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Research Article

The Exact Solutions for Fractional-Stochastic Drinfel'd–Sokolov–Wilson Equations Using a Conformable Operator

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The fractional-stochastic Drinfel'd–Sokolov–Wilson equations (FSDSWEs) perturbed by the multiplicative Wiener process are studied. The mapping method is used to obtain rational, hyperbolic, and elliptic stochastic solutions for FSDSWEs. Due to the importance of FSDSWEs in describing the propagation of shallow water waves, the derived solutions are significantly more useful and effective in understanding various important challenging physical phenomena. In addition, we use the MATLAB Package to generate 3D graphs for specific FSDSWE solutions in order to discuss the impact of fractional order and the Wiener process on the solutions of FSDSWEs.

1. Introduction

Partial differential equations (PDEs) have grown in popularity because of their broad spectrum of applications in nonlinear science including engineering [1], civil engineering [2], quantum mechanics [3], soil mechanics [4], statistical mechanics [5], population ecology [6], economics [7], and biology [8, 9]. Therefore, finding exact solutions is critical for a better understanding of nonlinear phenomena. To acquire exact solutions to these equations, a variety of methods such as Darboux transformation [10], Hirota's function [11], sine-cosine [12, 13], (G'/G)-expansion [14–16], perturbation [17, 18], Riccati-Bernoulli sub-ODE [19], $\exp(-\phi(\varsigma))$-expansion [20, 21], tanh-sech [22, 23], Jacobi elliptic function [24, 25], and Riccati equation method [26] have been used.

Recently, fractional derivatives are used to characterize a wide range of physical phenomena in mathematical biology, engineering disciplines, electromagnetic theory, signal processing, and other scientific research. These new fractional-order models are better than the previously used integer-order models because fractional-order derivatives and integrals allow for the modeling of distinct substances' memory and hereditary capabilities.

The conformable fractional derivative (CFD) helps us to develop an idea of how physical phenomena act. The CFD is very useful for modelling a variety of physical issues since differential equations with CFD are simpler to solve numerically than those with Caputo fractional derivative or the Riemann-Liouville. Currently, authors are focusing on fractional calculus and creating new operators such the Caputo

Fabrizio, Caputo, Riemann Liouville, and Atangana Baleanu derivatives. The conformable fractional operator [27–30] eliminates some of the restrictions of current fractional operators and provides standard calculus properties such as the derivative of the quotient of two functions, the product of two functions, Rolle's theorem, the chain rule, and the mean value theorem. Here, we use CFD stated in [29]. Therefore, let us state the definition of CFD and its properties as follows [29]:

The CFD of $\varphi : \mathbb{R}^+ \longrightarrow \mathbb{R}$ of order α is defined as

$$\mathbb{D}_y^\alpha \varphi(x) = \lim_{\varepsilon \longrightarrow 0} \frac{\varphi\left(y + \varepsilon y^{1-\alpha}\right) - \varphi(y)}{\varepsilon}. \tag{1}$$

The CFD satisfies

(1) $\mathbb{D}_y^\alpha[a\varphi(y) + b\psi(y)] = a\mathbb{D}_x^\alpha\varphi(y) + b\mathbb{D}_x^\alpha\psi(y),\ a, b \in \mathbb{R}$

(2) $\mathbb{D}_y^\alpha[C] = 0$, C is a constant

(3) $\mathbb{D}_y^\alpha(\varphi \circ \psi)(y) = x^{1-\alpha}\psi'(y)\varphi(\psi(y))$

(4) $\mathbb{D}_y^\alpha[x^y] = y y^{y-\alpha}, y \in \mathbb{R}$

(5) $\mathbb{D}_y^\alpha\psi(y) = y^{1-\alpha}(d\psi/dy)$

On the other hand, in the practically physical system, random perturbations emerge from a variety of natural sources. They cannot be avoided, because noise can cause statistical properties and significant phenomena. Consequently, stochastic differential equations emerged and they started to play a major role in modeling phenomena in oceanography, physics, biology, chemistry, atmosphere, fluid mechanics, and other fields.

