Research Article

Novel Analysis of Fuzzy Fractional Emden-Fowler Equations within New Iterative Transform Method

M. Mossa Al-Sawalha,¹ Naila Amir,² Rasool Shah,³ and Muhammad Yar⁴

¹Department of Mathematics, Faculty of Science, University of Ha’il, Ha’il 2440, Saudi Arabia
²Department of Humanities and Sciences, School of Electrical Engineering and Computer Science (SEECS), National University of Sciences and Technology (NUST), Islamabad, Pakistan
³Department of Mathematics, Abdul Wali Khan University Mardan, Pakistan
⁴Department of Mathematics, Kabul Polytechnic University, Kabul, Afghanistan

Correspondence should be addressed to Naila Amir; naila.amir@seecs.edu.pk and Muhammad Yar; myar@kpu.edu.af

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The analytical behavior of fractional differential equations is often puzzling and difficult to predict under uncertainty. It is crucial to develop a robust, extensive, and extremely successful theory to address these problems. An application of fuzzy fractional differential equations can be found in applied mathematics and engineering. Using the iterative transform technique, the study determines the analytic solution of fractional fuzzy Emden-Fowler equation in the sense of the Caputo operator, which is applied to evaluate the physical model range in several scientific and engineering disciplines. The derived solutions to the fractional fuzzy Emden-Fowler equations are more generic and applicable to a broader range of problems. Through the translation of fractional fuzzy differential equations into equivalent crisp systems of fractional differential equations, we obtain a parametric description of the solutions. The graphical and numerical representation demonstrates the symmetry among the upper and lower fuzzy solution representations in their simplest form, which may aid in the comprehension of artificial intelligence, control system models, computer science, image processing, quantum optics, medical science, physics, measure theory, stochastic optimization theory, biology, mathematical finance, and other domains, as well as nonfinancial evaluation.

1. Introduction

Physical models of real-world events sometimes contain significant uncertainty due to a number of variables. Fuzzy sets seem to be a decent way to simulate the uncertainty that comes from being vague and not being clear. We use it in fields like the environment, medicine, economics, sociology, and physics where there is uncertainty in the data. Zadeh examined these difficulties in 1965 when he introduced fuzziness to set theory. Over the last two decades, fractional calculus has become increasingly important due to its many implementations in applied science. The behavior of specified system processes contains numerous instances of fuzzy rather than stochastic uncertainty. Numerous authors have been focused in the theory foundations of fuzzy problems recent decades. Modeling population models, appraising civil engineering, weapon systems, and modeling electrohydraulics are examples of issues where fractional fuzzy differential equations come in handy. In addition to fractional calculus and the use of fuzzy concept, it is a powerful tool for deal with uncertainty, identifying affective or ambiguous condition, and offering further general answers in mathematics. This has been discussed in a number of real-life scenarios, components of the golden means [1], medicine [2], practical schemes [3], engineering phenomena, gravity, and quantum optics [4]. Zadeh [5] became acquainted with first time use fuzzy set theory. Now there is research on intuitionistic fuzzy and their application in fuzzy controls [6] and approximation reasoning issues [7]. It is tough to depict numerous scenarios using actual numbers in data analysis adequately [8, 9]. Later, Shah et al. and Mizumoto and Tanaka [10, 11], Dubois and Dubois and Prade [12, 13], Ralescu [14], and Nahmias [15] defined
the basics of fuzzy number arithmetic. They calculated the fuzzy number as a series of intervals, i.e., ρ-levels, 0 < ρ ≤ 1, [16]. It contains the basic ideas on noncrisp sets and details on fuzzy differential equation. The suggested concepts are differential generalization equations. This is a new concept that has piqued the interest of numerous scholars. Real-life situations involving fractional-order differential equations are important; applications may be found in physics, chemistry, engineering, etc.

