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*Research Article*

# Analysis of Risks and Security of E-Commerce by Using the Novel Concepts of Complex Cubic Picture Fuzzy Information

**Jeonghwan Gwak**,[1,2,3,4] **Naeem Jan**,[5] **Rabia Maqsood**,[5] **and Abdul Nasir**[5]

[1]*Department of Software, Korea National University of Transportation, Chungju 27469, Republic of Korea*
[2]*Department of Biomedical Engineering, Korea National University of Transportation, Chungju 27469, Republic of Korea*
[3]*Department of AI Robotics Engineering, Korea National University of Transportation, Chungju 27469, Republic of Korea*
[4]*Department of IT & Energy Convergence (BK21 FOUR), Korea National University of Transportation, Chungju 27469, Republic of Korea*
[5]*Department of Mathematics, Institute of Numerical Sciences, Gomal University, Dera Ismail Khan, 29050 KPK, Pakistan*

Correspondence should be addressed to Jeonghwan Gwak; jgwak@ut.ac.kr

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Digital technology is so broad today as to encompass almost everything. It eases the difficulties of day-to day life with internet. E-commerce plays an important role in digital age of technology. Instead of many benefits of online system, it faces many types of threats. Sometime, choosing the best e-commerce security has a lot of uncertainties and ambiguities to reduce the effect of threats. To reduce these uncertainties, this article developed a new concept of complex cubic picture fuzzy set (CoCPFS) of fuzzy algebra that explains the positive effect, neutral effect, and negative effect of any object and additionally discussed the Cartesian product between two CoCPFSs, CoCPF relations, and their types. The relationship among the e-commerce securities and threats is investigated for the first time in the history of fuzzy algebra. These relations show the ways of disabling the effects of a threat by an effective security technique. Furthermore, the advantages and benefits of CoCPFS are explained through comparison tests with preexisting frameworks of fuzzy sets.

## 1. Introduction

Uncertainty is a common feature of many every day decisions. A situation of uncertainty arises when there can be more than one possible consequence of selecting any course of action/decision. Mostly, human decisions are uncertain and unclear. It is an inevitable part of our lives. Meanwhile, in mathematics, successfully detecting, processing, and resolving uncertainty by using the theory of fuzzy sets (FSs) in 1965 were proposed by Zadeh [1], which deal with the uncertainty and ambiguousness of fuzziness. An FS assigns a membership function to each element whose values ranging between $[0, 1]$ interval. Fuzzy set used to solve real life problems easily with uncertainty and ambiguities, like Chen et al. [2] proposed an application of fuzzy set in economics. Adlassnig [3] used fuzzy set in field of medication and Lu and Ruan [4] in 2007 proposed an application of multiobjective group decision-making. The invention of fuzzy set theory opened up new ways to handle and model uncertainty. The relation of crisp sets was presented by Klir and Wierman [5]; these sets just described two possibilities yes or no. The crisp theory only deals with exact information and cannot deal with uncertainty. Mendel [6] proposed the concept of fuzzy relations (FRs) that are not limited to tell just two possibilities yes or no but also describes the strength, grade, and level of good relations between any pair of FSs. If the value of membership is closer to 1, this implies that the relation is good. If the value of membership is closer to 0, it specifies that relation is in bad situation. Yu et al. [7] measured uncertainty and gave some applications by using FRs. In 1975, Zadeh [8] introduced the interval-valued fuzzy set (IVFSs) which replaced the single value of membership with an interval of membership to reduce uncertainty in decision-making. The extremes of interval belong to [0,1], i.e., the membership is subinterval of [0,1]. Bustince [9] gave an application of fuzzy techniques to approximate reasoning. Bustince and Burillo [10] presented the concept of interval-valued fuzzy relations (IVFRs), which is an extension of FRs. Gehrke et al. [11] commented on IVFSs. Roy and Biswas [12] used IVFRs and Sanchez's approach in medical diagnosis. Atanassov [13] proposed an idea of intuitionistic fuzzy set (IFSs) which discuss the membership as well as nonmembership of any element. It is a broader term than FSs. Dengfeng and Chuntian [14] used IFS to measure similarities and applied them to pattern recognition. Szmidt and Kacprzyk [15] proposed an application of IFS in medical diagnosis. Burillo and Bustince [16] introduced the concept of intuitionistic fuzzy relations (IFRs) which explain effectiveness and ineffectiveness of any relation at the same time. It is an extension of FRs. Wang et al. [17] described the IFRs with compositional operators. Cuong and Kreinovich [18] invented a new notion of picture fuzzy set (PFSs) which is an extension of FSs and IFSs. In this structure, three stages of an element were discussed; one is level of membership, second is level of indeterminacy, and level of nonmembership and sum of membership, indeterminacy, and nonmembership belong to [0,1]. They also introduced the concept of picture fuzzy relations (PFRs). Ganie et al. [19] invented some new correlation coefficients of PFS and presented their applications.

After FS, a new innovation in fuzzy set which is an extended form of fuzzy set was introduced by Ramot et al. [20] known as the concept of complex fuzzy set (CoFS) which represents the membership grades in the form of a complex number, i.e., $\vec{\mathcal{D}}(u) = \breve{\alpha}(u)e^{\xi(u)(2\pi i)}$ where $\breve{\alpha}(u)$ is called amplitude term and $\xi(u)$ is called phase term. A CoFS is capable of modeling a problem with periodicity and tells both amplitude and phase term of an element. It reduces the chances of errors and ambiguities. Further, he also defines complex fuzzy relations (CoFRs). Li and Chiang [21] comprehensively worked on the application of CoFSs. Zhang et al. [22] established the data qualities of CoFRs. Greenfield et al. [23] described the notion of complex interval-valued FSs (CoIVFSs). In 2021, Nasir et al. [24] described the complex interval-valued fuzzy relations (CoIVFRs) as being used for studying the relationships between two or more CoIVFSs and also proposed an application of medical diagnosis. Complex intuitionistic fuzzy set (CoIFS) was introduced by Rani and Garg [25] in which membership and nonmembership are indicated in the form of complex numbers. Jan et al. [26] introduced the complex intuitionistic fuzzy relations (CoIFRs) with an application to investigate the cybersecurity and cybercrimes in oil and gas sector. Ngan et al. [27] represented CIFS by quaternion numbers and application to decision-making. Akram et al. [28] invented a new concept of complex picture fuzzy sets (CoPFSs) which is an extension of CoIFS and contains an extra level of indeterminacy expressing the neutral effect of any element. All the three stages represent in form of complex number and sum of right extremes of interval and sum of left extremes of interval ranging between [0,1]. Nasir et al. [29] proposed an application of CoPFRs in communication and network security.

Cubic fuzzy set tells present and future of any object firstly introduced by Jun et al. [30]. It is the generalization of fuzzy set and interval-valued fuzzy set. Kim et al. [31] revealed the idea of cubic fuzzy relations (CFRs) which is the extended form of FRs and IVFRs. It describes the present and future effect of an object. Kaur and Garg et al. [32] described the notion of generalized cubic fuzzy set with t-norm. In 2019, Garg and Kaur [33] give the notion of cubic intuitionistic fuzzy set (CIFS) which is a broader term than the cubic fuzzy set because it explains the effectiveness and ineffectiveness of an object with present and future forecasting. Jun et al. [34] introduced cubic interval-valued intuitionistic fuzzy set and its application in BCK/BCI. Chinnadurai et al. [35] introduce the notion of complex cubic fuzzy sets (CoCFS). CoCFS is a combination of the complex interval-valued fuzzy set (CoIVFS) and complex fuzzy set (CoFS). In CoCFS, there is an advantage to provide the membership grade in complex numbers to solve problems with periodicity. Zhou et al. [36] used CoCF aggregation operators in group decision-making. Chinnadurai et al. [37] presented the notion of complex cubic intuitionistic fuzzy set (CoCIFS) which is a combination of complex cubic membership values and complex cubic nonmembership values and define some related operations on these sets. Rani and Garg [38] proposed an application of complex intuitionistic fuzzy preference relation and their application in individual and group decision-making. As a new extension of a cubic fuzzy set, Gumaei and Hussain [39] proposed the concept of cubic picture fuzzy sets, which is an extension of cubic sets, picture fuzzy sets, and interval-valued picture fuzzy sets.

This article develops a new concept of complex cubic picture fuzzy set (CoCPFS), complex cubic picture fuzzy relation (CoCPFR), and Cartesian product of two COCPFSs. Moreover, the types of CoCPFRs have been defined, including converse, reflexive, symmetric, transitive, and equivalence relation. Every definition is followed by a suitable example for better understanding, and some interesting results of CoCPFR have also been proved. The innovative concept proposed in this paper is superior to all existing structures of FSs, CoFSs, IVFSs, CoIVFSs, PFSs, CoPFSs, IVPFSs, CoIVFSs, CFSs, CoCFSs, CIFSs, CoIFSs, and CPFSs. Since, CoCPFRs are the analyzing the relations between CoCPFSs, so they are composed of degree of membership, indeterminacy, and nonmembership. This structure explains the present and future impact of one object on the other. Due to complex-valued mappings, it is capable to solve multiple variable problems with phase term easily. It gives more accurate results to reduce the uncertainty and ambiguity. This article also aimed to study the effect of e-commerce threats with e-commerce securities in digital business system. For this reason, a better framework is used because it can cover all aspects of the problem, i.e., positive effects, no effects, and negative effects with time phase. By using CoCPFRs, investigate the relations among the set of e-commerce securities and threats. Additionally, this application problem is solved using preexisting structures which are fail to used and reduce uncertainty. These structures can extend to more superior and better structures to produce good modeling techniques which can be used in economics, statistics, medical fields and information technology and computer sciences, etc.

The arrangement of the remaining paper is as follows: Section 2 consists of all preexisting and basic concepts of fuzzy algebra that are used as basis of current research. Section 3 contains the newly defined concepts of CoCPFSs, Cartesian product of two CoCPFSs, CoCPFRs, and their types. Some results of CoCPFRs have also been proved. Section 4 contains an application of investigating the e-commerce securities and threats. Section 5 studies the comparison between CoCPFSs and preexisting frameworks of fuzzy set. Section 6 concludes the results.

## 2. Preliminaries

Here, we explained some fundamental concepts of fuzzy algebra. We define the FS, CFS, IVFS, CoIVFS, IFS, CoIFS, IVIFS, PFS, CoPFS, IVPFS, CoIVPFS, CFS, CoCFS, CIFS, CoIFS and CPFS.

*Definition 1* (see [1]). A fuzzy set (FS) $\dot{F}$ on a universe $\check{\dot{U}}$ with a mapping $\ddot{\varpi} : \check{\dot{U}} \longrightarrow [0,1]$ can be expressed as

$$\dot{F} = \left\{ \left( \sigma, \ddot{\varpi}(\sigma) \right) : \sigma \in \check{\dot{U}} \right\}, \quad (1)$$

where $\ddot{\varpi}(\sigma)$ denotes the degree of membership of $\sigma$.

*Definition 2* (see [20]). A complex fuzzy set (CoFS) Ḟ on universe $\check{\dot{U}}$ with mappings $\breve{\alpha}_{\ddot{\varpi}}, \xi_{\ddot{\varpi}} : \check{\dot{U}} \longrightarrow [0,1]$ is defined as

$$\dot{F} = \left\{ \left( \sigma, \breve{\alpha}_{\ddot{\varpi}}(\sigma) e^{(2\pi i)\xi_{\ddot{\varpi}}(\sigma)} \right) : \sigma \in \check{\dot{U}} \right\}, \quad (2)$$

where $\breve{\alpha}_{\ddot{\varpi}}$ and $\xi_{\ddot{\varpi}}$ denote the amplitude and phase terms of degree of membership of $\sigma$.

*Definition 3.* (see [23]). A complex interval-valued fuzzy set (CoIVFS) Ḟ on a universe $\check{\dot{U}}$ with mappings $\breve{\alpha}^-_{\ddot{\varpi}}, \breve{\alpha}^+_{\ddot{\varpi}}, \xi^-_{\ddot{\varpi}}, \xi^+_{\ddot{\varpi}} : \check{\dot{U}} \longrightarrow [0,1]$ can be written as

$$\dot{F} = \left\{ \left( \sigma, , [\breve{\alpha}^-_{\ddot{\varpi}}(\sigma), \breve{\alpha}^+_{\ddot{\varpi}}(\sigma)] e^{(2\pi i)\left[\xi^-_{\ddot{\varpi}}(\sigma), \xi^+_{\ddot{\varpi}}(\sigma)\right]} : \sigma \in \check{\dot{U}} \right) \right\}, \quad (3)$$

where $\breve{\alpha}^-_{\ddot{\varpi}}(\sigma)$ and $\breve{\alpha}^+_{\ddot{\varpi}}(\sigma)$ are right and left ends of amplitude term of membership grade of an interval, respectively, and $\xi^-_{\ddot{\varpi}}(\sigma)$ and $\xi^+_{\ddot{\varpi}}(\sigma)$ are right and left ends of amplitude term of membership grade of the interval, respectively. Moreover, $\breve{\alpha}^-_{\ddot{\varpi}}(\sigma) \le \breve{\alpha}^+_{\ddot{\varpi}}(\sigma)$ and $\xi^-_{\ddot{\varpi}}(\sigma) \le \xi^+_{\ddot{\varpi}}(\sigma)$.

*Definition 4* (see [13]). An intuitionistic fuzzy set (IFS) Ḟ on a universe $\check{\dot{U}}$ with a mapping $\ddot{\varpi}, \vartheta : \check{\dot{U}} \longrightarrow [0,1]$ can be defined as

$$\dot{F} = \left\{ \left( \sigma, \ddot{\varpi}(\sigma), \vartheta(\sigma) \right) : \sigma \in \check{\dot{U}} \right\}. \quad (4)$$

Given that $0 \le \ddot{\varpi}(\sigma) + \vartheta(\sigma) \le 1$, where $\ddot{\varpi}(\sigma)$ and $\vartheta(\sigma)$ are membership and nonmembership grades.

*Definition 5* (see [25]). A set on a universe $\check{\dot{U}}$ is said to be complex intuitionistic fuzzy set (CoIFS) Ḟ such as

$$\dot{F} = \left\{ \left( \sigma, \breve{\alpha}_{\ddot{\varpi}(\sigma)} e^{(2\pi i)\xi_{\ddot{\varpi}}(\sigma)}, \breve{\alpha}_{\vartheta}(\sigma) e^{(2\pi i)\xi_{\vartheta}(\sigma)} \right) : \sigma \in \check{\dot{U}} \right\}, \quad (5)$$

where $\breve{\alpha}_{\ddot{\varpi}}, \breve{\alpha}_{\vartheta} : \check{\dot{U}} \longrightarrow [0,1]$ denote the amplitude terms of membership and nonmembership grades, respectively, and $\xi_{\ddot{\varpi}}, \xi_{\vartheta} : \check{\dot{U}} \longrightarrow [0,1]$ denote the phase terms of membership and nonmembership grades, given that $\breve{\alpha}_{\ddot{\varpi}} + \breve{\alpha}_{\vartheta} \le 1$ and $\xi_{\ddot{\varpi}} + \xi_{\vartheta} \le 1$.

*Definition 6* (see [18]). A picture fuzzy set (PFS) Ḟ on a universe $\check{\dot{U}}$ with real-valued mappings $\ddot{\varpi}, \emptyset, \vartheta : \check{\dot{U}} \longrightarrow [0,1]$ can be defined as

$$\dot{F} = \left\{ \sigma, \ddot{\varpi}(\sigma), \emptyset(\sigma), \vartheta(\sigma) : \sigma \in \check{\dot{U}} \right\}, \quad (6)$$

where a condition $0 \le \ddot{\varpi}(\sigma) + \emptyset(\sigma) + \vartheta(\sigma) \le 1$ and $\ddot{\varpi}(\sigma)$, $\emptyset(\sigma), \vartheta(\sigma)$ are membership, indeterminacy, and nonmembership grades of $\sigma$.

*Definition 7* (see [28]). A complex picture fuzzy set (CoPFS) Ḟ on a universe $\check{\dot{U}}$ can be expressed as

$$\dot{F} = \left\{ \sigma, \breve{\alpha}_{\ddot{\varpi}}(\sigma) e^{(2\pi i)\xi_{\ddot{\varpi}}(\sigma)}, \breve{\alpha}_{\emptyset}(\sigma) e^{(2\pi i)\xi_{\emptyset}(\sigma)}, \breve{\alpha}_{\vartheta}(\sigma) e^{(2\pi i)\xi_{\vartheta}(\sigma)} \right\}, \quad (7)$$

where $\breve{\alpha}_{\ddot{\varpi}}, \breve{\alpha}_{\emptyset}, \breve{\alpha}_{\vartheta} : \check{\dot{U}} \longrightarrow [0,1]$ are amplitude terms and $\xi_{\ddot{\varpi}}, \xi_{\emptyset}, \xi_{\vartheta} : \check{\dot{U}} \longrightarrow [0,1]$ are phase terms of membership indeterminacy and nonmembership grades, respectively. Moreover, $\breve{\alpha}_{\ddot{\varpi}} + \breve{\alpha}_{\emptyset} + \breve{\alpha}_{\vartheta} \le 1$ and $\xi_{\ddot{\varpi}} + \xi_{\emptyset} + \xi_{\vartheta} \le 1$.

*Definition 8* (see [30]). A cubic fuzzy set (CFS) Ḟ on a universe $\check{\dot{U}}$ can be defined as

$$\dot{F} = \left\{ (\sigma, A(\sigma), B(\sigma)) : \sigma \in \check{\dot{U}} \right\}, \quad (8)$$

where $A : \check{\dot{U}} \longrightarrow [0,1]$ denotes the membership and $B(\sigma) = [\ddot{\varpi}^+, \ddot{\varpi}^-] \ni \ddot{\varpi}^+, \ddot{\varpi}^- : \check{\dot{U}} \longrightarrow [0,1]$ denotes the interval of membership.

*Definition 9* (see [35]). On a universe $\check{\dot{U}}$, a complex cubic fuzzy set (CCFS) Ḟ can be defined as

$$\dot{F} = \left\{ \left( \sigma, \ddot{\varpi}_c(\sigma), \left[ \ddot{\varpi}^-_c(\sigma), \ddot{\varpi}^+_c(\sigma) \right] \right) : \sigma \in \check{\dot{U}} \right\}, \quad (9)$$

where $\ddot{\varpi}_c(\sigma)$ denotes the complex membership and $B[\ddot{\varpi}^-_c(\sigma), \ddot{\varpi}^+_c(\sigma)]$ denotes the interval of complex membership.

*Definition 10* (see [33]). A cubic intuitionistic fuzzy set (CIFS) $\dot{\mathrm{F}}$ on a universe $\check{U}$ is defined by

$$\dot{\mathrm{F}} = \left\{ \left(\sigma, \left(\bar{\ddot{\varpi}}(\sigma), \vartheta(\sigma)\right), \left(\left[\bar{\ddot{\varpi}}^{-}(\sigma), \bar{\ddot{\varpi}}^{+}(\sigma)\right], \left[\vartheta^{-}(\sigma), \vartheta^{+}(\sigma)\right]\right)\right) : \sigma \in \check{U} \right\}, \tag{10}$$

where $\bar{\ddot{\varpi}}(\sigma)$ and $\vartheta(\sigma)$ are membership and nonmembership, respectively, and $[\bar{\ddot{\varpi}}^{-}(\sigma), \bar{\ddot{\varpi}}^{+}(\sigma)]$ and $[\vartheta^{-}(\sigma), \vartheta^{+}(\sigma)]$ are intervals of membership and nonmembership, respectively.

*Definition 11* (see [37]). A complex cubic intuitionistic fuzzy set (CoCIFS) $\dot{\mathrm{F}}$ on a universe $\check{U}$ is defined by

$$\dot{\mathrm{F}} = \left\{ \left(\sigma, \left(\bar{\ddot{\varpi}}_c(\sigma), \vartheta_c(\sigma)\right), \left(\left[\bar{\ddot{\varpi}}_c^{-}(\sigma), \bar{\ddot{\varpi}}_c^{+}(\sigma)\right], \left[\vartheta_c^{-}(\sigma), \vartheta_c^{+}(\sigma)\right]\right)\right) : \sigma \in \check{U} \right\}, \tag{11}$$

where $\bar{\ddot{\varpi}}(\sigma)$ and $\vartheta(\sigma)$ are complex membership and complex nonmembership, respectively, and $[\bar{\ddot{\varpi}}^{-}(\sigma), \bar{\ddot{\varpi}}^{+}(\sigma)]$ and $[\vartheta^{-}(\sigma), \vartheta^{+}(\sigma)]$ are intervals of complex membership and complex nonmembership, respectively.

Hence, a CoCIFS can be expressed as

$$\dot{\mathrm{F}} = \left\{ \begin{array}{l} \sigma, \left(\breve{\alpha}_{\bar{\ddot{\varpi}}}(\sigma) e^{(2\pi i)\xi_{\bar{\ddot{\varpi}}}(\sigma)}, \breve{\alpha}_{\vartheta}(\sigma) e^{(2\pi i)\xi_{\vartheta}(\sigma)}\right), \\ \left( \begin{array}{l} \left[\breve{\alpha}^{-}_{\bar{\ddot{\varpi}}}(\sigma) e^{(2\pi i)\xi^{-}_{\bar{\ddot{\varpi}}}(\sigma)}, \breve{\alpha}^{+}_{\bar{\ddot{\varpi}}}(\sigma) e^{(2\pi i)\xi^{+}_{\bar{\ddot{\varpi}}}(\sigma)}\right], \\ \left[\breve{\alpha}^{-}_{\vartheta}(\sigma) e^{(2\pi i)\xi^{-}_{\vartheta}(\sigma)}, \breve{\alpha}^{+}_{\vartheta}(\sigma) e^{(2\pi i)\xi^{+}_{\vartheta}(\sigma)}\right] \end{array} \right) : \sigma \in \check{U} \end{array} \right\}, \tag{12}$$

such that $\breve{\alpha}_{\bar{\ddot{\varpi}}}(\sigma) + \breve{\alpha}_{\vartheta}(\sigma) \leq 1$ and $\xi_{\bar{\ddot{\varpi}}}(\sigma) + \xi_{\vartheta}(\sigma) \leq 1$. Moreover, $\breve{\alpha}^{+}_{\bar{\ddot{\varpi}}}(\sigma) + \breve{\alpha}^{+}_{\vartheta}(\sigma) \leq 1$ and $\xi^{+}_{\bar{\ddot{\varpi}}}(\sigma) + \xi^{+}_{\vartheta}(\sigma) \leq 1$.

*Definition 12* (see [39]). A cubic picture fuzzy set (CPFS) $\dot{\mathrm{F}}$ on a universe $\check{U}$ can be defined as

$$\begin{aligned} \dot{\mathrm{F}} = \Big\{ & \sigma, \left(\bar{\ddot{\varpi}}(\sigma), \O(\sigma), \vartheta(\sigma)\right), \left(\left[\bar{\ddot{\varpi}}^{-}(\sigma), \bar{\ddot{\varpi}}^{+}(\sigma)\right], [\O^{-}(\sigma), \O^{+}(\sigma)], \right. \\ & \left. \left[\vartheta^{-}(\sigma), \vartheta^{+}(\sigma)\right]\right) : \sigma \in \check{U} \Big\}, \end{aligned} \tag{13}$$

where $\bar{\ddot{\varpi}}(\sigma), \O(\sigma)$, and $\vartheta(\sigma)$ are membership, indeterminacy, and nonmembership, respectively, and $[\bar{\ddot{\varpi}}^{-}(\sigma), \bar{\ddot{\varpi}}^{+}(\sigma)]$, $[\O^{-}(\sigma), \O^{+}(\sigma)]$, and $[\vartheta^{-}(\sigma), \vartheta^{+}(\sigma)]$ are intervals of membership, indeterminacy, and nonmembership, respectively.

Hence, a CPFS can be expressed as

$$\dot{\mathrm{F}} = \left\{ \begin{array}{l} \left( \sigma, \left( \begin{array}{c} \breve{\alpha}_{\bar{\ddot{\varpi}}}(\sigma) e^{(2\pi i)\xi_{\bar{\ddot{\varpi}}}(\sigma)}, \\ \breve{\alpha}_{\O}(\sigma) e^{(2\pi i)\xi_{\O}(\sigma)}, \\ \breve{\alpha}_{\vartheta}(\sigma) e^{(2\pi i)\xi_{\vartheta}(\sigma)} \end{array} \right) \right), \\ \left( \begin{array}{l} \left[\breve{\alpha}^{-}_{\bar{\ddot{\varpi}}}(\sigma) e^{(2\pi i)\xi^{-}_{\bar{\ddot{\varpi}}}(\sigma)}, \breve{\alpha}^{+}_{\bar{\ddot{\varpi}}}(\sigma) e^{(2\pi i)\xi^{+}_{\bar{\ddot{\varpi}}}(\sigma)}\right], \\ \left[\breve{\alpha}^{-}_{\O}(\sigma) e^{(2\pi i)\xi^{-}_{\O}(\sigma)}, \breve{\alpha}^{+}_{\O}(\sigma) e^{(2\pi i)\xi^{+}_{\O}(\sigma)}\right], \\ \left[\breve{\alpha}^{-}_{\vartheta}(\sigma) e^{(2\pi i)\xi^{-}_{\vartheta}(\sigma)}, \breve{\alpha}^{+}_{\vartheta}(\sigma) e^{(2\pi i)\xi^{+}_{\vartheta}(\sigma)}\right] \end{array} \right) : \sigma \in \check{U} \end{array} \right\}. \tag{14}$$

## 3. Main Results

By using previously defined basic concepts, the CoCPFSs, Cartesian product of two CoCPFSs, CoCPFRs, and their types are defined.

