Hindawi
Journal of Function Spaces
Volume 2022, Article ID 8165019, 8 pages
https://doi.org/10.1155/2022/8165019

Research Article

Novel Evaluation of Fuzzy Fractional Helmholtz Equations

Meshari Alesemi,[1] Naveed Iqbal ,[2] and Noorolhuda Wyal [3]

[1]*Department of Mathematics, College of Science, University of Bisha, Bisha, Saudi Arabia*
[2]*Department of Mathematics, College of Science, University of Ha'il, Ha'il 2440, Saudi Arabia*
[3]*Department of Mathematics, Polytechnical University of Kabul, Kabul, Afghanistan*

Correspondence should be addressed to Naveed Iqbal; n.iqbal@uoh.edu.sa and Noorolhuda Wyal; noorolhuda.wyal@kpu.edu.af

Received 17 March 2022; Accepted 11 April 2022; Published 29 April 2022

Academic Editor: Hanan Alolaiyan

The current article discusses the fuzzy new iterative transform approach, which is a combination of a fuzzy hybrid methodology and an iterative transformation technique. We establish the consistence of our strategy by obtaining fractional fuzzy Helmholtz equations with the initial fuzzy condition using the Caputo derivative under generalized Hukuhara differentiability. The series obtained result was calculated and compared to the proposed equations of the actual result. Three challenges were provided to validate our method, and the outcomes were approximated in fuzzy form. In each of the three examples, the upper and bottom halves of the fuzzy solution were approximated utilizing two various fractional order between 0 and 1. Due to the fact that it globalizes the dynamical behavior of the specified equation, it produces all forms of fuzzy results at any fractional order between 0 and 1. Due to the fact that the fuzzy numbers presents the result in a lower and upper branches fuzzy type, the unknown quantity incorporates fuzziness as well. It is critical to emphasize that the purpose of the proposed fuzziness approach is to demonstrate the efficiency and superiority of numerical solutions to nonlinear fractional fuzzy partial differential equations that arise in complex and physical structures.

1. Introduction

Modeling uncertain issues with the theory of fuzzy sets are a great technique. As a result, fuzzy concepts have been used to model a broad variety of natural events. The fuzzy fractional differential equation, in particular, is frequent models in several scientific domains, such as assessing weapon systems, population models, electrohydraulics, and civil engineering models. As a solution, the fuzzy calculus idea of the fractional derivative is critical [1–4]. As a solution, fractional fuzzy in the domains of mathematics and engineering, differential equations have gotten a lot of attention. The very first is the study by Agarwal et al. [5] devoted to fuzzy fractional differential equations. They established the Riemann-Liouville differentiability concept under the Hukuhara differentiability to analysis fractional fuzzy differential equations [6–8].

Recently, fractional calculus has been offered as a valuable topic to produce precise outcomes of engineering and mathematics issues like signal processing, aerodynamics and control [9–12] systems, biomathematical difficulties, and others. Furthermore, several writers have researched fractional differential equations in fuzzy situations [5], solved using various approaches [13–16]. Hoa investigated fractional fuzzy differential equation under Caputo gH-differentiability in [17]. At the same time, Agarwal et al. conducted a study on the same subject in [18] to highlight its connection to optimal control issues. Long et al. [19] demonstrated the solvability of fractional fuzzy differential problems, while Salahshour et al. [20] solved the issue using Laplace fuzzy transformation.

The Helmholtz equation (HE) is an elliptic partial differential equation (PDE) developed from a wave model. It is also known as the reduced wave equation. The HE is a partial differential equation representing mechanical growth that is time-independent throughout the cosmos. The HE is fundamental in applied physics and mathematics [21–24]. The HE findings, which are commonly obtained by separating variables, deal with crucial scientific phenomena [25]. Equations include magnetic fields in plates, fluid, wall, vibrating lines, nuclear power reactors, electromagnetic field, geology's Lamb equation, and acoustics. Consider a nonhomogeneous isotropic material in three dimensions with a velocity of c in Euclidean space [26, 27]. The resulting wave is $\mu(\wp,\zeta)$ in phase with the harmonic origin. The HE for the given field R is satisfied when $\phi(\wp,\zeta)$ vibrates at the recognized fixed frequency $\omega > 0$:

$$\frac{\partial^2 v(\wp,\varphi)}{\partial \wp^2} + \frac{\partial^2 v(\wp,\varphi)}{\partial \varphi^2} + \chi v(\wp,\varphi) = -g(\wp,\varphi), \tag{1}$$

where $v(\wp,\varphi)$ is an appropriately differentiable function at the boundary of R, $g(\wp,\varphi)$ is a specified function, $\chi > 0$ is a constant value, and $\sqrt{\chi} = \omega/c$ is a wave number with a wave length of $2\pi/\sqrt{\chi}$. If $\phi(\wp,\varphi) = 0$ is necessary, then equation (1) the HE is homogeneous. When the positive sign (in front of the χ term) is improved to a negative sign, most steady-state oscillations (thermoelectric, acoustic, hydraulic, electromagnetic) lead to a two-dimensional HE describing mass transport systems using chemical mixtures of first-order volume. In linear acoustics, for example, $\phi(\wp,\varphi)$ might represent a disruption of the reference state pressure (Thompson and Pinsky, 1995). Conservation equations, also known as HEs, are used to solve a wide range of physical issues, including fluid restricted or shear viscosity streams inside thermophysical barriers [26–29], such as Laplace variational iteration method [30], homotopy perturbation method [31], q-homotopy analysis transform method [32], reduced differential transform method [33], He's variational iteration method [34], and a spectral method [35] have all been used to solve fractional-order HEs in recent years [36, 37].