Therefore, we consider in this paper the following fractional-stochastic Drinfel'd–Sokolov–Wilson equations (FSDSWEs):

$$d\Psi + [\gamma_1 \Phi \mathbb{D}_x^\alpha \Phi]dt = \sigma\Psi d\beta, \tag{2}$$

$$d\Phi + [\gamma_2 \mathbb{D}_{xxx}^\alpha \Phi + \gamma_3 \Psi \mathbb{D}_x^\alpha \Phi + \gamma_4 \Phi \mathbb{D}_x^\alpha \Psi]dt = \sigma\Phi d\beta, \tag{3}$$

where γ_k for $k = 1, 2, 3, 4$ are nonzero parameters. $\mathbb{D}^\alpha$, for $0 < \alpha \leq 1$, is CFD [29]. $\beta(t)$ is a standard Wiener process (SWP), and σ is the noise strength.

The Drinfel'd–Sokolov–Wilson equations (DSWEs) ((2) and (3)), with $\alpha = 1$ and $\sigma = 0$, evolved from shallow water wave models initially given by Drinfel'd and Sokolov [31, 32] and later refined by Wilson [33]. Due to the importance of DSWEs, several authors have created analytical solutions for this system using a variety of methods, including exp-function method [34], truncated Painlevè method [35], F – expansion method [36], Bäcklund transformation of Riccati equation [37], homotopy analysis method [38], and tanh and extended tanh methods [39]. Furthermore, a few authors obtained exact solutions for fractional DSW using various methods such as Jacobi elliptical function method [40] and complete discrimination system for polynomial method [41], while the analytical fractional-stochastic solutions of FSDSWEs ((2) and (3)) have never been obtained before.

Our aim of this paper is to attain a wide range of solutions including rational, hyperbolic, and elliptic functions for FSDSWEs ((2) and (3)) by using the mapping method. This is the first study to obtain exact solutions to FSDSWEs with combination of a stochastic term and fractional derivative. Also, we utilize MATLAB to generate 3D diagrams for a number of the FSDSWEs ((2) and (3)) developed in this study to demonstrate how the SWP affects these solutions.

This paper will be formatted as follows. In Section 2, the mapping method is used to generate analytic solutions for FSDSWEs ((2) and (3)). In Section 3, we investigate the effect of the SWP and fractional order on the derived solutions. Section 4 presents the paper's conclusion.

2. Analytical Solutions of FSDSWEs

First, let us derive the wave equation of FSDSWEs as follows.

2.1. Wave Equation for FSDSWEs. Let us apply the following wave transformation

$$\begin{aligned}\Psi(x, t) &= \psi(\mu)e^{\left(\sigma\beta(t)-(1/2)\sigma^2 t\right)}, \Phi(x, t)\\ &= \varphi(\mu)e^{\left(\sigma\beta(t)-(1/2)\sigma^2 t\right)},\ \mu\\ &= \frac{1}{\alpha}x^\alpha + \omega t,\end{aligned} \tag{4}$$

to attain the wave equation of FSDSWEs ((2) and (3)), where ψ and φ are real deterministic functions and ω is a constant. Putting Equation (4) into Equations (2) and (3) and using

$$\begin{aligned}d\Psi &= \left[\omega\psi' dt + \sigma\psi d\beta\right]e^{\left(\sigma\beta(t)-(1/2)\sigma^2 t\right)},\\ d\Phi &= \left[\omega\varphi' dt + \sigma\varphi d\beta\right]e^{\left(\sigma\beta(t)-(1/2)\sigma^2 t\right)},\\ \mathbb{D}_x^\alpha\Phi &= \varphi' e^{\left(\sigma\beta(t)-(1/2)\sigma^2 t\right)},\\ \mathbb{D}_x^\alpha\Psi &= \psi' e^{\left(\sigma\beta(t)-(1/2)\sigma^2 t\right)},\\ \mathbb{D}_{xxx}^\alpha\Phi &= \varphi''' e^{\left(\sigma\beta(t)-(1/2)\sigma^2 t\right)},\end{aligned} \tag{5}$$

we attain

$$\omega\psi' + \gamma_1\varphi\varphi' e^{\left(\sigma\beta(t)-(1/2)\sigma^2 t\right)} = 0, \tag{6}$$

$$\omega\varphi' + \gamma_2\varphi''' + \gamma_3\psi\varphi' e^{\left(\sigma\beta(t)-(1/2)\sigma^2 t\right)} + \gamma_4\varphi\psi' e^{\left(\sigma\beta(t)-(1/2)\sigma^2 t\right)} = 0. \tag{7}$$

