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*Research Article*

# Ostrowski Type Inequalities for $s$-Convex Functions via $q$-Integrals

**Khuram Ali Khan**,[1] **Allah Ditta**,[2] **Ammara Nosheen**,[1] **Khalid Mahmood Awan**,[1] **and Rostin Matendo Mabela**[3]

[1]*Department of Mathematics, University of Sargodha, Sargodha 40100, Pakistan*
[2]*Department of Mathematics, University of Jhang, Jhang, Pakistan*
[3]*Department of Maths and Computer Science, Faculty of Science, University of Kinshasa, Democratic Republic of the Congo*

Correspondence should be addressed to Rostin Matendo Mabela; rostin.mabela@unikin.ac.cd

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The new outcomes of the present paper are $q$-analogues ($q$ stands for quantum calculus) of Hermite-Hadamard type inequality, Montgomery identity, and Ostrowski type inequalities for $s$-convex mappings. Some new bounds of Ostrowski type functionals are obtained by using Hölder, Minkowski, and power mean inequalities via quantum calculus. Special cases of new results include existing results from the literature.

## 1. Introduction

Integral inequalities provide a notable role in both pure and applied mathematics in the light of their wide applications in numerous regular and human sociologies, while convexity hypothesis has stayed a significant apparatus in the foundation of the theory of integral inequalities. The classical inequalities are helpful in numerous down-to-earth issues. In recent years, many authors (see [1–12]) proved numerous inequalities associated with the functions of bounded variation, Lipschitzian, monotone, absolutely continuous, convex functions, $s$-convex, $h$-convex, and $n$-times differentiable mappings with error estimates. Integral inequalities have been studied extensively by several researchers either in classical analysis or in the quantum one. In many practical problems, it is important to bound one quantity by another quantity. The classical inequalities including Hermite-Hadamard and Ostrowski type inequalities are very useful for this purpose (see [13–24]). Ostrowski type inequalities are well known to study the upper bounds for approximation of the integral average by the value of the function. In [25], Dragomir and Fitzpatrick have constructed Hermite-Hadamard's inequality which is specified to $s$-convex functions in the second sense as follows:

**Theorem 1.** *Suppose $\Phi : \mathbb{R}^+ \longrightarrow \mathbb{R}$ is an s-convex function in second sense, $s \in (0, 1)$, and suppose $\wp, \upsilon \in \mathbb{R}^+, \wp<\upsilon$. If $\Phi' \in L^1([\wp,\upsilon])$, then the integral inequality is valid:*

$$2^{s-1}\Phi\left(\frac{\wp+\upsilon}{2}\right) \leq \frac{1}{\upsilon-\wp}\int_{\wp}^{\upsilon}\Phi(w)dw \leq \frac{\Phi(\wp)+\Phi(\upsilon)}{s+1}, \tag{1}$$

*where $\mathbb{R}^+ = \{w \in \mathbb{R} \mid w \geq 0\}$.*

The following Montgomery equality is established by Alomari (see [26]):

**Lemma 2.** *Assume that $\Phi : J \subset \mathbb{R}^+ \longrightarrow \mathbb{R}$ is differentiable function on $(\wp, \upsilon)$ in which $\wp, \upsilon \in J$ for $\wp<\upsilon$. If $\Phi' \in L[\wp,\upsilon]$, then we have the equality:*

$$\Phi(w) - \frac{1}{\upsilon-\wp}\int_{\wp}^{\upsilon}\Phi(\zeta)d\zeta = \frac{(w-\wp)^2}{\upsilon-\wp}\int_0^1 \zeta\Phi'(\zeta w + (1-\zeta)\wp) \cdot d\zeta - \frac{(\upsilon-w)^2}{\upsilon-\wp}\int_0^1 \zeta\Phi'(\zeta w + (1-\zeta)\upsilon)d\zeta, \tag{2}$$

*for each $w \in [\wp,\upsilon]$.*

*By using Lemma 2, Alomari et al. in [26] had proved the Ostrowski type inequality, which holds for s-convex mappings in second sense as follows:*

**Theorem 3.** *Assume $\Phi : J \subset \mathbb{R}^{+} \longrightarrow \mathbb{R}$ is a differentiable on $(\wp, \upsilon)$ and $\Phi' \in L[\wp,\upsilon]$ such that $\wp, \upsilon \in J$ for $\wp<\upsilon$. If $|\Phi'|$ is s-convex mapping in the second sense on $[\wp, \upsilon]$ unique $s \in (0, 1]$ and $|\Phi'(w)| \le M, w \in [\wp,\upsilon]$, then the following result holds:*

$$\left|\Phi(w) - \frac{1}{\upsilon-\wp}\int_{\wp}^{\upsilon}\Phi(\zeta)d\zeta\right| \le \frac{M}{\upsilon-\wp}\left[\frac{(w-\wp)^2 + (\upsilon-w)^2}{s+1}\right], \quad (3)$$

*for each $w \in [\wp,\upsilon]$.*

**Theorem 4.** *Suppose that $\Phi : J \subset \mathbb{R}^{+} \longrightarrow \mathbb{R}$ is the differentiable on $(\wp, \upsilon)$ and $\Phi' \in L[\wp,\upsilon]$, where $\wp, \upsilon \in J$ with $\wp<\upsilon$. If absolute value of $(\Phi')^m$ is s-convex function in the second sense in $[\wp, \upsilon]$ for unique $s \in (0, 1]$, $m > 1$, $n = m/m - 1$ and $|\Phi'(w)| \le M, w \in [\wp,\upsilon]$, then following integral inequality holds:*

$$\left|\Phi(w) - \frac{1}{\upsilon-\wp}\int_{\wp}^{\upsilon}\Phi(\zeta)d\zeta\right| \le \frac{M}{(1+n)^{1/n}}\left(\frac{2}{s+1}\right)^{1/m}\left[\frac{(w-\wp)^2 + (\upsilon-w)^2}{\upsilon-\wp}\right], \quad (4)$$

