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Research Article

Fixed Points of Proinov Type Multivalued Mappings on Quasimetric Spaces

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In this paper, we obtain new results which have not been encountered before in the literature, in multivalued quasimetric spaces, inspired by Proinov type contractions. We use admissible function as proving theorems. We also give an example that supports our theorems.

1. Introduction and Preliminaries

Fixed point theory has become an important research topic after the famous mathematician Banach's definition of the metric fixed point [1]. Many theoretical and applied studies have been done on fixed point theory. In the 21st century, the fixed point is still a popular and dynamic research topic. The concept of metric space, which forms the basis of the fixed point theory, is generalized by many researchers and new spaces (b-metric, quasimetric, partial metric, fuzzy metric, etc.) are introduced. One of the important generalizations is quasimetric space proved in 1931 as follows.

Definition 1 (see [2–4]). Let $\mathcal{X} \neq \varnothing$. A function $q : \mathcal{X} \times \mathcal{X} \longrightarrow \mathbb{R}_0^+$ is a quasimetric on $\mathcal{X}$ if it satisfies the following:

$$\begin{aligned} q(t,u) = q(u,t) = 0 \Leftrightarrow t = u, \\ q(t,w) \le q(t,u) + q(u,w), \end{aligned} \tag{1}$$

for all $t, u, w \in \mathcal{X}$ in this case, the pair $(\mathcal{X}, q)$ is a quasimetric space.

Let q be a quasimetric on $\mathcal{X}$, and the set $\mathfrak{B}_q(t,e) = \{w \in \mathcal{X} : q(t,w) < e\}$. Thus, the family $\{\mathfrak{B}_q(t,e) \colon t \in \mathcal{X}, e > 0\}$ forms a base for a T_0 topology τ_q on $\mathcal{X}$. Moreover, if A is a subset of $\mathcal{X}$, we denote by $cl_q(A)$ the closure of A with respect to T_0 topology; we say that the subset A is τ_q-closed if it is closed with respect to τ_q.

A sequence (t_r) in a quasimetric space converges to $t \in \mathcal{X}$, (in τ_q) if and only if $q(t, t_r) \longrightarrow 0$ as $r \longrightarrow \infty$. Moreover, we say that the sequence (t_r) is

(1) left-Cauchy if for every $e > 0$ there exists $r_e \in \mathbb{N}$ such that $q(t_r, t_m) < e$, whenever $r_e \le r \le m$

(2) right-Cauchy if for every $e > 0$ there exists $r_e \in \mathbb{N}$ such that $q(t_m, t_r) < e$, whenever $r_e \le r \le m$

Thereupon, a quasimetric space is called to be left (resp., right) complete if every left (resp., right) Cauchy sequence converges (to respect τ_q) (see, e.g., [5, 6, 40, 41]).

Nadler [7] is the first who introduced the framework for multivalued contraction mappings. The author proved the important theorem generalized Banach principle using the Hausdorff metric for multivalued mappings. After the proof of Nadler theorem, the theory of multivalued contraction mappings attracted great attention and is used in various branches of mathematics. Multivalent mappings in different spaces are introduced. One of them is multivalued mapping introduced in quasimetric-spaces by Shoaib [8] (see also [9, 10]).

Let $(\mathcal{X}, q)$ be a quasimetric space. We shall denote by $\mathcal{P}(\mathcal{X})$ the set of all nonempty subsets of $\mathcal{X}$, by $\mathcal{C}l_q(\mathcal{X})$ the set of all nonempty closed bounded subsets of $\mathcal{X}$, and let $\mathcal{K}_q(\mathcal{X})$ be the set of all compact subsets of $\mathcal{X}$.

Definition 2. Let $\mathcal{X} \neq \varnothing$ and $Z : \mathcal{X} \longrightarrow \mathcal{P}(\mathcal{X})$ be a multivalued map. A point $t \in \mathcal{X}$ is said to be a fixed point of Z if $t \in Z(t)$.

The set of the fixed point of a mapping Z is denoted by $\mathcal{F}(Z)$.

Lemma 3 is an important condition in the following main results.

Lemma 3 (see [8]). *Let A and B be nonempty closed bounded subsets of a quasimetric space $(\mathcal{X}, q)$ and let $\delta > 1$. Then, for all $t \in A$, there exists $u \in B$ such that $q(t, u) \leq \delta H_q(A, B)$.*

Nadler [7] stated that if $A, B \in K(\mathcal{X})$ in the metric spaces it is also provided for $\delta \geq 1$. With similarly thinking, the following lemma can be written.

Lemma 4. *Let A and B be nonempty compact subsets of a quasimetric space $(\mathcal{X}, q)$, and let $\delta \geq 1$. Then, for all $t \in A$, there exists $u \in B$ such that $q(t, u) \leq \delta H_q(A, B)$.*

Many researchers have stated different studies on well-known quasimetric spaces, see e.g., [11–13]. In recent years, Alqahtani et al. [14] introduced a new generalization in quasimetric spaces and defined Δ-symmetric quasimetric spaces. This definition is as follows.

Definition 5 (see [14]). Assume that $(\mathcal{X}, q)$ is a quasimetric space. If there exists a positive real number $\Delta > 0$ such that

$$q(t, u) \leq \Delta \cdot q(u, t), \tag{2}$$

for all $t, u \in \mathcal{X}$, then, the pair $(\mathcal{X}, \mathrm{q})$ is called a Δ -symmetric quasimetric space.

To simplify the notations, in the following, we will mark by $(\mathcal{X}, q)_\Delta$ a Δ-symmetric quasimetric space.

It is clear that if $\Delta = 1$, thus $(\mathcal{X}, \mathrm{q})_1$ becomes a metric space.

Definition 6 (see [8]). Let $(\mathcal{X}, q)_\Delta$ and $A, B \in \mathcal{P}(\mathcal{X})$. A function $H_q : \mathcal{P}(\mathcal{X}) \times \mathcal{P}(\mathcal{X}) \longrightarrow [0,\infty)$, defined by

$$\mathrm{H_q}(A, B) = \max\left\{\sup_{t\in A} \mathrm{q}(t, B), \sup_{u\in B} \mathrm{q}(A, u)\right\}, \tag{3}$$

where $q(t, A) = \inf_{u\in A} q(t, u)$ and $q(A, t) = \inf_{u\in A} q(u, t)$), satisfies all the axioms of quasimetric and is known as the Hausdorff quasimetric induced by the quasimetric q.

Example 7. Let $(\mathbb{R}, d)$ be a metric space and a function $q : \mathbb{R} \times \mathbb{R} \longrightarrow \mathbb{R}^+$, where

$$\mathrm{q}(t, u) = \begin{cases} 3d(t, u), & \text{if } t \geq u, \\ d(t, u), & \text{otherwise.} \end{cases} \tag{4}$$

Then, $(\mathcal{X}, q)$ is a 3-symmetric quasimetric space, but it is not a metric space.

In the following, we shall collect some main properties of a Δ-symmetric quasimetric space.

