Hindawi
Journal of Function Spaces
Volume 2022, Article ID 6795978, 10 pages
https://doi.org/10.1155/2022/6795978

Research Article

Ulam-Hyers Stability Results of λ-Quadratic Functional Equation with Three Variables in Non-Archimedean Banach Space and Non-Archimedean Random Normed Space

Ly Van An,[1] Kandhasamy Tamilvanan,[2] R. Udhayakumar,[3] Masho Jima Kabeto,[4] and Ly Van Ngoc[5]

[1]*Faculty of Mathematics Teacher Education, Tay Ninh University, Ninh Trung, Ninh Son, Tay Ninh Province, Vietnam*
[2]*Department of Mathematics, School of Advanced Sciences, Kalasalingam Academy of Research and Education, Srivilliputhur, 626 126 Tamil Nadu, India*
[3]*Department of Mathematics, School of Advanced Sciences, Vellore Institute of Technology, Vellore 632 014, India*
[4]*Department of Mathematics, College of Natural Sciences, Jimma University, Ethiopia*
[5]*Faculty of Mathematics Teacher Education, Quang Nam Phan Chau Trinh University, Cam Thanh, Cam Ha, Quang Nam Province, Vietnam*

Correspondence should be addressed to Kandhasamy Tamilvanan; tamiltamilk7@gmail.com and Masho Jima Kabeto; masho.jima@ju.edu.et

Received 27 March 2022; Revised 20 April 2022; Accepted 22 April 2022; Published 20 May 2022

Academic Editor: Santosh Kumar

In this paper, we introduce the λ-quadratic functional equation with three variables and obtain its general solution. The main aim of this work is to examine the Ulam-Hyers stability of this functional equation in non-Archimedean Banach space by using direct and fixed point techniques and examine the stability results in non-Archimedean random normed space.

1. Introduction

One of the most important areas of research in mathematics is the investigation of stability issues for functional equations, which has its origins in concerns of applied mathematics. The first question about the stability of homomorphisms was given by Ulam [1] as follows.

Given a group $(M, *)$, a metric group $(M', \cdot)$ with the metric d, and a function ϕ from B and B', does there exists $\delta > 0$ satisfying

$$d(\phi(u * v), \phi(u) \cdot \phi(v)) \leq \delta, \tag{1}$$

for all $u, v \in B$, then there exists a homomorphism $h : B \longrightarrow B'$ such that

$$d(\phi(u), h(u)) \leq \varepsilon, \tag{2}$$

for all $u \in B$?

Ulam's question on Banach spaces was partially answered affirmatively by Hyers [2]. By assuming an infinite Cauchy difference, Aoki [3] expanded Hyers' and Rassias' theorems for additive and linear mappings, respectively. Using the same method as Rassias [4], Gajda [5] discovered a positive solution to the question $p > 1$. Rassias and Šemrl [6], as well as Gajda [5], have proved that a Rassias' type theorem cannot be formed for $p = 1$.

The functional equation

$$\phi(u + v) = \phi(u) + \phi(v) \tag{3}$$

is known as the Cauchy additive equation.

Since the function $\phi(u) = u$ is the solution of the functional equation (3), every solution of the additive functional equation (3) is called as an additive function. Every solution of the functional equation (3), in particular, is called as an additive function.

The functional equation

$$\phi(u+v) + \phi(u-v) = 2\phi(u) + 2\phi(v) \tag{4}$$

is known as the quadratic functional equation.

Since the function $\phi(u) = u^2$ is the solution of the functional equation (4), each solution of the functional equation (4) is called as a quadratic function. Every solution of functional equation (4), in particular, is called as a quadratic mapping.

Skof [7] established the stability of the quadratic functional equation for the function f between normed space and complete normed space. The authors [8–14] recently examined the Ulam-Hyers stability results for the following α-functional equation

$$2f(x) - 2f(y) = f(x+y) + \alpha^{-2} f(\alpha(x-y)), \tag{5}$$

in non-Archimedean Banach spaces.

The Skof theorem still applies when the relevant domain B is replaced by an Abelian group, according to Cholewa [15]. See [15–21] for other functional equations. A survey of the Ulam-Hyers stability results of functional equations was conducted by Brillouët-Belluot [22]. Park and Kim [11] demonstrated the Ulam-Hyers stability of quadratic α-functional equation.

In this paper, the authors present a new λ-quadratic functional equation with three variables as

$$2\xi\left(\frac{\vartheta_1+\vartheta_2}{2}\right) + 2\xi(\vartheta_3) = \xi\left(\frac{\vartheta_1+\vartheta_2}{2} + \vartheta_3\right) + \lambda^{-2}\xi\left(\lambda\left(\frac{\vartheta_1+\vartheta_2}{2} - \vartheta_3\right)\right), \tag{6}$$

where λ is a fixed non-Archimedean number with $\lambda^{-2} \neq 3$, and its general solution was obtained. The motivation behind this study is to investigate the Ulam-Hyers stability results for the above functional equation (6) in non-Archimedean Banach space by using direct and fixed point methods and non-Archimedean random normed space.

The following is the structure of this paper: in Section 2, we recall some fundamental notions and definitions, in Section 3, we look at the general solution of the equation (6), where V and W are two vector spaces. We investigate the Ulam-Hyers stability in non-Archimedean Banach space by using fixed point method and direct method in Sections 4 and 5, where V is a non-Archimedean normed space, W is a non-Archimedean Banach space, and $|2| \neq 1$ is a non-Archimedean Banach space. In Section 6, we recall some fundamental notions and results and investigate the Ulam-Hyers stability in non-Archimedean random normed space.

2. Preliminaries

To reach our major results, we use certain fundamental notations in [8, 10, 11].

A map $|\cdot|: \mathbb{K} \longrightarrow [0,\infty)$ is a valuation such that zero is the only one element having the zero valuation, $|k_1 k_2| = |k_1||k_2|$, and the inequality of the triangle holds true, that is, $|k_1 + k_2| \leq |k_1| + |k_2|$, for all $k_1, k_2 \in \mathbb{K}$.

We call a field $\mathbb{K}$ valued if $\mathbb{K}$ holds a valuation. Examples of valuations include the typical absolute values of $\mathbb{R}$ and $\mathbb{C}$.

Consider a valuation that satisfies a criterion that is stronger than the triangle inequality. A $|\cdot|$ is called a *non-Archimedean valuation* if the triangle inequality is replaced by $|k_1 + k_2| \leq \max\{|k_1|, |k_2|\}$, for all $k_1, k_2 \in \mathbb{K}$, and a field is called a *non-Archimedean field*. Evidently, $|-1| = 1 = |1|$ and $|n|$ are greater than or equal to 1, for all n in $\mathbb{N}$. The map $|\cdot|$ takes everything except 0 for 1, and $|0| = 0$ is a basic example of a non-Archimedean valuation.

