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Research Article

A New Strategy for the Approximate Solution of Hyperbolic Telegraph Equations in Nonlinear Vibration System

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This study examines a new approach for the approximate solution of hyperbolic telegraph equations emerging in magnetic fields and electrical impulse transmissions. We introduce a Laplace-Carson transform coupled with the homotopy perturbation method which is called the Laplace-Carson homotopy perturbation method ($\mathscr{L}_c$-HPM). The most significant feature of this approach is that we do not require any restriction of variables and hypotheses to find the results of nonlinear problems. Further, HPM using He's is applied to reduce the number of computations in nonlinear terms. We demonstrate some graphical results to show that $\mathscr{L}_c$-HPM is a simple and suitable approach for linear and nonlinear problems.

1. Introduction

Most of the nonlinear vibration phenomena are described by unsteady reactions, chaos, splitting processes, and some other multiple norms of motion. This vibration study starts from a large number of components such as high elastic deflection, electrical charge force, and complex absorption [1]. In this manner, a more proper comprehensive knowledge of the nonlinear vibration phenomena is important for the investigation of vibratory incidents. Recently, numerous researchers have paid much attention for the study of the applications of hyperbolic equations. Azab and Gamel [2] constructed a new approach built on a numerical strategy for the study of telegraph equations. Pandit et al. [3] applied a finite difference scheme to find the results of the hyperbolic telegraph problem. Evans and Bulut [4] proposed a new approach to determine the precise results of the telegraph problems in explicit form. Srinivasa and Rezazadeh [5] obtained the numerical solution of the one-dimensional telegraph equation via the wavelet technique. Ding et al. [6] used a nonpolynomial cubic spline approach in space direction for the study of the telegraph equation. Saadatmandi and Dehghan [7] used the Chebyshev tau method to achieve the numerical solution of the hyperbolic telegraph equation. Lakestani and Saray [8] applied scaling functions for the solution of the telegraph equation. Later, Sharifi and Rashidinia [9] applied extended cubic B-spline for the solution of the hyperbolic telegraph equation and also showed the convergence and stability of the method. Khater and Lu [10] investigated the stable analytical solutions of the nonlinear fractional nonlinear time–space telegraph equation by applying the trigonometric-quantic-B-spline method. Das and Gupta [11] used the homotopy analysis method to find the explicit solutions of the telegraph equations. A broad study of hyperbolic telegraph equation can be studied in [12–15].

The basic concept of the homotopy perturbation method (HPM) was suggested by He [16–18] to obtain the solution of some differential equations. Later, many researchers [19, 20] constructed a scheme coupled with Laplace transform and HPM to examine the solution of differential equations. Recently, Aggarwal et al. [21] used Laplace-Carson transform for the first kind of Volterra integrodifferential equation. Later, Kumar and Qureshi [22] obtained the exact solutions of non-integer-order initial value problems with the Caputo operator and confirmed the accuracy of this approach. Thange and Gade [23] introduced some properties of the Laplace-Carson transform with fractional order with the help of convolution theorem. In this paper, we introduce a new approach Laplace-Carson homotopy perturbation method ($\mathscr{L}_c$-HPM) built on Laplace-Carson transform and HPM for the study of hyperbolic telegraph equation. We observe that this strategy is simple to handle and produces the results in the form of series only after a few iterations. This article is arranged as follows: in Section 2, we define the Laplace-Carson transform and its basic properties. In Section 3, we introduce the basic idea of HPM to decompose the nonlinear terms. In Section 4, we illustrate some applications to indicate the competence of $\mathscr{L}_c$-PTM, and at last, some results are discussed with conclusion in Sections 5 and 6, respectively.

2. Fundamental Concepts of Laplace-Carson Transform

Definition 1. Let $f(t)$ be a function precise for $t \geq 0$; then,

$$\mathscr{L}\{f(t)\} = F(s) = \theta \int_0^\infty f(t)e^{-st}dt \tag{1}$$

is called the Laplace transform and s is the independent variable of the transformed function t.