Fractional partial differential equations (FPDEs) are useful for modelling a wide range of biological, economic, and dynamical phenomena. As a result, throughout the previous several decades, this topic has received a lot of intriguing and contemporary research. FPDEs can also be used to simulate a wide range of phenomena, including heat, sound, electromagnetism, electrodynamics, elastic, and hydrodynamics. The applicable research was focused on the numerical and analytical results of the FPDEs (see [17–19]). The well-known FPDEs, such as heat, wave, and Laplace equations, have been thoroughly researched, and there is a wealth of literature on these topics (see [20–22]).

During the thermal behavior research of a classical law connected to thermodynamics and the spherical cloud of gas, the nonlinear historical issue known as the Lane-Emden equation was developed by the well-known American scientific astrophysicist J. H. Lane and meteorologist J. R. Emden and Swiss astrophysicist. These singular equations described by Lane-Emden equations have since been used in a variety of applications in applied science [23–25]. The Emden-Fowler model is a differential equation that occurs in astrophysics and mathematical physics. It is numerically difficult to solve the Emden-Fowler problem and other different linear and nonlinear singular initial value problems in quantum mechanics and astrophysics because of the singularity behavior at the point (y = 0). This article studies with the approximate result for the singular, linear, and nonlinear fractional-order Emden-Fowler equation using the variational iteration transform method. The different analytical and numerical methods have been solved by Emden-Fowler equation, such as homotopy-perturbation method [26], Haar wavelet collocation method [27], residual power series method [28], Sumudu transform method [29], and modified differential transform method [30].

2. Preliminaries

**Definition 1.** Fractional fuzzy Riemann-Liouville integral is expressed in the presence of a fuzzy continuous function ϕ, ν on the interval [0, β] ∈ R.

\[ \mathcal{I}^{\nu} \nu = \int_{0}^{\nu} \frac{(\eta - \eta)^{\nu-1} \nu(\eta)}{\Gamma(\nu)} d\eta, \eta, \eta \in (0, \infty). \] (1)

In addition, if \( \nu \in C^{\nu}[0, \beta] \cap L^{\nu}[0, \beta], \) where \( C^{\nu}[0, \beta] \) is the universe of continuous fuzzy functions and \( L^{\nu}[0, \beta] \) is the Lebesgue integral, then the fractional fuzzy integral can be expressed as

\[ [\mathcal{I}^{\nu} \nu(\varphi)] = [\mathcal{I}^{\nu} \nu_{t}, \mathcal{I}^{\nu} \nu_{\xi}], 0 \leq \xi \leq 1 \] (2)

such that

\[ \mathcal{I}^{\nu} \nu_{t} = \int_{0}^{\nu} \frac{(\nu - \eta)^{\nu-1} \nu(\eta)}{\Gamma(\nu)} d\eta, q, \eta \in (0, \infty), \]

(3)

\[ \mathcal{I}^{\nu} \nu_{\xi} = \int_{0}^{\nu} \frac{(\nu - \eta)^{\nu-1} \nu(\eta)}{\Gamma(\nu)} d\eta, q, \eta \in (0, \infty). \]

**Definition 2.** For a term of \( \hat{\nu} \in C^{\nu}[0, \beta] \cap L^{\nu}[0, \beta], \) such that \( \nu = [\nu_{t}(\varphi), \nu_{\xi}(\varphi)], \xi \in [0, 1] \) and \( \varphi_{0} \in (0, \beta), \) then the fuzzy fractional Caputo operator is defined as

\[ [D_{\nu} \hat{\nu}(\varphi_{0})] = [D_{\nu} \nu(\varphi_{0}), D_{\nu} \nu(\varphi_{0})], 0 < \rho \leq 1, \] (4)

where

\[ D^{\nu} \nu_{t}(\varphi_{0}) = \left[ \frac{\varphi_{0}^{\mu m - 1} (d^{\mu m} / d\varphi^{\mu m}) \nu_{t}(\eta)}{\Gamma(\nu)} \right]_{\varphi = \varphi_{0}}, \]

