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Retraction

Retracted: Analysis of Eccentricity-Based Topological Invariants with Zero-Divisor Graphs

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This article has been retracted by Hindawi following an investigation undertaken by the publisher [1]. This investigation has uncovered evidence of one or more of the following indicators of systematic manipulation of the publication process:

(1) Discrepancies in scope

(2) Discrepancies in the description of the research reported

(3) Discrepancies between the availability of data and the research described

(4) Inappropriate citations

(5) Incoherent, meaningless and/or irrelevant content included in the article

(6) Peer-review manipulation

The presence of these indicators undermines our confidence in the integrity of the article's content and we cannot, therefore, vouch for its reliability. Please note that this notice is intended solely to alert readers that the content of this article is unreliable. We have not investigated whether authors were aware of or involved in the systematic manipulation of the publication process.

Wiley and Hindawi regrets that the usual quality checks did not identify these issues before publication and have since put additional measures in place to safeguard research integrity.

We wish to credit our own Research Integrity and Research Publishing teams and anonymous and named external researchers and research integrity experts for contributing to this investigation.

The corresponding author, as the representative of all authors, has been given the opportunity to register their agreement or disagreement to this retraction. We have kept a record of any response received.

References

[1] Z.-h. Hui, A. Rauf, M. M. Abbas, and A. Aslam, "Analysis of Eccentricity-Based Topological Invariants with Zero-Divisor Graphs," *Journal of Function Spaces*, vol. 2022, Article ID 6911654, 10 pages, 2022.

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Research Article

Analysis of Eccentricity-Based Topological Invariants with Zero-Divisor Graphs

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Let $R = Z_{\flat_1 \flat_2 \flat_3} \times Z_{q^2}$ be a commutative ring, where $\flat_1, \flat_2, \flat_3$ are distinct primes, and q is any prime integer. A zero divisor graph $J(R)$ of ring R is a graph with vertex set consist of zero divisors elements of R and any two vertices a, b are adjacent if and only if $ab = 0$. A topological index is a numerical number associated with the graph and may be helpful to correlate the graph with certain of its physical/chemical properties. In this paper, we have computed some eccentricity based topological indices of $J(R)$, namely, atom-bond connectivity index (ABC_5), eccentricity-based harmonic index of fourth type ($H_4(J)$), geometric-arithmetic eccentricity index ($GA_4(J)$), eccentricity-based third Zagreb index, and eccentricity-based first Zagreb index.

1. Introduction

There has been done wide investigation in combinatorics because of their strong ties to number theory and representation theory regarding algebraic structures [1, 2]. Due to their applications, finite rings and finite fields have attracted concentration in coding theory, cryptography along with vast theoretical study in these domains. The important functions known as molecular descriptors treat molecules as actual models as well as convert these molecules into numerals. These numerals are known as topological indices and are graph invariants.

Computing topological indices for different structures have been a study focus of recent work. In mathematical chemistry, the graph structure form of a chemical formula is called molecular graph. The compound's atoms are considered as vertices, and chemical bonds between the vertices are considered as edges. A topological index can be defined as a numeric number that describes the topological structure of a chemical graph in a chemical graph while being unchanged under graph automorphism. As a result, there are several applications of these indices in nanotube structures, chemistry, and medical sciences [3, 4].

Topological indices are mainly characterized in to three categories: distance-based, degree-based, and counting-related topological indices [5–8]. Atom-bond connectivity index (ABC), Randic connectivity index ($R_{-1/2}$), geometric-arithmetic index (GA), and harmonic index (H) are some famous degree-based topological indices. The Estrada index, Wiener index, and Hosaya index are the topological indices which are based on distance [9, 10]. The eccentricity-based connectivity atom-bond index [11], eccentricity-based harmonic index of fourth type [12, 13], geometric-arithmetic eccentricity index [14], and Zagreb eccentricity index [15, 16] are few examples of eccentricity-based topological indices. Topological indices may be used to broaden interdisciplinary study through mathematical operations on graphs as well as define conditions under which chemical structures are formed.

These topological indices attached to a molecular graph are useful to predict certain of its physical and chemical properties. For instance, the ABC index is used to study the stability of branched and linear alkanes. This index is utilized to calculate the shear energy of cycloalkanes [17, 18]. The geometric-arithmetic index has strong ability of predict certain physical and chemical properties of chemical structure as compared to the Randic connectivity index [19, 20]. First and second Zagreb indices are were used to estimate the total π-electron energy of alternate hydrocarbons [21]. For investigation of the chemical properties of different molecular structures, degree-based topological indices have great importance. Such application of degree-based indices offers motivation to study eccentricity-based topological indices. Eccentricity-based topological indices can be used to assess a compound's pharmacological, physicochemical, and toxicological qualities based on its molecular structure [22, 23]. To get more knowledge about topological indices and their applications, see [24–28].

2. Preliminary Definitions

Let $G = (V, E)$ be a graph, where V denotes the vertex set, and E denotes the edge set. Any two vertices $x, y \in V$ are adjacent if there is an edge between x and y. The degree of a vertex x is denoted by d_x and is defined as the number of vertices adjacent to x. The distance between two vertices x, $y \in V$ is the length of the shortest path joining them. Eccentricity of a vertex x is denoted by ε_x and is the maximum of the distances of all vertices from x. Mathematically $\varepsilon_x = \max \{d(x, y): y \in V\}$. To read more about the basic terminologies related to graph theory, see [29].

Let R be a commutative ring with identity. Any two nonzero elements $x, y \in R$ are called zero divisors if $x.y = 0$. Let $Z(R)$ be set of zero divisors of R. I. Beck [2] established the idea of the zero divisor graph $J(R)$ by considering $Z(R)$ as vertex set, and any two vertices $x, y \in Z(R)$ are connected by an edge if and only if $x.y = 0$. His fundamental goal was to show how a commutative ring can be coloured [2]. Livingston and Anderson [30] proved that $J(R)$ is a connected graph. For more results on this topic, see [24, 31, 32].

In general, any eccentricity-based topological invariant, denoted by $T(G)$, is defined as

$$T(G) = \sum_{xy \in E(G)} \varnothing(\varepsilon_x, \varepsilon_y), \quad (1)$$

where $\varnothing(\varepsilon_x, \varepsilon_y)$ is a real function with the property that $\varnothing(\varepsilon_x, \varepsilon_y) = \varnothing(\varepsilon_y, \varepsilon_x)$. From Equation (1), we can get some eccentricity based topological indices in the following way:

(a) Atom-bond connectivity eccentricity index ABC_5 if $\varnothing(\varepsilon_x, \varepsilon_y) = \sqrt{\varepsilon_x + \varepsilon_y - 2/\varepsilon_x \times \varepsilon_y}$ [11]

(b) Eccentricity-based harmonic index of fourth type H_4 if $\varnothing(\varepsilon_x, \varepsilon_y) = 2/\varepsilon_x + \varepsilon_y$ [12, 13]

(c) Geometric-arithmetic eccentricity index GA_4 if $\varnothing(\varepsilon_x, \varepsilon_y) = 2\sqrt{\varepsilon_x \times \varepsilon_y}/\varepsilon_x + \varepsilon_y$ [14].

(d) First Zagreb eccentricity index M_1^* if $\varnothing(\varepsilon_x, \varepsilon_y) = (\varepsilon_x + \varepsilon_y)^\beta$, where $\beta = 1$ [15, 16]

(e) Third Zagreb eccentricity index M_3^* if $\varnothing(\varepsilon_x, \varepsilon_y) = (\varepsilon_x \times \varepsilon_y)^\alpha$, where $\alpha = 1$ [15, 16]

3. Main Results

In this section, we compute the eccentricity based topological indices of a commutative ring $Z_{\flat_1\flat_2\flat_3} \times Z_{q^2}$, where $\flat_1, \flat_2, \flat_3$ are distinct primes, and q is any prime integer.

Theorem 1. *Let $\flat_1, \flat_2, \flat_3$ be distinct prime integers and q be any prime number. Then for the zero divisor graph $J(R)$ of a ring $R = Z_{\flat_1\flat_2\flat_3} \times Z_{q^2}$, we have $|V(J_R)| = \flat_1\flat_2\flat_3 q + \flat_1\flat_3 q^2 + \flat_1\flat_2 q^2 + \flat_2\flat_3 q^2 + \flat_1 q + 2\flat_2 + \flat_3 q + q^2 - \flat_1 q^2 - \flat_2 q^2 - \flat_3 q^2 - \flat_1\flat_3 q - \flat_2\flat_3 q + \flat_1^2 q - 3$.*

Proof. We can partition the vertex set of $R = Z_{\flat_1\flat_2\flat_3} \times Z_{q^2}$ as follows:

(1) $\varpi_1 = \{(0, v) \mid v \in Z_{q^2} : v \neq kq, 0 \leq k \leq q-1\}$. The vertices adjacent to $x \in \varpi_1$ are of the form $(u', 0)$ with $u' \in Z_{\flat_1\flat_2\flat_3} \setminus \{0\}$. Hence, $d_x = \flat_1\flat_2\flat_3 - 1$ for all $x \in \varpi_1$ and $|\varpi_1| = q^2 - q$

(2) $\varpi_2 = \{(0, v) \mid v = kq, 1 \leq k \leq q-1\}$. So we have $|\varpi_2| = q - 1$. The vertices adjacent to $x \in \varpi_2$ are of the form:

(i) $(u', 0)$: $u' \in Z_{\flat_1\flat_2\flat_3} \setminus \{0\}$

(ii) (u', v'): $u' \in Z_{\flat_1\flat_2\flat_3}, v' = t'q, 1 \leq t' \leq q-1$

Hence, for any $x \in \varpi_2$, $d_x = (\flat_1\flat_2\flat_3 - 1) + (\flat_1\flat_2\flat_3 - 1)(q-1) = \flat_1\flat_2\flat_3 q - q$.