Definition 1. Let V be a linear space over $\mathbb{K}$ with $|\cdot|$. A mapping $\|\cdot\|: V \longrightarrow [0,\infty)$ is known as a non-Archimedean norm if it satisfies

(i) $\|v\| = 0$ if and only if $v = 0$.

(ii) $\|rv\| = |r|\|v\|$, $v \in V$, and $r \in \mathbb{K}$.

(iii) the strong triangle inequality.

$$\|v_1 + v_2\| \leq \max\{\|v_1\|, \|v_2\|\},\ v_1, v_2 \in V. \tag{7}$$

Then, $(V, \|\cdot\|)$ is called a non-Archimedean normed space. Every Cauchy sequence converges in a complete non-Archimedean normed space, which we call a complete non-Archimedean normed space.

Definition 2. Let V be a non-Archimedean normed space and a sequence $\{v_p\}$ in V. Then,

(1) a sequence $\{v_p\}_{p=1}^{\infty}$ in V is a Cauchy sequence if $\{v_{p+1} - v_p\}_{p=1}^{\infty}$ converges to 0.

(2) $\{v_p\}$ is called convergent if, for any $\varepsilon > 0$, there is an integer $p > 0$ in $\mathbb{N}$ and $v \in V$ satisfies

$$\|v_p - v\| \leq \varepsilon \text{ for all } p \geq \mathbb{N}, \tag{8}$$

for every $p, q \geq \mathbb{N}$. Then, we called as v is a limit of the sequence $\{v_p\}$ and denoted by $\lim_{p\longrightarrow\infty} v_p = v$.

(3) if every Cauchy sequence in a non-Archimedean normed space V converges, it is called a non-Archimedean Banach space.

Theorem 3 (alternative fixed point theorem). *Let (V, d) be a generalized complete metric space and a strictly contractive mapping $M : V \longrightarrow V$ with Lipschitz constant $0 < L < 1$.*

Then, for all $v_1 \in V$, either

$$d\left(M^m v_1, M^{m+1} v_1\right) = \infty, m \geq m_0, \tag{9}$$

or there exists a positive integer m_0 such that

(i) $d(M^m v_1, M^{m+1} v_1) < \infty, m \geq m_0$.

(ii) the sequence $\{M^m v_1\}_{m\in\mathbb{N}}$ converges to a fixed point v_1^* of M.

(iii) v_1^* is the unique fixed point of M in $V^* = \{v_2 \in V | d(M^{m_0} v_1, v_2) < \infty\}$.

(iv) $d(v_2, v_1^*) \leq (1/1 - L) d(Mv_2, v_2)$, for all $v_2 \in V^*$.

3. Solution

Lemma 4. *If a mapping $\xi : V \longrightarrow W$ satisfies the functional equation (6) for all $\vartheta_1, \vartheta_2, \vartheta_3 \in V$, then the function ξ is quadratic.*

Proof. A mapping $\xi : V \longrightarrow W$ satisfies the functional equation (6). Replacing $(\vartheta_1, \vartheta_2, \vartheta_3)$ by $(0, 0, 0)$ in (6), we obtain

$$3\xi(0) = \lambda^{-2}\xi(0). \tag{10}$$

This implies that $\xi(0) = 0$. Replacing $(\vartheta_1, \vartheta_2, \vartheta_3)$ by $(\vartheta, \vartheta, 0)$ in (6), we obtain

$$\xi(\vartheta) = \lambda^{-2}\xi(\lambda(\vartheta)), \tag{11}$$

and so

$$\xi(\lambda v) = \lambda^2 \xi(\vartheta), \tag{12}$$

for all $\vartheta \in V$. Thus, equation (6) is reduced as

$$2\xi\left(\frac{\vartheta_1 + \vartheta_2}{2}\right) + 2\xi(\vartheta_3) = \xi\left(\frac{\vartheta_1 + \vartheta_2}{2} + \vartheta_3\right) + \xi\left(\frac{\vartheta_1 + \vartheta_2}{2} - \vartheta_3\right), \tag{13}$$

for all $\vartheta_1, \vartheta_2, \vartheta_3 \in V$. Now, replacing $\vartheta_1 = \vartheta_2 = \vartheta_3 = \vartheta$ in (13), we get

$$\xi(2\vartheta) = 2^2 \xi(\vartheta), \tag{14}$$

for all $\vartheta \in V$. Again, replacing ϑ by 2ϑ in (14), we have

$$\xi\left(2^2 \vartheta\right) = 2^4 \xi(\vartheta), \tag{15}$$

for all $\vartheta \in V$. From equalities (14) and (15), we can conclude that for any integer $p > 0$, we get

$$\xi(2^p \vartheta) = 2^{2p} \xi(\vartheta), \tag{16}$$

for all $\vartheta \in V$. Now, replacing $(\vartheta_1, \vartheta_2, \vartheta_3)$ by $(\vartheta_1, \vartheta_1, \vartheta_2)$ in (13), we reach (3) for all $\vartheta_1, \vartheta_2 \in V$. Hence, the function ξ is quadratic. □

For our notational simplicity, we use the following abbreviation:

$$\begin{aligned}\Delta\xi(\vartheta_1, \vartheta_2, \vartheta_3) = {} & 2\xi\left(\frac{\vartheta_1 + \vartheta_2}{2}\right) + 2\xi(\vartheta_3) - \xi\left(\frac{\vartheta_1 + \vartheta_2}{2} + \vartheta_3\right) \\ & - \lambda^{-2}\xi\left(\lambda\left(\frac{\vartheta_1 + \vartheta_2}{2} - \vartheta_3\right)\right).\end{aligned} \tag{17}$$

4. Stability of (6) in Non-Archimedean Banach Space: Direct Method

Theorem 5. *Let $\rho : V^3 \longrightarrow [0,\infty)$ be a mapping and a mapping $\xi : V \longrightarrow W$ such that $\xi(0) = 0$ and*

$$\lim_{j\longrightarrow\infty} \left|2^2\right|^j \rho\left(2^{-j}\vartheta_1, 2^{-j}\vartheta_2, 2^{-j}\vartheta_3\right) = 0, \tag{18}$$

$$\|\Delta\xi(\vartheta_1, \vartheta_2, \vartheta_3)\| \leq \rho(\vartheta_1, \vartheta_2, \vartheta_3), \tag{19}$$

for all $\vartheta_1, \vartheta_2, \vartheta_3 \in V$. Then, there exists a unique quadratic mapping $Q : V \longrightarrow W$ satisfying

$$\|\xi(\vartheta) - Q(\vartheta)\| \leq \sup_{j\in\mathbb{N}} \left\{\left|2^2\right|^{j-1} \rho\left(\frac{\vartheta_1}{2^j}, \frac{\vartheta_2}{2^j}, \frac{\vartheta_3}{2^j}\right)\right\}, \tag{20}$$

for all $\vartheta \in V$.