Taking expectation $\mathbb{E}(\cdot)$ for Equations (6) and (7), we get

$$\omega\psi' + \gamma_1\varphi\varphi' e^{-(1/2)\sigma^2 t}\mathbb{E}\left(e^{\sigma\beta(t)}\right) = 0, \tag{8}$$

$$\omega\varphi' + \gamma_2\varphi''' + \left[\gamma_3\psi\varphi' + \gamma_4\varphi\psi'\right]e^{-(1/2)\sigma^2 t}\mathbb{E}\left(e^{\sigma\beta(t)}\right) = 0. \tag{9}$$

Since $\beta(t)$ is a normal distribution, then $\mathbb{E}(e^{\sigma\beta(t)}) = e^{(\sigma^2/2)t}$. Now, Equations (8) and (9) take the type

$$\omega\psi' + \gamma_1\varphi\varphi' = 0, \tag{10}$$

$$\omega\varphi' + \gamma_2\varphi''' + \gamma_3\psi\varphi' + \gamma_4\varphi\psi' = 0. \tag{11}$$

Integrating Equation (10) and putting the constants of integration equal zero, we get

$$\psi = -\frac{\gamma_1}{\omega}\varphi^2 + C, \tag{12}$$

where C is the integral constant. Plugging Equation (12) into (11) and using Equation (10), we have

$$\gamma_2\varphi''' - \left[\frac{\gamma_1\gamma_3}{2\omega} + \frac{\gamma_1\gamma_4}{\omega}\right]\varphi^2\varphi' + [\omega + C\gamma_3]\varphi' = 0. \tag{13}$$

Integrating Equation (13), we obtain

$$\varphi'' - \ell_1\varphi^3 + \ell_2\varphi = 0, \tag{14}$$

where

$$\begin{aligned} \ell_1 &= \frac{\gamma_1\gamma_3}{6\gamma_2\omega} + \frac{\gamma_1\gamma_4}{3\gamma_2\omega}, \\ \ell_2 &= \frac{\omega}{\gamma_2} + \frac{C\gamma_3}{\gamma_2}. \end{aligned} \tag{15}$$

2.2. The Mapping Method Description. Here, let us describe the mapping method stated in [42]. Assuming the solutions of Equation (14) have the form

$$\varphi(\mu) = \sum_{i=0}^{N} a_i\chi^i, \tag{16}$$

where N is fixed by balancing the linear term of the highest order derivative φ'' with nonlinear term φ^3, a_i, for $i = 1, 2, \cdots a_N$, are constants to be calculated and χ satisfies the first kind of elliptic equation

$$\chi' = \sqrt{\frac{1}{2}p\chi^4 + q\chi^2 + r}, \tag{17}$$

where p, q, and r are real parameters.

We notice that Equation (17) has a variety of solutions depending on p, q, and r as follows (Table 1).

$sn(\mu) = sn(\mu, m)$, $cn(\mu) = cn(\mu, m)$, $dn(\mu, m) = dn(\mu, m)$ are the Jacobi elliptic functions (JEFs) for $0 < m < 1$. When $m \longrightarrow 1$, the JEFs are converted into the hyperbolic functions shown below:

$$\begin{gathered} cn(\mu) \longrightarrow \text{sech}\,(\mu),\ sn(\mu) \longrightarrow \tanh\,(\mu),\ cs(\mu) \longrightarrow \text{csch}\,(\mu), \\ ds \longrightarrow \text{csch}\,(\mu), \qquad dn(\mu) \longrightarrow \text{sech}\,(\mu). \end{gathered} \tag{18}$$

2.3. Solutions of FSDSWEs. Now, let us determine the parameter N by balancing φ'' with φ^3 in Equation (14) as

$$N + 2 = 3N \Longrightarrow N = 1. \tag{19}$$

Rewriting Equation (17) with $N = 1$ as

$$\varphi = a_0 + a_1\chi. \tag{20}$$

Differentiating Equation (20) twice, we have, by using (17),

$$\varphi'' = a_1q\chi + a_1p\chi^3. \tag{21}$$

Substituting Equations (20) and (21) into Equation (14), we obtain

$$\left(a_1p - \ell_1a_1^3\right)\chi^3 - 3a_0a_1^2\ell_1\chi^2 + \left(a_1q - 3\ell_1a_0^2a_1 + \ell_2a_1\right)\chi - \left(\ell_1a_0^3 - \ell_2a_0\right) = 0. \tag{22}$$

Putting each coefficient of χ^k for $k = 0, 1, 2, 3$ equal zero, we get

$$\begin{aligned} a_1p - \ell_1a_1^3 &= 0, \\ 3a_0a_1^2\ell_1 &= 0, \\ a_1q - 3\ell_1a_0^2a_1 + \ell_2a_1 &= 0, \\ \ell_1a_0^3 - \ell_2a_0 &= 0. \end{aligned} \tag{23}$$

Solving these equations, we obtain

$$a_0 = 0, a_1$$

$$= \pm\sqrt{\frac{p}{\ell_1}}, q = -\ell_2. \tag{24}$$

Table 1: All possible solutions for Equation (17) for different values of p, q, and r.

