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Retraction

Retracted: On Development of Neutrosophic Cubic Graphs with Applications in Decision Sciences

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This article has been retracted by Hindawi following an investigation undertaken by the publisher [1]. This investigation has uncovered evidence of one or more of the following indicators of systematic manipulation of the publication process:

(1) Discrepancies in scope
(2) Discrepancies in the description of the research reported
(3) Discrepancies between the availability of data and the research described
(4) Inappropriate citations
(5) Incoherent, meaningless and/or irrelevant content included in the article
(6) Manipulated or compromised peer review

The presence of these indicators undermines our confidence in the integrity of the article's content and we cannot, therefore, vouch for its reliability. Please note that this notice is intended solely to alert readers that the content of this article is unreliable. We have not investigated whether authors were aware of or involved in the systematic manipulation of the publication process.

Wiley and Hindawi regrets that the usual quality checks did not identify these issues before publication and have since put additional measures in place to safeguard research integrity.

We wish to credit our own Research Integrity and Research Publishing teams and anonymous and named external researchers and research integrity experts for contributing to this investigation.

The corresponding author, as the representative of all authors, has been given the opportunity to register their agreement or disagreement to this retraction. We have kept a record of any response received.

References

[1] F. Sultana, M. Gulistan, P. Liu et al., "On Development of Neutrosophic Cubic Graphs with Applications in Decision Sciences," *Journal of Function Spaces*, vol. 2022, Article ID 8597666, 24 pages, 2022.

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Research Article

On Development of Neutrosophic Cubic Graphs with Applications in Decision Sciences

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In this study, the neutrosophic cubic graphs are further developed. We discussed and explored the open and the closed neighborhood for any vertex in neutrosophic cubic graphs, regular and totally regular neutrosophic cubic graphs, complete neutrosophic cubic graphs, balanced and strictly balanced neutrosophic cubic graphs, irregular and totally irregular neutrosophic cubic graphs, complement of a neutrosophic cubic graph, neighborly irregular and neighborly totally irregular neutrosophic cubic graphs, and highly irregular neutrosophic cubic graphs. It has been demonstrated that the proposed neutrosophic cubic graphs are associated with specific conditions. The comparison study of the proposed graphs with the existing cubic graphs has been carried out. Eventually, decision-making approaches for handling daily life problems such as effects of different factors on the neighboring countries of Pakistan and selection of a house based on the notions of proposed graphs are presented.

1. Introduction

A human being has a higher position among all the creatures due to his ability to analyze and make decisions. The decisions are made by carefully scrutinizing the problem based on the experience and the current situation. In the past, this used to be a mental activity with its successful execution. With the advancement of science and technology, it is now possible to use some modern techniques to address this problem better. These methodologies rely on traditional knowledge virtually. The ability of humans has been mimicked out effectively by making use of artificial intelligence. Some artificial intelligence techniques have been used successfully to chalk out good decisions. In this approach, various instruments related to decision-making are used. There is a well-known approach called graph theory. Graph theory is the systematic and logical way to analyze and model many applications related to science and other social issues. Graph theory is an essential tool and has played a significant role in developing graph algorithms in computer-related applications. These algorithms are quite helpful in solving theoretical aspects of the problems. These techniques help in solving geometry, algebra, number theory, topology, and many other fields. But many issues are not practically solvable due to the crisp nature of the classical sets. So in 1965, Zadeh [1] introduced the notion of the fuzzy subset of a set. Many other extensions of fuzzy sets have been developed so far like interval-valued fuzzy sets [2] by Zadeh, intuitionistic fuzzy sets [3, 4] by Atanassov, and cubic sets [5, 6] by Jun et al. In [7], Akram et al. developed cubic KU-subalgebras. Smarandache extended the concept of Atanassov and gave the idea of neutrosophic sets [8, 9], and interval neutrosophic sets were introduced by Wang et al. [10]. Jun et al. gave the idea of a neutrosophic cubic set [11]. Rosenfeld [12] developed the fuzzy graphs in 1975. Bhattacharya [13] had started contributing in fuzzy graphs in 1987. Arya and Hazarika [14] developed functions with closed fuzzy graphs. Bhattacharya and Suraweera [15] had developed an algorithm to compute the supremum of max-min powers and a property of fuzzy graphs. Bhutani [16] studied automorphisms of fuzzy graphs. Cerruti [17] used graphs and fuzzy graphs in fuzzy information and decision processes. Chen [18] discuss matrix representations of fuzzy graphs. Crain [19] studied characterization of fuzzy interval graphs. After that, many others contributed to fuzzy graph's theory like Mordeson and Nair's contribution [20], Gani and Radha [21], Rashmanlou and Pal [22], Nandhini and Nandhini [23], Elmoasry et al. [24], and Akram et al. [25–27]. Other contributions are from Gani and Latha [28], Poulik et al. [29–32], Borzooei and Rashmanlou [33], Buckley [34], Rashmanlou and Pal [35], Mishra et al. [36], Pal et al. [37], Pramanik et al. [38, 39], Shannon and Atanassov [40], Parvathi et al. [41, 42], and Sahoo and Pal [43]. Akram [44, 45] initiated the concept of bipolar fuzzy graphs. Many others contributed on bipolar fuzzy graphs, like Rashmanlou et al. [46], Akram and Karunabigai [47], and Samanta and Pal [48]. Graphs in terms of neutrosophic's set have been studied by Huang et al. [49], Naz et al. [50], Dey et al. [51], Broumi et al. [52], and Karaaslan and Dawaz [53]. Zuo et al. [54] discussed picture fuzzy graphs. Kandasamy et al. and Smarandache [55, 56] developed neutrosophic graphs for the first time. Broumi et al. [57–61] discussed different versions of neutrosophic graphs. More development on the neutrosophic graphs can be seen in [50, 62–64]. After reading the extensive literature at neutrosophic graphs, recently, Gulistan et al. [65] discussed the cubic graphs with the application, neutrosophic graphs, and presented the idea of neutrosophic cubic graphs and their structures in their work [66, 67].

To further extend the work of Gulistan et al. [66, 67], in this paper we developed different types of neutrosophic cubic graphs including balanced, strictly balanced, complete, regular, totally regular, and irregular neutrosophic cubic graphs and complement of neutrosophic cubic graphs. Also, we established an open and close neighborhood of a vertex for neutrosophic cubic graphs and their application to the art of decision-making. The properties related to these newly suggested neutrosophic cubic graphs are also shown and how they are correlated. The arrangements of the paper are as follows: Section 2 is a review of basic concepts with their properties of neutrosophic cubic graphs. Section 3 describes different types of neutrosophic cubic graphs with examples. We also provide some results related to different types of neutrosophic cubic graphs. We present applications and a decision-making technique in Section 4. In Section 5, we provide a comparative analysis. Conclusions and suggested future work are presented in Section 6.

2. Preliminaries

This section consists of two parts: notations and predefined definitions.

2.1. Notations. Some notations with their descriptions are given in Table 1.

2.2. Predefined Definitions. In this subsection, we added some important definitions which are directly used in our work.

Definition 1 (see [66]). A neutrosophic cubic graph $\mathscr{L}_C^G = (\Gamma, \Lambda)$ for a crisp graph $\mathbf{G} = (A, B)$ is a pair with

$$\begin{aligned}\Gamma &= \left\{\Phi(g) = \left(\left(\widehat{\alpha_T}, \alpha_T\right), \left(\widehat{\alpha_I}, \alpha_I\right), \left(\widehat{\alpha_F}, \alpha_F\right)\right)(g) g \in A\right\},\\ \Gamma &= \left\{\left\langle g, \left(\left[\alpha_T^l, \alpha_T^r\right](g), \alpha_T(g)\right), \left(\left[\alpha_I^l, \alpha_I^r\right](g), \alpha_I(g)\right),\right.\right.\\ &\qquad \left.\left.\cdot \left(\left[\alpha_F^l, \alpha_F^r\right](g), \alpha_F(g)\right)\right\rangle \mid g \in A\right\},\end{aligned} \tag{1}$$

representing neutrosophic cubic vertex set A, and

$$\begin{aligned}\Lambda &= \left\{\Psi(g_1g_2) = \left(\left(\widehat{\beta_T}, \beta_T\right), \left(\widehat{\beta_I}, \beta_I\right), \left(\widehat{\beta_F}, \beta_F\right)\right)(g_1g_2) \mid g_1g_2 \in B\right\}\\ &= \left\{\left(\left[\beta_T^l, \beta_T^r\right](g_1g_2), \beta_T(g_1g_2)\right), \left(\left[\beta_I^l, \beta_I^r\right](g_1g_2), \beta_I(g_1g_2)\right),\right.\\ &\qquad \left.\cdot \left(\left[\beta_F^l, \beta_F^r\right](g_1g_2), \beta_F(g_1g_2)\right) \mid g_1g_2 \in B\right\}\end{aligned} \tag{2}$$

shows neutrosophic cubic edge set B such that

$$\begin{aligned}\widehat{\beta_T}(g_1g_2) &\leq r\min\left\{\widehat{\alpha_T}(g_1), \widehat{\alpha_T}(g_2)\right\}, \alpha_T(g_1g_2)\\ &\leq \max\left\{\alpha_T(g_1), \alpha_T(g_2)\right\},\\ \widehat{\beta_I}(g_1g_2) &\leq r\min\left\{\widehat{\alpha_I}(g_1), \widehat{\alpha_I}(g_2)\right\}, \alpha_I(g_1g_2)\\ &\leq \max\left\{\alpha_I(g_1), \alpha_I(g_2)\right\},\\ \widehat{\beta_F}(g_1g_2) &\leq r\max\left\{\widehat{\alpha_F}(g_1), \widehat{\alpha_F}(g_2)\right\}, \alpha_F(g_1g_2)\\ &\leq \min\left\{\alpha_F(g_1), \alpha_F(g_2)\right\},\end{aligned} \tag{3}$$

for every vertex $g_1, g_2 \in A$ and edge $g_1g_2 \in B$.