2. Basic Definition

Definition 1. Consider a continuous function fuzzy $\tilde{v}$ on $[0, \beta] \in$ R, we express fractional fuzzy Riemann-Liouvilli integral in the presence of $\mathfrak{I}$ as

$$\mathbf{I}^{\varrho}\tilde{v} = \int_0^{\mathfrak{I}} \frac{(\mathfrak{I} - \eta)^{\varrho-1}\tilde{v}(\eta)}{\Gamma(\varrho)} d\eta, \varrho, \eta \in (0, \infty). \tag{2}$$

Moreover, if $\tilde{v} \in C^F[0, \beta] \cap L^F[0, \beta]$, where $C^F[0, \beta]$ is the universe of fuzzy continuous function, and $L^F[0, \beta]$ is the space of continuous fuzzy functions. If the functions are Lebesgue integrable, then the fuzzy fractional integral is given as

$$[\mathbf{I}^{\varrho}\tilde{v}(\mathfrak{I})]_{\gamma} = \left[\mathbf{I}^{\varrho}\underline{v}_{\gamma}, \mathbf{I}^{\varrho}\bar{v}_{\gamma}\right], 0 \le \gamma \le 1, \tag{3}$$

such that

$$\begin{aligned} \mathbf{I}^{\varrho}\underline{v}_{\gamma} &= \int_0^{\mathfrak{I}} \frac{(\mathfrak{I} - n)^{\varrho-1}\underline{v}_{\gamma}(\eta)}{\Gamma(\varrho)} \eta, \varrho, \eta \in (0, \infty), \\ \mathbf{I}^{\varrho}\bar{v}_{\gamma} &= \int_0^{\mathfrak{I}} \frac{(\mathfrak{I} - n)^{\varrho-1}\bar{v}_{\gamma}(\eta)}{\Gamma(\varrho)} \eta, \varrho, \eta \in (0, \infty). \end{aligned} \tag{4}$$

Definition 2. For a function $\tilde{v} \in \mathrm{C^F}[0, \beta] \cap \mathrm{L_F}[0, \beta]$, such that $\tilde{v} = [\mathrm{w}_{\gamma}(\mathfrak{I}), \bar{v}_{\gamma}(\mathfrak{I})], \gamma \in [0, 1]$ and $\mathfrak{I}_0 \in (0, \beta)$, then the fractional Caputo fuzzy derivative is given as

$$\left[D_{\varrho}\tilde{v}(\mathfrak{I}_0)\right]_{\gamma} = \left[D_{\varrho}\underline{v}(\mathfrak{I}_0), D_{\varrho}\bar{v}(\mathfrak{I}_0)\right], 0 < \varrho \le 1, \tag{5}$$

where

$$\begin{aligned} D^{\varrho}\underline{v}_{\gamma}(\mathfrak{I}_0) &= \left[\int_0^{\mathfrak{I}} \frac{(\mathfrak{I} - n)^{m-\varrho-1}(d^m/d\eta^m)\underline{v}_{\gamma}(\eta)}{\Gamma(\varrho)} \eta\right]_{\mathfrak{I}=\mathfrak{I}_0}, \\ D^{\varrho}\bar{v}_{\gamma}(\mathfrak{I}_0) &= \left[\int_0^{\mathfrak{I}} \frac{(\mathfrak{I} - n)^{m-\varrho-1}(d^m/d\eta^m)\bar{v}_{\gamma}(\eta)}{\Gamma(\varrho)} \eta\right]_{\mathfrak{I}=\mathfrak{I}_0}, \end{aligned} \tag{6}$$

in such a way that the integrating on the right sides convergence and $m = \lceil \varrho \rceil$. Since $\varrho \in (0, 1] m = 1$.

Definition 3. The Laplace fuzzy transformation for $\mathrm{f}(\wp)$, where $\mathrm{f}(\wp)$ is value fuzzy term, is define as

$$G(\wp) = L[f(\wp)] = \int_0^{\infty} (\exp)^{-\wp\mathfrak{I}} f(\mathfrak{I}) d\mathfrak{I}, \mathfrak{I} > 0. \tag{7}$$

Definition 4. In terms of fuzzy convolution, a fuzzy Laplace transformation is described as

$$L[f_1 * f_2] = L[f_1] * L[f_2], \tag{8}$$

where $f_1 * f_2$ define the fuzzy convolution between f_1 and f_2, i.e.,

$$f_1 * f_2 = \int_0^{\wp} f_1(\mathfrak{I}) * f_2(\wp - \mathfrak{I}) d\mathfrak{I}. \tag{9}$$

Definition 5. The "Mittag-Leffler function" $E\rho(p)$ is expressed as

$$E_{\rho}(\mathfrak{I}) = \sum_{n=0}^{\infty} \frac{\mathfrak{I}^n}{\Gamma(n\rho + 1)}, \tag{10}$$

where $\rho > 0$.

Definition 6. Let $\kappa : \mathfrak{R} \longrightarrow [0, 1]$ be a number of fuzzy which have the specified properties

κ is an upper semicontinuous number

$\kappa\{\mu(\chi_1) + \mu(\chi_2)\} \ge \min\{\kappa(\chi_1), \kappa(\chi_2)\}$

$\exists \chi_0 \in R$ such that $\kappa(\chi_0) = 1$, i.e., ν is normal

$cl\{\chi \in \mathfrak{R}, \kappa(\chi) > 0\}$ is compact

The set of all fuzzy numbers is represented by the notation E.

Definition 7. The aforementioned number can be written in parametric form as $[\underline{\kappa}(\gamma), \bar{\kappa}(\gamma)]$, so that $\gamma \in [0,1]$ combined with the values

$\underline{\kappa}(\gamma)$ from left is continuous and bounded function increasing over $[0,1]$

$\underline{\kappa}(\gamma)$ from right is continuous and bounded function decreasing over $[0,1]$

(iii) $\underline{\kappa} \leq \bar{\kappa}$.

3. Main Work with Applications

$$D^{\varrho}_{\mathfrak{I}}\tilde{\upsilon}(\wp,\mathfrak{I}) = D^{2}_{\wp}\tilde{\upsilon}(\wp,\mathfrak{I}) + \tilde{\upsilon}(\wp,\mathfrak{I}) + \tilde{\kappa}(\gamma), 0 < \varrho \leq 1, \quad (11)$$

with the initial fuzzy condition

$$\tilde{\upsilon}(\wp,0) = \tilde{g}(\wp). \quad (12)$$

In this case, we use the Laplace transform on (12) as

$$\mathscr{L}\left[D^{\rho}_{\wp}\tilde{\upsilon}(\wp,\mathfrak{I})\right] = \mathscr{L}\left[D^{2}_{\wp}\tilde{\upsilon}(\wp,\mathfrak{I}) + \tilde{\upsilon}(\wp,I) + \tilde{\kappa}\right], \quad (13)$$

with initial condition using, and we get

$$\begin{aligned} s^{\varrho}\mathscr{L}[\tilde{\upsilon}(\wp,\mathfrak{I})] &= s^{\varrho-1}\tilde{g}(\wp) + \mathscr{L}\left[D^{2}_{\wp}\tilde{\upsilon}(\wp,\mathfrak{I}) + \tilde{\upsilon}(\wp,\mathfrak{I}) + \tilde{\kappa}\right], \\ \mathscr{L}[\tilde{\upsilon}(\wp,\mathfrak{I})] &= \frac{\tilde{g}(\wp)}{s} + \frac{1}{s^{\varrho}}\mathscr{L}\left[D^{2}_{\wp}\tilde{\upsilon}(\wp,\mathfrak{I}) + \tilde{\upsilon}(\wp,\mathfrak{I}) + \tilde{\kappa}\right]. \end{aligned} \quad (14)$$