*Definition 13.* A complex cubic picture fuzzy set (CoCPFS) $\dot{\mathrm{F}}$ on a universe $\check{U}$ can be defined as

$$\begin{aligned} U = \Big\{ & \sigma, \left(\bar{\ddot{\varpi}}_c(\sigma), \O_c(\sigma), \vartheta_c(\sigma)\right), \left(\left[\bar{\ddot{\varpi}}_c^{-}(\sigma), \bar{\ddot{\varpi}}_c^{+}(\sigma)\right], [\O_c^{-}(\sigma), \O_c^{+}(\sigma)], \right. \\ & \left. \left[\vartheta_c^{-}(\sigma), \vartheta_c^{+}(\sigma)\right]\right) : \sigma \in \check{U} \Big\}, \end{aligned} \tag{15}$$

where $\bar{\ddot{\varpi}}_c(\sigma)$ and $[\bar{\ddot{\varpi}}_c^{-}(\sigma), \bar{\ddot{\varpi}}_c^{+}(\sigma)]$ are complex-valued membership and complex interval-valued membership grades, $\O_c(\sigma)$ and $[\O_c^{-}(\sigma), \O_c^{+}(\sigma)]$ are complex-valued indeterminacy and interval-valued indeterminacy grades, and $\vartheta_c(\sigma)$ and $[\vartheta_c^{-}(\sigma), \vartheta_c^{+}(\sigma)]$ are complex-valued nonmembership and interval-valued nonmembership grades, respectively. CCPFS can also be defined as

$$\dot{\mathrm{F}} = \left\{ \begin{array}{l} \sigma, \left( \begin{array}{c} \breve{\alpha}_{\bar{\ddot{\varpi}}}(\sigma) e^{(2\pi i)\xi_{\bar{\ddot{\varpi}}}(\sigma)}, \\ \breve{\alpha}_{\O}(\sigma) e^{(2\pi i)\xi_{\O}(\sigma)}, \\ \breve{\alpha}_{\vartheta}(\sigma) e^{(2\pi i)\xi_{\vartheta}(\sigma)} \end{array} \right), \\ \left( \begin{array}{l} \left[\breve{\alpha}^{-}_{\bar{\ddot{\varpi}}}(\sigma) e^{(2\pi i)\xi^{-}_{\bar{\ddot{\varpi}}}(\sigma)}, \breve{\alpha}^{+}_{\bar{\ddot{\varpi}}}(\sigma) e^{(2\pi i)\xi^{+}_{\bar{\ddot{\varpi}}}(\sigma)}\right], \\ \left[\breve{\alpha}^{-}_{\O}(\sigma) e^{(2\pi i)\xi^{-}_{\O}(\sigma)}, \breve{\alpha}^{+}_{\O}(\sigma) e^{(2\pi i)\xi^{+}_{\O}(\sigma)}\right], \\ \left[\breve{\alpha}^{-}_{\vartheta}(\sigma) e^{(2\pi i)\xi^{-}_{\vartheta}(\sigma)}, \breve{\alpha}^{+}_{\vartheta}(\sigma) e^{(2\pi i)\xi^{+}_{\vartheta}(\sigma)}\right] \end{array} \right) : \sigma \in \check{U} \end{array} \right\}, \tag{16}$$

with condition $0 \leq \breve{\alpha}_{\bar{\ddot{\varpi}}}, \breve{\alpha}_{\O}, \breve{\alpha}_{\vartheta}, \xi_{\bar{\ddot{\varpi}}}, \xi_{\O}, \xi_{\vartheta} \leq 1$, $\breve{\alpha}^{-}_{\bar{\ddot{\varpi}}} \leq \breve{\alpha}^{+}_{\bar{\ddot{\varpi}}}$, $\breve{\alpha}^{-}_{\O} \leq \breve{\alpha}^{+}_{\O}$, $\breve{\alpha}^{-}_{\vartheta} \leq \breve{\alpha}^{+}_{\vartheta}$, and $0 \leq \xi^{-}_{\bar{\ddot{\varpi}}}, \xi^{+}_{\bar{\ddot{\varpi}}}, \xi^{-}_{\O}, \xi^{+}_{\O}, \xi^{-}_{\vartheta}, \xi^{+}_{\vartheta} \leq 1$ and $\xi^{-}_{\bar{\ddot{\varpi}}} \leq \xi^{+}_{\bar{\ddot{\varpi}}}$, $\xi^{-}_{\O} \leq \xi^{+}_{\O}$, $\xi^{-}_{\vartheta} \leq \xi^{+}_{\vartheta}$.

*Definition 14.* Take two CoCPFSs

$$
\begin{aligned}
U &= \left\{ \sigma, \begin{pmatrix} \check{\alpha}_{\mho}(\sigma)e^{(2\pi i)\xi_{\mho}(\sigma)}, \\ \check{\alpha}_{\emptyset}(\sigma)e^{(2\pi i)\xi_{\emptyset}(\sigma)}, \\ \check{\alpha}_{\vartheta}(\sigma)e^{(2\pi i)\xi_{\vartheta}(\sigma)} \end{pmatrix}, \begin{pmatrix} \left[\check{\alpha}^-_{\mho}(\sigma)e^{(2\pi i)\xi^-_{\mho}(\sigma)}, \check{\alpha}^+_{\mho}(\sigma)e^{(2\pi i)\xi^+_{\mho}(\sigma)}\right], \\ \left[\check{\alpha}^-_{\emptyset}(\sigma)e^{(2\pi i)\xi^-_{\emptyset}(\sigma)}, \check{\alpha}^+_{\emptyset}(\sigma)e^{(2\pi i)\xi^+_{\emptyset}(\sigma)}\right], \\ \left[\check{\alpha}^-_{\vartheta}(\sigma)e^{(2\pi i)\xi^-_{\vartheta}(\sigma)}, \check{\alpha}^+_{\vartheta}(\sigma)e^{(2\pi i)\xi^+_{\vartheta}(\sigma)}\right] \end{pmatrix} : \sigma \in \check{U} \right\}, \\
V &= \left\{ \tau, \begin{pmatrix} \check{\alpha}_{\mho}(\tau)e^{(2\pi i)\xi_{\mho}(\tau)}, \\ \check{\alpha}_{\emptyset}(\tau)e^{(2\pi i)\xi_{\emptyset}(\tau)}, \\ \check{\alpha}_{\vartheta}(\tau)e^{(2\pi i)\xi_{\vartheta}(\tau)} \end{pmatrix}, \begin{pmatrix} \left[\check{\alpha}^-_{\mho}(\tau)e^{(2\pi i)\xi^-_{\mho}(\tau)}, \check{\alpha}^+_{\mho}(\tau)e^{(2\pi i)\xi^+_{\mho}(\tau)}\right], \\ \left[\check{\alpha}^-_{\emptyset}(\tau)e^{(2\pi i)\xi^-_{\emptyset}(\tau)}, \check{\alpha}^+_{\emptyset}(\tau)e^{(2\pi i)\xi^+_{\emptyset}(\tau)}\right], \\ \left[\check{\alpha}^-_{\vartheta}(\tau)e^{(2\pi i)\xi^-_{\vartheta}(\tau)}, \check{\alpha}^+_{\vartheta}(\tau)e^{(2\pi i)\xi^+_{\vartheta}(\tau)}\right] \end{pmatrix} : \tau \in \check{U} \right\}.
\end{aligned}
\tag{17}
$$

Then, their Cartesian product is

$$
\mathrm{U} \times \mathrm{V} = \left\{ (\sigma, \tau), \begin{pmatrix} \check{\alpha}_{\mho}(\sigma,\tau)e^{(2\pi i)\xi_{\mho}(\sigma,\tau)}, \\ \check{\alpha}_{\emptyset}(\sigma,\tau)e^{(2\pi i)\xi_{\emptyset}(\sigma,\tau)}, \\ \check{\alpha}_{\vartheta}(\sigma,\tau)e^{(2\pi i)\xi_{\vartheta}(\sigma,\tau)} \end{pmatrix}, \begin{pmatrix} \left[\check{\alpha}^-_{\mho}(\sigma,\tau)e^{(2\pi i)\xi^-_{\mho}(\sigma,\tau)}, \check{\alpha}^+_{\mho}(\sigma,\tau)e^{(2\pi i)\xi^+_{\mho}(\sigma,\tau)}\right], \\ \left[\check{\alpha}^-_{\emptyset}(\sigma,\tau)e^{(2\pi i)\xi^-_{\emptyset}(\sigma,\tau)}, \check{\alpha}^+_{\emptyset}(\sigma,\tau)e^{(2\pi i)\xi^+_{\emptyset}(\sigma,\tau)}\right], \\ \left[\check{\alpha}^-_{\vartheta}(\sigma,\tau)e^{(2\pi i)\xi^-_{\vartheta}(\sigma,\tau)}, \check{\alpha}^+_{\vartheta}(\sigma,\tau)e^{(2\pi i)\xi^+_{\vartheta}(\sigma,\tau)}\right] \end{pmatrix} : \sigma, \tau \in \check{U} \right\}, \tag{18}
$$

where $\check{\alpha}_{\mho}(\sigma,\tau) = \min\{\check{\alpha}_{\mho}(\sigma), \check{\alpha}_{\mho}(\tau)\}$, $\check{\alpha}_{\emptyset}(\sigma,\tau) = \min\{\check{\alpha}_{\emptyset}(\sigma), \check{\alpha}_{\emptyset}(\tau)\}$, $\check{\alpha}_{\vartheta}(\sigma,\tau) = \max\{\check{\alpha}_{\vartheta}(\sigma), \check{\alpha}_{\vartheta}(\tau)\}$, $\xi_{\mho}(\sigma,\tau) = \min\{\xi_{\mho}(\sigma), \xi_{\mho}(\tau)\}$, $\xi_{\emptyset}(\sigma,\tau) = \min\{\xi_{\emptyset}(\sigma), \xi_{\emptyset}(\tau)\}$, $\xi_{\vartheta}(\sigma,\tau) = \max\{\xi_{\vartheta}(\sigma), \xi_{\vartheta}(\tau)\}$, $\check{\alpha}^-_{\mathrm{m}}(\sigma,\tau) = \min\{\check{\alpha}^-_{\mathrm{m}}(\sigma), \check{\alpha}^-_{m}(\tau)\}$, $\check{\alpha}^+_{\mho}(\sigma,\tau) = \{\check{\alpha}^+_{\mho}(\sigma), \check{\alpha}^+_{\mho}(\tau)\}$, $\check{\alpha}^-_{\emptyset}(\sigma,\tau) = \min\{\check{\alpha}^-_{\emptyset}(\sigma), \check{\alpha}^-_{\emptyset}(\tau)\}$, $\check{\alpha}^+_{\emptyset}(\sigma,\tau) = \min\{\check{\alpha}^+_{\emptyset}(\sigma), \check{\alpha}^+_{\emptyset}(\tau)\}$, $\check{\alpha}^-_{\vartheta}(\sigma,\tau) = \max\{\check{\alpha}^-_{\vartheta}(\sigma), \check{\alpha}^-_{\vartheta}(\tau)\}$, $\check{\alpha}^+_{\vartheta}(\sigma,\tau) = \max\{\check{\alpha}^+_{\vartheta}(\sigma), \check{\alpha}^+_{\vartheta}(\tau)\}$, $\xi^-_{\mho}(\sigma,\tau) = \min\{\xi^-_{\mho}(\sigma), \xi^-_{\mho}(\tau)\}$, $\xi^+_{\mho}(\sigma,\tau) = \min\{\xi^+_{\mho}(\sigma), \xi^+_{\mho}(\tau)\}$, $\xi^-_{\emptyset}(\sigma,\tau) = \min\{\xi^-_{\emptyset}(\sigma), \xi^-_{\emptyset}(\tau)\}$, $\xi^+_{\emptyset}(\sigma,\tau) = \min\{\xi^+_{\emptyset}(\sigma), \xi^+_{\emptyset}(\tau)\}$, $\xi^-_{\vartheta}(\sigma,\tau) = \max\{\xi^-_{\vartheta}(\sigma), \xi^-_{\vartheta}(\tau)\}$, and $\xi^+_{\vartheta}(\sigma,\tau) = \max\{\xi^+_{\vartheta}(\sigma), \xi^+_{\vartheta}(\tau)\}$.

*Example 1.*

Let O be a CoCPFS defined as

$$
O = \left\{ \begin{array}{l}
\left( o_1, \begin{pmatrix} 0.30e^{0.21(2\pi i)}, 0.31e^{0.10(2\pi i)}, \\ 0.32e^{0.57(2\pi i)} \end{pmatrix}, \begin{pmatrix} \left[0.10e^{0.23(2\pi i)}, 0.20e^{0.36(2\pi i)}\right], \left[0.33e^{0.14(2\pi i)}, 0.49e^{0.21(2\pi i)}\right], \\ \left[0.11e^{0.34(2\pi i)}, 0.29e^{0.41(2\pi i)}\right] \end{pmatrix} \right), \\
\left( o_2, \begin{pmatrix} 0.50e^{0.20(2\pi i)}, 0.12e^{0.25(2\pi i)}, \\ 0.32e^{0.55(2\pi i)} \end{pmatrix}, \begin{pmatrix} \left[0.30e^{0.23(2\pi i)}, 0.43e^{0.26(2\pi i)}\right], \left[0.13e^{0.14(2\pi i)}, 0.19e^{0.18(2\pi i)}\right] \\ \left[0.21e^{0.45(2\pi i)}, 0.29e^{0.56(2\pi i)}\right] \end{pmatrix} \right), \\
\left( o_3, \begin{pmatrix} 0.11e^{0.09(2\pi i)}, 0.38e^{0.33(2\pi i)}, \\ 0.51e^{0.47(2\pi i)} \end{pmatrix}, \begin{pmatrix} \left[0.37e^{0.03(2\pi i)}, 0.49e^{0.16(2\pi i)}\right], \left[0.23e^{0.14(2\pi i)}, 0.26e^{0.31(2\pi i)}\right], \\ \left[0.11e^{0.31(2\pi i)}, 0.15e^{0.39(2\pi i)}\right] \end{pmatrix} \right)
\end{array} \right\}. \tag{19}
$$

The Cartesian product $\mathrm{O}\times\mathrm{O}$ is

$$O\times O=\left\{\begin{array}{l}
\left((o_1,o_1),\left(\begin{array}{c}0.30e^{0.21(2\pi i)},0.31e^{0.10(2\pi i)},\\0.32e^{0.57(2\pi i)}\end{array}\right),\left(\begin{array}{c}\left[0.10e^{0.23(2\pi i)},0.20e^{0.36(2\pi i)}\right],\left[0.33e^{0.14(2\pi i)},0.49e^{0.21(2\pi i)}\right],\\\left[0.11e^{0.34(2\pi i)},0.29e^{0.41(2\pi i)}\right]\end{array}\right)\right),\\
\left((o_1,o_2),\left(\begin{array}{c}0.30e^{0.20(2\pi i)},0.12e^{0.10(2\pi i)},\\0.32e^{0.57(2\pi i)}\end{array}\right),\left(\begin{array}{c}\left[0.10e^{0.23(2\pi i)},0.20e^{0.26(2\pi i)}\right],\left[0.13e^{0.14(2\pi i)},0.19e^{0.18(2\pi i)}\right]\\\left[0.21e^{0.45(2\pi i)},0.29e^{0.56(2\pi i)}\right]\end{array}\right)\right),\\
\left((o_1,o_3),\left(\begin{array}{c}0.11e^{0.09(2\pi i)},0.31e^{0.10(2\pi i)},\\0.51e^{0.57(2\pi i)}\end{array}\right),\left(\begin{array}{c}\left[0.10e^{0.03(2\pi i)},0.20e^{0.16(2\pi i)}\right],\left[0.23e^{0.14(2\pi i)},0.26e^{0.21(2\pi i)}\right],\\\left[0.11e^{0.34(2\pi i)},0.29e^{0.41(2\pi i)}\right]\end{array}\right)\right),\\
\left((o_2,o_1),\left(\begin{array}{c}0.30e^{0.20(2\pi i)},0.12e^{0.10(2\pi i)},\\0.32e^{0.57(2\pi i)}\end{array}\right),\left(\begin{array}{c}\left[0.10e^{0.23(2\pi i)},0.20e^{0.26(2\pi i)}\right],\left[0.13e^{0.14(2\pi i)},0.19e^{0.18(2\pi i)}\right]\\\left[0.21e^{0.45(2\pi i)},0.29e^{0.56(2\pi i)}\right]\end{array}\right)\right),\\
\left((o_2,o_2),\left(\begin{array}{c}0.50e^{0.20(2\pi i)},0.12e^{0.25(2\pi i)},\\0.32e^{0.55(2\pi i)}\end{array}\right),\left(\begin{array}{c}\left[0.30e^{0.23(2\pi i)},0.43e^{0.26(2\pi i)}\right],\left[0.13e^{0.14(2\pi i)},0.19e^{0.18(2\pi i)}\right]\\\left[0.21e^{0.45(2\pi i)},0.29e^{0.56(2\pi i)}\right]\end{array}\right)\right),\\
\left((o_2,o_3),\left(\begin{array}{c}0.11e^{0.09(2\pi i)},0.12e^{0.25(2\pi i)},\\0.51e^{0.55(2\pi i)}\end{array}\right),\left(\begin{array}{c}\left[0.30e^{0.03(2\pi i)},0.43e^{0.26(2\pi i)}\right],\left[0.13e^{0.14(2\pi i)},0.19e^{0.18(2\pi i)}\right],\\\left[0.21e^{0.45(2\pi i)},0.29e^{0.56(2\pi i)}\right]\end{array}\right)\right),\\
\left((o_3,o_1),\left(\begin{array}{c}0.11e^{0.09(2\pi i)},0.31e^{0.10(2\pi i)},\\0.51e^{0.57(2\pi i)}\end{array}\right),\left(\begin{array}{c}\left[0.10e^{0.03(2\pi i)},0.20e^{0.16(2\pi i)}\right],\left[0.23e^{0.14(2\pi i)},0.26e^{0.21(2\pi i)}\right],\\\left[0.11e^{0.34(2\pi i)},0.29e^{0.41(2\pi i)}\right]\end{array}\right)\right),\\
\left((o_3,o_2),\left(\begin{array}{c}0.11e^{0.09(2\pi i)},0.12e^{0.25(2\pi i)},\\0.51e^{0.55(2\pi i)}\end{array}\right),\left(\begin{array}{c}\left[0.30e^{0.03(2\pi i)},0.43e^{0.26(2\pi i)}\right],\left[0.13e^{0.14(2\pi i)},0.19e^{0.18(2\pi i)}\right],\\\left[0.21e^{0.45(2\pi i)},0.29e^{0.56(2\pi i)}\right]\end{array}\right)\right),\\
\left((o_3,o_3),\left(\begin{array}{c}0.11e^{0.09(2\pi i)},0.38e^{0.33(2\pi i)},\\0.51e^{0.47(2\pi i)}\end{array}\right),\left(\begin{array}{c}\left[0.37e^{0.03(2\pi i)},0.49e^{0.16(2\pi i)}\right],\left[0.23e^{0.14(2\pi i)},0.26e^{0.31(2\pi i)}\right],\\\left[0.11e^{0.31(2\pi i)},0.15e^{0.39(2\pi i)}\right]\end{array}\right)\right)
\end{array}\right\}. \tag{20}$$

*Definition 15.* Any subcollection of the Cartesian products of CoCPFSs is known as CoCPF relation (CoCPFR) and denoted by $R$.

*Example 2.* From Equation (20), the subset R is a CoCPFR on CoCPFS O.

$$R=\left\{\begin{array}{l}
\left((o_1,o_1),\left(\begin{array}{c}0.30e^{0.21(2\pi i)},0.31e^{0.10(2\pi i)},\\0.32e^{0.57(2\pi i)}\end{array}\right),\left(\begin{array}{c}\left[0.10e^{0.23(2\pi i)},0.20e^{0.36(2\pi i)}\right],\left[0.33e^{0.14(2\pi i)},0.49e^{0.21(2\pi i)}\right],\\\left[0.11e^{0.34(2\pi i)},0.29e^{0.41(2\pi i)}\right]\end{array}\right)\right),\\
\left((o_1,o_2),\left(\begin{array}{c}0.30e^{0.20(2\pi i)},0.12e^{0.10(2\pi i)},\\0.32e^{0.57(2\pi i)}\end{array}\right),\left(\begin{array}{c}\left[0.10e^{0.23(2\pi i)},0.20e^{0.26(2\pi i)}\right],\left[0.13e^{0.14(2\pi i)},0.19e^{0.18(2\pi i)}\right]\\\left[0.21e^{0.45(2\pi i)},0.29e^{0.56(2\pi i)}\right]\end{array}\right)\right),\\
\left((o_1,o_3),\left(\begin{array}{c}0.11e^{0.09(2\pi i)},0.31e^{0.10(2\pi i)},\\0.51e^{0.57(2\pi i)}\end{array}\right),\left(\begin{array}{c}\left[0.10e^{0.03(2\pi i)},0.20e^{0.16(2\pi i)}\right],\left[0.23e^{0.14(2\pi i)},0.26e^{0.21(2\pi i)}\right],\\\left[0.11e^{0.34(2\pi i)},0.29e^{0.41(2\pi i)}\right]\end{array}\right)\right),
\end{array}\right\}. \tag{21}$$

*Definition 16.* A CoCPF relation $R$ is said to be complex cubic picture reflexive fuzzy relation (CoCP reflexive FR) on a CCPFS $\check{U}$, if

$$\forall\begin{pmatrix}\sigma,\begin{pmatrix}\check{\alpha}_{\varpi}(\sigma)e^{(2\pi i)\xi_{\varpi}(\sigma)},\\ \check{\alpha}_{\emptyset}(\sigma)e^{(2\pi i)\xi_{\emptyset}(\sigma)},\\ \check{\alpha}_{\vartheta}(\sigma)e^{(2\pi i)\xi_{\vartheta}(\sigma)}\end{pmatrix},\begin{pmatrix}\left[\check{\alpha}^{-}_{\varpi}(\sigma)e^{(2\pi i)\xi^{-}_{\varpi}(\sigma)},\check{\alpha}^{+}_{\varpi}(\sigma)e^{(2\pi i)\xi^{+}_{\varpi}(\sigma)}\right],\\ \left[\check{\alpha}^{-}_{\emptyset}(\sigma)e^{(2\pi i)\xi^{-}_{\emptyset}(\sigma)},\check{\alpha}^{+}_{\emptyset}(\sigma)e^{(2\pi i)\xi^{+}_{\emptyset}(\sigma)}\right],\\ \left[\check{\alpha}^{-}_{\vartheta}(\sigma)e^{(2\pi i)\xi^{-}_{\vartheta}(\sigma)},\check{\alpha}^{+}_{\vartheta}(\sigma)e^{(2\pi i)\xi^{+}_{\vartheta}(\sigma)}\right]\end{pmatrix}\end{pmatrix}\in\check{U}. \tag{22}$$

This implies

$$\forall\begin{pmatrix}(\sigma,\sigma),\begin{pmatrix}\check{\alpha}_{\varpi}(\sigma,\sigma)e^{(2\pi i)\xi_{\varpi}(\sigma,\sigma)},\\ \check{\alpha}_{\emptyset}(\sigma,\sigma)e^{(2\pi i)\xi_{\emptyset}(\sigma,\sigma)},\\ \check{\alpha}_{\vartheta}(\sigma,\sigma)e^{(2\pi i)\xi_{\vartheta}(\sigma,\sigma)}\end{pmatrix},\begin{pmatrix}\left[\check{\alpha}^{-}_{\varpi}(\sigma,\sigma)e^{(2\pi i)\xi^{-}_{\varpi}(\sigma,\sigma)},\check{\alpha}^{+}_{\varpi}(\sigma,\sigma)e^{(2\pi i)\xi^{+}_{\varpi}(\sigma,\sigma)}\right],\\ \left[\check{\alpha}^{-}_{\emptyset}(\sigma,\sigma)e^{(2\pi i)\xi^{-}_{\emptyset}(\sigma,\sigma)},\check{\alpha}^{+}_{\emptyset}(\sigma,\sigma)e^{(2\pi i)\xi^{+}_{\emptyset}(\sigma,\sigma)}\right],\\ \left[\check{\alpha}^{-}_{\vartheta}(\sigma,\sigma)e^{(2\pi i)\xi^{-}_{\vartheta}(\sigma,\sigma)},\check{\alpha}^{+}_{\vartheta}(\sigma,\sigma)e^{(2\pi i)\xi^{+}_{\vartheta}(\sigma,\sigma)}\right]\end{pmatrix}\end{pmatrix}\in R. \tag{23}$$