*for each $w \in [\wp,\upsilon]$.*

**Theorem 5.** *Suppose that $\Phi : J \subset \mathbb{R}^{+} \longrightarrow \mathbb{R}$ is differentiable on $(\wp, \upsilon)$ and $\Phi' \in L[\wp,\upsilon]$, in which $\wp, \upsilon \in J$ for $\wp<\upsilon$. If the absolute value of $(\Phi')^m$ is s-convex function in $[\wp, \upsilon]$ for static $s \in (0, 1]$, $m \ge 1$ and $|\Phi'(w)| \le M, w \in [\wp,\upsilon]$, then the following integral inequality holds:*

$$\left|\Phi(w) - \frac{1}{\upsilon-\wp}\int_{\wp}^{\upsilon}\Phi(\zeta)d\zeta\right| \le M\left(\frac{2}{s+1}\right)^{1/m}\left[\frac{(w-\wp)^2 + (\upsilon-w)^2}{2(\upsilon-\wp)}\right], \quad (5)$$

*for each $w \in [\wp,\upsilon]$.*

**Theorem 6.** *Suppose $\Phi : J \subset \mathbb{R}^{+} \longrightarrow \mathbb{R}$ be the differentiable on $(\wp, \upsilon)$ and $\Phi' \in L[\wp,\upsilon]$, in which $\wp, \upsilon \in J$ for $\wp<\upsilon$. If absolute value of $(\Phi')^m$ is a s-convex mapping in second sense on $[\wp, \upsilon]$ for static $s \in (0, 1]$, $m > 1$ and $n = m/m - 1$, we have*

$$\left|\Phi(w) - \frac{1}{\upsilon-\wp}\int_{\wp}^{\upsilon}\Phi(\zeta)d\zeta\right| \le \frac{2^{(s-1)/m}}{(1+n)^{1/n}(\upsilon-\wp)} \cdot \left[(w-\wp)^2\left|\Phi'\left(\frac{w+\wp}{2}\right)\right| + (\upsilon-w)^2\left|\Phi'\left(\frac{\upsilon+w}{2}\right)\right|\right], \quad (6)$$

*for each $w \in [\wp,\upsilon]$.*

The renowned mathematician Euler started the investigation of $q$-calculus in the eighteenth century by presenting Newton's work of limitless series. This subject has gotten extraordinary consideration by numerous specialists, and consequently, it is considered an in-corporative subject among math and material science. In the mid-20th century, Jackson (1910) has begun a symmetric investigation of calculus and presented $q$-distinct integrals. The subject of quantum analytic has various applications in different spaces of arithmetic and physical science like number hypothesis, combinatorics, symmetrical polynomials, essential hypermathematical functions, quantum theory, and mechanics and in the hypothesis of relativity. Quantum calculus can be seen as a scaffold among arithmetic and material science. It has been shown that quantum calculus is a subfield of the more general mathematical field of time scales calculus. Time scales provide a unified framework for studying dynamic equations on both discrete and continuous domains. In [27, 28], $q$-Bernoulli and dynamic inequalities associated with Leibniz integral rule on time scales were studied. In studying quantum calculus, we are concerned with a specific time scale, called the $q$-time scale. The study of $q$-integral inequalities is also of great importance. Integral inequalities have been studied extensively by several researchers either in classical analysis or in the quantum one.

The following $q$-Hermite-Hadamard and $q$-Ostrowski type integral inequalities were proved by Tariboon and Ntouyas (see Theorems 3.2 and 3.5 [29]):

**Theorem 7.** *Let $\Phi : J \longrightarrow \mathbb{R}$ be a q-differentiable function with $D_q\Phi$ continuous on $[\wp, \upsilon]$ and $0 < q < 1$. Then, we have*

$$\Phi\left(\frac{\wp+\upsilon}{2}\right) \le \frac{1}{\upsilon-\wp}\int_{\wp}^{\upsilon}\Phi(\zeta)_{\wp}d_q\zeta \le \frac{\Phi(\wp) + q\Phi(\upsilon)}{q+1}. \quad (7)$$

**Theorem 8.** *Suppose $\Phi : J \longrightarrow \mathbb{R}$, where $[\wp,\upsilon] \subseteq \mathbb{R}$ is an interval, be a q-differentiable in open interval $\wp, \upsilon$ belonging to interior I for $\wp<\upsilon$. If $|D_q\Phi(w)| \le M$ for all $w \in [\wp,\upsilon]$ and $0 < q < 1$, then the integral inequality is valid:*

$$\left|\Phi(w) - \frac{1}{\upsilon-\wp}\int_{\wp}^{\upsilon}\Phi(\zeta)_{\wp}d_q\zeta\right| \le M\left[\frac{2q}{1+q}\left(\frac{w - (((3q-1)\wp+(1+q)\upsilon)/4q)}{\upsilon-\wp}\right)^2 + \left(\frac{(-q^2+6q-1)}{8q(1+q)}\right)\right], \quad (8)$$

*for all $w \in [\wp,\upsilon]$. The least value of constant on RHS of inequality (8) is $(-q^2 + 6q - 1)/8q(1 + q)$.*

*The following q-Ostrowski type integral inequalities for convex functions were proved by Noor et al. (see [30]):*

**Theorem 9.** *Let $\Phi : J \subset \mathbb{R}^{+} \longrightarrow \mathbb{R}$ be q-differentiable mapping for $(\wp, \upsilon)$ and $D_q\Phi \in L[\wp,\upsilon]$, in which $\wp, \upsilon \in J$ for $\wp<\upsilon$. If $|D_q\Phi|$ is convex mapping $[\wp, \upsilon]$ for some static $q \in (0, 1)$ and $|D_q\Phi(w)| \le M, w \in [\wp,\upsilon]$, then we have the following q*

-*integral inequality:*

$$\left|\frac{1}{q}\left(\Phi(w)-\frac{1}{\upsilon-\wp}\int_{\wp}^{\upsilon}\Phi(\zeta)_{\wp}d_q\zeta\right)\right|\leq\frac{M}{\upsilon-\wp}\left[\frac{(w-\wp)^2+(\upsilon-w)^2}{q+1}\right],\tag{9}$$

*for each* $w\in[\wp,\upsilon]$.

