Hindawi
Journal of Function Spaces
Volume 2022, Article ID 7922001, 13 pages
https://doi.org/10.1155/2022/7922001

Research Article

Study of Fuzzy Fractional Third-Order Dispersive KdV Equation in a Plasma under Atangana-Baleanu Derivative

Mounirah Areshi,[1] **S. A. El-Tantawy**,[2,3] **B. M. Alotaibi**,[4] **and Shamsullah Zaland**[5]

[1]*Department of Mathematics, Faculty of Science, University of Tabuk, Tabuk 71491, Saudi Arabia*
[2]*Department of Physics, Faculty of Science, Port Said University, Port Said 42521, Egypt*
[3]*Research Center for Physics (RCP), Department of Physics, Faculty of Science and Arts, Al-Mikhwah, Al-Baha University, Saudi Arabia*
[4]*Department of Physics, College of Science, Princess Nourah Bint Abdulrahman University, P.O. Box 84428, Riyadh 11671, Saudi Arabia*
[5]*Faculty of Mathematics, Kabul Polytechnic University, Afghanistan*

Correspondence should be addressed to Shamsullah Zaland; shamszaland@kpu.edu.af

Received 2 January 2022; Accepted 14 February 2022; Published 16 March 2022

Academic Editor: Ganesh Ghorai

Motivated by the wide-spread of both integer and fractional third-order dispersive Korteweg-de Vries (KdV) equations in explaining many nonlinear phenomena in a plasma and many other fluid models, thus, in this article, we constructed a system for calculating an analytical solution to a fractional fuzzy third-order dispersive KdV problems. We implemented the Shehu transformation and the iterative transformation technique under the Atangana-Baleanu fractional derivative. The achieved series result was contacted and determined the analytic value of the suggested models. For the confirmation of our system, three various problems have been represented, and the fuzzy type solution was determined. The fuzzy results of upper and lower section of all three problems are simulate applying two different fractional orders among zero and one. Because it globalises the dynamic properties of the specified equation, it delivers all forms of fuzzy solutions occurring at any fractional order among zero and one. The present results can help many researchers to explain the nonlinear phenomena that can create and propagate in several plasma models.

1. Introduction

In recent years, researchers and scientists have been particularly interested in fractional calculus (FC), which encompasses fractional-order integrals and derivatives. FC offers a wide range of applications, with accurate and precise solutions in various modern biological and physical phenomena. The integral differential operators in fractional differential calculus have a higher freedom of degree. The solution studies have given a keen attraction in this area, and papers, monographs, and book, among other things, have been published in current history, containing numerous inquiries into various aspects such as existing theories and analytic conclusions [1–7]. FC can be used to a variety of fields in both practical and pure mathematics. Physical models of real-world phenomenon usually contain considerable uncertainty due to a variety of variables. Fuzzy sets also actually be a good method for modelling the unpredictability brought up by imprecision and vagueness. We implement it here to areas where needs to be explained ambiguity, such as medical, ecologic, social, financial, and sciences [8–15]. In 1965, Zadeh proposed the set theory of fuzzy to investigate these issues [16]. The fuzzy set approach has been applied in other areas, i.e., topology, control system, fuzzy automata, fixed-point theories, etc. Chang and Zadeh expanded on the fuzzy set notion by introducing fuzzy control and mapping [17]. Several academics are generalized the concept of fuzzy mapping and control to develop basic fuzzy calculus [18–20]. In the last many centuries, fractional fuzzy integral and differential equations have attracted much attention from researchers in applied sciences. The fundamental concept of integral fuzzy equations was first described by Dubois and Prade [21]. However, in situations where information is ambiguous and inaccurate, the parameters are expressed by fuzzy numbers rather than crisp numbers. Fractional fuzzy integral and differential equations can be used to simulate these types of problems. The mathematical modelling of particular real-world models maintaining uncertainty in data has given rise to fuzzy PDEs. PDEs have important implementations in several areas of engineering, plasma physics, and many science branches, as we will observe in future. Because numerous schemes in mechanical and machines and aerospace technology areas of study are applicable to heat, heat transfer is an important area of aerospace and mechanical engineering investigation [22]. As a result, numerous scholars investigated the solution of fuzzy FDEs using these models [23]. The researchers [24] used an effective mathematical technique to analyze analytical solutions to nonlinear Lane-Emden models and an efficient numerical method for the fractional advection dispersion model and vibration equations that arise in porous media. Many fractional PDEs applying in hydro-magnetic waves in a cold plasma, ion-acoustic waves in plasma, and magneto acoustic wave numerical simulation and analytical approaches are used [25, 26]. Nonlinear effects occur in many applied science areas, such as plasma physics, mathematical biology, nonlinear optics, quantum mechanics, chemical kinetics, solid-state physics, and fluid dynamics [27–34]. These processes are modeled on nonlinear partial differential equations (PDEs) of a various higher order. PDEs are commonly used in the description of physical processes. The nonlinear nature of most of the essential physical systems is hidden. The exact result of such nonlinear phenomena may not be possible for some physical problems. For instance in a plasma physics, there are many nonintegrable PDEs that can not support exact analytic solutions such as the integer and fractional damped third-order KdV-type equations and the damped integer and fractional fifth-order KdV-type equations (the family of damped Kawahara equation) and many other equations related to plasma physics [35–40]. Moreover, in non-Maxwellian plasma models that have trapped particles follow nonisothermal or Schamel distribution in addition to the particle kinematic viscosity, in this case, the fluid equations of the plasma model can be reduced to a nonintegrable damped Schamel KdV-Burgers equation [41, 42]. On the other side, there are other types of nonintegrable PDEs that can be used for describing modulated envelope structures in a plasma like a damped integer and fractional cubic nonlinear Schrödinger-type equations (CNLSE). This family also does not have exact analytic solutions but can support some approximate analytical and numerical solutions. Anyway, most nonlinear phenomena that can exist in plasma physics, nonlinear optics, quantum physics, etc. can only be investigated using useful techniques to solve their evolution equations [43–49]. In 1895, Korteweg and de Vries proposed a KdV model to design Russells soliton phenomenon, such as small and huge water waves. Solitons are steady waves of solitary; this implies that these lone waves are particles. The mathematical model for exploring dispersive wave phenomena in several research areas is the KdV equations, such as quantum mechanics, fluid dynamics, optics, and plasma physics [44, 45]. Fifth-order KdV/Kawahara form equations utilized to analyze different nonlinear phenomena in particle physics and in plasma physics [38–40]. It plays a vital function in the distribution of waves [50]. In their analysis, the KdV form equation has dispersive terms of the third and fifth-order relevant to the magneto acoustic wave problem in near-critical angle propagation that appears to be cool collision-free plasma and dispersive terms [51]. Numerous number of researchers used this family of differential equations to model many nonlinear phenomena that arise and propagate in various plasma model. For example, El-Tantawy research group applied the Poincaré-Lighthill-Kuo (PLK) method for reducing the fluid equation of an ultracold neutral plasma (UNP) to two-coupled planar and nonplanar KdV equations for studying the face-to-face planar and nonplanar soliton collisions and corresponding phase shifts after collisions. Moreover, the head-on collisions between the two-counterpart KdV and modified KdV (mKdV) planar solitons were investigated in different plasma models [52–54]. In addition to, the planar and nonplanar Gardner equations were used for modeling the several types of acoustic waves (AWs) in different plasma models [55–58]. All the abovementioned equations succeeded in giving a good description to many nonlinear phenomena that arise and propagate within different plasma systems and many other branches of science. On the other hand, many researchers tended to model most of the mysterious phenomena in plasma physics by describing them using fractional differential equations such as fractional KdV-type equations and many related equations in higher order because they give a more accurate and comprehensive description better than the integer differential equations [59–61]. As a result, numerous scholars investigated the solution of many fuzzy FDEs using these models of plasma physics. The researchers used an effective mathematical technique to analyze analytical solutions to nonlinear Lane-Emden models and an efficient numerical method for the fractional advection dispersion model and vibration equations that arise in porous media. Many fractional PDEs applying in hydro-magnetic waves in a cold plasma, ion-acoustic waves in plasma, and magneto acoustic wave numerical simulation and analytical approaches are used. Which as innovation is concerned, we suggest an initial value solitary wave solutions for the KdV equation under the fuzzy Caputo fractional operator. Because of the fuzzy number, we use an unidentified quantity's fuzziness and initial condition to estimate its result in fuzzy type with two section. Many researchers have been working on integer and fractional order diffusion equations, as well as the fuzzy heat equation. For the study of the given equation with various external source components, we examine both fuzziness and fractional order [62–66].

2. Basic Concept of Fuzzy and Fractional Calculus

This section demonstrates various key components related to fractional calculus and fuzzy sets, as well as some major research on the Shehu transformation.

Definition 1. We assume that $\varphi : \mathscr{R} \mapsto [0,1]$ is a fuzzy number; then, it is identified to be fuzzy set if it maintains the successive hypotheses [67–70]:

(1) φ is regular (in several $\eta_0 \in \mathscr{R}$; $\varphi(\wp_0) = 1$)

(2) φ is semicontinuous upper

(3) $\varphi(\wp_1\omega + (1-\omega)\wp_2) \geq (\varphi(\wp_1) \wedge \varphi(\wp_2)) \forall \omega \in [0,1], \wp_1, \wp_2 \in \mathscr{R}$, i.e., φ is convex

(4) $cl\{\wp \in \mathscr{R}, \varphi(\wp) > 0\}$ is compact

Definition 2. Assume that a set of fuzzy φ is ℓ-level set discussed [67–70].

$$[\varphi]^{\ell} = \{\Phi \in \mathscr{R} : \varphi(\Phi) \geq 1\}, \tag{1}$$

where $\ell \in [0,1]$ and $\Phi \in \mathscr{R}$.