Lemma 8 (see [15]). *Let $(\mathcal{X}, q)_\Delta$, $\{t_r\}$ be a sequence in $\mathcal{X}$ and $t \in \mathcal{X}$. Then,*

(i) $\{t_r\}$ is right-Cauchy $\Leftrightarrow \{t_r\}$ is left-Cauchy $\Leftrightarrow \{t_r\}$ is Cauchy

(ii) if $\{u_r\}$ is a sequence in $\mathcal{X}$ and $q(t_r, u_r) \longrightarrow 0$ then $q(u_r, t_r) \longrightarrow 0$

Recall the notion of α-admissibility introduced in [16, 17].

Definition 9. A map $Z : \mathcal{X} \longrightarrow \mathcal{X}$ is defined α-admissible if for every $t, u \in \mathcal{X}$, we have

$$\alpha(t, u) \geq 1 \Rightarrow \alpha(Zt, Zu) \geq 1, \tag{5}$$

where $\alpha : \mathcal{X} \times \mathcal{X} \longrightarrow [0,\infty)$ is an offered function.

Some authors [18–21] introduced by slightly modifying this definition.

Definition 10. Let $(\mathcal{X}, q)_\Delta$ and $w : \mathcal{X} \times \mathcal{X} \longrightarrow [0,\infty)$. A multivalued mapping $Z : \mathcal{X} \longrightarrow \mathcal{C}l_q(\mathcal{X})$ is called to be strictly $*$ -triangular-admissible on $\mathcal{X}$ if the following conditions are satisfied:

($\mathrm{w_t}$) for each $t, u, v \in \mathcal{X}$, $w(t, u) > 1$ and $w(u, v) > 1$ implies $w(t, v) > 1$

($\mathrm{w_a}$) for each $t, u \in \mathcal{X}$, $w(t, u) > 1$ implies $w^*(Zt, Zu) > 1$ where $w^*(Zt, Zu) = \inf\{w(x, y) : x \in Zt, y \in Zu\}$.

Definition 11. Let $(\mathcal{X}, q)$ be a Δ-symmetric quasimetric space, and let $w : \mathcal{X} \times \mathcal{X} \longrightarrow [0,\infty)$. The space $(\mathcal{X}, q)$ is said to be strictly w^* -regular if for any sequence $\{t_r\} \subset \mathcal{X}$ such that $w(t_r, t_{r+1}) > 1$ for all $r \in \mathbb{N}$ and $t_r \longrightarrow t$ as $r \longrightarrow \infty$, we have $w(t_r, t) > 1$ for all $r \in \mathbb{N}$.

In recent years, researchers working on the fixed point theory seem to focus on introducing new contractions in known spaces. These new contractions are also accepted by many researchers and there are important studies, for example, F-contraction ([22–26]), θ-contraction [27], and interpolation contraction [28].

In 2020, Proinov [29] introduced new and interesting contractions in metric spaces. Proinov proved that several fixed point results (Wardowski [22]; Jleli and Samet [27]) observed in recent years are the result of Skof's fixed point theorem [30], and he introduced a very general fixed point theorem containing the main result of Skof.

Theorem 12 (see [29]). *Let $(\mathcal{X}, d)$ be a complete metric space and $Z : \mathcal{X} \longrightarrow \mathcal{X}$ a map which satisfies the contractive type condition:*

$$\psi(d(Zt, Zu)) \le \varphi(d(t, u)) \text{ for all } t, u \in \mathcal{X} \text{ with } d(Zt, Zu) > 0, \tag{6}$$

where $\psi, \varphi : \mathbb{R}^+ \longrightarrow \mathbb{R}$ are two functions such that

(i) $\varphi(m) < \psi(m)$ for all $m > 0$

(ii) ψ is nondecreasing

(iii) $\limsup_{m \longrightarrow \varepsilon+} \varphi(m) < \psi(\varepsilon+)$ for each $m > 0$

Hence, Z has a unique fixed point $w \in \mathcal{X}$ and $Z^r(t_0) \longrightarrow w$ for all $t_0 \in \mathcal{X}$, as $r \longrightarrow \infty$.

There are several studies using Proinov's contractions; some interesting ones are as follows: Alqahtani et al. [31] proposed the Proinov type mappings by involving certain rational expression in dislocated b-metrics. Alqahtani et al. [32] introduced the common fixed point of Proinov type contraction via simulation function. Roldán López de Hierro et al. [33] examined multiparametric contractions in b-metric spaces, inspired by Proinov results. Alghamdi et al. [34], on the other hand, introduced a new type of contraction using admissible mappings, inspired by Proinov and E-contraction.

Besides these, Karapnar et al. [35] combined contractions of Proinov [29] and Górnicki [36] in complete metric spaces and proved new fixed point theorems using admissible functions. Later, Ahmed and Fulga [37] generalized the Górnicki-Proinov type contraction to quasimetric spaces. Erdal et al. [38] published the notion of $(\alpha, \beta, \psi, \phi)$-interpolative contraction using a combine of interpolative contractions, Proinov type contractions, and ample spectrum contraction. Roldán López de Hierro et al. [39] proposed a new class of contractions in non-Archimedean fuzzy metric spaces, based on the Proinov fixed point results.

2. Main Results

Let us now give an important lemma that we will use in our main results.

Lemma 13 (see [37]). *Let $\{t_r\}$ be a sequence on $(\mathcal{X}, q)_\Delta$ such that $\lim_{r \longrightarrow \infty} q(t_r, t_{r+1}) = 0$. If the sequence $\{t_r\}$ is not left-Cauchy sequence thus there exists an $e > 0$ and two subsequences $\{t_{m_l}\}$, $\{t_{r_l}\}$ of $\{t_r\}$ such that*

$$\lim_{k \longrightarrow \infty} q\left(t_{r_{k+1}}, t_{m_{k+1}}\right) = \lim_{k \longrightarrow \infty} q\left(t_{r_k}, t_{m_k}\right) = e+. \tag{7}$$

Proof. Supposing that the sequence $\{t_r\}$ is not left-Cauchy, we can find $e > 0$ and the sequences of positive integers $\{n_l\}$, $\{r_l\}$, with $l \le r_l < n_l$ for every $l \ge 0$, such that

$$q\left(t_{r_l}, t_{n_l}\right) > 2e. \tag{8}$$

□

On the other hand, since $\lim_{r \longrightarrow \infty} q(t_r, t_{r+1}) = 0$, we can find $r_0 \ge 1$ such that

$$q(t_r, t_{r+1}) < \frac{e}{2a}, \tag{9}$$

for every $r \ge r_0$, where $a = \max\{1, \Delta\}$. Moreover, since the space is supposed to be Δ symmetric,

$$q(t_{r+1}, t_r) \le \Delta q(t_r, t_{r+1}) < \frac{e}{2}, \tag{10}$$

for every $r \ge r_0$. Therefore,

$$\begin{aligned} 2e &< q\left(t_{r_l}, t_{n_l}\right) \le q\left(t_{r_l}, t_{r_l+1}\right) + q\left(t_{r_l+1}, t_{n_l}\right) \le q\left(t_{r_l}, t_{r_l+1}\right) \\ &+ q\left(t_{r_l+1}, t_{n_l+1}\right) + q\left(t_{n_l+1}, t_{n_l}\right) < \frac{e}{2} + q\left(t_{r_l+1}, t_{n_l+1}\right) \\ &+ \frac{e}{2a} \le e + q\left(t_{r_l+1}, t_{n_l+1}\right), \end{aligned} \tag{11}$$

for every $l \ge r_0$. Consequently, we have

$$q\left(t_{r_l+1}, t_{n_l+1}\right) > e, \tag{12}$$

for every $l \ge r_0$. Now, let m_l be the smallest positive integer, greater than n_l, such that

$$q\left(t_{r_l+1}, t_{m_l+1}\right) > e, q\left(t_{r_l}, t_{m_l}\right) > e. \tag{13}$$