Proof. Setting $\vartheta_1 = \vartheta_2 = \vartheta_3 = \vartheta$ in (19), we have

$$\left\|\xi(2\vartheta) - 2^2\xi(\vartheta)\right\| \leq \rho(\vartheta, \vartheta, \vartheta), \vartheta \in V. \tag{21}$$

Thus, from inequality (21), it implies that

$$\left\|\xi(\vartheta) - 2^2\xi\left(\frac{\vartheta}{2}\right)\right\| \leq \rho\left(\frac{\vartheta}{2}, \frac{\vartheta}{2}, \frac{\vartheta}{2}\right), \tag{22}$$

for all $\vartheta \in V$. Replacing ϑ by $\vartheta/2$ in (22), we obtain

$$\left\|2^2\xi\left(\frac{\vartheta}{2}\right) - 2^4\xi\left(\frac{\vartheta}{2^2}\right)\right\| \leq \left|2^2\right| \rho\left(\frac{\vartheta}{2^2}, \frac{\vartheta}{2^2}, \frac{\vartheta}{2^2}\right), \tag{23}$$

for all $\vartheta \in V$. Hence,

$$\begin{aligned}&\left\|2^{2l}\xi\left(\frac{\vartheta^l}{2}\right) - 2^{2m}\xi\left(\frac{\vartheta}{2^m}\right)\right\| \\ &\leq \max\left\{\left\|2^{2l}\xi\left(\frac{\vartheta}{2^l}v\right) - 2^{2(l+1)}\xi\left(\frac{\vartheta}{2^{l+1}}\right)\|, \cdots, \|2^{2(m-1)}\xi\left(\frac{\vartheta}{2^{m-1}}\right) - 2^{2m}\xi\left(\frac{\vartheta}{2^m}\right)\right\|\right\} \\ &\leq \max\left\{\left|2^2\right|^l \|\xi\left(\frac{\vartheta}{2^l}\right) - 2^2\xi\left(\frac{\vartheta}{2^{l+1}}\right)\|, \cdots, \left|2^2\right|^{m-1}\left\|\xi\left(\frac{\vartheta}{2^{m-1}}\right) - 2^2\xi\left(\frac{\vartheta}{2^m}\right)\|\right\} \\ &\leq \sup_{j\in\{l,l+1,\cdots\}}\left\{\left|2^2\right|^j \rho\left(\frac{\vartheta}{2^{j+1}}, \frac{\vartheta}{2^{j+1}}, \frac{\vartheta}{2^{j+1}}\right)\right\},\end{aligned} \tag{24}$$

for all $m > l > 0$ and all $\vartheta \in V$. From inequality (24), the sequence $\{2^{2n}\xi(\vartheta/2^n)\}$ is a Cauchy sequence for all $\vartheta \in V$. Since W is complete, thus the sequence $\{2^{2n}\xi(\vartheta/2^n)\}$ is convergent. Now, we can define a mapping $Q : V \longrightarrow W$ by

$$Q(\vartheta) := \lim_{l\longrightarrow\infty} 2^{2l}\xi\left(\frac{\vartheta}{2^l}\right),\ \vartheta \in V. \tag{25}$$

Taking $l = 0$ and passing the limit $m \longrightarrow \infty$ in (24), we obtain (20). From inequalities (18) and (19), we have

$$\begin{aligned}
&\|\Delta Q(\vartheta_1, \vartheta_2, \vartheta_3)\| \\
&\quad = \lim_{j\longrightarrow\infty} \left|2^2\right|^j \|\Delta\xi\left(2^{-j}\vartheta_1, 2^{-j}\vartheta_2, 2^{-j}\vartheta_3\right)\| \\
&\quad \le \lim_{j\longrightarrow\infty} \left|2^2\right|^j \rho\left(2^{-j}\vartheta_1, 2^{-j}\vartheta_2, 2^{-j}\vartheta_3\right) = 0.
\end{aligned} \tag{26}$$

From above, we conclude that $\Delta Q(\vartheta_1, \vartheta_2, \vartheta_3) = 0$ for all $\vartheta_1, \vartheta_2, \vartheta_3 \in V$. By using Lemma 4, the function Q is quadratic. Consider another quadratic mapping $T : V \longrightarrow W$ satisfying (20). Then, we have

$$\begin{aligned}
&\|Q(\vartheta) - T(\vartheta)\| \\
&\quad = \|2^{2q}Q\left(\frac{\vartheta}{2^q}\right) - 2^{2q}T\left(\frac{\vartheta}{2^q}\right)\| \\
&\quad \le \max\left\{\left\|2^{2q}Q\left(\frac{\vartheta}{2^q}\right) - 2^{2q}\xi\left(\frac{\vartheta}{2^q}\right)\|, \|2^{2q}T\left(\frac{\vartheta}{2^q}\right) - 2^{2q}\xi\left(\frac{\vartheta}{2^q}\right)\right\|\right\} \\
&\quad \le \sup_{j\in\mathbb{N}}\left\{\left|2^2\right|^{q+j-1}\rho\left(\frac{\vartheta}{2^{j+1}}, \frac{\vartheta}{2^{j+1}}, \frac{\vartheta}{2^{j+1}}\right)\right\} \longrightarrow 0 \text{ as } q \longrightarrow \infty,
\end{aligned} \tag{27}$$

for all $\vartheta \in V$. Thus, we can conclude that $T(\vartheta) = Q(\vartheta)$, $\vartheta \in V$. Hence, the function Q is unique. Thus, the unique quadratic mapping $Q : V \longrightarrow W$ satisfies (20). Hence, the proof of the theorem is now completed. □

Theorem 6. *Let $\rho : V^3 \longrightarrow [0,\infty)$ be a mapping and a mapping $\xi : V \longrightarrow W$ such that $\xi(0) = 0$ and*

$$\lim_{j\longrightarrow\infty}\left\{\frac{1}{\left|2^2\right|^j}\rho\left(2^{j-1}\vartheta_1, 2^{j-1}\vartheta_2, 2^{j-1}\vartheta_3\right)\right\} = 0, \tag{28}$$

and (19) for all $\vartheta_1, \vartheta_2, \vartheta_3 \in V$. Then, there exists a unique quadratic mapping $Q : V \longrightarrow W$ satisfying

$$\|\xi(\vartheta) - Q(\vartheta)\| \le Sup_{j\in\mathbb{N}}\left\{\frac{1}{\left|2^2\right|^{j-1}}\rho\left(2^{j-1}\vartheta_1, 2^{j-1}\vartheta_2, 2^{j-1}\vartheta_3\right)\right\}, \tag{29}$$

for all $\vartheta \in V$.