(3) $\varpi_3 = \{(u, 0) \mid u = k\flat_1\flat_2 ; 1 \leq k \leq \flat_3 - 1\}$ with $|\varpi_3| = \flat_3 - 1$. The vertices adjacent to $x \in \varpi_3$ are of the form:

(i) $(0, v_1)$: $v_1 \in Z_{q^2} \setminus \{0\}$

(ii) $(u', 0)$: $u' = t_1\flat_3 ; 1 \leq t_1 \leq \flat_1\flat_2 - 1$

(iii) (u', v_2): $u' = t_1\flat_3, v_2 = t_2 q ; 1 \leq t_1 \leq \flat_1\flat_2 - 1, 1 \leq t_2 \leq q-1$

(iv) (u', v_3): $u' = t_1\flat_3, v_3 \neq t_3 q ; 1 \leq t_1 \leq \flat_1\flat_2 - 1, 1 \leq t_3 \leq q-1$

Hence, for any $x \in \varpi_3$, $d_x = q^2 - 1 + \flat_1\flat_2 - 1 + (\flat_1\flat_2 - 1)(q-1) + (\flat_1\flat_2 - 1)(q^2 - q) = \flat_1\flat_2 q^2 - 1$.

(4) $\varpi_4 = \{(u, 0): u = k\flat_1\flat_3 ; 1 \leq k \leq \flat_2 - 1\}$. with $|\varpi_4| = \flat_2 - 1$. The vertices adjacent to $x \in \varpi_4$ are of the form:

(i) $(0, v_1)$: $v_1 \in Z_{q^2} \setminus \{0\}$

(ii) $(u', 0)$: $u' = t_1\flat_2 ; 1 \leq t_1 \leq \flat_1\flat_3 - 1$

(iii) (u', v_2): $u' = t_1\flat_2, v_2 = t_2 q ; 1 \leq t_1 \leq \flat_1\flat_3 - 1, 1 \leq t_2 \leq q-1$

(iv) (u', v_3): $u' = t_1\flat_2, v_3 \neq t_3 q\,; 1 \leq t_1 \leq \flat_1\flat_3 - 1, 1 \leq t_3 \leq q - 1$

Hence, for any $x \in \varpi_4$, $d_x = q^2 - 1 + \flat_1\flat_3 - 1 + (\flat_1\flat_3 - 1)(q - 1) + (\flat_1\flat_3 - 1)(q^2 - q) = \flat_1\flat_3 q^2 - 1$.

(5) $\varpi_5 = \{(u, 0)\colon u = k_1\flat_1\,; 1 \leq k_1 \leq \flat_2\flat_3 - 1\,; u \neq k_2\flat_1\flat_2, 1 \leq k_2 \leq \flat_3 - 1\,; u \neq k_3\flat_1\flat_3, 1 \leq k_3 \leq \flat_2 - 1\}$ with $|\varpi_5| = (\flat_2 - 1)(\flat_3 - 1)$. The vertices adjacent to $x \in \varpi_5$ are of the form:

(i) $(0, v_1)$: $v_1 \in Z_{q^2} \setminus \{0\}$

(ii) $(u', 0)$: $u' = t_1\flat_2\flat_3\,; 1 \leq t_1 \leq \flat_1 - 1$

(iii) (u', v_2): $u' = t_1\flat_2\flat_3, v_2 = t_2 q\,; 1 \leq t_1 \leq \flat_1 - 1, 1 \leq t_2 \leq q - 1$

(iv) (u', v_3): $u' = t_1\flat_2\flat_3, v_3 \neq t_3 q\,; 1 \leq t_1 \leq \flat_1 - 1, 1 \leq t_3 \leq q - 1$

Hence, for any $x \in \varpi_5$, $d_x = q^2 - 1 + \flat_1 - 1 + (\flat_1 - 1)(q - 1) + (\flat_1 - 1)(q^2 - q) = \flat_1 q^2 - 1$.

(6) $\varpi_6 = \{(u, 0)\,; u = k\flat_2\flat_3, 1 \leq k \leq \flat_1 - 1\}$ with $|\varpi_6| = \flat_1 - 1$. The vertices adjacent to $x \in \varpi_6$ are of the form:

(i) $(0, v_1)$: $v_1 \in Z_{q^2} \setminus \{0\}$

(ii) $(u', 0)$: $u' = t_1\flat_1\,; 1 \leq t_1 \leq \flat_2\flat_3 - 1$

(iii) (u', v_2): $u' = t_1\flat_1, v_2 = t_2 q\,; 1 \leq t_1 \leq \flat_2\flat_3 - 1, 1 \leq t_2 \leq q - 1$

(iv) (u', v_3): $u' = t_1\flat_1, v_3 \neq t_3 q\,; 1 \leq t_1 \leq \flat_2\flat_3 - 1, 0 \leq t_3 \leq q - 1$

Hence, for any $x \in \varpi_6$, $d_x = q^2 - 1 + \flat_2\flat_3 - 1 + (\flat_2\flat_3 - 1)(q - 1) + (\flat_2\flat_3 - 1)(q^2 - q) = \flat_2\flat_3 q^2 - 1$.

(7) $\varpi_7 = \{(u, 0)\,; u = k_1\flat_2, 1 \leq k_1 \leq \flat_1\flat_3 - 1\,; u \neq k_2\flat_1\flat_2, 1 \leq k_2 \leq \flat_3 - 1\,; u \neq k_3\flat_2\flat_3, 1 \leq k_3 \leq \flat_1 - 1\}$. So, $|\varpi_7)| = (\flat_1 - 1)(\flat_3 - 1)$. The vertices adjacent to $x \in \varpi_7$ are of the form:

(i) $(0, v_1)$: $v_1 \in Z_{q^2} \setminus \{0\}$

(ii) $(u', 0)$: $u' = t_1\flat_1\flat_3\,; 1 \leq t_1 \leq \flat_2 - 1$

(iii) (u', v_2): $u' = t_1\flat_1\flat_3, v_2 = t_2 q\,; 1 \leq t_1 \leq \flat_2 - 1, 1 \leq t_2 \leq q - 1$

(iv) (u', v_3): $u' = t_1\flat_1\flat_3, v_3 \neq t_3 q\,; 1 \leq t_1 \leq \flat_2 - 1, 0 \leq t_3 \leq q - 1$

Hence, for any $x \in \varpi_7$, $d_x = q^2 - 1 + \flat_2 - 1 + (\flat_2 - 1)(q - 1) + (\flat_2 - 1)(q^2 - q) = \flat_2 q^2 - 1$.

(8) $\varpi_8 = \{(u, 0)\,; u = k_1\flat_3, 1 \leq k_1 \leq \flat_1\flat_2 - 1\,; u \neq k_2\flat_1\flat_3 1 \leq k_2 \leq \flat_2 - 1\,; u \neq k_3\flat_2\flat_3, 1 \leq k_3 \leq \flat_1 - 1\}$, with $|\varpi_8| = (\flat_1 - 1)(\flat_2 - 1)$. The vertices adjacent to $x \in \varpi_8$ are of the form:

(i) $(0, v_1)$: $v_1 \in Z_{q^2} \setminus \{0\}$

(ii) $(u', 0)$: $u' = t_1\flat_1\flat_2\,; 1 \leq t_1 \leq \flat_3 - 1$

(iii) (u', v_2): $u' = t_1\flat_1\flat_2, v_2 = t_2 q\,; 1 \leq t_1 \leq \flat_3 - 1, 1 \leq t_2 \leq q - 1$

(iv) (u', v_3): $u' = t_1\flat_1\flat_2, v_3 \neq t_3 q\,; 1 \leq t_1 \leq \flat_3 - 1, 0 \leq t_3 \leq q - 1$

Hence, for any $x \in \varpi_7$, $d_x = q^2 - 1 + \flat_3 - 1 + (\flat_3 - 1)(q - 1) + (\flat_3 - 1)(q^2 - q) = \flat_3 q^2 - 1$.

(9) $\varpi_9 = \{(u, 0)\,; u \neq k_1\flat_1, 1 \leq k_1 \leq \flat_2\flat_3 - 1\,; u \neq k_2\flat_2, 1 \leq k_2 \leq \flat_1\flat_3 - 1\,; u \neq k_3\flat_3, 1 \leq k_3 \leq \flat_1\flat_2 - 1\}$ with $|\varpi_9| = (\flat_1 - 1)(\flat_2 - 1)(\flat_3 - 1)$. In this case $d_x = q^2 - 1$ for all $x \in \varpi_9$

(10) $\varpi_{10} = \{(u, v) \mid u = k_1\flat_1\flat_2, 1 \leq k_1 \leq \flat_3 - 1\,; v \neq k_2 q, 0 \leq k_2 \leq q - 1\}$ with $|\varpi_{10}| = (\flat_3 - 1)(q^2 - q)$. In this case $d_x = \flat_1\flat_2 - 1$ for all $x \in \varpi_{10}$

(11) $\varpi_{11} = \{(u, v) \mid u = k_1\flat_1\flat_3, 1 \leq k_1 \leq \flat_2 - 1\,; v \neq k_2 q, 0 \leq k_2 \leq q - 1,\}$ with $|\varpi_{11}| = (\flat_2 - 1)(q^2 - q)$. In this case $d_x = \flat_1\flat_3 - 1$ for all $x \in \varpi_{11}$

(12) $\varpi_{12} = \{(u, v) \mid u = k_1\flat_1, 1 \leq k_1 \leq \flat_2\flat_3 - 1\,; u \neq k_2\flat_1\flat_2, 1 \leq k_2 \leq \flat_3 - 1\,; u \neq k_3\flat_1\flat_3, 1 \leq k_3 \leq \flat_2 - 1\,; v \neq k_4 q, 0 \leq k_4 \leq q - 1\}$ with $|\varpi_{12}| = (\flat_2 - 1)(\flat_3 - 1)(q^2 - q)$. The vertices adjacent to $x \in \varpi_{12}$ are of the form $(u', 0)\,; u' = t'\flat_2\flat_3, 1 \leq t' \leq \flat_1 - 1$. Hence $d_x = \flat_1 - 1$ for all $x \in \varpi_{12}$

