varrho)\eta^{\varrho}}{\Gamma(\varrho+1)} + \frac{\varrho^2 \eta^{2\varrho}}{\Gamma(2\varrho+1)} \right). \tag{27}$$

In the same way, we can get the higher functions. Equation (25) is used to solve the series, so we write it down

$$\tilde{\Psi}(\vartheta, \eta) = \tilde{\Psi}_0(\vartheta, \eta) + \tilde{\Psi}_1(\vartheta, \eta) + \tilde{\Psi}_2(\vartheta, \eta) + \tilde{\Psi}_3(\vartheta, \eta) + \tilde{\Psi}_4(\vartheta, \eta) + \cdots. \tag{28}$$

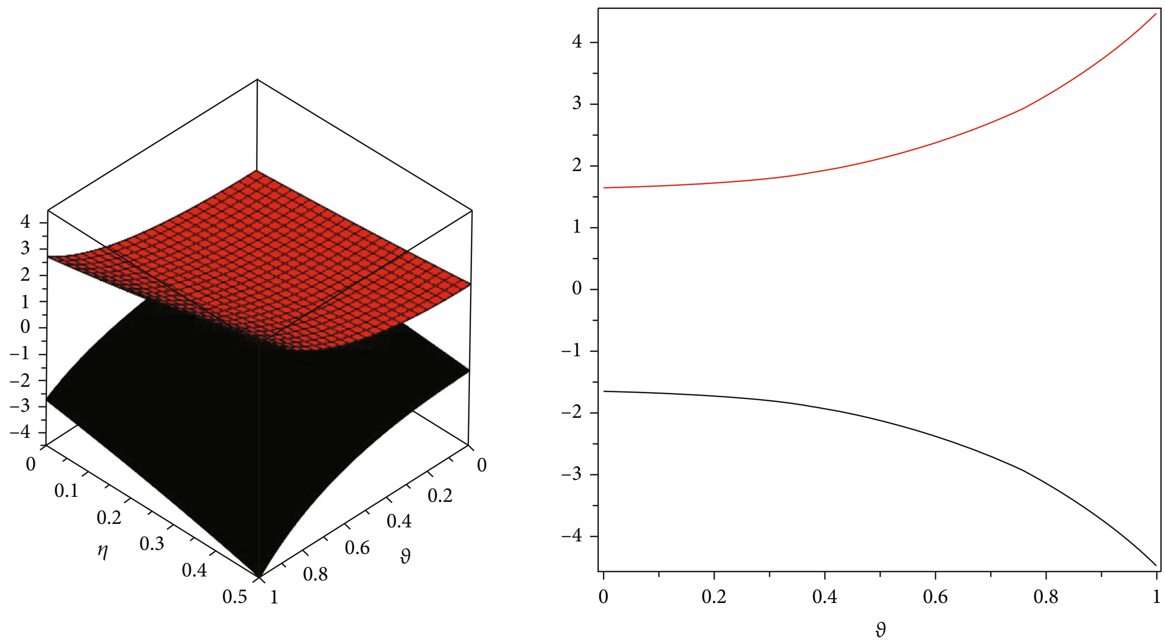

Figure 3: The first figure shows the 3D fuzzy upper and lower branches of analytical solution at $\varrho = 1$ and the 2nd figure shows the 2D fuzzy figure at $\varrho = 1$.

In the upper and lower portion form, it is

$$\underline{\Psi}(\vartheta,\eta) = \underline{\Psi}_0(\vartheta,\eta) + \underline{\Psi}_1(\vartheta,\eta) + \underline{\Psi}_2(\vartheta,\eta) + \underline{\Psi}_3(\vartheta,\eta) + \underline{\Psi}_4(\vartheta,\eta) + \cdots,$$

$$\bar{\Psi}(\vartheta,\eta) = \bar{\Psi}_0(\vartheta,\eta) + \bar{\Psi}_1(\vartheta,\eta) + \bar{\Psi}_2(\vartheta,\eta) + \bar{\Psi}_3(\vartheta,\eta) + \bar{\Psi}_4(\vartheta,\eta) + \cdots,$$