*Definition 17.* A CoCPF relation $R$ is said to be complex cubic picture symmetric fuzzy relation (CoCP symmetric FR) on CoCPFS $\check{U}$, if

$$\begin{aligned}&\forall\begin{pmatrix}\sigma,\begin{pmatrix}\check{\alpha}_{\varpi}(\sigma)e^{(2\pi i)\xi_{\varpi}(\sigma)},\\ \check{\alpha}_{\emptyset}(\sigma)e^{(2\pi i)\xi_{\emptyset}(\sigma)},\\ \check{\alpha}_{\vartheta}(\sigma)e^{(2\pi i)\xi_{\vartheta}(\sigma)}\end{pmatrix},\begin{pmatrix}\left[\check{\alpha}^{-}_{\varpi}(\sigma)e^{(2\pi i)\xi^{-}_{\varpi}(\sigma)},\check{\alpha}^{+}_{\varpi}(\sigma)e^{(2\pi i)\xi^{+}_{\varpi}(\sigma)}\right],\\ \left[\check{\alpha}^{-}_{\emptyset}(\sigma)e^{(2\pi i)\xi^{-}_{\emptyset}(\sigma)},\check{\alpha}^{+}_{\emptyset}(\sigma)e^{(2\pi i)\xi^{+}_{\emptyset}(\sigma)}\right],\\ \left[\check{\alpha}^{-}_{\vartheta}(\sigma)e^{(2\pi i)\xi^{-}_{\vartheta}(\sigma)},\check{\alpha}^{+}_{\vartheta}(\sigma)e^{(2\pi i)\xi^{+}_{\vartheta}(\sigma)}\right]\end{pmatrix}\end{pmatrix},\\ &\begin{pmatrix}\tau,\begin{pmatrix}\check{\alpha}_{\varpi}(\tau)\mathrm{e}^{(2\pi \mathrm{i})\xi_{\varpi}(\tau)},\\ \check{\alpha}_{\emptyset}(\tau)\mathrm{e}^{(2\pi \mathrm{i})\xi_{\emptyset}(\sigma)},\\ \check{\alpha}_{\vartheta}(\tau)\mathrm{e}^{(2\pi \mathrm{i})\xi_{\vartheta}(\tau)}\end{pmatrix},\begin{pmatrix}\left[\check{\alpha}^{-}_{\varpi}(\tau)\mathrm{e}^{(2\pi \mathrm{i})\xi^{-}_{\varpi}(\tau)},\check{\alpha}^{+}_{\varpi}(\tau)\mathrm{e}^{(2\pi \mathrm{i})\xi^{+}_{\varpi}(\tau)}\right],\\ \left[\check{\alpha}^{-}_{\emptyset}(\tau)\mathrm{e}^{(2\pi \mathrm{i})\xi^{-}_{\emptyset}(\tau)},\check{\alpha}^{+}_{\emptyset}(\tau)\mathrm{e}^{(2\pi \mathrm{i})\xi^{+}_{\emptyset}(\tau)}\right],\\ \left[\check{\alpha}^{-}_{\vartheta}(\tau)\mathrm{e}^{(2\pi \mathrm{i})\xi^{-}_{\vartheta}(\tau)},\check{\alpha}^{+}_{\vartheta}(\tau)\mathrm{e}^{(2\pi \mathrm{i})\xi^{+}_{\vartheta}(\tau)}\right]\end{pmatrix}\end{pmatrix}\in\check{U}.\end{aligned} \tag{24}$$

If

$$\forall\begin{pmatrix}(\sigma,\tau),\begin{pmatrix}\check{\alpha}_{\varpi}(\sigma,\tau)e^{(2\pi i)\xi_{\varpi}(\sigma,\tau)},\\ \check{\alpha}_{\emptyset}(\sigma,\tau)e^{(2\pi i)\xi_{\emptyset}(\sigma,\tau)},\\ \check{\alpha}_{\vartheta}(\sigma,\tau)e^{(2\pi i)\xi_{\vartheta}(\sigma,\tau)}\end{pmatrix},\begin{pmatrix}\left[\check{\alpha}^{-}_{\varpi}(\sigma,\tau)e^{(2\pi i)\xi^{-}_{\varpi}(\sigma,\tau)},\check{\alpha}^{+}_{\varpi}(\sigma,\tau)e^{(2\pi i)\xi^{+}_{\varpi}(\sigma,\tau)}\right],\\ \left[\check{\alpha}^{-}_{\emptyset}(\sigma,\tau)e^{(2\pi i)\xi^{-}_{\emptyset}(\sigma,\tau)},\check{\alpha}^{+}_{\emptyset}(\sigma,\tau)e^{(2\pi i)\xi^{+}_{\emptyset}(\sigma,\tau)}\right],\\ \left[\check{\alpha}^{-}_{\vartheta}(\sigma,\tau)e^{(2\pi i)\xi^{-}_{\vartheta}(\sigma,\tau)},\check{\alpha}^{+}_{\vartheta}(\sigma,\tau)e^{(2\pi i)\xi^{+}_{\vartheta}(\sigma,\tau)}\right]\end{pmatrix}\end{pmatrix}\in \mathrm{R}. \tag{25}$$

Then, there exists

$$\left( (\tau,\sigma), \begin{pmatrix} \check{\alpha}_{\ddot{\varpi}}(\tau,\sigma) e^{(2\pi i)\xi_{\ddot{\varpi}}(\tau,\sigma)}, \\ \check{\alpha}_{\emptyset}(\tau,\sigma) e^{(2\pi i)\xi_{\emptyset}(\tau,\sigma)}, \\ \check{\alpha}_{\vartheta}(\tau,\sigma) e^{(2\pi i)\xi_{\vartheta}(\tau,\sigma)} \end{pmatrix}, \begin{pmatrix} \left[\check{\alpha}^{-}_{\ddot{\varpi}}(\tau,\sigma) e^{(2\pi i)\xi^{-}_{\ddot{\varpi}}(\tau,\sigma)}, \check{\alpha}^{+}_{\ddot{\varpi}}(\tau,\sigma) e^{(2\pi i)\xi^{+}_{\ddot{\varpi}}(\tau,\sigma)}\right], \\ \left[\check{\alpha}^{-}_{\emptyset}(\tau,\sigma) e^{(2\pi i)\xi^{-}_{\emptyset}(\tau,\sigma)}, \check{\alpha}^{+}_{\emptyset}(\tau,\sigma) e^{(2\pi i)\xi^{+}_{\emptyset}(\tau,\sigma)}\right], \\ \left[\check{\alpha}^{-}_{\vartheta}(\tau,\sigma) e^{(2\pi i)\xi^{-}_{\vartheta}(\tau,\sigma)}, \check{\alpha}^{+}_{\vartheta}(\tau,\sigma) e^{(2\pi i)\xi^{+}_{\vartheta}(\tau,\sigma)}\right] \end{pmatrix} \right) \in R. \tag{26}$$

*Definition 18.* A CoCPF relation $R$ is said to be complex cubic picture transitive fuzzy relation (CoCP transitive FR) on CoCPFS $\check{U}$, if

$$\forall \left( (\sigma,\tau), \begin{pmatrix} \check{\alpha}_{\ddot{\varpi}}(\sigma,\tau) e^{(2\pi i)\xi_{\ddot{\varpi}}(\sigma,\tau)}, \\ \check{\alpha}_{\emptyset}(\sigma,\tau) e^{(2\pi i)\xi_{\emptyset}(\sigma,\tau)}, \\ \check{\alpha}_{\vartheta}(\sigma,\tau) e^{(2\pi i)\xi_{\vartheta}(\sigma,\tau)} \end{pmatrix}, \begin{pmatrix} \left[\check{\alpha}^{-}_{\ddot{\varpi}}(\sigma,\tau) e^{(2\pi i)\xi^{-}_{\ddot{\varpi}}(\sigma,\tau)}, \check{\alpha}^{+}_{\ddot{\varpi}}(\sigma,\tau) e^{(2\pi i)\xi^{+}_{\ddot{\varpi}}(\sigma,\tau)}\right], \\ \left[\check{\alpha}^{-}_{\emptyset}(\sigma,\tau) e^{(2\pi i)\xi^{-}_{\emptyset}(\sigma,\tau)}, \check{\alpha}^{+}_{\emptyset}(\sigma,\tau) e^{(2\pi i)\xi^{+}_{\emptyset}(\sigma,\tau)}\right], \\ \left[\check{\alpha}^{-}_{\vartheta}(\sigma,\tau) e^{(2\pi i)\xi^{-}_{\vartheta}(\sigma,\tau)}, \check{\alpha}^{+}_{\vartheta}(\sigma,\tau) e^{(2\pi i)\xi^{+}_{\vartheta}(\sigma,\tau)}\right] \end{pmatrix} \right) \text{and} \left( (\tau,\text{ɰ}), \begin{pmatrix} \check{\alpha}_{\ddot{\varpi}}(\tau,\text{ɰ}) e^{(2\pi i)\xi_{\ddot{\varpi}}(\tau,\text{ɰ})}, \\ \check{\alpha}_{\emptyset}(\tau,\text{ɰ}) e^{(2\pi i)\xi_{\emptyset}(\tau,\text{ɰ})}, \\ \check{\alpha}_{\vartheta}(\tau,\text{ɰ}) e^{(2\pi i)\xi_{\vartheta}(\tau,\text{ɰ})} \end{pmatrix}, \begin{pmatrix} \left[\check{\alpha}^{-}_{\ddot{\varpi}}(\tau,\text{ɰ}) e^{(2\pi i)\xi^{-}_{\ddot{\varpi}}(\tau,\text{ɰ})}, \check{\alpha}^{+}_{\ddot{\varpi}}(\tau,\text{ɰ}) e^{(2\pi i)\xi^{+}_{\ddot{\varpi}}(\tau,\text{ɰ})}\right], \\ \left[\check{\alpha}^{-}_{\emptyset}(\tau,\text{ɰ}) e^{(2\pi i)\xi^{-}_{\emptyset}(\tau,\text{ɰ})}, \check{\alpha}^{+}_{\emptyset}(\tau,\text{ɰ}) e^{(2\pi i)\xi^{+}_{\emptyset}(\tau,\text{ɰ})}\right], \\ \left[\check{\alpha}^{-}_{\vartheta}(\tau,\text{ɰ}) e^{(2\pi i)\xi^{-}_{\vartheta}(\tau,\text{ɰ})}, \check{\alpha}^{+}_{\vartheta}(\tau,\text{ɰ}) e^{(2\pi i)\xi^{+}_{\vartheta}(\tau,\text{ɰ})}\right] \end{pmatrix} \right) \in R \Longrightarrow \left( (\sigma,\text{ɰ}), \begin{pmatrix} \check{\alpha}_{\ddot{\varpi}}(\sigma,\text{ɰ}) e^{(2\pi i)\xi_{\ddot{\varpi}}(\sigma,\text{ɰ})}, \\ \check{\alpha}_{\emptyset}(\sigma,\text{ɰ}) e^{(2\pi i)\xi_{\emptyset}(\sigma,\text{ɰ})}, \\ \check{\alpha}_{\vartheta}(\sigma,\text{ɰ}) e^{(2\pi i)\xi_{\vartheta}(\sigma,\text{ɰ})} \end{pmatrix}, \begin{pmatrix} \left[\check{\alpha}^{-}_{\ddot{\varpi}}(\sigma,\text{ɰ}) e^{(2\pi i)\xi^{-}_{\ddot{\varpi}}(\sigma,\text{ɰ})}, \check{\alpha}^{+}_{\ddot{\varpi}}(\sigma,\text{ɰ}) e^{(2\pi i)\xi^{+}_{\ddot{\varpi}}(\sigma,\text{ɰ})}\right], \\ \left[\check{\alpha}^{-}_{\emptyset}(\sigma,\text{ɰ}) e^{(2\pi i)\xi^{-}_{\emptyset}(\sigma,\text{ɰ})}, \check{\alpha}^{+}_{\emptyset}(\sigma,\text{ɰ}) e^{(2\pi i)\xi^{+}_{\emptyset}(\sigma,\text{ɰ})}\right], \\ \left[\check{\alpha}^{-}_{\vartheta}(\sigma,\text{ɰ}) e^{(2\pi i)\xi^{-}_{\vartheta}(\sigma,\text{ɰ})}, \check{\alpha}^{+}_{\vartheta}(\sigma,\text{ɰ}) e^{(2\pi i)\xi^{+}_{\vartheta}(\sigma,\text{ɰ})}\right] \end{pmatrix} \right) \in R. \tag{27}$$

*Definition 19.* A CoCPF relation $R$ is said to be complex cubic picture fuzzy equivalence relation, if it is

(i) CoCP reflexive FR

(ii) CoCP symmetric FR

(iii) CoCP transitive FR

*Definition 20.* A CoCPF relation is known as complex cubic picture antisymmetric fuzzy relation (CoCP anti-symmetric FR), if

$$\forall \left( (\sigma,\tau), \begin{pmatrix} \check{\alpha}_{\ddot{\varpi}}(\sigma,\tau) e^{(2\pi i)\xi_{\ddot{\varpi}}(\sigma,\tau)}, \\ \check{\alpha}_{\emptyset}(\sigma,\tau) e^{(2\pi i)\xi_{\emptyset}(\sigma,\tau)}, \\ \check{\alpha}_{\vartheta}(\sigma,\tau) e^{(2\pi i)\xi_{\vartheta}(\sigma,\tau)} \end{pmatrix}, \begin{pmatrix} \left[\check{\alpha}^{-}_{\ddot{\varpi}}(\sigma,\tau) e^{(2\pi i)\xi^{-}_{\ddot{\varpi}}(\sigma,\tau)}, \check{\alpha}^{+}_{\ddot{\varpi}}(\sigma,\tau) e^{(2\pi i)\xi^{+}_{\ddot{\varpi}}(\sigma,\tau)}\right], \\ \left[\check{\alpha}^{-}_{\emptyset}(\sigma,\tau) e^{(2\pi i)\xi^{-}_{\emptyset}(\sigma,\tau)}, \check{\alpha}^{+}_{\emptyset}(\sigma,\tau) e^{(2\pi i)\xi^{+}_{\emptyset}(\sigma,\tau)}\right], \\ \left[\check{\alpha}^{-}_{\vartheta}(\sigma,\tau) e^{(2\pi i)\xi^{-}_{\vartheta}(\sigma,\tau)}, \check{\alpha}^{+}_{\vartheta}(\sigma,\tau) e^{(2\pi i)\xi^{+}_{\vartheta}(\sigma,\tau)}\right] \end{pmatrix} \right) \in R. \tag{28}$$

Then,

$$\left( (\tau,\sigma), \begin{pmatrix} \check{\alpha}_{\ddot{\varpi}}(\tau,\sigma) e^{(2\pi i)\xi_{\ddot{\varpi}}(\tau,\sigma)}, \\ \check{\alpha}_{\emptyset}(\tau,\sigma) e^{(2\pi i)\xi_{\emptyset}(\tau,\sigma)}, \\ \check{\alpha}_{\vartheta}(\tau,\sigma) e^{(2\pi i)\xi_{\vartheta}(\tau,\sigma)} \end{pmatrix}, \begin{pmatrix} \left[\check{\alpha}^{-}_{\ddot{\varpi}}(\tau,\sigma) e^{(2\pi i)\xi^{-}_{\ddot{\varpi}}(\tau,\sigma)}, \check{\alpha}^{+}_{\ddot{\varpi}}(\tau,\sigma) e^{(2\pi i)\xi^{+}_{\ddot{\varpi}}(\tau,\sigma)}\right], \\ \left[\check{\alpha}^{-}_{\emptyset}(\tau,\sigma) e^{(2\pi i)\xi^{-}_{\emptyset}(\tau,\sigma)}, \check{\alpha}^{+}_{\emptyset}(\tau,\sigma) e^{(2\pi i)\xi^{+}_{\emptyset}(\tau,\sigma)}\right], \\ \left[\check{\alpha}^{-}_{\vartheta}(\tau,\sigma) e^{(2\pi i)\xi^{-}_{\vartheta}(\tau,\sigma)}, \check{\alpha}^{+}_{\vartheta}(\tau,\sigma) e^{(2\pi i)\xi^{+}_{\vartheta}(\tau,\sigma)}\right] \end{pmatrix} \right) \notin R. \tag{29}$$

*Definition 21.* A CoCPF relation $R$ is said to be complex cubic partial order fuzzy relation, if

(i) CoCP reflexive FR

(ii) CoCP antisymmetric FR

(iii) CoCP transitive FR

*Definition 22.* A CCPF relation $R$ is said to be complex cubic picture complete fuzzy relation (CCP complete FR), if

$$\forall\begin{pmatrix}\sigma, \begin{pmatrix}\breve{\alpha}_{\circleddash}(\sigma)e^{(2\pi i)\xi_{\circleddash}(\sigma)},\\ \breve{\alpha}_{\emptyset}(\sigma)e^{(2\pi i)\xi_{\emptyset}(\sigma)},\\ \breve{\alpha}_{\vartheta}(\sigma)e^{(2\pi i)\xi_{\vartheta}(\sigma)}\end{pmatrix}, \begin{pmatrix}\left[\breve{\alpha}^{-}_{\circleddash}(\sigma)e^{(2\pi i)\xi^{-}_{\circleddash}(\sigma)}, \breve{\alpha}^{+}_{\circleddash}(\sigma)e^{(2\pi i)\xi^{+}_{\circleddash}(\sigma)}\right],\\ \left[\breve{\alpha}^{-}_{\emptyset}(\sigma)e^{(2\pi i)\xi^{-}_{\emptyset}(\sigma)}, \breve{\alpha}^{+}_{\emptyset}(\sigma)e^{(2\pi i)\xi^{+}_{\emptyset}(\sigma)}\right],\\ \left[\breve{\alpha}^{-}_{\vartheta}(\sigma)e^{(2\pi i)\xi^{-}_{\vartheta}(\sigma)}, \breve{\alpha}^{+}_{\vartheta}(\sigma)e^{(2\pi i)\xi^{+}_{\vartheta}(\sigma)}\right]\end{pmatrix}\end{pmatrix},$$
$$\begin{pmatrix}\tau, \begin{pmatrix}\breve{\alpha}_{\circleddash}(\tau)e^{(2\pi i)\xi_{\circleddash}(\tau)},\\ \breve{\alpha}_{\emptyset}(\tau)e^{(2\pi i)\xi_{\emptyset}(\sigma)},\\ \breve{\alpha}_{\vartheta}(\tau)e^{(2\pi i)\xi_{\vartheta}(\tau)}\end{pmatrix}, \begin{pmatrix}\left[\breve{\alpha}^{-}_{\circleddash}(\tau)e^{(2\pi i)\xi^{-}_{\circleddash}(\tau)}, \breve{\alpha}^{+}_{\circleddash}(\tau)e^{(2\pi i)\xi^{+}_{\circleddash}(\tau)}\right],\\ \left[\breve{\alpha}^{-}_{\emptyset}(\tau)e^{(2\pi i)\xi^{-}_{\emptyset}(\tau)}, \breve{\alpha}^{+}_{\emptyset}(\tau)e^{(2\pi i)\xi^{+}_{\emptyset}(\tau)}\right],\\ \left[\breve{\alpha}^{-}_{\vartheta}(\tau)e^{(2\pi i)\xi^{-}_{\vartheta}(\tau)}, \breve{\alpha}^{+}_{\vartheta}(\tau)e^{(2\pi i)\xi^{+}_{\vartheta}(\tau)}\right]\end{pmatrix}\end{pmatrix} \in \breve{U}. \quad (30)$$

Then, there exist

$$\begin{pmatrix}(\sigma,\tau), \begin{pmatrix}\breve{\alpha}_{\circleddash}(\sigma,\tau)e^{(2\pi i)\xi_{\circleddash}(\sigma,\tau)},\\ \breve{\alpha}_{\emptyset}(\sigma,\tau)e^{(2\pi i)\xi_{\emptyset}(\sigma,\tau)},\\ \breve{\alpha}_{\vartheta}(\sigma,\tau)e^{(2\pi i)\xi_{\vartheta}(\sigma,\tau)}\end{pmatrix}, \begin{pmatrix}\left[\breve{\alpha}^{-}_{\circleddash}(\sigma,\tau)e^{(2\pi i)\xi^{-}_{\circleddash}(\sigma,\tau)}, \breve{\alpha}^{+}_{\circleddash}(\sigma,\tau)e^{(2\pi i)\xi^{+}_{\circleddash}(\sigma,\tau)}\right],\\ \left[\breve{\alpha}^{-}_{\emptyset}(\sigma,\tau)e^{(2\pi i)\xi^{-}_{\emptyset}(\sigma,\tau)}, \breve{\alpha}^{+}_{\emptyset}(\sigma,\tau)e^{(2\pi i)\xi^{+}_{\emptyset}(\sigma,\tau)}\right],\\ \left[\breve{\alpha}^{-}_{\vartheta}(\sigma,\tau)e^{(2\pi i)\xi^{-}_{\vartheta}(\sigma,\tau)}, \breve{\alpha}^{+}_{\vartheta}(\sigma,\tau)e^{(2\pi i)\xi^{+}_{\vartheta}(\sigma,\tau)}\right]\end{pmatrix}\end{pmatrix} \in R,$$
$$\text{or}\begin{pmatrix}(\tau,\sigma), \begin{pmatrix}\breve{\alpha}_{\circleddash}(\tau,\sigma)e^{(2\pi i)\xi_{\circleddash}(\tau,\sigma)},\\ \breve{\alpha}_{\emptyset}(\tau,\sigma)e^{(2\pi i)\xi_{\emptyset}(\tau,\sigma)},\\ \breve{\alpha}_{\vartheta}(\tau,\sigma)e^{(2\pi i)\xi_{\vartheta}(\tau,\sigma)}\end{pmatrix}, \begin{pmatrix}\left[\breve{\alpha}^{-}_{\circleddash}(\tau,\sigma)e^{(2\pi i)\xi^{-}_{\circleddash}(\tau,\sigma)}, \breve{\alpha}^{+}_{\circleddash}(\tau,\sigma)e^{(2\pi i)\xi^{+}_{\circleddash}(\tau,\sigma)}\right],\\ \left[\breve{\alpha}^{-}_{\emptyset}(\tau,\sigma)e^{(2\pi i)\xi^{-}_{\emptyset}(\tau,\sigma)}, \breve{\alpha}^{+}_{\emptyset}(\tau,\sigma)e^{(2\pi i)\xi^{+}_{\emptyset}(\tau,\sigma)}\right],\\ \left[\breve{\alpha}^{-}_{\vartheta}(\tau,\sigma)e^{(2\pi i)\xi^{-}_{\vartheta}(\tau,\sigma)}, \breve{\alpha}^{+}_{\vartheta}(\tau,\sigma)e^{(2\pi i)\xi^{+}_{\vartheta}(\tau,\sigma)}\right]\end{pmatrix}\end{pmatrix} \in R. \quad (31)$$

*Definition 23.* A CoCPF relation $R$ is said to be complex cubic picture preorder fuzzy relation (CoCP preorder FR), if it is

(i) CoCP reflexive FR

(ii) CoCP transitive FR

*Definition 24.* A CoCPF relation $R$ is said to be complex cubic picture strict order fuzzy relation (CoCP strict order FR), if it is

(i) CoCP irreflexive FR

(ii) CoCP transitive FR

*Definition 25.* A CoCPF relation $R$ is said to be complex cubic picture linear order fuzzy relation (CoCP linear order FR), if it is

(i) CoCP reflexive FR

(ii) CoCP anti symmetric FR

(iii) CoCP transitive FR

(iv) CoCP complete FR

*Example 3.* Considering the Cartesian product in Equation (20), the following relations are given as