**Theorem 10.** *Assume that* $\Phi : J\subset\mathbb{R}^+\longrightarrow\mathbb{R}$ *is q-differentiable mapping on* $(\wp,\upsilon)$ *and* $D_q\Phi\in L[\wp,\upsilon]$, *in which* $\wp,\upsilon\in J$ *for* $\wp<\upsilon$. *If* $|D_q\Phi|^m$ *is a convex function in second sense on* $[\wp,\upsilon]$ *unique* $q\in(0,1)$, $m>1$, $n=m/m-1$, *and* $|D_q\Phi(w)|\leq M$, $w\in[\wp,\upsilon]$, *then we have the q-integral inequality:*

$$\left|\frac{1}{q}\left(\Phi(w)-\frac{1}{\upsilon-\wp}\int_{\wp}^{\upsilon}\Phi(\zeta)_{\wp}d_q\zeta\right)\right|\leq\frac{M}{([n+1])^{1/n}}\left[\frac{(w-\wp)^2+(\upsilon-w)^2}{\upsilon-\wp}\right],\tag{10}$$

*for each* $w\in[\wp,\upsilon]$.

The aim of this work is to find $q$-analogues of Hermite-Hadmard and Ostrowski type integral inequalities for functions whose $q$-derivatives are $s$-convex in the second sense. An interesting feature of our results is that they provide new estimates and good approximation on such types of inequalities involving $q$-integrals.

## 2. Basic Essentials

*2.1. Convex Function.* Let $\Phi$ be the function; it is said to be convex function on interval$J$ if

$$\Phi(\Omega w+(1-\Omega)\rho)\leq\Omega\Phi(w)+(1-\Omega)\Phi(\rho)\tag{11}$$

holds for all $w,\rho\in J$ and $\Omega\in[0,1]$.

In [31], $s$-convex functions in the second sense have been introduced by Hudzik and Maligranda as follows:

*2.2. s-Convex Function.* A mapping $\Phi:\mathbb{R}^+\longrightarrow\mathbb{R}$ is said to be $s$-convex if

$$\Phi(\Omega w+(1-\Omega)\rho)\leq\Omega^s\Phi(w)+(1-\Omega)^s\Phi(\rho),\tag{12}$$

for each $w,\rho\in\mathbb{R}^+$, $\Omega\in 0,1]$ and for unique $s\in(0,1]$.

*2.3. q-Derivative [32].* For a continuous mapping $\Phi:[\wp,\upsilon]\longrightarrow\mathbb{R}q$-derivative at $w\in[\wp,\upsilon]$ is

$${}_{\wp}D_q\Phi(w)=\frac{\Phi(w)-\Phi(qw+(1-q)\wp)}{(1-q)(w-\wp)}\quad w\neq\wp.\tag{13}$$

Also, for $n\geq1$, one may find the following evaluations:

$$\begin{aligned}&(w-\wp)_q^n=(w-\wp)(w-q\wp)\left(w-q^2\wp\right)\cdots\left(w-q^{n-1}\wp\right),\\&(\wp-w)_q^n=(\wp-qw)\left(\wp-q^2w\right)\cdots\left(\wp-q^{n-1}w\right),\\&D_q(w-\wp)_q^n=[n](w-\wp)_q^{n-1},\\&D_q(\wp-w)_q^n=-[n](\wp-qw)_q^{n-1},\\&(\wp-qw)_q^n=-\frac{1}{[n+1]}D_q(\wp-w)_q^{n+1},\\&D_q(\wp-w)_q^n=-[n](\wp-qw)_q^{n-1},\\&\int(\wp-w)_q^nd_qw=-\frac{q\left(\wp-q^{-1}w\right)_q^{n+1}}{[n+1]}\quad(\wp\neq-1).\end{aligned}\tag{14}$$

Here,

$$[n]=\frac{q^n-1}{q-1},\tag{15}$$

and also, we have

$$(1-\wp)_q^n=\prod_{j=0}^{n}\left(1-q^j\wp\right).\tag{16}$$

*2.4. q-Antiderivative [32].* Suppose that $\Phi:[\wp,\upsilon]\longrightarrow\mathbb{R}$ be the continuous mapping. Then, $q$-definite integral on $[\wp,\upsilon]$ is stated as

$$\int_{\wp}^{w}\Phi(\zeta)_{\wp}d_q\zeta=(1-q)(w-\wp)\sum_{n=0}^{\infty}q^n\Phi(q^nw+(1-q^n)\wp),\tag{17}$$

for $w\in[\wp,\upsilon]$.

*2.5. The Formula of q-Integration by Parts [29].* Let $\Phi,g:[\wp,\upsilon]\longrightarrow\mathbb{R}$ be the continuous functions $\wp\in\mathbb{R}$ and $w,c\in[\wp,\upsilon]$, Then, the formula of $q$-integration by parts is stated as

$$\begin{aligned}\int_c^w\Phi(\zeta)_{\wp}D_qg(\zeta)d_q\zeta&=\Phi(w)g(w)-\Phi(c)g(c)\\&\quad-\int_c^wg(q\zeta+(1-q)\wp)_{\wp}D_q\Phi(\zeta)d_q\zeta.\end{aligned}\tag{18}$$

**Theorem 11.** **$q$-Hölder Inequality** ([4], Theorem 2). *Let* $\Phi$ *and* $g$ *be q-integrable on* $[\wp,\upsilon]$ *and* $0<q<1$ *and* $(1/n)+(1/m)=1$ *with* $m>1$; *then, one may obtain the following:*

$$\int_{\wp}^{\upsilon}|\Phi(\zeta)g(\zeta)|_{\wp}d_q\zeta\leq\left\{\int_{\wp}^{\upsilon}|\Phi(\zeta)|^n{}_{\wp}d_q\zeta\right\}^{1/n}\left\{\int_{\wp}^{\upsilon}|g(\zeta)|^m{}_{\wp}d_q\zeta\right\}^{1/m}.\tag{19}$$