(13) $\varpi_{13} = \{(u, v) \mid u = k_1\flat_2\flat_3, 1 \leq k_1 \leq \flat_1 - 1\,; v \neq k_2 q, 0 \leq k_2 \leq q - 1\}$ with $|\varpi_{13}| = (\flat_1 - 1)(q^2 - q)$. The vertices adjacent to $x \in \varpi_{12}$ are of the form $(u', 0)\,; u' = t'\flat_1, 1 \leq t' \leq \flat_2\flat_3 - 1$. Hence $d_x = \flat_2\flat_3 - 1$ for all $x \in \varpi_{13}$

(14) $\varpi_{14} = \{(u, v)\,; u = k_1\flat_2, 1 \leq k_1 \leq \flat_1\flat_3 - 1\,; u \neq k_2\flat_1\flat_2, 1 \leq k_2 \leq \flat_3 - 1\,; u \neq k_3\flat_2\flat_3, 1 \leq k_3 \leq \flat_1 - 1\,; v \neq k_4 q, 0 \leq k_4 \leq q - 1\}$ with $|\varpi_{14})| = (\flat_1 - 1)(\flat_3 - 1)(q^2 - q)$. Then, each vertex $x = \in \varpi_{14}$ has degree $\flat_2 - 1$

(15) $\varpi_{15} = \{(u, v) \mid u = k_1\flat_3, 1 \leq k_1 \leq \flat_1\flat_2 - 1\,; u \neq k_2\flat_1\flat_3 1 \leq k_2 \leq \flat_2 - 1\,; u \neq k_3\flat_2\flat_3, 1 \leq k_3 \leq \flat_1 - 1\,; v \neq k_4 q, 0 \leq k_4 \leq q - 1\}$, with $|\varpi_{15}| = (\flat_1 - 1)(\flat_2 - 1)(q^2 - q)$. The vertices adjacent to $x \in \varpi_{15}$ are of the form $(u', 0)\,; u' = t'\flat_1\flat_2, 1 \leq t' \leq \flat_3 - 1$. Then each vertex $x = \in \varpi_{15}$ has degree $\flat_3 - 1$

(16) $\varpi_{16} = \{(u, v) \mid u = k_1\flat_1\flat_2\,; 1 \leq k_1 \leq \flat_3 - 1\,; v = k_2 q, 1 \leq k_2 \leq q - 1\}$ with $|\varpi_{16}| = (\flat_3 - 1)(q - 1)$. The vertices adjacent to $x \in \varpi_{16}$ are of the form:

(i) $(0, v_1)$: $v_1 = t_1 q, 1 \leq t_1 \leq q - 1$

(ii) $(u', 0)$: $u' = t_2\flat_3\,; 1 \leq t_2 \leq \flat_1\flat_2 - 1$

(iii) (u', v_1): $u' = t_2\flat_3, v_1 = t_1 q\,; 1 \leq t_1 \leq q - 1, 1 \leq t_2 \leq \flat_1\flat_2 - 1$

Hence, for any $x \in \varpi_{16}$, $d_x = q - 1 + \flat_1\flat_2 - 1 + (\flat_1\flat_2 - 1)(q - 1) = \flat_1\flat_2 q - 1$.

(17) $\varpi_{17} = \{(u, v) \mid u = k_1\flat_1\flat_3, 1 \le k_1 \le \flat_2 - 1\,; v = k_2 q, 1 \le k_2 \le q - 1\}$ with $|\varpi_{17}| = \flat_2 - 1$. The vertices adjacent to $x \in \varpi_{17}$ are of the form:

(i) $(0, v_1)$: $v_1 = t_1 q, 1 \le t_1 \le q - 1$

(ii) $(u', 0)$: $u' = t_2\flat_2, 1 \le t_2 \le \flat_1\flat_3 - 1$

(iii) (u', v_1): $v_1 = t_1 q, u' = t_2\flat_2 : 1 \le t_1 \le q - 1, 1 \le t_2 \le \flat_1\flat_3 - 1$

Hence, for any $x \in \varpi_{17}$, $d_x = q - 1 + \flat_1\flat_3 - 1 + (\flat_1\flat_3 - 1)(q - 1) = \flat_1\flat_3 q - 1$.

(18) $\varpi_{18} = \{(u, v) \mid u = k_1\flat_1, 1 \le k_1 \le \flat_2\flat_3 - 1\,; u \ne k_2\flat_1\flat_2, 1 \le k_2 \le \flat_3 - 1\,; u \ne k_3\flat_1\flat_3, 1 \le k_3 \le \flat_2 - 1\,; v = k_4 q, 1 \le k_4 \le q - 1\}$ with $|\varpi_{18}| = (\flat_2 - 1)(\flat_3 - 1)(q - 1)$. The vertices adjacent to $x \in \varpi_{18}$ are of the form:

(i) (u', v'): $u' = t_1\flat_2\flat_3, 1 \le t_1 \le \flat_1 - 1\,; v' = t_2 q, 0 \le t_2 \le q - 1$

(ii) $(0, v_1)$: $v_1 = t_3 q, 1 \le t_3 \le q - 1$

Hence, for any $x \in \varpi_{18}$, $d_x = (\flat_1 - 1)(q^2 - q) + q - 1 = (q - 1)(\flat_1 q - q + 1)$.

(19) $\varpi_{19} = \{(u, v) \mid u = k_1\flat_2\flat_3, 1 \le k_1 \le \flat_1 - 1\,; v = k_2 q, 1 \le k_2 \le q - 1\}$ with $|\varpi_{19}| = (\flat_1 - 1)(q - 1)$. The vertices adjacent to $x \in \varpi_{19}$ are of the form:

(i) $(0, v_1)$: $v_1 = t_1 q, 1 \le t_1 \le q - 1$

(ii) $(u', 0)$: $u' = t_2\flat_1, 1 \le t_2 \le \flat_2\flat_3 - 1$

(iii) (u', v_1): $u' = t_2\flat_1, v_1 = t_1 q\,; 1 \le t_1 \le q - 1, 1 \le t_2 \le \flat_2\flat_3 - 1$

Hence, for any $x \in \varpi_{19}$, $d_x = q - 1 + \flat_2\flat_3 - 1 + (\flat_2\flat_3 - 1)(q - 1) = \flat_2\flat_3 q - 1$.

(20) $\varpi_{20} = \{(u, v) \mid u = k_1\flat_2, 1 \le k_1 \le \flat_1\flat_3 - 1\,; u \ne k_2\flat_1\flat_2, 1 \le k_2 \le \flat_3 - 1\,; u \ne k_3\flat_2\flat_3, 1 \le k_3 \le \flat_1 - 1\,; v = k_4 q, 1 \le k_4 \le q - 1\}$ with $|\varpi_{20})| = (\flat_1 - 1)(\flat_3 - 1)(q - 1)$. The vertices adjacent to $x \in \varpi_{20}$ are of the form:

(i) $(0, v_1)$: $v_1 = t_1 q, 1 \le t_1 \le q - 1$

(ii) $(u', 0)$: $u' = t_2\flat_1\flat_3\,; 1 \le t_2 \le \flat_2 - 1$

(iii) (u', v_1): $u' = t_2\flat_1\flat_3, v_1 = t_1 q\,; 1 \le t_1 \le q - 1, 1 \le t_2 \le \flat_2 - 1$

Hence, for any $x \in \varpi_{20}$, $d_x = q - 1 + \flat_2 - 1 + (\flat_2 - 1)(q - 1) = \flat_2 q - 1$.

(21) $\varpi_{21} = \{(u, v)\,; u = k_1\flat_3, 1 \le k_1 \le \flat_1\flat_2 - 1\,; u \ne k_2\flat_1\flat_3\ 1 \le k_2 \le \flat_2 - 1\,; u \ne k_3\flat_2\flat_3, 1 \le k_3 \le \flat_1 1\,; v = k_4 q, 1 \le k_4 \le q - 1\}$ with $|\varpi_{21}| = (\flat_1 - 1)(\flat_2 - 1)(q - 1)$. The vertices adjacent to $x \in \varpi_{21}$ are of the form:

(i) $(0, v_1)$: $v_1 = t_1 q, 1 \le t_1 \le q - 1$

(ii) $(u', 0)$: $u' = t_2\flat_1\flat_2, 1 \le t_2 \le \flat_3 - 1$

(iii) (u', v_1): $u' = t_2\flat_1\flat_2, v_1 = t_1 q\,; 1 \le t_1 \le q - 1, 1 \le t_2 \le \flat_3 - 1$

Hence, for any $x \in \varpi_{21}$, $d_x = q - 1 + \flat_3 - 1 + (\flat_3 - 1)(q - 1) = \flat_3 q - 1$.

(22) $\varpi_{22} = \{(u, v) \mid u \ne k_1\flat_1, 1 \le k_1 \le \flat_2\flat_3 - 1\,; u \ne k_2\flat_2, 1 \le k_2 \le \flat_1\flat_3 - 1\,; u \ne k_3\flat_3, 1 \le k_3 \le \flat_1\flat_2 - 1\,; v = k_4 q, 1 \le k_4 \le q - 1\}$. with $|\varpi_{22}| = (\flat_1 - 1)(\flat_2 - 1)(\flat_3 - 1)(q - 1)$. Then each vertex $x = \in\varpi_{22}$ has degree $q - 1$

Now, the cardinality of vertex set $V(J_R)$ can be calculated as $|V(J_R)| = (q^2 - q) + (q - 1) + (\flat_3 - 1) + (\flat_2 - 1) + (\flat_2 - 1)(\flat_3 - 1) + (\flat_1 - 1) + (\flat_1 - 1)(\flat_3 - 1) + (\flat_1 - 1)(\flat_2 - 1) + (\flat_1 - 1)(\flat_2 - 1)(\flat_3 - 1) + (\flat_3 - 1)(q^2 - q) + (\flat_2 - 1)(q^2 - q) + (\flat_2 - 1)(\flat_3 - 1)(q^2 - q) + (\flat_1 - 1)(q^2 - q) + (\flat_1 - 1)(\flat_3 - 1)(q^2 - q) + (\flat_1 - 1)(\flat_2 - 1)(q^2 - q) + (\flat_3 - 1)(q - 1) + (\flat_2 - 1) + (\flat_2 - 1)(\flat_3 - 1)(q - 1) + (\flat_1 - 1)(q - 1) + (\flat_1 - 1)(\flat_3 - 1)(q - 1) + (\flat_1 - 1)(\flat_2 - 1)(q - 1) + (\flat_1 - 1)(\flat_2 - 1)(\flat_3 - 1)(q - 1) = \flat_1\flat_2\flat_3 q + \flat_1\flat_3 q^2 + \flat_1\flat_2 q^2 + \flat_2\flat_3 q^2 + \flat_1 q + 2\flat_2 + \flat_3 q + q^2 - \flat_1 q^2 - \flat_2 q^2 - \flat_3 q^2 - \flat_1\flat_3 q - \flat_2\flat_3 q + {\flat_1}^2 q - 3$.