(5)

\[ D^{\nu} \nu_{\xi}(\varphi_{0}) = \left[ \frac{\varphi_{0}^{\mu m - 1} (d^{\mu m} / d\varphi^{\mu m}) \nu_{\xi}(\eta)}{\Gamma(\nu)} \right]_{\varphi = \varphi_{0}}, \]

in such a way that the right side integration converges and \( m = [\nu]. \) Since \( \rho \in (0, 1), \) \( m = 1. \)

**Definition 3.** The Laplace fuzzy transformation for \( h(\varphi), \) where \( h(\varphi) \) is fuzzy value term, is given as

\[ G(\varphi) = L[h(\varphi)] = \int_{0}^{\infty} e^{\nu \varphi} h(\varphi) d\varphi, \varphi > 0. \] (6)

**Definition 4.** In terms of convolution fuzzy, a Laplace fuzzy transform is defined as

\[ L[h_{1} \ast h_{2}] = L[h_{1}] \ast L[h_{2}], \]

(7)

where \( h_{1} \ast h_{2} \) express the convolution fuzzy among \( h_{1} \) and \( h_{2}, \) i.e.,

\[ h_{1} \ast h_{2} = \int_{0}^{\nu} h_{1}(\varphi) \ast h_{2}(\varphi - \varphi) d\varphi. \] (8)

**Definition 5.** The “Mittag-Leffler function” \( E_{\rho}(\varphi) \) is defined as

\[ E_{\rho}(\varphi) = \sum_{n=0}^{\infty} \frac{\varphi^{n}}{(\rho \varphi + 1)^{n}}, \]

(9)

where \( \rho > 0. \)

**Definition 6.** Let \( \kappa : \mathbb{R} \rightarrow [0, 1] \) be a count of fuzzy with the appropriate qualities.

(i) \( \kappa \) is an upper semi-continuous number
Theorem 8. Let \( h' (p) \) be an integrable fuzzy-valued function, and \( h(p) \) is the primitive of \( h'(p) \) on \([0, \infty)\). Then, \( \mathcal{L} \left[ h'(p) \right] = p \circ \mathcal{L}[h(p)] - \hat{h}(0) \) where \( h \) is \( (i) \)-differentiable or \( \mathcal{L}[h'(p)] = (-h(0)) - \theta (-p \circ \mathcal{L}[h(p)]) \) where \( h \) is \( (ii) \)-differentiable.

Proof. For arbitrary fixed \( r \in [0, 1] \) we have

\[
(p \circ \mathcal{L}[h(p)]) - \hat{h}(0) = (p* \hat{h}(p,r) - \hat{h}(0,r), p* \hat{h}(p,r)).
\]

Since \( \ell [h'(p,r)] = p* \hat{h}(p,r) - \hat{h}(0,r) \), then

\[
(p \circ \mathcal{L}[h(p)]) - \hat{h}(0) = \ell [h'(p,r), \hat{h}'(p,r)].
\]

By linearity of \( L \),

\[
(p \circ \mathcal{L}[h(p)]) - \hat{h}(0) = \ell [h'(p,r), \hat{h}'(p,r)] = \ell [\hat{h}'(p,r)].
\]

Since \( h \) is \( (i) \)-differentiable, it follows that

\[
(p \circ \mathcal{L}[h(p)]) - \hat{h}(0) = \mathcal{L} [h'(p)].
\]

Now, we assume that \( h \) is the \( (ii) \)-differentiable; for arbitrary fixed \( r \in [0, 1] \), we have

\[
(-h(0)) - (-p \circ \mathcal{L}[h(p)]) = (-h(0,r) + p* \hat{h}(p,r), -h(0,r) + p* \hat{h}(p,r)).
\]