(a) the CoCP equivalence FR $R_1$ on O is

$$R_1 = \left\{ \begin{array}{l} \left( (o_1, o_1), \left( 0.30e^{0.21(2\pi i)}, 0.31e^{0.10(2\pi i)}, 0.32e^{0.57(2\pi i)} \right), \left( \left[0.10e^{0.23(2\pi i)}, 0.20e^{0.36(2\pi i)}\right], \left[0.33e^{0.14(2\pi i)}, 0.49e^{0.21(2\pi i)}\right], \left[0.11e^{0.34(2\pi i)}, 0.29e^{0.41(2\pi i)}\right] \right) \right), \\ \left( (o_1, o_2), \left( 0.30e^{0.20(2\pi i)}, 0.12e^{0.10(2\pi i)}, 0.32e^{0.57(2\pi i)} \right), \left( \left[0.10e^{0.23(2\pi i)}, 0.20e^{0.26(2\pi i)}\right], \left[0.13e^{0.14(2\pi i)}, 0.19e^{0.18(2\pi i)}\right], \left[0.21e^{0.45(2\pi i)}, 0.29e^{0.56(2\pi i)}\right] \right) \right), \\ \left( (o_2, o_1), \left( 0.30e^{0.20(2\pi i)}, 0.12e^{0.10(2\pi i)}, 0.32e^{0.57(2\pi i)} \right), \left( \left[0.10e^{0.23(2\pi i)}, 0.20e^{0.26(2\pi i)}\right], \left[0.13e^{0.14(2\pi i)}, 0.19e^{0.18(2\pi i)}\right], \left[0.21e^{0.45(2\pi i)}, 0.29e^{0.56(2\pi i)}\right] \right) \right), \\ \left( (o_2, o_2), \left( 0.50e^{0.20(2\pi i)}, 0.12e^{0.25(2\pi i)}, 0.32e^{0.55(2\pi i)} \right), \left( \left[0.30e^{0.23(2\pi i)}, 0.43e^{0.26(2\pi i)}\right], \left[0.13e^{0.14(2\pi i)}, 0.19e^{0.18(2\pi i)}\right], \left[0.21e^{0.45(2\pi i)}, 0.29e^{0.56(2\pi i)}\right] \right) \right), \\ \left( (o_3, o_3), \left( 0.11e^{0.09(2\pi i)}, 0.38e^{0.33(2\pi i)}, 0.51e^{0.47(2\pi i)} \right), \left( \left[0.37e^{0.03(2\pi i)}, 0.49e^{0.16(2\pi i)}\right], \left[0.23e^{0.14(2\pi i)}, 0.26e^{0.31(2\pi i)}\right], \left[0.11e^{0.31(2\pi i)}, 0.15e^{0.39(2\pi i)}\right] \right) \right) \end{array} \right\}. \tag{32}$$

(b) the CoCP partial order FR $R_2$ on O is

$$R_2 = \left\{ \begin{array}{l} \left( (o_1, o_1), \left( 0.30e^{0.21(2\pi i)}, 0.31e^{0.10(2\pi i)}, 0.32e^{0.57(2\pi i)} \right), \left( \left[0.10e^{0.23(2\pi i)}, 0.20e^{0.36(2\pi i)}\right], \left[0.33e^{0.14(2\pi i)}, 0.49e^{0.21(2\pi i)}\right], \left[0.11e^{0.34(2\pi i)}, 0.29e^{0.41(2\pi i)}\right] \right) \right), \\ \left( (o_1, o_2), \left( 0.30e^{0.20(2\pi i)}, 0.12e^{0.10(2\pi i)}, 0.32e^{0.57(2\pi i)} \right), \left( \left[0.10e^{0.23(2\pi i)}, 0.20e^{0.26(2\pi i)}\right], \left[0.13e^{0.14(2\pi i)}, 0.19e^{0.18(2\pi i)}\right], \left[0.21e^{0.45(2\pi i)}, 0.29e^{0.56(2\pi i)}\right] \right) \right), \\ \left( (o_1, o_3), \left( 0.11e^{0.09(2\pi i)}, 0.31e^{0.10(2\pi i)}, 0.51e^{0.57(2\pi i)} \right), \left( \left[0.10e^{0.03(2\pi i)}, 0.20e^{0.16(2\pi i)}\right], \left[0.23e^{0.14(2\pi i)}, 0.26e^{0.21(2\pi i)}\right], \left[0.11e^{0.34(2\pi i)}, 0.29e^{0.41(2\pi i)}\right] \right) \right), \\ \left( (o_2, o_2), \left( 0.50e^{0.20(2\pi i)}, 0.12e^{0.25(2\pi i)}, 0.32e^{0.55(2\pi i)} \right), \left( \left[0.30e^{0.23(2\pi i)}, 0.43e^{0.26(2\pi i)}\right], \left[0.13e^{0.14(2\pi i)}, 0.19e^{0.18(2\pi i)}\right], \left[0.21e^{0.45(2\pi i)}, 0.29e^{0.56(2\pi i)}\right] \right) \right), \\ \left( (o_2, o_3), \left( 0.11e^{0.09(2\pi i)}, 0.12e^{0.25(2\pi i)}, 0.51e^{0.55(2\pi i)} \right), \left( \left[0.30e^{0.03(2\pi i)}, 0.43e^{0.26(2\pi i)}\right], \left[0.13e^{0.14(2\pi i)}, 0.19e^{0.18(2\pi i)}\right], \left[0.21e^{0.45(2\pi i)}, 0.29e^{0.56(2\pi i)}\right] \right) \right), \\ \left( (o_3, o_3), \left( 0.11e^{0.09(2\pi i)}, 0.38e^{0.33(2\pi i)}, 0.51e^{0.47(2\pi i)} \right), \left( \left[0.37e^{0.03(2\pi i)}, 0.49e^{0.16(2\pi i)}\right], \left[0.23e^{0.14(2\pi i)}, 0.26e^{0.31(2\pi i)}\right], \left[0.11e^{0.31(2\pi i)}, 0.15e^{0.39(2\pi i)}\right] \right) \right) \end{array} \right\}. \tag{33}$$

(c) the CoCP preorder FR $R_2$ on O is

$$R_3=\left\{\begin{array}{l}\left((o_1,o_1),\left(\begin{array}{c}0.30e^{0.21(2\pi i)},0.31e^{0.10(2\pi i)},\\0.32e^{0.57(2\pi i)}\end{array}\right),\left(\begin{array}{c}\left[0.10e^{0.23(2\pi i)},0.20e^{0.36(2\pi i)}\right],\left[0.33e^{0.14(2\pi i)},0.49e^{0.21(2\pi i)}\right],\\\left[0.11e^{0.34(2\pi i)},0.29e^{0.41(2\pi i)}\right]\end{array}\right)\right),\\\left((o_1,o_2),\left(\begin{array}{c}0.30e^{0.20(2\pi i)},0.12e^{0.10(2\pi i)},\\0.32e^{0.57(2\pi i)}\end{array}\right),\left(\begin{array}{c}\left[0.10e^{0.23(2\pi i)},0.20e^{0.26(2\pi i)}\right],\left[0.13e^{0.14(2\pi i)},0.19e^{0.18(2\pi i)}\right]\\\left[0.21e^{0.45(2\pi i)},0.29e^{0.56(2\pi i)}\right]\end{array}\right)\right),\\\left((o_1,o_3),\left(\begin{array}{c}0.11e^{0.09(2\pi i)},0.31e^{0.10(2\pi i)},\\0.51e^{0.57(2\pi i)}\end{array}\right),\left(\begin{array}{c}\left[0.10e^{0.03(2\pi i)},0.20e^{0.16(2\pi i)}\right],\left[0.23e^{0.14(2\pi i)},0.26e^{0.21(2\pi i)}\right],\\\left[0.11e^{0.34(2\pi i)},0.29e^{0.41(2\pi i)}\right]\end{array}\right)\right),\\\left((o_2,o_2),\left(\begin{array}{c}0.50e^{0.20(2\pi i)},0.12e^{0.25(2\pi i)},\\0.32e^{0.55(2\pi i)}\end{array}\right),\left(\begin{array}{c}\left[0.30e^{0.23(2\pi i)},0.43e^{0.26(2\pi i)}\right],\left[0.13e^{0.14(2\pi i)},0.19e^{0.18(2\pi i)}\right]\\\left[0.21e^{0.45(2\pi i)},0.29e^{0.56(2\pi i)}\right]\end{array}\right)\right),\\\left((o_2,o_3),\left(\begin{array}{c}0.11e^{0.09(2\pi i)},0.12e^{0.25(2\pi i)},\\0.51e^{0.55(2\pi i)}\end{array}\right),\left(\begin{array}{c}\left[0.30e^{0.03(2\pi i)},0.43e^{0.26(2\pi i)}\right],\left[0.13e^{0.14(2\pi i)},0.19e^{0.18(2\pi i)}\right],\\\left[0.21e^{0.45(2\pi i)},0.29e^{0.56(2\pi i)}\right]\end{array}\right)\right),\\\left((o_3,o_3),\left(\begin{array}{c}0.11e^{0.09(2\pi i)},0.38e^{0.33(2\pi i)},\\0.51e^{0.47(2\pi i)}\end{array}\right),\left(\begin{array}{c}\left[0.37e^{0.03(2\pi i)},0.49e^{0.16(2\pi i)}\right],\left[0.23e^{0.14(2\pi i)},0.26e^{0.31(2\pi i)}\right],\\\left[0.11e^{0.31(2\pi i)},0.15e^{0.39(2\pi i)}\right]\end{array}\right)\right)\end{array}\right\}. \tag{34}$$

*Definition 26*. The converse of a CoCPF relation $R$ is defined as

$$R^c=\left\{\begin{array}{l}\left((\tau,\sigma),\left(\begin{array}{c}\breve{\alpha}_{\ddot{\varpi}}(\tau,\sigma)e^{(2\pi i)\xi_{\ddot{\varpi}}(\tau,\sigma)},\\\breve{\alpha}_{\varnothing}(\tau,\sigma)e^{(2\pi i)\xi_{\varnothing}(\tau,\sigma)},\\\breve{\alpha}_{\vartheta}(\tau,\sigma)e^{(2\pi i)\xi_{\vartheta}(\tau,\sigma)}\end{array}\right),\left(\begin{array}{c}\left[\breve{\alpha}^-_{\ddot{\varpi}}(\tau,\sigma)e^{(2\pi i)\xi^-_{\ddot{\varpi}}(\tau,\sigma)},\breve{\alpha}^+_{\ddot{\varpi}}(\tau,\sigma)e^{(2\pi i)\xi^+_{\ddot{\varpi}}(\tau,\sigma)}\right],\\\left[\breve{\alpha}^-_{\varnothing}(\tau,\sigma)e^{(2\pi i)\xi^-_{\varnothing}(\tau,\sigma)},\breve{\alpha}^+_{\varnothing}(\tau,\sigma)e^{(2\pi i)\xi^+_{\varnothing}(\tau,\sigma)}\right],\\\left[\breve{\alpha}^-_{\vartheta}(\tau,\sigma)e^{(2\pi i)\xi^-_{\vartheta}(\tau,\sigma)},\breve{\alpha}^+_{\vartheta}(\tau,\sigma)e^{(2\pi i)\xi^+_{\vartheta}(\tau,\sigma)}\right]\end{array}\right)\right):\\\forall\left((\sigma,\tau),\left(\begin{array}{c}\breve{\alpha}_{\ddot{\varpi}}(\sigma,\tau)e^{(2\pi i)\xi_{\ddot{\varpi}}(\sigma,\tau)},\\\breve{\alpha}_{\varnothing}(\sigma,\tau)e^{(2\pi i)\xi_{\varnothing}(\sigma,\tau)},\\\breve{\alpha}_{\vartheta}(\sigma,\tau)e^{(2\pi i)\xi_{\vartheta}(\sigma,\tau)}\end{array}\right),\left(\begin{array}{c}\left[\breve{\alpha}^-_{\ddot{\varpi}}(\sigma,\tau)e^{(2\pi i)\xi^-_{\ddot{\varpi}}(\sigma,\tau)},\breve{\alpha}^+_{\ddot{\varpi}}(\sigma,\tau)e^{(2\pi i)\xi^+_{\ddot{\varpi}}(\sigma,\tau)}\right],\\\left[\breve{\alpha}^-_{\varnothing}(\sigma,\tau)e^{(2\pi i)\xi^-_{\varnothing}(\sigma,\tau)},\breve{\alpha}^+_{\varnothing}(\sigma,\tau)e^{(2\pi i)\xi^+_{\varnothing}(\sigma,\tau)}\right],\\\left[\breve{\alpha}^-_{\vartheta}(\sigma,\tau)e^{(2\pi i)\xi^-_{\vartheta}(\sigma,\tau)},\breve{\alpha}^+_{\vartheta}(\sigma,\tau)e^{(2\pi i)\xi^+_{\vartheta}(\sigma,\tau)}\right]\end{array}\right)\right)\in R\end{array}\right\}. \tag{35}$$

*Definition 27.* Let $R_1$and $R_2$ be two CoCPF relations on a CoCPFS $\check{U}$. Then, the complex cubic picture composite fuzzy relation $R_1 \circ R_2$ is defined as follows:

For any

$$\begin{gathered}\left( (\sigma,\tau), \begin{pmatrix} \check{\alpha}_{\ddot{\varpi}}(\sigma,\tau)e^{(2\pi i)\xi_{\ddot{\varpi}}(\sigma,\tau)}, \\ \check{\alpha}_{\emptyset}(\sigma,\tau)e^{(2\pi i)\xi_{\emptyset}(\sigma,\tau)}, \\ \check{\alpha}_{\vartheta}(\sigma,\tau)e^{(2\pi i)\xi_{\vartheta}(\sigma,\tau)} \end{pmatrix}, \begin{pmatrix} \left[\check{\alpha}^-_{\ddot{\varpi}}(\sigma,\tau)e^{(2\pi i)\xi^-_{\ddot{\varpi}}(\sigma,\tau)}, \check{\alpha}^+_{\ddot{\varpi}}(\sigma,\tau)e^{(2\pi i)\xi^+_{\ddot{\varpi}}(\sigma,\tau)}\right], \\ \left[\check{\alpha}^-_{\emptyset}(\sigma,\tau)e^{(2\pi i)\xi^-_{\emptyset}(\sigma,\tau)}, \check{\alpha}^+_{\emptyset}(\sigma,\tau)e^{(2\pi i)\xi^+_{\emptyset}(\sigma,\tau)}\right], \\ \left[\check{\alpha}^-_{\vartheta}(\sigma,\tau)e^{(2\pi i)\xi^-_{\vartheta}(\sigma,\tau)}, \check{\alpha}^+_{\vartheta}(\sigma,\tau)e^{(2\pi i)\xi^+_{\vartheta}(\sigma,\tau)}\right] \end{pmatrix} \right) \in R_{1.} \\ \left( (\tau,\text{ɰ}), \begin{pmatrix} \check{\alpha}_{\ddot{\varpi}}(\tau,\sigma)e^{(2\pi i)\xi_{\ddot{\varpi}}(\tau,\sigma)}, \\ \check{\alpha}_{\emptyset}(\tau,\sigma)e^{(2\pi i)\xi_{\emptyset}(\tau,\sigma)}, \\ \check{\alpha}_{\vartheta}(\tau,\sigma)e^{(2\pi i)\xi_{\vartheta}(\tau,\sigma)} \end{pmatrix}, \begin{pmatrix} \left[\check{\alpha}^-_{\ddot{\varpi}}(\tau,\sigma)e^{(2\pi i)\xi^-_{\ddot{\varpi}}(\tau,\sigma)}, \check{\alpha}^+_{\ddot{\varpi}}(\tau,\sigma)e^{(2\pi i)\xi^+_{\ddot{\varpi}}(\tau,\sigma)}\right], \\ \left[\check{\alpha}^-_{\emptyset}(\tau,\sigma)e^{(2\pi i)\xi^-_{\emptyset}(\tau,\sigma)}, \check{\alpha}^+_{\emptyset}(\tau,\sigma)e^{(2\pi i)\xi^+_{\emptyset}(\tau,\sigma)}\right], \\ \left[\check{\alpha}^-_{\vartheta}(\tau,\sigma)e^{(2\pi i)\xi^-_{\vartheta}(\tau,\sigma)}, \check{\alpha}^+_{\vartheta}(\tau,\sigma)e^{(2\pi i)\xi^+_{\vartheta}(\tau,\sigma)}\right] \end{pmatrix} \right) \in R_2. \end{gathered} \tag{36}$$

Then,

$$\left( (\sigma,\text{ɰ}), \begin{pmatrix} \check{\alpha}_{\ddot{\varpi}}(\sigma,\text{ɰ})e^{(2\pi i)\xi_{\ddot{\varpi}}(\sigma,\text{ɰ})}, \\ \check{\alpha}_{\emptyset}(\sigma,\text{ɰ})e^{(2\pi i)\xi_{\emptyset}(\sigma,\text{ɰ})}, \\ \check{\alpha}_{\vartheta}(\sigma,\text{ɰ})e^{(2\pi i)\xi_{\vartheta}(\sigma,\text{ɰ})} \end{pmatrix}, \begin{pmatrix} \left[\check{\alpha}^-_{\ddot{\varpi}}(\sigma,\text{ɰ})e^{(2\pi i)\xi^-_{\ddot{\varpi}}(\sigma,\text{ɰ})}, \check{\alpha}^+_{\ddot{\varpi}}(\sigma,\text{ɰ})e^{(2\pi i)\xi^+_{\ddot{\varpi}}(\sigma,\text{ɰ})}\right], \\ \left[\check{\alpha}^-_{\emptyset}(\sigma,\text{ɰ})e^{(2\pi i)\xi^-_{\emptyset}(\sigma,\text{ɰ})}, \check{\alpha}^+_{\emptyset}(\sigma,\text{ɰ})e^{(2\pi i)\xi^+_{\emptyset}(\sigma,\text{ɰ})}\right], \\ \left[\check{\alpha}^-_{\vartheta}(\sigma,\text{ɰ})e^{(2\pi i)\xi^-_{\vartheta}(\sigma,\text{ɰ})}, \check{\alpha}^+_{\vartheta}(\sigma,\text{ɰ})e^{(2\pi i)\xi^+_{\vartheta}(\sigma,\text{ɰ})}\right] \end{pmatrix} \right) \in R_1 \circ R_2. \tag{37}$$

*Definition 28.* For CoCPF equivalence fuzzy relation $R$, the CoCPF equivalence class of $\sigma$ modulo $R$ is defined as

$$R[\sigma] = \left\{ \begin{array}{c} \left( \tau, \begin{pmatrix} \check{\alpha}_{\ddot{\varpi}}(\tau)e^{(2\pi i)\xi_{\ddot{\varpi}}(\tau)}, \\ \check{\alpha}_{\emptyset}(\tau)e^{(2\pi i)\xi_{\emptyset}(\sigma)}, \\ \check{\alpha}_{\vartheta}(\tau)e^{(2\pi i)\xi_{\vartheta}(\tau)} \end{pmatrix}, \begin{pmatrix} \left[\check{\alpha}^-_{\ddot{\varpi}}(\tau)e^{(2\pi i)\xi^-_{\ddot{\varpi}}(\tau)}, \check{\alpha}^+_{\ddot{\varpi}}(\tau)e^{(2\pi i)\xi^+_{\ddot{\varpi}}(\tau)}\right], \\ \left[\check{\alpha}^-_{\emptyset}(\tau)e^{(2\pi i)\xi^-_{\emptyset}(\tau)}, \check{\alpha}^+_{\emptyset}(\tau)e^{(2\pi i)\xi^+_{\emptyset}(\tau)}\right], \\ \left[\check{\alpha}^-_{\vartheta}(\tau)e^{(2\pi i)\xi^-_{\vartheta}(\tau)}, \check{\alpha}^+_{\vartheta}(\tau)e^{(2\pi i)\xi^+_{\vartheta}(\tau)}\right] \end{pmatrix} \right) : \\ \left( (\tau,\sigma), \begin{pmatrix} \check{\alpha}_{\ddot{\varpi}}(\tau,\sigma)e^{(2\pi i)\xi_{\ddot{\varpi}}(\tau,\sigma)}, \\ \check{\alpha}_{\emptyset}(\tau,\sigma)e^{(2\pi i)\xi_{\emptyset}(\tau,\sigma)}, \\ \check{\alpha}_{\vartheta}(\tau,\sigma)e^{(2\pi i)\xi_{\vartheta}(\tau,\sigma)} \end{pmatrix}, \begin{pmatrix} \left[\check{\alpha}^-_{\ddot{\varpi}}(\tau,\sigma)e^{(2\pi i)\xi^-_{\ddot{\varpi}}(\tau,\sigma)}, \check{\alpha}^+_{\ddot{\varpi}}(\tau,\sigma)e^{(2\pi i)\xi^+_{\ddot{\varpi}}(\tau,\sigma)}\right], \\ \left[\check{\alpha}^-_{\emptyset}(\tau,\sigma)e^{(2\pi i)\xi^-_{\emptyset}(\tau,\sigma)}, \check{\alpha}^+_{\emptyset}(\tau,\sigma)e^{(2\pi i)\xi^+_{\emptyset}(\tau,\sigma)}\right], \\ \left[\check{\alpha}^-_{\vartheta}(\tau,\sigma)e^{(2\pi i)\xi^-_{\vartheta}(\tau,\sigma)}, \check{\alpha}^+_{\vartheta}(\tau,\sigma)e^{(2\pi i)\xi^+_{\vartheta}(\tau,\sigma)}\right] \end{pmatrix} \right) \in R \end{array} \right\}. \tag{38}$$

**Theorem 29.** *A CoCPFR R is a CoCP symmetric FR on a CoCPFS $\check{U}$ if $R = R^{c}$.*

*Proof.* Necessity condition:

Suppose that $R$ is a CoCP symmetric FR on a CoCPFS $\check{U}$. Then,

$$\left((\sigma,\tau),\left(\begin{array}{c}\breve{\alpha}_{\bar{\ddot{\varpi}}}(\sigma,\tau)e^{(2\pi i)\xi_{\bar{\ddot{\varpi}}}(\sigma,\tau)},\\ \breve{\alpha}_{\emptyset}(\sigma,\tau)e^{(2\pi i)\xi_{\emptyset}(\sigma,\tau)},\\ \breve{\alpha}_{\vartheta}(\sigma,\tau)e^{(2\pi i)\xi_{\vartheta}(\sigma,\tau)}\end{array}\right),\left(\begin{array}{c}\left[\breve{\alpha}^{-}_{\bar{\ddot{\varpi}}}(\sigma,\tau)e^{(2\pi i)\xi^{-}_{\bar{\ddot{\varpi}}}(\sigma,\tau)},\breve{\alpha}^{+}_{\bar{\ddot{\varpi}}}(\sigma,\tau)e^{(2\pi i)\xi^{+}_{\bar{\ddot{\varpi}}}(\sigma,\tau)}\right],\\ \left[\breve{\alpha}^{-}_{\emptyset}(\sigma,\tau)e^{(2\pi i)\xi^{-}_{\emptyset}(\sigma,\tau)},\breve{\alpha}^{+}_{\emptyset}(\sigma,\tau)e^{(2\pi i)\xi^{+}_{\emptyset}(\sigma,\tau)}\right],\\ \left[\breve{\alpha}^{-}_{\vartheta}(\sigma,\tau)e^{(2\pi i)\xi^{-}_{\vartheta}(\sigma,\tau)},\breve{\alpha}^{+}_{\vartheta}(\sigma,\tau)e^{(2\pi i)\xi^{+}_{\vartheta}(\sigma,\tau)}\right]\end{array}\right)\right)$$
$$\cdot \in R \Longleftrightarrow \left((\tau,\sigma),\left(\begin{array}{c}\breve{\alpha}_{\bar{\ddot{\varpi}}}(\tau,\sigma)e^{(2\pi i)\xi_{\bar{\ddot{\varpi}}}(\tau,\sigma)},\\ \breve{\alpha}_{\emptyset}(\tau,\sigma)e^{(2\pi i)\xi_{\emptyset}(\tau,\sigma)},\\ \breve{\alpha}_{\vartheta}(\tau,\sigma)e^{(2\pi i)\xi_{\vartheta}(\tau,\sigma)}\end{array}\right),\left(\begin{array}{c}\left[\breve{\alpha}^{-}_{\bar{\ddot{\varpi}}}(\tau,\sigma)e^{(2\pi i)\xi^{-}_{\bar{\ddot{\varpi}}}(\tau,\sigma)},\breve{\alpha}^{+}_{\bar{\ddot{\varpi}}}(\tau,\sigma)e^{(2\pi i)\xi^{+}_{\bar{\ddot{\varpi}}}(\tau,\sigma)}\right],\\ \left[\breve{\alpha}^{-}_{\emptyset}(\tau,\sigma)e^{(2\pi i)\xi^{-}_{\emptyset}(\tau,\sigma)},\breve{\alpha}^{+}_{\emptyset}(\tau,\sigma)e^{(2\pi i)\xi^{+}_{\emptyset}(\tau,\sigma)}\right],\\ \left[\breve{\alpha}^{-}_{\vartheta}(\tau,\sigma)e^{(2\pi i)\xi^{-}_{\vartheta}(\tau,\sigma)},\breve{\alpha}^{+}_{\vartheta}(\tau,\sigma)e^{(2\pi i)\xi^{+}_{\vartheta}(\tau,\sigma)}\right]\end{array}\right)\right) \in R. \tag{39}$$