3. Different Types of Neutrosophic Cubic Graphs

This section contains definitions for different neutrosophic cubic graphs with a good discussion on some of their related results.

3.1. The Open and the Closed Neighborhood for Any Vertex in $\mathscr{L}_C^G$. In this subsection, we present the idea of open neighborhood $\mathbb{N}_{ncg}(g)$, degree of open neighborhood $\theta(\mathbb{N}_{ncg}(g))$, and closed neighborhood degree of neutrosophic cubic graph $\mathscr{L}_C^G$.

Table 1: Notations and their descriptions.

S.no	Notation	Description
1	$\mathbf{G}$	An arbitrary graph
2	A	Vertex set for $\mathbf{G}$ graph
3	B	Edge set for $\mathbf{G}$ graph
4	$F_{\mathbf{G}}$	Fuzzy graph
5	θ	Degree
6	$\wp$	Density
7	α	Membership function for vertex set in $F_{\mathbf{G}}$
8	β	Membership function for edge set in $F_{\mathbf{G}}$
9	$\mathbf{N}$	Neighborhood
10	$\hat{I}$	Represents an interval $[a^l, a^r]$ with $0 \le a^l \le a^r \le 1$
11	$\mathscr{L}_C^S$	Neutrosophic cubic set
12	$\mathscr{L}_C^G$	Neutrosophic cubic graph
13	Φ	Membership function for vertex set in $\mathscr{L}_C^G$
14	Ψ	Membership function for edge set in $\mathscr{L}_C^G$.
15	$N_f^{\mathbf{G}}(g)$	Fuzzy neighborhood of a vertex g in $F_{\mathbf{G}}$
16	$\mathbb{N}_{ncg}(g)$	Open neighborhood for any vertex g in $\mathscr{L}_C^G$
17	$\mathbb{N}_{ncg}[g]$	Closed neighborhood for any vertex g in $\mathscr{L}_C^G$

Definition 2. An open neighborhood $\mathbb{N}_{ncg}(g)$ for any vertex g in $\mathscr{L}_C^G = (\Gamma, \Lambda)$ is given by

$$\mathbb{N}_{ncg}(g) = \left\{ \left(\left[N_T^l(g), N_T^r(g) \right], N_T(g) \right), \left(\left[N_I^l(g), N_I^r(g) \right], \cdot N_I(g) \right), \left(\left[N_F^l(g), N_F^r(g) \right], N_F(g) \right) \right\}, \tag{4}$$

where

$$\begin{aligned}
N_T^l(g) &= \left\{ s \in A : \beta_T^l(gs) \le \min \left\{ \left(\alpha_T^l(g), \alpha_T^l(s) \right) \right\}, g \ne s \right\}, \\
N_T^r(g) &= \{ s \in A : \beta_T^r(gs) \le \min \{ (\alpha_T^r(g), \alpha_T^r(s)) \}, g \ne s \}, \\
N_T(g) &= \{ s \in A : \beta_T(gs) \le \min \{ (\alpha_T(g), \alpha_T(s)) \}, g \ne s \}, \\
N_I^l(g) &= \left\{ s \in A : \beta_I^l(gs) \le \min \left\{ \left(\alpha_I^l(g), \alpha_I^l(s) \right) \right\}, g \ne s \right\}, \\
N_I^r(g) &= \left\{ s \in A : \beta_I^+(gs) \le \min \{ (\alpha_I^+(g), \alpha_I^+(s)) \}, g \ne s \right\}, \\
N_I(g) &= \{ s \in A : \beta_T(gs) \le \min \{ (\alpha_T(g), \alpha_T(s)) \}, g \ne s \}, \\
N_F^l(g) &= \{ s \in A : \beta_F^-(gs) \le \max \{ (\alpha_F^-(g), \alpha_F^-(s)) \}, g \ne s \}, \\
N_F^r(g) &= \left\{ s \in A : \beta_F^+(gs) \le \max \{ (\alpha_F^-(g), \alpha_F^-(s)) \}, g \ne s \right\}, \\
N_F(g) &= \{ s \in A : \beta_F(gs) \le \max \{ (\alpha_F(g), \alpha_F(s)) \}, g \ne s \}.
\end{aligned} \tag{5}$$

It consists of membership functions for all vertices adjacent to g, excluding s.

Definition 3. The degree of open neighborhood $\theta(\mathbb{N}_{ncg}(g))$ for any vertex g in $\mathscr{L}_C^G$ is defined by

$$\begin{aligned}
\theta\left(\mathbb{N}_{ncg}(g)\right) &= \sum_{s \in \left\{ \mathbb{N}_{ncg}(g) | s \ne g \right\}} \Phi(s) = \left\{ \left(\left[\theta_T^l(g), \theta_T^r(g) \right], \theta_T(g) \right), \right. \\
&\quad \cdot \left. \left(\left[\theta_I^l(g), \theta_I^r(g) \right], \theta_I(g) \right), \left(\left[\theta_F^l(g), \theta_F^r(g) \right], \theta_F(g) \right) \right\} \\
&= \left\{ \left(\left[\sum_{s \in \left\{ \mathbb{N}_{ncg}(g) | s \ne g \right\}} \alpha_T^l(s), \sum_{s \in \left\{ \mathbb{N}_{ncg}(g) | s \ne g \right\}} \alpha_T^r(y) \right], \right. \right. \\
&\quad \cdot \sum_{s \in \left\{ \mathbb{N}_{ncg}(g) | s \ne g \right\}} \alpha_T(y)), \\
&\quad \cdot \left(\left[\sum_{s \in \left\{ \mathbb{N}_{ncg}(g) | s \ne g \right\}} \alpha_I^l(s), \sum_{s \in \left\{ \mathbb{N}_{ncg}(g) | s \ne g \right\}} \alpha_I^r(s) \right], \right. \\
&\quad \cdot \left. \sum_{s \in \left\{ \mathbb{N}_{ncg}(g) | s \ne g \right\}} \alpha_I(s) \right), \left(\left[\sum_{s \in \left\{ \mathbb{N}_{ncg}(g) | s \ne g \right\}} \alpha_F^l(s), \right. \right. \\
&\quad \cdot \left. \left. \left. \sum_{s \in \left\{ \mathbb{N}_{ncg}(g) | s \ne g \right\}} \alpha_F^r(s)], \sum_{s \in \left\{ \mathbb{N}_{ncg}(g) | s \ne g \right\}} \alpha_F(s) \right) \right\}.
\end{aligned} \tag{6}$$

Example 4. Let $\mathbf{G} = (A, B)$ with vertices $A = \{g_1, g_2, g_3\}$ and edges $B = \{g_1 g_2, g_2 g_3, g_1 g_3\}$. Also, $\mathscr{L}_C^G = (\Gamma, \Lambda)$ such that

$$\begin{aligned}
\Gamma &= \langle \{ g_1, ([.1, .2], .5), ([.4, .5], .3), ([.6, .7], .2) \}, \\
&\quad \cdot \{ g_2, ([.2, .4], .1), ([.5, .6], .4), ([.1, .2], .3) \}, \\
&\quad \cdot \{ g_3, ([.3, .4], .2), ([.1, .3], .7), ([.4, .6], .3) \} \rangle, \\
\Lambda &= \langle \{ g_1 g_2, ([.1, .2], .5), ([.4, .5], .4), ([.6, .7], .2) \}, \\
&\quad \cdot \{ g_2 g_3, ([.2, .4], .2), ([.1, .3], .7), ([.4, .6], .3) \}, \\
&\quad \cdot \{ g_1 g_3, ([.1, .2], .5), ([.1, .3], .7), ([.6, .7], .2) \} \rangle,
\end{aligned} \tag{7}$$

then clearly, $\mathscr{L}_C^G = (\Gamma, \Lambda)$ is a neutrosophic cubic graph as shown in Figure 1.