Thus, we have either

$$q\left(t_{r_l}, t_{m_l-1}\right) \le e, \tag{14}$$

$$\text{or } q\left(t_{r_l+1}, t_{m_l}\right) \le e. \tag{15}$$

In the case of the first inequality holds,

$$e < q\left(t_{r_l}, t_{m_l}\right) \le q\left(t_{r_l}, t_{m_l-1}\right) + q\left(t_{m_l-1}, t_{m_l}\right) \le e + q\left(t_{m_l-1}, t_{m_l}\right), \tag{16}$$

and letting $l \longrightarrow \infty$, we get $\lim_{l\longrightarrow\infty} q(t_{r_l}, t_{m_l}) = e+$. Similarly, in case of the second inequality holds, we can consider

$$e < q\left(t_{r_l}, t_{m_l}\right) \le q\left(t_{r_l}, t_{r_l+1}\right) + q\left(t_{r_l+1}, t_{m_l}\right) \le q\left(t_{r_l}, t_{r_l+1}\right) + e, \tag{17}$$

so, we also obtain $\lim_{l\longrightarrow\infty} q(t_{r_l}, t_{m_l}) = e+$. Now, by the triangle inequality, and taking into account the above considerations, we have

$$\begin{aligned} e < q\left(t_{r_l+1}, t_{m_l+1}\right) &\le q\left(t_{r_l}, t_{r_l+1}\right) + q\left(t_{r_l+1}, t_{m_l+1}\right) \\ &+ q\left(t_{m_l+1}, t_{m_l}\right) \le q\left(t_{r_l}, t_{r_l+1}\right) + q\left(t_{r_l+1}, t_{m_l+1}\right) \\ &+ \Delta \cdot q\left(t_{m_l}, t_{m_l+1}\right), \end{aligned} \tag{18}$$

and as $l \longrightarrow \infty$, we get

$$\lim_{l\longrightarrow\infty} q\left(t_{r_l+1}, t_{m_l+1}\right) = e+. \tag{19}$$

We will give multivalued (w, ψ, φ)-contractive mappings.

Definition 14. Let $(\mathcal{X}, q)_\Delta$, be a Δ-symmetric quasimetric space, a mapping $w : \mathcal{X} \times \mathcal{X} \longrightarrow [0,\infty)$ and $Z : \mathcal{X} \longrightarrow \mathrm{CB}(\mathcal{X})$ be a multivalued operator. We say that Z is a multivalued (w, ψ, φ)-contractive mapping if there exist two functions $\psi, \varphi : (0,\infty) \longrightarrow \mathbb{R}$ such that

$$\psi\left(w(t, u) H_q(Z(t), Z(u))\right) \le \varphi(q(t, u)), \tag{20}$$

for every $t, u \in \mathcal{X}$ with $w(t, u) > 1$ and $H_q(Z(t), Z(u)) > 0$.

Theorem 15. *Let $(\mathcal{X}, q)_\Delta$ be a complete Δ-symmetric quasimetric space, and $Z : \mathcal{X} \longrightarrow CB(\mathcal{X})$ be a multivalued (w, ψ, φ)-contractive mapping. Assume that following conditions are satisfied:*

$(\mathcal{K}_1)$ Z is strictly $*$-admissible and there exist $t_0 \in \mathcal{X}$ and $t_1 \in Z(t_0)$ such that $w(t_0, t_1) > 1$

$(\mathcal{K}_2)$ if $\{t_r\}$ is a sequence in $\mathcal{X}$ such that $w(t_r, t_{r+1}) > 1$ for all $r \in \mathbb{N}$ and $t_r \longrightarrow t$ as $r \longrightarrow \infty$, we have $w(t_r, t) > 1$

$(\mathcal{K}_3)$ ψ is nondecreasing and $\varphi(v) < \psi(v)$ for all $v > 0$

$(\mathcal{K}_4)$ $\limsup_{v\longrightarrow j+} \varphi(v) < \psi(j+)$ for all $j > 0$

Therefore, Z has a fixed point in $\mathcal{X}$.

Proof. Let t_0 be an arbitrary point in $\mathcal{X}$ and $t_1 \in \mathcal{X}$ such that $q(t_0, Zt_0) = q(t_0, t_1)$ and $q(Zt_0, t_0) = q(t_1, t_0)$. Let now $t_2 \in Zt_1$ be such that $q(t_1, Zt_1) = q(t_1, t_2)$ and $q(Zt_1, t_1) = q(t_2, t_1)$. Continuing in this way, we can build the sequence $\{t_r\}$ of points in $\mathcal{X}$, such that $t_{r+1} \in Zt_r$, with $q(t_r, Zt_r) = q(t_r, t_{r+1})$ and $q(Zt_r, t_r) = q(t_{r+1}, t_r)$, for $r \in \mathbb{N}_0$. Moreover, by condition $(\mathcal{K}_1)$, we have that there exist $t_0 \in \mathcal{X}$ and $t_1 \in Z(t_0)$ such that $w(t_0, t_1) > 1$. Supposing that $r_0 \ne r_1$, if $r_1 \in Z r_1$, we get that t_1 is a fixed point of Z. Then, let $t_1 \notin Zt_1$. As Z is a strictly $*$-admissible map, we have that $*(Zt_0, Zt_1) > 1$. Thus, there exists $t_2 \in Z(t_1)$ such that $w(t_1, t_2) > 1$ which implies $*(Zt_1, Zt_2) > 1$. By continuing this process, we can construct a sequence $\{t_r\}$ in $\mathcal{X}$ such that $t_{r+1} \in Z(t_r)$ where $t_{r+1} \ne t_r$ for every $r \ge 0$ (as otherwise, if $t_r \in Z(t_r)$, thus t_r is a fixed point of Z) and $w(t_r, t_{r+1}) > 1$. Therefore, $H_q(Zt_{r-1}, Zt_r) > 0$. From Lemma 3 with $w(t_r, t_{r+1}) > 1$, we obtain

$$q(t_r, t_{r+1}) \le w(t_{r-1}, t_r) H_q(Z(t_{r-1}), Z(t_r)), \tag{21}$$

for each $r \ge 1$. Keeping in mind $(\mathcal{K}_3)$ and (20) and we get

$$\psi(q(t_r, t_{r+1})) \le \psi\left(w(t_{r-1}, t_r) H_q(Z(t_{r-1}), Z(t_r))\right) \le \varphi(q(t_{r-1}, t_r)). \tag{22}$$