This is equivalent to the following:

\[
(p* \hat{h}(p,r) - \hat{h}(0,r), p* \hat{h}(p,r) - \hat{h}(0,r)).
\]
\[ \int_0^\infty (h(p) \otimes g(p)) \otimes e^{-p\rho} d\rho = \left( \int_0^\infty g(p) h(p, r) e^{-p\rho} d\rho \right) \int_0^\infty (h(p, r) e^{-p\rho} d\rho). \] (19)

**Theorem 12.** Let \( h \) be continuous fuzzy-valued function and \( \mathcal{L}[h(p)] = H(p) \); then,

\[ \mathcal{L}[e^{\rho p} \otimes h(p)] = H(p - a), \] (20)

where \( e^{\rho p} \) is real value function and \( p - a > 0 \).

**Proof.**

\[ \mathcal{L}[e^{\rho p} \otimes h(p)] = \int_0^\infty e^{p\rho - p\rho} \otimes h(p) \]
\[ = \left( \int_0^\infty e^{p\rho - p\rho} h(p, r) d\rho \right) \int_0^\infty (h(p, r) e^{p\rho - p\rho} d\rho) \]
\[ = \int_0^\infty e^{(p-a)p} \otimes h(p) = H(p - a). \] (21)

\[ \square \]

3. **General Implementation of the Proposed Method**

\[ D^q_\rho \nu^-(p, \varphi) = D^q_\rho \nu^-(p, \varphi) + \nu^-(p, \varphi) + \tilde{\kappa}(\zeta), \] \( 0 < q \leq 1, \nu^-(p, 0) = \tilde{\gamma}(p), \] (22)

where \( D \) stands for the Caputo fractional derivative and

\[ \nu^-(p, \varphi) = \tilde{\gamma}(\varphi). \] (23)

Apply the Laplace transformation on (22) as

\[ \mathcal{L}\left[D^q_\rho \nu^-(p, \varphi)\right] = \mathcal{L}\left[D^q_\rho \nu^-(p, \varphi) + \nu^-(p, \varphi) + \tilde{\kappa}\right]. \] (24)

Using the initial condition, we have

\[ s^q \mathcal{L}\left[\nu^-(p, \varphi)\right] = s^q - 1 \tilde{\gamma}(\varphi) + \mathcal{L}\left[D^q_\rho \nu^-(p, \varphi) + \nu^-(p, \varphi) + \tilde{\kappa}\right], \]
\[ \mathcal{L}\left[\nu^-(p, \varphi)\right] = \frac{\tilde{\gamma}(\varphi)}{s^q} + \frac{1}{s^q} \mathcal{L}\left[D^q_\rho \nu^-(p, \varphi) + \nu^-(p, \varphi) + \tilde{\kappa}\right]. \] (25)

Suppose that the solution is \( \nu^-(p, \varphi) = \sum_{n=0}^\infty U_n(p, \varphi) \); then, (25) expressed

\[ \mathcal{L}\left[\sum_{n=0}^\infty U_n(p, \varphi)\right] = \frac{\tilde{\gamma}(p)}{s^q} + \frac{1}{s^q} \mathcal{L}\left[D^q_\rho \sum_{n=0}^\infty U_n(p, \varphi) + \sum_{n=0}^\infty V_n(p, \varphi) + \tilde{\kappa}\right]. \] (26)

In terms of comparison on both sides, we obtain

\[ \mathcal{L}\left[\nu^-(0, \varphi)\right] = \frac{\tilde{\gamma}(p)}{s^q} + \frac{1}{s^q} \mathcal{L}[\tilde{\kappa}], \]
\[ \mathcal{L}\left[\nu^-(1, \varphi)\right] = \frac{1}{s^q} \mathcal{L}\left[D^q_\rho \nu^-(0, \varphi) + \nu^-(0, \varphi)\right], \]
\[ \mathcal{L}\left[\nu^-(2, \varphi)\right] = \frac{1}{s^q} \mathcal{L}\left[D^q_\rho \nu^-(1, \varphi) + \nu^-(1, \varphi)\right], \]
\[ \vdots \]
\[ \mathcal{L}\left[\nu^-(n, \varphi)\right] = \frac{1}{s^q} \mathcal{L}\left[D^q_\rho \nu^-(n-1, \varphi) + \nu^-(n-1, \varphi)\right], n \geq 0. \] (27)