However,

$$\left((\tau,\sigma),\left(\begin{array}{c}\breve{\alpha}_{\bar{\ddot{\varpi}}}(\tau,\sigma)e^{(2\pi i)\xi_{\bar{\ddot{\varpi}}}(\tau,\sigma)},\\ \breve{\alpha}_{\emptyset}(\tau,\sigma)e^{(2\pi i)\xi_{\emptyset}(\tau,\sigma)},\\ \breve{\alpha}_{\vartheta}(\tau,\sigma)e^{(2\pi i)\xi_{\vartheta}(\tau,\sigma)}\end{array}\right),\left(\begin{array}{c}\left[\breve{\alpha}^{-}_{\bar{\ddot{\varpi}}}(\tau,\sigma)e^{(2\pi i)\xi^{-}_{\bar{\ddot{\varpi}}}(\tau,\sigma)},\breve{\alpha}^{+}_{\bar{\ddot{\varpi}}}(\tau,\sigma)e^{(2\pi i)\xi^{+}_{\bar{\ddot{\varpi}}}(\tau,\sigma)}\right],\\ \left[\breve{\alpha}^{-}_{\emptyset}(\tau,\sigma)e^{(2\pi i)\xi^{-}_{\emptyset}(\tau,\sigma)},\breve{\alpha}^{+}_{\emptyset}(\tau,\sigma)e^{(2\pi i)\xi^{+}_{\emptyset}(\tau,\sigma)}\right],\\ \left[\breve{\alpha}^{-}_{\vartheta}(\tau,\sigma)e^{(2\pi i)\xi^{-}_{\vartheta}(\tau,\sigma)},\breve{\alpha}^{+}_{\vartheta}(\tau,\sigma)e^{(2\pi i)\xi^{+}_{\vartheta}(\tau,\sigma)}\right]\end{array}\right)\right) \in R^{c}$$
$$\Longrightarrow R = R^{c}. \tag{40}$$

Sufficient condition:

Let $R = R^{c}$, then

$$\left((\sigma,\tau),\left(\begin{array}{c}\breve{\alpha}_{\bar{\ddot{\varpi}}}(\sigma,\tau)e^{(2\pi i)\xi_{\bar{\ddot{\varpi}}}(\sigma,\tau)},\\ \breve{\alpha}_{\emptyset}(\sigma,\tau)e^{(2\pi i)\xi_{\emptyset}(\sigma,\tau)},\\ \breve{\alpha}_{\vartheta}(\sigma,\tau)e^{(2\pi i)\xi_{\vartheta}(\sigma,\tau)}\end{array}\right),\left(\begin{array}{c}\left[\breve{\alpha}^{-}_{\bar{\ddot{\varpi}}}(\sigma,\tau)e^{(2\pi i)\xi^{-}_{\bar{\ddot{\varpi}}}(\sigma,\tau)},\breve{\alpha}^{+}_{\bar{\ddot{\varpi}}}(\sigma,\tau)e^{(2\pi i)\xi^{+}_{\bar{\ddot{\varpi}}}(\sigma,\tau)}\right],\\ \left[\breve{\alpha}^{-}_{\emptyset}(\sigma,\tau)e^{(2\pi i)\xi^{-}_{\emptyset}(\sigma,\tau)},\breve{\alpha}^{+}_{\emptyset}(\sigma,\tau)e^{(2\pi i)\xi^{+}_{\emptyset}(\sigma,\tau)}\right],\\ \left[\breve{\alpha}^{-}_{\vartheta}(\sigma,\tau)e^{(2\pi i)\xi^{-}_{\vartheta}(\sigma,\tau)},\breve{\alpha}^{+}_{\vartheta}(\sigma,\tau)e^{(2\pi i)\xi^{+}_{\vartheta}(\sigma,\tau)}\right]\end{array}\right)\right) \in R. \tag{41}$$

We have

$$\left((\tau,\sigma),\left(\begin{array}{c}\breve{\alpha}_{\bar{\ddot{\varpi}}}(\tau,\sigma)e^{(2\pi i)\xi_{\bar{\ddot{\varpi}}}(\tau,\sigma)},\\ \breve{\alpha}_{\emptyset}(\tau,\sigma)e^{(2\pi i)\xi_{\emptyset}(\tau,\sigma)},\\ \breve{\alpha}_{\vartheta}(\tau,\sigma)e^{(2\pi i)\xi_{\vartheta}(\tau,\sigma)}\end{array}\right),\left(\begin{array}{c}\left[\breve{\alpha}^{-}_{\bar{\ddot{\varpi}}}(\tau,\sigma)e^{(2\pi i)\xi^{-}_{\bar{\ddot{\varpi}}}(\tau,\sigma)},\breve{\alpha}^{+}_{\bar{\ddot{\varpi}}}(\tau,\sigma)e^{(2\pi i)\xi^{+}_{\bar{\ddot{\varpi}}}(\tau,\sigma)}\right],\\ \left[\breve{\alpha}^{-}_{\emptyset}(\tau,\sigma)e^{(2\pi i)\xi^{-}_{\emptyset}(\tau,\sigma)},\breve{\alpha}^{+}_{\emptyset}(\tau,\sigma)e^{(2\pi i)\xi^{+}_{\emptyset}(\tau,\sigma)}\right],\\ \left[\breve{\alpha}^{-}_{\vartheta}(\tau,\sigma)e^{(2\pi i)\xi^{-}_{\vartheta}(\tau,\sigma)},\breve{\alpha}^{+}_{\vartheta}(\tau,\sigma)e^{(2\pi i)\xi^{+}_{\vartheta}(\tau,\sigma)}\right]\end{array}\right)\right) \in R^{c}. \tag{42}$$

This implies that

$$\left( (\tau,\sigma), \begin{pmatrix} \check{\alpha}_{\bar{\omega}}(\tau,\sigma)e^{(2\pi i)\xi_{\bar{\omega}}(\tau,\sigma)}, \\ \check{\alpha}_{\emptyset}(\tau,\sigma)e^{(2\pi i)\xi_{\emptyset}(\tau,\sigma)}, \\ \check{\alpha}_{\vartheta}(\tau,\sigma)e^{(2\pi i)\xi_{\vartheta}(\tau,\sigma)} \end{pmatrix}, \begin{pmatrix} \left[\check{\alpha}^-_{\bar{\omega}}(\tau,\sigma)e^{(2\pi i)\xi^-_{\bar{\omega}}(\tau,\sigma)}, \check{\alpha}^+_{\bar{\omega}}(\tau,\sigma)e^{(2\pi i)\xi^+_{\bar{\omega}}(\tau,\sigma)}\right], \\ \left[\check{\alpha}^-_{\emptyset}(\tau,\sigma)e^{(2\pi i)\xi^-_{\emptyset}(\tau,\sigma)}, \check{\alpha}^+_{\emptyset}(\tau,\sigma)e^{(2\pi i)\xi^+_{\emptyset}(\tau,\sigma)}\right], \\ \left[\check{\alpha}^-_{\vartheta}(\tau,\sigma)e^{(2\pi i)\xi^-_{\vartheta}(\tau,\sigma)}, \check{\alpha}^+_{\vartheta}(\tau,\sigma)e^{(2\pi i)\xi^+_{\vartheta}(\tau,\sigma)}\right] \end{pmatrix} \right) \in R. \tag{43}$$

This is required result. □

**Theorem 30.** *A CoCPFR $R$ is a CoCP transitive FR on a CoCPFS $\check{U}$ if $R \circ R \subseteq R$.*

*Proof.* Necessity condition

Assume that $R$ is an CoCP transitive FR on CoCPFS $\check{U}$. Let

$$\left( (\sigma,ɯ), \begin{pmatrix} \check{\alpha}_{\bar{\omega}}(\sigma,ɯ)e^{(2\pi i)\xi_{\bar{\omega}}(\sigma,ɯ)}, \\ \check{\alpha}_{\emptyset}(\sigma,ɯ)e^{(2\pi i)\xi_{\emptyset}(\sigma,ɯ)}, \\ \check{\alpha}_{\vartheta}(\sigma,ɯ)e^{(2\pi i)\xi_{\vartheta}(\sigma,ɯ)} \end{pmatrix}, \begin{pmatrix} \left[\check{\alpha}^-_{\bar{\omega}}(\sigma,ɯ)e^{(2\pi i)\xi^-_{\bar{\omega}}(\sigma,ɯ)}, \check{\alpha}^+_{\bar{\omega}}(\sigma,ɯ)e^{(2\pi i)\xi^+_{\bar{\omega}}(\sigma,ɯ)}\right], \\ \left[\check{\alpha}^-_{\emptyset}(\sigma,ɯ)e^{(2\pi i)\xi^-_{\emptyset}(\sigma,ɯ)}, \check{\alpha}^+_{\emptyset}(\sigma,ɯ)e^{(2\pi i)\xi^+_{\emptyset}(\sigma,ɯ)}\right], \\ \left[\check{\alpha}^-_{\vartheta}(\sigma,ɯ)e^{(2\pi i)\xi^-_{\vartheta}(\sigma,ɯ)}, \check{\alpha}^+_{\vartheta}(\sigma,ɯ)e^{(2\pi i)\xi^+_{\vartheta}(\sigma,ɯ)}\right] \end{pmatrix} \right) \in R \circ R. \tag{44}$$

Then by definition of transitivity

$$\begin{aligned} &\left( (\sigma,\tau), \begin{pmatrix} \check{\alpha}_{\bar{\omega}}(\sigma,\tau)e^{(2\pi i)\xi_{\bar{\omega}}(\sigma,\tau)}, \\ \check{\alpha}_{\emptyset}(\sigma,\tau)e^{(2\pi i)\xi_{\emptyset}(\sigma,\tau)}, \\ \check{\alpha}_{\vartheta}(\sigma,\tau)e^{(2\pi i)\xi_{\vartheta}(\sigma,\tau)} \end{pmatrix}, \begin{pmatrix} \left[\check{\alpha}^-_{\bar{\omega}}(\sigma,\tau)e^{(2\pi i)\xi^-_{\bar{\omega}}(\sigma,\tau)}, \check{\alpha}^+_{\bar{\omega}}(\sigma,\tau)e^{(2\pi i)\xi^+_{\bar{\omega}}(\sigma,\tau)}\right], \\ \left[\check{\alpha}^-_{\emptyset}(\sigma,\tau)e^{(2\pi i)\xi^-_{\emptyset}(\sigma,\tau)}, \check{\alpha}^+_{\emptyset}(\sigma,\tau)e^{(2\pi i)\xi^+_{\emptyset}(\sigma,\tau)}\right], \\ \left[\check{\alpha}^-_{\vartheta}(\sigma,\tau)e^{(2\pi i)\xi^-_{\vartheta}(\sigma,\tau)}, \check{\alpha}^+_{\vartheta}(\sigma,\tau)e^{(2\pi i)\xi^+_{\vartheta}(\sigma,\tau)}\right] \end{pmatrix} \right) \in R. \\ &\left( (\tau,ɯ), \begin{pmatrix} \check{\alpha}_{\bar{\omega}}(\tau,\sigma)e^{(2\pi i)\xi_{\bar{\omega}}(\tau,\sigma)}, \\ \check{\alpha}_{\emptyset}(\tau,\sigma)e^{(2\pi i)\xi_{\emptyset}(\tau,\sigma)}, \\ \check{\alpha}_{\vartheta}(\tau,\sigma)e^{(2\pi i)\xi_{\vartheta}(\tau,\sigma)} \end{pmatrix}, \begin{pmatrix} \left[\check{\alpha}^-_{\bar{\omega}}(\tau,\sigma)e^{(2\pi i)\xi^-_{\bar{\omega}}(\tau,\sigma)}, \check{\alpha}^+_{\bar{\omega}}(\tau,\sigma)e^{(2\pi i)\xi^+_{\bar{\omega}}(\tau,\sigma)}\right], \\ \left[\check{\alpha}^-_{\emptyset}(\tau,\sigma)e^{(2\pi i)\xi^-_{\emptyset}(\tau,\sigma)}, \check{\alpha}^+_{\emptyset}(\tau,\sigma)e^{(2\pi i)\xi^+_{\emptyset}(\tau,\sigma)}\right], \\ \left[\check{\alpha}^-_{\vartheta}(\tau,\sigma)e^{(2\pi i)\xi^-_{\vartheta}(\tau,\sigma)}, \check{\alpha}^+_{\vartheta}(\tau,\sigma)e^{(2\pi i)\xi^+_{\vartheta}(\tau,\sigma)}\right] \end{pmatrix} \right) \in R. \end{aligned} \tag{45}$$

This implies

$$\left( (\sigma,ɯ), \begin{pmatrix} \check{\alpha}_{\bar{\omega}}(\sigma,ɯ)e^{(2\pi i)\xi_{\bar{\omega}}(\sigma,ɯ)}, \\ \check{\alpha}_{\emptyset}(\sigma,ɯ)e^{(2\pi i)\xi_{\emptyset}(\sigma,ɯ)}, \\ \check{\alpha}_{\vartheta}(\sigma,ɯ)e^{(2\pi i)\xi_{\vartheta}(\sigma,ɯ)} \end{pmatrix}, \begin{pmatrix} \left[\check{\alpha}^-_{\bar{\omega}}(\sigma,ɯ)e^{(2\pi i)\xi^-_{\bar{\omega}}(\sigma,ɯ)}, \check{\alpha}^+_{\bar{\omega}}(\sigma,ɯ)e^{(2\pi i)\xi^+_{\bar{\omega}}(\sigma,ɯ)}\right], \\ \left[\check{\alpha}^-_{\emptyset}(\sigma,ɯ)e^{(2\pi i)\xi^-_{\emptyset}(\sigma,ɯ)}, \check{\alpha}^+_{\emptyset}(\sigma,ɯ)e^{(2\pi i)\xi^+_{\emptyset}(\sigma,ɯ)}\right], \\ \left[\check{\alpha}^-_{\vartheta}(\sigma,ɯ)e^{(2\pi i)\xi^-_{\vartheta}(\sigma,ɯ)}, \check{\alpha}^+_{\vartheta}(\sigma,ɯ)e^{(2\pi i)\xi^+_{\vartheta}(\sigma,ɯ)}\right] \end{pmatrix} \right) \in R \Longrightarrow R \circ R \subseteq R. \tag{46}$$

Conversely, assume that $R\circ R\subseteq R$, then

$$\left((\sigma,\tau),\begin{pmatrix}\check{\alpha}_{\varpi}(\sigma,\tau)e^{(2\pi i)\xi_{\varpi}(\sigma,\tau)},\\ \check{\alpha}_{\emptyset}(\sigma,\tau)e^{(2\pi i)\xi_{\emptyset}(\sigma,\tau)},\\ \check{\alpha}_{\vartheta}(\sigma,\tau)e^{(2\pi i)\xi_{\vartheta}(\sigma,\tau)}\end{pmatrix},\begin{pmatrix}\left[\check{\alpha}^{-}_{\varpi}(\sigma,\tau)e^{(2\pi i)\xi^{-}_{\varpi}(\sigma,\tau)},\check{\alpha}^{+}_{\varpi}(\sigma,\tau)e^{(2\pi i)\xi^{+}_{\varpi}(\sigma,\tau)}\right],\\ \left[\check{\alpha}^{-}_{\emptyset}(\sigma,\tau)e^{(2\pi i)\xi^{-}_{\emptyset}(\sigma,\tau)},\check{\alpha}^{+}_{\emptyset}(\sigma,\tau)e^{(2\pi i)\xi^{+}_{\emptyset}(\sigma,\tau)}\right],\\ \left[\check{\alpha}^{-}_{\vartheta}(\sigma,\tau)e^{(2\pi i)\xi^{-}_{\vartheta}(\sigma,\tau)},\check{\alpha}^{+}_{\vartheta}(\sigma,\tau)e^{(2\pi i)\xi^{+}_{\vartheta}(\sigma,\tau)}\right]\end{pmatrix}\right)\in R. \tag{47}$$

And

$$\left((\tau,\text{ɰ}),\begin{pmatrix}\check{\alpha}_{\varpi}(\tau,\sigma)e^{(2\pi i)\xi_{\varpi}(\tau,\sigma)},\\ \check{\alpha}_{\emptyset}(\tau,\sigma)e^{(2\pi i)\xi_{\emptyset}(\tau,\sigma)},\\ \check{\alpha}_{\vartheta}(\tau,\sigma)e^{(2\pi i)\xi_{\vartheta}(\tau,\sigma)}\end{pmatrix},\begin{pmatrix}\left[\check{\alpha}^{-}_{\varpi}(\tau,\sigma)e^{(2\pi i)\xi^{-}_{\varpi}(\tau,\sigma)},\check{\alpha}^{+}_{\varpi}(\tau,\sigma)e^{(2\pi i)\xi^{+}_{\varpi}(\tau,\sigma)}\right],\\ \left[\check{\alpha}^{-}_{\emptyset}(\tau,\sigma)e^{(2\pi i)\xi^{-}_{\emptyset}(\tau,\sigma)},\check{\alpha}^{+}_{\emptyset}(\tau,\sigma)e^{(2\pi i)\xi^{+}_{\emptyset}(\tau,\sigma)}\right],\\ \left[\check{\alpha}^{-}_{\vartheta}(\tau,\sigma)e^{(2\pi i)\xi^{-}_{\vartheta}(\tau,\sigma)},\check{\alpha}^{+}_{\vartheta}(\tau,\sigma)e^{(2\pi i)\xi^{+}_{\vartheta}(\tau,\sigma)}\right]\end{pmatrix}\right)\in R. \tag{48}$$

This implies

$$\left((\sigma,\text{ɰ}),\begin{pmatrix}\check{\alpha}_{\varpi}(\sigma,\text{ɰ})e^{(2\pi i)\xi_{\varpi}(\sigma,\text{ɰ})},\\ \check{\alpha}_{\emptyset}(\sigma,\text{ɰ})e^{(2\pi i)\xi_{\emptyset}(\sigma,\text{ɰ})},\\ \check{\alpha}_{\vartheta}(\sigma,\text{ɰ})e^{(2\pi i)\xi_{\vartheta}(\sigma,\text{ɰ})}\end{pmatrix},\begin{pmatrix}\left[\check{\alpha}^{-}_{\varpi}(\sigma,\text{ɰ})e^{(2\pi i)\xi^{-}_{\varpi}(\sigma,\text{ɰ})},\check{\alpha}^{+}_{\varpi}(\sigma,\text{ɰ})e^{(2\pi i)\xi^{+}_{\varpi}(\sigma,\text{ɰ})}\right],\\ \left[\check{\alpha}^{-}_{\emptyset}(\sigma,\text{ɰ})e^{(2\pi i)\xi^{-}_{\emptyset}(\sigma,\text{ɰ})},\check{\alpha}^{+}_{\emptyset}(\sigma,\text{ɰ})e^{(2\pi i)\xi^{+}_{\emptyset}(\sigma,\text{ɰ})}\right],\\ \left[\check{\alpha}^{-}_{\vartheta}(\sigma,\text{ɰ})e^{(2\pi i)\xi^{-}_{\vartheta}(\sigma,\text{ɰ})},\check{\alpha}^{+}_{\vartheta}(\sigma,\text{ɰ})e^{(2\pi i)\xi^{+}_{\vartheta}(\sigma,\text{ɰ})}\right]\end{pmatrix}\right)\in R\circ R\subseteq R. \tag{49}$$

So,

$$\left((\sigma,\text{ɰ}),\begin{pmatrix}\check{\alpha}_{\varpi}(\sigma,\text{ɰ})e^{(2\pi i)\xi_{\varpi}(\sigma,\text{ɰ})},\\ \check{\alpha}_{\emptyset}(\sigma,\text{ɰ})e^{(2\pi i)\xi_{\emptyset}(\sigma,\text{ɰ})},\\ \check{\alpha}_{\vartheta}(\sigma,\text{ɰ})e^{(2\pi i)\xi_{\vartheta}(\sigma,\text{ɰ})}\end{pmatrix},\begin{pmatrix}\left[\check{\alpha}^{-}_{\varpi}(\sigma,\text{ɰ})e^{(2\pi i)\xi^{-}_{\varpi}(\sigma,\text{ɰ})},\check{\alpha}^{+}_{\varpi}(\sigma,\text{ɰ})e^{(2\pi i)\xi^{+}_{\varpi}(\sigma,\text{ɰ})}\right],\\ \left[\check{\alpha}^{-}_{\emptyset}(\sigma,\text{ɰ})e^{(2\pi i)\xi^{-}_{\emptyset}(\sigma,\text{ɰ})},\check{\alpha}^{+}_{\emptyset}(\sigma,\text{ɰ})e^{(2\pi i)\xi^{+}_{\emptyset}(\sigma,\text{ɰ})}\right],\\ \left[\check{\alpha}^{-}_{\vartheta}(\sigma,\text{ɰ})e^{(2\pi i)\xi^{-}_{\vartheta}(\sigma,\text{ɰ})},\check{\alpha}^{+}_{\vartheta}(\sigma,\text{ɰ})e^{(2\pi i)\xi^{+}_{\vartheta}(\sigma,\text{ɰ})}\right]\end{pmatrix}\right)\in R. \tag{50}$$

Hence, $R$ is CoCP transitive FR on $\check{U}$. □

**Theorem 31.** *Suppose $R$ is a CoCPF equivalence FR on CoCPFS $\check{U}$, then $R\circ R=R$.*

*Proof.* Assume that

$$\left((\sigma,\tau),\begin{pmatrix}\check{\alpha}_{\varpi}(\sigma,\tau)e^{(2\pi i)\xi_{\varpi}(\sigma,\tau)},\\ \check{\alpha}_{\emptyset}(\sigma,\tau)e^{(2\pi i)\xi_{\emptyset}(\sigma,\tau)},\\ \check{\alpha}_{\vartheta}(\sigma,\tau)e^{(2\pi i)\xi_{\vartheta}(\sigma,\tau)}\end{pmatrix},\begin{pmatrix}\left[\check{\alpha}^{-}_{\varpi}(\sigma,\tau)e^{(2\pi i)\xi^{-}_{\varpi}(\sigma,\tau)},\check{\alpha}^{+}_{\varpi}(\sigma,\tau)e^{(2\pi i)\xi^{+}_{\varpi}(\sigma,\tau)}\right],\\ \left[\check{\alpha}^{-}_{\emptyset}(\sigma,\tau)e^{(2\pi i)\xi^{-}_{\emptyset}(\sigma,\tau)},\check{\alpha}^{+}_{\emptyset}(\sigma,\tau)e^{(2\pi i)\xi^{+}_{\emptyset}(\sigma,\tau)}\right],\\ \left[\check{\alpha}^{-}_{\vartheta}(\sigma,\tau)e^{(2\pi i)\xi^{-}_{\vartheta}(\sigma,\tau)},\check{\alpha}^{+}_{\vartheta}(\sigma,\tau)e^{(2\pi i)\xi^{+}_{\vartheta}(\sigma,\tau)}\right]\end{pmatrix}\right)\in R. \tag{51}$$

Then by symmetry of CoCP equivalence FR,

$$\left((\tau,\sigma),\begin{pmatrix}\breve{\alpha}_{\ddot{\varpi}}(\tau,\sigma)e^{(2\pi i)\xi_{\ddot{\varpi}}(\tau,\sigma)},\\ \breve{\alpha}_{\emptyset}(\tau,\sigma)e^{(2\pi i)\xi_{\emptyset}(\tau,\sigma)},\\ \breve{\alpha}_{\vartheta}(\tau,\sigma)e^{(2\pi i)\xi_{\vartheta}(\tau,\sigma)}\end{pmatrix},\begin{pmatrix}\left[\breve{\alpha}^{-}_{\ddot{\varpi}}(\tau,\sigma)e^{(2\pi i)\xi^{-}_{\ddot{\varpi}}(\tau,\sigma)},\breve{\alpha}^{+}_{\ddot{\varpi}}(\tau,\sigma)e^{(2\pi i)\xi^{+}_{\ddot{\varpi}}(\tau,\sigma)}\right],\\ \left[\breve{\alpha}^{-}_{\emptyset}(\tau,\sigma)e^{(2\pi i)\xi^{-}_{\emptyset}(\tau,\sigma)},\breve{\alpha}^{+}_{\emptyset}(\tau,\sigma)e^{(2\pi i)\xi^{+}_{\emptyset}(\tau,\sigma)}\right],\\ \left[\breve{\alpha}^{-}_{\vartheta}(\tau,\sigma)e^{(2\pi i)\xi^{-}_{\vartheta}(\tau,\sigma)},\breve{\alpha}^{+}_{\vartheta}(\tau,\sigma)e^{(2\pi i)\xi^{+}_{\vartheta}(\tau,\sigma)}\right]\end{pmatrix}\right)\in R^{c}\in R. \tag{52}$$

Now by using transitive property,

$$\left((\sigma,\sigma),\begin{pmatrix}\breve{\alpha}_{\ddot{\varpi}}(\sigma,\sigma)e^{(2\pi i)\xi_{\ddot{\varpi}}(\sigma,\sigma)},\\ \breve{\alpha}_{\emptyset}(\sigma,\sigma)e^{(2\pi i)\xi_{\emptyset}(\sigma,\sigma)},\\ \breve{\alpha}_{\vartheta}(\sigma,\sigma)e^{(2\pi i)\xi_{\vartheta}(\sigma,\sigma)}\end{pmatrix},\begin{pmatrix}\left[\breve{\alpha}^{-}_{\ddot{\varpi}}(\sigma,\sigma)e^{(2\pi i)\xi^{-}_{\ddot{\varpi}}(\sigma,\sigma)},\breve{\alpha}^{+}_{\ddot{\varpi}}(\sigma,\sigma)e^{(2\pi i)\xi^{+}_{\ddot{\varpi}}(\sigma,\sigma)}\right],\\ \left[\breve{\alpha}^{-}_{\emptyset}(\sigma,\sigma)e^{(2\pi i)\xi^{-}_{\emptyset}(\sigma,\sigma)},\breve{\alpha}^{+}_{\emptyset}(\sigma,\sigma)e^{(2\pi i)\xi^{+}_{\emptyset}(\sigma,\sigma)}\right],\\ \left[\breve{\alpha}^{-}_{\vartheta}(\sigma,\sigma)e^{(2\pi i)\xi^{-}_{\vartheta}(\sigma,\sigma)},\breve{\alpha}^{+}_{\vartheta}(\sigma,\sigma)e^{(2\pi i)\xi^{+}_{\vartheta}(\sigma,\sigma)}\right]\end{pmatrix}\right)\in R\in R. \tag{53}$$