Suppose that the result as $\tilde{\upsilon}(\wp,\mathfrak{I}) = \sum_{n=0}^{\infty} U_n(\wp,\mathfrak{I})$, then (15) defines

$$\begin{aligned} \mathscr{L}\left[\sum_{n=0}^{\infty}\tilde{\upsilon}_n(\wp,\mathfrak{I})\right] &= \frac{\tilde{g}(\wp)}{s} + \frac{1}{s^{\rho}}\mathscr{L}\left[D^{2}_{\wp}\sum_{n=0}^{\infty}\tilde{\upsilon}_n(\wp,\mathfrak{I})\right. \\ &\quad \left. + \sum_{n=0}^{\infty}\tilde{\upsilon}_n(\wp,\mathfrak{I}) + \tilde{\kappa}\right]. \end{aligned} \quad (15)$$

On both sides terms comparisons, we get

$$\begin{aligned} \mathscr{L}[\tilde{\upsilon}_0(\wp,\mathfrak{I})] &= \frac{\tilde{g}(\wp)}{s} + \frac{1}{s^{\rho}}\mathscr{L}[\tilde{\kappa}], \\ \mathscr{L}[\tilde{\upsilon}_1(\wp,\mathfrak{I})] &= \frac{1}{s^{\varrho}}\mathscr{L}\left[D^{2}_{\wp}\tilde{\upsilon}_0(\wp,\mathfrak{I}) + \tilde{\upsilon}_0(\wp,\mathfrak{I})\right], \\ \mathscr{L}[\tilde{\upsilon}_2(\wp,\mathfrak{I})] &= \frac{1}{s^{\varrho}}\mathscr{L}\left[D^{2}_{\wp}\tilde{\upsilon}_1(\wp,\mathfrak{I}) + \tilde{\upsilon}_1(\wp,\mathfrak{I})\right], \\ &\vdots \\ \mathscr{L}[\tilde{\upsilon}_{n+1}(\wp,\mathfrak{I})] &= \frac{1}{s^{\varrho}}\mathscr{L}\left[D^{2}_{\wp}\tilde{\upsilon}_n(\wp,\mathfrak{I}) + \tilde{\upsilon}_n(\wp,\mathfrak{I})\right], n \geq 0. \end{aligned} \quad (16)$$

Using inverse Laplace transformation, we get

$$\begin{aligned} \tilde{\upsilon}_0(\wp,\mathfrak{I}) &= \tilde{g}(\wp) + \mathscr{L}^{-1}\left[\frac{1}{s^{\rho}}\mathscr{L}[\tilde{\kappa}]\right], \\ \tilde{\upsilon}_1(\wp,\mathfrak{I}) &= \mathscr{L}^{-1}\left[\frac{1}{s^{\varrho}}\mathscr{L}\left[D^{2}_{\wp}\tilde{\upsilon}_0(\wp,\mathfrak{I}) + \tilde{\upsilon}_0(\wp,\mathfrak{I})\right]\right], \\ &\vdots \\ \tilde{\upsilon}_{n+1}(\wp,\mathfrak{I}) &= \mathscr{L}^{-1}\left[\frac{1}{s^{\varrho}}\mathscr{L}\left[D^{2}_{\wp}\tilde{\upsilon}_n(\wp,\mathfrak{I}) + \tilde{\upsilon}_n(\wp,\mathfrak{I})\right]\right], n \geq 0. \end{aligned} \quad (17)$$

As a consequence, the needed series result is obtained by

$$\tilde{\upsilon}(\wp,\mathfrak{I}) = \tilde{\upsilon}_0(\wp,\mathfrak{I}) + \tilde{\upsilon}_1(\wp,\mathfrak{I}) + \tilde{\upsilon}_2(\wp,\mathfrak{I}) + \cdots \quad (18)$$

3.1. Examples

Example 1. Consider fuzzy fractional homogeneous Helmholtz equation with x-space with the fuzzy initial condition

$$\begin{aligned} D^{\varrho}_{\wp}\tilde{\upsilon}(\wp,\mathfrak{I}) + D^{2}_{\mathfrak{I}}\tilde{\upsilon}(\wp,\mathfrak{I}) - \tilde{\upsilon}(\wp,\mathfrak{I}) = 0, 0 < \varrho \leq 1, \mathfrak{I} > 0, \\ \tilde{\upsilon}(0,\mathfrak{I}) = \tilde{\kappa}(\gamma)\mathfrak{I}, 0 < \mathfrak{I} < 1, \end{aligned} \quad (19)$$

where $\tilde{\kappa}(\gamma) = [\underline{\kappa}(\gamma), \bar{\kappa}(\gamma)] = [\gamma - 1, 1 - \gamma], 0 \leq \gamma \leq 1$. Using the abovementioned procedure as described in (18), we obtain the following results.

$$\begin{aligned} &\underline{\upsilon}_0(\wp,\mathfrak{I}) = \underline{\kappa}(\gamma)\mathfrak{I}, \bar{\upsilon}_0(\wp,\mathfrak{I}) = \bar{\kappa}(\gamma)\mathfrak{I}, \\ &\underline{\upsilon}_1(\wp,\mathfrak{I}) = \underline{\kappa}(\gamma)\mathfrak{I}\frac{\wp^{\varrho}}{\Gamma(\varrho+1)}, \bar{\upsilon}_1(\wp,\mathfrak{I}) = \bar{\kappa}(\gamma)\mathfrak{I}\frac{\wp^{\varrho}}{\Gamma(\varrho+1)}, \\ &\underline{\upsilon}_2(\wp,\mathfrak{I}) = \underline{\kappa}(\gamma)\mathfrak{I}\frac{\wp^{2\varrho}}{\Gamma(2\varrho+1)}, \bar{\upsilon}_2(\wp,\mathfrak{I}) = \bar{\kappa}(\gamma)\mathfrak{I}\frac{\wp^{2\varrho}}{\Gamma(2\varrho+1)}, \\ &\underline{\upsilon}_3(\wp,\mathfrak{I}) = \underline{\kappa}(\gamma)\mathfrak{I}\frac{\wp^{3\varrho}}{\Gamma(3\varrho+1)}, \bar{\upsilon}_3(\wp,\mathfrak{I}) = \bar{\kappa}(\gamma)\mathfrak{I}\frac{\wp^{3\varrho}}{\Gamma(3\varrho+1)}, \end{aligned} \quad (20)$$