Definition 3. The parameter version a set of fuzzy is designated by $[\underline{\varphi}\%(\ell), \bar{\varphi}(\ell)]$ so that $\ell \in [0,1]$ gratify the subsequent presumption [67–70]:

(1) $\varphi(\ell)$ is left continuous, nondecreasing, left continuous at 0, and over bounded $(0,1]$

(2) $\varphi(\ell)$ is right continuous, nonincreasing, right continuous at 0, and over bounded $(0,1]$

(3) $\underline{\varphi}(\ell) \leq \bar{\varphi}(\ell)$

Definition 4. For $\ell \in [0,1]$ and scalar of Y, assume they are two fuzzy sets [67–70] $\widetilde{v_1} = (\underline{v}_1, \overline{\mu_1})$, $\widetilde{v_2} = (\underline{v_2}, \overline{\mu_2})$; then, the subtraction, multiplication, and addition, respectively, are stated as follows:

(1) $\widetilde{v_1} \ominus \widetilde{v_2} = (\overline{v_1}(\%\ell) - \underline{v_2}(\ell), \overline{v_1}(\%\ell) - \overline{v_2}(\ell))$

(2) $\widetilde{v_1} \oplus \widetilde{v_2} = (\underline{v_1}(\%\ell) + \underline{v_2}(\ell), \overline{v_1}(\ell) + \%\overline{v_2}(\ell))$

(3) $Y \odot \widetilde{v_1} = \{(Y\underline{\%v_1}, Y\overline{v_1})Y \geq 0, (\%Y\overline{v_1}, Y\underline{v_1})Y < 0$

Definition 5. Assume a mapping of fuzzy $\Theta : \tilde{E} \times \tilde{E} \mapsto \mathscr{R}$ have two fuzzy sets [67–70] $\widetilde{v_1} = (\%\underline{v_1}, \overline{v_1})$, $\widetilde{v_2} = (\%\underline{v_2}, \overline{v_2})$; then, Θ-distance among $\widetilde{v_1}$ and $\widetilde{v_2}$ is define as follows:

$$\Theta(\widetilde{v_1}, \widetilde{v_2}) = \sup_{\ell \in [0,1]} \left[\max\left\{\left|\underline{v_1}(\ell) - \underline{v_2}(\ell)\right|, \left|\overline{v_1}(\ell) - \overline{v_2}(\ell)\right|\right\}\right]. \tag{2}$$

Definition 6. Suppose that $\mathbf{E} : (b_1, b_2) \mapsto \tilde{E}$ and $\%\eta_0 \in (b_1, b_2)$.. Then, $\mathbf{E}$ is defined as a heavily generalized differentiable variable at η_0 if $\mathbf{E}\%'(\eta_0) \in \tilde{E}$ exists such that [67–70]:

(i) $\mathbf{E}'(\eta_0) = \lim_{\hbar \mapsto 0}(\mathbf{E}(\eta_0 + \hbar) \ominus^{g\mathscr{H}} \mathbf{E}\%(\eta_0)/\hbar) = \lim_{\hbar \mapsto 0}(\mathbf{E}\%(\eta_0) \ominus^{g\mathscr{H}} \mathbf{E}(\eta_0 - \hbar\%)/\hbar)$

(ii) $\mathbf{E}'(\eta_0) = \lim_{\hbar \mapsto 0}(\mathbf{E}(\eta_0) \ominus^{g\mathscr{H}} \mathbf{E}\%(\eta_0 + \hbar)/-\hbar) = \lim_{\hbar \mapsto 0}(\mathbf{E}\%(\eta_0 - \hbar) \ominus^{g\mathscr{H}} \mathbf{E}\%(\eta_0)/-\hbar)$

Definition 7. Suppose a mapping of fuzzy $\Omega : \mathscr{R} \mapsto \tilde{E}$, if for any $\epsilon > 0 \exists \delta > 0$ and fixed values of $v_0 \in [a_1, a_2]$, we have [67–70]:

$$\Theta(\mathbf{E}(v), \mathbf{E}(v_0)) < \epsilon \,; \text{whenever } |v - v_0| < \delta, \tag{3}$$

then $\mathbf{E}$ is known to be continuous.

Theorem 8. *Suppose that fuzzy value term $\mathbf{E} : \mathscr{R} \mapsto \tilde{E}$ such that $\mathbf{E}(\eta_0;\ell) = [\underline{\mathbf{E}}(\eta_0;\ell), \bar{\mathbf{E}}(\eta_0;\ell)]$ and $\ell \in [0,1]$. Then [67–70]:*

(1) $(\eta_0;\ell)$ and $\mathbf{E}(\eta_0;\ell)$ are differentiable, if $\mathbf{E}$ is a (21)-differentiable, and

$$\left[\mathbf{E}'(\eta_0)\right]^{\ell} = \left[\underline{\mathbf{E}}'(\eta_0;\ell), \bar{\mathbf{E}}'(\eta_0;\ell)\right]. \tag{4}$$

(2) $\underline{\mathbf{E}}(\eta_0;\ell)$ and $\bar{\mathbf{E}}(\eta_0;\ell)$ are differentiable, if $\mathbf{E}$ is a (23)-differentiable, and

$$\left[\mathbf{E}'(\eta_0)\right]^{\ell} = \left[\bar{\mathbf{E}}'(\eta_0;\ell), \underline{\mathbf{E}}'(\eta_0;\ell)\right]. \tag{5}$$

Definition 9. Suppose that a mapping of fuzzy $\Phi^{(\ell)}_{g\mathscr{H}} = \Phi^{(\ell)} \in \mathbb{C}^F[0,s] \cap \mathbb{L}^F[0,s]$. Then, the fuzzy $g\mathscr{H}$-fractional differentiability of Caputo fuzzy number mappings Φ is expressed as [67–70]

$$\begin{aligned}(g\mathscr{H}\mathscr{D}^{v}\Phi)(\mathfrak{J}) &= \mathscr{I}_{a_1}^{\ell-v} \odot \left(\Phi^{(\ell)}\right)(\eta) \\ &= \frac{1}{\Gamma(\ell-v)} \odot \int_{a_1}^{\mathfrak{J}} (\mathfrak{J}_1 - \wp)^{\ell-v-1} \odot \Phi^{(\ell)}(\wp)d\wp, \quad v \in (\ell-1,\ell], \ell \in \mathbb{N}, \mathfrak{J} > a_1.\end{aligned} \tag{6}$$

Consequently, the parameter varieties of $\Phi = [\underline{\Phi}_{\ell}(\mathfrak{J}), \bar{\Phi}_{\ell}(\mathfrak{J}\%)]$, $\ell \in [0,1]$ and $\mathfrak{J}_{10} \in (0,s)$, and CFD in the sense of fuzzy are defined as

$$\left[\mathscr{D}^{v}_{(i)-g\mathscr{H}}\Phi(\mathfrak{J}_{10})\right]_{\ell} = \left[\mathscr{D}^{v}_{(i)-g\mathscr{H}}\underline{\Phi}(\mathfrak{J}_{10}), \mathscr{D}^{\theta}_{(i)-g\mathscr{H}}\bar{\Phi}(\mathfrak{J}_{10})\right], \ell \in [0,1]. \tag{7}$$

where $\ell = [\ell]$:

$$\left[\mathscr{D}^{\nu}_{(i)-g\mathscr{H}}\underline{\Phi}(\mathfrak{I}_{10})\right] = \frac{1}{\Gamma(\ell-\nu)}\left[\int_0^{\mathfrak{I}}(\mathfrak{I}-\mathbf{x})^{\ell-\nu-1}\frac{d^{\ell}}{d\mathbf{x}^{\ell}}\underline{\Phi}_{(i)-g\mathscr{H}}(\mathbf{x})d\mathbf{x}\right]_{\mathfrak{I}=\mathfrak{I}_{10}}.$$

$$\left[\mathscr{D}^{\theta}_{(i)-g\mathscr{H}}\bar{\Phi}(\mathfrak{I}_{10})\right] = \frac{1}{\Gamma(\ell-\nu)}\left[\int_0^{\mathfrak{I}}(\mathfrak{I}-\mathbf{x})^{\ell-\nu-1}\frac{d^{\ell}}{d\mathbf{x}^{\ell}}\bar{\Phi}_{(i)-g\mathscr{H}}(\mathbf{x})d\mathbf{x}\right]_{\mathfrak{I}=\mathfrak{I}_{10}}. \tag{8}$$

Definition 10. Suppose that a mapping of fuzzy $\tilde{\Phi}(\mathfrak{I}) \in \widetilde{\mathbb{H}}^1(0, T)$ and $\nu \in [0, 1]$, then the fuzzy $g\mathscr{H}$-fractional differentiability Atangana-Baleanu of fuzzy number mappings is defined as

$$(g\mathscr{H}\mathscr{D}^{\nu}\Phi)(\mathfrak{I}) = \frac{\mathbb{B}(\nu)}{1-\nu}\odot\left[\int_0^{t_1}\underline{\Phi}'(\mathbf{x})\odot E_{\nu}\left[\frac{-\nu(\mathfrak{I}-\mathbf{x})^{\nu}}{1-\nu}\right]d\mathbf{x}\right]. \tag{9}$$