Hence, by definition of CoCP composite FR,

$$\left((\sigma,\sigma),\begin{pmatrix}\breve{\alpha}_{\ddot{\varpi}}(\sigma,\sigma)e^{(2\pi i)\xi_{\ddot{\varpi}}(\sigma,\sigma)},\\ \breve{\alpha}_{\emptyset}(\sigma,\sigma)e^{(2\pi i)\xi_{\emptyset}(\sigma,\sigma)},\\ \breve{\alpha}_{\vartheta}(\sigma,\sigma)e^{(2\pi i)\xi_{\vartheta}(\sigma,\sigma)}\end{pmatrix},\begin{pmatrix}\left[\breve{\alpha}^{-}_{\ddot{\varpi}}(\sigma,\sigma)e^{(2\pi i)\xi^{-}_{\ddot{\varpi}}(\sigma,\sigma)},\breve{\alpha}^{+}_{\ddot{\varpi}}(\sigma,\sigma)e^{(2\pi i)\xi^{+}_{\ddot{\varpi}}(\sigma,\sigma)}\right],\\ \left[\breve{\alpha}^{-}_{\emptyset}(\sigma,\sigma)e^{(2\pi i)\xi^{-}_{\emptyset}(\sigma,\sigma)},\breve{\alpha}^{+}_{\emptyset}(\sigma,\sigma)e^{(2\pi i)\xi^{+}_{\emptyset}(\sigma,\sigma)}\right],\\ \left[\breve{\alpha}^{-}_{\vartheta}(\sigma,\sigma)e^{(2\pi i)\xi^{-}_{\vartheta}(\sigma,\sigma)},\breve{\alpha}^{+}_{\vartheta}(\sigma,\sigma)e^{(2\pi i)\xi^{+}_{\vartheta}(\sigma,\sigma)}\right]\end{pmatrix}\right)\in R\in R\circ R. \tag{54}$$

Thus,

$$R\subseteq R\circ R. \tag{55}$$

Conversely, suppose that $(\sigma,\tau)\in R\circ R$. Then, $\exists w\in Y\ni(\sigma,\text{ɰ})\in R$ and $(\text{ɰ},\tau)\in R$. Since, $R$ is a CoCP equivalence FR, $R$ is also a CoCP transitive FR. Thus,

$$\left((\sigma,\tau),\begin{pmatrix}\breve{\alpha}_{\ddot{\varpi}}(\sigma,\tau)e^{(2\pi i)\xi_{\ddot{\varpi}}(\sigma,\tau)},\\ \breve{\alpha}_{\emptyset}(\sigma,\tau)e^{(2\pi i)\xi_{\emptyset}(\sigma,\tau)},\\ \breve{\alpha}_{\vartheta}(\sigma,\tau)e^{(2\pi i)\xi_{\vartheta}(\sigma,\tau)}\end{pmatrix},\begin{pmatrix}\left[\breve{\alpha}^{-}_{\ddot{\varpi}}(\sigma,\tau)e^{(2\pi i)\xi^{-}_{\ddot{\varpi}}(\sigma,\tau)},\breve{\alpha}^{+}_{\ddot{\varpi}}(\sigma,\tau)e^{(2\pi i)\xi^{+}_{\ddot{\varpi}}(\sigma,\tau)}\right],\\ \left[\breve{\alpha}^{-}_{\emptyset}(\sigma,\tau)e^{(2\pi i)\xi^{-}_{\emptyset}(\sigma,\tau)},\breve{\alpha}^{+}_{\emptyset}(\sigma,\tau)e^{(2\pi i)\xi^{+}_{\emptyset}(\sigma,\tau)}\right],\\ \left[\breve{\alpha}^{-}_{\vartheta}(\sigma,\tau)e^{(2\pi i)\xi^{-}_{\vartheta}(\sigma,\tau)},\breve{\alpha}^{+}_{\vartheta}(\sigma,\tau)e^{(2\pi i)\xi^{+}_{\vartheta}(\sigma,\tau)}\right]\end{pmatrix}\right)\in R \tag{56}$$

This implies

$$R\circ R\subseteq R. \tag{57}$$

From (55) to (57), $R\circ R=R$ □

## 4. Application

In this section, an application of newly proposed concepts CoCPFSs, CoCPFRs and theirs types is discussed.

*4.1. Protection of E-Commerce from Digital Extortions.* E-commerce is buying and selling of goods over internet. In

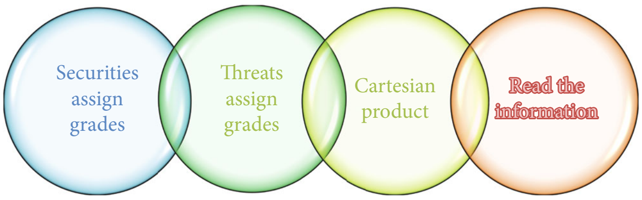


FIGURE 1: Algorithm of application.

TABLE 1: Summary of securities.

| Securities | Abbreviations |
|---|---|
| Strong cybersecurity | SCS |
| Securing payment gateway | SPG |
| Mobile devices | MD |
| Solid rock firewall | SRF |
| Antimalware software | AMS |

such transactions, these products and goods are sold through an electronic medium without using any paper document. Almost everything can be purchased through e-commerce. It focuses on the use of information and communication technology to enable the external activities and relationships of the business with individuals, groups, and other businesses. It facilitates the human lives to buy things at home. The recent outbreak of COVID-19 has dramatically changed the current business climate. For stores that have closed their doors, for the time being, having a reliable e-commerce platform can help to create stable revenue and save business. This pandemic has affected consumer behavior. People are now choosing to shop online to avoid contacts with other. This rapid grow of e-commerce is facing many types of threats and risks. These threats can put negative impact on e-commerce, which results in losses of money. Some threats and methods used for security purposes are discussed below. Figure 1 presents the algorithm of application.

Here, each threat has been assigned membership, indeterminacy, and nonmembership grades. These all grades are set by experts according to their work. The membership grade of threats indicates its weakness; indeterminacy indicates neutral effect, indeterminacy indicates the neutral effect of threat, and nonmembership grades show the strength of threats. Amplitude term indicates the term which indicates the level of strength or weakness, and phase term show the time duration of level of strength or weakness.

*4.1.1. Securities.* The method use to secure e-commerce is discussed below and assigned membership, indeterminacy, and nonmembership to each security. Table 1 contains all the abbreviations for security measures.

*(1) Strong Cybersecurity.* A strong cybersecurity is the application of technologies, processes, and controls to protest system, devices, and data from cyberattacks. It protects against the unauthorized exploitation of system. A robust cybersecurity strategy is the best defense against attack but many organizations do not know where to begin.

$$\left(\text{SCS}, \begin{pmatrix} 0.41e^{0.56(2\pi i)}, 0.09e^{0.74(2\pi i)}, \\ 0.344e^{055(2\pi i)} \end{pmatrix}, \begin{pmatrix} \left[0.21e^{0.‘5(2\pi i)}, 0.73e^{0.67(2\pi i)}\right], \left[0.56e^{0.41(2\pi i)}, 0.78e^{0.64(2\pi i)}\right], \\ \left[0.37e^{0.45(2\pi i)}, 0.93e^{0.85(2\pi i)}\right] \end{pmatrix}\right). \quad (58)$$

*(2) Securing Payment Gateway.* It is a payment gateway as a merchant service that processes credit cards payments for e-commerce sites and traditional brick and motor stores. Popular payment gateways include PayPal, Stripe, and Square.

$$\left(\text{SPG}, \begin{pmatrix} 0.05e^{0.47(2\pi i)}, 0.39e^{0.51(2\pi i)}, \\ 0.22e^{051(2\pi i)} \end{pmatrix}, \begin{pmatrix} \left[0.25e^{0.31(2\pi i)}, 0.45e^{0.37(2\pi i)}\right], \left[0.51e^{0.42(2\pi i)}, 0.67e^{0.51(2\pi i)}\right], \\ \left[0.61e^{0.54(2\pi i)}, 0.85e^{0.89(2\pi i)}\right] \end{pmatrix}\right). \quad (59)$$

*(3) Mobile Devices.* Mobility is an essential part of business these days. However, while it is necessary, the use of personal phones and mobile devices for work can open your business to additional security risks. If you plan to allow your employees to use their personal devices for business purposes, you need to be sure you have security policies in place, including requirements that devices be password-protected to prevent outside access to confidential data in the event that devices are lost or stolen.

$$\left(\text{MD}, \begin{pmatrix} 0.04e^{0.51(2\pi i)}, 0.45e^{0.37(2\pi i)}, \\ 0.53e^{0.67(2\pi i)} \end{pmatrix}, \begin{pmatrix} \left[0.17e^{0.27(2\pi i)}, 0.19e^{039(2\pi i)}\right], \left[0.47e^{0.32(2\pi i)}, 0.67e^{0.55(2\pi i)}\right], \\ \left[0.73e^{0.41(2\pi i)}, 0.75e^{0.78(2\pi i)}\right] \end{pmatrix}\right). \tag{60}$$

*(4) Solid Rock Firewalls*. Use effective e-commerce software and plugins to bar untrusted networks and regulate the inflow and outflow of website traffic. They should provide selective permeability, only permitting trusted traffic to go through. You can trust the Astra firewall to stop spam, malware, and many other attacks on your website.

$$\left(\text{SR}\dot{\text{F}}, \begin{pmatrix} 0.55e^{042(2\pi i)}, 009e^{0.38(2\pi i)}, \\ 0.39e^{0.48(2\pi i)} \end{pmatrix}, \begin{pmatrix} \left[0.03e^{0.55(2\pi i)}, 0.19e^{0.57(2\pi i)}\right], \left[0.43e^{0.14(2\pi i)}, 0.89e^{0.81(2\pi i)}\right], \\ \left[0.51e^{0.62(2\pi i)}, 0.80e^{0.87(2\pi i)}\right] \end{pmatrix}\right). \tag{61}$$

*(5) Antimalware Software*. The electronic devices, computer systems, and web system need a program or software that detects and block malicious software, otherwise known as malware. Such protective software is called antimalware software. An effective antimalware should render all the hidden malware on your website.

$$\left(\text{AMS}, \begin{pmatrix} 0.66e^{0.58(2\pi i)}, 0.33e^{0.43(2\pi i)}, \\ 0.11e^{0.32(2\pi i)} \end{pmatrix}, \begin{pmatrix} \left[0.33e^{0.23(2\pi i)}, 0.41e^{0.31(2\pi i)}\right], \left[0.52e^{0.42(2\pi i)}, 0.63e^{0.69(2\pi i)}\right], \\ \left[0.77e^{0.54(2\pi i)}, 0.96e^{0.95(2\pi i)}\right] \end{pmatrix}\right). \tag{62}$$

Now the set of securities $I$ is

$$I = \left\{ \begin{array}{l} \left(SCS, \begin{pmatrix} 0.41e^{0.56(2\pi i)}, 0.09e^{0.74(2\pi i)}, \\ 0.34e^{0.55(2\pi i)} \end{pmatrix}, \begin{pmatrix} \left[0.21e^{0.55(2\pi i)}, 0.73e^{0.67(2\pi i)}\right], \left[0.56e^{0.41(2\pi i)}, 0.78e^{0.64(2\pi i)}\right], \\ \left[0.37e^{0.45(2\pi i)}, 0.93e^{0.85(2\pi i)}\right] \end{pmatrix}\right), \\ \left(SPG, \begin{pmatrix} 0.05e^{0.47(2\pi i)}, 0.39e^{0.51(2\pi i)}, \\ 0.22e^{0.51(2\pi i)} \end{pmatrix}, \begin{pmatrix} \left[0.25e^{0.31(2\pi i)}, 0.45e^{0.37(2\pi i)}\right], \left[0.51e^{0.42(2\pi i)}, 0.67e^{0.51(2\pi i)}\right], \\ \left[0.61e^{0.54(2\pi i)}, 0.85e^{0.89(2\pi i)}\right] \end{pmatrix}\right), \\ \left(MD, \begin{pmatrix} 0.04e^{0.51(2\pi i)}, 0.45e^{0.37(2\pi i)}, \\ 0.53e^{0.67(2\pi i)} \end{pmatrix}, \begin{pmatrix} \left[0.17e^{0.27(2\pi i)}, 0.19e^{039(2\pi i)}\right], \left[0.47e^{0.32(2\pi i)}, 0.67e^{0.55(2\pi i)}\right], \\ \left[0.73e^{0.41(2\pi i)}, 0.75e^{0.78(2\pi i)}\right] \end{pmatrix}\right), \\ \left(SR\dot{F}, \begin{pmatrix} 0.55e^{042(2\pi i)}, 0.09e^{0.38(2\pi i)}, \\ 0.39e^{0.48(2\pi i)} \end{pmatrix}, \begin{pmatrix} \left[0.03e^{0.55(2\pi i)}, 0.19e^{0.57(2\pi i)}\right], \left[0.43e^{0.14(2\pi i)}, 0.89e^{0.81(2\pi i)}\right], \\ \left[0.51e^{0.62(2\pi i)}, 0.80e^{0.87(2\pi i)}\right] \end{pmatrix}\right), \\ \left(AMS, \begin{pmatrix} 0.66e^{0.58(2\pi i)}, 0.33e^{0.43(2\pi i)}, \\ 0.11e^{0.32(2\pi i)} \end{pmatrix}, \begin{pmatrix} \left[0.33e^{0.23(2\pi i)}, 0.41e^{0.31(2\pi i)}\right], \left[0.52e^{0.42(2\pi i)}, 0.63e^{0.69(2\pi i)}\right], \\ \left[0.77e^{0.54(2\pi i)}, 0.96e^{0.95(2\pi i)}\right] \end{pmatrix}\right) \end{array} \right\}. \tag{63}$$

*4.1.2. Threats.* Some common threats faced in e-commerce are explained below and assigned the level of membership, level of abstinence, and level of nonmembership to them. Table 2 contains all the abbreviations for security threats.

*(1) Phishing.* It is cyberattack that uses disguised email as a weapon. The goal is to steal sensitive data like credit cards and login information.

$$\left(\text{P}, \begin{pmatrix} 0.30e^{0.21(2\pi i)}, 0.41e^{0.90(2\pi i)}, \\ 0.32e^{0.57(2\pi i)} \end{pmatrix}, \begin{pmatrix} \left[0.50e^{0.23(2\pi i)}, 0.43e^{0.76(2\pi i)}\right], \left[0.43e^{0.14(2\pi i)}, 0.89e^{0.81(2\pi i)}\right], \\ \left[0.61e^{0.45(2\pi i)}, 0.79e^{0.91(2\pi i)}\right] \end{pmatrix}\right). \tag{64}$$

*(2) Spamming.* Some hackers can leave infected links in their comments and messages on blog post and contact forms. Once you click on such links, they will direct you to their spam websites, where you may end up being victim.

$$\left(\text{S}, \begin{pmatrix} 0.11e^{0.21(2\pi i)}, 0.251e^{0.41(2\pi i)}, \\ 0.17e^{0.67(2\pi i)} \end{pmatrix}, \begin{pmatrix} \left[0.31e^{0.11(2\pi i)}, 0.43e^{0.89(2\pi i)}\right], \left[0.43e^{0.56(2\pi i)}, 0.89e^{0.91(2\pi i)}\right], \\ \left[0.61e^{0.36(2\pi i)}, 0.67e^{0.54(2\pi i)}\right] \end{pmatrix}\right). \tag{65}$$

*(3) Bots.* Some attackers develop special bots that can scrape your websites to get information about inventory and prices. Such hackers can then use the data to lower and modify the prices in their websites in an attempt to lower your sales and revenue.

$$\left(\text{B}, \begin{pmatrix} 0.22e^{0.59(2\pi i)}, 0.78e^{0.90(2\pi i)}, \\ 0.81e^{0.57(2\pi i)} \end{pmatrix}, \begin{pmatrix} \left[0.17e^{0.35(2\pi i)}, 0.39e^{0.76(2\pi i)}\right], \left[0.71e^{0.45(2\pi i)}, 0.79e^{0.64(2\pi i)}\right], \\ \left[0.81e^{0.45(2\pi i)}, 0.89e^{0.55(2\pi i)}\right] \end{pmatrix}\right). \tag{66}$$

*(4) Man, in the Middle.* A hacker may listen in on the communication taking place between your e-commerce store and a user. If the user is connected to a vulnerable Wi-Fi or network, then attackers can take advantage of that.

$$\left(\text{MIM}, \begin{pmatrix} 0.07e^{0.09(2\pi i)}, 0.51e^{0.98(2\pi i)}, \\ 0.22e^{0.43(2\pi i)} \end{pmatrix}, \begin{pmatrix} \left[0.61e^{0.09(2\pi i)}, 0.63e^{0.21(2\pi i)}\right], \left[0.54e^{0.23(2\pi i)}, 0.76e^{0.41(2\pi i)}\right], \\ \left[0.81e^{0.49(2\pi i)}, 0.92e^{0.71(2\pi i)}\right] \end{pmatrix}\right). \tag{67}$$

*(5) e-Skimming.* It involves infecting a website's checkout pages with malicious software. The intention is to steal the client's personal data and payment details.

$$\left(\text{eS}, \begin{pmatrix} 0.90e^{0.20(2\pi i)}, 0.87e^{0.87(2\pi i)}, \\ 0.31e^{0.67(2\pi i)} \end{pmatrix}, \begin{pmatrix} \left[0.21e^{0.56(2\pi i)}, 0.29e^{0.59(2\pi i)}\right], \left[0.41e^{0.14(2\pi i)}, 0.49e^{0.81(2\pi i)}\right], \\ \left[0.11e^{0.32(2\pi i)}, 0.71e^{0.91(2\pi i)}\right] \end{pmatrix}\right). \tag{68}$$

Table 2: Summary of threats.

| Threats | Abbreviations |
|---|---|
| Phishing | P |
| Spamming | S |
| Bots | B |
| Man in the middle | MIM |
| e-Skimming | Es |

Then the set of threats $J$ is

$$J=\left\{\begin{array}{l}\left(P,\begin{pmatrix}0.30e^{0.21(2\pi i)},0.41e^{0.90(2\pi i)},\\ 0.32e^{0.57(2\pi i)}\end{pmatrix},\begin{pmatrix}\left[0.50e^{0.23(2\pi i)},0.43e^{0.76(2\pi i)}\right],\left[0.43e^{0.14(2\pi i)},0.89e^{0.81(2\pi i)}\right],\\ \left[0.61e^{0.45(2\pi i)},0.79e^{0.91(2\pi i)}\right]\end{pmatrix}\right),\\ \left(S,\begin{pmatrix}0.11e^{0.21(2\pi i)},0.25e^{0.41(2\pi i)},\\ 0.17e^{0.67(2\pi i)}\end{pmatrix},\begin{pmatrix}\left[0.31e^{0.11(2\pi i)},0.43e^{0.89(2\pi i)}\right],\left[0.43e^{0.56(2\pi i)},0.89e^{0.91(2\pi i)}\right],\\ \left[0.61e^{0.36(2\pi i)},0.67e^{0.54(2\pi i)}\right]\end{pmatrix}\right),\\ \left(B,\begin{pmatrix}0.22e^{0.59(2\pi i)},0.78e^{0.90(2\pi i)},\\ 0.81e^{0.57(2\pi i)}\end{pmatrix},\begin{pmatrix}\left[0.17e^{0.35(2\pi i)},0.39e^{0.76(2\pi i)}\right],\left[0.71e^{0.45(2\pi i)},0.79e^{0.64(2\pi i)}\right],\\ \left[0.81e^{0.45(2\pi i)},0.89e^{0.55(2\pi i)}\right]\end{pmatrix}\right),\\ \left(MIM,\begin{pmatrix}0.07e^{0.09(2\pi i)},0.51e^{0.98(2\pi i)},\\ 0.22e^{0.43(2\pi i)}\end{pmatrix},\begin{pmatrix}\left[0.61e^{0.09(2\pi i)},0.63e^{0.21(2\pi i)}\right],\left[0.54e^{0.23(2\pi i)},0.76e^{0.41(2\pi i)}\right],\\ \left[0.81e^{0.49(2\pi i)},0.92e^{0.71(2\pi i)}\right]\end{pmatrix}\right),\\ \left(eS,\begin{pmatrix}0.90e^{0.20(2\pi i)},0.87e^{0.87(2\pi i)},\\ 0.31e^{0.67(2\pi i)}\end{pmatrix},\begin{pmatrix}\left[0.21e^{0.56((2\pi i))},0.29e^{0.59(2\pi i)}\right],\left[0.41e^{0.14(2\pi i)},0.49e^{0.81(2\pi i)}\right],\\ \left[0.11e^{0.32(2\pi i)},0.71e^{0.91(2\pi i)}\right]\end{pmatrix}\right)\end{array}\right\}. \tag{69}$$

Now the determining the effectiveness of each security against each threat, calculate the Cartesian product $I \times J$ as given in Table 3:

Each member of $I \times J$ of Table 3 is an ordered pair, which indicates the relation between that pair, i.e., the effect of one factor on the other. The degree of membership indicates the effectiveness of security to overcome a specific threat in particular time duration. The degree of indeterminacy shows the neutral effect of one parameter on the other parameter over the time. The degree of nonmembership describes the noneffectiveness of the security to overcome any threat with phase duration. For example, any ordered pair $\left((\dot{SRF}, B),\begin{pmatrix}0.22e^{0.42(2\pi i)},0.09e^{0.38(2\pi i)},\\ 0.81e^{057(2\pi i)}\end{pmatrix},\begin{pmatrix}\left[0.03e^{0.35(2\pi i)},0.19e^{0.57(2\pi i)}\right],\\ \left[0.43e^{0.14(2\pi i)},0.79e^{0.64(2\pi i)}\right],\\ \left[0.81e^{0.62(2\pi i)},0.89e^{0.87(2\pi i)}\right]\end{pmatrix}\right)$ explains that bots can easily overcome the solid rock firewalls. Further it explains the present and future effect of the ordered pair. Bots overcome the threat of SRF, in present, and have low efficacy with short time, low indeterminacy with short time period, but a high ineffectiveness for a long period. Given ordered pair predicts the future forecasting in the form of interval to reduce the uncertainty. In future, bots have a lower effectiveness with a normal time period, indeterminacy with long time duration, and higher level of ineffectiveness for longer time period. As for as the security concerns, the membership with long time period is considered better, while nonmembership with less time period is good.

## 5. Comparative Analysis

In this section, compare the proposed concept of CoCPFRs with preexisting concepts, i.e., cubic fuzzy relations (CFRs), complex CFRs (CoCFRs), cubic intuitionistic fuzzy relations (CIFRs), complex cubic intuitionistic fuzzy relations (CoCIRs), and cubic picture fuzzy relations (CPFRs).