Applying Laplace inverse transform, we have

\[ \nu^-(0, \varphi) = \tilde{\gamma}(\varphi) + \mathcal{L}^{-1}\left[\frac{1}{s^q} \mathcal{L}[\tilde{\kappa}]\right], \]
\[ \nu^-(1, \varphi) = \mathcal{L}^{-1}\left[\frac{1}{s^q} \mathcal{L}\left[D^q_\rho \nu^-(0, \varphi) + \nu^-(0, \varphi)\right]\right], \]
\[ \vdots \]
\[ \tilde{\nu}_{n+1}(\varphi) = \mathcal{L}^{-1}\left[\frac{1}{s^q} \mathcal{L}\left[D^q_\rho \nu^-(n, \varphi) + \nu^-(n, \varphi)\right]\right], n \geq 0. \] (28)

Therefore, the required series solution is achieved by

\[ \nu^-(p, \varphi) = \nu^-(0, \varphi) + \nu^-(1, \varphi) + \nu^-(2, \varphi) + \cdots. \] (29)

3.1. **Examples**

**Example 1.** Consider linear nonhomogeneous time-fractional Emden-Fowler heat equation

\[ D^q_\rho \nu^-(p, \varphi) = \frac{\partial^2 \nu^-(p, \varphi)}{\partial \rho^2} \]
\[ + \frac{2}{\rho} \frac{\partial \nu^-(p, \varphi)}{\partial \rho} - (5 + 4\rho^2) \nu^-(p, \varphi) - (6 - 5\rho^2 - 4\rho^4) ; 0 < \rho \leq 1, \] (30)

with the fuzzy initial condition
\[ v^-(\rho, 0) = \tilde{k}(\varsigma) \left( \phi^2 + e^{q^2} \right), \]  

(31) the above-mentioned procedures as described in (28), we obtain the following results.

where \[ \tilde{k}(\varsigma) = [k(\varsigma), \tilde{k}(\varsigma)] = [\varsigma - 1, 1 - \varsigma], 0 \leq \varsigma \leq 1. \] Applying

\[ \psi_0(\rho, \varphi) = k(\varsigma) \left( \phi^2 + e^{q^2} \right), \]

\[ \psi_1(\rho, \varphi) = k(\varsigma) \left( \frac{4e^{q^2}}{\varphi} - 3e^{q^2} - 8\rho^4 - 10\rho^2 \right) \frac{q^q}{\Gamma(q + 1)}, \]

\[ \psi_2(\rho, \varphi) = k(\varsigma) \left( 32e^{q^2} \varphi - \frac{8e^{q^2}}{\varphi} + 24e^{q^2} \rho^3 + 32e^{q^2} - 110\rho^2 + 80\rho^4 + 32\rho^6 - 20 \right) \frac{q^q}{\Gamma(q + 1)} - k(\varsigma) \left( 6 - 5\phi^2 - 4\phi^4 \right) \frac{q^q}{\Gamma(q + 1)} \cdots, \]

(32)

Similarly, more term may be constructed in this way. As a result of (29), the required series solution can be expressed as an infinite series.

\[ v^-(\rho, \varphi) = v_0^-(\rho, \varphi) + v_1^-(\rho, \varphi) + v_2^-(\rho, \varphi) + \cdots, \]

(33)

such that

\[ \psi(\rho, \varphi) = \psi_0(\rho, \varphi) + \psi_1(\rho, \varphi) + \psi_2(\rho, \varphi) + \cdots, \]

(34)