As a result, the parametric approach is used of $\Phi = [\underline{\%\Phi}_{\ell}(\mathfrak{I}), \bar{\Phi}_{\ell}(\%\mathfrak{I})]$, $\ell \in [0, 1]$, and $\mathfrak{I}_0 \in (0, s)$ is defined as

$$\left[{}^{ABC}\mathscr{D}^{\nu}_{(i)-g\mathscr{H}}\tilde{\Phi}(\mathfrak{I}_0;\ell)\right] = \left[{}^{ABC}\mathscr{D}^{\nu}_{(i)-g\mathscr{H}}\underline{\Phi}(\mathfrak{I}_0;\ell), {}^{ABC}\mathscr{D}^{\theta}_{(i)-g\mathscr{H}}\Phi(\mathfrak{I}_0;\ell)\right], \ell \in [0, 1]. \tag{10}$$

where

$${}^{ABC}\mathscr{D}^{\theta}_{(i)-g\mathscr{H}}\underline{\Phi}(\mathfrak{I}_0;\ell) = \frac{\mathbb{B}(\nu)}{1-\nu}\left[\int_0^{t_1}\underline{\Phi}'_{(i)-g\mathscr{H}}(\mathbf{x})E_{\theta}\left[\frac{-\nu(\mathfrak{I}-\mathbf{x})^{\theta}}{1-\nu}\right]d\mathbf{x}\right]_{t_1=\mathfrak{I}_0},$$

$${}^{ABC}\mathscr{D}^{\theta}_{(i)-g\mathscr{H}}\bar{\Phi}(\mathfrak{I}_0;\ell) = \frac{\mathbb{B}(\nu)}{1-\nu}\left[\int_0^{\mathfrak{I}}\bar{\Phi}'_{(i)-g\mathscr{H}}(\mathbf{x})E_{\theta}\left[\frac{-\nu(\mathfrak{I}-\mathbf{x})^{\theta}}{1-\nu}\right]d\mathbf{x}\right]_{\mathfrak{I}=\mathfrak{I}_0}, \tag{11}$$

where $\mathbb{B}(\nu)$ represents a normalised function with the value of 1 when ν is supposed to be zero and one.

Definition 11. Consider a continuous real-valued mapping Ψ and there is an improper fuzzy Riemann-integrable mapping $\exp(-\omega/\sigma)\odot\tilde{\Phi}(\mathfrak{I})$ on $[0, +\infty)$. Then, the integral $\int_0^{+\infty}\exp(-\omega/\sigma)\odot\tilde{\Phi}(\mathfrak{I})d\mathfrak{I}$ is identified to be the fuzzy Shehu transform, and it is stated over the set of mappings [67–70]:

$$\mathscr{S} = \left\{\tilde{\Phi}(\mathbf{g}) : \exists\mathscr{A}, p_1, p_2 > 0, \left|\tilde{\Phi}(\mathfrak{I})\right| < \mathscr{A}\exp\left(\frac{|\mathfrak{I}|}{\zeta_j}\right), \text{if } \mathfrak{I} \in (-1)^J \times [0,+\infty)\right\}, \tag{12}$$

as

$$\mathscr{S}\left[\tilde{\Phi}(\mathfrak{I})\right] = \mathscr{S}(\omega, \sigma) = \int_0^{+\infty}\exp\left(\frac{-\omega}{\sigma}\mathfrak{I}\right)\odot\tilde{\Phi}(\mathfrak{I})d\mathfrak{I}, \omega, \sigma > 0. \tag{13}$$

Remark 12. In (34), $\tilde{\Phi}$ completes the hypothesis of the reducing diameter $\underline{\Phi}$ and tiameter $\bar{\Phi}$ of a fuzzy mapping Φ. If $\sigma = 1$, then fuzzy the Shehu transformation is red * Laplace transformation [67–70].

$$\int_0^{+\infty}\exp\left(\frac{-\omega}{\sigma}\mathfrak{I}\right)\odot\tilde{\Phi}(\mathfrak{I})d\mathfrak{I} = \left(\int_0^{+\infty}\exp\left(\frac{-\omega}{\sigma}\mathfrak{I}\right)\underline{\Phi}(\mathfrak{I};\ell)d\mathfrak{I}, \int_0^{+\infty}\exp\left(\frac{-\omega}{\sigma}\mathfrak{I}\right)\bar{\Phi}(\mathfrak{I};\ell)d\mathfrak{I}\right). \tag{14}$$

Moreover, consider Shehu transformation of classical form [67–70], we get:

$$\mathscr{S}[\underline{\Phi}(\mathfrak{I};\ell)] = \int_0^{+\infty}\exp\left(\frac{-\omega}{\sigma}\mathfrak{I}\right)\underline{\Phi}(\mathfrak{I};\ell)d\mathfrak{I}, \tag{15}$$

$$\mathscr{S}\left[\bar{\Phi}(\mathfrak{I};\ell)\right] = \int_0^{+\infty}\exp\left(\frac{-\omega}{\sigma}\mathfrak{I}\right)\bar{\Phi}(\mathfrak{I};\ell)d\mathfrak{I}. \tag{16}$$

The aforementioned expressions can then be expressed as

$$\mathscr{S}\left[\tilde{\Phi}(\mathfrak{I})\right] = \left(\mathbb{S}[\underline{\Phi}(\mathfrak{I};\ell)], \mathbb{S}\left[\bar{\Phi}(\mathfrak{I};\ell)\right]\right) = \left(\underline{\mathscr{S}}(\omega,\sigma), \bar{\mathscr{S}}(\omega,\sigma)\right). \tag{17}$$

Definition 13. Assume that it is an integral fuzzy mapping value ${}^{c}_{g\mathscr{H}}\mathscr{D}^{\nu}_{\mathfrak{I}}\tilde{\Phi}(\mathfrak{I})$, and $\Phi(\mathfrak{I})$ is the primitive of ${}^{c}_{g\mathscr{H}}\mathscr{D}^{\nu}_{\mathfrak{I}}\tilde{\Phi}(\mathfrak{I})$ on $[0, +\infty)$; then, the Caputo fractional order ν is shown as [67–70]

$$\begin{aligned}
\mathscr{S}\left[{}^{c}_{g\mathscr{H}}\mathscr{D}^{\nu}_{\mathfrak{I}}\tilde{\Phi}(\mathfrak{I})\right] &= \left(\frac{\omega}{\sigma}\right)^{\nu}\odot\mathscr{S}\left[\tilde{\Phi}(\mathfrak{I})\right]\ominus\sum_{\kappa=0}^{\ell-1}\left(\frac{\omega}{\sigma}\right)^{\nu-\kappa-1}\odot\tilde{\Phi}^{(\kappa)}(0), \nu \in (\ell-1, \ell], \\
\left(\frac{\omega}{\sigma}\right)^{\nu}\odot\mathscr{S}\left[\tilde{\Phi}(\mathfrak{I})\right]\ominus\sum_{\kappa=0}^{\ell-1}\left(\frac{\omega}{\sigma}\right)^{\nu-\kappa-1}\odot\tilde{f}^{(\kappa)}(0) &= \left(\left(\frac{\omega}{\sigma}\right)^{\nu}\mathscr{S}[\underline{\Phi}(\mathfrak{I};\ell)] - \sum_{\kappa=0}^{\ell-1}\left(\frac{\omega}{\sigma}\right)^{\nu-\kappa-1}\odot\underline{\Phi}^{(\kappa)}(0;\ell),\right. \\
&\quad \left.\times\left(\frac{\omega}{\sigma}\right)^{\nu}\mathscr{S}\left[\bar{\Phi}(\mathfrak{I};\ell)\right] - \sum_{\kappa=0}^{\ell-1}\left(\frac{\omega}{\sigma}\right)^{\nu-\kappa-1}\bar{\Phi}^{(\kappa)}(0;\ell)\right).
\end{aligned} \tag{18}$$

Bokhari et al. described the fractional derivative ABC operator in the sense Shehu transform. Moreover, we apply the concept of fuzzy fractional derivative of ABC in a Shehu fuzzy transformation sense as follows.