Definition 2. Aggarwal et al. [21] introduced Laplace-Carson transform for the solution of first kind of Volterra integro-differential problem; then,

$$\mathscr{L}_c\{g(t)\} = R(\theta) = \theta \int_0^\infty g(t)e^{-\theta t}dt, \quad k_1 \leq \theta \leq k_2, \tag{2}$$

where $\mathscr{L}_c$ is denoted as Laplace-Carson transform and θ is the independent variable of the transformed function t. On the other hand, let $R(\theta)$ be the Laplace-Carson transform of a function $g(t)$; then, $g(t)$ is the inverse of $R(\theta)$ so that

$$\mathscr{L}_c^{-1}\{R(\theta)\} = g(t), \tag{3}$$

where $\mathscr{L}_c^{-1}$ is called inverse Laplace-Carson transform.

Definition 3. If $g(t) = t^m$, then the Laplace-Carson transform is applied as

$$\mathscr{L}_c\{g(t)\} = R(\theta) = \frac{m!}{\theta^m}. \tag{4}$$

Properties 4. If $\mathscr{L}_c\{g(t)\} = R(\theta)$, then it has the following differential properties [21, 23]:

(a) $\mathscr{L}_c\{g'(t)\} = \theta R(\theta) - \theta G(0)$

(b) $\mathscr{L}_c\{g''(t)\} = \theta^2 R(\theta) - \theta^2 G(0) - \theta G'(0)$

(c) $\mathscr{L}_c\{g^m(t)\} = \theta^m R(\theta) - \theta^m G(0) - \theta^{m-1} G'(0) - \cdots - \theta G^{m-1}(0)$

3. Basic Idea of HPM

In this segment, we illustrate a nonlinear functional equation to explain the basic view HPM [24, 25]. Consider

$$T(u) - g(h) = 0, \quad h \in \Omega, \tag{5}$$

with conditions

$$S\left(u, \frac{\partial u}{\partial n}\right) = 0, \quad h \in \Gamma, \tag{6}$$

where T and S are known as general functional operator and boundary operator, respectively, and $g(h)$ is known function with Γ as a interval of the domain Ω. We now divide T into two units such as T_1 which represents a linear and T_2 a nonlinear operator. As a result, we can express Equation (6) such as

$$T_1(u) + T_2(u) - g(h) = 0. \tag{7}$$

Assume a homotopy $v(h, \theta)$: $\Omega \times [0, 1] \longrightarrow \mathbb{H}$ in such a way that it is appropriate for

$$H(v, \theta) = (1-\theta)[T_1(v) - T_1(u_0)] + \theta[T_1(v) - T_2(v) - g(h)] \tag{8}$$

or

$$H(v, \theta) = T_1(v) - T_1(u_0) + qL(u_0) + \theta[T_2(v) - g(h)] = 0, \tag{9}$$

where $\theta \in [0, 1]$ is embedding parameter and u_0 is an initial guess of Equation (5), which is suitable for the boundary conditions. The theory of HPM states that θ is considered as a slight variable and the solution of Equation (5) in the resulting form of θ.

$$v = v_0 + \theta v_1 + \theta^2 v_2 + \theta^3 v_3 + \cdots = \sum_{i=0}^{\infty} \theta^i v_i. \tag{10}$$

Let $\theta = 1$; then, the particular of Equation (6) is written as

$$u = \lim_{\theta \longrightarrow 1} v = v_0 + v_1 + v_2 + v_3 + \cdots = \sum_{i=0}^{\infty} v_i. \tag{11}$$

The nonlinear terms can be calculated as

$$T_2 u(x, t) = \sum_{n=0}^{\infty} \theta^n H_n(u). \tag{12}$$

Then, He's polynomials $H_n(u)$ can be obtained using the following expression:

$$H_n(u_0 + u_1 + \cdots + u_n) = \frac{1}{n!}\frac{\partial^n}{\partial \theta^n}\left(T_2\left(\sum_{i=0}^{\infty}\theta^i u_i\right)\right)_{\theta=0}, \quad n = 0, 1, 2, \cdots. \tag{13}$$

The series solution in Equation (12) is mostly convergent due to and the convergence rate of the series depending on the nonlinear operator T_2.

4. Numerical Applications

In this section, we incorporate the concept of $\mathscr{L}_c$-PTM for obtaining the approximate solution of linear and nonlinear telegraph equations. We observe that only after iteration, this scheme produces excellent accuracy. Mathematical Software 11.0.1 is used to perform the calculations. We present some 2D and 3D graphs for better understanding the behavior of this scheme.