□

By hypothesis $(\mathcal{K}_3)$, we have

$$\psi(q(t_r, t_{r+1})) \le \varphi(q(t_{r-1}, t_r)) < \psi(q(t_{r-1}, t_r)). \tag{23}$$

Thus, since ψ is a nondecreasing map, $q(t_r, t_{r+1}) < q(t_{r-1}, t_r)$ for each $r \ge 1$. So, the sequence $\{q(t_{r-1}, t_r)\}$ is positively decreasing. Then, there exists $G \ge 0$ such that $\lim_{r\longrightarrow\infty} q(t_{r-1}, t_r) = G+$.

Assuming that $G > 0$ on account of (23), we get a contradiction to supposition $(\mathcal{K}_4)$ as follows:

$$\psi(G+) = \lim_{r\longrightarrow\infty} \psi(q(t_r, t_{r+1})) \le \lim_{r\longrightarrow\infty} \sup \varphi(q(t_{r-1}, t_r)) \le \lim_{v\longrightarrow G+} \sup \varphi(v). \tag{24}$$

Therefore, $G = 0$, as a result, $\lim_{r\longrightarrow\infty} q(t_{r-1}, t_r) = 0$.

We prove that the sequence $\{t_r\}$ is left-Cauchy. Let us suppose by contradiction that the sequence $\{t_r\}$ is not left-Cauchy. Thus, by using Lemma 13, there exist $e > 0$ and two subsequences $\{t_{r_k}\}, \{t_{m_k}\}$, $(t_{m_k} > t_{r_k} \ge k,)$ of $\{t_r\}$ such that (7) is fulfilled. From (7), we conclude that $q(t_{r_k+1}, t_{m_k+1}) > \varepsilon$ and since the mapping Z is strictly triungular admissible, $w(t_{r_k}, t_{m_k}) > 1$ for every $k \ge 1$. Substituting $t = t_{r_k}$ and $u = t_{m_k}$ in (7), we obtain for each $k \ge 1$,

$$\psi\left(q\left(t_{r_k+1}, t_{m_k+1}\right)\right) \le \psi\left(w\left(t_{r_k}, t_{m_k}\right) H_q\left(Zt_{r_k}, Zt_{m_k}\right)\right) \le \varphi\left(q\left(t_{r_k}, t_{m_k}\right)\right), \tag{25}$$

then,

$$\psi\left(q\left(t_{r_k+1}, t_{m_k+1}\right)\right) \le \varphi\left(q\left(t_{r_k}, t_{m_k}\right)\right) < \psi\left(q\left(t_{r_k}, t_{m_k}\right)\right), \tag{26}$$

for any $k \ge 1$, so that is $q(t_{r_k+1}, t_{m_k+1}) < q(t_{r_k}, t_{m_k})$ Because of $\lim_{k\longrightarrow\infty} q(t_{r_k+1}, t_{m_k+1}) = \varepsilon+$, we obtain $\lim_{k\longrightarrow\infty} q(t_{r_k}, t_{m_k}) = \varepsilon+$. Therefore, we can write

$$\psi(\varepsilon+) = \lim_{k\longrightarrow\infty} \psi\left(q\left(t_{r_k+1}, t_{m_k+1}\right)\right) \le \lim_{k\longrightarrow\infty} sup\varphi\left(q\left(t_{r_k}, t_{m_k}\right)\right) \le \lim_{\gamma\longrightarrow\varepsilon+} \varphi(\gamma), \tag{27}$$

which contradicts the supposition $(\mathcal{K}_4)$; then, $\{t_r\}$ is left-Cauchy sequence in $(\mathcal{X}, q)$, so that it is Cauchy sequence using Lemma 8. Therefore, the sequence $\{t_r\}$ is Cauchy in the complete Δ-symmetric quasimetric space and so converges to limit $t^* \in \mathcal{X}$. Now, we consider the following cases.

Case 1. If $q(t_{r+1}, Z(t^*)) = 0$ for some $r \in \mathbb{N}$, so by triangle inequality of Δ-symmetric quasimetric space

$$q(t^*, Z(t^*)) \leq q(t^*, t_{r+1}) + q(t_{r+1}, Z(t^*)) < q(t^*, t_{r+1}), \quad (28)$$

and thus, letting $r \longrightarrow \infty$, we conclude that $q(t^*, Z(t^*)) \leq 0$, that is,

$q(t^*, Z(t^*)) = 0$. As $Z(t^*)$ is closed, we obtain $t^* \in Z(t^*)$.

Case 2. On the contrary, if $q(t_{r+1}, Z(t^*)) > 0$ for every $r \in \mathbb{N}$ from $(\mathcal{K}_2)$, we have $w(t_r, t^*) > 1$ for all $r \in \mathbb{N}$. We claim that $q(t^*, Z(t^*)) = 0$. Supposing, on the contrary, $q(t^*, Z(t^*)) > 0$, there exists $r \in \mathbb{N}$ such that $q(t_r, Z(t^*)) > 0$. Therefore, we obtain

$$\begin{aligned} \psi(q(t_{r+1}, Z(t^*))) &\leq \psi\left(w(t_r, t^*) H_q(Z(t_r), Z(t^*))\right) \\ &\leq \varphi(q(t_r, t^*)) < \psi(q(t_r, t^*)). \end{aligned} \quad (29)$$

Taking into account the condition $(\mathcal{K}_3)$, we get $q(t_{r+1}, Z(t^*)) < q(t_r, t^*)$. Passing to limit as $r \longrightarrow \infty$, we obtain $q(t^*, Z(t^*)) < 0$. Therefore,

$q(t^*, Z(t^*)) = 0$, as $Z(t^*)$ is closed, $t^* \in Z(t^*)$.

Example 16. Let $\mathcal{X} = [0,\infty)$ be endowed with the 2-symmetric quasimetric $q : \mathcal{X} \times \mathcal{X} \longrightarrow [0,+\infty)$, where

$$q(t, u) = \begin{cases} 2(t-u), & \text{if } t \geq u, \\ u - t, & \text{otherwise,} \end{cases} \quad (30)$$

and a mapping $Z : \mathcal{X} \longrightarrow CB(\mathcal{X})$, defined as

$$Zt = \begin{cases} \left\{0, \dfrac{t}{8}\right\}, & \text{if } t \in [0,1], \\ \{2, 3\}, & \text{otherwise.} \end{cases} \quad (31)$$

We choose two functions $\psi, \varphi : (0,\infty) \longrightarrow \mathbb{R}$ with ψ is nondecreasing, and $\varphi(m) < \psi(m)$ for all $m > 0$ where $\psi(m) = m$ and $\varphi(m) = m/2$. Let also

$$w(t, u) = \begin{cases} 2, & \text{if } t, u \in [0,1], \\ 0, & \text{otherwise.} \end{cases} \quad (32)$$

We check that Z is a multivalued (w, ψ, φ)-contractive mapping of (20). Actually, if taking into account the way the function w is defined, we have consider the case $u, t \in [0, 1]$.