TABLE 3: Cartesian product $I \times J$.

| Ordered pair | Complex picture fuzzy set | Complex interval-valued picture fuzzy set |
|---|---|---|
| (SCS, P) | $\left(0.30e^{0.21(2\pi i)}, 0.09e^{0.74(2\pi i)}, 0.34e^{057(2\pi i)}\right)$ | $\left(\left[0.21e^{0.23(2\pi i)}, 0.43e^{0.67(2\pi i)}\right], \left[0.43e^{0.14(2\pi i)}, 0.78e^{0.64(2\pi i)}\right], \left[0.61e^{0.45(2\pi i)}, 0.93e^{0.91(2\pi i)}\right]\right)$ |
| (SPG, P) | $\left(0.05e^{0.21(2\pi i)}, 0.39e^{0.51(2\pi i)}, 0.32e^{0.57(2\pi i)}\right)$ | $\left(\left[0.25e^{0.31(2\pi i)}, 0.43e^{0.37(2\pi i)}\right], \left[0.43e^{0.42(2\pi i)}, 0.67e^{0.51(2\pi i)}\right], \left[0.61e^{0.54(2\pi i)}, 0.85e^{0.89(2\pi i)}\right]\right)$ |
| (MD, P) | $\left(0.04e^{0.21(2\pi i)}, 0.41e^{0.37(2\pi i)}, 0.53e^{0.67(2\pi i)}\right)$ | $\left(\left[0.17e^{0.27(2\pi i)}, 0.19e^{0.39(2\pi i)}\right], \left[0.47e^{0.32(2\pi i)}, 0.67e^{0.55(2\pi i)}\right], \left[0.61e^{0.45(2\pi i)}, 0.79e^{0.91(2\pi i)}\right]\right)$ |
| (SRF, P) | $\left(0.30e^{0.21(2\pi i)}, 0.09e^{0.38(2\pi i)}, 0.39e^{0.57(2\pi i)}\right)$ | $\left(\left[0.03e^{0.23(2\pi i)}, 0.19e^{0.57(2\pi i)}\right], \left[0.43e^{0.14(2\pi i)}, 0.89e^{0.81(2\pi i)}\right], \left[0.61e^{0.62(2\pi i)}, 0.80e^{0.91(2\pi i)}\right]\right)$ |
| (AMS, P) | $\left(0.30e^{0.21(2\pi i)}, 0.33e^{0.43(2\pi i)}, 0.32e^{0.57(2\pi i)}\right)$ | $\left(\left[0.33e^{0.23(2\pi i)}, 0.41e^{0.31(2\pi i)}\right], \left[0.43e^{0.14(2\pi i)}, 0.63e^{0.69(2\pi i)}\right], \left[0.77e^{0.54(2\pi i)}, 0.96e^{0.95(2\pi i)}\right]\right)$ |
| (SCS, S) | $\left(0.11e^{0.21(2\pi i)}, 0.09e^{0.41(2\pi i)}, 0.34e^{0.67(2\pi i)}\right)$ | $\left(\left[0.21e^{0.11(2\pi i)}, 0.43e^{0.67(2\pi i)}\right], \left[0.43e^{0.41(2\pi i)}, 0.78e^{0.64(2\pi i)}\right], \left[0.61e^{0.45(2\pi i)}, 0.93e^{0.85(2\pi i)}\right]\right)$ |
| (SPG, S) | $\left(0.05e^{0.21(2\pi i)}, 0.25e^{0.41(2\pi i)}, 0.22e^{0.67(2\pi i)}\right)$ | $\left(\left[0.25e^{0.11(2\pi i)}, 0.43e^{0.37(2\pi i)}\right], \left[0.43e^{0.42(2\pi i)}, 0.67e^{0.51(2\pi i)}\right], \left[0.61e^{0.54(2\pi i)}, 0.85e^{0.89(2\pi i)}\right]\right)$ |
| (MD, S) | $\left(0.04e^{0.21(2\pi i)}, 0.25e^{0.37(2\pi i)}, 0.53e^{0.67(2\pi i)}\right)$ | $\left(\left[0.17e^{0.11(2\pi i)}, 0.19e^{0.39(2\pi i)}\right], \left[0.43e^{0.32(2\pi i)}, 0.67e^{0.55(2\pi i)}\right], \left[0.73e^{0.41(2\pi i)}, 0.75e^{0.78(2\pi i)}\right]\right)$ |
| (SRF, S) | $\left(0.11e^{0.21(2\pi i)}, 0.09e^{0.38(2\pi i)}, 0.39e^{0.67(2\pi i)}\right)$ | $\left(\left[0.03e^{0.11(2\pi i)}, 0.19e^{0.57(2\pi i)}\right], \left[0.43e^{0.14(2\pi i)}, 0.89e^{0.91(2\pi i)}\right], \left[0.51e^{0.62(2\pi i)}, 0.80e^{0.87(2\pi i)}\right]\right)$ |
| (AMS, S) | $\left(0.11e^{0.21(2\pi i)}, 0.25e^{0.41(2\pi i)}, 0.17e^{0.67(2\pi i)}\right)$ | $\left(\left[0.31e^{0.11(2\pi i)}, 0.41e^{0.31(2\pi i)}\right], \left[0.43e^{0.42(2\pi i)}, 0.63e^{0.69(2\pi i)}\right], \left[0.77e^{0.54(2\pi i)}, 0.96e^{0.95(2\pi i)}\right]\right)$ |
| (SCS, B) | $\left(0.22e^{0.56(2\pi i)}, 0.09e^{0.74(2\pi i)}, 0.81e^{0.57(2\pi i)}\right)$ | $\left(\left[0.17e^{0.35(2\pi i)}, 0.39e^{0.67(2\pi i)}\right], \left[0.56e^{0.41(2\pi i)}, 0.78e^{0.64(2\pi i)}\right], \left[0.81e^{0.45(2\pi i)}, 0.93e^{0.85(2\pi i)}\right]\right)$ |
| (SPG, B) | $\left(0.22e^{0.59(2\pi i)}, 0.39e^{0.51(2\pi i)}, 0.81e^{0.51(2\pi i)}\right)$ | $\left(\left[0.17e^{0.31(2\pi i)}, 0.39e^{0.37(2\pi i)}\right], \left[0.51e^{0.42(2\pi i)}, 0.67e^{0.51(2\pi i)}\right], \left[0.81e^{0.54(2\pi i)}, 0.85e^{0.89(2\pi i)}\right]\right)$ |
| (MD, B) | $\left(0.04e^{0.51(2\pi i)}, 0.45e^{0.37(2\pi i)}, 0.81e^{0.67(2\pi i)}\right)$ | $\left(\left[0.17e^{0.27(2\pi i)}, 0.19e^{0.39(2\pi i)}\right], \left[0.47e^{0.32(2\pi i)}, 0.67e^{0.55(2\pi i)}\right], \left[0.81e^{0.45(2\pi i)}, 0.89e^{0.78(2\pi i)}\right]\right)$ |
| (SRF, B) | $\left(0.22e^{0.42(2\pi i)}, 0.09e^{0.38(2\pi i)}, 0.81e^{057(2\pi i)}\right)$ | $\left(\left[0.03e^{0.35(2\pi i)}, 0.19e^{0.57(2\pi i)}\right], \left[0.43e^{0.14(2\pi i)}, 0.79e^{0.64(2\pi i)}\right], \left[0.81e^{0.62(2\pi i)}, 0.89e^{0.87(2\pi i)}\right]\right)$ |
| (AMS, B) | $\left(0.22e^{0.58(2\pi i)}, 0.33e^{0.43(2\pi i)}, 0.81e^{0.57(2\pi i)}\right)$ | $\left(\left[0.17e^{0.23(2\pi i)}, 0.39e^{0.31(2\pi i)}\right], \left[0.52e^{0.42(2\pi i)}, 0.63e^{0.64(2\pi i)}\right], \left[0.77e^{0.54(2\pi i)}, 0.96e^{0.95(2\pi i)}\right]\right)$ |
| (SCS, MIM) | $\left(0.07e^{0.09(2\pi i)}, 0.09e^{0.74(2\pi i)}, 0.34e^{0.55(2\pi i)}\right)$ | $\left(\left[0.21e^{0.09(2\pi i)}, 0.63e^{0.21(2\pi i)}\right], \left[0.54e^{0.23(2\pi i)}, 0.76e^{0.41(2\pi i)}\right], \left[0.81e^{049(2\pi i)}, 0.93e^{0.85(2\pi i)}\right]\right)$ |
| (SPG, MIM) | $\left(0.05e^{0.09(2\pi i)}, 0.9e^{0.51(2\pi i)}, 0.22e^{0.51(2\pi i)}\right)$ | $\left(\left[0.25e^{0.09(2\pi i)}, 0.45e^{0.21(2\pi i)}\right], \left[0.51e^{0.23(2\pi i)}, 0.67e^{0.41(2\pi i)}\right], \left[0.81e^{0.54(2\pi i)}, 0.92e^{0.89(2\pi i)}\right]\right)$ |
| (MD, MIM) | $\left(0.04e^{0.09(2\pi i)}, 0.45e^{0.37(2\pi i)}, 0.53e^{0.67(2\pi i)}\right)$ | $\left(\left[0.17e^{0.09(2\pi i)}, 0.19e^{0.21(2\pi i)}\right], \left[0.47e^{0.23(2\pi i)}, 0.67e^{0.41(2\pi i)}\right], \left[0.81e^{0.41(2\pi i)}, 0.92e^{0.78(2\pi i)}\right]\right)$ |

Table 3: Continued.

| Ordered pair | Complex picture fuzzy set | Complex interval-valued picture fuzzy set |
|---|---|---|
| (SRF, MIM) | $\left(0.07e^{0.09(2\pi i)}, 0.09e^{0.38(2\pi i)}, 0.39e^{0.48(2\pi i)}\right)$ | $\left(\left[0.03e^{0.09(2\pi i)}, 0.19e^{0.21(2\pi i)}\right], \left[0.43e^{0.14(2\pi i)}, 0.76e^{0.41(2\pi i)}\right], \left[0.81e^{0.62(2\pi i)}, 0.92e^{0.87(2\pi i)}\right]\right)$ |
| (AMS, MIM) | $\left(0.07e^{0.09(2\pi i)}, 0.33e^{0.43(2\pi i)}, 0.22e^{0.43(2\pi i)}\right)$ | $\left(\left[0.33e^{0.09(2\pi i)}, 0.41e^{0.21(2\pi i)}\right], \left[0.52e^{0.23(2\pi i)}, 0.63e^{0.41(2\pi i)}\right], \left[0.81e^{0.54(2\pi i)}, 0.96e^{0.95(2\pi i)}\right]\right)$ |
| (SCS, eS) | $\left(0.41e^{0.20(2\pi i)}, 0.09e^{0.74(2\pi i)}, 0.34e^{0.55(2\pi i)}\right)$ | $\left(\left[0.21e^{0.55(2\pi i)}, 0.29e^{0.59(2\pi i)}\right], \left[0.41e^{0.14(2\pi i)}, 0.49e^{0.64(2\pi i)}\right], \left[0.37e^{0.45(2\pi i)}, 0.93e^{0.91(2\pi i)}\right]\right)$ |
| (SPG, eS) | $\left(0.05e^{0.20(2\pi i)}, 0.39e^{0.51(2\pi i)}, 0.31e^{0.67(2\pi i)}\right)$ | $\left(\left[0.21e^{0.31(2\pi i)}, 0.29e^{0.37(2\pi i)}\right], \left[0.41e^{0.14(2\pi i)}, 0.49e^{0.51(2\pi i)}\right], \left[0.61e^{0.54(2\pi i)}, 0.85e^{0.91(2\pi i)}\right]\right)$ |
| (MD, eS) | $\left(0.04e^{0.20(2\pi i)}, 0.45e^{0.37(2\pi i)}, 0.53e^{0.67(2\pi i)}\right)$ | $\left(\left[0.17e^{0.27(2\pi i)}, 0.19e^{0.39(2\pi i)}\right], \left[0.41e^{0.14(2\pi i)}, 0.49e^{0.55(2\pi i)}\right], \left[0.73e^{0.41(2\pi i)}, 0.75e^{0.91(2\pi i)}\right]\right)$ |
| (SRF, eS) | $\left(0.55e^{0.20(2\pi i)}, 0.09e^{0.38(2\pi i)}, 0.39e^{0.67(2\pi i)}\right)$ | $\left(\left[0.03e^{0.55(2\pi i)}, 0.19e^{0.57(2\pi i)}\right], \left[0.41e^{0.14(2\pi i)}, 0.49e^{0.81(2\pi i)}\right], \left[0.51e^{0.62(2\pi i)}, 0.80e^{0.87(2\pi i)}\right]\right)$ |
| (AMS, eS) | $\left(0.66e^{0.58(2\pi i)}, 0.33e^{0.43(2\pi i)}, 0.31e^{0.67(2\pi i)}\right)$ | $\left(\left[0.21e^{0.23(2\pi i)}, 0.29e^{0.31(2\pi i)}\right], \left[0.41e^{0.14(2\pi i)}, 0.49e^{0.69(2\pi i)}\right], \left[0.77e^{0.54(2\pi i)}, 0.96e^{0.95(2\pi i)}\right]\right)$ |

*5.1. CoCPFRs with CoFRs, CoIFRs, and CoPFRs.* The CoFRs only describe the membership grade with respect to amplitude and phase terms of an entity. Although it possesses the properties to model the uncertain problems with periodicity, but it fails to cover the future aspects of a problem. Moreover, the lack of nonmembership and indeterminacy also makes it limited in practicality, whereas CoIFRs and CoPFRs are more generalized structure. They not only possess the property of amplitude and phase terms but also define an object with membership, nonmembership, and indeterminacy. But they also fall short when dealing with the future aspects. Let us analyze this mathematically by solving the application problem using complex picture fuzzy information.

Consider the following CoPFSs for the sets of securities and threats:

$$\begin{aligned} I &= \left\{ \begin{array}{c} \left(\text{SCS}, \left(0.41e^{0.56(2\pi i)}, 0.09e^{0.74(2\pi i)}, 0.34e^{0.55(2\pi i)}\right)\right), \left(\text{SPG}, \left(0.05e^{0.47(2\pi i)}, 0.39e^{0.51(2\pi i)}, 0.22e^{0.51(2\pi i)}\right)\right), \\ \left(\text{MD}, \left(0.04e^{0.51(2\pi i)}, 0.45e^{0.37(2\pi i)}, 0.53e^{0.67(2\pi i)}\right)\right), \left(\dot{\text{SRF}}, \left(0.55e^{042(2\pi i)}, 0.09e^{0.38(2\pi i)}, 0.39e^{0.48(2\pi i)}\right)\right), \\ \left(\text{AMS}, \left(0.66e^{0.58(2\pi i)}, 0.33e^{0.43(2\pi i)}, 0.11e^{0.32(2\pi i)}\right)\right) \end{array} \right\}, \\ J &= \left\{ \begin{array}{c} \left(\text{P}, \left(0.30e^{0.21(2\pi i)}, 0.41e^{0.90(2\pi i)}, 0.32e^{0.57(2\pi i)}\right)\right), \left(\text{S}, \left(0.11e^{0.21(2\pi i)}, 0.25e^{0.41(2\pi i)}, 0.17e^{0.67(2\pi i)}\right)\right), \\ \left(\text{B}, \left(0.22e^{0.59(2\pi i)}, 0.78e^{0.90(2\pi i)}, 0.81e^{0.57(2\pi i)}\right)\right), \left(\text{MIM}, \left(0.07e^{0.09(2\pi i)}, 0.51e^{0.98(2\pi i)}, 0.22e^{0.43(2\pi i)}\right)\right), \\ \left(\text{eS}, \left(0.90e^{0.20(2\pi i)}, 0.87e^{0.87(2\pi i)}, 0.31e^{0.67(2\pi i)}\right)\right) \end{array} \right\}. \end{aligned} \tag{70}$$

Table 4: Cartesian product $I \times J$.

| Ordered pair | Membership | Indeterminacy | Nonmembership |
|---|---|---|---|
| (SCS, P) | $0.30e^{0.21(2\pi i)}$ | $0.09e^{0.74(2\pi i)}$ | $0.34e^{057(2\pi i)}$ |
| (SPG, P) | $0.05e^{0.21(2\pi i)}$ | $0.39e^{0.51(2\pi i)}$ | $0.32e^{0.57(2\pi i)}$ |
| (MD, P) | $0.04e^{0.21(2\pi i)}$ | $0.41e^{0.37(2\pi i)}$ | $0.53e^{0.67(2\pi i)}$ |
| (SRF, P) | $0.30e^{0.21(2\pi i)}$ | $0.09e^{0.38(2\pi i)}$ | $0.39e^{0.57(2\pi i)}$ |
| (AMS, P) | $0.30e^{0.21(2\pi i)}$ | $0.33e^{0.43(2\pi i)}$ | $0.32e^{0.57(2\pi i)}$ |
| (SCS, S) | $0.11e^{0.21(2\pi i)}$ | $0.09e^{0.41(2\pi i)}$ | $0.34e^{0.67(2\pi i)}$ |
| (SPG, S) | $0.05e^{0.21(2\pi i)}$ | $0.25e^{0.41(2\pi i)}$ | $0.22e^{0.67(2\pi i)}$ |
| (MD, S) | $0.04e^{0.21(2\pi i)}$ | $0.25e^{0.37(2\pi i)}$ | $0.53e^{0.67(2\pi i)}$ |
| (SRF, S) | $0.11e^{0.21(2\pi i)}$ | $0.09e^{0.38(2\pi i)}$ | $0.39e^{0.67(2\pi i)}$ |
| (AMS, S) | $0.11e^{0.21(2\pi i)}$ | $0.25e^{0.41(2\pi i)}$ | $0.17e^{0.67(2\pi i)}$ |
| (SCS, B) | $0.22e^{0.56(2\pi i)}$ | $0.09e^{0.74(2\pi i)}$ | $0.81e^{0.57(2\pi i)}$ |
| (SPG, B) | $0.22e^{0.59(2\pi i)}$ | $0.39e^{0.51(2\pi i)}$ | $0.81e^{0.51(2\pi i)}$ |
| (MD, B) | $0.04e^{0.51(2\pi i)}$ | $0.45e^{0.37(2\pi i)}$ | $0.81e^{0.67(2\pi i)}$ |
| (SRF, B) | $0.22e^{0.42(2\pi i)}$ | $0.09e^{0.38(2\pi i)}$ | $0.81e^{057(2\pi i)}$ |
| (AMS, B) | $0.22e^{0.58(2\pi i)}$ | $0.33e^{0.43(2\pi i)}$ | $0.81e^{0.57(2\pi i)}$ |
| (SCS, MIM) | $0.07e^{0.09(2\pi i)}$ | $0.09e^{0.74(2\pi i)}$ | $0.34e^{0.55(2\pi i)}$ |
| (SPG, MIM) | $0.05e^{0.09(2\pi i)}$ | $0.9e^{0.51(2\pi i)}$ | $0.22e^{0.51(2\pi i)}$ |
| (MD, MIM) | $0.04e^{0.09(2\pi i)}$ | $0.45e^{0.37(2\pi i)}$ | $0.53e^{0.67(2\pi i)}$ |
| (SRF, MIM) | $0.07e^{0.09(2\pi i)}$ | $0.09e^{0.38(2\pi i)}$ | $0.39e^{0.48(2\pi i)}$ |
| (AMS, MIM) | $0.07e^{0.09(2\pi i)}$ | $0.33e^{0.43(2\pi i)}$ | $0.22e^{0.43(2\pi i)}$ |
| (SCS, eS) | $0.41e^{0.20(2\pi i)}$ | $0.09e^{0.74(2\pi i)}$ | $0.34e^{0.55(2\pi i)}$ |
| (SPG, eS) | $0.05e^{0.20(2\pi i)}$ | $0.39e^{0.51(2\pi i)}$ | $0.31e^{0.67(2\pi i)}$ |
| (MD, eS) | $0.04e^{0.20(2\pi i)}$ | $0.45e^{0.37(2\pi i)}$ | $0.53e^{0.67(2\pi i)}$ |
| (SRF, eS) | $0.55e^{0.20(2\pi i)}$ | $0.09e^{0.38(2\pi i)}$ | $0.39e^{0.67(2\pi i)}$ |
| (AMS, eS) | $0.66e^{0.58(2\pi i)}$ | $0.33e^{0.43(2\pi i)}$ | $0.31e^{0.67(2\pi i)}$ |

Their Cartesian product is in Table 4:

Each ordered pair of $I \times J$ in Table 4 shows the membership, indeterminacy, and nonmembership grades with amplitude term and phase term. Moreover, the interval part is missing that is responsible to represent the future aspect or prediction for a relationship, whereas CoCPFRs are broader than CoFRs, CoIFRs, and CoPFRs. So, the above-mentioned structures give limited information.

*5.2. CoCPFRs with CFRs, CIFRs, and CPFRs.* CFR is a collection of FS and IVFS. It explains only the amplitude term of membership and interval-valued membership grades of any object. CIFRs explain the amplitude term of membership and nonmembership with single variable. CPFR discussed membership, indeterminacy, and nonmembership with single variable. They are limited to solve only one-dimensional problems. These structures are not capable to solve the periodicity of the problem. On the other hand, CoCPFSs discuss all the three stages, i.e., membership, indeterminacy, and nonmembership with multivariable.

Table 5: Cartesian product $I \times J$.

| Ordered pair | Picture fuzzy grades | Interval-valued picture fuzzy grades |
|---|---|---|
| (SCS, P) | ( 0.30,0.09,0.34 ) | ( [0.21,0.43], [0.43,0.78], [0.61,0.93] ) |
| (SPG, P) | ( 0.05,0.39,0.32 ) | ( [0.25,0.43], [0.43,0.67], [0.61,0.85] ) |
| (MD, P) | ( 0.04,0.41,0.53 ) | ( [0.17,0.19], [0.47,0.67], [0.61,0.79] ) |
| (SRF, P) | ( 0.30,0.09,0.39 ) | ( [0.03,0.19], [0.43,0.89], [0.61,0.80] ) |
| (AMS, P) | ( 0.30,0.33,0.32 ) | ( [0.33,0.41], [0.43,0.63], [0.77,0.96] ) |
| (SCS, S) | ( 0.11,0.09,0.34 ) | ( [0.21,0.43], [0.43,0.78], [0.61,0.93] ) |
| (SPG, S) | ( 0.05,0.25,0.22 ) | ( [0.25,0.43], [0.43,0.67], [0.61,0.85] ) |
| (MD, S) | ( 0.04,0.25,0.53 ) | ( [0.17,0.19], [0.73,0.75], [0.43,0.67] ) |
| (SRF, S) | ( 0.11,0.09,0.39 ) | ( [0.03,0.19], [0.43,0.89], [0.51,0.80] ) |
| (AMS, S) | ( 0.11,0.25,0.17 ) | ( [0.31,0.41], [0.43,0.63], [0.77,0.96] ) |
| (SCS, B) | ( 0.22,0.09,0.81 ) | ( [0.17,0.39], [0.56,0.78], [0.81,0.93] ) |
| (SPG, B) | ( 0.22,0.39,0.81 ) | ( [0.17,0.39], [0.51,0.67], [0.81,0.85] ) |
| (MD, B) | ( 0.04,0.45,0.81 ) | ( [0.17,0.19], [0.47,0.67], [0.81,0.89] ) |
| (SRF, B) | ( 0.22,0.09,0.81 ) | ( [0.03,0.19], [0.43,0.79], [0.81,0.89] ) |
| (AMS, B) | ( 0.22,0.33,0.81 ) | ( [0.17,0.39], [0.52,0.63], [0.77,0.96] ) |
| (SCS, MIM) | ( 0.07,0.09,0.34 ) | ( [0.21,0.63], [0.54,0.76], [0.81,0.93] ) |
| (SPG, MIM) | ( 0.05,0.9,0.22 ) | ( [0.25,0.45], [0.51,0.67], [0.81,0.92] ) |
| (MD, MIM) | ( 0.04,0.45,0.53 ) | ( [0.17,0.19], [0.47,0.67], [0.81,0.92] ) |
| (SRF, MIM) | ( 0.07,0.09,0.39 ) | ( [0.03,0.19], [0.43,0.76], [0.81,0.92] ) |
| (AMS, MIM) | ( 0.07,0.33,0.22 ) | ( [0.33,0.41], [0.52,0.63], [0.81,0.96] ) |
| (SCS, eS) | ( 0.41,0.09,0.34 ) | ( [0.21,0.29], [0.41,0.49], [0.37,0.93] ) |
| (SPG, eS) | ( 0.05,0.39,0.31 ) | ( [0.21,0.29], [0.41,0.49], [0.61,0.85] ) |
| (MD, eS) | ( 0.04,0.45,0.53 ) | ( [0.17,0.19], [0.41,0.49], [0.73,0.75] ) |
| (SRF, eS) | ( 0.55,0.09,0.39 ) | ( [0.03,0.19], [0.41,0.49], [0.51,0.80] ) |
| (AMS, eS) | ( 0.66,0.33,0.31 ) | ( [0.21,0.29], [0.41,0.49], [0.77,0.96] ) |

Let consider the previous problem of e-commerce with CPFSs:

$$I = \left\{ \begin{array}{c} (\mathrm{SCS}, (0.41,0.09,0.34), ([0.21,0.73], [0.56,0.78], [0.37,0.93])), \\ (\mathrm{SPG}, (0.05,0.39,0.22), ([0.25,0.45], [0.51,0.67], [0.61,0.85])), \\ (\mathrm{MD}, (0.04,0.45,0.53), ([0.17,0.19], [0.47,0.6], [0.73,0.75])), \\ (\mathrm{SR\dot{F}}, (0.55,0.09,0.39), ([0.03,0.19], [0.43,0.89], [0.51,0.80])), \\ (\mathrm{AMS}, (0.66,0.33,0.11), ([0.33,0.41], [0.52,0.63], [0.77,0.96])) \end{array} \right\},$$
$$J = \left\{ \begin{array}{c} (\mathrm{P}, (0.30,0.41,0.32), ([0.50,0.43], [0.43,0.89], [0.61,0.79])), \\ (\mathrm{S}, (0.11,0.25,0.17), ([0.31,0.43], [0.43,0.89], [0.61,0.67])), \\ (\mathrm{B}, (0.22,0.78,0.81), ([0.1,0.39], [0.7,0.79], [0.81,0.89])), \\ (\mathrm{MIM}, (0.07,0.51,0.22), ([0.61,0.63], [0.54,0.76], [0.81,0.92])), \\ (\mathrm{eS}, (0.90,0.87,0.31), ([0.21,0.29], [0.41,0.49], [0.11,0.71])) \end{array} \right\}. \quad (71)$$