$$\begin{aligned}\underline{\Psi}(\vartheta,\eta) &= \underline{\kappa}(\delta)e^{\vartheta^2} + \underline{\kappa}(\delta)\frac{e^{\vartheta^2}}{\mathrm{ABC}(\varrho)}\left[1-\varrho+\frac{\varrho\eta^{\varrho}}{\Gamma(\varrho+1)}\right] \\ &\quad + \underline{\kappa}(\delta)\frac{e^{\vartheta^2}}{(\mathrm{ABC}(\varrho))^2}\left((1-\varrho)^2+\frac{2\varrho(1-\varrho)\eta^{\varrho}}{\Gamma(\varrho+1)}+\frac{\varrho^2\eta^{2\varrho}}{\Gamma(2\varrho+1)}\right) \\ &\quad + \cdots, \\ \bar{\Psi}(\vartheta,\eta) &= \bar{\kappa}(\delta)e^{\vartheta^2} + \bar{\kappa}(\delta)\frac{e^{\vartheta^2}}{\mathrm{ABC}(\varrho)}\left[1-\varrho+\frac{\varrho\eta^{\varrho}}{\Gamma(\varrho+1)}\right] \\ &\quad + \bar{\kappa}(\delta)\frac{e^{\vartheta^2}}{(\mathrm{ABC}(\varrho))^2}\left((1-\varrho)^2+\frac{2\varrho(1-\varrho)\eta^{\varrho}}{\Gamma(\varrho+1)}+\frac{\varrho^2\eta^{2\varrho}}{\Gamma(2\varrho+1)}\right) \\ &\quad + \cdots.\end{aligned} \tag{29}$$

The exact result is

$$\tilde{\Psi}(\vartheta,\eta) = \tilde{\kappa}\left(e^{\vartheta^2+\eta}\right). \tag{30}$$

In Figure 3, the first figure shows the 3D fuzzy upper and lower branches of analytical solution at $\rho = 1$ and the 2nd figure shows the 2D fuzzy figure at $\varrho = 1$. In Figure 4 are the various fractional order ρ figures of upper and lower branches of analytical solution.

4.3. Case III. Consider fractional-order fuzzy Cauchy reaction-diffusion equation:

$$^{\mathrm{ABC}}D_{\eta}^{\varrho}\tilde{\Psi}(\vartheta,\eta) = D_{\eta}^{2}\tilde{\Psi}(\vartheta,\eta) + 2\eta\tilde{\Psi}(\vartheta,\eta), \quad 0<\rho\le 1, \tag{31}$$

with initial condition

$$\tilde{\Psi}(\vartheta,0) = \tilde{\kappa}e^{\vartheta}, \quad \tilde{\kappa} = (\underline{\kappa}(\delta)\bar{\kappa}(\delta)) = (\delta-1, 1-\delta). \tag{32}$$

Using the proposed method, we have

$$\begin{aligned}\underline{\Psi}_0(\vartheta,\eta) &= \underline{\kappa}(\delta)e^{\vartheta}, \\ \bar{\Psi}_0(\vartheta,\eta) &= \bar{\kappa}(\delta)e^{\vartheta}, \\ \underline{\Psi}_1(\vartheta,\eta) &= \underline{\kappa}(\delta)\frac{e^{\vartheta}}{\mathrm{ABC}(\varrho)}\left[\frac{2\varrho\eta^{\varrho+1}}{\Gamma(\varrho+2)}+\frac{\varrho\eta^{\varrho}}{\Gamma(\varrho+1)}+2\eta(1-\varrho)+(1-\varrho)\right], \\ \bar{\Psi}_1(\vartheta,\eta) &= \bar{\kappa}(\delta)\frac{e^{\vartheta}}{\mathrm{ABC}(\varrho)}\left[\frac{2\varrho\eta^{\varrho+1}}{\Gamma(\varrho+2)}+\frac{\varrho\eta^{\varrho}}{\Gamma(\varrho+1)}+2\eta(1-\varrho)+(1-\varrho)\right], \\ \underline{\Psi}_2(\vartheta,\eta) &= \underline{\kappa}(\delta)\frac{e^{\vartheta}}{(\mathrm{ABC}(\varrho))^2}\left[4\varrho(\varrho+2)\frac{\eta^{2\varrho+2}}{\Gamma(2\varrho+3)}\right. \\ &\quad +2\varrho(\varrho+2)\frac{\eta^{2\varrho+1}}{\Gamma(2\varrho+1)}+\varrho\frac{\eta^{2\varrho}}{\Gamma(2\rho+1)}+8(1-\varrho)\frac{\eta^{\varrho+2}}{\Gamma(\varrho+3)} \\ &\quad \left.+4(1-\varrho)\frac{\eta^{\varrho+1}}{\Gamma(\varrho+2)}+(1-\varrho)\frac{\eta^{\varrho}}{\Gamma(\varrho+1)}\right], \\ \bar{\Psi}_2(\vartheta,\eta) &= \bar{\kappa}(\delta)\frac{e^{\vartheta}}{(\mathrm{ABC}(\varrho))^2}\left[4\varrho(\varrho+2)\frac{\eta^{2\varrho+2}}{\Gamma(2\varrho+3)}\right. \\ &\quad +2\varrho(\varrho+2)\frac{\eta^{2\varrho+1}}{\Gamma(2\varrho+1)}+\varrho\frac{\eta^{2\varrho}}{\Gamma(2\varrho+1)}+8(1-\varrho)\frac{\eta^{\varrho+2}}{\Gamma(\varrho+3)} \\ &\quad \left.+4(1-\varrho)\frac{\eta^{\varrho+1}}{\Gamma(\varrho+2)}+(1-\varrho)\frac{\eta^{\varrho}}{\Gamma(\varrho+1)}\right].\end{aligned} \tag{33}$$