Definition 14. Consider $\Phi \in \mathbb{C}^F[0, s] \cap \mathbb{L}^F[0, s]$ such that $\tilde{\Phi}(\mathfrak{I}) = [\underline{\Phi}(\mathfrak{I}, \ell), \bar{\Phi}(\mathfrak{I}, \ell)]$, $\ell \in [0, 1]$; then, the fuzzy of Shehu transformation of ABC of order $\nu \in [0, 1]$ is describe given as:

$$\mathscr{S}\left[g\mathscr{H}\mathscr{D}^{\nu}_{\mathfrak{I}}\tilde{\Phi}(\mathfrak{I})\right] = \frac{\mathbb{B}(\nu)}{1-\nu+\nu(\sigma/\omega)^{\nu}}\odot\left(\tilde{\mathbf{V}}(\sigma,\omega)\ominus\frac{\sigma}{\omega}\tilde{\Phi}(0)\right). \tag{19}$$

Moreover, applying the fact of Salahshour et al. [67], we get

$$\frac{\mathbb{B}(\nu)}{1-\nu+\nu(\sigma/\tilde{\omega})^{\nu}} \odot \left(\tilde{\mathbf{V}}(\sigma,\omega) \ominus \frac{\omega}{\sigma}\tilde{\Phi}(0)\right) = \left(\frac{\mathbb{B}(\nu)}{1-\nu+\nu(\sigma/\tilde{\omega})^{\nu}}\left(\underline{\mathbf{V}}(\sigma,\omega;\ell) - \frac{\sigma}{\omega}\Phi(0;\ell)\right), \frac{\mathbb{B}(\nu)}{1-\nu+\nu(\sigma/\zeta)^{\theta}}\left(\bar{\mathbf{V}}(\sigma,\omega;\ell) - \frac{\sigma}{\tilde{\omega}}\bar{\Phi}(0;\ell)\right)\right). \quad (20)$$

3. Main Result

Consider the general fuzzy fractional partial differential equation

$$\mathcal{S}\left[{}^{ABC}D_{\mathfrak{I}}^{\wp}\tilde{\Phi}(\zeta,\xi,\mathfrak{I})\right] = \mathcal{S}\left[D_{\zeta}^{2}\tilde{\Phi}(\zeta,\xi,\mathfrak{I}) + D_{\xi}^{2}\tilde{\Phi}(\zeta,\xi,\mathfrak{I}) + \tilde{k}(r)\mathcal{F}(\zeta,\xi,\mathfrak{I})\right], \quad (21)$$

where $\wp\in(0,1]$; therefore, the Shehu transform of (21) is

$$\frac{\mathbb{B}(\wp)}{1-\wp+\wp(\sigma/\omega)^{\wp}}\mathcal{S}\left[\tilde{\Phi}(\zeta,\xi,\mathfrak{I})\right] - \frac{\mathbb{B}(\wp)}{1-\wp+\wp(\sigma/\omega)^{\wp}}\left(\frac{\nu}{\omega}\right)\tilde{\Phi}(\zeta,\xi,\mathfrak{I}) = \mathcal{S}\left[D_{\zeta}^{2}\tilde{\Phi}(\zeta,\xi,\mathfrak{I}) + D_{\xi}^{2}\tilde{\Phi}(\zeta,\xi,\mathfrak{I}) + \tilde{k}(r)\mathcal{F}(\zeta,\xi,\mathfrak{I})\right], \quad (22)$$

applying the initial condition, we achieved as

$$\mathcal{S}\left[\tilde{\Phi}(\zeta,\xi,\mathfrak{I})\right] = \frac{g(\zeta,\xi)}{\omega} + \frac{1-\wp+\wp(\sigma/\omega)^{\wp}}{\mathbb{B}(\wp)}\mathcal{S} \cdot \left[D_{\zeta}^{2}\tilde{\Phi}(\zeta,\xi,\mathfrak{I}) + D_{\xi}^{2}\tilde{\Phi}(\zeta,\xi,\mathfrak{I}) + \tilde{k}(r)\mathcal{F}(\zeta,\xi,\mathfrak{I})\right]. \quad (23)$$

Decompose the result as $\tilde{\Phi}(\zeta,\xi,\mathfrak{I}) = \sum_{n=0}^{\infty}\tilde{\Phi}_n(\zeta,\xi,\mathfrak{I})$; then, (23) applies

$$\mathcal{S}\sum_{n=0}^{\infty}\tilde{\Phi}_n(\zeta,\xi,\mathfrak{I}) = \frac{g(\zeta,\xi)}{\omega} + \frac{1-\wp+\wp(\sigma/\omega)^{\wp}}{\mathbb{B}(\wp)}\mathcal{S} \cdot \left[D_{\zeta}^{2}\sum_{n=0}^{\infty}\tilde{\Phi}_n(\zeta,\xi,\mathfrak{I}) + D_{\xi}^{2}\sum_{n=0}^{\infty}\tilde{\Phi}_n(\zeta,\xi,\mathfrak{I}) + \tilde{k}(r)\mathcal{F}(\zeta,\xi,\mathfrak{I})\right]. \quad (24)$$

Parts of the result can be taken as a comparison

$$\begin{aligned}
\mathcal{S}\left[\tilde{\Phi}_0(\zeta,\xi,\mathfrak{I})\right] &= \frac{g(\zeta,\xi)}{\omega} + \frac{1-\wp+\wp(\sigma/\omega)^{\wp}}{\mathbb{B}(\wp)}\mathcal{S}\left[\tilde{k}(r)\mathcal{F}(\zeta,\xi,\mathfrak{I})\right],\\
\mathcal{S}\left[\tilde{\Phi}_1(\zeta,\xi,\mathfrak{I})\right] &= \frac{1-\wp+\wp(\sigma/\omega)^{\wp}}{\mathbb{B}(\wp)}\mathcal{S}\left[D_{\zeta}^{2}\tilde{\Phi}_0(\zeta,\xi,\mathfrak{I}) + D_{\xi}^{2}\tilde{\Phi}_0(\zeta,\xi,\mathfrak{I})\right],\\
\mathcal{S}\left[\tilde{\Phi}_2(\zeta,\xi,\mathfrak{I})\right] &= \frac{1-\wp+\wp(\sigma/\omega)^{\wp}}{\mathbb{B}(\wp)}\mathcal{S}\left[D_{\zeta}^{2}\tilde{\Phi}_1(\zeta,\xi,\mathfrak{I}) + D_{\xi}^{2}\tilde{\Phi}_1(\zeta,\xi,\mathfrak{I})\right],\\
&\vdots\\
\mathcal{S}\left[\tilde{\Phi}_{n+1}(\zeta,\xi,\mathfrak{I})\right] &= \frac{1-\wp+\wp(\sigma/\omega)^{\wp}}{\mathbb{B}(\wp)}\mathcal{S}\left[D_{\zeta}^{2}\tilde{\Phi}_n(\zeta,\xi,\mathfrak{I}) + D_{\xi}^{2}\tilde{\Phi}_n(\zeta,\xi,\mathfrak{I})\right].
\end{aligned} \quad (25)$$

Taking the inverse Shehu transform, we obtain

$$\begin{aligned}
\underline{\Phi}_0(\zeta,\xi,\mathfrak{I}) &= g(\zeta,\xi) + \mathcal{S}^{-1}\left[\frac{1-\wp+\wp(\sigma/\omega)^{\wp}}{\mathbb{B}(\wp)}\mathcal{S}[\underline{k}(r)\mathcal{F}(\zeta,\xi,\mathfrak{I})]\right],\\
\bar{\Phi}_0(\zeta,\xi,\mathfrak{I}) &= g(\zeta,\xi) + \mathcal{S}^{-1}\left[\frac{1-\wp+\wp(\sigma/\omega)^{\wp}}{\mathbb{B}(\wp)}\mathcal{S}[\bar{k}(r)\mathcal{F}(\zeta,\xi,\mathfrak{I})]\right],\\
\underline{\Phi}_1(\zeta,\xi,\mathfrak{I}) &= \mathcal{S}^{-1}\left[\frac{1-\wp+\wp(\sigma/\omega)^{\wp}}{\mathbb{B}(\wp)}\mathcal{S}\left[D_{\zeta}^{2}\underline{\Phi}_0(\zeta,\xi,\mathfrak{I}) + D_{\xi}^{2}\underline{\Phi}_0(\zeta,\xi,\mathfrak{I})\right]\right],\\
\bar{\Phi}_1(\zeta,\xi,\mathfrak{I}) &= \mathcal{S}^{-1}\left[\frac{1-\wp+\wp(\sigma/\omega)^{\wp}}{\mathbb{B}(\wp)}\mathcal{S}\left[D_{\zeta}^{2}\bar{\Phi}_0(\zeta,\xi,\mathfrak{I}) + D_{\xi}^{2}\bar{\Phi}_0(\zeta,\xi,\mathfrak{I})\right]\right],\\
\underline{\Phi}_2(\zeta,\xi,\mathfrak{I}) &= \mathcal{S}^{-1}\left[\frac{1-\wp+\wp(\sigma/\omega)^{\wp}}{\mathbb{B}(\wp)}\mathcal{S}\left[D_{\zeta}^{2}\underline{\Phi}_1(\zeta,\xi,\mathfrak{I}) + D_{\xi}^{2}\underline{\Phi}_1(\zeta,\xi,\mathfrak{I})\right]\right],\\
\bar{\Phi}_2(\zeta,\xi,\mathfrak{I}) &= \mathcal{S}^{-1}\left[\frac{1-\wp+\wp(\sigma/\omega)^{\wp}}{\mathbb{B}(\wp)}\mathcal{S}\left[D_{\zeta}^{2}\bar{\Phi}_1(\zeta,\xi,\mathfrak{I}) + D_{\xi}^{2}\bar{\Phi}_1(\zeta,\xi,\mathfrak{I})\right]\right],\\
&\vdots\\
\underline{\Phi}_{n+1}(\zeta,\xi,\mathfrak{I}) &= \mathcal{S}^{-1}\left[\frac{1-\wp+\wp(\sigma/\omega)^{\wp}}{\mathbb{B}(\wp)}\mathcal{S}\left[D_{\zeta}^{2}\underline{\Phi}_n(\zeta,\xi,\mathfrak{I}) + D_{\xi}^{2}\underline{\Phi}_n(\zeta,\xi,\mathfrak{I})\right]\right],\\
\bar{\Phi}_{n+1}(\zeta,\xi,\mathfrak{I}) &= \mathcal{S}^{-1}\left[\frac{1-\wp+\wp(\sigma/\omega)^{\wp}}{\mathbb{B}(\wp)}\mathcal{S}\left[D_{\zeta}^{2}\bar{\Phi}_n(\zeta,\xi,\mathfrak{I}) + D_{\xi}^{2}\bar{\Phi}_n(\zeta,\xi,\mathfrak{I})\right]\right].
\end{aligned} \quad (26)$$

Thus, the solution becomes

$$\underline{\Phi}(\zeta,\xi,\mathfrak{I}) = \underline{\Phi}_0(\zeta,\xi,\mathfrak{I}) + \underline{\Phi}_1(\zeta,\xi,\mathfrak{I}) + \underline{\Phi}_2(\zeta,\xi,\mathfrak{I}) + \cdots, \quad (27)$$

$$\bar{\Phi}(\zeta, \xi, \mathfrak{T}) = \bar{\Phi}_0(\zeta, \xi, \mathfrak{T}) + \bar{\Phi}_1(\zeta, \xi, \mathfrak{T}) + \bar{\Phi}_2(\zeta, \xi, \mathfrak{T}) + \cdots. \quad (28)$$

Equation (27) is the series type solution.