Proof. Setting $\vartheta_1 = \vartheta_2 = \vartheta_3 = \vartheta$ in (19), we have

$$\left\|\xi(2\vartheta) - 2^2\xi(\vartheta)\right\| \le \rho(\vartheta, \vartheta, \vartheta),\ \vartheta \in V. \tag{30}$$

From inequality (30), we obtain

$$\left\|\xi(\vartheta) - \frac{1}{2^2}\xi(2\vartheta)\right\| \le \frac{1}{\left|2^2\right|}\rho(\vartheta, \vartheta, \vartheta),\ \vartheta \in V. \tag{31}$$

Replacing ϑ by 2ϑ in (31), we get

$$\left\|\frac{\xi(2\vartheta)}{2^2} - \frac{1}{2^4}\xi\left(2^2\vartheta\right)\right\| \le \frac{1}{\left|2^2\right|^2}\rho(2\vartheta, 2\vartheta, 2\vartheta), \tag{32}$$

for all $\vartheta \in V$. Hence,

$$\begin{aligned}
&\left\|\frac{1}{2^{2l}}\xi\left(2^l\vartheta\right) - \frac{1}{2^{2m}}\xi(2^m\vartheta)\right\| \\
&\quad \le \max\left\{\left\|\frac{1}{2^{2l}}\xi\left(2^l\vartheta\right) - \frac{1}{2^{2(l+1)}}\xi\left(2^{l+1}\vartheta\right)\|, \cdots, \|\frac{1}{2^{2(m-1)}}\xi\left(2^{m-1}\vartheta\right) - \frac{1}{2^{2m}}\xi(2^m\vartheta)\right\|\right\} \\
&\quad \le \max\left\{\frac{1}{\left|2^2\right|^l}\left\|\xi\left(2^l\vartheta\right) - \frac{1}{\left|2^2\right|^{m-1}}\xi\left(2^{l+1}\vartheta\right)\right\|, \cdots, \frac{1}{\left|2^2\right|^{m-1}}\left\|\xi\left(2^{m-1}\vartheta\right) - \frac{1}{2^2}\xi(2^m\vartheta)\right\|\right\} \\
&\quad \le \sup_{j\in\{l,l+1,\cdots\}}\left\{\frac{1}{\left|2^2\right|^{j+1}}\rho\left(2^j\vartheta_1, 2^j\vartheta_2, 2^j\vartheta_3\right)\right\},
\end{aligned} \tag{33}$$

for all $m > l > 0$ and all $\vartheta \in V$. From inequality (33), the sequence $\{(1/2^{2n})\xi(2^n\vartheta)\}$ is a Cauchy sequence for all $\vartheta \in V$. Since W is complete, the sequence $\{(1/2^{2n})\xi(2^n\vartheta)\}$ is convergent. Now, we can define a mapping $Q : V \longrightarrow W$ by

$$Q(\vartheta) := \lim_{n\longrightarrow\infty}\frac{1}{2^{2n}}\xi(2^n\vartheta),\ \vartheta \in V. \tag{34}$$

The remaining proof is the same as the proof of Theorem 5. □

Corollary 7. *Let $\xi : V \longrightarrow W$ be a mapping such that $\xi(\vartheta) = 0$ and*

$$\|\Delta(\vartheta_1, \vartheta_2, \vartheta_3)\| \le \theta(\|\vartheta_1\|^r + \|\vartheta_2\|^r + \|\vartheta_3\|^r), \tag{35}$$

for all $\vartheta_1, \vartheta_2, \vartheta_3 \in V$, where r and θ are in $\mathbb{R}^+$ with $r < 2$. Then, there exists a unique quadratic mapping $Q : V \longrightarrow W$ satisfying

$$\|\xi(\vartheta) - Q(\vartheta)\| \le \frac{3\theta}{|2|^r}\|\vartheta\|^r, \tag{36}$$

for all $\vartheta \in V$.

Corollary 8. *Let $\xi : V \longrightarrow W$ be a mapping such that $\xi(\vartheta) = 0$ and*

$$\|\Delta(\vartheta_1, \vartheta_2, \vartheta_3)\| \le \theta(\|\vartheta_1\|^r + \|\vartheta_2\|^r + \|\vartheta_3\|^r), \tag{37}$$

for all $\vartheta_1, \vartheta_2, \vartheta_3 \in V$, where r and θ are in $\mathbb{R}^+$ with $r > 2$. Then, there exists a unique quadratic mapping $Q : V \longrightarrow$

W satisfying

$$\|\xi(\vartheta) - Q(\vartheta)\| \leq \frac{3\theta}{|2^2|}\|\vartheta\|^r, \tag{38}$$

for all $\vartheta \in V$.

5. Stability of (6) in Non-Archimedean Banach Space: Fixed Point Method

Theorem 9. *Let* $\rho : V^3 \longrightarrow [0,\infty)$ *be a mapping such that there exists* $L < 1$ *with*

$$\rho\left(2^{-1}\vartheta_1, 2^{-1}\vartheta_2, 2^{-1}\vartheta_3\right) \leq \frac{L}{|4|}\rho\left(2^{-1}\vartheta_1, 2^{-1}\vartheta_2, 2^{-1}\vartheta_3\right), \tag{39}$$

for all $\vartheta_1, \vartheta_2, \vartheta_3 \in V$. *If a mapping* $\xi : V \longrightarrow W$ *such that* $\xi(0) = 0$ *and (19) for all* $\vartheta_1, \vartheta_2, \vartheta_3 \in V$, *then there exists a unique quadratic mapping* $Q : V \longrightarrow W$ *satisfying*

$$\|\xi(\vartheta) - Q(\vartheta)\| \leq \frac{L}{|2^2|(1-L)}\rho(\vartheta, \vartheta, \vartheta), \tag{40}$$

for all $\vartheta \in V$.

Proof. Setting $\vartheta_1 = \vartheta_2 = \vartheta_3 = \vartheta$ in (19), we obtain

$$\|\xi(2\vartheta) - 4\xi(\vartheta)\| \leq \rho(\vartheta, \vartheta, \vartheta), \tag{41}$$

for all $\vartheta \in V$. Consider

$$S := \{q : V \longrightarrow W, q(0) = 0\}, \tag{42}$$

and the generalized metric d defined by

$$d(p, q) := \inf\{\varepsilon \in \mathbb{R} : \|p(\vartheta) - q(\vartheta)\| \leq \varepsilon\rho(\vartheta, \vartheta, \vartheta), \forall\vartheta \in V\}, \tag{43}$$

here, as usual, $\inf \xi = +\infty$. Clearly, (S, q) is complete (see [23]). Next, consider a mapping $J : S \longrightarrow S$ defined by

$$Jp(\vartheta) := 2^2 p\left(\frac{\vartheta}{2}\right), \ \vartheta \in V \tag{44}$$