*Using (19), the following is valid.*

*2.6. q-Minkowski's Inequality.* Let $\wp, \upsilon \in \mathbb{R}$ and $n > 1$ be a real number then for continuous functions $\Phi, g : [\wp,\upsilon] \longrightarrow \mathbb{R}$,

$$\left\{\int_{\wp}^{\upsilon} |(\Phi(\zeta) + g(\zeta))|^n {}_{\wp}d_q\zeta\right\}^{1/n} \leq \left\{\int_{\wp}^{\upsilon} |\Phi(\zeta)|^n {}_{\wp}d_q\zeta\right\}^{1/n} + \left\{\int_{\wp}^{\upsilon} |g(\zeta)|^n {}_{\wp}d_q\zeta\right\}^{1/n}. \tag{20}$$

*Proof.*

$$\begin{aligned}
&\int_{\upsilon}^{\wp} |(\Phi + g)(\zeta)|^n d_q\zeta = \int_{\upsilon}^{\wp} |(\Phi + g)(\zeta)|^{n-1} |(\Phi + g)(\zeta)| d_q\zeta \\
&\leq \int_{\upsilon}^{\wp} |(\Phi + g)(\zeta)|^{n-1} |\Phi(\zeta)| d_q\zeta + \int_{\upsilon}^{\wp} |(\Phi + g)(\zeta)|^{n-1} \\
&\quad \cdot |g(\zeta)| d_q\zeta \leq \left\{\int_{\upsilon}^{\wp} |\Phi(\zeta)|^n d_q\zeta\right\}^{1/n} \left\{\int_{\upsilon}^{\wp} |(\Phi + g)(\zeta)|^{m(n-1)} d_q\zeta\right\}^{1/n} \\
&\quad + \left\{\int_{\upsilon}^{\wp} |g(\zeta)|^n d_q\zeta\right\}^{1/n} \left\{\int_{\upsilon}^{\wp} |(\Phi + g)(\zeta)|^{m(n-1)} d_q\zeta\right\}^{1/m} \\
&= \left[\left\{\int_{\upsilon}^{\wp} |\Phi(\zeta)|^n d_q\zeta\right\}^{1/n} + \left\{\int_{\upsilon}^{\wp} |g(\zeta)|^n d_q\zeta\right\}^{1/n}\right] \\
&\quad \cdot \left[\left\{\int_{\upsilon}^{\wp} |(\Phi + g)(\zeta)|^{m(n-1)} d_q\zeta\right\}^{1/m}\right],
\end{aligned} \tag{21}$$

which gives the required result for positive real numbers $m, n$ such that $(1/m) + (1/n) = 1$.

The classical power mean inequality for integrals has the following form for $q$-integral. □

*2.7. q-Power Mean Inequality.* Let $(1/n) + (1/m) = 1$ for real numbers $n, m > 1$. Let $\wp, \upsilon \in \mathbb{R}$ and $\Phi, g : [\wp,\upsilon] \longrightarrow \mathbb{R}$ be continuous functions; then,

$$\int_{\wp}^{\upsilon} |\Phi(\zeta) g(\zeta)|_{\wp} d_q\zeta \leq \left\{\int_{\wp}^{\upsilon} |\Phi(\zeta)|_{\wp} d_q\zeta\right\}^{1-(1/m)} \cdot \left\{\int_{\wp}^{\upsilon} |\Phi(\zeta)| |g(\zeta)|^m {}_{\wp}d_q\zeta\right\}^{1/m}. \tag{22}$$

**Proposition 12.** [33]. *For each $k, r \in \mathbb{N}(or\, \mathbb{Z}\, q \in \mathbb{R}^{\times})$, we have*

$$[k + r]_q = [k]_q + q^k [r]_q. \tag{23}$$

## 3. Main Results

*3.1. q-Hermite-Hadamard Inequality*

**Theorem 13.** *Suppose $\Phi : \mathbb{R}^+ \longrightarrow \mathbb{R}$ is a s-convex mapping in the second sense, in which $s, q \in (0, 1)$, and let $\wp, \upsilon \in \mathbb{R}^+$, $\wp < \upsilon$. If $D_q\Phi \in L([a, b])$, then the integral inequality is valid:*

$$2^{s-1}\Phi\left(\frac{\wp+\upsilon}{2}\right) \leq \frac{1}{\upsilon-\wp}\int_{\wp}^{\upsilon} \Phi(\zeta)_{\wp} d_q\zeta \leq \frac{\Phi(\upsilon) q\left(1 - \left(1 - q^{-1}\right)^{s+1}\right) + \Phi(\wp)}{[s + 1]}. \tag{24}$$

*Proof.* By definition of s-convex functions,

$$\begin{aligned}
&\Phi(\zeta\wp + (1 - \zeta)\upsilon) \leq \zeta^s \Phi(\wp) + (1 - \zeta)^s \Phi(\upsilon) \\
&\int_0^1 \Phi(\zeta\wp + (1 - \zeta)\upsilon)_0 d_q\zeta \leq \Phi(\wp) \int_0^1 \zeta^s {}_0d_q\zeta + \Phi(\upsilon) \int_0^1 (1 - \zeta)^s {}_0d_q\zeta, \\
&\int_0^1 \Phi(\zeta\wp + (1 - \zeta)\upsilon)_0 d_q\zeta = \frac{(1 - q)(\upsilon-\wp)}{\upsilon-\wp} \sum_{n=0}^{\infty} q^n \Phi(q^n\wp + (1 - q^n)\upsilon) \\
&= \frac{1}{\upsilon-\wp}\int_{\wp}^{\upsilon} \Phi(\zeta)_0 d_q\zeta \leq \Phi(\wp) \int_0^1 \zeta^s {}_0d_q\zeta + \Phi(\upsilon) \int_0^1 (1 - \zeta)^s {}_0d_q\zeta \\
&= \frac{\Phi(\wp) + q\left(1 - \left(1 - q^{-1}\right)^{s+1}\right)\Phi(\upsilon)}{[s + 1]}.
\end{aligned} \tag{25}$$