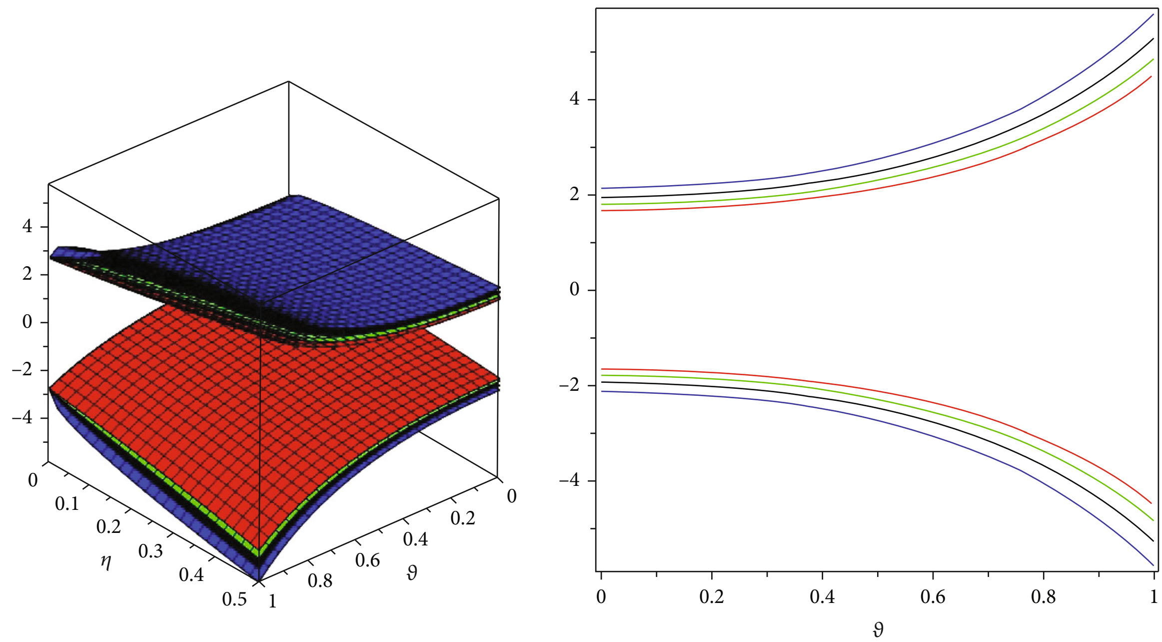

Figure 4: The various fractional order ϱ figures of upper and lower branches of analytical solution.