4.1. Example 1. Consider one-dimensional linear hyperbolic telegraph equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2} + \frac{\partial u}{\partial t} + u, \tag{14}$$

with conditions

$$\begin{aligned} u(x,0) &= e^x, \\ u_t(x,0) &= -e^x, \\ u(0,t) &= e^{-t}, \\ u_x(0,t) &= e^{-t}. \end{aligned} \tag{15}$$

Applying Laplace-Carson transform to Equation (14), we get

$$\mathscr{L}_c\left[\frac{\partial^2 u}{\partial x^2}\right] = \mathscr{L}_c\left[\frac{\partial^2 u}{\partial t^2} + \frac{\partial u}{\partial t} + u\right]. \tag{16}$$

Using the properties of Laplace-Carson transform, we get

$$\theta^2 u(\theta,t) - \theta^2 u(0,t) - \theta u'(0,t) = \mathscr{L}_c\left[\frac{\partial^2 u}{\partial t^2} + \frac{\partial u}{\partial t} + u\right], \tag{17}$$

which may be solved further as

$$u(\theta,t) = u(0,t) + \frac{1}{\theta}u'(0,t) + \frac{1}{\theta^2}\mathscr{L}_c\left\{\frac{\partial^2 u}{\partial t^2} + \frac{\partial u}{\partial t} + u\right\}. \tag{18}$$

Applying inverse Laplace-Carson transform, we get

$$u(x,t) = u(0,t) + xu'(0,t) + \mathscr{L}_c^{-1}\left[\frac{1}{\theta^2}\mathscr{L}_c\left\{\frac{\partial^2 u}{\partial t^2} + \frac{\partial u}{\partial t} + u\right\}\right]. \tag{19}$$

Now, we introduce HPM on Equation (38); we get

$$\sum_{n=0}^{\infty}\theta^n u_n(x,t) = u(0,t) + xu'(0,t) + \theta\mathscr{L}_c^{-1} \cdot \left[\frac{1}{\theta^2}\mathscr{L}\left\{\frac{\partial^2}{\partial t^2}\sum_{n=0}^{\infty}\theta^n u_n(x,t) + \frac{\partial}{\partial t}\sum_{n=0}^{\infty}\theta^n u_n(x,t) + \sum_{n=0}^{\infty}\theta^n u_n(x,t)\right\}\right]. \tag{20}$$

On comparing, the following iterations can be obtained:

$$\begin{aligned} \theta^0 &: u_0(x,t) = e^{-t} + xe^{-t}, \\ \theta^1 &: u_1(x,t) = \mathscr{L}_c^{-1}\left[\frac{1}{\theta^2}\mathscr{L}_c\left\{\frac{\partial^2 u_0}{\partial t^2} + \frac{\partial u_0}{\partial t} + u_0\right\}\right] = e^{-t}\frac{x^2}{2!} + e^{-t}\frac{x^3}{3!}, \\ \theta^2 &: u_2(x,t) = \mathscr{L}_c^{-1}\left[\frac{1}{\theta^2}\mathscr{L}_c\left\{\frac{\partial^2 u_1}{\partial t^2} + \frac{\partial u_1}{\partial t} + u_1\right\}\right] = e^{-t}\frac{x^4}{4!} + e^{-t}\frac{x^5}{5!}, \\ \theta^3 &: u_3(x,t) = \mathscr{L}_c^{-1}\left[\frac{1}{\theta^2}\mathscr{L}_c\left\{\frac{\partial^2 u_2}{\partial t^2} + \frac{\partial u_2}{\partial t} + u_2\right\}\right] = e^{-t}\frac{x^6}{6!} + e^{-t}\frac{x^7}{7!}. \\ &\vdots \end{aligned} \tag{21}$$

Hence, the solution can be expressed as

$$\begin{aligned} u(x,t) &== u_1(x,t) + u_2(x,t) + u_3(x,t) + \cdots, \\ u(x,t) &== e^{-t} + xe^{-t} + \frac{x^2}{2!}e^{-t} + \frac{x^3}{3!}e^{-t} + \frac{x^4}{4!}e^{-t} + \frac{x^5}{5!}e^{-t} + \frac{x^6}{6!}e^{-t} + \frac{x^7}{7!}e^{-t} + \cdots, \\ u(x,t) &= e^{x-t}. \end{aligned} \tag{22}$$

4.2. Example 2. Consider another linear hyperbolic telegraph equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2} + 4\frac{\partial u}{\partial t} + 4u, \tag{23}$$

with initial conditions

$$\begin{aligned} u(x,0) &= 1+e^{2x}, \\ u_t(x,0) &= -2, \\ u(0,t) &= 1+e^{-2t}, \\ u_x(0,t) &= 2. \end{aligned} \tag{24}$$