Let then, $t, u \in [0, 1]$, $u \geq t$. We get

$$q(0, Zu) = \inf \left\{0, \frac{u}{8}\right\} = 0, q(0, Zt) = \inf \left\{0, \frac{t}{8}\right\} = 0, \quad (33)$$

$$\begin{aligned} q\left(\frac{t}{8}, Zu\right) &= \inf_u \left\{2\left|0 - \frac{t}{8}\right|, 2\left|\frac{t}{8} - \frac{u}{8}\right|\right\}, q\left(\frac{u}{8}, Zt\right) \\ &= \inf_t \left\{2\left|0 - \frac{u}{8}\right|, 2\left|\frac{t}{8} - \frac{u}{8}\right|\right\}, \end{aligned}$$

$$\begin{aligned} H_q(Zt, Zu) &= \max\left\{\sup_{t\in Zt} q(t, Zu), \sup_{u\in Zu} q(u, Zt)\right\} \\ &= \max\left\{\sup_{t\in Zt} \inf_u \left\{\left|\frac{t}{4}\right|, \left|\frac{t}{4} - \frac{u}{4}\right|\right\}, \sup_{u\in Zu} \inf_t \right. \\ &\quad \left. \cdot \left\{\left|\frac{u}{4}\right|, \left|\frac{u}{4} - \frac{t}{4}\right|\right\}\right\} = \left|\frac{t}{4} - \frac{u}{4}\right|. \end{aligned} \quad (34)$$

So, we obtain

$$\psi\left(w(t, u) H_q(Z(t), Z(u))\right) = 2\left|\frac{t}{4} - \frac{u}{4}\right| = \left|\frac{t}{2} - \frac{u}{2}\right| \leq |t - u| = \varphi(q(t, u)). \quad (35)$$

Therefore, (20) fulfilled. Further, all other cases are satisfying, from $w(u, t) = 0$. Consequently, by Theorem 15, map Z has a fixed point, this being $t = 0$.

Definition 17. Let $(\mathcal{X}, q)_\Delta w : \mathcal{X} \times \mathcal{X} \longrightarrow [0,\infty)$ and $Z : \mathcal{X} \longrightarrow CB(\mathcal{X})$ be a multivalued operator. Z is said to be a multivalued C′iric′ type (w, ψ, φ)-contractive mapping if there exist two functions $\psi, \varphi : (0,\infty) \longrightarrow \mathbb{R}$ such that

$$\psi\left(w(t, u) H_q(Z(t), Z(u))\right) \leq \varphi(\Theta(t, u)), \quad (36)$$

for every $t, u \in \mathcal{X}$ with $w(t, u) > 1$ and $H_q(Z(t), Z(u)) > 0$ where

$$\Theta(t, u) = \max\left\{q(t, u), q(t, Zt), q(u, Zu), \frac{(q(t, Zu) + q(Zt, u))}{2}\right\}. \quad (37)$$

Theorem 18. *Let $(\mathcal{X}, q)$ be a complete Δ-symmetric quasimetric space, $w : \mathcal{X} \times \mathcal{X} \longrightarrow \mathbb{R}^+ \setminus \{0\}$ and $Z : \mathcal{X} \longrightarrow K(\mathcal{X})$ be a multivalued C′iric′ type (w, ψ, φ)-contractive mapping. Assume that following conditions are satisfied:*

$(\mathcal{K}_1)$ Z is strictly $*$-triangular-admissible and there exists $t_0 \in \mathcal{X}$ and $t_1 \in Z(t_0)$ such that $w(t_0, t_1) > 1$

$(\mathcal{K}_2)$ if $\{t_r\}$ is a sequence in $\mathcal{X}$ such that $w(t_r, t_{r+1}) > 1$ for all $r \in \mathbb{N}$ and $t_r \longrightarrow t$ as $r \longrightarrow \infty$, we have $w(t_r, t) > 1$

$(\mathcal{K}_3)$ ψ is nondecreasing, and $\varphi(v) < \psi(v)$ for all $v > 0$

$(\mathcal{K}_4)$ $\limsup_{v\longrightarrow j+} \varphi(v) < \psi(j+)$ for all $j > 0$

Therefore, Z has a fixed point in $\mathcal{X}$.

Proof. By condition $(\mathcal{K}_1)$, and following the lines of the proof of the previous theorem, we have that $w(t_r, t_m) > 1$, for every $m > r \geq 1$. Moreover, $H_q(Zt_{r-1}, Zt_r) > 0$ and from Lemma 3 with $w(t_r, t_{r+1}) > 1$, we obtain

$$q(t_r, t_{r+1}) \leq w(t_{r-1}, t_r) H_q(Z(t_{r-1}), Z(t_r)), \quad (38)$$

for each $r \geq 1$. Keeping in mind $(\mathscr{K}_3)$ and (36), we get

$$\psi(q(t_r, t_{r+1})) \leq \psi(w(t_{r-1}, t_r)H_q(Z(t_{r-1}), Z(t_r))) \leq \varphi(\Theta(t_{r-1}, t_r)). \quad (39)$$

As $Z(t)$ is closed for every $t \in \mathscr{X}$, we get that $t_r \in Z(t_{r-1})$ such that $q(t_{r-1}, t_r) = q(t_{r-1}, Z(t_{r-1}))$,

$$\psi(q(t_r, t_{r+1})) \leq \varphi(\Theta(t_{r-1}, t_r)) = \varphi(\max \{q(t_{r-1}, t_r), q \cdot (t_{r-1}, Z(t_{r-1})), q(t_r, Z(t_r)),$$

$$q(t_{r-1}, Z(t_r)) + \frac{q(Zt_{r-1}, t_r)}{2}\Big) = \varphi(\max \{q(t_{r-1}, t_r), q(t_r, t_{r+1})\}), \quad (40)$$

for every $r \geq 1$. □

If $\max \{q(t_{r-1}, t_r), q(t_r, t_{r+1})\} = q(t_r, t_{r+1})$ so $\psi(q(t_r, t_{r+1})) \leq \varphi(q(t_r, t_{r+1}))$, from assumption $(\mathscr{K}_3)$, this is a contradiction. Hence, we obtain $q(t_{r-1}, t_r) > q(t_r, t_{r+1})$, and

$$\psi(q(t_r, t_{r+1})) \leq \varphi(q(t_{r-1}, t_r)). \quad (41)$$

Similarly, again using $(\mathscr{K}_3)$, we get

$$\psi(q(t_r, t_{r+1})) \leq \varphi(q(t_{r-1}, t_r)) < \psi(q(t_{r-1}, t_r)). \quad (42)$$