$\theta(\mathbb{N}_{ncg}(g))$ for each element $g \in A$ is given by

$$\begin{aligned}
\theta\left(\mathbb{N}_{ncg}(g_1)\right) &= \Phi(g_2) + \Phi(g_3) \\
&= \{ ([.2, .4], .1), ([.5, .6], .4), ([.1, .2], .3) \} \\
&\quad + \{ ([.3, .4], .2), ([.1, .3], .7), ([.4, .6], .3) \} \\
&= \{ ([.5, .8], .3), ([.6, .9], 1.1), ([.5, .8], .6) \}, \\
\theta\left(\mathbb{N}_{ncg}(g_2)\right) &= \Phi(g_1) + \Phi(g_3) \\
&= \{ ([.1, .2], .5), ([.4, .5], .3), ([.6, .7], .2) \} \\
&\quad + \{ ([.3, .4], .2), ([.1, .3], .7), ([.4, .6], .3) \} \\
&= \{ ([.4, .7], .6), ([.5, .8], 1), ([1, 1.3], .5) \}, \theta\left(\mathbb{N}_{ncg}(g_3)\right) \\
&= \Phi(g_1) + \Phi(g_2) = \{ ([.1, .2], .5), ([.4, .5], .3), ([.6, .7], .2) \} \\
&\quad + \{ ([.2, .4], .1), ([.5, .6], .4), ([.1, .2], .3) \} \\
&= \{ ([.3, .6], .6), ([.9, 1.1], .7), ([.7, .9], .5) \}.
\end{aligned} \tag{8}$$

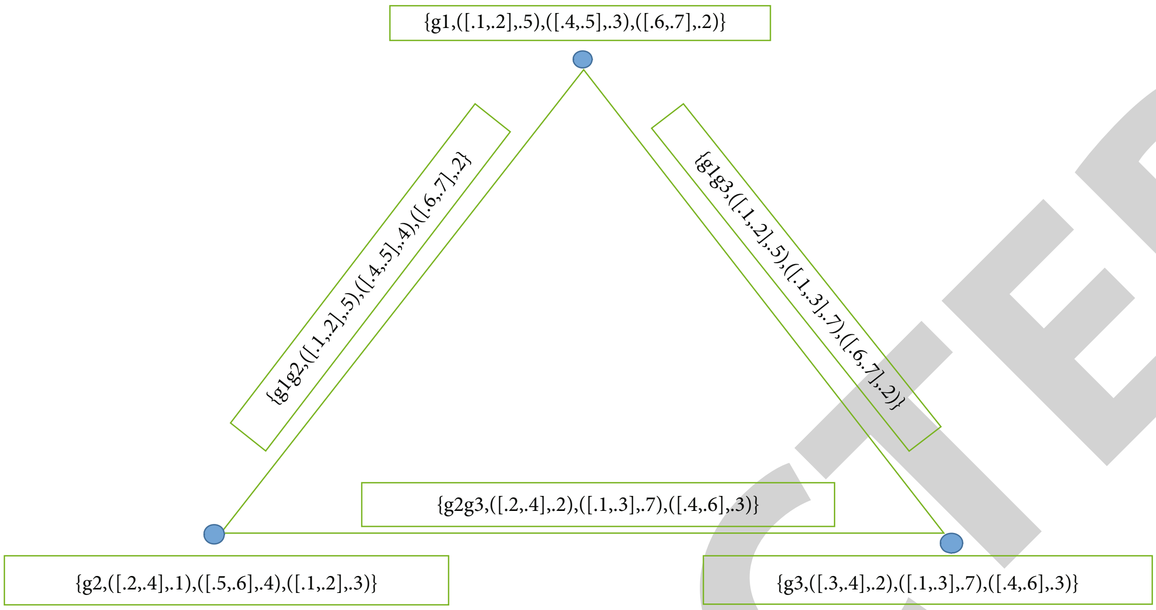

FIGURE 1: Represents a neutrosophic cubic graph.

Definition 5. A closed neighborhood degree for a vertex g in $\mathscr{L}_C^G = (\Gamma, \Lambda)$ is given by

$$\begin{aligned}\theta(\mathbb{N}_{ncg}[g]) = & \sum_{s\in\{\mathbb{N}_{ncg}(g)\cup\{g\}\}} \Phi(s)\Big\{\Big(\Big[\theta_T^l[g], \theta_T^r[g]\Big], \theta_T[g]\Big), \\ & \cdot \Big(\Big[\theta_T^l[g], \theta_I^r[g]\Big], \theta_I[g]\Big), \Big(\Big[\theta_F^l[g], \theta_F^r[g]\Big], \theta_F[g]\Big)\Big\} \\ = & \Bigg\{\Bigg(\Bigg[\sum_{s\in\{\mathbb{N}_{ncg}(g)\cup\{g\}\}} \alpha_T^l(s), \sum_{s\in\{\mathbb{N}_{ncg}(g)\cup\{g\}\}} \alpha_T^r(s)\Bigg], \\ & \cdot \sum_{s\in\{\mathbb{N}_{ncg}(g)\cup\{g\}\}} \alpha_T(s)\Bigg), \\ & \cdot \Bigg(\Bigg[\sum_{s\in\{\mathbb{N}_{ncg}(g)\cup\{g\}\}} \alpha_T^l(s), \sum_{s\in\{\mathbb{N}_{ncg}(g)\cup\{g\}\}} \alpha_I^r(s)\Bigg], \\ & \cdot \sum_{s\in\{\mathbb{N}_{ncg}(g)\cup\{g\}\}} \alpha_I(s)), \Bigg(\Bigg[\sum_{s\in\{\mathbb{N}_{ncg}(g)\cup\{g\}\}} \alpha_F^l(y), \\ & \cdot \sum_{s\in\{\mathbb{N}_{ncg}(g)\cup\{g\}\}} \alpha_F^r(y)], \sum_{s\in\{\mathbb{N}_{ncg}(g)\cup\{g\}\}} \alpha_F(y)\Bigg)\Bigg\}.\end{aligned} \tag{9}$$

It consists of the sum of membership functions of all vertices adjacent to x and membership function of x.

Example 6. Consider Example 4, closed neighborhood degree $\theta(\mathbb{N}_{ncg}[g])$ for each element $g \in A$ in $\mathscr{L}_C^G$ is given by

$$\begin{aligned}\theta(\mathbb{N}_{ncg}[g_1]) &= (\Phi(g_2) + \Phi(g_3)) + \Phi(g_1) \\ &= (\{([.2, .4], .1), ([.5, .6], .4), ([.1, .2], .3)\} \\ &\quad + \{([.3, .4], .2), ([.1, .3], .7), ([.4, .6], .3)\}) \\ &\quad + \{([.1, .2], .5), ([.4, .5], .3), ([.6, .7], .2)\} \\ &= \{([.6, 1], .8), ([1, 1.4], 1.4), ([1.1, 1.5], .8)\};\end{aligned} \tag{10}$$

similarly,

$$\theta(\mathbb{N}_{ncg}[g_2]) = (\Phi(g_1) + \Phi(g_3)) + \Phi(g_2); \tag{11}$$

also, we have

$$\theta(\mathbb{N}_{ncg}[g_3]) = (\Phi(g_1) + \Phi(g_2)) + \Phi(g_3). \tag{12}$$

3.2. Regular and Totally Regular Neutrosophic Cubic Graphs.

In this subsection, we present the idea of regular and totally regular neutrosophic cubic graphs based on the open neighborhood degree and closed neighborhood degree.

Definition 7. If every vertex in $\mathscr{L}_C^G$ has the same open neighborhood degree n, i.e., if $\theta(\mathbb{N}_{ncg}(g)) = n$, for all $g \in A$, then $\mathscr{L}_C^G$ is called an n regular neutrosophic cubic graph.

Definition 8. If closed neighborhood degree is the same for all vertices in $\mathscr{L}_C^G$, i.e., if $\theta(\mathbb{N}_{ncg}[g]) = m$, for all $g \in A$, then $\mathscr{L}_C^G$ is called an m-totally regular neutrosophic cubic graph.

Example 9. Consider Example 4; here, $\mathscr{L}_C^G$ is a totally regular but not a regular neutrosophic cubic graph.