Applying Laplace-Carson transform to Equation (23), we get

$$\mathscr{L}_c\left[\frac{\partial^2 u}{\partial x^2}\right] = \mathscr{L}_c\left[\frac{\partial^2 u}{\partial t^2} + 4\frac{\partial u}{\partial t} + 4u\right]. \tag{25}$$

Using the properties of Laplace-Carson transform, we get

$$\theta^2 u(\theta,t) - \theta^2 u(0,t) - \theta u'(0,t) = \mathscr{L}_c\left[\frac{\partial^2 u}{\partial t^2} + 4\frac{\partial u}{\partial t} + 4u\right], \tag{26}$$

which may be solved further as

$$u(\theta,t) = u(0,t) + \frac{1}{\theta}u'(0,t) + \frac{1}{\theta^2}\mathscr{L}_c\left\{\frac{\partial^2 u}{\partial t^2} + 4\frac{\partial u}{\partial t} + 4u\right\}. \tag{27}$$

Applying inverse Laplace-Carson transform, we get

$$\begin{aligned} u(x,t) = u(0,t) + xu'(0,t) + \mathscr{L}_c^{-1} \\ \cdot \left[\frac{1}{\theta^2}\mathscr{L}_c\left\{\frac{\partial^2 u}{\partial t^2} + 4\frac{\partial u}{\partial t} + 4u\right\}\right]. \end{aligned} \tag{28}$$

Now, we introduce HPM on Equation (28); we get

$$\begin{aligned} \sum_{n=0}^{\infty}\theta^n u_n(x,t) = u(0,t) + xu'(0,t) + \theta\mathscr{L}_c^{-1} \\ \cdot \left[\frac{1}{\theta^2}\mathscr{L}\left\{\frac{\partial^2}{\partial t^2}\sum_{n=0}^{\infty}\theta^n u_n(x,t)\right.\right. \\ \left.\left. + 4\frac{\partial}{\partial t}\sum_{n=0}^{\infty}\theta^n u_n(x,t) + 4\sum_{n=0}^{\infty}\theta^n u_n(x,t)\right\}\right]. \end{aligned} \tag{29}$$

On comparing, the following iterations can be obtained:

$$\begin{aligned} \theta^0 &: u_0(x,t) = 1+e^{-2t}+2x, \\ \theta^1 &: u_1(x,t) = \mathscr{L}_c^{-1}\left[\frac{1}{\theta^2}\mathscr{L}_c\left\{\frac{\partial^2 u_0}{\partial t^2} + 4\frac{\partial u_0}{\partial t} + 4u_0\right\}\right] \\ &= 4\frac{x^2}{2!} + 8\frac{x^3}{3!}, \\ \theta^2 &: u_2(x,t) = \mathscr{L}_c^{-1}\left[\frac{1}{\theta^2}\mathscr{L}_c\left\{\frac{\partial^2 u_1}{\partial t^2} + 4\frac{\partial u_1}{\partial t}4 + u_1\right\}\right] \\ &= 16\frac{x^4}{4!} + 32\frac{x^5}{5!}, \\ \theta^3 &: u_3(x,t) = \mathscr{L}_c^{-1}\left[\frac{1}{\theta^2}\mathscr{L}_c\left\{\frac{\partial^2 u_2}{\partial t^2} + 4\frac{\partial u_2}{\partial t} + 4u_2\right\}\right] \\ &= 64\frac{x^6}{6!} + 128\frac{x^7}{7!}. \\ &\vdots \end{aligned} \tag{30}$$

Hence, the solution can be expressed as

$$\begin{aligned} u(x,t) &= u_1(x,t) + u_2(x,t) + u_3(x,t) + \cdots, \\ u(x,t) &= 1+e^{-2t}+2x+4\frac{x^2}{2!}+8\frac{x^3}{3!}+16\frac{x^4}{4!} \\ &\quad + 32\frac{x^5}{5!} + 64\frac{x^6}{6!} + 128\frac{x^7}{7!}, \\ u(x,t) &= e^{2x}+e^{-2t}. \end{aligned} \tag{31}$$