4. Numerical Problems

Problem 15. Consider the fractional fuzzy KdV equation is given as

$$^{\mathrm{ABC}}D^{\wp}_{\mathfrak{T}}\tilde{\Phi}(\zeta, \mathfrak{T}) + 2\frac{\partial\tilde{\Phi}(\zeta, \mathfrak{T})}{\partial\zeta} + \frac{\partial^3\tilde{\Phi}(\zeta, \tau)}{\partial\zeta^3} = 0, \quad 0 < \wp \le 1, \quad (29)$$

with the initial condition

$$\tilde{\Phi}(\zeta, 0) = \tilde{k}\sin\zeta. \quad (30)$$

Applying the proposed method of Equation (29), we get

$$\begin{aligned}
\underline{\Phi}_0(\zeta, \mathfrak{T}) &= \underline{k}(r)\sin\zeta,\\
\bar{\Phi}_0(\zeta, \mathfrak{T}) &= \bar{k}(r)\sin\zeta,\\
\underline{\Phi}_1(\zeta, \mathfrak{T}) &= -\underline{k}(r)\cos\zeta\frac{1}{\mathbb{B}(\wp)}\left\{\frac{\wp\mathfrak{T}^{\wp}}{\Gamma(\wp+1)} + (1-\wp)\right\},\\
\bar{\Phi}_1(\zeta, \mathfrak{T}) &= -\bar{k}(r)\cos\zeta\frac{1}{\mathbb{B}(\wp)}\left\{\frac{\wp\mathfrak{T}^{\wp}}{\Gamma(\wp+1)} + (1-\wp)\right\},\\
\underline{\Phi}_2(\zeta, \mathfrak{T}) &= -\underline{k}(r)\sin\zeta\frac{1}{\mathbb{B}^2(\wp)}\\
&\quad\cdot\left\{\frac{\wp^2\mathfrak{T}^{2\wp}}{\Gamma(2\wp+1)} + 2\wp(1-\wp)\frac{\mathfrak{T}^{\wp}}{\Gamma(\wp+1)} + (1-\wp)^2\right\},\\
\bar{\Phi}_2(\zeta, \mathfrak{T}) &= -\bar{k}(r)\sin\zeta\frac{1}{\mathbb{B}^2(\wp)}\\
&\quad\cdot\left\{\frac{\wp^2\mathfrak{T}^{2\wp}}{\Gamma(2\wp+1)} + 2\wp(1-\wp)\frac{\mathfrak{T}^{\wp}}{\Gamma(\wp+1)} + (1-\wp)^2\right\},\\
\underline{\Phi}_3(\zeta, \mathfrak{T}) &= \underline{k}(r)\cos\zeta\frac{1}{\mathbb{B}^3(\wp)}\\
&\quad\cdot\left\{\frac{\wp^3\mathfrak{T}^{3\theta}}{\Gamma(3\wp+1)} + 3\wp^2(1-\wp)\frac{\mathfrak{T}^{2\wp}}{\Gamma(2\wp+1)} + 3\wp(1-\wp)^2\frac{\mathfrak{T}^{\theta}}{\Gamma(\wp+1)}\right\},\\
\bar{\Phi}_3(\zeta, \mathfrak{T}) &= \bar{k}(r)\cos\zeta\frac{1}{\mathbb{B}^3(\wp)}\\
&\quad\cdot\left\{\frac{\wp^3\mathfrak{T}^{3\theta}}{\Gamma(3\wp+1)} + 3\wp^2(1-\wp)\frac{\mathfrak{T}^{2\wp}}{\Gamma(2\wp+1)} + 3\wp(1-\wp)^2\frac{\mathfrak{T}^{\theta}}{\Gamma(\wp+1)}\right\}.
\end{aligned} \quad (31)$$

Using Equation (27) to achieve the series form solution, we get

$$\tilde{\Phi}(\zeta, \mathfrak{T}) = \tilde{\Phi}_0(\zeta, \mathfrak{T}) + \tilde{\Phi}_1(\zeta, \mathfrak{T}) + \tilde{\Phi}_2(\zeta, \mathfrak{T}) + \tilde{\Phi}_3(\zeta, \mathfrak{T}) + \tilde{\Phi}_4(\zeta, \mathfrak{T}) + \cdots. \quad (32)$$

In lower and upper portion types can be written as

$$\begin{aligned}
\underline{\Phi}(\zeta, \mathfrak{T}) &= \underline{\Phi}_0(\zeta, \mathfrak{T}) + \underline{\Phi}_1(\zeta, \mathfrak{T}) + \underline{\Phi}_2(\zeta, \mathfrak{T})\\
&\quad + \underline{\Phi}_3(\zeta, \mathfrak{T}) + \underline{\Phi}_4(\zeta, \mathfrak{T}) + \cdots,\\
\bar{\Phi}(\zeta, \mathfrak{T}) &= \bar{\Phi}_0(\zeta, \mathfrak{T}) + \bar{\Phi}_1(\zeta, \mathfrak{T}) + \bar{\Phi}_2(\zeta, \mathfrak{T})\\
&\quad + \bar{\Phi}_3(\zeta, \mathfrak{T}) + \bar{\Phi}_4(\zeta, \mathfrak{T}) + \cdots,\\
\underline{\Phi}(\zeta, \mathfrak{T}) &= \underline{k}(r)\sin\zeta - \underline{k}(r)\cos\zeta\frac{1}{\mathbb{B}(\wp)}\left\{\frac{\wp\mathfrak{T}^{\wp}}{\Gamma(\wp+1)} + (1-\wp)\right\}\\
&\quad - \underline{k}(r)\sin\zeta\frac{1}{\mathbb{B}^2(\wp)}\\
&\quad\cdot\left\{\frac{\wp^2\mathfrak{T}^{2\wp}}{\Gamma(2\wp+1)} + 2\wp(1-\wp)\frac{\mathfrak{T}^{\wp}}{\Gamma(\wp+1)} + (1-\wp)^2\right\}\\
&\quad + \underline{k}(r)\cos\zeta\frac{1}{\mathbb{B}^3(\wp)}\\
&\quad\cdot\left\{\frac{\wp^3\mathfrak{T}^{3\theta}}{\Gamma(3\wp+1)} + 3\wp^2(1-\wp)\frac{\mathfrak{T}^{2\wp}}{\Gamma(2\wp+1)} + 3\wp(1-\wp)^2\frac{\mathfrak{T}^{\theta}}{\Gamma(\wp+1)}\right\} + \cdots,\\
\bar{\Phi}(\zeta, \mathfrak{T}) &= \bar{k}(r)\sin\zeta - \bar{k}(r)\cos\zeta\frac{1}{\mathbb{B}(\wp)}\left\{\frac{\wp\mathfrak{T}^{\wp}}{\Gamma(\wp+1)} + (1-\wp)\right\}\\
&\quad - \bar{k}(r)\sin\zeta\frac{1}{\mathbb{B}^2(\wp)}\left\{\frac{\wp^2\mathfrak{T}^{2\wp}}{\Gamma(2\wp+1)} + 2\wp(1-\wp)\frac{\mathfrak{T}^{\wp}}{\Gamma(\wp+1)} + (1-\wp)^2\right\}\\
&\quad + \bar{k}(r)\cos\zeta\frac{1}{\mathbb{B}^3(\wp)}\\
&\quad\cdot\left\{\frac{\wp^3\mathfrak{T}^{3\theta}}{\Gamma(3\wp+1)} + 3\wp^2(1-\wp)\frac{\mathfrak{T}^{2\wp}}{\Gamma(2\wp+1)} + 3\wp(1-\wp)^2\frac{\mathfrak{T}^{\theta}}{\Gamma(\wp+1)}\right\} + \cdots.
\end{aligned} \quad (33)$$

The exact result is

$$\tilde{\Phi}(\zeta, \mathfrak{T}) = \tilde{k}\sin(\zeta + \mathfrak{T}). \quad (34)$$

Figure 1 shows that the accuracy of this method by lower and upper branches of fuzzy solution of problem 15 link with the fuzzy Shehu transformation and Atangana-Baleanu operator is shown in this article. Figure 2 shows the two-dimensional upper and lower branch plots. The behaviour defines the variance in the mappings $\widetilde{\%\Phi}(\zeta, \mathfrak{T})$ on the space coordinate ξ with the consider of $\mathfrak{T}$ and the unpredictability parameters $r\varepsilon[0, 1]$. The graph shows that, as the passage of time, the mappings $\tilde{\Phi}(\zeta, \mathfrak{T})$ will become much intricate.