Example 10. Consider $\mathscr{L}_C^G = (\Gamma, \Lambda)$ for any graph $\mathbf{G} = (A, B)$ with $A = \{v_1, v_2, v_3\}, B = \{v_1v_2, v_2v_3, v_1v_3\}$ and let

$$\begin{aligned}\Gamma = \langle & \{v_1, ([.3, .5], .6), ([.4, .5], .3), ([.6, .7], .2)\}, \\ & \cdot \{v_2, ([.3, .5], .6), ([.4, .5], .3), ([.6, .7], .2)\}, \\ & \cdot \{v_3, ([.3, .5], .6), ([.4, .5], .3), ([.6, .7], .3)\}\rangle, \\ \Lambda = \langle & \{v_1v_2, ([.2, .4], .6), ([.4, .5], .3), ([.6, .7], .2)\}, \\ & \cdot \{v_2v_3, ([.1, .2], .6), ([.4, .5], .3), ([.6, .7], .3)\}, \\ & \cdot \{v_1v_3, ([.3, .4], .6), ([.4, .5], .3), ([.6, .7], .3)\}\rangle,\end{aligned} \tag{13}$$

then the neutrosophic cubic open neighborhood degree of each vertex is given by

$$\begin{gathered}\theta\left(\mathbb{N}_{ncg}(v_1)\right) = \Phi(v_2) + \Phi(v_3) = \{([.6, 1], 1.2), ([.8, 1], .6), ([1.2, 1.4], .4)\},\\ \theta\left(\mathbb{N}_{ncg}(v_2) = \Phi(v_1) + \Phi(v_3) = \{([.6, 1], 1.2), ([.8, 1], .6), ([1.2, 1.4], .4)\},\\ \theta\left(\mathbb{N}_{ncg}(v_3) = \Phi(v_1) + \Phi(v_2) = \{([.6, 1], 1.2), ([.8, 1], .6), ([1.2, 1.4], .4)\};\end{gathered} \tag{14}$$

similarly, the neutrosophic cubic closed neighborhood degree of each vertex is given by

$$\begin{gathered}\theta\left(\mathbb{N}_{ncg}[v_1]\right) = (\Phi(v_2) + \Phi(v_3)) + \Phi(v_1) = \{([.9, 1.5], 1.8), ([1.2, 1.5], .9), ([1.8, 2.1], .7)\},\\ \theta\left(\mathbb{N}_{ncg}[v_2]\right) = (\Phi(v_1) + \Phi(v_3)) + \Phi(v_2) = \{([.9, 1.5], 1.8), ([1.2, 1.5], .9), ([1.8, 2.1], .7)\},\\ \theta\left(\mathbb{N}_{ncg}[v_3]\right) = (\Phi(v_2) + \Phi(v_1)) + \Phi(v_3) = \{([.9, 1.5], 1.8), ([1.2, 1.5], .9), ([1.8, 2.1], .7)\}.\end{gathered} \tag{15}$$

As $\theta(\mathbb{N}_{ncg}(v_1)) = \theta(\mathbb{N}_{ncg}(v_2)) = \theta(\mathbb{N}_{ncg}(v_3))$ also $\theta(\mathbb{N}_{ncg}[v_1]) = \theta(\mathbb{N}_{ncg}[v_2]) = \theta(\mathbb{N}_{ncg}[v_3])$. Hence, $\mathscr{L}_C^G$ is regular, also totally regular neutrosophic cubic graph, as shown in Figure 2.

Example 11. Let $\mathscr{L}_C^G = (\Gamma, \Lambda)$ be a neutrosophic cubic graph of $\mathbf{G} = (A, B)$ with $A = \{v_1, v_2, v_3\}, B = \{v_1 v_2, v_2 v_3, v_1 v_3\}$ such that Γ and Λ are given by Tables 2 and 3.

Then,

$$\begin{aligned}\theta\left(\mathbb{N}_{ncg}(v_1)\right. &= \sum_{\alpha \in \mathbb{N}_{ncg}(v_1), \alpha \neq v_1} \Phi(\alpha) = \Phi(v_2)\\ &= \{([.2, .4], .3), ([.1, .4], .3), ([.6, .7], .7)\},\end{aligned} \tag{16}$$

$$\begin{aligned}\theta\left(\mathbb{N}_{ncg}(v_2)\right) &= \sum_{\alpha \in \mathbb{N}_{ncg}(v_2), \alpha \neq v_2} \Phi(\alpha) = \Phi(v_1) + \Phi(v_3)\\ &= \{([.4, .8], .6), ([.3, .6], .3), ([1.2, 1.4], 1.1)\}.\end{aligned} \tag{17}$$

Since $\theta(\mathbb{N}_{ncg}(v_1) \neq \theta(\mathbb{N}_{ncg}(v_2))$ although $\theta(\mathbb{N}_{ncg}(v_1)) = \theta(\mathbb{N}_{ncg}(v_3))$, so $\mathscr{L}_C^G$ is not regular as well as totally regular as shown in Figure 3.

Theorem 12. *Let $\mathscr{L}_C^G = (\Gamma, \Lambda)$ be a neutrosophic cubic graph of $\mathbf{G}$, with Γ showing $\mathscr{L}_C^S$ for vertex set A and Λ is $\mathscr{L}_C^S$ for edge set B. Then,*

$$\begin{aligned}\Gamma = \Phi(g) = \Big\{ \Big\langle g\left(\left[\alpha_T^l, \alpha_T^r\right](g), \alpha_T(g)\right),\\ \cdot \left(\left[\alpha_I^l, \alpha_I^r\right], \alpha_I(g)\right), \left(\left[\alpha_F^l, \alpha_F^r\right](g), \alpha_F(g)\right) \Big\rangle | g \in A \Big\}\end{aligned} \tag{18}$$

is a constant if and only if we have equivalence in the following:

(I) $\mathscr{L}_C^G$ is regular

(II) $\mathscr{L}_C^G$ is totally regular

Proof. Suppose

$$\begin{aligned}\Gamma = \Phi(g) = \Big\{ \left(\left[\alpha_T^l, \alpha_T^r\right](g), \alpha_T(g)\right), \left(\left[\alpha_I^l, \alpha_I^r\right](g), \alpha_I(g)\right),\\ \cdot \left(\left[\alpha_F^l, \alpha_F^r\right](g), \alpha_F(g)\right) \Big\} = k\end{aligned} \tag{19}$$

for all $g \in A$, where k is some constant, then

$$\begin{gathered}\alpha_T^l(g) = t_1, \alpha_T^r(g) = t^2, \alpha_T(g) = t,\\ \alpha_I^l(g) = i_1, \alpha_I^r(g) = i^2, \alpha_I(g) = i,\\ \alpha_F^l(g) = f_1, \alpha_F^r(g) = f^2, \alpha_F(g) = f\end{gathered} \tag{20}$$

for all $g \in A$ and for some constants $t_1, t_2, t, i_1, i_2, i, f_1, f_2, f$.

(I) ⇒ (II) Let $\mathscr{L}_C^G$ be a regular, then $\theta(\mathbb{N}_{ncg}(g)) = n$ for all $g \in A$. So,

$$\begin{aligned}\theta\left(\mathbb{N}_{ncg}(g)\right) &= \sum_{s \in \mathbb{N}_{ncg}(g), s \neq g} \Phi(s) = n\\ &= \Big\{ \left(\theta_T^l(g), \theta_T(g)\right), \left(\left[\theta_I^l(g), \theta_I^r(g)\right], \theta_I(g)\right)\\ &\quad \cdot \left(\left[\theta_F^l(g), \theta_F^r(g)\right], \theta_F(g)\right) \Big\}.\end{aligned} \tag{21}$$

Hence, for all $g \in A$, we have

$$\begin{gathered}\theta_T^l(g) = n_{1t}, \theta_T^r(g) = n_{2t}, \theta_T(g) = n_t,\\ \theta_I^l(g) = n_{1i}, \theta_I^r(g) = n_{2i}, \theta_I(g) = n_i,\\ \theta_F^l(g) = n_{1f}, \theta_F^r(g) = n_{2f}, \theta_F(g) = n_f.\end{gathered} \tag{22}$$

Thus,

$$\begin{aligned}\theta\left(\mathbb{N}_{ncg}[g]\right) &= \sum_{s \in \left\{\mathbb{N}_{ncg}(g) \cup \{g\}\right\}} \Phi(s) = \Big\{ \left(\left[\theta_T^l[g], \theta_T^r[g]\right], \theta_T[g]\right),\\ &\quad \cdot \left(\left[\theta_I^l[g], \theta_I^r[g]\right], \theta_I[g]\right), \left(\left[\theta_F^l[g], \theta_F^r[g]\right], \theta_F[g]\right) \Big\}\\ &= \Big\{ \left(\left[\theta_T^l(g), \theta_T^r(g)\right], \theta_T(g)\right), \left(\left[\theta_I^l(g), \theta_I^r(g)\right], \theta_I(g)\right),\\ &\quad \cdot \left(\left[\theta_F^l(g), \theta_F^r(g)\right], \theta_F(g)\right) \Big\} + \Big\{ \left(\left[\alpha_T^l, \alpha_T^r\right](g), \alpha_T(g)\right),\\ &\quad \cdot \left(\left[\alpha_I^l, \alpha_I^r\right](g), \alpha_I(x)\right), \left(\left[\alpha_F^l, \alpha_F^r\right](g), \alpha_F(g)\right) \Big\}\\ &= \{([n_{1t}, n_{2t}], n_t), ([n_{1i}, n_{2i}], n_i), ([n_{1f}, n_{2f}], n_f)\}\\ &\quad + \{([t_1, t_2], t), ([i_1, i_2], i), ([f_1, f_2], f)\},\end{aligned} \tag{23}$$

i.e.,

$$\theta\left(\mathbb{N}_{ncg}[g]\right) = \theta\left(\mathbb{N}_{ncg}(g)\right) + \Phi(g) = n + k, \tag{24}$$