4.3. Example 3. Consider nonlinear hyperbolic telegraph equation

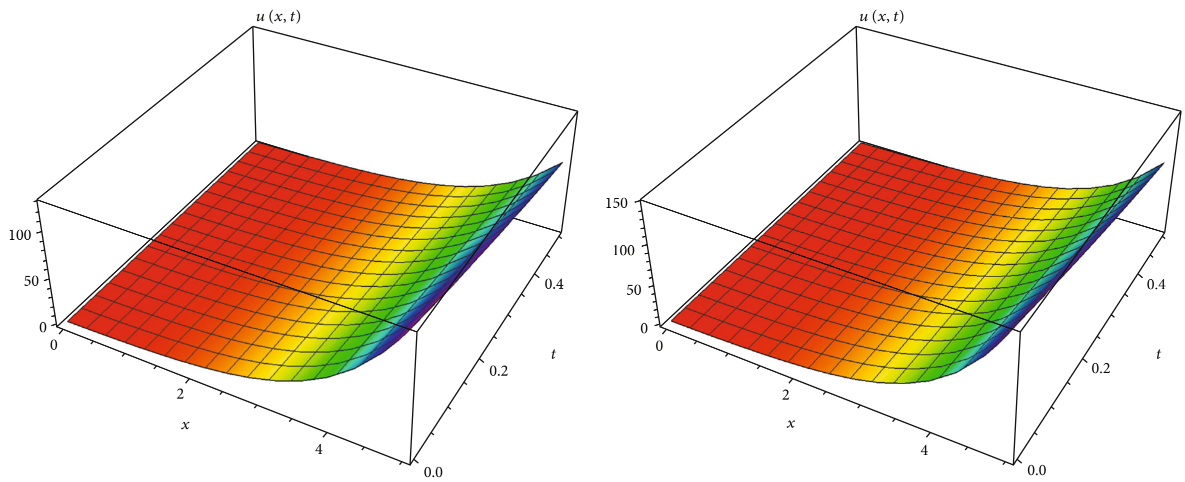
$$\frac{\partial^2 u}{\partial t^2} + 2\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + u^3 - u, \tag{32}$$

with conditions

$$\begin{aligned} u(x,0) &= \frac{1}{2} + \frac{1}{2}\tanh\left(\frac{x}{8}+5\right), \\ u_t(x,0) &= \frac{3}{16}\operatorname{sec h}^2\left(\frac{x}{8}+5\right), \\ u(0,t) &= \frac{1}{2} + \frac{1}{2}\tanh\left(\frac{3t}{8}+5\right), \\ u_x(0,t) &= \frac{1}{16}\operatorname{sec h}^2\left(\frac{3t}{8}+5\right). \end{aligned} \tag{33}$$

Applying Laplace-Carson transform on Equation (32), we get

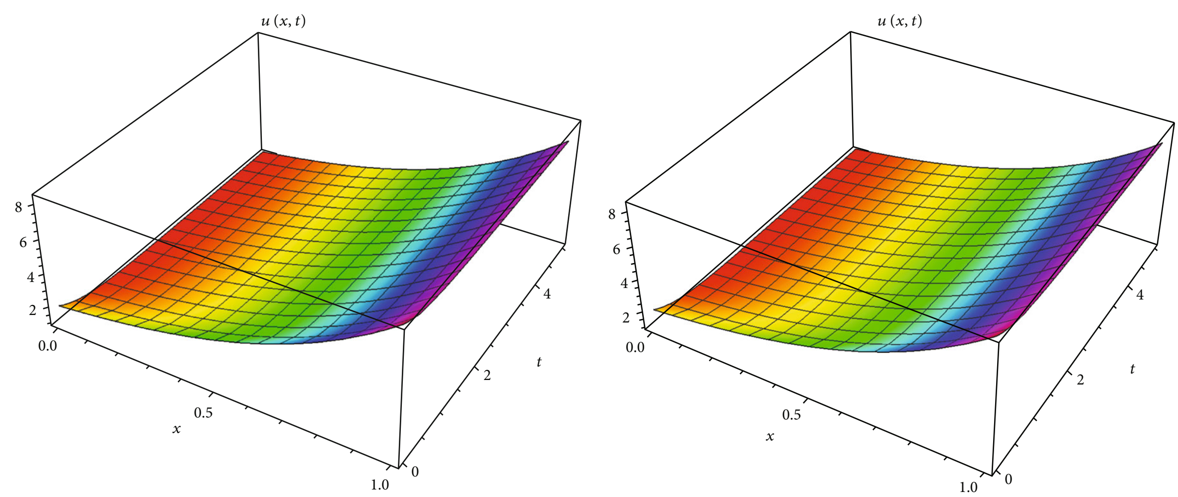
$$\mathscr{L}_c\left[\frac{\partial^2 u}{\partial x^2}\right] = \mathscr{L}_c\left[\frac{\partial^2 u}{\partial t^2} + 2\frac{\partial u}{\partial t} - u^3 + u\right]. \tag{34}$$