From the above Theorem, we can compute the number of edges $E(J_R)$ by using hand shaking lemma.

$$\begin{aligned} E(J_R) = {} & (q^2 - q)(\flat_1\flat_2\flat_3 - 1) + (q - 1)(\flat_1\flat_2\flat_3 q - q) \\ & + (\flat_3 - 1)(\flat_1\flat_2 q^2 - 1) + (\flat_2 - 1)(\flat_1\flat_3 q^2 - 1) \\ & + (\flat_2 - 1)(\flat_3 - 1)(\flat_1 q^2 - 1) + (\flat_1 - 1)(\flat_2\flat_3 q^2 - 1) \\ & + (\flat_1 - 1)(\flat_3 - 1)(\flat_2 q^2 - 1) + (\flat_1 - 1)(\flat_2 - 1) \\ & \cdot (\flat_3 q^2 - 1) + (\flat_1 - 1)(\flat_2 - 1)(\flat_3 - 1)(q^2 - 1) \\ & + (\flat_3 - 1)(q^2 - q)(\flat_1\flat_2 - 1) + (\flat_2 - 1)(q^2 - q) \\ & \cdot (\flat_1\flat_3 - 1) + (\flat_2 - 1)(\flat_3 - 1)(q^2 - q)(\flat_1 - 1) \\ & + (\flat_1 - 1)(q^2 - q)(\flat_2\flat_3 - 1) + (\flat_1 - 1)(\flat_3 - 1) \\ & \cdot (q^2 - q)(\flat_2 - 1) + (\flat_1 - 1)(\flat_2 - 1)(q^2 - q)(\flat_3 - 1) \\ & + (\flat_3 - 1)(q - 1)(\flat_1\flat_2 q - 1) + (\flat_2 - 1)(\flat_1\flat_3 q - 1) \\ & + (\flat_2 - 1)(\flat_3 - 1)(q - 1)(q - 1)(\flat_1 q - q + 1) \\ & + (\flat_1 - 1)(q - 1)(\flat_2\flat_3 q - 1) + (\flat_1 - 1)(\flat_3 - 1) \\ & \cdot (q - 1)(\flat_2 q - 1) + (\flat_1 - 1)(\flat_2 - 1)(q - 1)(\flat_3 q - 1) \\ & + (\flat_1 - 1)(\flat_2 - 1)(\flat_3 - 1)(q - 1)(q - 1). \\ = {} & 2 - q(-1 + q + q^2) + (-2 + q(-1 + q(3 + q)))\flat_2 \\ & + q(-2 + q(3 + q) - (-6 + q(9 + q))\flat_2)\flat_3 \\ & + q\flat_1(-2 + q(3 + q) - (-4 + q(8 + q))\flat_3 \\ & + \flat_2(6 - q(9 + q) + (-12 + q(18 + q))\flat_2)). \end{aligned} \tag{2}$$

Anderson and Livingston [30] proved that the diameter of the graph $J(R)$ is atmost three. It follows that the eccentricity of any vertex $x \in V(J(R))$ is atmost three. Figure 1 reflects this fact for the case $R = Z_{\flat_1\flat_2\flat_3} \times Z_{q^2}$. □

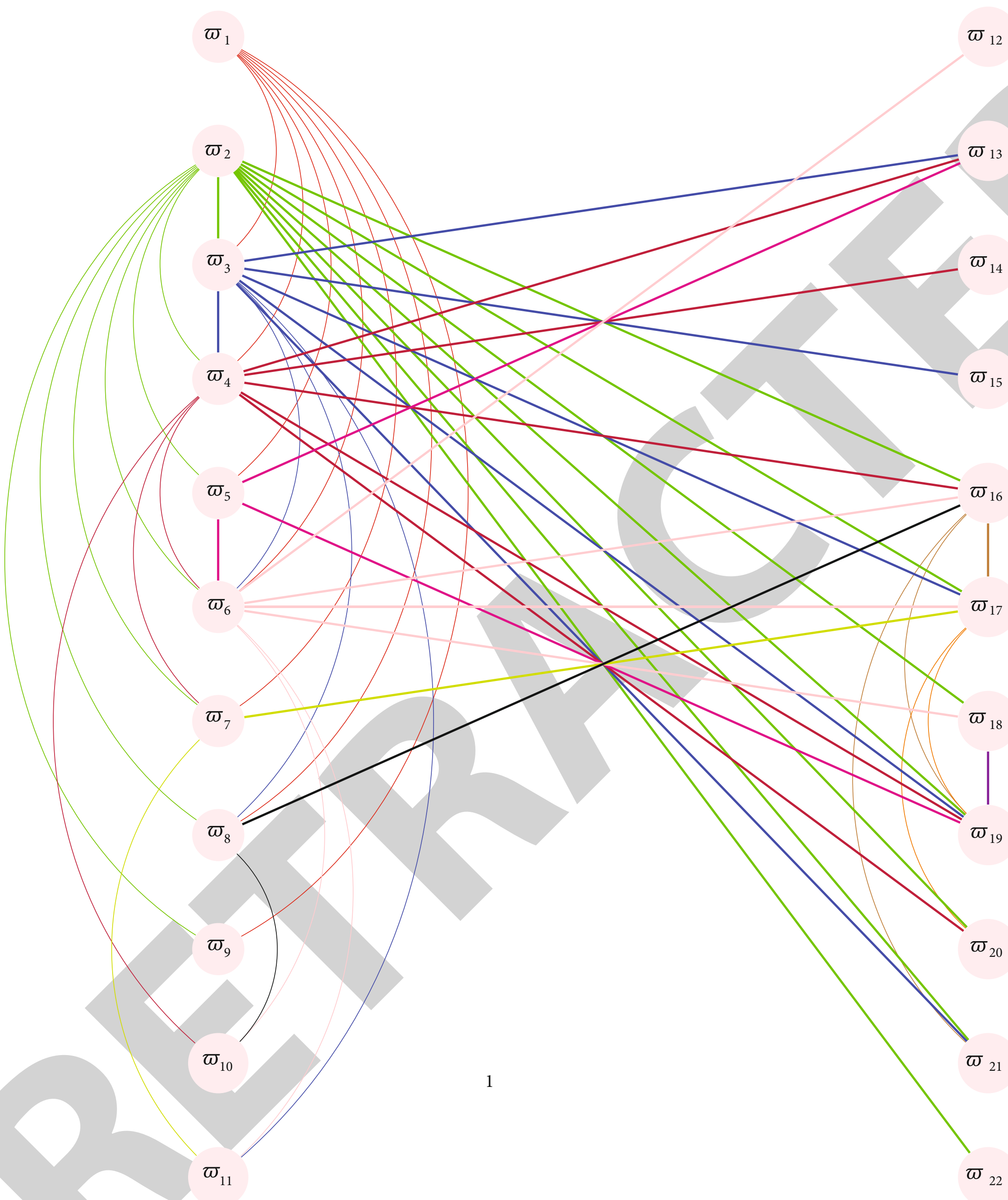

FIGURE 1: Structure of $Z_{\flat_1\flat_2\flat_3\times q^2}$.

Theorem 2 (see [30]). *Let $R = Z_{\flat_1\flat_2\flat_3} \times Z_{q^2}$, where $\flat_1, \flat_2, \flat_3$ be distinct prime integers and q is any prime number. Then, eccentricity of a vertex $x \in V(J(R))$ is either 2 or 3.*

In the next Theorem, we find the exact expressions for the ABC_5, H_4, GA_4, $M_1^* M_3^*$ indices of $R = Z_{\flat_1\flat_2\flat_3} \times Z_{q^2}$, where $\flat_1, \flat_2, \flat_3$ be distinct prime integers and q is any prime number. For simplicity, we fix some notations. Let $\alpha = \{6 - 3q + (-3+q)\flat_3 + \flat_2(-3+q+\flat_3) + \flat_1(-3+q+\flat_2+\flat_3)\}$,

$$\beta = -4 - (-2+q)q\left(1+q^2\right) + (-1+q)^2 q^2 \flat_1{}^2 (-1+\flat_2)^2 \cdot (-1+\flat_3)^3 + (-1+q)^2 q^2 \flat_2{}^2 (-1+\flat_3)^3, \quad (3)$$

$$\gamma = \flat_3\left(4 + q\left(-1 + 3(-1+q)^2 q\right) + (-1+q)^2 q^2 (-3+\flat_3)\flat_3\right) + \flat_2\left(2q\left(2+(-2+q)q^2\right) + \flat_3\left(-1 - 2q\left(1+3(-1+q)^2 q\right) - 2(-1+q)^2 q^2 (-3+\flat_3)\flat_3\right)\right), \quad (4)$$

$$\delta = \flat_1\left(3 + q\left(-1 + 2(-1+q)^2 q\right) - 2\flat_3 + q\left(-2(-1+q)^2 \cdot q\flat_2^{\,2}(-1+\flat_3)^3 + q\flat_3\left(-7 - 6(-2+q)q - 2(-1+q)^2 \cdot (-3+\flat_3)\flat_3\right) + \flat_2\left(-2 - 4(-1+q)^2 q + q\flat_3\left(3(5+4(-2+q)q) + 4(-1+q)^2(-3+\flat_3)\flat_3\right)\right)\right)\right), \tag{5}$$

$$\zeta = -1 - 2(-1+q)q + q(-2+3q)\flat_3 - \flat_2(-2 + (5-4q)q + (1 + 5(-1+q)q)\flat_3), \tag{6}$$

$$\mu = \flat_1(q(-2+3q) + (-1 - 5(-1+q)q)\flat_2 + \flat_3 + q(2 - 4q + (-5+6q)\flat_2)\flat_3). \tag{7}$$