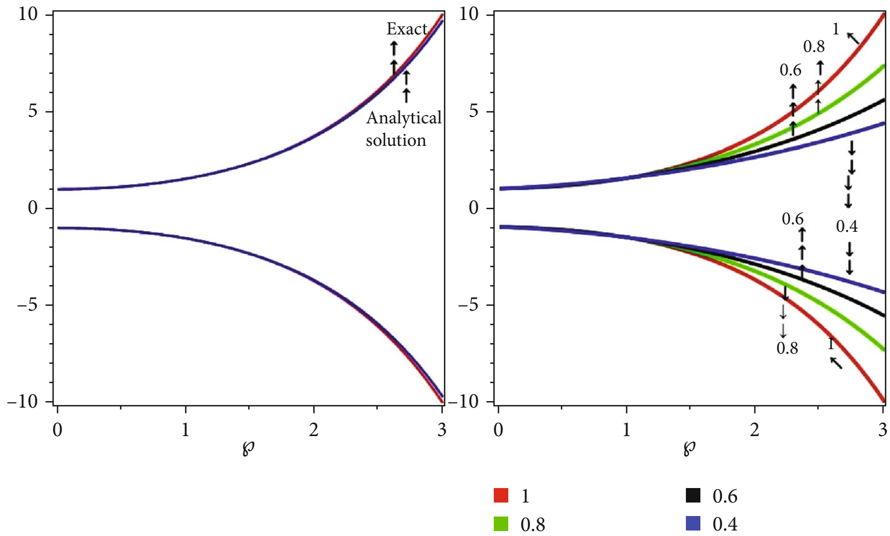

Figure 1: The first graph depicts a two-dimensional fuzzy upper and bottom branch graph of an analytic series result, while the second depicts various fractional of ρ.

and so forth, and more terms can be determined in this manner. As a result of (19), we can write the needed series result as an infinite series.

$$\tilde{\upsilon}(\wp,\mathfrak{I}) = \tilde{\upsilon}_0(\wp,\mathfrak{I}) + \tilde{\upsilon}_1(\wp,\mathfrak{I}) + \tilde{\upsilon}_2(\wp,\mathfrak{I}) + \cdots, \tag{21}$$

such that

$$\begin{aligned} \underline{\upsilon}(\wp,\mathfrak{I}) &= \underline{\upsilon}_0(\wp,\mathfrak{I})+\underline{\upsilon}_1(\wp,\mathfrak{I})+\underline{\upsilon}_2(\wp,\mathfrak{I}) + \cdots, \\ \bar{\upsilon}(\wp,\mathfrak{I}) &= \bar{\upsilon}_0(\wp,\mathfrak{I})+\bar{\upsilon}_1(\wp,\mathfrak{I})+\bar{\upsilon}_2(\wp,\mathfrak{I}) + \cdots, \end{aligned} \tag{22}$$

In general, we can write as follows:

$$\begin{aligned} \underline{\upsilon}(\wp,\mathfrak{I}) &= \underline{\kappa}(\gamma)\mathfrak{I} + \underline{\kappa}(\gamma)\mathfrak{I}\frac{\wp^{\varrho}}{\Gamma(\varrho+1)} + \underline{\kappa}(\gamma)\mathfrak{I}\frac{\wp^{2\varrho}}{\Gamma(2\varrho+1)} \\ &\quad + \underline{\kappa}(\gamma)\mathfrak{I}\frac{\wp^{3\varrho}}{\Gamma(3\varrho+1)}+\cdots, \\ \bar{\upsilon}(\wp,\mathfrak{I}) &= \bar{\kappa}(\gamma)\mathfrak{I} + \bar{\kappa}(\gamma)\mathfrak{I}\frac{\wp^{\varrho}}{\Gamma(\varrho+1)} + \bar{\kappa}(\gamma)\mathfrak{I}\frac{\wp^{2\varrho}}{\Gamma(2\varrho+1)} \\ &\quad + \bar{\kappa}(\gamma)\mathfrak{I}\frac{\wp^{3\varrho}}{\Gamma(3\varrho+1)}+\cdots. \end{aligned} \tag{23}$$

The exact result is

$$\tilde{\upsilon}(\wp,\mathfrak{I}) = \tilde{\kappa}(\gamma)\mathfrak{I}\cosh\wp. \tag{24}$$

Figure 1 shows the fuzzy result comparison for lower and upper branches of Example 1 by Laplace decomposition method. The red color shows that the exact solution of lower and upper branch of fuzzy solution at integer order and second cure shows the analytical solution of example 1. In Figure 1, second graph shows two dimensional figure of fuzzy result at four other various fractional order of ρ of Example 1. The two similar color legends represent lower and upper portions of fuzzy solution, respectively.

Example 2. Consider fuzzy fractional homogeneous Helmholtz equation with x-space with the fuzzy initial condition

$$\begin{gathered} D_{\wp}^{\varrho}\tilde{\upsilon}(\wp,\mathfrak{I}) + D_{\mathfrak{I}}^{2}\tilde{\upsilon}(\wp,y) + 5\tilde{\upsilon}(\wp,\mathfrak{I}) = 0, 0 < \varrho \le 1, \mathfrak{I} > 0, \\ \tilde{\upsilon}(0,\mathfrak{I}) = \tilde{\kappa}(\gamma)\mathfrak{I}, 0 < \mathfrak{I} < 1, \end{gathered} \tag{25}$$

where $\tilde{\kappa}(\gamma) = [\underline{\kappa}(\gamma), \bar{\kappa}(\gamma)] = [\gamma-1, 1-\gamma], 0 \le \gamma \le 1$. Using the abovementioned procedure as described in (18), we obtain the following results:

$$\begin{aligned} \underline{\upsilon}_0(\wp,\mathfrak{I}) &= \underline{\kappa}(\gamma)\mathfrak{I}, \bar{\upsilon}_0(\wp,\mathfrak{I}) = \bar{\kappa}(\gamma)\mathfrak{I}, \\ \underline{\upsilon}_1(\wp,\mathfrak{I}) &= -5\underline{\kappa}(\gamma)\mathfrak{I}\frac{\wp^{\varrho}}{\Gamma(\varrho+1)}, \bar{\upsilon}_1(\wp,\mathfrak{I}) \\ &= -5\bar{\kappa}(\gamma)\mathfrak{I}\frac{\wp^{\varrho}}{\Gamma(\varrho+1)}, \\ \underline{\upsilon}_2(\wp,\mathfrak{I}) &= 25\underline{\kappa}(\gamma)\mathfrak{I}\frac{\wp^{2\varrho}}{\Gamma(2\varrho+1)}, \bar{\upsilon}_2(\wp,\mathfrak{I}) \\ &= 25\bar{\kappa}(\gamma)\mathfrak{I}\frac{\wp^{2\varrho}}{\Gamma(2\varrho+1)}, \\ \underline{\upsilon}_3(\wp,\mathfrak{I}) &= -125\underline{\kappa}(\gamma)\mathfrak{I}\frac{\wp^{3\varrho}}{\Gamma(3\varrho+1)}, \bar{\upsilon}_3(\wp,\mathfrak{I}) \\ &= -125\bar{\kappa}(\gamma)\mathfrak{I}\frac{\wp^{3\varrho}}{\Gamma(3\varrho+1)}, \end{aligned} \tag{26}$$

and so forth, and more terms can be determined in this manner. As a result of (19), we can write the needed series

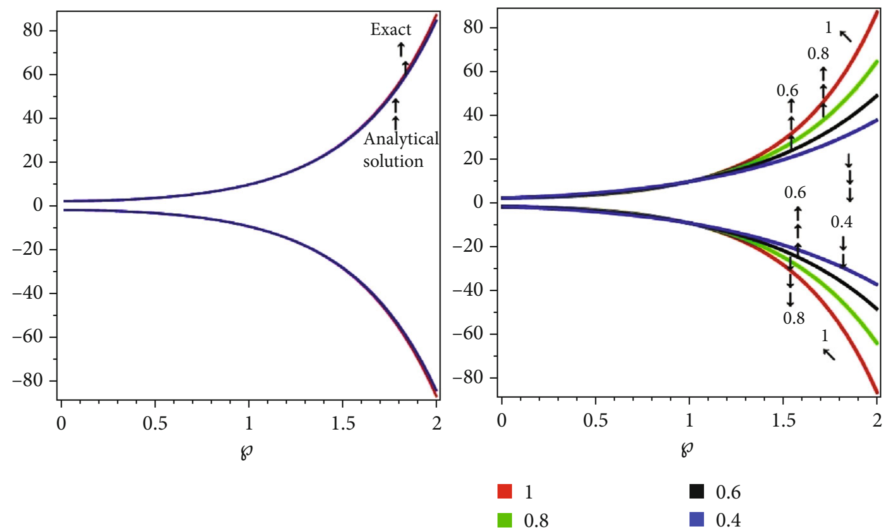

FIGURE 2: The first graph depicts a two-dimensional fuzzy upper and bottom branch graph of an analytic series result, while the second depicts various fractional of ρ.

result as an infinite series.

$$\tilde{\upsilon}(\wp,\mathfrak{I}) = \tilde{\upsilon}_0(\wp,\mathfrak{I}) + \tilde{\upsilon}_1(\wp,\mathfrak{I}) + \tilde{\upsilon}_2(\wp,\mathfrak{I}) + \cdots, \quad (27)$$

such that

$$\begin{aligned} \underline{\upsilon}(\wp,\mathfrak{I}) &= \underline{\upsilon}_0(\wp,\mathfrak{I}) + \underline{\upsilon}_1(\wp,\mathfrak{I}) + \underline{\upsilon}_2(\wp,\mathfrak{I}) + \cdots, \\ \bar{\upsilon}(\wp,\mathfrak{I}) &= \bar{\upsilon}_0(\wp,\mathfrak{I}) + \bar{\upsilon}_1(\wp,\mathfrak{I}) + \bar{\upsilon}_2(\wp,\mathfrak{I}) + \cdots \end{aligned} \quad (28)$$

In general, we can write as follows:

$$\begin{aligned} \underline{\upsilon}(\wp,\mathfrak{I}) &= \underline{\kappa}(\gamma)\mathfrak{I} + 5\underline{\kappa}(\gamma)\mathfrak{I}\frac{\wp^{\varrho}}{\Gamma(\varrho+1)} - 25\underline{\kappa}(\gamma)\mathfrak{I}\frac{\wp^{2\varrho}}{\Gamma(2\varrho+1)} \\ &\quad - 125\underline{\kappa}(\gamma)\mathfrak{I}\frac{\wp^{3\varrho}}{\Gamma(3\varrho+1)} + \cdots, \\ \bar{\upsilon}(\wp,\mathfrak{I}) &= \bar{\kappa}(\gamma)\mathfrak{I} + 5\bar{\kappa}(\gamma)\mathfrak{I}\frac{\wp^{\varrho}}{\Gamma(\varrho+1)} - 25\bar{\kappa}(\gamma)\mathfrak{I}\frac{\wp^{2\varrho}}{\Gamma(2\varrho+1)} \\ &\quad - 125\bar{\kappa}(\gamma)\mathfrak{I}\frac{\wp^{3\varrho}}{\Gamma(3\varrho+1)} + \cdots. \end{aligned} \quad (29)$$

The exact result is

$$\tilde{\upsilon}(\wp,\mathfrak{I}) = \tilde{\kappa}(\gamma)\mathfrak{I}\cos\sqrt{5}\wp. \quad (30)$$

Figure 2 shows the fuzzy result comparison for lower and upper branches of Example 1 by Laplace decomposition method. The red color shows that the exact solution of lower and upper branch of fuzzy solution at integer order and second cure shows the analytical solution of Example 2. In Figure 2, second graph shows two dimensional figure of fuzzy result at four other various fractional order of ρ of Example 2. The two similar color legends represent lower and upper portions of fuzzy solution, respectively.