(a) Analytical solution of $u(x, t)$ for Equation (14)

(b) Particular solution of $u(x, t)$ for Equation (14)

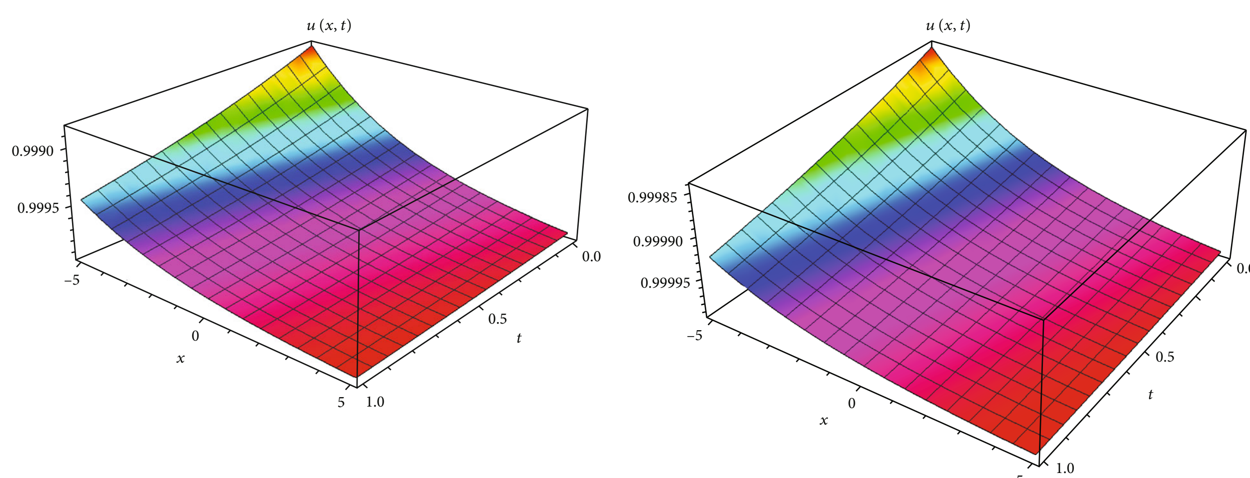
FIGURE 1: Surface solutions for nonlinear hyperbolic telegraph equation.



(a) Analytical solution of $u(x, t)$ for Equation (23)

(b) Particular solution of $u(x, t)$ for Equation (23)

FIGURE 2: Surface solutions for linear hyperbolic telegraph equation.



(a) Analytical solution of $u(x, t)$ for Equation (32)

(b) Particular solution of $u(x, t)$ for Equation (32)

FIGURE 3: Surface solutions for nonlinear hyperbolic telegraph equation.

Using the properties of Laplace-Carson transform, we get

$$\theta^2 u(\theta, t) - \theta^2 u(0, t) - \theta u'(0, t) = \mathscr{L}_c\left[\frac{\partial^2 u}{\partial t^2} + 2\frac{\partial u}{\partial t} - u^3 + u\right], \tag{35}$$

which may be solved further as,

$$u(\theta, t) = u(0, t) + \frac{1}{\theta}u'(0, t) + \frac{1}{\theta^2}\mathscr{L}_c\left\{\frac{\partial^2 u}{\partial t^2} + 2\frac{\partial u}{\partial t} - u^3 + u\right\}. \tag{36}$$

Applying inverse Laplace-Carson transform,

$$u(x, t) = u(0, t) + xu'(0, t) + \mathscr{L}_c^{-1} \cdot \left[\frac{1}{\theta^2}\mathscr{L}_c\left\{\frac{\partial^2 u}{\partial t^2} + 2\frac{\partial u}{\partial t} - u^3 + u\right\}\right]. \tag{37}$$