But, the function ψ is nondecreasing map, so that we get $q(t_r, t_{r+1}) < q(t_{r-1}, t_r)$ for all $r \geq 1$. Therefore, the sequence $\{q(t_{r-1}, t_r)\}$ is positively decreasing, and then, there exists $G \geq 0$ such that $\lim_{r \longrightarrow \infty} q(t_{r-1}, t_r) = G+$. If $G > 0$, from (42), we obtain

$$\psi(G+) = \lim_{r \longrightarrow \infty} \psi(q(t_r, t_{r+1})) \leq \lim_{r \longrightarrow \infty} \sup \varphi(q(t_{r-1}, t_r)) \leq \lim_{\rho \longrightarrow G+} \sup \varphi(\rho), \quad (43)$$

which contradictions $(\mathscr{K}_4)$. Therefore, $G = 0$ and, as a result,

$$\lim_{r \longrightarrow \infty} q(t_{r-1}, t_r) = 0. \quad (44)$$

We claim that $\{t_r\}$ is Cauchy sequence. Let us assume by contradiction that the sequence $\{t_r\}$ is not left-Cauchy. Then, by Lemma 13, we can find $e > 0$ and two subsequences $\{t_{r_k}\}, \{t_{m_k}\}$, (with $m_k > r_k \geq k$) of $\{t_r\}$ such that (7) holds. Thereupon, we have that $w(t_{r_k}, t_{m_k}) > 1$ for all $m_k > r_k > k \geq 1$. Letting $t = t_{r_k}$ and $u = t_{m_k}$ in (9), we get

$$\psi(q(t_{r_k+1}, t_{m_k+1})) \leq \psi(w(t_{r_k}, t_{m_k})H_q(Zt_{r_k}, Zt_{m_k})) \leq \varphi(\Theta(t_{r_k}, t_{m_k})), \quad (45)$$

for every $k \geq 1$, where

$$\Theta(t_{r_k}, t_{m_k}) = \max \left\{ \begin{array}{c} q(t_{r_k}, t_{m_k}), q(t_{r_k}, Zt_{r_k}), q(t_{m_k}, Zt_{m_k}), \\ \frac{q(t_{r_k}, Zt_{m_k}) + q(Zt_{r_k}, t_{m_k})}{2} \end{array} \right\}. \quad (46)$$

Keeping in mind the way the sequence $\{t_r\}$ was define, let $t_{r_k+1} \in Zt_{r_k}$ and $t_{m_k+1} \in Zt_{m_k}$. Thus,

$$q(t_{r_k}, t_{m_k}) \leq \Theta(t_{r_k}, t_{m_k}) = \max \left\{ \begin{array}{c} q(t_{r_k}, t_{m_k}), q(t_{r_k}, t_{r_k+1}), q(t_{m_k}, t_{m_k+1}) \\ \frac{q(t_{r_k}, t_{m_k+1}) + q(t_{r_k+1}, t_{m_k})}{2} \end{array} \right\}$$
$$\leq \max \left\{ \begin{array}{c} q(t_{r_k}, t_{m_k}), q(t_{r_k}, t_{r_k+1}), q(t_{m_k}, t_{m_k+1}), \\ \frac{q(t_{r_k}, t_{r_k+1}) + q(t_{r_k+1}, t_{m_k+1}) + q(t_{r_k+1}, t_{r_k}) + q(t_{r_k}, t_{m_k})}{2} \end{array} \right\}. \quad (47)$$

Letting $k \longrightarrow \infty$ in the above inequality, and taking into account (44), respectively (7), we get

$$\lim_{k \longrightarrow \infty} \Theta(t_{r_k}, t_{m_k}) = e+. \quad (48)$$

Moreover, since the function ψ is nondecreasing, taking the limit superior when $k \longrightarrow \infty$ in (45) we get

$$\psi(\varepsilon+) = \lim_{k \longrightarrow \infty} \psi(q(t_{r_k+1}, t_{m_k+1})) \leq \limsup_{k \longrightarrow \infty} \varphi(\Theta(t_{r_k}, t_{m_k})) \leq \limsup_{\rho \longrightarrow \varepsilon+} \varphi(\rho), \quad (49)$$

which contradicts the supposition $(\mathscr{K}_4)$; then, $\{t_r\}$ is left Cauchy sequence in $(\mathscr{X}, q)$, so that it is Cauchy sequence using Lemma 8. Therefore, the sequence $\{t_r\}$ is Cauchy in the complete Δ-symmetric quasimetric space and so converges to a point $t^* \in \mathscr{X}$. Now, we consider following cases:

Case 1. If $q(t_{r+1}, Z(t^*)) = 0$ for some $r \in \mathbb{N}$, so by triangle inequality of Δ-symmetric quasimetric space

$$q(t^*, Z(t^*)) \leq q(t^*, t_{r+1}) + q(t_{r+1}, Z(t^*)) < q(t^*, t_{r+1}), \quad (50)$$

and thus, letting $r \longrightarrow \infty$, we conclude that $q(t^*, Z(t^*)) \leq 0$, that is,

$$q(t^*, Z(t^*)) = 0. As\, Z(t^*) \text{ is closed, we obtain } t^* \in Z(t^*). \quad (51)$$

Case 2. If we suppose the contrary, that is, $q(t_{r+1}, Z(t^*)) = 0$ for any r, from $(\mathscr{K}_2)$ we know that $w(t_r, t^*) > 1$ for all $r \in \mathbb{N}$. We assert that $q(t^*, Z(t^*)) = 0$. Suppose, on the contrary, $q(t^*, Z(t^*)) > 0$. Thus, there exists $r \in \mathbb{N}$ such that $q(t_r, Z(t^*)) > 0$ for every r. Using (36), we obtain

$$\psi(q(t_{r+1}, Z(t^*))) \leq \psi(w(t_r, t^*)H_q(Z(t_r), Z(t^*))) \leq \varphi(\Theta(t_r, t^*)) < \psi(\Theta(t_r, t^*)), \quad (52)$$

where

$$\Theta(t_r, t^*) = \max\left\{\begin{array}{c} q(t_r, t^*)), q(t_r, Z(t_r)), q(t^*, Z(t^*)), \\ \dfrac{q(t_r, Z(t^*)) + q(Zt_r, t^*))}{2} \end{array}\right\} = \max\left\{\begin{array}{c} q(t_r, t^*)), q(t_r, t_{r+1})), q(t^*, Z(t^*)), \\ \dfrac{q(t_r, Z(t^*)) + q(t_{r+1}, t^*))}{2} \end{array}\right\}. \tag{53}$$

Taking into account the condition $(\mathscr{K}_3)$, we get $q(t_{r+1}, Z(t^*)) < \Theta(t^*, Z(t^*))$. Passing to limit as $r \longrightarrow \infty$, we obtain $q(t^*, Z(t^*)) < q(t^*, Z(t^*))$ a contradiction, then $q(t^*, Z(t^*)) = 0$. As $Z(t^*)$ is compact, $t^* \in Z(t^*)$.