Hence,

$$\frac{1}{\upsilon-\wp}\int_{\wp}^{\upsilon} \Phi(\zeta)_0 d_q\zeta \leq \frac{\Phi(\wp) + q\left(1 - \left(1 - q^{-1}\right)^{s+1}\right)\Phi(\upsilon)}{[s + 1]}. \tag{26}$$

Let $w = \zeta\wp + (1 - \zeta)\upsilon$ and $\zeta = \zeta\upsilon + (1 - \zeta)\wp$ in $\Phi((w + \zeta)/2) \leq ((\Phi(w) + \Phi(\zeta))/2^s)$ to get

$$\begin{aligned}
&\Phi\left(\frac{\zeta\wp + (1 - \zeta)\upsilon + \zeta\upsilon + (1 - \zeta)\wp}{2}\right) \\
&\quad \leq \frac{\Phi(\zeta\wp + (1 - \zeta)\upsilon) + \Phi(\zeta\upsilon + (1 - \zeta)\wp)}{2^s}, \\
&\Phi\left(\frac{\wp+\upsilon}{2}\right) \leq \frac{1}{2^s}\left(\int_0^1 \Phi(\zeta\wp + (1 - \zeta)\upsilon)_0 d_q\zeta + \int_0^1 \Phi(\zeta\upsilon + (1 - \zeta)\wp)_0 d_q\zeta\right) \\
&\quad = \frac{1}{2^s}\left(\frac{1}{\wp-\upsilon}\int_{\upsilon}^{\wp} \Phi(\zeta)_0 d_q\zeta + \frac{1}{\upsilon-\wp}\int_{\wp}^{\upsilon} \Phi(\zeta)_0 d_q\zeta\right),
\end{aligned} \tag{27}$$

$$2^{s-1}\Phi\left(\frac{\wp+\upsilon}{2}\right) \leq \frac{1}{\upsilon-\wp}\int_{\wp}^{\upsilon} \Phi(\zeta)_0 d_q\zeta, \tag{28}$$

From (26) and (28), the desired result is

$$2^{s-1}\Phi\left(\frac{\wp+\upsilon}{2}\right) \leq \frac{1}{\upsilon-\wp}\int_{\wp}^{\upsilon} \Phi(\zeta)_{\wp} d_q\zeta \leq \frac{\Phi(\wp) + q\left(1 - \left(1 - q^{-1}\right)^{s+1}\right)\Phi(\upsilon)}{[s + 1]}. \tag{29}$$

□

*3.2. q-Ostrowski Type Inequalities.* To prove some $q$-Ostrowski type inequalities, it needs to establish the following Montgomery identity for $q$-integrals:

**Lemma 14.** *Let $\Phi : J \subset \mathbb{R} \longrightarrow \mathbb{R}$ be a q-differentiable on $J^{\circ}$ in which $\wp, \upsilon \in J$ for $\wp<\upsilon$. If $D_q\Phi \in L[\wp,\upsilon]$, we have the following q-integral equality which is valid:*

$$\frac{1}{q}\left[\Phi(w) - \frac{1}{\upsilon-\wp}\int_{\wp}^{\upsilon}\Phi(\zeta)_{\wp}d_q\zeta\right] = \frac{(w-\wp)^2}{\upsilon-\wp}\int_0^1 \zeta D_q\Phi(\zeta w + (1-\zeta)\wp) \cdot_0 d_q\zeta - \frac{(\upsilon-w)^2}{\upsilon-\wp}\int_0^1 \zeta D_q\Phi(\zeta w + (1-\zeta)\upsilon)_0 d_q\zeta, \tag{30}$$

*for each $w \in [\wp,\upsilon]$.*

*By using Lemma 14, we have constructed the following Ostrowski type inequalities, which hold for s-convex functions in the second sense:*

**Theorem 15.** *Let $\Phi : J \subset \mathbb{R}^+ \longrightarrow \mathbb{R}$ is a q-differentiable mapping on $J^{\circ}$ and $D_q\Phi \in L[\wp,\upsilon]$, in which $\wp, \upsilon \in J$ for $\wp<\upsilon$. If the absolute value of $D_q\Phi(w)$ is s-convex in second sense on $[\wp,\upsilon]$ for unique $s \in (0,1]$ and $D_q\Phi(w)$ is bounded by M, $w \in [\wp,\upsilon]$, we have been seeing that the following q-integral inequality is valid:*

$$\left|\frac{1}{q}\left[\Phi(w) - \frac{1}{\upsilon-\wp}\int_{\wp}^{\upsilon}\Phi(\zeta)_{\wp}d_q\zeta\right]\right| \leq M\frac{(w-\wp)^2 + (\upsilon-w)^2}{\upsilon-\wp} \cdot \left[-\frac{q}{[s+1]}\left(\left(1-q^{-1}\right)_q^{s+1} + \frac{q\left(1-q^{-1}\right)_q^{s+2}}{[s+2]} - \frac{q}{[s+2]}\right) + \frac{1}{[s+2]}\right], \tag{31}$$

*for each $w \in [\wp,\upsilon]$.*

*Proof.* Since $|D_q\Phi|$ is $s$-convex function in the second sense on $[\wp,\upsilon]$, therefore, Lemma 14 gives the following:

$$\begin{aligned}
&\left|\frac{1}{q}\left[\Phi(w) - \frac{1}{\upsilon-\wp}\int_{\wp}^{\upsilon}\Phi(\zeta)_{\wp}d_q\zeta\right]\right| \\
&\leq \frac{(w-\wp)^2}{\upsilon-\wp}\int_0^1 \zeta\left|D_q\Phi(\zeta w + (1-\zeta)\wp)\right|_0 d_q\zeta + \frac{(\upsilon-w)^2}{\upsilon-\wp}\int_0^1 \zeta\left|D_q\Phi(\zeta w + (1-\zeta)\upsilon)\right|_0 d_q\zeta \\
&\leq \frac{(w-\wp)^2}{\upsilon-\wp}\int_0^1 \zeta_q^{s+1}\left|D_q\Phi(w)\right|_0 d_q\zeta + \int_0^1 \zeta(1-\zeta)_q^s\left|D_q\Phi(\wp)\right|_0 d_q\zeta \\
&\quad + \frac{(\upsilon-w)^2}{\upsilon-\wp}\int_0^1 \zeta_q^{s+1}\left|D_q\Phi(w)\right|_0 d_q\zeta + \int_0^1 \zeta(1-\zeta)_q^s\left|D_q\Phi(\upsilon)\right|_0 d_q\zeta \\
&= \frac{M(w-\wp)^2}{\upsilon-\wp}\left[\int_0^1 \zeta_q^{s+1}{}_0 d_q\zeta + \int_0^1 \zeta(1-\zeta)_{q0}^s d_q\zeta\right] + \frac{(\upsilon-w)^2}{\upsilon-\wp}\left[\int_0^1 \zeta_q^{s+1}{}_0 d_q\zeta + \int_0^1 \zeta(1-\zeta)_{q0}^s d_q\zeta\right] \\
&= M\frac{(w-\wp)^2 + (\upsilon-w)^2}{\upsilon-\wp}\left[\int_0^1 \zeta^{s+1}{}_0 d_q\zeta + \int_0^1 \zeta(1-\zeta)_{q0}^s d_q\zeta\right],
\end{aligned}$$

$$\int_0^1 \zeta_q^{s+1}{}_0 d_q\zeta = \frac{1}{[s+2]}, \tag{32}$$

$$\begin{aligned}
&= -\frac{q}{[s+1]}\int_0^1 \zeta D_q\left(1-q^{-1}\zeta\right)_q^{s+1}{}_0 d_q\zeta \\
&= -\frac{1}{q[s+1]}\left[\left|\zeta\left(1-q^{-1}\zeta\right)_q^{s+1}\right|_0^1 - \int_0^1 (1-\zeta)_q^{s+1}.1_0 d_q\zeta\right] \\
&= -\frac{q}{[s+1]}\left[\left(1-q^{-1}\right)_q^{s+1} + \frac{q\left(1-q^{-1}\right)_q^{s+2}}{[s+2]} - \frac{q}{[s+2]}\right] \\
&= -\frac{q}{[s+1]}\left[\left(1-q^{-1}\right)_q^{s+1} + \frac{q\left(1-q^{-1}\right)_q^{s+2}}{[s+2]} - \frac{q}{[s+2]}\right] + \frac{1}{[s+2]} \\
&= M\frac{(w-\wp)^2 + (\upsilon-w)^2}{\upsilon-\wp} \cdot \left[-\frac{q}{[s+1]}\left(\left(1-q^{-1}\right)_q^{s+1} + \frac{q\left(1-q^{-1}\right)_q^{s+2}}{[s+2]} - \frac{q}{[s+2]}\right) + \frac{1}{[s+2]}\right].
\end{aligned} \tag{33}$$

□

**Theorem 16.** *Suppose that $\Phi : J \subset \mathbb{R}^+ \longrightarrow \mathbb{R}$ is a q-differentiable on $J^{\circ}$ and $D_q\Phi \in L[\wp,\upsilon]$, in which $\wp, \upsilon \in J$ for $\wp<\upsilon$. If $|D_q\Phi|^m$ is a s-convex function in second sense on $[\wp,\upsilon]$ for some static $s \in (0,1]$, $m > 1$, $n = m/m-1$ and $D_q\Phi(w)$ is bounded by M, $w \in [\wp,\upsilon]$, then the q-integral inequality is valid:*

$$\left|\frac{1}{q}\left[\Phi(w) - \frac{1}{\upsilon-\wp}\int_{\wp}^{\upsilon}\Phi(\zeta)_{\wp}d_q\zeta\right]\right| \leq \frac{M}{[n+1]^{1/n}}\left[\frac{1 + q\left(1-\left(1-q^{-1}\right)^{s+1}\right)}{[s+1]}\right]^{1/m} \times \left[\frac{(w-\wp)^2 + (\upsilon-w)^2}{\upsilon-\wp}\right], \tag{34}$$

*for each $w \in [\wp,\upsilon]$.*

*Proof.* From Lemma 14 and keeping in view the well-known $q$-analogue of Hölder inequality, we have

$$\begin{aligned}
&\left|\frac{1}{q}\left[\Phi(w) - \frac{1}{\upsilon-\wp}\int_{\wp}^{\upsilon}\Phi(\zeta)_{\wp}d_q\zeta\right]\right| \leq \frac{(w-\wp)^2}{\upsilon-\wp}\int_0^1 \zeta\left|D_q\Phi(\zeta w + (1-\zeta)\wp)\right| \\
&\cdot_0 d_q\zeta + \frac{(\upsilon-w)^2}{\upsilon-\wp}\int_0^1 \zeta\left|D_q\Phi(\zeta w + (1-\zeta)\upsilon)\right|_0 d_q\zeta \\
&\leq \frac{(w-\wp)^2}{\upsilon-\wp}\left(\int_0^1 \zeta_{q0}^n d_q\zeta\right)^{1/n}\left(\int_0^1 \left|D_q\Phi(\zeta w + (1-\zeta)\wp)\right|^m{}_0 d_q\zeta\right)^{1/m} \\
&\quad + \frac{(\upsilon-w)^2}{\upsilon-\wp}\int_0^1\left(\int_0^1 \zeta_{q0}^n d_q\zeta\right)^{1/n}\left(\int_0^1 \left|D_q\Phi(\zeta w + (1-\zeta)\upsilon)\right|^m{}_0 d_q\zeta\right)^{1/m},
\end{aligned}$$

$$\int_0^1 \left|D_q\Phi(\zeta w+(1-\zeta)\wp)\right|^m {}_0d_q\zeta \le \int_0^1 \zeta_q^s \left|D_q\Phi(w)\right|^m {}_0d_q\zeta + \int_0^1 (1-\zeta)_q^s \left|D_q\Phi(\wp)\right|^m {}_0d_q\zeta \le M^m\left(\left|\frac{\zeta_q^{s+1}}{[s+1]}\right|_0^1 - \left|\frac{q\left(1-q^{-1}\zeta\right)_q^{s+1}}{[s+1]}\right|_0^1\right) = M^m\left(\frac{1}{[s+1]} - \frac{q\left(1-q^{-1}\right)_q^{s+1}}{[s+1]} + \frac{q}{[s+1]}\right) = M^m\left(\frac{1+q\left(1-\left(1-q^{-1}\right)_q^{s+1}\right)}{[s+1]}\right),$$