Now, we introduce HPM on Equation (32); we get

$$\sum_{n=0}^{\infty}\theta^n u_n(x, t) = u(0, t) + xu'(0, t) + \theta\mathscr{L}_c^{-1} \cdot \left[\frac{1}{\theta^2}\mathscr{L}\left\{\frac{\partial^2}{\partial t^2}\sum_{n=0}^{\infty}\theta^n u_n(x, t) + 2\frac{\partial}{\partial t}\sum_{n=0}^{\infty}\theta^n u_n \cdot (x, t) - \sum_{n=0}^{\infty}\theta^n u_n^3(x, t) + \sum_{n=0}^{\infty}\theta^n u_n(x, t)\right\}\right]. \tag{38}$$

On comparing, the following iterations can be obtained:

$$\begin{aligned}
\theta^0 &: u_0(x, t) = \frac{1}{2} + \frac{1}{2}\tanh\left(\frac{3t}{8} + 5\right) + x\frac{1}{2}\sec\mathrm{h}^2\left(\frac{3t}{8} + 5\right),\\
\theta^1 &: u_1(x, t) = \mathscr{L}_c^{-1}\left[\frac{1}{\theta^2}\mathscr{L}_c\left\{\frac{\partial^2 u_0}{\partial t^2} + 2\frac{\partial u_0}{\partial t} - u_0^3 + u_0\right\}\right],\\
\theta^2 &: u_2(x, t) = \mathscr{L}_c^{-1}\left[\frac{1}{\theta^2}\mathscr{L}_c\left\{\frac{\partial^2 u_1}{\partial t^2} + \frac{\partial u_1}{\partial t} + u_1 - 3u_0^2 u_1\right\}\right],\\
\theta^3 &: u_3(x, t) = \mathscr{L}_c^{-1}\left[\frac{1}{\theta^2}\mathscr{L}_c\left\{\frac{\partial^2 u_2}{\partial t^2} + \frac{\partial u_2}{\partial t} + u_2 - 3u_0 u_1^2 - 3u_0^2 u_2\right\}\right].\\
&\vdots
\end{aligned} \tag{39}$$

The other iterations are computed with the help of Wolfram Mathematica to obtain $u_1, u_2, u_3, \cdots$, which turns to the particular solution such as

$$u(x, t) = \frac{1}{2} + \frac{1}{2}\tanh\left(\frac{x}{8} + \frac{3t}{8} + 5\right). \tag{40}$$

5. Results and Discussion

This segment presents the discussion of the solution behaviors for the hyperbolic telegraph equations. Figure 1 represents the physical behavior at $0 \le x \le 5$ and $0 \le t \le 0.5$, whereas Figure 2 shows the physical behavior at $0 \le x \le 1$ and $0 \le t \le 5$ for the linear telegraph equations. We observe that the solution graphs turn to the particular solution very rapidly only after a few computations of iterations. Figure 3 represents the solution behavior of nonlinear hyperbolic telegraph equation at $0 \le x \le 5$ and $0 \le t \le 0.5$. The solution graph of the approximate solution is computed only for one iteration which coincides with the exact solation. Graphical representation and physical behavior of the linear and nonlinear hyperbolic telegraph equations demonstrate that the results obtained by $\mathscr{L}_c$-HPM are accurate and agreed with the results of exact solutions which confirm the authenticity of this approach.

6. Conclusion

In this article, we successfully conducted $\mathscr{L}_c$-HPM for finding the approximate solution of hyperbolic telegraph equations. We provided the results in the form of series without any discretization, linearization, or assumptions. The proposed strategy predicts the following fruitful remarks:

(i) $\mathscr{L}_c$-HPM is a direct approach to find the approximate solution of the problems

(ii) This scheme has less computational work, and there is no restriction of variables to obtain the solution

(iii) $\mathscr{L}_c$-HPM is applicable for both linear and nonlinear problems that provides the series solution only after a few iterations

(iv) We made all calculations with the help of Mathematica Software 11.0.1

(v) This approach is also applicable for other nonlinear fractional partial differential equations in science and engineering for future problems

Data Availability

We have provided all the data within the article.

Conflicts of Interest

The authors report that they have no conflicts of interest.

Authors' Contributions

Jiao Zeng worked in investigation, methodology, software, and writing original draft of the manuscript. Asma Idrees did work in validation, editing, and improvement of the English language during the revision of the manuscript.

Mohammed S. Abdo supervised and approved the manuscript during the initial and final submission.

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