Corollary 19. *Let $(\mathscr{X}, q)$ be a Δ-symmetric quasimetric space and $Z : \mathscr{X} \longrightarrow K(\mathscr{X})$ be a multivalued mapping satisfying the condition:*

$$\psi\big(H_q(Z(t), Z(u))\big) < \varphi(q(t, u)), \tag{54}$$

for every $t, u \in \mathscr{X}$, where the functions $\psi, \varphi : (0,\infty) \longrightarrow \mathbb{R}$ and $H_q(Z(t), Z(u)) > 0$. The map Z admits a fixed point in $\mathscr{X}$ provided that following conditions hold:

(K_1) ψ is nondecreasing, and $\varphi(v) < \psi(v)$ for all $v > 0$

(K_2) $\limsup_{v \longrightarrow j+} \varphi(v) < \psi(j+)$ for all $j > 0$

Letting $\varphi(a) = \delta\psi(a)$, in Corollary 19, we obtain the following result.

Corollary 20. *Let $(\mathscr{X}, q)$ be a Δ-symmetric quasimetric space and $Z : \mathscr{X} \longrightarrow K(\mathscr{X})$ be a multivalued mapping satisfying the condition:*

$$\psi\big(H_q(Z(t), Z(u))\big) < \delta\psi(q(t, u)), \tag{55}$$

for every $t, u \in \mathscr{X}$, where the functions $\psi, \varphi : (0,\infty) \longrightarrow \mathbb{R}$ and $H_q(Z(t), Z(u)) > 0$. The map Z admits a fixed point in $\mathscr{X}$ provided that following conditions hold:

(K_1) ψ is nondecreasing, and $\varphi(v) < \psi(v)$ for all $v > 0$;

(K_2) $\limsup_{v \longrightarrow j+} \varphi(v) < \psi(j+)$ for all $j > 0$.

Corollary 21. *Let $(\mathscr{X}, q)$ be a Δ-symmetric quasimetric space and $Z : \mathscr{X} \longrightarrow K(\mathscr{X})$ be a multivalued mapping satisfying the condition:*

$$\psi\big(H_q(Z(t), Z(u))\big) < \varphi(\Theta(t, u)), \tag{56}$$

for every $t, u \in \mathscr{X}$ and $H_q(Z(t), Z(u)) > 0$, where the functions $\psi, \varphi : (0,\infty) \longrightarrow \mathbb{R}$ and

$$\Theta(t, u) = \max\left\{q(t, u), q(t, Zt), q(u, Zu), \frac{(q(t, Zu) + q(Zt, u))}{2}\right\}. \tag{57}$$

The map Z admits a fixed point in $\mathscr{X}$ provided that following conditions:

(K_1) ψ is nondecreasing, and $\varphi(v) < \psi(v)$ for all $v > 0$

(K_2) $\limsup_{v \longrightarrow j+} \varphi(v) < \psi(j+)$ for all $j > 0$

Taking $\varphi(a) = \delta\psi(a)$, in Corollary 21, we get the following result.

Corollary 22. *Let $(\mathscr{X}, q)$ be a Δ-symmetric quasimetric space and $Z : \mathscr{X} \longrightarrow K(\mathscr{X})$ be a multivalued mapping satisfying the condition:*

$$\psi\big(w(t, u)H_q(Z(t), Z(u))\big) \leq \delta\psi(\Theta(t, u)), \tag{58}$$

for every $t, u \in \mathscr{X}$ and $H_q(Z(t), Z(u)) > 0$, where the functions $\psi, \varphi : (0,\infty) \longrightarrow \mathbb{R}$ and

$$\Theta(t, u) = \max\left\{q(t, u), q(t, Zt), q(u, Zu), \frac{(q(t, Zu) + q(Zt, u))}{2}\right\}. \tag{59}$$

The map Z admits a fixed point in $\mathscr{X}$ provided that following conditions hold:

(K_1) ψ is nondecreasing, and $\varphi(v) < \psi(v)$ for all $v > 0$

(K_2) $\limsup_{v \longrightarrow j+} \varphi(v) < \psi(j+)$ for all $j > 0$

3. Conclusion

In this paper, we expand the very interesting results of Proinov [29] in several ways: First, we involve a more general form of the function by considering multivalued mapping. Secondly, we refine the structure of the considered abstract space with Δ-symmetric quasimetric space. Indeed, quasimetric space is one of the novel extensions of metric space. Besides, Δ-symmetric quasimetric space is more reasonable to work since almost all quasimetric space form Δ-symmetric quasimetric spaces. There are still rooms for the fixed point results in the context of Δ-symmetric quasimetric spaces.

Data Availability

No data are used.

Disclosure

The authors declare that the study was realized in collaboration with equal responsibility.

Conflicts of Interest

The authors declare that they have no competing interests.

Authors' Contributions

All authors read and approved the final manuscript.

References

[1] S. Banach, "Sur les opérations dans les ensembles abstraits et leur application aux équations intégrales," *Fund. math*, vol. 3, no. 1, pp. 133–181, 1922.

[2] H. P. A. Künzi, "A note on sequentially compact quasi-pseudo-metric spaces," *Monatshefte für Mathematik*, vol. 95, no. 3, pp. 219-220, 1983.

[3] N. A. Secelean, S. Mathew, and D. Wardowski, "New fixed point results in quasi-metric spaces and applications in fractals theory," *Advances in Difference Equations*, vol. 2019, no. 1, 2019.

[4] W. A. Wilson, "On quasi-metric spaces," *American Journal of Mathematics*, vol. 53, no. 3, pp. 675–684, 1931.

[5] S. B. Chen, S. P. Tian, and Z. Y. Mao, "On Caristi's fixed point theorem in quasi-metric spaces," in *Impulsive dynamical systems & applications*, vol. 3, pp. 1150–1157, Watam Press, Waterloo, 2005.

[6] H. P. Kunzi and M. P. Schellekens, "On the Yoneda completion of a quasi-metric space," *Theoretical Computer Science*, vol. 278, no. 1-2, pp. 159–194, 2002.

[7] S. B. Nadler, "Multivalued contraction mappings," *Pacific Journal of Mathematics*, vol. 30, no. 2, pp. 475–488, 1969.

[8] A. Shoaib, "Fixed point results for $\alpha * -\psi$-mutivalued mappings," *Bulletin of Mathematical Analysis Applications*, vol. 8, no. 4, pp. 43–55, 2016.

[9] M. Cvetkovic and V. Rakocevic, "α-Admissible contractions on quasi-metric-like space," *Advances in the Theory of Nonlinear Analysis and Its Application*, vol. 1, no. 2, pp. 113–124, 2017.

[10] B. E. Rhoades, "Some fixed point theorems in quasi-metric spaces," *Demonstratio Mathematica*, vol. 30, no. 2, pp. 301–306, 1997.