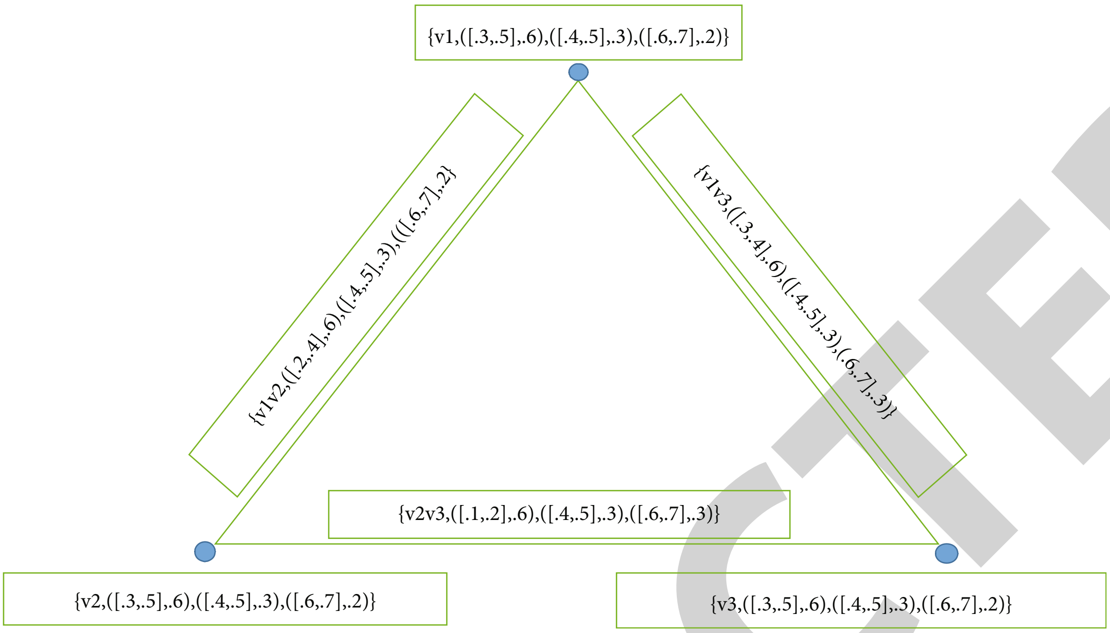

Figure 2: Represents a regular and totally regular neutrosophic cubic graph.

Table 2: A neutrosophic cubic membership for vertices.

	A	α_T^l, α_T^r	α_T	α_I^l, α_I^r	α_I	α_F^l, α_F^r	α_F
$\Gamma =$	v_1	$[.3, .5]$	$.4$	$[.1, .3]$	$.2$	$[.5, .6]$	$.5$
	v_2	$[.2, .4]$	$.3$	$[: 1, .4]$	$.3$	$[.6, .7]$	$.7$
	v_3	$[.1, .3]$	$.2$	$[: 2, .3]$	$.7$	$[.7, .8]$	$.6$

Table 3: A neutrosophic cubic membership for edges.

	B	β_T^l, β_T^r	β_T	β_T^l, β_I^r	β_I	β_F^l, β_F^r	β_F
$\Lambda =$	$v_1 v_2$	$[.2, .4]$	$.4$	$[.1, .3]$	$.2$	$[.3, .5]$	$.6$
	$v_2 v_3$	$[.1, .3]$	$.3$	$[.1, .3]$	$.3$	$[.4, .6]$	$.5$

a constant number, for all $g \in A$. Thus, $\mathscr{L}_C^G$ is totally regular.

(II) $\Rightarrow$ (I) Suppose that $\mathscr{L}_C^G$ is totally regular. Then, for all $g \in A$,

$$\begin{gathered} \theta\left(\mathbb{N}_{ncg}[g]\right) = m, \\ \theta\left(\mathbb{N}_{ncg}[g]\right) = \theta\left(\mathbb{N}_{ncg}(g)\right) + \Phi(g), \end{gathered} \tag{25}$$

i.e.,

$$\begin{aligned} m = \Big\{ & \left(\left[\theta_T^l[g], \theta_T^r[g]\right], \theta_T[g]\right), \left(\left[\theta_I^l[g], \theta_I^r[g]\right], \theta_I[g]\right), \\ & \cdot \left(\left[\theta_F^l[g], \theta_F^r[g]\right], \theta_F[g]\right) \Big\}. \end{aligned} \tag{26}$$

Let

$$m = \left\{ ([m_{1t}, m_{2t}], m_t), ([m_{1i}, m_{2i}], m_i), \left(\left[m_{1f}, m_{2f}\right], m_f\right) \right\}, \tag{27}$$

where $m_{1t}, m_{2t}, m_t, m_{1i}, m_{2i}, m_i, m_{1f}, m_{2f}, m_f$ are all constants. Also, given that for every $g \in A$,

$$\begin{aligned} \Gamma = \Phi(g) = \Big\{ & \left(\left[\alpha_T^l, \alpha_T^r\right](g), \alpha_T(g)\right), \left(\left[\alpha_I^l, \alpha_I^r\right](g), \alpha_I(g)\right), \\ & \cdot \left(\left[\alpha_F^l, \alpha_F^r\right](g), \alpha_F(g)\right) \Big\} = k, \end{aligned} \tag{28}$$

where k is a constant; also, let

$$\Phi(g) = k = \left\{ ([t_1, t_2], t), ([i_1, i_2], i), ([f_1, f]_2, f) \right\}. \tag{29}$$

Then,

$$\begin{aligned} & \left\{ ([m_{1t}, m_{2t}], m_t), ([m_{1i}, m_{2i}], m_i), \left(\left[m_{1f}, m_{2f}\right], m_f\right) \right\} \\ & = \theta\left(\mathbb{N}_{ncg}(g)\right) + \left\{ ([t_1, t_2], t), ([i_1, i_2], i), ([f_1, f_2], f) \right\} \end{aligned} \tag{30}$$

for all $g \in A$. Hence, for all $g \in A$,

$$\begin{aligned} \theta\left(\mathbb{N}_{ncg}(g)\right) &= \left\{ ([m_{1t}, m_{2t}], m_t), ([m_{1i}, m_{2i}], m_i), \left(\left[m_{1f}, m_{2f}\right], m_f\right) \right\} \\ &\quad - \left\{ ([t_1, t_2], t), ([i_1, i_2], i), ([f_1, f_2], f) \right\} \\ &= \{ ([m_{1t} - t_1, m_{2t} - t_2], m_t - t), \\ &\quad \cdot ([m_{1i} - i_1, m_{2i} - i_2], m_i - i), \\ &\quad \cdot \left(\left[m_{1f} - f_1, m_{2f} - f_2, m_f - f\right)\right\}, \end{aligned} \tag{31}$$

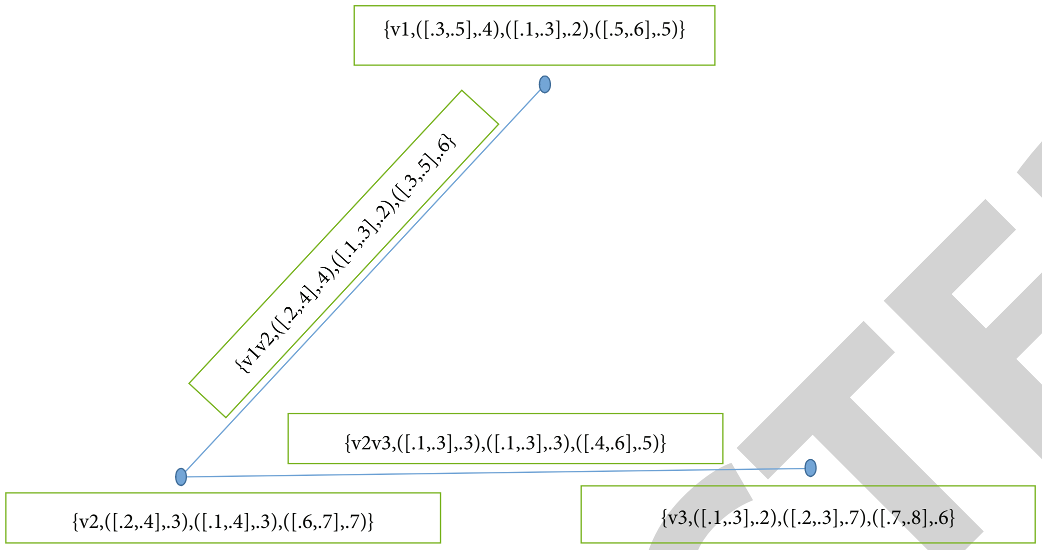

FIGURE 3: Shows a neutrosophic cubic graph, which is not regular as well as totally regular.