$$\int_0^1 \left|D_q\Phi(\zeta w+(1-\zeta)\upsilon)\right|^m {}_0d_q\zeta \le \int_0^1 \zeta_q^s \left|D_q\Phi(w)\right|^m {}_0d_q\zeta + \int_0^1 (1-\zeta)_q^s \left|D_q\Phi(\upsilon)\right|^m {}_0d_q\zeta \le M^m\left(\left|\frac{\zeta_q^{s+1}}{[s+1]}\right|_0^1 - \left|\frac{q\left(1-q^{-1}\zeta\right)_q^{s+1}}{[s+1]}\right|_0^1\right) = M^m\left(\frac{1}{[s+1]} - \frac{q\left(1-q^{-1}\right)_q^{s+1}}{[s+1]} + \frac{q}{[s+1]}\right) = M^m\left(\frac{1+q\left(1-\left(1-q^{-1}\right)_q^{s+1}\right)}{[s+1]}\right) \le M\left(\frac{1}{[1+n]}\right)^{1/n}\left(\frac{1+q\left(1-\left(1-q^{-1}\right)_q^{s+1}\right)}{[s+1]}\right)^{1/m}\left[\frac{(w-\wp)^2+(\upsilon-w)^2}{\upsilon-\wp}\right]. \tag{35}$$

It completes the proof. □

**Theorem 17.** *Let $\Phi : J \subset \mathbb{R}^+ \longrightarrow \mathbb{R}$ is a q-differentiable mapping on $J^\circ$ such as $D_q\Phi \in L[\wp,\upsilon]$, in which $\wp, \upsilon \in J$ for $\wp<\upsilon$. If the absolute value of $(D_q\Phi(w))^m$ is a s -convex mapping in the second sense on $[\wp,\upsilon]$ for unique $s \in (0,1]$, $m \ge 1$, and $|D_q\Phi(w)| \le M$, $w \in [\wp,\upsilon]$, we have seen that the q-integral inequality is valid:*

$$\left|\frac{1}{q}\left[\Phi(w) - \frac{1}{\upsilon-\wp}\int_\wp^\upsilon \Phi(\zeta)_\wp d_q\zeta\right]\right| \le M\left(\frac{(w-\wp)^2+(\upsilon-w)^2}{\upsilon-\wp}\right) \cdot \left(\frac{1}{[2]}\right)^{1-(1/m)}\left[-\frac{q}{[s+1]}\left(\left(1-q^{-1}\right)_q^{s+1} + \frac{q\left(1-q^{-1}\right)_q^{s+2}}{[s+2]} - \frac{q}{[s+2]}\right)\right]^{1/m}, \tag{36}$$

*for each $w \in [\wp,\upsilon]$.*

*Proof.* Lemma 14 and keeping in view the well-known $q$ -analogue of power-mean inequality, we have

$$\left|\frac{1}{q}\left[\Phi(w) - \frac{1}{\upsilon-\wp}\int_\wp^\upsilon \Phi(\zeta)_\wp d_q\zeta\right]\right| \le \frac{(w-\wp)^2}{\upsilon-\wp}\int_0^1 \zeta\left|D_q\Phi(\zeta w+(1-\zeta)\wp)\right|_0 d_q\zeta + \frac{(\upsilon-w)^2}{\upsilon-\wp}\int_0^1 \zeta\left|D_q\Phi(\zeta w+(1-\zeta)\upsilon)\right|_0 d_q\zeta \le \frac{(w-\wp)^2}{\upsilon-\wp}\left(\int_0^1 \zeta_0 d_q\zeta\right)^{1-(1/m)} \cdot \left(\int_0^1 \zeta\left|D_q\Phi(\zeta w+(1-\zeta)\wp)\right|^m {}_0d_q\zeta\right)^{1/m} + \frac{(\upsilon-w)^2}{\upsilon-\wp}\left(\int_0^1 \zeta_0 d_q\zeta\right)^{1-(1/m)} \cdot \left(\int_0^1 \zeta\left|D_q\Phi(\zeta w+(1-\zeta)\upsilon)\right|^m {}_0d_q\zeta\right)^{1/m},$$

$$\int_0^1 \zeta\left|D_q\Phi(\zeta w+(1-\zeta)\wp)\right|^m {}_0d_q\zeta \le \int_0^1 \zeta_q^{s+1}\left|D_q\Phi(w)\right|^m {}_0d_q\zeta + \int_0^1 \zeta(1-\zeta)_q^s\left|D_q\Phi(\wp)\right|^m {}_0d_q\zeta \le M^m\left(\int_0^1 \zeta_0 d_q\zeta\right)^{1-(1/m)}\left[\int_0^1 \zeta_q^{s+1}{}_0d_q\zeta + \int_0^1 \zeta(1-\zeta)_{q0}^s d_q\zeta\right] = M\frac{(w-\wp)^2+(\upsilon-w)^2}{\upsilon-\wp} \cdot \left(\int_0^1 \zeta_0 d_q\zeta\right)^{1-(1/m)}\left[\int_0^1 \zeta^{s+1}{}_0d_q\zeta + \int_0^1 \zeta(1-\zeta)_{q0}^s d_q\zeta\right],$$

$$\int_0^1 \zeta_q^{s+1}{}_0d_q\zeta = \frac{1}{[s+2]},$$

$$-\frac{q}{[s+1]}\int_0^1 \zeta D_q\left(1-q^{-1}\zeta\right)_q^{s+1}{}_0d_q\zeta = -\frac{q}{[s+1]} \cdot \left[\left|\zeta\left(1-q^{-1}\zeta\right)_q^{s+1}\right|_0^1 - \int_0^1 (1-\zeta)_q^{s+1}.1_0 d_q\zeta\right] = -\frac{q}{[s+1]}\left[\left(1-q^{-1}\right)_q^{s+1} + \frac{q\left(1-q^{-1}\right)_q^{s+2}}{[s+2]} - \frac{q}{[s+2]}\right] = -\frac{q}{[s+1]}\left[\left(1-q^{-1}\right)_q^{s+1} + \frac{q\left(1-q^{-1}\right)_q^{s+2}}{[s+2]} - \frac{q}{[s+2]}\right] = M\left(\frac{(w-\wp)^2+(\upsilon-w)^2}{\upsilon-\wp}\right)\left(\frac{1}{[2]}\right)^{1-(1/m)} \cdot \left[-\frac{q}{[s+1]}\left(\left(1-q^{-1}\right)_q^{s+1} + \frac{q\left(1-q^{-1}\right)_q^{s+2}}{[s+2]} - \frac{q}{[s+2]}\right)\right]^{1/m}. \tag{37}$$