Case	p	q	r	$\chi(\mu)$
1	$2m^2$	$-(1+m^2)$	1	$sn(\mu)$
2	2	$2m^2-1$	$-m^2(1-m^2)$	$ds(\mu)$
3	2	$2-m^2$	$(1-m^2)$	$cs(\mu)$
4	$-2m^2$	$2m^2-1$	$(1-m^2)$	$cn(\mu)$
5	-2	$2-m^2$	(m^2-1)	$dn(\mu)$
6	$\frac{m^2}{2}$	$\frac{(m^2-2)}{2}$	$\frac{1}{4}$	$\frac{sn(\mu)}{1\pm dn(\mu)}$
7	$\frac{m^2}{2}$	$\frac{(m^2-2)}{2}$	$\frac{m^2}{4}$	$\frac{sn(\mu)}{1\pm dn(\mu)}$
8	$\frac{-1}{2}$	$\frac{(m^2+1)}{2}$	$\frac{-(1-m^2)^2}{4}$	$mcn(\mu)\pm dn(\mu)$
9	$\frac{m^2-1}{2}$	$\frac{(m^2+1)}{2}$	$\frac{(m^2-1)}{4}$	$\frac{dn(\mu)}{1\pm sn(\mu)}$
10	$\frac{1-m^2}{2}$	$\frac{(1-m^2)}{2}$	$\frac{(1-m^2)}{4}$	$\frac{cn(\mu)}{1\pm sn(\mu)}$
11	$\frac{(1-m^2)^2}{2}$	$\frac{(1-m^2)^2}{2}$	$\frac{1}{4}$	$\frac{sn(\mu)}{dn\pm cn(\mu)}$
12	2	0	0	$\frac{c}{\mu}$
13	0	1	0	ce^{μ}

Table 2: All possible solutions for wave Equation (14) when $p>0$.

Case	p	q	r	$\chi(\mu)$	$\varphi(\mu)$
1	$2m^2$	$-(1+m^2)$	1	$sn(\mu)$	$\pm\sqrt{\frac{p}{\ell_1}}sn(\mu)$
2	2	$2m^2-1$	$-m^2(1-m^2)$	$ds(\mu)$	$\pm\sqrt{\frac{p}{\ell_1}}ds(\mu)$
3	2	$2-m^2$	$(1-m^2)$	$cs(\mu)$	$\pm\sqrt{\frac{p}{\ell_1}}cs(\mu)$
4	$\frac{m^2}{2}$	$\frac{(m^2-2)}{2}$	$1/4$ or $m^2/4$	$\frac{sn(\mu)}{1\pm dn(\mu)}$	$\pm\sqrt{\frac{p}{\ell_1}}\frac{sn(\mu)}{1\pm dn(\mu)}$
5	$\frac{1-m^2}{2}$	$\frac{(1-m^2)}{2}$	$\frac{(1-m^2)}{4}$	$\frac{cn(\mu)}{1\pm sn(\mu)}$	$\pm\sqrt{\frac{p}{\ell_1}}\frac{cn(\mu)}{1\pm sn(\mu)}$
6	$\frac{(1-m^2)^2}{2}$	$\frac{(1-m^2)^2}{2}$	$\frac{1}{4}$	$\frac{sn(\mu)}{dn\pm cn(\mu)}$	$\pm\sqrt{\frac{p}{\ell_1}}\frac{sn(\mu)}{dn\pm cn(\mu)}$
7	2	0	0	$\frac{c}{\mu}$	$\pm\sqrt{\frac{p}{\ell_1}}\frac{c}{\mu}$

Hence, the solution of Equation (14) is

$$\varphi(\mu)=\pm\sqrt{\frac{p}{\ell_1}}\chi(\mu), \tag{25}$$

for $p/\ell_1>0$. There are two sets depending only on p and ℓ_1 as follows.