For all $p, q \in S$ such that $d(p, q) = \varepsilon$, then

$$\|p(\vartheta) - q(\vartheta)\| \leq \varepsilon\rho(\vartheta, \vartheta, \vartheta), \tag{45}$$

for all $\vartheta \in V$. Hence,

$$\begin{aligned}\|Jp(\vartheta) - Jq\xi(\vartheta)\| &= \left\|2^2 p\left(2^{-1}\vartheta\right) - 2^2 q\xi\left(2^{-1}\vartheta\right)\right\| \\ &\leq \left|2^2\right|\varepsilon\rho\left(2^{-1}\vartheta, 2^{-1}\vartheta, 2^{-1}\nu\right) \\ &\leq \left|2^2\right|\varepsilon\frac{L}{|2^2|}\rho(\vartheta, \vartheta, \vartheta) \leq \left|2^2\right|L\varepsilon\rho(\vartheta, \vartheta, \vartheta),\end{aligned} \tag{46}$$

for all $\vartheta \in V$. Thus,

$$d(p, q) = \varepsilon \Rightarrow d(Jp, Jq) \leq L\varepsilon. \tag{47}$$

This concludes that

$$d(Jp, Jq) \leq Ld(p, q), \ p, q \in S. \tag{48}$$

From inequality (41),

$$\left\|\xi(\vartheta) - 2^2\xi\left(\frac{\vartheta}{2}\right)\right\| \leq \rho\left(2^{-1}\vartheta, 2^{-1}\vartheta, 2^{-1}\vartheta\right) \leq \frac{L}{|2^2|}\rho(\vartheta, \vartheta, \vartheta), \ \vartheta \in V. \tag{49}$$

Therefore,

$$d(\xi, J\xi) \leq \left|\frac{1}{2^2}\right|L, \ \vartheta \in V. \tag{50}$$

By using Theorem 3, there exists a mapping $Q : V \longrightarrow W$ satisfying the following conditions:

(1) Q is a fixed point of J, i.e.,

$$Q(\vartheta) = 2^2 Q\left(2^{-1}\vartheta\right)\forall\vartheta \in V. \tag{51}$$

In the set below, the function Q is the unique fixed point J.

$$M = \{p \in S : d(\xi, p) < \infty\}. \tag{52}$$

This proves that the uniqueness of the function Q satisfies (51) such that there exists $\varepsilon \in [0,\infty)$ such that

$$\|\xi(\vartheta) - Q(\vartheta)\| \leq \varepsilon\rho(\vartheta, \vartheta, \vartheta), \ \vartheta \in V. \tag{53}$$

(2) $d(J^l\xi, Q)$ tends to 0 as taking the limit $l \longrightarrow \infty$. This implies

$$\lim_{l \longrightarrow \infty} 4^n\xi(2^{-n}\vartheta) = Q(\vartheta), \text{ for all } \vartheta \in V. \tag{54}$$

(3) $d(\xi, Q) \leq (1/1-L)d(\xi, J\xi)$, which implies

$$\|\xi(\vartheta) - Q(\vartheta)\| \leq \frac{L}{|2^2|(1-L)}\rho(\vartheta, \vartheta, \vartheta), \text{ for all } \vartheta \in V. \tag{55}$$

From (39) and (51),

$$\begin{aligned}\|\Delta Q(\vartheta_1, \vartheta_2, \vartheta_3)\| &= \lim_{j\longrightarrow\infty} |2^2|^j \|\Delta\xi(2^{-j}\vartheta_1, 2^{-j}\vartheta_2, 2^{-j}\vartheta_3)\| \\ &\leq \lim_{j\longrightarrow\infty} |2^2|^j \rho(2^{-j}\vartheta_1, 2^{-j}\vartheta_2, 2^{-j}\vartheta_3) = 0,\end{aligned} \tag{56}$$

for all $\vartheta_1, \vartheta_2, \vartheta_3 \in V$. Thus,

$$\Delta Q(\vartheta_1, \vartheta_2, \vartheta_3) = 0, \tag{57}$$

for all $\vartheta_1, \vartheta_2, \vartheta_3 \in V$. By using Lemma 4, the function Q is quadratic. Hence, the proof of the theorem is now completed. □

Theorem 10. *Let $\rho : V^3 \longrightarrow [0,\infty)$ be a mapping such that there exists $L < 1$ with*

$$\rho(\vartheta_1, \vartheta_2, \vartheta_3) \leq L|2^2|\rho(2^{-1}\vartheta_1, 2^{-1}\vartheta_2, 2^{-1}\vartheta_3),\ \vartheta_1, \vartheta_2, \vartheta_3 \in V. \tag{58}$$

If a mapping $\xi : V \longrightarrow W$ such that $\xi(0) = 0$ and (19) for all $\vartheta_1, \vartheta_2, \vartheta_3 \in V$, then there exists a unique quadratic mapping $Q : V \longrightarrow W$ satisfying

$$\|\xi(\vartheta) - Q(\vartheta)\| \leq \frac{1}{|2^2|(1-L)}\rho(\vartheta, \vartheta, \vartheta), \tag{59}$$

for all $\vartheta \in V$.

Proof. Setting $\vartheta_1 = \vartheta_2 = \vartheta_3 = \vartheta$ in (19), we have

$$\|\xi(2\vartheta) - 4\xi(\vartheta)\| \leq \rho(\vartheta, \vartheta, \vartheta), \tag{60}$$

for all $\vartheta \in V$. From the inequality (60), we get

$$\left\|\xi(\vartheta) - \frac{1}{2^2}\xi(\vartheta)\right\| \leq \frac{1}{|2^2|}\rho(\vartheta, \vartheta, \vartheta),\ \vartheta \in V. \tag{61}$$

The generalized metric space (S, d) is defined in the proof of Theorem 9. Consider a mapping $J : S \longrightarrow S$ defined by

$$Jp(\vartheta) := \frac{1}{2^2}p(2\vartheta),\ \vartheta \in V. \tag{62}$$

From inequality (61),

$$d(\xi, J\xi) \leq \frac{1}{|2^2|}. \tag{63}$$

Hence,

$$\|\xi(\vartheta) - Q(\vartheta)\| \leq \frac{1}{|2^2|(1-L)}\rho(\vartheta, \vartheta, \vartheta),\ \vartheta \in V. \tag{64}$$

The remaining proof is the same as in the proof of Theorem 9. □

Corollary 11. *Let $\xi : V \longrightarrow W$ be a mapping such that $\xi(0) = 0$ and*

$$\|\Delta\xi(\vartheta_1, \vartheta_2, \vartheta_3)\| \leq \theta\left(\sum_{i=1}^{3}\|\vartheta_i\|^r\right), \tag{65}$$

for all $\vartheta_1, \vartheta_2, \vartheta_3 \in V$, where r and θ are in $\mathbb{R}^+$ with $r < 2$; then there exists a unique quadratic mapping $Q : V \longrightarrow W$ satisfying

$$\|\xi(\vartheta) - Q(\vartheta)\| \leq \frac{2\theta\|\vartheta\|^r}{|2|^r - |2^2|}, \tag{66}$$

for all $\vartheta \in V$.