In general, we can say

\[ \psi(\rho, \varphi) = k(\varsigma) \left( \phi^2 + e^{q^2} \right) + k(\varsigma) \left( \frac{4e^{q^2}}{\varphi} - 3e^{q^2} - 8\rho^4 - 10\rho^2 \right) \frac{q^q}{\Gamma(q + 1)} + k(\varsigma) \left( 32e^{q^2} \varphi - \frac{8e^{q^2}}{\varphi} + 24e^{q^2} \rho^3 + 32e^{q^2} - 110\rho^2 + 80\rho^4 + 32\rho^6 - 20 \right) \frac{q^q}{\Gamma(q + 1)} - k(\varsigma) \left( 6 - 5\phi^2 - 4\phi^4 \right) \frac{q^q}{\Gamma(q + 1)} \cdots, \]

\[ \tilde{\psi}(\rho, \varphi) = \tilde{k}(\varsigma) \left( \phi^2 + e^{q^2} \right) + \tilde{k}(\varsigma) \left( \frac{4e^{q^2}}{\varphi} - 3e^{q^2} - 8\rho^4 - 10\rho^2 \right) \frac{q^q}{\Gamma(q + 1)} + \tilde{k}(\varsigma) \left( 32e^{q^2} \varphi - \frac{8e^{q^2}}{\varphi} + 24e^{q^2} \rho^3 + 32e^{q^2} - 110\rho^2 + 80\rho^4 + 32\rho^6 - 20 \right) \frac{q^q}{\Gamma(q + 1)} - \tilde{k}(\varsigma) \left( 6 - 5\phi^2 - 4\phi^4 \right) \frac{q^q}{\Gamma(q + 1)} \cdots. \]

(35)
The exact solution is

$$v'(p, \varphi) = \tilde{\kappa}(\varsigma) \left( p^2 + e^{\varphi^2} \right).$$  \hfill (36)

Example 2. Consider nonlinear nonhomogeneous time-fractional Emden-Fowler heat equation

$$D^\rho v'(p, \varphi) = \frac{\partial^2 v'(p, \varphi)}{\partial p^2} + 6 \frac{\partial v'(p, \varphi)}{\partial p} + (14p + \rho^4) \tilde{v} + 4p \tilde{v} \ln (\tilde{v}); 1 < \rho \leq 2,$$

(37)

with the fuzzy initial conditions

$$v'(p, 0) = \tilde{\kappa}(\varsigma), \quad v'(0, \varphi) = -\tilde{\kappa}(\varsigma) p^2,$$  \hfill (38)

where $\tilde{\kappa}(\varsigma) = [\kappa(\varsigma), \kappa(\varsigma)] = [\varsigma - 1, 1 - \varsigma], 0 \leq \varsigma \leq 1$. Applying the above-mentioned procedures as described in (28), we obtain the following results.

$$v_0(p, \varphi) = \kappa(\varsigma) \left( 1 - \varphi^2 p^2 \right),$$

$$v_1(p, \varphi) = \kappa(\varsigma) \left( -28p^2 \Gamma(q + 3) + p^4 \Gamma(q + 1) - p^2 \Gamma(q + 2) \right),$$

$$v_2(p, \varphi) = \kappa(\varsigma) \left( -28p^2 \Gamma(q + 3) + p^4 \Gamma(q + 1) - p^2 \Gamma(q + 2) \right) + \frac{p^4}{\gamma(2q + 1)} + \frac{p^{10}}{\gamma(2q + 2)} \right),$$

$$v_3(p, \varphi) = \kappa(\varsigma) \left( -392p^2 \Gamma(q + 3) + 36p^2 \Gamma(q + 2) - 66p^4 \frac{\varphi^2}{\Gamma(q + 2)} + 14p \Gamma(q + 2) \right),$$