First set: if $p>0$ and $\ell_1>0$, then the solutions $\varphi(\mu)$, from Table 1, of wave Equation (14) are as follows (Table 2).

If $m\longrightarrow 1$, then Table 2 degenerates to Table 3.

Now, using Table 2 (or Table 3 when $m\longrightarrow 1$) and Equations (25) and (12), we get the solutions of FSDSWEs ((2) and (3)), for $p/\ell_1>0$, as follows:

$$\Phi(x,t)=\varphi(\mu)e^{\left(\sigma\beta(t)-(1/2)\sigma^2 t\right)}, \tag{26}$$

TABLE 3: All possible solutions for wave Equation (14) when $p > 0$ and $m \longrightarrow 1$.

Case	p	q	r	$\chi(\mu)$	$\varphi(\mu)$
1	2	−2	1	$\tanh(\mu)$	$\pm\sqrt{\frac{p}{\ell_1}}\tanh(\mu)$
2	2	1	0	$\operatorname{sech}(\mu)$	$\pm\sqrt{p/\ell_1}\operatorname{sech}(\mu)$
3	2	1	0	$\operatorname{csch}(\mu)$	$\pm\sqrt{p/\ell_1}\operatorname{csch}(\mu)$
4	$\frac{1}{2}$	$\frac{-1}{2}$	$\frac{1}{4}$	$\frac{\tanh(\mu)}{1\pm\operatorname{sech}(\mu)}$	$\pm\sqrt{\frac{p}{\ell_1}}\frac{\tanh(\mu)}{1\pm\operatorname{sech}(\mu)}$
5	2	0	0	$\frac{c}{\mu}$	$\pm\sqrt{\frac{p}{\ell_1}}\frac{c}{\mu}$

TABLE 4: All possible solutions for wave Equation (14) when $p < 0$ and $m \longrightarrow 1$.

Case	p	q	r	$\chi(\mu)$	$\varphi(\mu)$
1	−2	1	0	$\operatorname{sech}(\mu)$	$\pm\sqrt{\frac{p}{\ell_1}}\operatorname{sech}(\mu)$
2	$\frac{-1}{2}$	2	0	$2\operatorname{sech}(\mu)$	$\pm 2\sqrt{\frac{p}{\ell_1}}\operatorname{sech}(\mu)$

TABLE 5: All possible solutions for wave Equation (14) when $p < 0$.

Case	p	q	r	$\chi(\mu)$	$\varphi(\mu)$
1	$-2m^2$	$2m^2-1$	$(1-m^2)$	$cn(\mu)$	$\pm\sqrt{\frac{p}{\ell_1}}cn(\mu)$
2	−2	$2-m^2$	(m^2-1)	$dn(\mu)$	$\pm\sqrt{\frac{p}{\ell_1}}dn(\mu)$
3	$\frac{-1}{2}$	$\frac{(m^2+1)}{2}$	$\frac{-(1-m^2)^2}{4}$	$mcn(\mu)\pm dn(\mu)$	$\pm\sqrt{\frac{p}{\ell_1}}[mcn(\mu)\pm dn(\mu)]$
4	$\frac{m^2-1}{2}$	$\frac{(m^2+1)}{2}$	$\frac{(m^2-1)}{4}$	$\frac{dn(\mu)}{1\pm sn(\mu)}$	$\pm\sqrt{\frac{p}{\ell_1}}\frac{dn(\mu)}{1\pm sn(\mu)}$