Theorem 3. *Let $R = Z_{\flat_1\flat_2\flat_3} \times Z_{q^2}$, where $\flat_1, \flat_2, \flat_3$ be distinct prime integers, and q is any prime number. Then, $T(G)$ has the following expression $T(J_R) = \alpha\phi(2,2) + \{\beta + \gamma + \delta\}\phi(2,3) + \{\zeta + \mu\}\phi(3,3)$,*

Proof. Let

$$\Xi_{m,n} = \left\{u'v' \in E(J_R) \colon \varepsilon_{u'} = m, \varepsilon_{v'} = n\right\} \tag{8}$$

Then, from Theorem 1 and Figure 1, we have

$$\Xi_{2,2} = (q-1)(\flat_3-1) + (q-1)(\flat_2-1) + (q-1)(p-1) + (\flat_3-1)(\flat_2-1) + (\flat_3-1)(\flat_1-1) + (\flat_2-1)(\flat_1-1), \tag{9}$$

$$\begin{aligned}\Xi_{2,3} ={}& (q-1)(\flat_2-1)(\flat_3-1) + (q-1)(\flat_1-1)(\flat_3-1) + (q-1)(\flat_1-1)(\flat_2-1) + (q-1)(\flat_1-1)(\flat_2-1) \\ &\cdot (\flat_3-1) + (q-1)^2(\flat_3-1) + (q-1)(\flat_2-1) + (q-1)^2(\flat_2-1)(\flat_3-1) + (q-1)^2(\flat_1-1) \\ &+ (q-1)^2(\flat_1-1)(\flat_3-1) + (q-1)^2(\flat_1-1)(\flat_2-1) + (q-1)^2(\flat_1-1)(\flat_2-1)(\flat_3-1) + (\flat_3-1)\left(q^2-q\right) \\ &+ (\flat_1-1)(\flat_2-1)(\flat_3-1) + \left(q^2-q\right)(\flat_2-1)(\flat_3-1) + (\flat_1-1)(\flat_3-1)\left(q^2-q\right) + (\flat_1-1)(\flat_2-1)(\flat_3-1) \\ &\cdot \left(q^2-q\right) + (\flat_2-1)(\flat_3-1) + (\flat_1-1)(\flat_3-1)(q-1) + (\flat_1-1)(\flat_2-1)(\flat_3-1)(q-1) + (\flat_2-1)\left(q^2-q\right) \\ &+ (\flat_1-1)(\flat_2-1)(\flat_3-1) + (\flat_2-1)(\flat_3-1)\left(q^2-q\right) + (\flat_1-1)(\flat_2-1)\left(q^2-q\right) + (\flat_1-1)(\flat_2-1)(\flat_3-1) \\ &\cdot \left(q^2-q\right) + (\flat_2-1)(\flat_3-1)(q-1) + (\flat_1-1)(\flat_2-1) \cdot (q-1) + (\flat_1-1)(\flat_2-1)(\flat_3-1)(q-1) + (\flat_1-1) \\ &\cdot \left(q^2-q\right) + (\flat_1-1)(\flat_2-1)(\flat_3-1) + (\flat_1-1)(\flat_3-1) \cdot \left(q^2-q\right) + (\flat_1-1)(\flat_2-1)\left(q^2-q\right) + (\flat_1-1)(\flat_2-1) \\ &\cdot (\flat_3-1)\left(q^2-q\right) + (\flat_1-1)(\flat_3-1)(q-1) + (\flat_1-1) \cdot (\flat_2-1)(\flat_3-1)(q-1) + (\flat_1-1)(\flat_2-1)\end{aligned} \tag{10}$$

and

$$\begin{aligned}\Xi_{3,3} ={}& (\flat_2-1)(\flat_3-1)\left(q^2-q\right) + (\flat_1-1)(\flat_3-1)\left(q^2-q\right) + (\flat_1-1)(\flat_2-1)\left(q^2-q\right) + 4(\flat_1-1)(\flat_2-1)(\flat_3-1) \\ &\cdot \left(q^2-q\right) + 3(\flat_1-1)(\flat_2-1)(\flat_3-1)(q-1) + (\flat_1-1) \cdot (\flat_2-1)(\flat_3-1) + (\flat_2-1)(\flat_3-1)(q-1) + (\flat_1-1) \\ &\cdot (\flat_3-1)(q-1)^2 + 2(\flat_1-1)(\flat_2-1)(\flat_3-1)(q-1)^2 + (\flat_1-1)(\flat_2-1)(q-1).\end{aligned} \tag{11}$$

Finally,

$$\begin{aligned}\mathrm{T}(\mathrm{J_R}) ={}& \sum_{u'v' \in E(J_R)} \varnothing(\varepsilon_{u'}, \varepsilon_{v'}) = \sum_{u'v' \in \Xi_{2,2}} \varnothing(2,2) + \sum_{u'v' \in \Xi_{2,3}} \varnothing(2,3) + \sum_{u'v' \in \Xi_{3,3}} \varnothing(3,3) = \{(q-1)(\flat_3-1) + (q-1)(\flat_2-1) \\ &+ (q-1)(p-1) + (\flat_3-1)(\flat_2-1) + (\flat_3-1)(\flat_1-1) + (\flat_2-1)(\flat_1-1)\varnothing(2,2) + (q-1)(\flat_2-1)(\flat_3-1) \\ &+ (q-1)(\flat_1-1)(\flat_3-1) + (q-1)(\flat_1-1)(\flat_2-1) + (q-1)(\flat_1-1)(\flat_2-1)(\flat_3-1) + (q-1)^2(\flat_3-1) \\ &+ (q-1)(\flat_2-1) + (q-1)^2(\flat_2-1)(\flat_3-1) + (q-1)^2(\flat_1-1) + (q-1)^2(\flat_1-1)(\flat_3-1) \\ &+ (q-1)^2(\flat_1-1)(\flat_2-1) + (q-1)^2(\flat_1-1)(\flat_2-1)(\flat_3-1) + (\flat_3-1)\left(q^2-q\right) + (\flat_1-1)(\flat_2-1)(\flat_3-1) \\ &+ \left(q^2-q\right)(\flat_2-1)(\flat_3-1) + (\flat_1-1)(\flat_3-1)\left(q^2-q\right) + (\flat_1-1)(\flat_2-1)(\flat_3-1)\left(q^2-q\right) + (\flat_2-1)(\flat_3-1) \\ &+ (\flat_1-1)(\flat_3-1)(q-1) + (\flat_1-1)(\flat_2-1)(\flat_3-1)(q-1) + (\flat_2-1)\left(q^2-q\right) + (\flat_1-1)(\flat_2-1)(\flat_3-1) \\ &+ (\flat_2-1)(\flat_3-1)\left(q^2-q\right) + (\flat_1-1)(\flat_2-1)\left(q^2-q\right) + (\flat_1-1)(\flat_2-1)(\flat_3-1)\left(q^2-q\right) + (\flat_2-1)(\flat_3-1)(q-1) \\ &+ (\flat_1-1)(\flat_2-1)(q-1) + (\flat_1-1)(\flat_2-1)(\flat_3-1)(q-1) + (\flat_1-1)\left(q^2-q\right) + (\flat_1-1)(\flat_2-1)(\flat_3-1) \\ &+ (\flat_1-1)(\flat_3-1)\left(q^2-q\right) + (\flat_1-1)(\flat_2-1)\left(q^2-q\right) + (\flat_1-1)(\flat_2-1)(\flat_3-1)\left(q^2-q\right) + (\flat_1-1)(\flat_3-1)(q-1) \\ &+ (\flat_1-1)(\flat_2-1) + (\flat_1-1)(\flat_2-1)(\flat_3-1)(q-1)\}\varnothing(2,3) \\ &+ \{(\flat_2-1)(\flat_3-1)\left(q^2-q\right) + (\flat_1-1)(\flat_3-1)\left(q^2-q\right) + (\flat_1-1)(\flat_2-1)\left(q^2-q\right) + 4(\flat_1-1)(\flat_2-1)(\flat_3-1)\left(q^2-q\right) \\ &+ 3(\flat_1-1)(\flat_2-1)(\flat_3-1)(q-1) + (\flat_1-1)(\flat_2-1)(\flat_3-1) + (\flat_2-1)(\flat_3-1)(q-1) + (\flat_1-1)(\flat_3-1)(q-1)^2 \\ &+ 2(\flat_1-1)(\flat_2-1)(\flat_3-1)(q-1)^2 + (\flat_1-1)(\flat_2-1)(q-1)\}\varnothing(3,3). = \{6 - 3q + (-3+q)\flat_3 \\ &+ \flat_2(-3+q+\flat_3) + \flat_1(-3+q+\flat_2+\flat_3)\}\phi(2,2) \\ &+ \left\{-4 - (-2+q)q\left(1+q^2\right) + (-1+q)^2q^2\flat_1^{\,2}(-1+\flat_2)^2(-1+\flat_3)^3 + (-1+q)^2q^2\flat_2^{\,2}(-1+\flat_3)^3 + \flat_3\left(4 + q\left(-1+3(-1+q)^2q\right)\right.\right. \\ &\left. + (-1+q)^2q^2(-3+\flat_3)\flat_3\right) + \flat_2\left(2q\left(2+(-2+q)q^2\right) + \flat_3\left(-1 - 2q\left(1+3(-1+q)^2q\right) - 2(-1+q)^2q^2(-3+\flat_3)\flat_3\right)\right) \\ &+ \flat_1\left(3 + q\left(-1+2(-1+q)^2q\right) - 2\flat_3 + q\left(-2(-1+q)^2q\flat_2^{\,2}(-1+\flat_3)^3 + q\flat_3\left(-7 - 6(-2+q)q - 2(-1+q)^2(-3+\flat_3)\flat_3\right)\right.\right. \\ &\left.\left.\left. + \flat_2\left(-2 - 4(-1+q)^2q + q\flat_3(3(5+4(-2+q)q) + 4(-1+q)^2(-3+\flat_3)\flat_3)\right)\right)\right)\right\}\phi(2,3) + \{-1 - 2(-1+q)q \\ &+ q(-2+3q)\flat_3 - \flat_2(-2+(5-4q)q+(1+5(-1+q)q)\flat_3) + \flat_1(q(-2+3q) + (-1-5(-1+q)q)\flat_2 + \flat_3 + q(2-4q \\ &+ (-5+6q)\flat_2)\flat_3)\}\phi(3,3), = \alpha\phi(2,2) + \{\beta+\gamma+\delta\}\phi(2,3) + \{\zeta+\mu\}\phi(3,3).\end{aligned} \tag{12}$$