Corollary 12. *Let $\xi : V \longrightarrow W$ be a mapping such that $\xi(0) = 0$ and*

$$\|\Delta\xi(\vartheta_1, \vartheta_2, \vartheta_3)\| \leq \theta\left(\sum_{i=1}^{3}\|\vartheta_i\|^r\right), \tag{67}$$

for all $\vartheta_1, \vartheta_2, \vartheta_3 \in V$, where r and θ are in $\mathbb{R}^+$ with $r > 2$; then there exists a unique quadratic mapping $Q : V \longrightarrow W$ satisfying

$$\|\xi(\vartheta) - Q(\vartheta)\| \leq \frac{2\theta}{|2^2|^r - |2|}\|\vartheta\|^r, \tag{68}$$

for all $\vartheta \in V$.

6. Stability of (6) in Non-Archimedean Random Normed Space

Definition 13 [24]. A random normed space is triple (V, μ, T), where V is a vector space, T is a continuous $t-$norm, and a mapping $\mu : V \longrightarrow D^+$ satisfies

(RN1) $\mu_\vartheta(t) = \varepsilon_0(t), \forall t > 0$ if and only if $\vartheta = 0$.

(RN2) $\mu_{\lambda\vartheta}(t) = \mu_\vartheta(t/|\lambda|)$ for all $\vartheta \in V$, $\lambda \neq 0$.

(RN3) $\mu_{\vartheta_1+\vartheta_2}(t_1 + t_2) \geq T(\mu_{\vartheta_1}(t_1), \mu_{\vartheta_2}(t_2))$ for all $\vartheta_1, \vartheta_2 \in V$ and $t_1, t_2 \geq 0$.

Definition 14 [25]. A random normed space (V, μ, T) is said to be non-Archimedean random normed space if it satisfies

(NAR1) $\mu_\vartheta(t) = \varepsilon_0(t)$ for all $t > 0$ if and only if $\vartheta = 0$.

(NAR2) $\mu_{\lambda\vartheta}(t) = \mu_\vartheta(1/|\lambda|)$ for all $\vartheta \in V$, $t > 0$, $\lambda \neq 0$.

(NAR3) $\mu_{\vartheta_1+\vartheta_2}(\max\{t_1, t_2\}) \geq T(\mu_{\vartheta_1}(t_1), \mu_{\vartheta_2}(t_2))$ for all $\vartheta_1, \vartheta_2 \in V$ and $t_1, t_2 \geq 0$.

It is clear that if (NAR3) holds, then so

$$(RN3)\mu_{\vartheta_1+\vartheta_2}(t+s) \geq T\left(\mu_{\vartheta_1}(t), \mu_{\vartheta_2}(s)\right). \tag{69}$$

Example 1 [25]. Let a non-Archimedean normed space $(V, \|\cdot\|)$ and we define

$$\mu_\vartheta(t) = \frac{t}{t+\|\vartheta\|}, \tag{70}$$

for all $\vartheta \in V$ and all $t > 0$. Then, the triple (V, μ, T_M) is a non-Archimedean random normed space.

Definition 15 [25]. Let (V, μ, T) be a non-Archimedean random normed space and a sequence $\{\vartheta_n\}$ in V. Then, the sequence $\{\vartheta_n\}$ is called as convergent if there exist $\vartheta \in V$ such that

$$\lim_{n\longrightarrow\infty} \mu_{n-\vartheta}(t) = 1, \tag{71}$$

for all $t > 0$. In particular, ϑ is called the limit of the sequence $\{\vartheta_n\}$.

Here, let V be a vector space over a non-Archimedean field $\mathbb{K}$ and (W, μ, T) be a non-Archimedean random Banach space over $\mathbb{K}$. And consider that $2 \neq 0$ in $\mathbb{K}$.

Next, we define a random approximately quadratic function. Let a distribution mapping $\psi : V \times V \longrightarrow [0,\infty)$ satisfies $\psi(\vartheta_1, \vartheta_2, \vartheta_3, \cdot)$ which is symmetric and nondecreasing and

$$\psi(\lambda\vartheta, \lambda\vartheta, \lambda\vartheta, t) \geq \psi\left(\vartheta, \vartheta, \vartheta, \frac{t}{|\lambda|}\right), \tag{72}$$

for all $\vartheta \in V$ and all $\lambda \neq 0$.

Definition 16. A function $\xi : V \longrightarrow W$ is called as a ψ-approximately quadratic if

$$\mu_{2\xi(\vartheta_1+\vartheta_2/2)+2\xi(\vartheta_3)-\xi((\vartheta_1+\vartheta_2/2)+\vartheta_3)-\lambda^{-2}\xi(\lambda((\vartheta_1+\vartheta_2/2)-\vartheta_3))} \geq \psi(\vartheta_1, \vartheta_2, \vartheta_3, t), \tag{73}$$

for all $\vartheta_1, \vartheta_2, \vartheta_3 \in V$ and $t > 0$.

Theorem 17. *Let a function $\xi : V \longrightarrow W$ be a ψ-approximately quadratic mapping. If for some real number $\alpha > 0$, and some integer k, $k > 1$ with $\alpha > |2^k|$,*

$$\psi\left(2^{-k}\vartheta_1, 2^{-k}\vartheta_2, 2^{-k}\vartheta_3, t\right) \geq \psi(\vartheta_1, \vartheta_2, \vartheta_3, \alpha t), \tag{74}$$

for all $\vartheta_1, \vartheta_2, \vartheta_3 \in V$ and $t > 0$, and

$$\lim_{n\longrightarrow\infty} T_{j=n}^{\infty} M\left(\vartheta, \frac{\alpha^j t}{|2|^{kj}}\right) = 1, \tag{75}$$

for all $\vartheta \in V$ and every $t > 0$; then there exists a unique quadratic mapping $Q : V \longrightarrow W$ such that

$$\mu_{\xi(\vartheta)-Q(\vartheta)}(t) \geq T_{i=1}^{\infty} M\left(\vartheta, \frac{\alpha^{i+1} t}{|2|^{ki}}\right), \tag{76}$$

where

$$M(\vartheta, t) := T\left(\psi(\vartheta, \vartheta, \vartheta, t)\psi(2\vartheta, 2\vartheta, 2\vartheta, t), \cdots, \psi\left(2^{k-1}\vartheta, 2^{k-1}\vartheta, 2^{k-1}\vartheta, t\right)\right), \tag{77}$$

for all $\vartheta \in V$ and all $t > 0$.