It completes the proof. □

**Theorem 18.** *Suppose that $\Phi : J \subset \mathbb{R}^+ \longrightarrow \mathbb{R}$ is a q-differentiable mapping on $J^\circ$ such that $D_q\Phi \in L[\wp,\upsilon]$, in which $\wp, \upsilon \in J$ for $\wp<\upsilon$. If $|D_q\Phi|^m$ is s-convex function in second sense on $[\wp,\upsilon]$ for some $s \in (0,1]$, $q>1$ and $m>1$ and $n = m/m-1$, then the q-integral inequality is valid:*

$$\left|\frac{1}{q}\left[\Phi(w) - \frac{1}{\upsilon-\wp}\int_\wp^\upsilon \Phi(\zeta)_\wp d_q\zeta\right]\right| \le \frac{2^{(s-1/m)}}{[1+n]^{1/n}(\upsilon-\wp)}\left[(w-\wp)^2\left|D_q\Phi\left(\frac{w+\wp}{2}\right)\right| + (\upsilon-w)^2\left|D_q\Phi\left(\frac{\upsilon+w}{2}\right)\right|\right], \tag{38}$$

*for each $w \in [\wp,\upsilon]$.*

*Proof.* Lemma 3.1 and keeping in view the familiar $q$-analogue of Hölder inequality, we have

$$
\begin{aligned}
&\left|\frac{1}{q}\left[\Phi(w)-\frac{1}{\wp+\upsilon}\int_{\wp}^{\upsilon}\Phi(\zeta)_{\wp}d_q\zeta\right]\right| \\
&\quad\leq \frac{(w-\wp)^2}{\upsilon-\wp}\int_0^1 \zeta\left|D_q\Phi(\zeta w+(1-\zeta)\wp)\right|_0 d_q\zeta \\
&\qquad+ \frac{(\upsilon-w)^2}{\upsilon-\wp}\int_0^1 \zeta\left|D_q\Phi(\zeta w+(1-\zeta)\upsilon)\right|_0 d_q\zeta \\
&\quad\leq \frac{(w-\wp)^2}{\upsilon-\wp}\left(\int_0^1 \zeta^n_{q0} d_q\zeta\right)^{1/n}\left(\int_0^1 \left|D_q\Phi(\zeta w+(1-\zeta)\wp)\right|^m{}_0 d_q\zeta\right)^{1/m} \\
&\qquad+ \frac{(\upsilon-w)^2}{\upsilon-\wp}\int_0^1\left(\int_0^1 \zeta^n_{q0} d_q\zeta\right)^{1/n}\left(\int_0^1 \left|D_q\Phi(\zeta w+(1-\zeta)\upsilon)\right|^m{}_0 d_q\zeta\right)^{1/m}, \\
&\qquad\qquad \int_0^1\left|D_q\Phi(\zeta w+(1-\zeta)\wp)\right|^m d\zeta \leq 2^{s-1}\left|D_q\Phi\left(\frac{w+\wp}{2}\right)\right|^m \\
&\int_0^1\left|D_q\Phi(\zeta w+(1-\zeta)\upsilon)\right|^m d\zeta \leq 2^{s-1}\left|D_q\Phi\left(\frac{\upsilon+w}{2}\right)\right|^m \\
&\quad\leq \frac{2^{(s-1/m)}}{[1+n]^{1/n}(\upsilon-\wp)}\left[(w-\wp)^2\left|D_q\Phi\left(\frac{w+\wp}{2}\right)\right|+(\upsilon-w)^2\left|D_q\Phi\left(\frac{\upsilon+w}{2}\right)\right|\right].
\end{aligned}
\tag{39}
$$

□

*Remark 19.* In Theorem 13, if we choose $q=1$, then (24) diminishes the inequality (1) of Theorem 1.

*Remark 20.* In Theorem 13, if we choose $s=1$, then (24) diminishes the inequality (7) of Theorem 7.

*Remark 21.* In Theorem 15, if we fixed $q=1$, then (31) reduces the inequality (3) of Theorem 3.

*Remark 22.* In Theorem 15, if we take $s=1$, then (31) diminishes the inequality (9) of Theorem 9.

*Remark 23.* In Theorem 16, if we take $q=1$, then (34) reduces the inequality (4) of Theorem 4.

*Remark 24.* In Theorem 16, if we choose $s=1$, then (34) diminishes the inequality (10) of Theorem 10.

*Remark 25.* In Theorem 17, if we take $q=1$, then (36) diminishes the inequality (5) of Theorem 5.

*Remark 26.* In Theorem 18, if we take $q=1$, then (38) diminishes the inequality (6) of Theorem 6.

## 4. Conclusion

By the virtue of $q$-calculus, some integral inequalities are proved, which provides a method to study more properties of $q$-integrals via other classes of integral inequalities. $q$-Hermite-Hadmard and $q$-Ostrowski type integral inequalities have provided new estimates and good approximations in comparison with existing Hermite-Hadamard and Ostrowski inequalities. In similar fashion, the same methods can be applied to other inequalities, including Simpson's and trapezoidal inequalities for different classes of $s$-convex functions.

## Data Availability

Not applicable.

## Conflicts of Interest

The authors declare that there is no conflict of interest.

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