Problem 16. Consider the fractional fuzzy KdV equation is given as

$$^{\mathrm{ABC}}D^{\wp}_{\mathfrak{T}}\tilde{\Phi}(\zeta, \xi, \mathfrak{T}) + 2\frac{\partial^3\tilde{\Phi}(\zeta, \xi, \tau)}{\partial\zeta^3} + \frac{\partial^3\tilde{\Phi}(\zeta, \xi, \tau)}{\partial\xi^3} = 0, \quad 0 < \wp \le 1, \quad (35)$$

with the initial condition

$$\tilde{\Phi}(\zeta, \xi, 0) = \tilde{k}\cos(\zeta + \xi). \quad (36)$$

Applying the proposed method of Equation (35), we

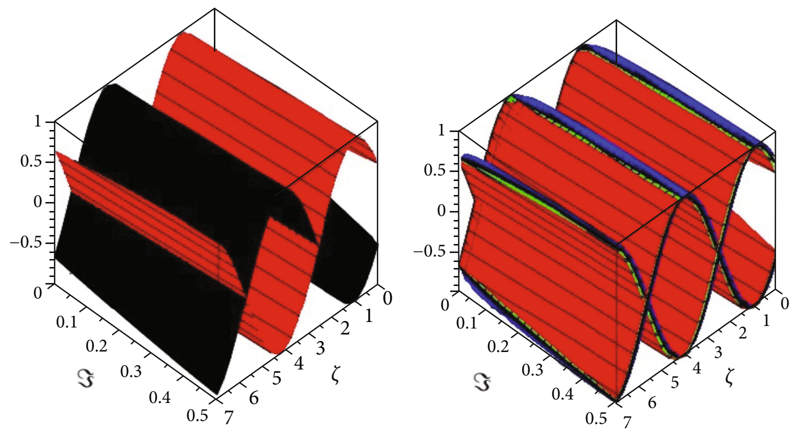

FIGURE 1: The first graph shows the approximate solution of fuzzy upper and lower portions, and the second figure is the various fractional order of ℘.

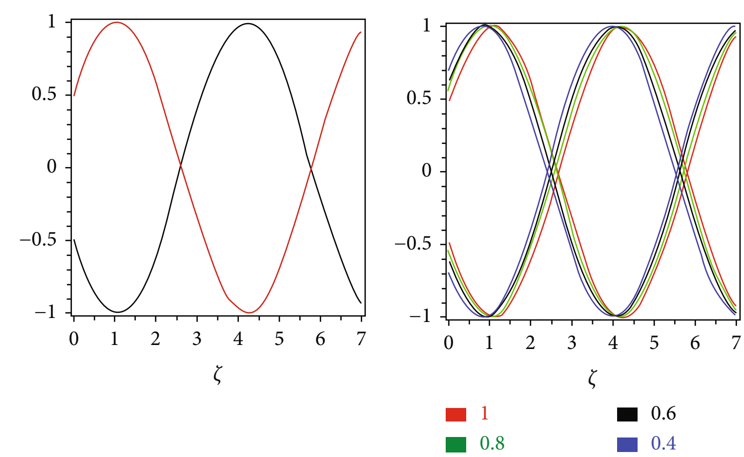

FIGURE 2: The approximate solution graph of fuzzy upper and lower branches and the 2nd figure is the various fractional order of ℘.

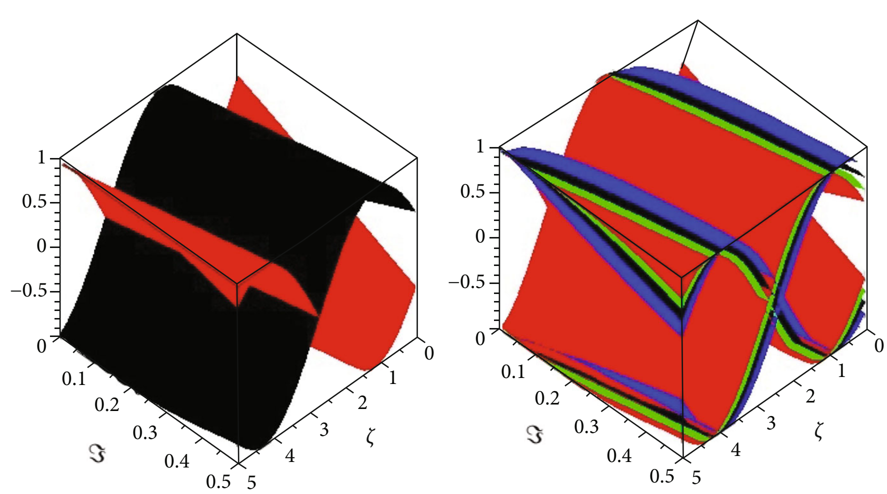

FIGURE 3: The first graph shows the approximate solution of fuzzy upper and lower portions, and the second figure is the various fractional order of ℘.

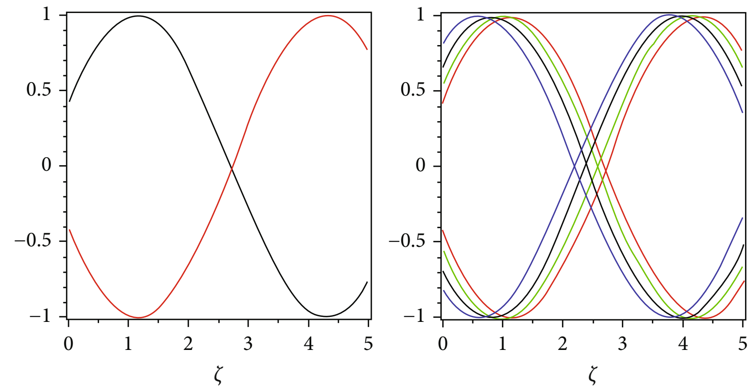

Figure 4: The approximate solution graph of fuzzy upper and lower branches and the 2nd figure is the various fractional order of $\wp$.

get

$$\begin{aligned}
&\underline{\Phi}_0(\zeta,\xi,\mathfrak{I}) = \underline{k}(r)\cos(\zeta+\xi),\\
&\bar{\Phi}_0(\zeta,\xi,\mathfrak{I}) = \bar{k}(r)\cos(\zeta+\xi),\\
&\underline{\Phi}_1(\zeta,\xi,\mathfrak{I}) = -2\underline{k}(r)\sin(\zeta+\xi)\frac{1}{\mathbb{B}(\wp)}\left\{\frac{\wp\mathfrak{I}^{\wp}}{\Gamma(\wp+1)}+(1-\wp)\right\},\\
&\bar{\Phi}_1(\zeta,\xi,\mathfrak{I}) = -2\bar{k}(r)\sin(\zeta+\xi)\frac{1}{\mathbb{B}(\wp)}\left\{\frac{\wp\mathfrak{I}^{\wp}}{\Gamma(\wp+1)}+(1-\wp)\right\},\\
&\underline{\Phi}_2(\zeta,\xi,\mathfrak{I}) = -4\underline{k}(r)\cos(\zeta+\xi)\frac{1}{\mathbb{B}^2(\wp)}\cdot\left\{\frac{\wp^2\mathfrak{I}^{2\wp}}{\Gamma(2\wp+1)}+2\wp(1-\wp)\frac{\mathfrak{I}^{\wp}}{\Gamma(\wp+1)}+(1-\wp)^2\right\},\\
&\bar{\Phi}_2(\zeta,\xi,\mathfrak{I}) = -4\bar{k}(r)\cos(\zeta+\xi)\frac{1}{\mathbb{B}^2(\wp)}\cdot\left\{\frac{\wp^2\mathfrak{I}^{2\wp}}{\Gamma(2\wp+1)}+2\wp(1-\wp)\frac{\mathfrak{I}^{\wp}}{\Gamma(\wp+1)}+(1-\wp)^2\right\},\\
&\underline{\Phi}_3(\zeta,\xi,\mathfrak{I}) = 8\underline{k}(r)\sin(\zeta+\xi)\frac{1}{\mathbb{B}^3(\wp)}\cdot\left\{\frac{\wp^3\mathfrak{I}^{3\theta}}{\Gamma(3\wp+1)}+3\wp^2(1-\wp)\frac{\mathfrak{I}^{2\wp}}{\Gamma(2\wp+1)}+3\wp(1-\wp)^2\frac{\mathfrak{I}^{\theta}}{\Gamma(\wp+1)}\right\},\\
&\bar{\Phi}_3(\zeta,\xi,\mathfrak{I}) = 8\bar{k}(r)\sin(\zeta+\xi)\frac{1}{\mathbb{B}^3(\wp)}\cdot\left\{\frac{\wp^3\mathfrak{I}^{3\theta}}{\Gamma(3\wp+1)}+3\wp^2(1-\wp)\frac{\mathfrak{I}^{2\wp}}{\Gamma(2\wp+1)}+3\wp(1-\wp)^2\frac{\mathfrak{I}^{\theta}}{\Gamma(\wp+1)}\right\}.
\end{aligned}\tag{37}$$