□

Theorem 4. *Let $R = Z_{\flat_1\flat_2\flat_3} \times Z_{q^2}$, where $\flat_1, \flat_2, \flat_3$ be distinct prime integers and q is any prime number. Then*

$$M_1^*(J_R) = 4\alpha + \{\beta+\gamma+\delta\}5 + \{\zeta+\mu\}6, \tag{13}$$

$$M_3^*(J_R) = 4\alpha + \{\beta+\gamma+\delta\}6 + \{\zeta+\mu\}9, \tag{14}$$

$$GA_4(J_R) = \alpha + \{\beta+\gamma+\delta\}\frac{2\sqrt{6}}{5} + \{\zeta+\mu\}, \tag{15}$$

$$ABC_5(J_R) = \frac{\alpha}{\sqrt{2}} + \{\beta+\gamma+\delta\}\frac{1}{\sqrt{2}} + \{\zeta+\mu\}\frac{2}{3}, \tag{16}$$

$$H_4(J_R) = \frac{\alpha}{2} + \{\beta+\gamma+\delta\}\frac{2}{5} + \{\zeta+\mu\}\frac{1}{3}. \tag{17}$$

Proof. For the first Zagreb eccentricity index, we have $\varnothing(\varepsilon_{u'}, \varepsilon_{v'}) = \varepsilon_{u'} + \varepsilon_{v'}$. Then, $\varnothing(2,2)=4$, $\varnothing(2,3)=5$, and $\varnothing(3,3)=6$. Substituting these values in Theorem 3, we get

$$\begin{aligned}
M_1^*(J_R) = {} & \{6 - 3q + (-3+q)\flat_3 + \flat_2(-3+q+\flat_3) \\
& + \flat_1(-3+q+\flat_2+\flat_3)\}\phi(2,2) \\
& +\{-4-(-2+q)q(1+q^2) \\
& + (-1+q)^2q^2\flat_1{}^2(-1+\flat_2)^2(-1+\flat_3)^3 \\
& + (-1+q)^2q^2\flat_2{}^2(-1+\flat_3)^3 \\
& + \flat_3(4+q(-1+3(-1+q(-1+\flat_3)^2q) \\
& + (-1+q)^2q^2(-3+\flat_3)\flat_3) \\
& + \flat_2(2q(2+(-2+q)q^2) \\
& + \flat_3(-1-2q(1+3(-1+q)^2q) \\
& - 2(-1+q)^2q^2(-3+\flat_3)\flat_3)) \\
& + \flat_1(3+q(-1+2(-1+q)^2q) \\
& - 2\flat_3 + q - 2(-1+q)^2q\flat_2{}^2(-1+\flat_3)^3 \\
& + q\flat_3(-7-6(-2+q)q \\
& - 2(-1+q)^2(-3+\flat_3)\flat_3) \\
& + \flat_2((-2-4(-1+q)^2q \\
& + q\flat_3(3(5+4(-2+q)q) \\
& + 4(-1+q)^2(-3+\flat_3)\flat_3)))))\}\phi(2,3) \\
& +\{-1-2(-1+q)q+q(-2+3q)\flat_3 \\
& - \flat_2(-2+(5-4q)q+(1+5(-1+q)q)\flat_3) \\
& + \flat_1(q(-2+3q)+(-1-5(-1+q)q)\flat_2 \\
& + \flat_3 + q(2-4q+(-5+6q)\flat_2)\flat_3)\}\phi(3,3). \\
= {} & \{6+\flat_3(-3+q)-3q+\flat_2(-3+\flat_3+q) \\
& + \flat_1(-3+\flat_2+\flat_3+q)\}4 \\
& +\{-4+\flat_1{}^2(-1+\flat_2)^2(-1+\flat_3)^3(-1+q)^2q^2 \\
& + \flat_2{}^2(-1+\flat_3)^3(-1+q)^2q^2 - (-2+q)q(1+q^2) \\
& + \flat_3(4+(-3+\flat_3)\flat_3(-1+q)^2q^2 \\
& + q(-1+3(-1+q)^2q)) \\
& + \flat_2(2q(2+(-2+q)q^2) \\
& + \flat_3(-1-2(-3+\flat_3)\flat_3(-1+q)^2q^2 \\
& - 2q(1+3(-1+q)^2q))) \\
& + \flat_1(3-2\flat_3+q(-1+2(-1+q)^2q) \\
& + q(-2\flat_2{}^2(-1+\flat_3)^3(-1+q)^2q \\
& + \flat_3q(-7-2(-3+\flat_3)\flat_3(-1+q)^2 \\
& - 6(-2+q)q) + \flat_2(-2-4(-1+q)^2q \\
& + \flat_3q(4(-3+\flat_3)\flat_3(-1+q)^2 \\
& + 3(5+4(-2+q)q))))))\}5 \\
& +\{-1-2(-1+q)q+\flat_3q(-2+3q) \\
& - \flat_2(-2+(5-4q)q+\flat_3(1+5(-1+q)q)) \\
& + \flat_1(\flat_3+q(-2+3q)+\flat_2(-1-5(-1+q)q) \\
& + \flat_3q(2-4q+\flat_2(-5+6q)))\}6, = 4\alpha \\
& + \{\beta+\gamma+\delta\}5 + \{\zeta+\mu\}6.
\end{aligned} \tag{18}$$

□

For third Zagreb eccentricity index, we have $\varnothing(\varepsilon_{u'}, \varepsilon_{v'}) = \varepsilon_{u'} \times \varepsilon_{v'}$ and

$$\begin{aligned}
M_3^* = {} & \{6-3q+(-3+q)\flat_3+\flat_2(-3+q+\flat_3) \\
& + \flat_1(-3+q+\flat_2+\flat_3)\}\phi(2,2) \\
& +\{-4-(-2+q)q(1+q^2) \\
& + (-1+q)^2q^2\flat_1{}^2(-1+\flat_2)^2(-1+\flat_3)^3 \\
& + (-1+q)^2q^2\flat_2{}^2(-1+\flat_3)^3 \\
& + \flat_3(4+q(-1+3(-1+q)^2q) \\
& + (-1+q)^2q^2(-3+\flat_3)\flat_3) \\
& + \flat_2(2q(2+(-2+q)q^2) \\
& + \flat_3(-1-2q(1+3(-1+q)^2q) \\
& - 2(-1+q)^2q^2(-3+\flat_3)\flat_3)) \\
& + \flat_1(3+q(-1+2(-1+q)^2q) \\
& - 2\flat_3 + q(-2(-1+q)^2q\flat_2{}^2(-1+\flat_3)^3 \\
& + q\flat_3(-7-6(-2+q)q \\
& - 2(-1+q)^2(-3+\flat_3)\flat_3) \\
& + \flat_2(-2-4(-1+q)^2q \\
& + q\flat_3(3(5+4(-2+q)q) \\
& + 4(-1+q)^2(-3+\flat_3)\flat_3)))))\}\phi(2,3) \\
& + \{-1-2(-1+q)q+q(-2+3q)\flat_3
\end{aligned}$$

$$
\begin{aligned}
&- \flat_2(-2 + (5 - 4q)q + (1 + 5(-1 + q)q)\flat_3) \\
&+ \flat_1(q(-2 + 3q) + (-1 - 5(-1 + q)q)\flat_2 \\
&+ \flat_3 + q(2 - 4q + (-5 + 6q)\flat_2)\flat_3)\}\phi(3, 3). \\
&= \{6 + \flat_3(-3 + q) - 3q + \flat_2(-3 + \flat_3 + q) \\
&+ \flat_1(-3 + \flat_2 + \flat_3 + q)\}4 \\
&+ \{-4 + {\flat_1}^2(-1 + \flat_2)^2(-1 + \flat_3)^3 \\
&\cdot(-1 + q)^2q^2 + {\flat_2}^2(-1 + \flat_3)^3 \\
&\cdot(-1 + q)^2q^2 - (-2 + q)q(1 + q^2) \\
&+ \flat_3(4 + (-3 + \flat_3)\flat_3(-1 + q)^2q^2 \\
&+ q(-1 + 3(-1 + q)^2q)) \\
&+ \flat_2(2q(2 + (-2 + q)q^2) \\
&+ \flat_3(-1 - 2(-3 + \flat_3)\flat_3(-1 + q)^2q^2 \\
&- 2q(1 + 3(-1 + q)^2q))) \\
&+ \flat_1(3 - 2\flat_3 + q(-1 + 2(-1 + q)^2q) \\
&+ q(-2{\flat_2}^2(-1 + \flat_3)^3(-1 + q)^2q \\
&+ \flat_3q(-7 - 2(-3 + \flat_3)\flat_3(-1 + q)^2 \\
&- 6(-2 + q)q) + \flat_2(-2 - 4(-1 + q)^2q \\
&+\flat_3q(4(-3 + \flat_3)\flat_3(-1 + q)^2 \\
&+ 3(5 + 4(-2 + q)q))))))\}6 \\
&+ \{-1 - 2(-1 + q)q + \flat_3q(-2 + 3q) \\
&- \flat_2(-2 + (5 - 4q)q + \flat_3(1 + 5(-1 + q)q)) \\
&+ \flat_1(\flat_3 + q(-2 + 3q) + \flat_2(-1 - 5(-1 + q)q) \\
&+ \flat_3q(2 - 4q + \flat_2(-5 + 6q)))\}9, \\
&= 4\alpha + \{\beta + \gamma + \delta\}6 + \{\zeta + \mu\}9. \qquad (19)
\end{aligned}
$$