Proof. First, we demonstrate by induction on j that for all $\vartheta \in V$, $t > 0$ and $j > 0$,

$$\mu_{\xi(2^j\vartheta)-2^{2j}\xi(\vartheta)}(t) \geq M_j(\vartheta, t) := T\left(\psi(\vartheta, \vartheta, \vartheta, t), \cdots, \psi\left(2^{j-1}\vartheta, 2^{j-1}\vartheta, 2^{j-1}\vartheta, t\right)\right). \tag{78}$$

Setting $\vartheta_1 = \vartheta_2 = \vartheta_3 = \vartheta$ in (73), we obtain

$$\mu_{\xi(2\vartheta)-2^2\xi(\vartheta)}(t) \geq \psi(\vartheta, \vartheta, \vartheta, t), \tag{79}$$

for all $\vartheta \in V$ and all $t > 0$. This proves that (78) for $j = 1$. Suppose that (78) holds for some $j > 0$. Replacing ϑ by $2^j\vartheta$ in (73), we get

$$\mu_{\xi\left(2^{j+1}\vartheta-2^2\xi\left(2^j\vartheta\right)\right.}(t) \geq \psi\left(2^j\vartheta, 2^j\vartheta, 2^j\vartheta, t\right), \tag{80}$$

for all $\vartheta \in V$ and all $t > 0$. Since $|2^2| \leq 1$,

$$\begin{aligned}
&\mu_{\xi\left(2^{j+1}\vartheta\right)-2^{2(j+1)}\xi(\vartheta)}(t) \\
&\quad \geq T\left(\mu_{\xi\left(2^{j+1}\vartheta\right)-2^2\xi\left(2^j\vartheta\right)}(t), \mu_{2^2\xi\left(2^j\vartheta\right)-2^{2(j+1)}\xi(\vartheta)}(t)\right) \\
&\quad = T\left(\mu_{\xi\left(2^{j+1}\vartheta\right)-2^2\xi\left(2^j\vartheta\right)}(t), \mu_{\xi\left(2^j\vartheta\right)-2^{2j}\xi(\vartheta)}\left(\frac{t}{|2^2|}\right)\right) T \\
&\qquad \cdot\left(\mu_{\xi\left(2^{j+1}\vartheta\right)-2^2\xi\left(2^j\vartheta\right)}(t), \mu_{\xi\left(2^j\vartheta\right)-2^{2j}\xi(\vartheta)}(t)\right) \\
&\quad \geq T\left(\psi\left(2^j\vartheta, 2^j\vartheta, 2^j\vartheta, t\right), M_j(\vartheta, t)\right) = M_{j+1}(\vartheta, t),
\end{aligned} \tag{81}$$

for all $\vartheta \in V$. Thus, condition (78) holds for all $j > 0$. In particular,

$$\mu_{\xi\left(2^k\vartheta\right)-2^{2k}\xi(\vartheta)}(t) \geq M(\vartheta, t), \tag{82}$$

for all $\vartheta \in V$ and all $t > 0$. Replacing ϑ by $2^{-(k+kn)}\vartheta$ in (82) and using the inequality (74), we have

$$\mu_{\xi(\vartheta/2^{kn})-2^{2k}\xi(\vartheta/2^{kn+k})} \geq M\left(\frac{\vartheta}{2^{kn+k}}, t\right) \geq M\left(\vartheta, \alpha^{n+1}t\right); n = 0, 1, 2, \cdots, \tag{83}$$

for all $\vartheta \in V$ and all $t > 0$. Then,

$$\mu_{(2^{2k})^n\xi(\vartheta/(2^k)^n)-(2^{2k})^{n+1}\xi(\vartheta/(2^k)^{n+1})}(t) \geq M\left(\vartheta, \frac{\alpha^{n+1}}{\left|(2^{2k})^n\right|}t\right); n = 0, 1, 2, \cdots, \tag{84}$$

for all $\vartheta \in V$ and all $t > 0$. Hence,

$$\begin{aligned}
&\mu_{(2^k)^n\xi(\vartheta/(2^k)^n)-(2^{2k})^{n+p}\xi(\vartheta/(2^k)^{n+p})}(t) \\
&\geq T_{j=n}^{n+p}\left(\mu_{(2^k)^j\xi(\vartheta/(2^k)^j)-(2^{2k})^{j+p}\xi(\vartheta/(2^k)^{j+p})}(t)\right) \\
&\geq T_{j=n}^{n+p}M\left(\vartheta, \frac{\alpha^{j+1}}{\left|(2^{2k})^j\right|}t\right) \geq T_{j=n}^{n+p}M\left(\vartheta, \frac{\alpha^{j+1}}{\left|(2^k)^j\right|}t\right).
\end{aligned} \tag{85}$$

Since $\lim_{n\longrightarrow\infty}T_{j=n}^{\infty}M(\vartheta, (\alpha^{j+1}/|(2^k)^j|)t) = 1$ for all $\vartheta \in V$ and all $t > 0$, $\{(2^{2k})^n\xi(\vartheta/(2^k)^n)\}_{n\in N}$ is a Cauchy sequence in (W, μ, T). Hence, we can define a mapping $Q : V \longrightarrow W$ such that

$$\lim_{n\longrightarrow\infty}\mu_{(2^{2k})^n\xi(\vartheta/(2^k)^n)-Q(\vartheta)}(t) = 1, \tag{86}$$

for all $\vartheta \in V$ and all $t > 0$. Now, for all $n \geq 1$,

$$\begin{aligned}
&\mu_{\xi(\vartheta)-(2^{2k})^n\xi(\vartheta/(2^k)^n)}(t) \\
&= \mu_{\sum_{i=0}^{n-1}\left(2^{2k}\right)^i\xi\left(\vartheta/\left(2^k\right)^i\right)-\left(2^{2k}\right)^{i+1}\xi\left(\vartheta/\left(2^k\right)^{i+1}\right)}(t) \\
&\geq T_{i=0}^{n-1}\left(\mu_{(2^{2k})^i\xi(\vartheta/(2^k)^i)-(2^{2k})^{i+1}\xi(\vartheta/(2^k)^{i+1})}(t)\right) \\
&\geq T_{i=0}^{n-1}M\left(\vartheta, \frac{\alpha^{i+1}t}{\left|2^{2k}\right|^i}\right),
\end{aligned} \tag{87}$$

for all $\vartheta \in V$ and $t > 0$. Thus,

$$\begin{aligned}
\mu_{\xi(\vartheta)-Q(\vartheta)}(t) &\geq T\left(\mu_{\xi(\vartheta)-(2^{2k})^n\xi(\vartheta/(2^k)^n)}, \mu_{(2^{2k})^n\xi(\vartheta/(2^k)^n)-Q(\vartheta)}(t)\right) \\
&\geq T\left(T_{i=0}^{n-1}M\left(\vartheta, \frac{\alpha^{i+1}t}{\left|2^{2k}\right|^i}\right), \mu_{(2^{2k})^n\xi(\vartheta/(2^k)^n)-Q(\vartheta)}(t)\right).
\end{aligned} \tag{88}$$

By taking the limit $n \longrightarrow \infty$, we have

$$\mu_{\xi(\vartheta)-Q(\vartheta)}(t) \geq T_{i=1}^{\infty}M\left(\vartheta, \frac{\alpha^{i+1}t}{\left|2^k\right|^i}\right). \tag{89}$$

This shows that (76) holds. Since T is continuous, by a well-known result in probabilistic metric space (see, e.g., [[26], Chapter 12]), that

$$\lim_{n\longrightarrow\infty}\mu_{(2^k)^n\Delta\xi(2^{-kn}\vartheta_1, 2^{-kn}\vartheta_2, 2^{-kn}\vartheta_3)}(t) = \mu_{\Delta Q(\vartheta_1,\vartheta_2,\vartheta_3)}(t), \tag{90}$$

for all $t > 0$.