Using Equation (27) to achieve the series form solution, we get

$$\begin{aligned}
\tilde{\Phi}(\zeta,\xi,\mathfrak{I}) &= \tilde{\Phi}_0(\zeta,\xi,\mathfrak{I})+\tilde{\Phi}_1(\zeta,\xi,\mathfrak{I})+\tilde{\Phi}_2(\zeta,\xi,\mathfrak{I})\\
&\quad+\tilde{\Phi}_3(\zeta,\xi,\mathfrak{I})+\tilde{\Phi}_4(\zeta,\xi,\mathfrak{I})+\cdots.
\end{aligned}\tag{38}$$

In lower and upper portion types can be written as

$$\begin{aligned}
&\underline{\Phi}(\zeta,\xi,\mathfrak{I}) = \underline{\Phi}_0(\zeta,\xi,\mathfrak{I})+\underline{\Phi}_1(\zeta,\xi,\mathfrak{I})+\underline{\Phi}_2(\zeta,\xi,\mathfrak{I})+\underline{\Phi}_3(\zeta,\xi,\mathfrak{I})+\underline{\Phi}_4(\zeta,\xi,\mathfrak{I})+\cdots,\\
&\bar{\Phi}(\zeta,\xi,\mathfrak{I}) = \bar{\Phi}_0(\zeta,\xi,\mathfrak{I})+\bar{\Phi}_1(\zeta,\xi,\mathfrak{I})+\bar{\Phi}_2(\zeta,\xi,\mathfrak{I})+\bar{\Phi}_3(\zeta,\xi,\mathfrak{I})+\bar{\Phi}_4(\zeta,\xi,\mathfrak{I})+\cdots.\\
&\underline{\Phi}(\zeta,\xi,\mathfrak{I}) = \underline{k}(r)\cos(\zeta+\xi)-2\underline{k}(r)\sin(\zeta+\xi)\frac{1}{\mathbb{B}(\wp)}\cdot\left\{\frac{\wp\mathfrak{I}^{\wp}}{\Gamma(\wp+1)}+(1-\wp)\right\}-4\underline{k}(r)\cos(\zeta+\xi)\frac{1}{\mathbb{B}^2(\wp)}\\
&\quad\cdot\left\{\frac{\wp^2\mathfrak{I}^{2\wp}}{\Gamma(2\wp+1)}+2\wp(1-\wp)\frac{\mathfrak{I}^{\wp}}{\Gamma(\wp+1)}+(1-\wp)^2\right\}+8\underline{k}(r)\sin(\zeta+\xi)\frac{1}{\mathbb{B}^3(\wp)}\\
&\quad\cdot\left\{\frac{\wp^3\mathfrak{I}^{3\theta}}{\Gamma(3\wp+1)}+3\wp^2(1-\wp)\frac{\mathfrak{I}^{2\wp}}{\Gamma(2\wp+1)}+3\wp(1-\wp)^2\frac{\mathfrak{I}^{\theta}}{\Gamma(\wp+1)}\right\}\cdots,\\
&\bar{\Phi}(\zeta,\xi,\mathfrak{I}) = \bar{k}(r)\cos(\zeta+\xi)-2\bar{k}(r)\sin(\zeta+\xi)\frac{1}{\mathbb{B}(\wp)}\cdot\left\{\frac{\wp\mathfrak{I}^{\wp}}{\Gamma(\wp+1)}+(1-\wp)\right\}-4\bar{k}(r)\cos(\zeta+\xi)\frac{1}{\mathbb{B}^2(\wp)}\\
&\quad\cdot\left\{\frac{\wp^2\mathfrak{I}^{2\wp}}{\Gamma(2\wp+1)}+2\wp(1-\wp)\frac{\mathfrak{I}^{\wp}}{\Gamma(\wp+1)}+(1-\wp)^2\right\}+8\bar{k}(r)\sin(\zeta+\xi)\frac{1}{\mathbb{B}^3(\wp)}\\
&\quad\cdot\left\{\frac{\wp^3\mathfrak{I}^{3\theta}}{\Gamma(3\wp+1)}+3\wp^2(1-\wp)\frac{\mathfrak{I}^{2\wp}}{\Gamma(2\wp+1)}+3\wp(1-\wp)^2\frac{\mathfrak{I}^{\theta}}{\Gamma(\wp+1)}\right\}\cdots.
\end{aligned}\tag{39}$$

The exact result is

$$\tilde{\Phi}(\zeta,\mathfrak{I}) = \tilde{k}\cos(\zeta+\xi+2\mathfrak{I}).\tag{40}$$

Figures 3 and 4 show that the accuracy of this method by upper and lower branches of fuzzy solutions of problem 16 link with the fuzzy Shehu transformation and

Atangana-Baleanu operator is represented in this article. Figure 5 the approximate solution graph of fuzzy upper and lower branches and the 2nd figure is the various fractional order of $\wp$ with respect to time. The behaviour defines the variance in the mappings $\tilde{\Phi}(\zeta,\mathfrak{T})$ on the space coordinate ξ with the consider of $\mathfrak{T}$ and the unpredictability parameters $r\varepsilon[0,1]$. The graph shows that, as the passage of time, the mappings $\tilde{\Phi}(\zeta,\mathfrak{T})$ will become much intricate.

Problem 17. Consider the fractional fuzzy KdV equation is given as

$$ {}^{\mathrm{ABC}}D_{\mathfrak{T}}^{\wp}\tilde{\Phi}(\zeta,\mathfrak{T})+\tilde{\Phi}(\zeta,\mathfrak{T})\frac{\partial\tilde{\Phi}(\zeta,\tau)}{\partial\zeta}+\frac{\partial^3\tilde{\Phi}(\zeta,\tau)}{\partial\zeta^3}=0,\quad 0<\wp\le 1, \tag{41}$$

with the initial condition

$$\tilde{\Phi}(\zeta,0)=\tilde{k}(1-\zeta). \tag{42}$$

Applying the proposed method of Equation (42), we get

$$\begin{aligned}
\underline{\Phi}_0(\zeta,\mathfrak{T})&=\underline{k}(r)(1-\zeta),\\
\bar{\Phi}_0(\zeta,\mathfrak{T})&=\bar{k}(r)(1-\zeta),\\
\underline{\Phi}_1(\zeta,\mathfrak{T})&=\underline{k}(r)(1-\zeta)\frac{1}{\mathbb{B}(\wp)}\left\{\frac{\wp\mathfrak{T}^{\wp}}{\Gamma(\wp+1)}+(1-\wp)\right\},\\
\bar{\Phi}_1(\zeta,\mathfrak{T})&=\bar{k}(r)(1-\zeta)\frac{1}{\mathbb{B}(\wp)}\left\{\frac{\wp\mathfrak{T}^{\wp}}{\Gamma(\wp+1)}+(1-\wp)\right\},\\
\underline{\Phi}_2(\zeta,\mathfrak{T})&=2\underline{k}(r)(1-\zeta)\frac{1}{\mathbb{B}^2(\wp)}\cdot\left\{\frac{\wp^2\mathfrak{T}^{2\wp}}{\Gamma(2\wp+1)}+2\wp(1-\wp)\frac{\mathfrak{T}^{\wp}}{\Gamma(\wp+1)}+(1-\wp)^2\right\},\\
\bar{\Phi}_2(\zeta,\mathfrak{T})&=2\bar{k}(r)(1-\zeta)\frac{1}{\mathbb{B}^2(\wp)}\cdot\left\{\frac{\wp^2\mathfrak{T}^{2\wp}}{\Gamma(2\wp+1)}+2\wp(1-\wp)\frac{\mathfrak{T}^{\wp}}{\Gamma(\wp+1)}+(1-\wp)^2\right\},\\
\underline{\Phi}_3(\zeta,\mathfrak{T})&=6\underline{k}(r)(1-\zeta)\frac{1}{\mathbb{B}^3(\wp)}\cdot\left\{\frac{\wp^3\mathfrak{T}^{3\theta}}{\Gamma(3\wp+1)}+3\wp^2(1-\wp)\frac{\mathfrak{T}^{2\wp}}{\Gamma(2\wp+1)}+3\wp(1-\wp)^2\frac{\mathfrak{T}^{\theta}}{\Gamma(\wp+1)}\right\},\\
\bar{\Phi}_3(\zeta,\mathfrak{T})&=6\bar{k}(r)(1-\zeta)\frac{1}{\mathbb{B}^3(\wp)}\cdot\left\{\frac{\wp^3\mathfrak{T}^{3\theta}}{\Gamma(3\wp+1)}+3\wp^2(1-\wp)\frac{\mathfrak{T}^{2\wp}}{\Gamma(2\wp+1)}+3\wp(1-\wp)^2\frac{\mathfrak{T}^{\theta}}{\Gamma(\wp+1)}\right\}.
\end{aligned} \tag{43}$$

Using Equation (27) to achieve the series form solution, we get

$$\tilde{\Phi}(\zeta,\mathfrak{T})=\tilde{\Phi}_0(\zeta,\mathfrak{T})+\tilde{\Phi}_1(\zeta,\mathfrak{T})+\tilde{\Phi}_2(\zeta,\mathfrak{T})+\tilde{\Phi}_3(\zeta,\mathfrak{T})+\tilde{\Phi}_4(\zeta,\mathfrak{T})+\cdots. \tag{44}$$