Then, their Cartesian product is in Table 5:

Each ordered pair of $I \times J$ in Table 5 only shows membership grade without phase terms. CPFR just represents the amplitude term, but complex fuzzy relations explain

Table 6: Cartesian product $I \times J$.

| Ordered pair | Complex picture fuzzy set | Complex interval-valued picture fuzzy set |
|---|---|---|
| (SCS, P) | $\left(0.30e^{0.21(2\pi i)}, 0.34e^{057(2\pi i)}\right)$ | $\left(\left[0.21e^{0.23(2\pi i)}, 0.43e^{0.67(2\pi i)}\right], \left[0.61e^{0.45(2\pi i)}, 0.93e^{0.91(2\pi i)}\right]\right)$ |
| (SPG, P) | $\left(0.05e^{0.21(2\pi i)}, 0.32e^{0.57(2\pi i)}\right)$ | $\left(\left[0.25e^{0.31(2\pi i)}, 0.43e^{0.37(2\pi i)}\right], \left[0.61e^{0.54(2\pi i)}, 0.85e^{0.89(2\pi i)}\right]\right)$ |
| (MD, P) | $\left(0.04e^{0.21(2\pi i)}, 0.53e^{0.67(2\pi i)}\right)$ | $\left(\left[0.17e^{0.27(2\pi i)}, 0.19e^{0.39(2\pi i)}\right], \left[0.61e^{0.45(2\pi i)}, 0.79e^{0.91(2\pi i)}\right]\right)$ |
| (SRF, P) | $\left(0.30e^{0.21(2\pi i)}, 0.39e^{0.57(2\pi i)}\right)$ | $\left(\left[0.03e^{0.23(2\pi i)}, 0.19e^{0.57(2\pi i)}\right], \left[0.61e^{0.62(2\pi i)}, 0.80e^{0.91(2\pi i)}\right]\right)$ |
| (AMS, P) | $\left(0.30e^{0.21(2\pi i)}, 0.32e^{0.57(2\pi i)}\right)$ | $\left(\left[0.33e^{0.23(2\pi i)}, 0.41e^{0.31(2\pi i)}\right], \left[0.77e^{0.54(2\pi i)}, 0.96e^{0.95(2\pi i)}\right]\right)$ |
| (SCS, S) | $\left(0.11e^{0.21(2\pi i)}, 0.34e^{0.67(2\pi i)}\right)$ | $\left(\left[0.21e^{0.11(2\pi i)}, 0.43e^{0.67(2\pi i)}\right], \left[0.61e^{0.45(2\pi i)}, 0.93e^{0.85(2\pi i)}\right]\right)$ |
| (SPG, S) | $\left(0.05e^{0.21(2\pi i)}, 0.22e^{0.67(2\pi i)}\right)$ | $\left(\left[0.25e^{0.11(2\pi i)}, 0.43e^{0.37(2\pi i)}\right], \left[0.61e^{0.54(2\pi i)}, 0.85e^{0.89(2\pi i)}\right]\right)$ |
| (MD, S) | $\left(0.04e^{0.21(2\pi i)}, 0.53e^{0.67(2\pi i)}\right)$ | $\left(\left[0.17e^{0.11(2\pi i)}, 0.19e^{0.39(2\pi i)}\right], \left[0.73e^{0.41(2\pi i)}, 0.75e^{0.78(2\pi i)}\right]\right)$ |
| (SRF, S) | $\left(0.11e^{0.21(2\pi i)}, 0.39e^{0.67(2\pi i)}\right)$ | $\left(\left[0.03e^{0.11(2\pi i)}, 0.19e^{0.57(2\pi i)}\right], \left[0.51e^{0.62(2\pi i)}, 0.80e^{0.87(2\pi i)}\right]\right)$ |
| (AMS, S) | $\left(0.11e^{0.21(2\pi i)}, 0.17e^{0.67(2\pi i)}\right)$ | $\left(\left[0.31e^{0.11(2\pi i)}, 0.41e^{0.31(2\pi i)}\right], \left[0.77e^{0.54(2\pi i)}, 0.96e^{0.95(2\pi i)}\right]\right)$ |
| (SCS, B) | $\left(0.22e^{0.56(2\pi i)}, 0.81e^{0.57(2\pi i)}\right)$ | $\left(\left[0.17e^{0.35(2\pi i)}, 0.39e^{0.67(2\pi i)}\right], \left[0.81e^{0.45(2\pi i)}, 0.93e^{0.85(2\pi i)}\right]\right)$ |
| (SPG, B) | $\left(0.22e^{0.59(2\pi i)}, 0.81e^{0.51(2\pi i)}\right)$ | $\left(\left[0.17e^{0.31(2\pi i)}, 0.39e^{0.37(2\pi i)}\right], \left[0.81e^{0.54(2\pi i)}, 0.85e^{0.89(2\pi i)}\right]\right)$ |
| (MD, B) | $\left(0.04e^{0.51(2\pi i)}, 0.81e^{0.67(2\pi i)}\right)$ | $\left(\left[0.17e^{0.27(2\pi i)}, 0.19e^{0.39(2\pi i)}\right], \left[0.81e^{0.45(2\pi i)}, 0.89e^{0.78(2\pi i)}\right]\right)$ |
| (SRF, B) | $\left(0.22e^{0.42(2\pi i)}, 0.81e^{057(2\pi i)}\right)$ | $\left(\left[0.03e^{0.35(2\pi i)}, 0.19e^{0.57(2\pi i)}\right], \left[0.81e^{0.62(2\pi i)}, 0.89e^{0.87(2\pi i)}\right]\right)$ |
| (AMS, B) | $\left(0.22e^{0.58(2\pi i)}, 0.81e^{0.57(2\pi i)}\right)$ | $\left(\left[0.17e^{0.23(2\pi i)}, 0.39e^{0.31(2\pi i)}\right], \left[0.77e^{0.54(2\pi i)}, 0.96e^{0.95(2\pi i)}\right]\right)$ |
| (SCS, MIM) | $\left(0.07e^{0.09(2\pi i)}, 0.34e^{0.55(2\pi i)}\right)$ | $\left(\left[0.21e^{0.09(2\pi i)}, 0.63e^{0.21(2\pi i)}\right], \left[0.81e^{049(2\pi i)}, 0.93e^{0.85(2\pi i)}\right]\right)$ |
| (SPG, MIM) | $\left(0.05e^{0.09(2\pi i)}, 0.22e^{0.51(2\pi i)}\right)$ | $\left(\left[0.25e^{0.09(2\pi i)}, 0.45e^{0.21(2\pi i)}\right], \left[0.81e^{0.54(2\pi i)}, 0.92e^{0.89(2\pi i)}\right]\right)$ |
| (MD, MIM) | $\left(0.04e^{0.09(2\pi i)}, 0.53e^{0.67(2\pi i)}\right)$ | $\left(\left[0.17e^{0.09(2\pi i)}, 0.19e^{0.21(2\pi i)}\right], \left[0.81e^{0.41(2\pi i)}, 0.92e^{0.78(2\pi i)}\right]\right)$ |
| (SRF, MIM) | $\left(0.07e^{0.09(2\pi i)}, 0.39e^{0.48(2\pi i)}\right)$ | $\left(\left[0.03e^{0.09(2\pi i)}, 0.19e^{0.21(2\pi i)}\right], \left[0.81e^{0.62(2\pi i)}, 0.92e^{0.87(2\pi i)}\right]\right)$ |
| (AMS, MIM) | $\left(0.07e^{0.09(2\pi i)}, 0.22e^{0.43(2\pi i)}\right)$ | $\left(\left[0.33e^{0.09(2\pi i)}, 0.41e^{0.21(2\pi i)}\right], \left[0.81e^{0.54(2\pi i)}, 0.96e^{0.95(2\pi i)}\right]\right)$ |
| (SCS, eS) | $\left(0.41e^{0.20(2\pi i)}, 0.34e^{0.55(2\pi i)}\right)$ | $\left(\left[0.21e^{0.55(2\pi i)}, 0.29e^{0.59(2\pi i)}\right], \left[0.37e^{0.45(2\pi i)}, 0.93e^{0.91(2\pi i)}\right]\right)$ |
| (SPG, eS) | $\left(0.05e^{0.20(2\pi i)}, 0.31e^{0.67(2\pi i)}\right)$ | $\left(\left[0.21e^{0.31(2\pi i)}, 0.29e^{0.37(2\pi i)}\right], \left[0.61e^{0.54(2\pi i)}, 0.85e^{0.91(2\pi i)}\right]\right)$ |
| (MD, eS) | $\left(0.04e^{0.20(2\pi i)}, 0.53e^{0.67(2\pi i)}\right)$ | $\left(\left[0.17e^{0.27(2\pi i)}, 0.19e^{0.39(2\pi i)}\right], \left[0.73e^{0.41(2\pi i)}, 0.75e^{0.91(2\pi i)}\right]\right)$ |
| (SRF, eS) | $\left(0.55e^{0.20(2\pi i)}, 0.39e^{0.67(2\pi i)}\right)$ | $\left(\left[0.03e^{0.55(2\pi i)}, 0.19e^{0.57(2\pi i)}\right], \left[0.51e^{0.62(2\pi i)}, 0.80e^{0.87(2\pi i)}\right]\right)$ |
| (AMS, eS) | $\left(0.66e^{0.58(2\pi i)}, 0.31e^{0.67(2\pi i)}\right)$ | $\left(\left[0.21e^{0.23(2\pi i)}, 0.29e^{0.31(2\pi i)}\right], \left[0.77e^{0.54(2\pi i)}, 0.96e^{0.95(2\pi i)}\right]\right)$ |

Table 7: A complete comparison among all the structure based on different fuzzy information.

| Structure | Membership | Indeterminacy | Nonmembership | Multidimensional | Dual memberships |
|---|---|---|---|---|---|
| FR | Yes | No | No | No | No |
| CFR | Yes | No | No | No | Yes |
| CoCFR | Yes | No | No | Yes | Yes |
| IFR | Yes | No | Yes | No | No |
| CIFR | Yes | No | Yes | No | Yes |
| CoCIFR | Yes | No | Yes | Yes | Yes |
| PFR | Yes | Yes | Yes | No | No |
| CPFR | Yes | Yes | Yes | No | Yes |
| CoCPFR | Yes | Yes | Yes | Yes | Yes |

both terms of membership. But CoCPFRs are broader than FRs, CFRs, CIFRs, and CPFRs. So, the above-mentioned structures give limited information.

*5.3. CoCPFRs with CoCFRs and CoCIFRs.* CoCFRs discuss complex membership, and CoCIFRs only explain the complex membership and complex nonmembership with multivariable. These structures are capable to solve multidimensional problems and also discuss the periodicity of the problem. But CoCPFRs also describes the neutral effect of one object on the other. It is the broader than CoCFRs and CoCIFRs.

Here, an example is taken of previously presented application problem and solve it with respect to CoCIFSs and carry out a comprehensive analysis.

$$I=\left\{\begin{array}{l}\left(\text{SCS},\left(0.41e^{0.56(2\pi i)},0.34e^{0.55(2\pi i)}\right),\left(\left[0.21e^{0.55(2\pi i)},0.73e^{0.67(2\pi i)}\right],\left[0.37e^{0.45(2\pi i)},0.93e^{0.85(2\pi i)}\right]\right)\right),\\ \left(\text{SPG},\left(0.05e^{0.47(2\pi i)},0.22e^{0.51(2\pi i)}\right),\left(\left[0.25e^{0.31(2\pi i)},0.45e^{0.37(2\pi i)}\right],\left[0.61e^{0.54(2\pi i)},0.85e^{0.89(2\pi i)}\right]\right)\right),\\ \left(\text{MD},\left(0.04e^{0.51(2\pi i)},0.53e^{0.67(2\pi i)}\right),\left(\left[0.17e^{0.27(2\pi i)},0.19e^{039(2\pi i)}\right],\left[0.73e^{0.41(2\pi i)},0.75e^{0.78(2\pi i)}\right]\right)\right),\\ \left(\text{SR}\dot{\text{F}},\left(0.55e^{042(2\pi i)},0.39e^{0.48(2\pi i)}\right),\left(\left[0.03e^{0.55(2\pi i)},0.19e^{0.57(2\pi i)}\right],\left[0.51e^{0.62(2\pi i)},0.80e^{0.87(2\pi i)}\right]\right)\right),\\ \left(\text{AMS},\left(0.66e^{0.58(2\pi i)},0.11e^{0.32(2\pi i)}\right),\left(\left[0.33e^{0.23(2\pi i)},0.41e^{0.31(2\pi i)}\right],\left[0.77e^{0.54(2\pi i)},0.96e^{0.95(2\pi i)}\right]\right)\right)\end{array}\right\},$$
$$J=\left\{\begin{array}{l}\left(\text{P},\left(0.30e^{0.21(2\pi i)},0.32e^{0.57(2\pi i)}\right),\left(\left[0.50e^{0.23(2\pi i)},0.43e^{0.76(2\pi i)}\right],\left[0.61e^{0.45(2\pi i)},0.79e^{0.91(2\pi i)}\right]\right)\right),\\ \left(\text{S},\left(0.11e^{0.21(2\pi i)},0.17e^{0.67(2\pi i)}\right),\left(\left[0.31e^{0.11(2\pi i)},0.43e^{0.89(2\pi i)}\right],\left[0.61e^{0.36(2\pi i)},0.67e^{0.54(2\pi i)}\right]\right)\right),\\ \left(\text{B},\left(0.22e^{0.59(2\pi i)},0.81e^{0.57(2\pi i)}\right),\left(\left[0.17e^{0.35(2\pi i)},0.39e^{0.76(2\pi i)}\right],\left[0.81e^{0.45(2\pi i)},0.89e^{0.55(2\pi i)}\right]\right)\right),\\ \left(\text{MIM},\left(0.07e^{0.09(2\pi i)},0.22e^{0.43(2\pi i)}\right),\left(\left[0.61e^{0.09(2\pi i)},0.63e^{0.21(2\pi i)}\right],\left[0.81e^{0.49(2\pi i)},0.92e^{0.71(2\pi i)}\right]\right)\right),\\ \left(\text{eS},\left(0.90e^{0.20(2\pi i)},0.31e^{0.67(2\pi i)}\right),\left(\left[0.21e^{0.56(2\pi i)},0.29e^{0.59(2\pi i)}\right],\left[0.11e^{0.32(2\pi i)},0.71e^{0.91(2\pi i)}\right]\right)\right)\end{array}\right\}. \tag{72}$$

Their Cartesian product is in Table 6:

Each ordered pair of $I\times J$ of Table 6 describes the effectiveness and ineffectiveness of e-commerce securities against threats through membership and nonmembership, respectively. But do not tell about the neutral effect of securities on threats. These structures give limited information and not providing required results of given problem. CoCPFRs produce better results that are required to obtained detailed information. In Table 7, the complete comparison among different structures is presented.

To summarize the above comparisons, the advantages of the proposed concepts are stated below based on the experimental results:

(a) CoCPFRs deal with three grades, which are membership, nonmembership, and indeterminacy

(b) Their structure is composed of complex-valued functions, so they model periodicity with the help of phase terms, that is, the imaginary part

(c) They have dual degrees comprising of single-valued and interval-valued membership, indeterminacy, and nonmembership grades. This allows them to present two different time periods or stages in a decision-making process

## 6. Conclusion

This research introduced the innovative concepts of complex cubic picture fuzzy sets (CoCPFSs), complex cubic picture fuzzy relations (CoCPFRs), and Cartesian product of two complex cubic picture fuzzy sets. Furthermore, various types of CoCPFRs are also defined, including CoCP-reflexive-FR, CoCP-symmetric-FR, CoCP-transitive-FR, CoCP-equivalence-FR, CoCP-partial order-FR, CoCP-strict order-FR, CoCP-linear order-FR, CoCP-composite-FR, and CoCPF equivalence class, and were also studied with the help of suitable examples, properties, and some results which are also proven. The idea of these newly defined concepts and novel modeling techniques is used to address the security concerns in e-commerce in digital business system. Online business systems have recently been targeted by the hackers and cybercriminals. The current study analyzes the relationships among the effectiveness of e-commerce security and the most serious and common risks to the digital business. Then, by using CoCPFRs investigate the impact of security on the threats. This innovative structure explains the present impact as well as the future forecasting of the impact in the form of interval to reduce ambiguity of securities on threats. The strength of this structure is it discussed all level of an object which is level of membership, level of indeterminacy, and level of nonmembership. As it is complex, it deals with periodicity of an object with multivariables. These concepts can be extended to other generalization of fuzzy sets which will give rise to many structures with vast range of applications.

## Data Availability

All the data used is given in the paper.

## Conflicts of Interest

The authors declare that there are no conflicts of interest among them.

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## References

[1] L. A. Zadeh, "Fuzzy sets," *Information and control*, vol. 8, no. 3, pp. 338–353, 1965.

[2] G. Q. Chen, S. C. Lee, and S. H. Eden, "Application of fuzzy set theory to economics," in *Advances in Fuzzy Sets, Possibility Theory, and Applications*, pp. 277–305, Springer, Boston, MA, 1983.

[3] K. P. Adlassnig, "Fuzzy set theory in medical diagnosis," *IEEE Transactions on Systems, Man, and Cybernetics*, vol. 16, no. 2, pp. 260–265, 1986.

[4] J. Lu and D. Ruan, *Multi-Objective Group Decision Making: Methods, Software and Applications with Fuzzy Set Techniques*, vol. 6, Imperial College Press, 2007.

[5] G. J. Klir and M. J. Wierman, "Uncertainty-based information: elements of generalized information theory," *Physica*, vol. 15, 2013.

[6] J. M. Mendel, "Fuzzy logic systems for engineering: a tutorial," *Proceedings of the IEEE*, vol. 83, no. 3, pp. 345–377, 1995.

[7] D. Yu, Q. Hu, and C. Wu, "Uncertainty measures for fuzzy relations and their applications," *Applied Soft Computing*, vol. 7, no. 3, pp. 1135–1143, 2007.

[8] L. A. Zadeh, "The concept of a linguistic variable and its application to approximate reasoning–I," *Information Sci*, vol. 8, no. 3, pp. 199–249, 1975.

[9] H. Bustince, "Indicator of inclusion grade for interval-valued fuzzy sets. Application to approximate reasoning based on interval-valued fuzzy sets," *International Journal of Approximate Reasoning*, vol. 23, no. 3, pp. 137–209, 2000.

[10] H. Bustince and P. Burillo, "Mathematical analysis of interval-valued fuzzy relations: application to approximate reasoning," *Fuzzy Sets and Systems*, vol. 113, no. 2, pp. 205–219, 2000.

[11] M. Gehrke, C. Walker, and E. Walker, "Some comments on interval valued fuzzy sets!," *Structure*, vol. 1, p. 2, 1996.

[12] M. K. Roy and R. Biswas, "I-v fuzzy relations and Sanchez's approach for medical diagnosis," *Fuzzy Sets and Systems*, vol. 47, no. 1, pp. 35–38, 1992.

[13] K. Atanassov, "Intuitionistic fuzzy sets," *International Journal Bioautomation*, vol. 20, p. 1, 2016.

[14] L. Dengfeng and C. Chuntian, "New similarity measures of intuitionistic fuzzy sets and application to pattern recognitions," *Pattern Recognition Letters*, vol. 23, no. 1-3, pp. 221–225, 2002.

[15] E. Szmidt and J. Kacprzyk, "Intuitionistic fuzzy sets in some medical applications," in *International conference on computational intelligence*, pp. 148–151, Springer, Berlin, Heidelberg, 2001.

[16] P. Burillo and H. Bustince, "Intuitionistic fuzzy relations (part I)," *Mathware and soft computing*, vol. 2, no. 1, pp. 5–38, 1995.

[17] Y. J. Lei, B. S. Wang, and Q. G. Miao, "On the intuitionistic fuzzy relations with compositional operations," *Systems Engineering-theory & Practice*, vol. 2, 2005.

[18] B. C. Cuong and V. Kreinovich, "Picture fuzzy sets-a new concept for computational intelligence problems," in *2013 third world congress on information and communication technologies (WICT 2013)*, pp. 1–6, IEEE, 2013.

[19] A. H. Ganie, S. Singh, and P. K. Bhatia, "Some new correlation coefficients of picture fuzzy sets with applications," *Neural Computing and Applications*, vol. 32, no. 16, pp. 12609–12625, 2020.

[20] D. Ramot, R. Milo, M. Friedman, and A. Kandel, "Complex fuzzy sets," *IEEE Transactions on Fuzzy Systems*, vol. 10, no. 2, pp. 171–186, 2002.

[21] C. Li and T. W. Chiang, "Complex neurofuzzy ARIMA forecasting—a new approach using complex fuzzy sets," *IEEE Transactions on Fuzzy Systems*, vol. 21, no. 3, pp. 567–584, 2013.

[22] G. Zhang, T. S. Dillon, K. Y. Cai, J. Ma, and J. Lu, "Operation properties and $\delta$-equalities of complex fuzzy sets," *International Journal of Approximate Reasoning*, vol. 50, no. 8, pp. 1227–1249, 2009.

[23] S. Greenfield, F. Chiclana, and S. Dick, "Interval-valued complex fuzzy logic," in *2016 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE)*, pp. 2014–2019, IEEE, 2016.

[24] A. Nasir, N. Jan, A. Gumaei, and S. U. Khan, "Medical diagnosis and life span of sufferer using interval valued complex fuzzy relations," *IEEE Access*, vol. 9, pp. 93764–93780, 2021.

[25] D. Rani and H. Garg, "Distance measures between the complex intuitionistic fuzzy sets and their applications to the decision-making process," *International Journal for Uncertainty Quantification*, vol. 7, no. 5, pp. 423–439, 2017.

[26] N. Jan, A. Nasir, M. S. Alhilal, S. U. Khan, D. Pamucar, and A. Alothaim, "Investigation of cyber-security and cyber-crimes in oil and gas sectors using the innovative structures of complex intuitionistic fuzzy relations," *Entropy*, vol. 23, no. 9, p. 1112, 2021.

[27] R. T. Ngan, M. Ali, D. E. Tamir, N. D. Rishe, and A. Kandel, "Representing complex intuitionistic fuzzy set by quaternion numbers and applications to decision making," *Applied Soft Computing*, vol. 87, p. 105961, 2020.

[28] M. Akram, A. Bashir, and H. Garg, "Decision-making model under complex picture fuzzy Hamacher aggregation operators," *Computational and Applied Mathematics*, vol. 39, no. 3, pp. 1–38, 2020.

[29] A. Nasir, N. Jan, S. U. Khan, A. Gumaei, and A. Alothaim, "Analysis of communication and network securities using the concepts of complex picture fuzzy relations," *Computational Intelligence and Neuroscience*, vol. 2021, 20 pages, 2021.

[30] Y. B. Jun, C. S. Kim, and K. O. Yang, "Cubic sets," *Ann. Fuzzy Math. Inform*, vol. 4, no. 1, pp. 83–98, 2012.

[31] J. Kim, P. K. Lim, J. G. Lee, and K. Hur, "Cubic relations," *Ann. Fuzzy Math. Inform*, vol. 19, no. 1, pp. 21–43, 2020.

[32] G. Kaur and H. Garg, "Generalized cubic intuitionistic fuzzy aggregation operators using t-norm operations and their applications to group decision-making process," *Arabian Journal for Science and Engineering*, vol. 44, no. 3, pp. 2775–2794, 2019.

[33] H. Garg and G. Kaur, "Cubic intuitionistic fuzzy sets and its fundamental properties," *Journal of Multiple-Valued Logic & Soft Computing*, vol. 33, no. 6, 2019.

[34] Y. B. Jun, S. Z. Song, and S. J. Kim, "Cubic interval-valued intuitionistic fuzzy sets and their application in BCK/BCI-algebras," *Axioms*, vol. 7, no. 1, p. 7, 2018.

[35] V. Chinnadurai, S. Thayalana, and A. Bobin, "Complex cubic set and their properties," *Advances in Mathematics*, vol. 9, pp. 1561–1567, 2020.

[36] X. Zhou, Y. Deng, Z. Huang, F. Yan, and W. Li, "Complex cubic fuzzy aggregation operators with applications in group decision-making," *IEEE Access*, vol. 8, pp. 223869–223888, 2020.

[37] V. Chinnadurai, S. Thayalana, and A. Bobin, "Complex cubic intuitionistic fuzzy set and its decision making," *Advances in Mathematics*, vol. 9, pp. 7933–7946, 2020.

[38] D. Rani and H. Garg, "Complex intuitionistic fuzzy preference relations and their applications in individual and group decision-making problems," *International Journal of Intelligent Systems*, vol. 36, no. 4, pp. 1800–1830, 2021.

[39] A. Gumaei and A. Hussain, "New operators of cubic picture fuzzy information with applications," *Journal of Mathematics*, vol. 2021, 16 pages, 2021.