which a constant number. Thus, $\mathscr{L}_C^G$ is regular. So, (I) and (II) have equivalence. Conversely, if $\mathscr{L}_C^G$ is totally regular, then $\theta(\mathbb{N}_{ncg}[g]) = m$ (a constant) for all $g \in A$; also, $\mathscr{L}_C^G$ is regular. So, $\theta(\mathbb{N}_{ncg}(g)) = n$ say for every $g \in A$. Hence, for every $g \in A$,

$$\theta(\mathbb{N}_{ncg}[g]) = \theta(\mathbb{N}_{ncg}(g)) + \Phi(g) \Rightarrow m = n + \Phi(g) \Rightarrow \Phi(g) = m - n, \tag{32}$$

a constant no for every $g \in A$. Hence,

$$\begin{aligned} \Gamma = \Phi(g) = \Big\{ &\Big(\big[\alpha_T^l, \alpha_T^r\big](g), \alpha_T(g)\Big), \Big(\big[\alpha_I^l, \alpha_I^r\big](g), \alpha_I(g)\Big), \\ &\cdot \Big(\big[\alpha_F^l, \alpha_F^r\big](g), \alpha_F(g)\Big)\Big\} \end{aligned} \tag{33}$$

for all $g \in A$ is a constant function. ☐

Theorem 13. *Consider $\mathscr{L}_C^G = (\Gamma, \Lambda)$ as a neutrosophic cubic graph with crisp graph $\mathbf{G}$ of an odd cycle. Then, $\mathscr{L}_C^G$ is regular if and only if Λ is a constant function.*

Proof. Let

$$\begin{aligned} \Lambda = \Psi(xy) = \Big\{ &\Big(\big[\beta_T^l, \beta_T^r\big](xy), \beta_T(xy)\Big), \\ &\cdot \Big(\big[\beta_I^l, \beta_I^r\big](xy), \beta_I(xy)\Big), \Big(\big[\beta_F^l, \beta_F^r\big](xy), \beta_F(xy)\Big)\Big\} = C \end{aligned} \tag{34}$$

be a constant function for all $xy \in B$. Then,

$$C = \left\{ ([c_{1t}, c_{2t}], c_t), ([c_{1i}, c_{2i}], c_i), \left([c_{1f}, c_{2f}], c_f\right) \right\}. \tag{35}$$

Now,

$$\begin{aligned} \theta(\mathbb{N}_{ncg}(x)) = \Big\{ &\Big(\big[\theta_T^l(x), \theta_T^r(x)\big], \theta_T(x)\Big), \Big(\big[\theta_I^l(x), \theta_I^r(x)\big], \theta_I(x)\Big), \\ &\cdot \Big(\big[\theta_F^l(x), \theta_F^r(x)\big], \theta_F(x)\Big)\Big\} \\ = \{ &([2c_{1t}, 2c_{2t}], 2c_t), ([2c_{1i}, 2c_{2i}], 2c_i), \\ &\cdot \left([2c_{1f}, 2c_{2f}], 2c_f\right)\Big\} = 2C, \quad \forall x \in A. \end{aligned} \tag{36}$$

Since $\mathbf{G}$ is an odd cycle, $\mathscr{L}_C^G$ is regular. Conversely, suppose that $\mathscr{L}_C^G$ is an n- regular, where

$$n = \left\{ ([n_{1t}, n_{2t}], n_t), ([n_{1i}, n_{2i}], n_i), \left([n_{1f}, n_{2f}], n_f\right) \right\}. \tag{37}$$

Let $e_1, e_2, e_3, \cdots, e_{2n+1}$ be edges of $\mathscr{L}_C^G$ in that order. Let

$$\begin{aligned} \beta_T^l(e_1) &= k_1, \upsilon_T^l(e_2) = n_{1t} - k_1, \\ \beta_T^l(e_3) &= n_{1t} - (n_{1t} - k_1) = k_1, \upsilon_T^l(e_4) = n_{1t} - k_1, \end{aligned} \tag{38}$$

and so on. Therefore,

$$\beta_T^l(e_i) = \begin{cases} k_1, & \text{if } i \text{ is odd}, \\ n_{1t} - k, & \text{if } i \text{ is even}. \end{cases} \tag{39}$$

This implies

$$\beta_T^l(e_1) = \upsilon_T^l(e_{2n+1}) = k_1. \tag{40}$$

So, if e_1 and e_{2n+1} are incident at a vertex v_1, then

$$\theta(v_1) = n_{1t}, \theta(e_1) + \theta(e_{2n+1}) = n_{1t}. \tag{41}$$

Hence,

$$k_1 + k_1 = n_{1t}, 2k_1 = n_{1t}, \tag{42}$$

and so, $k_1 = n_{1t}/2$, which shows that β^l_T is a constant function. Similarly, let

$$\begin{aligned}\beta^r_T(e_1) &= k_2, \beta^r_T(e_2) = n_{2t} - k_2, \beta^r_T(e_3) = n_{2t} - (n_{2t} - k_2)\\ &= k_2, \beta^r_T(e_4) = n_{2t} - k_2,\end{aligned} \tag{43}$$

and so on. Therefore,

$$\beta^r_T(e_i) = \begin{cases} k_2, & \text{if } i \text{ is odd,}\\ n_{2t} - k_2, & \text{if } i \text{ is even.}\end{cases} \tag{44}$$

Thus, $\beta^r_T(e_2) = \beta^r_T(e_{2n}) = k_2$. So, if e_2 and e_{2n} are incident at a vertex v_2, then

$$\theta(v_2) = n_{2t}, \theta(e_2) + \theta(e_{2n}) = n_{2t}. \tag{45}$$

Hence,

$$k_2 + k_2 = n_{2t}, 2k_2 = n_{2t}, \tag{46}$$

and so, $k_2 = n_{2t}/2$, which shows that υ^r_T is a constant function.

Similar results hold for membership functions $\beta(x)$, $\beta^-_I(x)$, $\beta^+_I(x)$, $\beta_I(x)$, $\beta^-_F(x)$, $\beta^+_F(x)$, $\beta_F(x)$. This shows that

$$\begin{aligned}\Lambda = \Psi(xy) = \Big\{&\Big(\Big[\beta^l_T, \beta^r_T\Big](xy), \beta_T(xy)\Big), \Big(\Big[\beta^l_I, \beta^r_I\Big](xy), \beta_I(xy)\Big),\\ &\cdot \Big(\Big[\beta^l_F, \beta^r_F\Big](xy), \beta_F(xy)\Big)\Big\}\end{aligned} \tag{47}$$

is a constant function. □

3.3. Complete Neutrosophic Cubic Graphs.

In this subsection, we present complete neutrosophic cubic graph $\mathscr{L}^G_C$.

Definition 14. Consider $\mathscr{L}^G_C = (\Gamma, \Lambda)$ be a neutrosophic cubic graph for any arbitrary graph $\mathbf{G} = (A, B)$. Then, $\mathscr{L}^G_C$ is complete if

$$\begin{gathered}\widehat{\beta_T}(v_1v_2) = r\min\left\{\widehat{\alpha_T}(v_1), \alpha_T(v_2)\right\},\\ \beta_T(v_1v_2) = \max\{\alpha_T(v_1), \alpha_T(v_2)\},\\ \widehat{\beta_I}(v_1v_2) = r\min\left\{\widehat{\alpha_I}(v_1), \alpha_I(v_2)\right\},\\ \beta_I(v_1v_2) = \max\{\alpha_I(v_1), \alpha_I(v_2)\},\\ \widehat{\beta_F}(v_1v_2) = r\max\left\{\widehat{\alpha_F}(v_1), \alpha_F(v_2)\right\},\\ \beta_F(v_1v_2) = \min\{\alpha_F(v_1), \alpha_F(v_2)\}\end{gathered} \tag{48}$$

for all vertices $v_1, v_2 \in A$ and for all edges $v_1v_2 \in B$.