In lower and upper portion types can be written as

$$\begin{aligned}
\underline{\Phi}(\zeta,\mathfrak{T})&=\underline{\Phi}_0(\zeta,\mathfrak{T})+\underline{\Phi}_1(\zeta,\mathfrak{T})+\underline{\Phi}_2(\zeta,\mathfrak{T})+\underline{\Phi}_3(\zeta,\mathfrak{T})+\underline{\Phi}_4(\zeta,\mathfrak{T})+\cdots,\\
\bar{\Phi}(\zeta,\mathfrak{T})&=\bar{\Phi}_0(\zeta,\mathfrak{T})+\bar{\Phi}_1(\zeta,\mathfrak{T})+\bar{\Phi}_2(\zeta,\mathfrak{T})+\bar{\Phi}_3(\zeta,\mathfrak{T})+\bar{\Phi}_4(\zeta,\mathfrak{T})+\cdots.\\
\underline{\Phi}(\zeta,\mathfrak{T})&=\underline{k}(r)(1-\zeta)+\underline{k}(r)(1-\zeta)\frac{1}{\mathbb{B}(\wp)}\cdot\left\{\frac{\wp\mathfrak{T}^{\wp}}{\Gamma(\wp+1)}+(1-\wp)\right\}+2\underline{k}(r)(1-\zeta)\frac{1}{\mathbb{B}^2(\wp)}\\
&\quad\cdot\left\{\frac{\wp^2\mathfrak{T}^{2\wp}}{\Gamma(2\wp+1)}+2\wp(1-\wp)\frac{\mathfrak{T}^{\wp}}{\Gamma(\wp+1)}+(1-\wp)^2\right\}+\underline{k}(r)\cos\zeta\frac{1}{\mathbb{B}^3(\wp)}\\
&\quad\cdot\left\{\frac{\wp^3\mathfrak{T}^{3\theta}}{\Gamma(3\wp+1)}+3\wp^2(1-\wp)\frac{\mathfrak{T}^{2\wp}}{\Gamma(2\wp+1)}+3\wp(1-\wp)^2\frac{\mathfrak{T}^{\theta}}{\Gamma(\wp+1)}\right\}+\cdots,\\
\bar{\Phi}(\zeta,\mathfrak{T})&=\bar{k}(r)(1-\zeta)+\bar{k}(r)(1-\zeta)\frac{1}{\mathbb{B}(\wp)}\cdot\left\{\frac{\wp\mathfrak{T}^{\wp}}{\Gamma(\wp+1)}+(1-\wp)\right\}+2\bar{k}(r)(1-\zeta)\frac{1}{\mathbb{B}^2(\wp)}\\
&\quad\cdot\left\{\frac{\wp^2\mathfrak{T}^{2\wp}}{\Gamma(2\wp+1)}+2\wp(1-\wp)\frac{\mathfrak{T}^{\wp}}{\Gamma(\wp+1)}+(1-\wp)^2\right\}+\bar{k}(r)\cos\zeta\frac{1}{\mathbb{B}^3(\wp)}\\
&\quad\cdot\left\{\frac{\wp^3\mathfrak{T}^{3\theta}}{\Gamma(3\wp+1)}+3\wp^2(1-\wp)\frac{\mathfrak{T}^{2\wp}}{\Gamma(2\wp+1)}+3\wp(1-\wp)^2\frac{\mathfrak{T}^{\theta}}{\Gamma(\wp+1)}\right\}+\cdots.
\end{aligned} \tag{45}$$

The exact result is

$$\tilde{\Phi}(\zeta,\mathfrak{T})=\frac{1-\zeta}{1-\mathfrak{T}}. \tag{46}$$

Figure 6 shows that the accuracy of this method by upper and lower branches of fuzzy solution of problem 17 link with the fuzzy Shehu transformation and Atangana-Baleanu operator is represented in this article. Figure 7 the approximate solution graph of fuzzy upper and lower branches and the 2nd figure is the various fractional order of $\wp$. The behaviour defines the variance in the mappings $\tilde{\Phi}(\zeta,\mathfrak{T})$ on the space coordinate ξ with the consider of $\mathfrak{T}$ and the unpredictability parameters $r\varepsilon[0,1]$. The graph shows that, as the passage of time, the mappings $\tilde{\Phi}(\zeta,\mathfrak{T})$ will become much intricate.

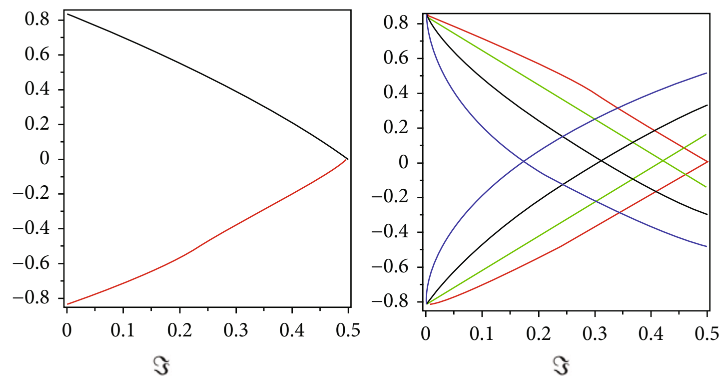

Figure 5: The approximate solution graph of fuzzy upper and lower branches and the 2nd figure is the various fractional order of ℘ with respect to time.

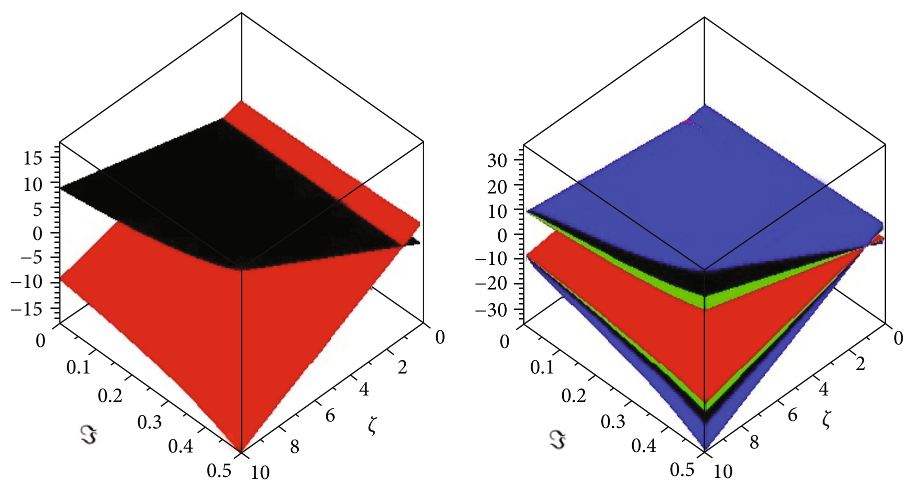

Figure 6: The first graph shows the approximate solution of fuzzy upper and lower portions, and the second figure is the various fractional order of ℘.

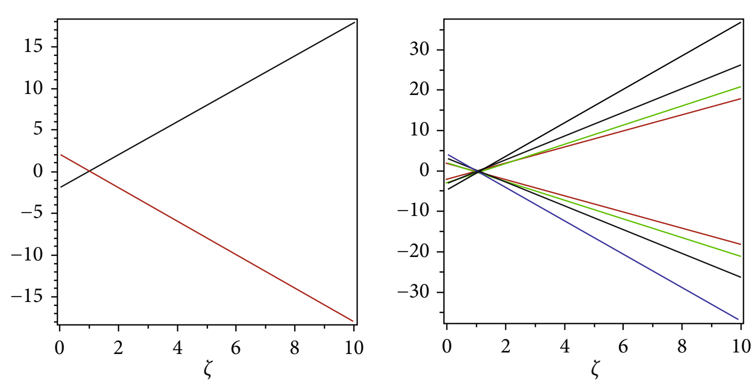

Figure 7: The approximate solution graph of fuzzy upper and lower branches and the 2nd figure is the various fractional order of ℘.

5. Conclusion

In this paper, an analytical result of fuzzy fractional third-order KdV equations has been successfully developed. The series form solution of the consider equation in sense of fractional the Atangana-Baleanu fractional derivatives has been introduced by Shehu transformation along with iterative method. By multiplying the number of fuzzy, we achieved the lower and upper portions of the required result. Figures of the analytical solutions at various noninteger order of the given models are also provided. Furthermore, the method used to solve the fuzzy fractional KdV equation in this article is novel and can be used to all areas of physical and natural sciences where uncertainty exists in various phenomena such as quantum mechanics and various theories of orbits or orbital around the nucleus. As a result, our study will open new doors in the fields of fuzzy fractional calculus and fuzzy calculus. Moreover, the suggested approach can be devoted for modeling acoustic waves in different plasma models by solving many fractional PDEs like fractional Schamel-KdV equation, fractional Schamel KdV-Burgers equation, fractional modified KdV and extended KdV equations, Schamel nonlinear Schrödinger equation, two-and three-dimensional nonlinear Schrödinger equations, and so on [41, 42].

Data Availability

The numerical data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

Acknowledgments

The authors express their gratitude to the Princess Nourah Bint Abdulrahman University Researchers Supporting Project (Grant No. PNURSP2022R32), Princess Nourah Bint Abdulrahman University, Riyadh, Saudi Arabia.

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