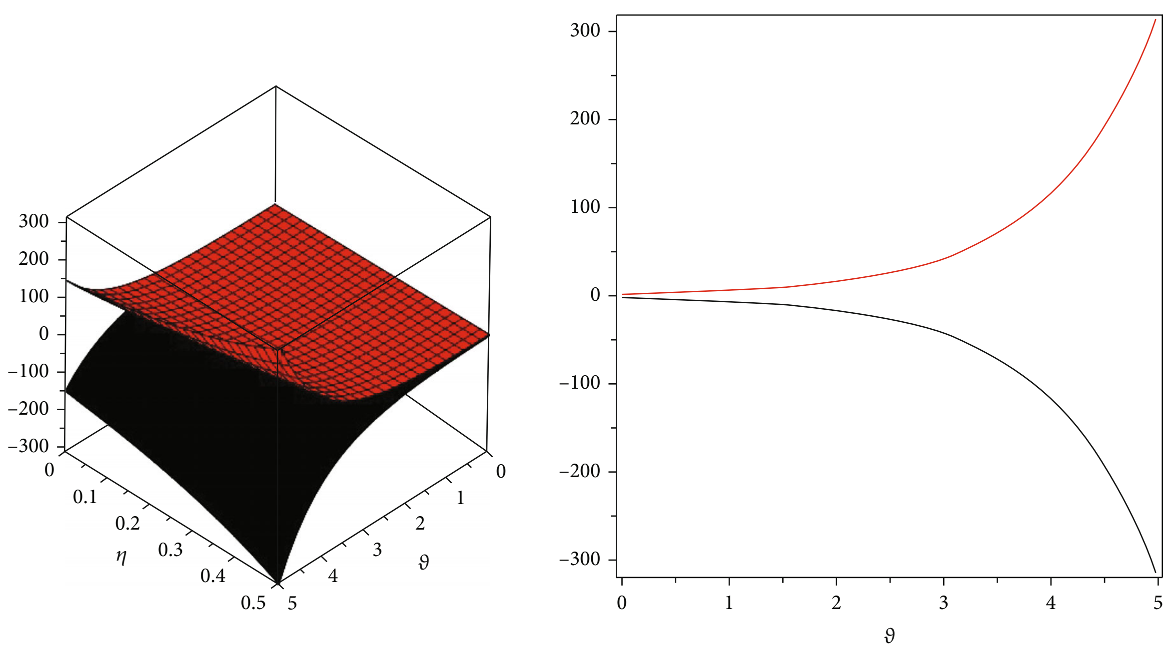

Figure 5: The first figure shows the 3D fuzzy upper and lower branches of analytical solution at $\rho = 1$ and the 2nd figure shows the 2D fuzzy figure at $\varrho = 1$.

In the same way, we can get the higher functions. Equation (31) is used to solve the series, so we write it down

$$\tilde{\Psi}(\vartheta, \eta) = \tilde{\Psi}_0(\vartheta, \eta) + \tilde{\Psi}_1(\vartheta, \eta) + \tilde{\Psi}_2(\vartheta, \eta) + \tilde{\Psi}_3(\vartheta, \eta) + \tilde{\Psi}_4(\vartheta, \eta) + \cdots. \tag{34}$$

In the upper and lower portion form, it is

$$\underline{\Psi}(\vartheta, \eta) = \underline{\Psi}_0(\vartheta, \eta) + \underline{\Psi}_1(\vartheta, \eta) + \underline{\Psi}_2(\vartheta, \eta) + \underline{\Psi}_3(\vartheta, \eta) + \underline{\Psi}_4(\vartheta, \eta) + \cdots,$$

$$\bar{\Psi}(\vartheta, \eta) = \bar{\Psi}_0(\vartheta, \eta) + \bar{\Psi}_1(\vartheta, \eta) + \bar{\Psi}_2(\vartheta, \eta) + \bar{\Psi}_3(\vartheta, \eta) + \bar{\Psi}_4(\vartheta, \eta) + \cdots,$$

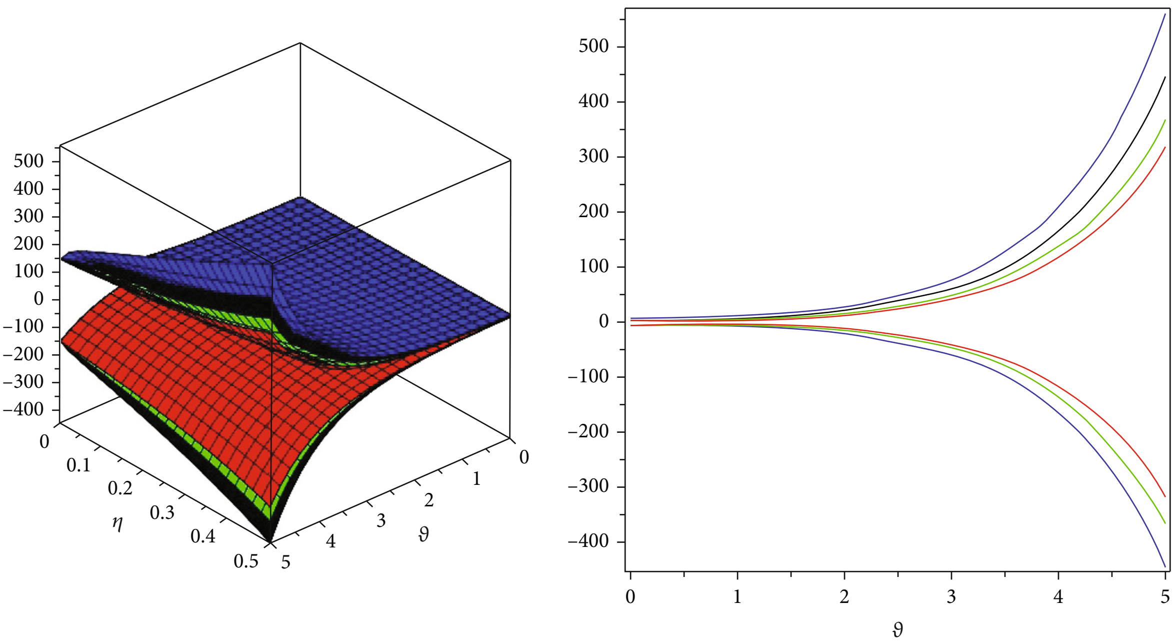

FIGURE 6: The various fractional order ϱ figures of upper and lower branches of analytical solution.