On the other hand, replacing $(\vartheta_1, \vartheta_2, \vartheta_3)$ by $(2^{-kn}\vartheta_1, 2^{-kn}\vartheta_2, 2^{-kn}\vartheta_3)$, respectively, in (73) and using (NAR2) and (74), we get

$$\begin{aligned}
\mu_{(2^k)^n\Delta\xi(2^{-kn}\vartheta_1, 2^{-kn}\vartheta_2, 2^{-kn}\vartheta_3)}(t) &\geq \psi\left(2^{-kn}\vartheta_1, 2^{-kn}\vartheta_2, 2^{-kn}\vartheta_3, \frac{t}{\left|2^k\right|^n}\right) \\
&\geq \psi\left(\vartheta_1, \vartheta_2, \vartheta_3, \frac{\alpha^n t}{\left|2^k\right|^n}\right).
\end{aligned} \tag{91}$$

Since $\lim_{n\longrightarrow\infty}\psi(\vartheta_1, \vartheta_2, \vartheta_3, \alpha^n t/|2^k|^n) = 1$, we can conclude that the function Q is quadratic. Consider another quadratic mapping $Q' : V \longrightarrow W$ such that $\mu_{Q'(\vartheta)-\xi(\vartheta)}(t) \geq M(\vartheta, t)$ for all $\vartheta \in V$ and all $t > 0$; then for all $n \in N$ and $\vartheta \in V$ and all $t > 0$,

$$\mu_{Q(\vartheta)-Q'(\vartheta)}(t) \geq T\left(\mu_{Q(\vartheta)-(2^{4k})^n\xi(\vartheta/(2^k)^n)}(t), \mu_{(2^{2k})^n\xi(\vartheta/(2^k)^n)-Q'(\vartheta)}(t), t\right). \tag{92}$$

From condition (86), we arrive at the conclusion that $Q = Q'$. □

Corollary 18. *Let a function $\xi : V \longrightarrow W$ be a ψ-approximately quadratic. If for some real number $\alpha > 0$ and some integer k, $k > 1$, with $|2^k| < \alpha$,*

$$\psi\left(2^{-k}\vartheta_1, 2^{-k}\vartheta_2, 2^{-k}\vartheta_3, t\right) \geq \psi(\vartheta_1, \vartheta_2, \vartheta_3, \alpha t), \tag{93}$$

for all $\vartheta_1, \vartheta_2, \vartheta_3 \in V$ and $t > 0$, then there exists a unique quadratic mapping $Q : V \longrightarrow W$ satisfying

$$\mu_{\xi(\vartheta)-Q(\vartheta)}(t) \geq T_{i=1}^{\infty}M\left(\vartheta, \frac{\alpha^{i+1}t}{|2|^{ki}}\right), \tag{94}$$

where

$$M(\vartheta, t) := T(\psi(\vartheta, \vartheta, \vartheta, t), \psi(2\vartheta, 2\vartheta, 2\vartheta, t), \cdots, \psi\left(2^{k-1}\vartheta, 2^{k-1}\vartheta, 2^{k-1}\vartheta, t\right), \tag{95}$$

for all $\vartheta \in V$ and all $t > 0$.

Proof. Since

$$\lim_{n\longrightarrow\infty} M\left(\vartheta, \frac{\alpha^j t}{|2|^{kj}}\right) = 1, \tag{96}$$

for all $\vartheta \in V$ and all $t > 0$ and T is of Hadzic type, from Proposition 2.1 in [25], it follows that

$$\lim_{n\longrightarrow\infty} T_{j=n}^{\infty} M\left(\vartheta, \frac{\alpha^j t}{|2|^{kj}}\right), \tag{97}$$

for all $\vartheta \in V$ and $t > 0$. Now, we can obtain our needed result by using Theorem 17 □

Example 2. Let a non-Archimedean random normed space (V, μ, T_M), in which

$$\mu_\vartheta(t) = \frac{t}{t+\|\vartheta\|}, \tag{98}$$

for all $\vartheta \in V$ and every $t > 0$, and let (W, μ, T_M) be a complete non-Archimedean random normed space (see Example 1). Now, we can define

$$\psi(\vartheta_1, \vartheta_2, \vartheta_3, t) = \frac{t}{1+t}. \tag{99}$$

It is obvious that (74) holds for $\alpha = 1$. Furthermore,

$$M(\vartheta, t) = \frac{t}{1+t}. \tag{100}$$

We obtain

$$\begin{aligned}\lim_{n\longrightarrow\infty} T_{M,j=n}^{\infty} M\left(\vartheta, \frac{\alpha^j t}{|2|^{kj}}\right) &= \lim_{n\longrightarrow\infty}\left(\lim_{m\longrightarrow\infty} T_{M,j=n}^{m} M\left(\vartheta, \frac{t}{|2|^{kj}}\right)\right)\\ &= \lim_{n\longrightarrow\infty}\lim_{m\longrightarrow\infty}\left(\frac{t}{t+|2^k|^n}\right) = 1,\end{aligned} \tag{101}$$

for all $\vartheta \in V$ and all $t > 0$.

7. Conclusion

In this paper, we introduced λ-quadratic functional equation and obtained its general solution. In Section 4 and Section 5, we investigated Ulam-Hyers stability of equation (6) by using direct method and fixed point method in non-Archimedean Banach space, and also in Section 6, we investigated the Ulam-Hyers stability results in non-Archimedean random normed space. The direct method requires us to find the Cauchy sequence and prove that every Cauchy sequence is convergent, as well as prove the uniqueness of the function; this method was introduced by Hyers [2], and the fixed point method requires us to use the Banach contraction principle and Lipschitz constant L to obtain the stability results of the functional equation; this method was introduced by Radu [27]. The fixed point method gives more accurate stability results when compared with the direct method. Finally, these stability results generalized the findings of [11].

Data Availability

No data were used to support the findings of the study.

Conflicts of Interest

The authors declare that they have no conflict interests.

Authors' Contributions

All authors contributed equally to this work. And all the authors have read and approved the final version of the manuscript.

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