[11] S. Cobzas, "Completeness in quasi-metric spaces and Ekeland variational principle," *Topology and its Applications*, vol. 158, no. 8, pp. 1073–1084, 2011.

[12] T. L. Hicks, "Fixed point theorems for quasi-metric spaces," *Mathematica Japonica*, vol. 33, no. 2, pp. 231–236, 1988.

[13] A. M. Patriciu and V. Popa, "A general fixed point theorem of Ciric type in quasi-partial metric spaces," *Novi Sad Journal of Mathematics*, vol. 50, no. 2, pp. 1–6, 2020.

[14] B. Alqahtani, A. Fulga, and E. Karapınar, "Fixed point results on δ-symmetric quasi-metric space via simulation function with an application to Ulam stability," *Mathematics*, vol. 6, no. 10, p. 208, 2018.

[15] E. Karapınar, A. F. Roldan-Lopez-de-Hierro, and B. Samet, "Matkowski theorems in the context of quasi-metric spaces and consequences on G-metric spaces," *Analele Universitatii" Ovidius" Constanta-Seria Matematica*, vol. 24, no. 1, pp. 309–333, 2016.

[16] E. Karapınar and B. Samet, "Generalized ?-? contractive type mappings and related fixed point theorems with applications," in *Abstract and Applied Analysis*, vol. 2012, Hindawi, 2012.

[17] B. Samet, C. Vetro, and P. Vetro, "Fixed point theorems for $\alpha - \psi$-contractive type mappings," *Nonlinear Analysis: Theory, Methods and Applications*, vol. 75, no. 4, pp. 2154–2165, 2012.

[18] M. U. Ali, T. Kamran, and E. Karapınar, "Discussion on α-strictly contractive nonself multivalued maps," *Vietnam Journal of Mathematics*, vol. 44, no. 2, pp. 441–447, 2016.

[19] C. M. Chen, E. Karapınar, and D. O'regan, "On (α– ϕ)-Meir-Keeler contractions on partial Hausdorff metric spaces," *University Politehnica Of Bucharest Scientific*, vol. 2018, pp. 101–110, 2018.

[20] E. Karapınar, K. Taş, and V. Rakočević, "Advances on fixed point results on partial metric spaces," in *Mathematical Methods in Engineering*, pp. 3–66, Springer, Cham, 2019.

[21] H. Monfared, M. Asadi, and A. Farajzadeh, "New generalization of Darbo's fixed point theorem via α-admissible simulation functions with application," *Sahand Communications in Mathematical Analysis*, vol. 17, no. 2, pp. 161–171, 2020.

[22] D. Wardowski, "Fixed points of a new type of contractive mappings in complete metric spaces," *Fixed point theory and applications*, vol. 2012, no. 1, 2012.

[23] M. Eshraghisamani, S. Mansour Vaezpour, and M. Asadi, "Some results by quasi-contractive mappings in f-orbitally complete metric space," *Communications in Nonlinear Analysis*, vol. 4, 2017.

[24] M. Asadi, "Discontinuity of control function in the (F, φ, θ)-contraction in metric spaces," *Filomat*, vol. 31, no. 17, pp. 5427–5433, 2017.

[25] H. Monfared, M. Asadi, M. Azhini, and D. O'Regan, "F (ψ, φ) \$ F (\psi,\varphi) \$-contractions for α-admissible mappings on M-metric spaces," *Fixed Point Theory and Applications*, vol. 2018, no. 1, pp. 1–17, 2018.

[26] H. Monfared, M. Azhini, and M. Asadi, "\$ C \$-class and \$ F (psi, varphi) \$-contractions on \$ M \$-metric spaces," *International Journal of Nonlinear Analysis and Applications*, vol. 8, no. 1, pp. 209–224, 2017.

[27] M. Jleli and A. Samet, "A new generalization of the Banach contraction principle," *Journal of inequalities and applications*, vol. 38, 8 pages, 2014.

[28] E. Karapinar, "Revisiting the Kannan type contractions via interpolation," *Advances in the Theory of Nonlinear Analysis and its Application*, vol. 2, no. 2, pp. 85–87, 2018.

[29] P. D. Proinov, "Fixed point theorems for generalized contractive mappings in metric spaces," *Journal of Fixed Point Theory and Applications*, vol. 22, no. 1, p. 21, 2020.

[30] F. Skof, "Theoremi di punto fisso per applicazioni negli spazi metrici," *Atti della Accademia delle Scienze di Torino. Classe di Scienze Fisiche, Matematiche e Naturali*, vol. 111, pp. 323–329, 1977.

[31] B. Alqahtani, S. S. Alzaid, A. Fulga, R. López, and A. F. de Hierro, "Proinov type contractions on dislocated b-metric spaces," *Advances in Difference Equations*, vol. 2021, no. 1, pp. 1–16, 2021.

[32] B. Alqahtani, S. S. Alzaid, A. Fulga, and S. S. Yesilkaya, "Common fixed point theorem on Proinov type mappings via simulation function," *Advances in Difference Equations*, vol. 2021, no. 1, pp. 1–17, 2021.

[33] A. F. R. L. de Hierro, E. Karapınar, and A. Fulga, "Multiparametric contractions and related Hardy-Roger type fixed point theorems," *Mathematics*, vol. 8, no. 6, p. 957, 2020.

[34] M. A. Alghamdi, S. Gulyaz-Ozyurt, and A. Fulga, "Fixed points of Proinov E-contractions," *Symmetry*, vol. 13, no. 6, p. 962, 2021.

[35] M. E Karapnar, De La Sen, and A. Fulga, "A note on the Gornicki-Proinov type contraction," *Journal of Function Spaces*, vol. 2021, Article ID 6686644, 8 pages, 2021.

[36] J. Górnicki, "Remarks on asymptotic regularity and fixed points," *Journal of Fixed Point Theory and Applications*, vol. 21, no. 1, p. 29, 2019.

[37] A. E. S. Ahmed and A. Fulga, “The Górnicki-Proinov type contraction on quasi-metric spaces,” *AIMS Mathematics*, vol. 6, no. 8, pp. 8815–8834, 2021.

[38] K. Erdal, F. Andreea, and R. L. D. H. A. Francisco, “Fixed point theory in the setting of $(\alpha, \beta, \psi, \phi)$-interpolative contractions,” *Advances in Difference Equations*, vol. 2021, no. 1, pp. 1–16, 2021.

[39] A. F. R. L. de Hierro, A. Fulga, E. Karapınar, and N. Shahzad, “Proinov-type fixed-point results in non-Archimedean fuzzy metric spaces,” *Mathematics*, vol. 9, no. 14, p. 1594, 2021.

[40] A. Fulga, “On (ψ, ϕ)-rational contractions,” *Symmetry*, vol. 12, no. 5, p. 723, 2020.

[41] S. Romaguera and P. Tirado, “The Meir-Keeler fixed point theorem for quasi-metric spaces and some consequences,” *Symmetry.*, vol. 11, no. 6, p. 741, 2019.