Example 3. Consider fuzzy fractional homogeneous Helmholtz equation with x-space with the fuzzy initial condition

$$\begin{aligned} &D_{\wp}^{\varrho}\tilde{\upsilon}(\wp,\mathfrak{I}) + D_{\mathfrak{I}}^{2}\tilde{\upsilon}(\wp,\mathfrak{I}) - 2\tilde{\upsilon}(\wp,\mathfrak{I}) \\ &\quad - \left(12\wp^2 - 3\wp^4\right)\sin\mathfrak{I} = 0, 1 < \varrho \le 2, \mathfrak{I} > 0, \\ &\quad \tilde{\upsilon}(0,\mathfrak{I}) = \tilde{\kappa}(\gamma), \text{and } \tilde{\upsilon}_{\wp}(0,\mathfrak{I}) = \tilde{\kappa}(\gamma)0, \end{aligned} \quad (31)$$

where $\tilde{\kappa}(\gamma) = [\underline{\kappa}(\gamma), \bar{\kappa}(\gamma)] = [\gamma - 1, 1 - \gamma], 0 \le \gamma \le 1$. Using the abovementioned procedure as described in (18), we obtain the following results:

$$\begin{aligned} \underline{\upsilon}_0(\wp,\mathfrak{I}) &= \underline{\kappa}(\gamma)\left(\wp^4 - \frac{\wp^6}{10}\right)\sin\mathfrak{I}, \bar{\upsilon}_0(\wp,\mathfrak{I}) \\ &= \bar{\kappa}(\gamma)\left(\wp^4 - \frac{\wp^6}{10}\right)\sin\mathfrak{I}, \\ \underline{\upsilon}_1(\wp,\mathfrak{I}) &= \underline{\kappa}(\gamma)\left(\frac{72\wp^{\varrho+4}}{\Gamma(\varrho+5)} - \frac{216\wp^{\varrho+6}}{\Gamma(\varrho+7)}\right)\sin\mathfrak{I}, \bar{\upsilon}_1(\wp,\mathfrak{I}) \\ &= \bar{\kappa}(\gamma)\left(\frac{72\wp^{\varrho+4}}{\Gamma(\varrho+5)} - \frac{216\wp^{\varrho+6}}{\Gamma(\varrho+7)}\right)\sin\mathfrak{I}, \\ \underline{\upsilon}_2(\wp,\gamma) &= \underline{\kappa}(\gamma)\left(\frac{216\wp^{2\varrho+4}}{\Gamma(2\varrho+5)} - \frac{648\wp^{2\varrho+6}}{\Gamma(2\varrho+7)}\right)\sin\mathfrak{I}, \bar{\upsilon}_2(\wp,\mathfrak{I}) \\ &= \bar{\kappa}(\gamma)\left(\frac{216\wp^{2\varrho+4}}{\Gamma(2\varrho+5)} - \frac{648\wp^{2\varrho+6}}{\Gamma(2\varrho+7)}\right)\sin\mathfrak{I}, \\ \underline{\upsilon}_3(\wp,\mathfrak{I}) &= \underline{\kappa}(\gamma)\left(\frac{648\wp^{3\varrho+4}}{\Gamma(3\varrho+5)} - \frac{1944\wp^{3\varrho+6}}{\Gamma(3\varrho+7)}\right)\sin\mathfrak{I}, \bar{\upsilon}_3(\wp,\mathfrak{I}) \\ &= \bar{\kappa}(\gamma)\left(\frac{648\wp^{3\varrho+4}}{\Gamma(3\varrho+5)} - \frac{1944\wp^{3\varrho+6}}{\Gamma(3\varrho+7)}\right)\sin\mathfrak{I}, \end{aligned} \quad (32)$$

and so forth, and more terms can be determined in this manner. As a result of (19), we can write the needed series

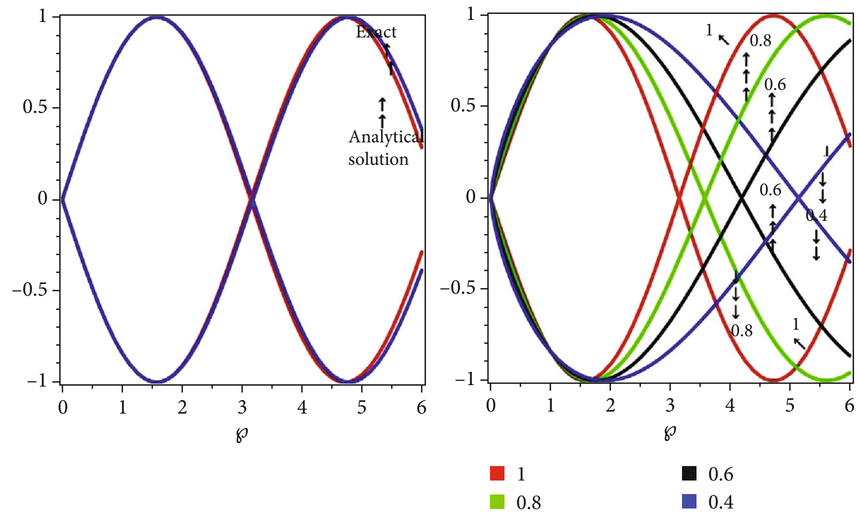

FIGURE 3: The first graph depicts a two-dimensional fuzzy upper and bottom branch graph of an analytic series result, while the second depicts various fractional of ρ.

result as an infinite series.

$$\tilde{\upsilon}(\wp,\mathfrak{I}) = \tilde{\upsilon}_0(\wp,\mathfrak{I}) + \tilde{\upsilon}_1(\wp,\mathfrak{I}) + \tilde{\upsilon}_2(\wp,\mathfrak{I}) + \cdots, \quad (33)$$

such that

$$\begin{aligned}\underline{\upsilon}(\wp,\mathfrak{I}) &= \underline{\upsilon}_0(\wp,\mathfrak{I})+\underline{\upsilon}_1(\wp,\mathfrak{I})+\underline{\upsilon}_2(\wp,\mathfrak{I})+\underline{\upsilon}_3(\wp,\mathfrak{I}) + \cdots,\\ \bar{\upsilon}(\wp,\mathfrak{I}) &= \bar{\upsilon}_0(\wp,\mathfrak{I})+\bar{\upsilon}_1(\wp,\mathfrak{I})+\bar{\upsilon}_2(\wp,\mathfrak{I})+\bar{\upsilon}_3(\wp,\mathfrak{I}) + \cdots.\end{aligned} \quad (34)$$