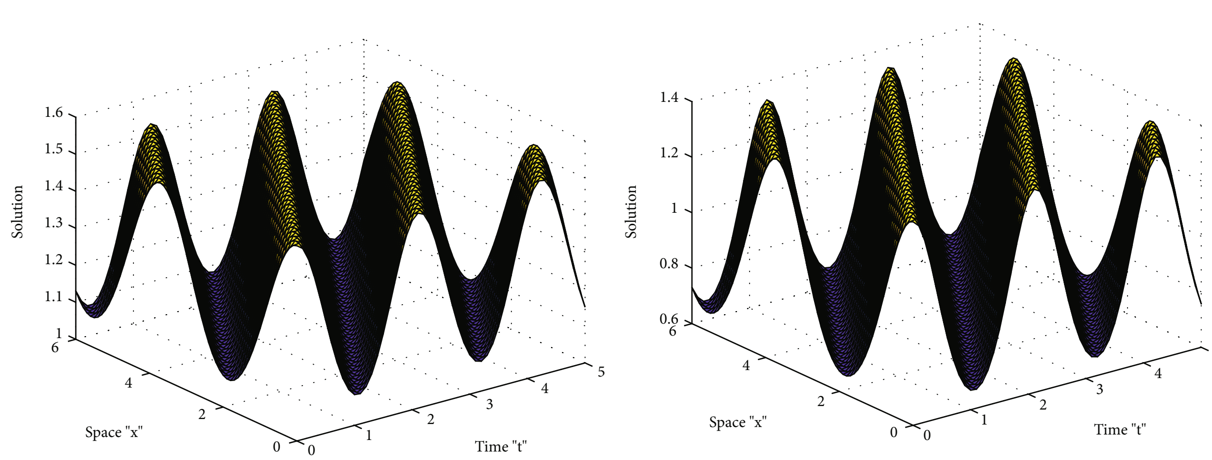

FIGURE 1: 3D plot of Equations (28) and (29) with $\sigma = 0$ and $\alpha = 1$.

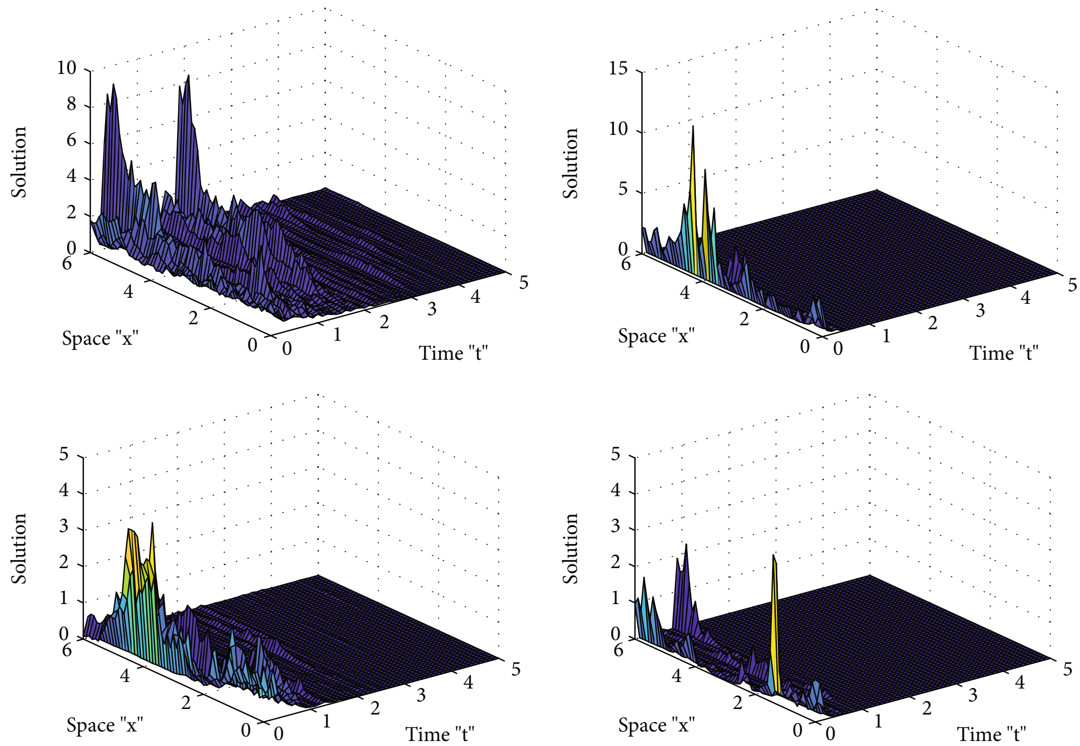

Figure 2: 3D plot of Equations (28) and (29) with $\sigma = 1, 2$ and $\alpha = 1$.