$$
\begin{aligned}
\underline{\Psi}(\vartheta,\eta) &= \underline{\kappa}(\delta)e^{\vartheta} + \underline{\kappa}(\delta)\frac{e^{\vartheta}}{\mathrm{ABC}(\varrho)}\left[\frac{2\varrho\eta^{\varrho+1}}{\Gamma(\varrho+2)} + \frac{\varrho\eta^{\varrho}}{\Gamma(\varrho+1)} + 2\eta(1-\varrho) + (1-\varrho)\right] \\
&\quad + \underline{\kappa}(\delta)\frac{e^{\vartheta}}{(\mathrm{ABC}(\varrho))^2}\left[4\varrho(\varrho+2)\frac{\eta^{2\varrho+2}}{\Gamma(2\varrho+3)} + 2\varrho(\varrho+2)\frac{\eta^{2\varrho+1}}{\Gamma(2\varrho+1)}\right. \\
&\quad + \rho\frac{\eta^{2\varrho}}{\Gamma(2\varrho+1)} + 8(1-\varrho)\frac{\eta^{\varrho+2}}{\Gamma(\varrho+3)} + 4(1-\varrho)\frac{\eta^{\varrho+1}}{\Gamma(\varrho+2)} \\
&\quad \left. + (1-\varrho)\frac{\eta^{\varrho}}{\Gamma(\varrho+1)}\right] + \cdots, \\
\bar{\Psi}(\vartheta,\eta) &= \bar{\kappa}(\delta)e^{\vartheta} + \bar{\kappa}(\delta)\frac{e^{\vartheta}}{\mathrm{ABC}(\varrho)}\left[\frac{2\varrho\eta^{\varrho+1}}{\Gamma(\varrho+2)} + \frac{\varrho\eta^{\varrho}}{\Gamma(\varrho+1)} + 2\eta(1-\varrho) + (1-\varrho)\right] \\
&\quad + \bar{\kappa}(\delta)\frac{e^{\vartheta}}{(\mathrm{ABC}(\varrho))^2}\left[4\varrho(\varrho+2)\frac{\eta^{2\varrho+2}}{\Gamma(2\varrho+3)} + 2\varrho(\varrho+2)\frac{\eta^{2\varrho+1}}{\Gamma(2\varrho+1)}\right. \\
&\quad + \varrho\frac{\eta^{2\varrho}}{\Gamma(2\varrho+1)} + 8(1-\varrho)\frac{\eta^{\varrho+2}}{\Gamma(\varrho+3)} + 4(1-\varrho)\frac{\eta^{\varrho+1}}{\Gamma(\varrho+2)} \\
&\quad \left. + (1-\varrho)\frac{\eta^{\varrho}}{\Gamma(\varrho+1)}\right] + \cdots.
\end{aligned}
\tag{35}
$$

The exact result is

$$\tilde{\Psi}(\vartheta,\eta) = \tilde{\kappa}\left(e^{\vartheta+\eta+\eta^2}\right). \tag{36}$$

In Figure 5, the first figure shows the 3D fuzzy upper and lower branches of analytical solution at $\rho = 1$ and the 2nd figure shows the 2D fuzzy figure at $\varrho = 1$. In Figure 6 are the various fractional order ρ figures of upper and lower branches of analytical solution.

5. Conclusion

This investigation is aimed at providing a semianalytical result to the fuzzy fractional Cauchy reaction-diffusion equation solution by considering the Atangana-Baleanu operator. Therefore, fuzzy operators are preferable to describe the physical phenomenon in such a scenario. We explored the Cauchy reaction-diffusion equation in a fuzzy approach, taking into account the uncertainty in the initial condition. In this research, we have generalized the fuzzy fractional of the Cauchy reaction-diffusion equation. We then used a new iterative transform method to obtain the approximate expression of the suggested problem in its parametric form. We identified numerous illustrations to support the intended methodology and achieved a parametric solution for each case. In the end, it is not simple to find analytical solutions for many types of fuzzy fractional partial differential equations. In the future, it is essential to look at and solve fractional fuzzy partial differential, dynamical, and integrodifferential equations based on the Atangana-Baleanu operator of fractional order $\varrho > 1$.

Data Availability

The numerical data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

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