$$v_4(p, \varphi) = \kappa(\varsigma) \left( -392p^2 \Gamma(q + 3) + 36p^2 \Gamma(q + 2) - 66p^4 \frac{\varphi^2}{\Gamma(q + 2)} + 14p \Gamma(q + 2) \right) + \frac{p^4}{\gamma(2q + 1)} + \frac{p^{10}}{\gamma(2q + 2)} \right),$$

$$v_5(p, \varphi) = \kappa(\varsigma) \left( -392p^2 \Gamma(q + 3) + 36p^2 \Gamma(q + 2) - 66p^4 \frac{\varphi^2}{\Gamma(q + 2)} + 14p \Gamma(q + 2) \right) + \frac{p^4}{\gamma(2q + 1)} + \frac{p^{10}}{\gamma(2q + 2)} \right),$$

$$v_6(p, \varphi) = \kappa(\varsigma) \left( -392p^2 \Gamma(q + 3) + 36p^2 \Gamma(q + 2) - 66p^4 \frac{\varphi^2}{\Gamma(q + 2)} + 14p \Gamma(q + 2) \right) + \frac{p^4}{\gamma(2q + 1)} + \frac{p^{10}}{\gamma(2q + 2)} \right).$$

Similarly, more term may be constructed in this way. As a result of (29), the required series solution can be expressed as an infinite series.

$$v'(p, \varphi) = v'_{0}(p, \varphi) + v'_{1}(p, \varphi) + v'_{2}(p, \varphi) + \cdots,$$  \hfill (40)

such that

$$v(p, \varphi) = v_{0}(p, \varphi) + v_{1}(p, \varphi) + v_{2}(p, \varphi) + \cdots,$$  \hfill (41)

In general, we can say

$$v(p, \varphi) = \kappa(\varsigma) \left( 1 - \varphi^2 p^2 \right) + \frac{p^4}{\gamma(2q + 1)} + \frac{p^{10}}{\gamma(2q + 2)} \right),$$

$$v'(p, \varphi) = \kappa(\varsigma) \left( 1 - \varphi^2 p^2 \right) + \frac{p^4}{\gamma(2q + 1)} + \frac{p^{10}}{\gamma(2q + 2)} \right),$$

The exact solution is

$$v'(p, \varphi) = \kappa(\varsigma) e^{-\varphi^2 p^2}. \hfill (43)$$

In Figures 1 and 2, we plot the fractional fuzzy findings for Examples 1 and 2, which correspond to distinct fractional-order amounts of uncertainty varrho. We have effectively established series type solutions to universal one-dimensional fuzzy fractional partial differential equations in the presence of an external source term. The found findings are complemented with a captivating image. In addition, graphs of the approximate solutions at various fractional orders have been supplied. We have provided estimates of illustrative solutions. As the fractional-order $\rho$ approaches its integer value, it is obvious from the charts that the curve will finish at classical order 1 as $h$ approaches its integer value. Consequently, we came to the conclusion that fractional calculus also reflects the global element of the dynamics of fuzzy concept concerns. In the future, we
will apply this technique to a wider variety of scenarios involving fuzzy dynamics.

4. Conclusion

This study was aimed at providing a semianalytical solution to the fuzzy fractional Emden-Fowler equations by incorporating the Caputo operator. Accordingly, fuzzy operators are more appropriate for describing the physical phenomenon in such a scenario. We investigated the Emden-Fowler equation using a fuzzy technique that took the uncertainty in the initial condition. The fuzzy fractional of the Emden-Fowler equations has been generalised in this research. We next employed a novel iterative transform method to generate the approximate solution.
parametric formulation of the proposed problem. We investigated different illustrations that bolstered the methodology’s intended application and developed a parametric solution for each scenario. Finally, it is not straightforward to find analytical solutions to a large number of various forms of fuzzy fractional partial differential equations.

Data Availability
The numerical data used to support the findings of this study are included within the article.

Conflicts of Interest
The authors declare that there are no conflicts of interest regarding the publication of this article.

References