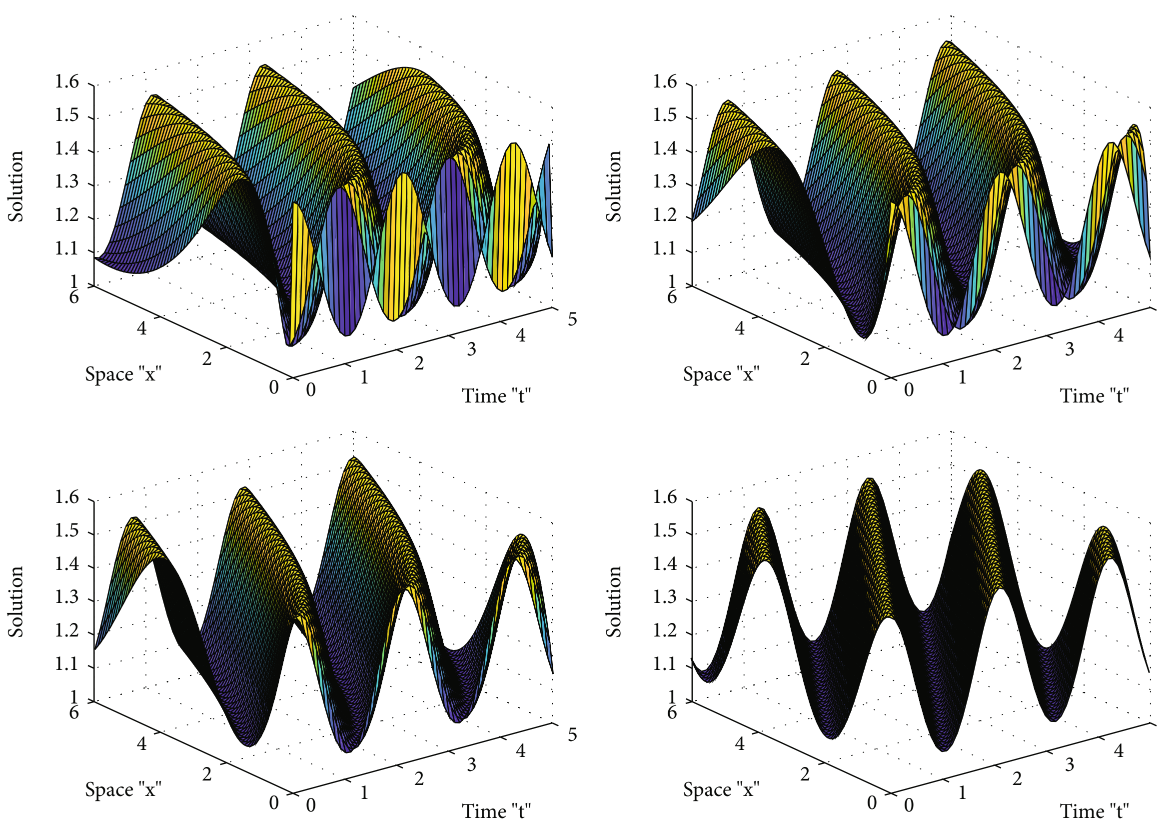

Figure 3: 3D plot of Equation (28) with $\sigma = 0$ and different α.