In general, we can write as follows:

$$\begin{aligned}\underline{\upsilon}(\wp,\mathfrak{I}) &= \underline{\kappa}(\gamma)\left(\wp^4 - \frac{\wp^6}{10}\right)\sin\mathfrak{I} + \underline{\kappa}(\gamma)\left(\frac{72\wp^{\varrho+4}}{\Gamma(\varrho+5)} - \frac{216\wp^{\varrho+6}}{\Gamma(\varrho+7)}\right)\sin\mathfrak{I} + \underline{\kappa}(\gamma)\left(\frac{216\wp^{2\varrho+4}}{\Gamma(2\varrho+5)} - \frac{648\wp^{2\varrho+6}}{\Gamma(2\varrho+7)}\right)\sin\mathfrak{I} \\ &\quad + \underline{\kappa}(\gamma)\left(\frac{648\wp^{3\varrho+4}}{\Gamma(3\varrho+5)} - \frac{1944\wp^{3\varrho+6}}{\Gamma(3\varrho+7)}\right)\sin\mathfrak{I}+\cdots,\\ \bar{\upsilon}(\wp,\mathfrak{I}) &= \bar{\kappa}(\gamma)\left(\wp^4 - \frac{\wp^6}{10}\right)\sin\mathfrak{I} + \bar{\kappa}(\gamma)\left(\frac{72\wp^{\varrho+4}}{\Gamma(\varrho+5)} - \frac{216\wp^{\varrho+6}}{\Gamma(\varrho+7)}\right)\sin\mathfrak{I} + \bar{\kappa}(\gamma)\left(\frac{216\wp^{2\varrho+4}}{\Gamma(2\varrho+5)} - \frac{648\wp^{2\varrho+6}}{\Gamma(2\varrho+7)}\right)\sin\mathfrak{I} \\ &\quad + \bar{\kappa}(\gamma)\left(\frac{648\wp^{3\varrho+4}}{\Gamma(3\varrho+5)} - \frac{1944\wp^{3\varrho+6}}{\Gamma(3\varrho+7)}\right)\sin\mathfrak{I}+\cdots.\end{aligned} \quad (35)$$

The exact result is

$$\tilde{\upsilon}(\wp,\mathfrak{I}) = \tilde{\kappa}(\gamma)\wp^4 \sin\mathfrak{I}. \quad (36)$$

Figure 3 shows the fuzzy result comparison for lower and upper branches of Example 1 by the Laplace decomposition method. The red color shows that the exact solution of lower and upper branch of fuzzy solution at integer order and second cure shows the analytical solution of Example 3. In Figure 3, second graph shows two dimensional figure of fuzzy result at four other various fractional order of ρ of Example 3. The two similar color legends represent lower and upper portions of fuzzy solution, respectively.

4. Conclusion

The aim of this paper was to propose a semianalytic solution to the fractional fuzzy Helmholtz equations using the Caputo derivative. An significant example has verified the solution achieved. We have also included plots of the analytical solution at various fractional orders. As seen in the graphs, when the fractional order nu approachs its integer value, the graphs will similar with the curve at integer order 1. Our study has shown that fractional calculus can identify the global nature of equations that are associated with fuzzy concepts. As future research continues, we will apply this approach to models with more dynamic features. In the future, this technique can be used to derive analytic and approximation solutions to perturbed fractional differential equations with fractional and classical initial conditions in the sense of the Caputo operator.

Data Availability

The numerical data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

References

[1] M. Alesemi, N. Iqbal, and T. Botmart, "Novel analysis of the fractional-order system of non-linear partial differential equations with the exponential-decay kernel," *Mathematics*, vol. 10, no. 4, p. 615, 2022.

[2] M. Naeem, H. Rezazadeh, A. A. Khammash, R. Shah, and S. Zaland, "Analysis of the fuzzy fractional-order solitary wave solutions for the KdV equation in the sense of Caputo-Fabrizio derivative," *Journal of Mathematics*, vol. 2022, Article ID 3688916, 12 pages, 2022.

[3] N. Iqbal, A. U. K. Niazi, R. Shafqat, and S. Zaland, "Existence and uniqueness of mild solution for fractional-order controlled fuzzy evolution equation," *Journal of Function Spaces*, vol. 2021, Article ID 5795065, 8 pages, 2021.

[4] N. A. Shah, H. A. Alyousef, S. A. El-Tantawy, R. Shah, and J. D. Chung, "Analytical investigation of fractional-order Korteweg-De-Vries-type equations under Atangana-Baleanu-Caputo operator: modeling nonlinear waves in a plasma and fluid," *Symmetry*, vol. 14, no. 4, p. 739, 2022.

[5] R. P. Agarwal, V. Lakshmikantham, and J. J. Nieto, "On the concept of solution for fractional differential equations with uncertainty," *Nonlinear Analysis: Theory Methods & Applications*, vol. 72, no. 6, pp. 2859–2862, 2010.

[6] W. W. Mohammed, N. Iqbal, and T. Botmart, "Additive noise effects on the stabilization of fractional-space diffusion equation solutions," *Mathematics*, vol. 10, no. 1, p. 130, 2022.

[7] N. Iqbal, A. U. K. Niazi, I. U. Khan, R. Shah, and T. Botmart, "Cauchy problem for non-autonomous fractional evolution equations with nonlocal conditions of order $(1, 2)$," *AIMS Mathematics*, vol. 7, no. 5, pp. 8891–8913, 2022.

[8] A. O. Almatroud, A. E. Matouk, W. W. Mohammed, N. Iqbal, and S. Alshammari, "Self-excited and hidden chaotic attractors in Matouk's hyperchaotic systems," vol. 2022, Article ID 6458027, pp. 1–14, 2022.

[9] P. Sunthrayuth, A. M. Zidan, S. W. Yao, R. Shah, and M. Inc, "The comparative study for solving fractional-order Fornberg–Whitham equation via ρ-Laplace transform," *Symmetry*, vol. 13, no. 5, p. 784, 2021.

[10] A. A. Kilbas, M. H. Srivastava, and J. J. Trujillo, *Theory and application of fractional differential equations*, vol. 204, Elsevier, Amsterdam, The Netherlands, 2006.

[11] M. Naeem, A. M. Zidan, K. Nonlaopon, M. I. Syam, Z. Al-Zhour, and R. Shah, "A new analysis of fractional-order equal-width equations via novel techniques," *Symmetry*, vol. 13, no. 5, p. 886, 2021.