Example 15. Consider $\mathscr{L}^G_C = (\Gamma, \Lambda)$ for a graph $\mathbf{G} = (A, B)$ with $A = \{v_1, v_2, v_3\}$, $B = \{v_1v_2, v_2v_3, v_1v_3\}$ and let

$$\begin{aligned}\Gamma = \langle\{&v_1, ([.3, .4], .6), ([.2, .6], 0.3), ([.6, .3], .3)\},\\ &\cdot\{v_2, ([.2, .5], .5), ([.3, .7], 0.4), ([.5, .4], .2)\},\\ &\cdot\{v_3, ([.1, .2], .6), ([.4, .5], 0.3), ([.6, .2], .3)\}\rangle,\end{aligned} \tag{49}$$

$$\begin{aligned}\Lambda = \langle\{&v_1v_2, ([.2, .4], .6), ([.2, .6], .3), ([.6, .4], .2)\},\\ &\cdot\{v_2v_3, ([.1, .2], .6), ([.3, .5], .3), ([.6, .4], .2)\},\\ &\cdot\{v_1v_3, ([.1, .2], .6), ([.2, .5], .3), ([.6, .3], .3)\}\rangle,\end{aligned} \tag{50}$$

then

$$\begin{gathered}\widehat{\beta_T}(v_1v_2) = [.2, .4], r\min\left\{\widehat{\alpha_T}(v_1), \widehat{\alpha_T}(v_2)\right\} = [.2, .4],\\ \beta_T(v_1v_2) = .6, \max\{\alpha_T(v_1), \alpha_T(v_2)\} = .6,\end{gathered} \tag{51}$$

$$\begin{gathered}\widehat{\beta_I}(v_1v_2) = [.2, .6], r\min\left\{\widehat{\alpha_I}(v_1), \widehat{\alpha_I}(v_2)\right\} = [.2, .6],\\ \beta_I(v_1v_2) = .3, \max\{\alpha_I(v_1), \alpha_I(v_2)\} = .3.\end{gathered} \tag{52}$$

Also,

$$\begin{gathered}\widehat{\beta_F}(v_1v_2) = r\max\left\{\widehat{\alpha_F}(v_1), \widehat{\alpha_F}(v_2)\right\} = [.6, .4],\\ \beta_F(v_1v_2) = \min\{\alpha_F(v_1), \alpha_F(v_2)\} = .2\,;\end{gathered} \tag{53}$$

similar holds for other edges. Hence, $\mathscr{L}^G_C$ is a complete neutrosophic cubic graph, as shown in Figure 4.

Definition 16. Let $\mathscr{L}^G_C = (\Gamma, \Lambda)$ be a neutrosophic cubic graph for some graph $G = (V, E)$. The density of $\mathscr{L}^G_C$ is defined as

$$\begin{aligned}\wp\left(\mathscr{L}^G_C\right) = &\left(\left[\wp^l_T\left(\mathscr{L}^G_C\right), \wp^r_T\left(\mathscr{L}^G_C\right)\right], \wp_T\left(\mathscr{L}^G_C\right)\right),\\ &\cdot\left(\left[\wp^l_I\left(\mathscr{L}^G_C\right), \wp^r_I\left(\mathscr{L}^G_C\right)\right], \wp_I\left(\mathscr{L}^G_C\right)\right),\\ &\cdot\left(\left[\wp^l_F\left(\mathscr{L}^G_C\right), \wp^r_F\left(\mathscr{L}^G_C\right)\right], \wp_F\left(\mathscr{L}^G_C\right)\right),\end{aligned} \tag{54}$$

where

$$\wp^l_T\left(\mathscr{L}^G_C\right) = \frac{2\left(\sum_{g_1,g_2\in V}\beta^l_T(g_1g_2)\right)}{\sum_{g_1,g_2\in V}\min\{\alpha^l_T(g_1), \alpha^l_T(g_2)\}},$$

$$\wp^r_T\left(\mathscr{L}^G_C\right) = \frac{2\left(\sum_{g_1,g_2\in V}\beta^r_T(g_1g_2)\right)}{\sum_{g_1,g_2\in V}\min\{\alpha^r_T(g_1), \alpha^r_T(g_2)\}},$$

$$\wp_T\left(\mathscr{L}^G_C\right) = \frac{2\left(\sum_{g_1,g_2\in V}\beta_T(g_1g_2)\right)}{\sum_{g_1,g_2\in V}\min\{\alpha_T(g_1), \alpha_T(g_2)\}},$$

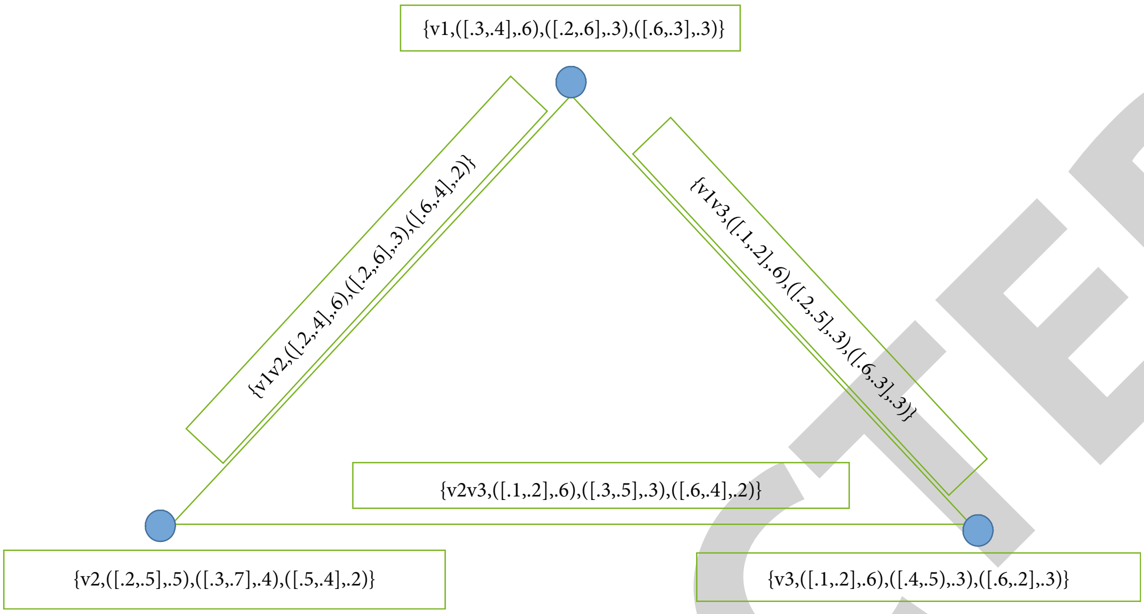

Figure 4: Represents a complete neutrosophic cubic graph.

$$\wp_I^l\left(\mathscr{L}_C^G\right)=\frac{2\left(\sum_{g_1,g_2\in V}\beta_I^l(g_1g_2)\right)}{\sum_{g_1,g_2\in V}\min\left\{\alpha_I^l(g_1),\alpha_I^l(g_2)\right\}},$$

$$\wp_I^r\left(\mathscr{L}_C^G\right)=\frac{2\left(\sum_{g_1,g_2\in V}\beta_I^r(g_1g_2)\right)}{\sum_{g_1,g_2\in V}\min\left\{\alpha_I^r(g_1),\alpha_I^r(g_2)\right\}},$$

$$\wp_I\left(\mathscr{L}_C^G\right)=\frac{2\left(\sum_{g_1,g_2\in V}\beta_I(g_1g_2)\right)}{\sum_{g_1,g_2\in V}\min\left\{\alpha_I(g_1),\alpha_I(g_2)\right\}},$$

$$\wp_F^l\left(\mathscr{L}_C^G\right)=\frac{2\left(\sum_{g_1,g_2\in V}\beta_F^l(g_1g_2)\right)}{\sum_{g_1,g_2\in V}\min\left\{\alpha_F^l(g_1),\alpha_F^l(g_2)\right\}},$$

$$\wp_F^r\left(\mathscr{L}_C^G\right)=\frac{2\left(\sum_{g_1,g_2\in V}\beta_F^r(g_1g_2)\right)}{\sum_{g_1,g_2\in V}\min\left\{\alpha_F^r(g_1),\alpha_F^r(g_2)\right\}},$$

$$\wp_F\left(\mathscr{L}_C^G\right)=\frac{2\left(\sum_{g_1,g_2\in V}\beta_T^l(g_1g_2)\right)}{\sum_{g_1,g_2\in V}\min\left\{\alpha_F^l(g_1),\alpha_F^l(g_2)\right\}}. \tag{55}$$

Example 17. Consider $\mathscr{L}_C^G=(\Gamma,\Lambda)$ for a graph $\mathbf{G}=(A,B)$, with vertex set $A=\{v_1,v_2,v_3\}$ and edge set $B=\{v_1v_2,v_2v_3,v_1v_3\}$. Also, let Γ and Λ be neutrosophic membership functions for vertices and edges, respectively, shown in Tables 4 and 5.