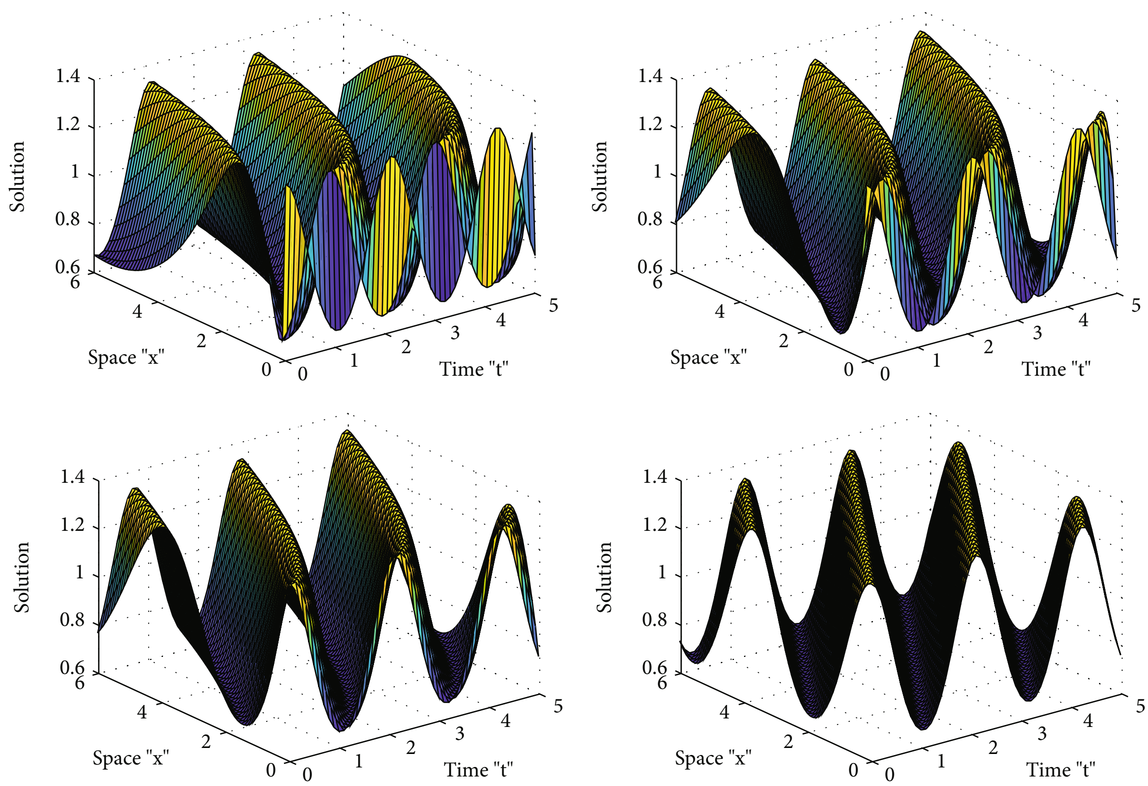

Figure 4: 3D plot of Equation (29) with $\sigma = 0$ and different α.

$$\Psi(x,t) = \left[-\frac{\gamma_1}{\omega}\varphi^2(\mu) + C\right] e^{\left(\sigma\beta(t) - (1/2)\sigma^2 t\right)}, \quad (27)$$

where $\mu = (x^\alpha/\alpha) + \omega t$.

Second set: if $p < 0$ and $\ell_1 < 0$, then the solutions $\varphi(\mu)$, from Table 1, of wave Equation (14) are as follows.

If $m \longrightarrow 1$, then Table 3 degenerates to Table 4.

In this case, using Table 5 (or Table 4 when $m \longrightarrow 1$), we can get the analytical solutions of FSDSWEs ((2) and (3)) as stated in Equations (26) and (27).

3. The Impact of Noise and Fractional Order on the Solutions

The impact of the noise and fractional order on the acquired solutions of FSDSWEs ((2) and (3)) is addressed. MATLAB tools are used to generate graphs for the following solutions:

$$\Phi(x,t) = \sqrt{\frac{p}{\ell_1}}cn\left(\frac{x^\alpha}{\alpha} + \omega t\right) e^{\left(\sigma\beta(t) - (1/2)\sigma^2 t\right)}, \quad (28)$$

$$\Psi(x,t) = \left[-\frac{\gamma_1 p}{\omega \ell_1}cn^2\left(\frac{x^\alpha}{\alpha} + \omega t\right)\right] e^{\left(\sigma\beta(t) - (1/2)\sigma^2 t\right)}, \quad (29)$$

with $C = 0$, $p = -2m^2$, $\gamma_1 = \gamma_2 = 1$, $\gamma_3 = \gamma_4 = 3$, $p = -2$, $q = 2 - m^2$, and $m = 0.5$. Then, $\ell_1 = -6/7$ and $\omega = 7/4$.

Firstly the impact of noise: in the absence of the noise, the surface is periodic (not flat) as we see in Figure 1.

While in Figure 2, if the noise is introduced and its strength σ is raised, the surface becomes substantially flatter as follows.

Secondly the impact of fractional order: in Figures 3 and 4, if $\sigma = 0$, we can see that the surface expands when α is increasing.

From the previous simulations, we may examine the nature of the solution as a double-periodic wave in physical form. We may conclude that it is critical to incorporate some fluctuation when modelling any phenomenon since the ignored terms may have an influence on the solutions.

4. Conclusions

In this paper, we considered the fractional-stochastic Drinfel'd–Sokolov–Wilson equations. This equation is well known in mathematical physics, population dynamics, surface physics, plasma physics, and applied sciences. The analytical solutions to FSDSWEs ((2) and (3)) were successfully attained by utilizing the mapping method. Due to the importance of FSDSWEs, these established solutions are significantly more useful and effective in understanding a variety of critical physical processes. In addition, we utilized the MATLAB software to demonstrate how multiplicative noise and fractional order affected the solutions of FSDSWEs. We may employ additive noise to address the FSDSWEs ((2) and (3)) in future study.

Data Availability

All data are available in this paper.

Conflicts of Interest

The authors declare that they have no competing interests.

Authors' Contributions

All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

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