For the geometric-arithmetic eccentricity index, we have $\varnothing(\varepsilon_{u'}, \varepsilon_{v'}) = 2\sqrt{\varepsilon_{u'} \times \varepsilon_{v'}}/\varepsilon_{u'} + \varepsilon_{v'}$ and

$$
\begin{aligned}
GA_4(J_R) = &\{6 - 3q + (-3 + q)\flat_3 + \flat_2(-3 + q + \flat_3) \\
&+ \flat_1(-3 + q + \flat_2 + \flat_3)\}\phi(2, 2) \\
&+\{-4 - (-2 + q)q(1 + q^2) \\
&+ (-1 + q)^2q^2{\flat_1}^2(-1 + \flat_2)^2(-1 + \flat_3)^3 \\
&+ (-1 + q)^2q^2{\flat_2}^2(-1 + \flat_3)^3 \\
&+ \flat_3(4 + q(-1 + 3(-1 + q)^2q) \\
&+ (-1 + q)^2q^2(-3 + \flat_3)\flat_3) \\
&+ \flat_2(2q(2 + (-2 + q)q^2) \\
&+\flat_3(-1 - 2q(1 + 3(-1 + q)^2q) \\
&- 2(-1 + q)^2q^2(-3 + \flat_3)\flat_3)) \\
&+ \flat_1(3 + q(-1 + 2(-1 + q)^2q) \\
&- 2\flat_3 + q(-2(-1 + q)^2q{\flat_2}^2(-1 + \flat_3)^3 \\
&+ q\flat_3(-7 - 6(-2 + q)q \\
&- 2(-1 + q)^2(-3 + \flat_3)\flat_3) \\
&+ \flat_2(-2 - 4(-1 + q)^2q \\
&+ q\flat_3(3(5 + 4(-2 + q)q) \\
&+4(-1 + q)^2(-3 + \flat_3)\flat_3))))\}\phi(2, 3) \\
&+ \{-1 - 2(-1 + q)q + q(-2 + 3q)\flat_3 \\
&- \flat_2(-2 + (5 - 4q)q + (1 + 5(-1 + q)q)\flat_3) \\
&+ \flat_1(q(-2 + 3q) + (-1 - 5(-1 + q)q)\flat_2 \\
&+ \flat_3 + q(2 - 4q + (-5 + 6q)\flat_2)\flat_3)\}\phi(3, 3). \\
= &5 + \flat_3(-3 + q) - 3q - 2(-1 + q)q \\
&+ \flat_2(-3 + \flat_3 + q) + \flat_1(-3 + \flat_2 + \flat_3 + q) \qquad (20) \\
&+ \{-4 + {\flat_1}^2(-1 + \flat_2)^2(-1 + \flat_3)^3(-1 + q)^2q^2 \\
&+ {\flat_2}^2(-1 + \flat_3)^3(-1 + q)^2q^2 - (-2 + q)q(1 + q^2) \\
&+ \flat_3(4 + (-3 + \flat_3)\flat_3(-1 + q)^2q^2 \\
&+ q(-1 + 3(-1 + q)^2q)) \\
&+ \flat_2(2q(2 + (-2 + q)q^2) \\
&+ \flat_3(-1 - 2(-3 + \flat_3)\flat_3(-1 + q)^2q^2 \\
&- 2q(1 + 3(-1 + q)^2q))) \\
&+ \flat_1(3 - 2\flat_3 + q(-1 + 2(-1 + q)^2q) \\
&+ q(-2{\flat_2}^2(-1 + \flat_3)^3(-1 + q)^2q \\
&+ \flat_3q(-7 - 2(-3 + \flat_3)\flat_3(-1 + q)^2 \\
&- 6(-2 + q)q) + \flat_2(-2 - 4(-1 + q)^2q \\
&+\flat_3q(4(-3 + \flat_3)\flat_3(-1 + q)^2 \\
&+ 3(5 + 4(-2 + q)q))))))\}\frac{2\sqrt{6}}{5} \\
&+ \flat_3q(-2 + 3q) - \flat_2(-2 + (5 - 4q)q \\
&+ \flat_3(1 + 5(-1 + q)q)) + \flat_1(\flat_3 + q(-2 + 3q) \\
&+ \flat_2(-1 - 5(-1 + q)q) \\
&+ \flat_3q(2 - 4q + \flat_2(-5 + 6q))), \\
= &\alpha + \{\beta + \gamma + \delta\}\frac{2\sqrt{6}}{5} + \{\zeta + \mu\}.
\end{aligned}
$$

For atom-bond connectivity eccentricity index, we have $\varnothing(\varepsilon_{u'}, \varepsilon_{v'}) = \sqrt{\varepsilon_{u'} + \varepsilon_{v'} - 2/\varepsilon_{u'} \times \varepsilon_{v'}}$ and

$$
\begin{aligned}
\mathrm{ABC}_5(J_R) = {} & \{6 - 3q + (-3+q)\flat_3 + \flat_2(-3+q+\flat_3) \\
& + \flat_1(-3+q+\flat_2+\flat_3)\}\phi(2,2) \\
& +\{-4 - (-2+q)q(1+q^2) + (-1+q)^2 q^2 \flat_1{}^2(-1+\flat_2)^2(-1+\flat_3)^3 \\
& + (-1+q)^2 q^2 \flat_2{}^2(-1+\flat_3)^3 + \flat_3(4 + q(-1+3(-1+q)^2 q) \\
& + (-1+q)^2 q^2(-3+\flat_3)\flat_3) + \flat_2(2q(2+(-2+q)q^2) \\
& + \flat_3(-1 - 2q(1+3(-1+q)^2 q) - 2(-1+q)^2 q^2(-3+\flat_3)\flat_3)) \\
& + \flat_1(3 + q(-1+2(-1+q)^2 q) \\
& - 2\flat_3 + q(-2(-1+q)^2 q\flat_2{}^2(-1+\flat_3)^3 \\
& + q\flat_3(-7 - 6(-2+q)q - 2(-1+q)^2 \\
& \cdot(-3+\flat_3)\flat_3) + \flat_2(-2 - 4(-1+q)^2 q \\
& +q\flat_3(3(5+4(-2+q)q) + 4(-1+q)^2 \\
& \cdot(-3+\flat_3)\flat_3)))))\}\phi(2,3) + \{-1 - 2(-1+q)q + q(-2+3q)\flat_3 \\
& - \flat_2(-2 + (5-4q)q + (1+5(-1+q)q)\flat_3) \\
& + \flat_1(q(-2+3q) + (-1-5(-1+q)q)\flat_2 \\
& + \flat_3 + q(2 - 4q + (-5+6q)\flat_2)\flat_3)\}\phi(3,3). \\
= {} & \frac{6 + \flat_3(-3+q) - 3q + \flat_2(-3 + \flat\flat_3 + q) + \flat_1(-3+\flat_2+\flat_3+q)}{\sqrt{2}} \\
& + \{-4 + \flat_1{}^2(-1+\flat_2)^2(-1+\flat_3)^3(-1+q)^2 q^2 \\
& + \flat_2{}^2(-1+\flat_3)^3(-1+q)^2 q^2 - (-2+q)q(1+q^2) \\
& + \flat_3(4 + (-3+\flat_3)\flat_3(-1+q)^2 q^2 + q(-1+3(-1+q)^2 q)) \\
& + \flat_2(2q(2+(-2+q)q^2) + \flat_3(-1 - 2(-3+\flat_3)\flat_3(-1+q)^2 q^2 \\
& - 2q(1+3(-1+q)^2 q))) + \flat_1(3 - 2\flat_3 + q(-1+2(-1+q)^2 q) \\
& + q(-2\flat_2{}^2(-1+\flat_3)^3(-1+q)^2 q \\
& + \flat_3 q(-7 - 2(-3+\flat_3)\flat_3(-1+q)^2 - 6(-2+q)q) \\
& + \flat_2(-2 - 4(-1+q)^2 q + \flat_3 q(4(-3+\flat_3)\flat_3(-1+q)^2 \\
& + 3(5+4(-2+q)q))))))\}\frac{1}{\sqrt{2}} + \{-1 - 2(-1+q)q \\
& + \flat_3 q(-2+3q) - \flat_2(-2 + (5-4q)q \\
& + \flat_3(1+5(-1+q)q)) + \flat_1(\flat_3 + q(-2+3q) \\
& + \flat_2(-1-5(-1+q)q) + \flat_3 q(2 - 4q + \flat_2(-5+6q)))\}\frac{2}{3}, \\
= {} & \frac{\alpha}{\sqrt{2}} + \{\beta+\gamma+\delta\}\frac{1}{\sqrt{2}} + \{\zeta+\mu\}\frac{2}{3}.
\end{aligned}
\tag{21}
$$