[12] K. Nonlaopon, A. M. Alsharif, A. M. Zidan, A. Khan, Y. S. Hamed, and R. Shah, "Numerical investigation of fractional-order Swift-Hohenberg equations via a novel transform," *Symmetry*, vol. 13, no. 7, p. 1263, 2021.

[13] V. H. Ngo, "Fuzzy fractional functional integral and differential equations," *Fuzzy Sets and Systems*, vol. 280, pp. 58–90, 2015.

[14] N. Iqbal, A. Akgul, R. Shah, A. Bariq, M. Mossa Al-Sawalha, and A. Ali, "On solutions of fractional-order gas dynamics equation by effective techniques," *Journal of Function Spaces*, vol. 2022, Article ID 3341754, 14 pages, 2022.

[15] T. Allahviranloo, Z. Gouyandeh, and A. Armand, "A full fuzzy method for solving differential equation based on Taylor expansion," *Journal of Intelligent Fuzzy Systems*, vol. 29, no. 3, pp. 1039–1055, 2015.

[16] M. Chehlabi and T. Allahviranloo, "Concreted solutions to fuzzy linear fractional differential equations," *Applied Soft Computing*, vol. 44, pp. 108–116, 2016.

[17] N. V. Hoa, "Fuzzy fractional functional differential equations under Caputo gH- differentiability," *Communications in Nonlinear Science and Numerical Simulation*, vol. 22, no. 1-3, pp. 1134–1157, 2015.

[18] R. P. Agarwal, D. Baleanu, J. J. Nieto, D. F. M. Torres, and Y. Zhou, "A survey on fuzzy fractional differential and optimal control nonlocal evolution equations," *Journal of Computational and Applied Mathematics*, vol. 339, pp. 3–29, 2018.

[19] A. U. K. Niazi, N. Iqbal, R. Shah, F. Wannalookkhee, and K. Nonlaopon, "Controllability for fuzzy fractional evolution equations in credibility space," *Fractal and Fractional*, vol. 5, no. 3, p. 112, 2021.

[20] S. Salahshour, T. Allahviranloo, and S. Abbasbandy, "Solving fuzzy fractional differential equations by fuzzy Laplace transforms," *Communications in Nonlinear Science and Numerical Simulation*, vol. 17, no. 3, pp. 1372–1381, 2012.

[21] R. Ali and K. Pan, "The new iteration methods for solving absolute value equations," *Applications of Mathematics*, pp. 1–14, 2021.

[22] N. A. Shah, S. J. Yook, and O. Tosin, "Analytic simulation of thermophoretic second grade fluid flow past a vertical surface with variable fluid characteristics and convective heating," *Scientific Reports*, vol. 12, no. 1, pp. 1–17, 2022.

[23] R. Ali and K. Pan, "The solution of the absolute value equations using two generalized accelerated overrelaxation methods," *Asian-European Journal of Mathematics*, no. article 2250154, 2021.

[24] R. Ali, I. Khan, A. Ali, and A. Mohamed, "Two new generalized iteration methods for solving absolute value equations using M -matrix," *AIMS Mathematics*, vol. 7, no. 5, pp. 8176–8187, 2022.

[25] N. A. Shah, I. Dassios, E. R. El-Zahar, and J. D. Chung, "An Efficient Technique of Fractional-Order Physical Models Involving ρ-Laplace Transform," *Mathematics*, vol. 10, no. 5, p. 816, 2022.

[26] Q. Zheng, F. Xie, and W. Lin, "Solution of two-dimensional Helmholtz equation by multipole theory method," *Journal of electromagnetic waves and applications*, vol. 13, no. 2, pp. 205–220, 1999.

[27] L. L. Thompson and P. M. Pinsky, "A Galerkin least-squares finite element method for the two-dimensional Helmholtz equation," *International Journal for Numerical Methods in Engineering*, vol. 38, no. 3, pp. 371–397, 1995.

[28] W. Zhang and Y. Dai, "Finite-difference solution of the Helmholtz equation based on two domain decomposition algorithms," *Journal of Applied Mathematics and Physics*, vol. 1, no. 4, pp. 18–24, 2013.

[29] M. Samuel and A. Thomas, "On fractional Helmholtz equations," *Fractional Calculus and Applied Analysis*, vol. 13, no. 3, pp. 295p–308p, 2010.

[30] D. Baleanu, H. K. Jassim, and M. Al Qurashi, "Solving Helmholtz equation with local fractional derivative operators," *Fractal and Fractional*, vol. 3, no. 3, p. 43, 2019.

[31] P. K. Gupta, A. H. M. E. T. Yildirim, and K. N. Rai, "Application of He's homotopy perturbation method for multi-dimensional fractional Helmholtz equation," *International Journal of Numerical Methods for Heat and Fluid Flow*, vol. 22, no. 4, pp. 424–435, 2012.

[32] N. Iqbal, H. Yasmin, A. Rezaiguia, J. Kafle, A. O. Almatroud, and T. S. Hassan, "Analysis of the fractional-order kaup–kupershmidt equation via novel transforms," *Journal of Mathematics*, vol. 2021, Article ID 2567927, 13 pages, 2021.

[33] S. Abuasad, K. Moaddy, and I. Hashim, "Analytical treatment of two-dimensional fractional Helmholtz equations," *Journal of King Saud University-Science*, vol. 31, no. 4, pp. 659–666, 2019.

[34] S. Momani and S. Abuasad, "Application of He's variational iteration method to Helmholtz equation," *Chaos, Solitons & Fractals*, vol. 27, no. 5, pp. 1119–1123, 2006.

[35] A. Prakash, M. Goyal, and S. Gupta, "Numerical simulation of space-fractional Helmholtz equation arising in seismic wave propagation, imaging and inversion," *Pramana*, vol. 93, no. 2, p. 28, 2019.

[36] R. Jan, H. Khan, P. Kumam, F. Tchier, R. Shah, and H. Bin Jebreen, "The investigation of the fractional-view dynamics of Helmholtz equations within Caputo operator," *CMC-COMPUTERS MATERIALS & CONTINUA*, vol. 68, no. 3, pp. 3185–3201, 2021.

[37] H. M. Srivastava, R. Shah, H. Khan, and M. Arif, "Some analytical and numerical investigation of a family of fractional-order Helmholtz equations in two space dimensions," *Mathematical Methods in the Applied Sciences*, vol. 43, no. 1, pp. 199–212, 2020.