For the harmonic index based on eccentricity of fourth type, we have $\varnothing(\varepsilon_{u'}, \varepsilon_{v'}) = 2/\varepsilon_{u'} + \varepsilon_{v'}$ and

$$
\begin{aligned}
H_4(J_R) = {} & \{6 - 3q + (-3+q)\flat_3 + \flat_2(-3+q+\flat_3) \\
& + \flat_1(-3+q+\flat_2+\flat_3)\}\phi(2,2) \\
& +\{-4 - (-2+q)q(1+q^2) + (-1+q)^2 q^2 \flat_1{}^2 \\
& \cdot(-1+\flat_2)^2(-1+\flat_3)^3 + (-1+q)^2 q^2 \flat_2{}^2(-1+\flat_3)^3 \\
& + \flat_3(4 + q(-1+3(-1+q)^2 q) + (-1+q)^2 q^2(-3+\flat_3)\flat_3) \\
& + \flat_2(2q(2+(-2+q)q^2) + \flat_3(-1 - 2q(1+3(-1+q)^2 q) \\
& - 2(-1+q)^2 q^2(-3+\flat_3)\flat_3)) + \flat_1(3 + q(-1+2(-1+q)^2 q) \\
& - 2\flat_3 + q(-2(-1+q)^2 q\flat_2{}^2(-1+\flat_3)^3 \\
& + q\flat_3(-7 - 6(-2+q)q - 2(-1+q)^2 \\
& \cdot(-3+\flat_3)\flat_3) + \flat_2(-2 - 4(-1+q)^2 q + q\flat_3(3(5+4(-2+q)q) \\
& + 4(-1+q)^2(-3+\flat_3)\flat_3)))))\}\phi(2,3) \\
& + \{-1 - 2(-1+q)q + q(-2+3q)\flat_3 \\
& - \flat_2(-2 + (5-4q)q + (1+5(-1+q)q)\flat_3) \\
& + \flat_1(q(-2+3q) + (-1-5(-1+q)q)\flat_2 \\
& + \flat_3 + q(2 - 4q + (-5+6q)\flat_2)\flat_3)\}\phi(3,3). \\
= {} & \frac{6 + \flat_3(-3+q) - 3q + \flat_2(-3+\flat_3+q) + \flat_1(-3+\flat_2+\flat_3+q)}{2} \\
& + \{-4 + \flat_1{}^2(-1+\flat_2)^2(-1+\flat_3)^3(-1+q)^2 q^2 \\
& + \flat_2{}^2(-1+\flat_3)^3(-1+q)^2 q^2 - (-2+q)q(1+q^2) \\
& + \flat_3(4 + (-3+\flat_3)\flat_3(-1+q)^2 q^2 \\
& + q(-1+3(-1+q)^2 q)) + \flat_2(2q(2+(-2+q)q^2) \\
& + \flat_3(-1 - 2(-3+\flat_3)\flat_3(-1+q)^2 q^2 \\
& - 2q(1+3(-1+q)^2 q))) \\
& + \flat_1(3 - 2\flat_3 + q(-1+2(-1+q)^2 q) \\
& + q(-2\flat_2{}^2(-1+\flat_3)^3(-1+q)^2 q \\
& + \flat_3 q(-7 - 2(-3+\flat_3)\flat_3(-1+q)^2 - 6(-2+q)q) \\
& + \flat_2(-2 - 4(-1+q)^2 q + \flat_3 q(4(-3+\flat_3)\flat_3(-1+q)^2 \\
& + 3(5+4(-2+q)q))))))\}\frac{2}{5} \\
& + \{-1 - 2(-1+q)q + \flat_3 q(-2+3q) \\
& - \flat_2(-2 + (5-4q)q + \flat_3(1+5(-1+q)q)) \\
& + \flat_1(\flat_3 + q(-2+3q) + \flat_2(-1-5(-1+q)q) \\
& + \flat_3 q(2 - 4q + \flat_2(-5+6q)))\}\frac{1}{3}, \\
= {} & \frac{\alpha}{2} + \{\beta+\gamma+\delta\}\frac{2}{5} + \{\zeta+\mu\}\frac{1}{3}.
\end{aligned}
\tag{22}
$$

4. Conclusion

For zero divisor graph of commutative ring $Z_{\flat_1\flat_2\flat_3} \times Z_{q^2}$, we calculated the eccentricity-based atom-bond index of connectivity, the harmonic index based on eccentricity of fourth type, the geometric-arithmetic eccentricity index, the eccentricity-based third Zagreb index, and the eccentricity-based first Zagreb index. This work is a part of the open problem to calculate the eccentricity based topological indices for zero divisor graph of commutative ring $Z_m \times Z_n$ for $m, n \in \mathbb{Z}$. In future, similar work can be done for other cases of commutative ring $Z_m \times Z_n$.

Data Availability

No data is required to support the study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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References

[1] A. Ahmed and A. Haider, "Computing the radio labeling associated with zero divisor graph of a commutative ring," *U.P. B. Scientific Bulletin, Series A*, vol. 81, no. 1, pp. 65–72, 2019.

[2] I. Beck, "Coloring of commutative rings," *Journal of Algebra*, vol. 116, no. 1, pp. 208–226, 1988.

[3] M. Baca, J. Horváthová, M. Mokrišová, and A. Suhányiová, "On topological indices of fullerenes," *Applied Mathematics and Computation*, vol. 251, pp. 154–161, 2015.

[4] I. Gutman and O. E. Polansky, *Mathematical Concepts in Organic Chemistry*, Springer-Verlag, New York, NY, USA, 1986.

[5] A. Ahmad, "On the degree-based topological indices of benzene ring embedded in P-type-surface in 2D network," *Hacettepe Journal of Mathematics and Statistics*, vol. 47, pp. 9–18, 2018.

[6] A. Ahmad, "Computation of certain topological properties of para-line graph of honeycomb networks and graphene," *Discrete Mathematics, Algorithms and Applications*, vol. 9, no. 5, article 1750064, 2017.

[7] S. Akhter, W. Gao, M. Imran, and M. R. Farahani, "On topological indices of honeycomb networks and graphene networks," *Hacettepe Journal of Mathematics and Statistics*, vol. 47, pp. 19–35, 2018.

[8] M. F. Nadeem, S. Zafar, and Z. Zahid, "On topological properties of the line graphs of subdivision graphs of certain nanostructures," *Applied Mathematics and Computation*, vol. 273, pp. 125–130, 2016.

[9] S. Wang, M. R. Farahani, M. R. R. Kanna, M. K. Jamil, and R. P. Kumar, "The Wiener index and the Hosoya polynomial of the Jahangir graphs," *Applied and Computational Mathematics*, vol. 5, no. 3, pp. 138–141, 2016.

[10] H. Wiener, "Structural determination of paraffin boiling points," *Journal of the American Chemical Society*, vol. 69, no. 1, pp. 17–20, 1947.

[11] M. R. Farahani, "Eccentricity version of atom bond connectivity index of benzenoid family ABC5(Hk)," *World Applied Sciences Journal*, vol. 21, pp. 1260–1265, 2013.

[12] M. R. Farahani, S. Ediz, and M. Imran, "On novel harmonic indices of certain nanotubes," *International Journal of Advanced Biotechnology and Research*, vol. 8, pp. 277–282, 2017.

[13] Y. Gao, S. Ediz, M. R. Farahani, and M. Imran, "On the second harmonic index of titania nanotubes," *Drug Designing & Intellectual Properties International Journal*, vol. 1, pp. 7–10, 2018.

[14] M. Ghorbani and A. Khaki, "A note on the fourth version of the geometric-arithmetic index," *Optoelectronics and Advanced Materials-Rapid Communications*, vol. 4, pp. 2212–2215, 2010.

[15] M. Ghorbani and M. A. Hosseinzadeh, "A new version of Zagreb indices," *Univerzitet u Nišu*, vol. 26, no. 1, pp. 93–100, 2012.

[16] D. Vukičević and A. Graovac, "Note on the comparison of the first and second normalized Zagreb eccentricity indices," *Acta Chimica Slovenica*, vol. 57, pp. 524–528, 2010.

[17] E. Estrada, L. Torres, L. Rodriguez, and I. Gutman, "An atom-bond connectivity index: modeling the enthalpy of formation of alkanes," *Indian Journal of Chemistry*, vol. 37A, pp. 849–855, 1998.

[18] M. K. Siddiqui, M. Naeem, N. A. Rehman, and M. Imran, "Computing topological indices of certain networks," *Journal of Optoelectronics and Advanced Materials*, vol. 18, pp. 884–892, 2016.

[19] Z. Shao, P. Wu, Y. Gao, I. Gutman, and X. Zhang, "On the maximum *ABC* index of graphs without pendent vertices," *Applied Mathematics and Computation*, vol. 315, pp. 298–312, 2017.

[20] M. K. Siddiqui, M. Imran, and A. Ahmad, "On Zagreb indices, Zagreb polynomials of some nanostar dendrimers," *Applied Mathematics and Computation*, vol. 280, pp. 132–139, 2016.

[21] Z. Shao, M. K. Siddiqui, and M. H. Muhammad, "Computing Zagreb indices and Zagreb polynomials for symmetrical nanotubes," *Symmetry*, vol. 10, no. 7, p. 244, 2018.

[22] S. Gupta, M. Singh, and A. K. Madan, "Application of graph theory: relationship of eccentric connectivity index and Wiener's index with anti-inflammatory activity," *Journal of Mathematical Analysis and Applications*, vol. 266, no. 2, pp. 259–268, 2002.

[23] A. Ahmad and S. C. Lopez, "Distance-based topological polynomials associated with zero-divisor graphs," *Mathematical Problems in Engineering*, vol. 2021, Article ID 4959559, 8 pages, 2021.

[24] K. Elahi, A. Ahmad, and R. Hasni, "Construction algorithm for zero divisor graphs of finite commutative rings and their vertex-based eccentric topological indices," *Mathematics*, vol. 6, no. 12, p. 301, 2018.

[25] M. Imran, M. K. Siddiqui, A. A. E. Abunamous, D. Adi, S. H. Rafique, and A. Q. Baig, "Eccentricity-based topological indices of an oxide network," *Mathematics*, vol. 6, no. 7, p. 126, 2018.

[26] A. Aslam, M. K. Jamil, W. Gao, and W. Nazeer, "Topological aspects of some dendrimer structures," *Nanotechnology Reviews*, vol. 7, no. 2, pp. 123–129, 2018.

[27] A. Aslam, Y. Bashir, S. Ahmed, and W. Gao, "On topological indices of certain dendrimer structures," *Zeitschrift für Naturforschung A*, vol. 72, no. 6, pp. 559–566, 2017.

[28] Y. Bashir, A. Aslam, M. Kamran et al., "On forgotten topological indices of some dendrimers structure," *Molecules*, vol. 22, no. 6, pp. 867–874, 2017.

[29] J. L. Gross and J. Yellen, *Graph Theory*, CRC Press, Boca Raton, FL, USA, 2000.

[30] D. F. Anderson and P. S. Livingston, "The zero-divisor graph of a commutative ring," *Journal of Algebra*, vol. 217, no. 2, pp. 434–447, 1999.

[31] S. Akbari and A. Mohammadian, "On the zero-divisor graph of a commutative ring," *Journal of Algebra*, vol. 274, no. 2, pp. 847–855, 2004.

[32] D. F. Anderson and S. B. Mulay, "On the diameter and girth of a zero-divisor graph," *Journal of Pure and Applied Algebra*, vol. 210, pp. 543–550, 2008.