Table 4: Neutrosophic membership functions for vertices.

	A	$[\alpha_T^l,\alpha_T^r]$	α_T	$[\alpha_I^l,\alpha_I^r]$	α_I	$[\alpha_F^l,\alpha_F^r]$	α_F
$\Gamma=$	v_1	[.1, .5]	.6	[.5, .7]	1	[.6, .7]	.9
	v_2	[.2, .6]	.5	[.6, .8]	.8	[.7, .8]	.8
	v_3	[.3, .7]	.8	[.7, .8]	.7	[.5, .6]	.7

Table 5: Neutrosophic membership functions for edges.

	B	$[\beta_T^l,\beta_T^r]$	β_T	$[\beta_I^l,\beta_I^r]$	β_I	$[\beta_F^l,\beta_F^r]$	β_F
$\Lambda=$	v_1v_2	[.1, .2]	.5	[.4, .5]	.4	[.5, .6]	.2
	v_2v_3	[.2, .4]	.2	[.1, .3]	.5	[.4, .5]	.3
	v_1v_3	[.1, .2]	.5	[.1, .3]	.6	[.4, .6]	.2

Since

$$\begin{aligned}2\left(\sum_{x,y\in A}\Psi(xy)\right)&=2(\Psi(v_1v_2)+\Psi(v_2v_3)+\Psi(v_1v_3))\\&=2\left(\sum_{x,y\in A}\left\{\left(\left[\beta_T^l,\beta_T^r\right](xy),\beta_T(xy)\right),\right.\right.\\&\quad\cdot\left(\left[\beta_I^l,\beta_I^r\right](xy),\beta_I(xy)\right),\\&\quad\cdot\left.\left(\left[\beta_F^l,\beta_F^r\right](xy),\beta_F(xy)\right)\right\}\\&=2\{([.4,.8],1.2),([.6,1.1],1.5),\\&\quad\cdot([1.3,1.7],.7)\}=\{([.8,1.6],2.4),\\&\quad\cdot([1.2,2.2],3),([2.6,3.4],1.4)\}.\end{aligned} \tag{56}$$

Also,

$$\begin{aligned}\min\{\Phi(v_1),\Phi(v_2)\}&=\{([.1,0.5],.5),([.5,.7],.8),([.6,.7],.8)\},\\ \min\{\Phi(v_2),\Phi(v_3)\}&=\{([.2,.6],.5),([.6,.8],.7),([.5,.6],.7)\},\\ \min\{\Phi(v_1),\Phi(v_3)\}&=\{([.1,.5],.6),([.5,0.7],.7),([.5,.6],.7)\}.\end{aligned}\tag{57}$$

So,

$$\sum_{x,y\in A}\min\{\Phi(x),\Phi(y)\}=\{([.4,1.6],1.6),([1.6,2.2],2.2),([1.6,1.9],2.2)\}.\tag{58}$$

Hence, $\wp(\mathscr{L}_C^G)$ is given by

$$\wp\left(\mathscr{L}_C^G\right)=\{([2,1],1.6),([.75,1],1.36),([1.625,1.78],.63)\}.\tag{59}$$

3.4. Balanced and Strictly Balanced Neutrosophic Cubic Graphs. In this subsection, we use the density function $\wp$ to discuss the idea of balanced and strictly balanced neutrosophic cubic graph $\mathscr{L}_C^G$.

Definition 18. $\mathscr{L}_C^G$ is balanced if $\wp(H)\leq\wp(\mathscr{L}_C^G)$ for all subgraphs H of $\mathscr{L}_C^G$.

Definition 19. $\mathscr{L}_C^G$ is strictly balanced if $\wp(H)=\wp(\mathscr{L}_C^G)$ for all nonempty subgraphs H of $\mathscr{L}_C^G$.

Example 20. Consider $\mathscr{L}_C^G$ as given in Example 17. Let $H_1=\{a,b\}$, $H_2=\{a,c\}$, $H_3=\{b,c\}$. Then,

$$\begin{aligned}\wp(H_1)&=\frac{2\Psi(ab)}{\min\{\Phi(a),\Phi(b)\}}\\ &=\frac{\{2([.1,.2],.5),([.4,.5],.4),([.5,.6],.2)\}}{\{([.1,.5],.5),([.5,.7],.8),([.6,.7],.8)\}}\\ &=\frac{\{([.2,.4],1),([.8,1],.8),([1,1.2],.4)\}}{\{([.1,.5],.5),([.5,.7],.8),([.6,0.7],.8)\}}\\ &=\{([2,.8],2),([1.6,1.4],1),([1.66,1.7],.5)\},\\ \wp(H_2)&=\frac{2\Psi(ac)}{\min\{\Phi(a),\Phi(c)\}}\\ &=\frac{\{2([.1,.2],.5),([.1,.3],.6),([.4,.6],.2)\}}{\{([.1,.5],.6),([.5,.7],.7),([.5,.6],.7)\}}\\ &=\frac{\{([.2,.4],1),([.2,.6],1.2),([.8,1.2],.4)\}}{\{([.1,.5],.6),([.5,.7],.7),([.5,.6],.7)\}}\\ &=\{([2,1.33],.4),([.33,.75],1.42),([1.6,1.66],.85)\};\end{aligned}\tag{60}$$

also,

$$\begin{aligned}\wp(H_3)&=\frac{2\Psi(bc)}{\min\{\Phi(b),\Phi(c)\}}\\ &=\frac{2\{([.2,.4],.2),([.1,.3],.5),([.4,.5],.3)\}}{\{([.2,.6],.5),([.6,.8],.7),([.5,.6],.7)\}}\\ &=\frac{\{([.4,.8],.2),([.2,.6],1),([.8,1],.6)\}}{\{([.2,.6],.5),([.6,.8],.7),([.5,.6],.7)\}}\\ &=\{([2,1.33],.4),([.33,.75],1.42),([1.6,1.66],.85)\}.\end{aligned}\tag{61}$$

Hence,

$$\begin{aligned}\wp(H_1)&=\{([2,.8],2),([1.6,1.4],1),([1.66,1.7],.5)\},\\ \wp(H_2)&=\{([2,.8],1.66),([.4,.85],1.7),([1.6,2],.57)\},\\ \wp(H_3)&=\{([2,1.33],.4),([.33,.75],1.42),([1.6,1.66],.85)\},\end{aligned}\tag{62}$$

as $\wp(H)\lneqq\wp(\mathscr{L}_C^G)$, for all nonempty subgraphs H of $\mathscr{L}_C^G$. Hence, $\mathscr{L}_C^G$ is not balanced.

Remark 21. All regular neutrosophic cubic graphs are not necessarily balanced.

3.5. Irregular and Totally Irregular Neutrosophic Cubic Graphs. In this subsection, we use the neighborhood degrees to discuss the idea of irregular and totally irregular neutrosophic cubic graph $\mathscr{L}_C^G$.

Definition 22. If there is at least one vertex in $\mathscr{L}_C^G$ adjacent to vertices having different open neighborhood degrees, then $\mathscr{L}_C^G$ is called irregular, i.e., if $\theta(\mathbb{N}_{ncg}(v))\neq n$ for all $v\in A$.

Example 23. Consider $\mathscr{L}_C^G=(\Gamma,\Lambda)$ for some graph $\mathbf{G}=(A,B)$, with $A=\{v_1,v_2,v_3,v_4\}$ and

$$B=\{v_1v_2,v_2v_3,v_3v_4,v_2v_4\},\tag{63}$$

and let

$$\begin{aligned}\Gamma=\langle&\{v_1,([.2,.5],.4),([.1,.3],.6),([.6,.7],.8)\},\\ &\cdot\{v_2,([.3,.4],.3),([.2,.5],.5),([.6,.8],.7)\},\\ &\cdot\{v_3,([.1,.3],.5),([.3,.4],.2),([.5,.7],.9)\},\\ &\cdot\{v_4,([.2,.4],.2),([.4,.5],.3),([.7,.8],.6)\}\rangle,\end{aligned}\tag{64}$$

$$\begin{aligned}\Lambda=\langle&\{v_1v_2,([.1,.4],.4),([.1,.5],.6),([.6,.7],.8)\},\\ &\cdot\{v_2v_3,([.1,.3],.3),([.2,.4],.5),([.6,.8],.9)\},\\ &\cdot\{v_3v_4,([.1,.3],.5),([.2,.4],.4),([.5,.7],.9)\},\\ &\cdot\{v_2v_4,([.1,.4],.3),([.2,.4],.5),([.5,.7],1)\}\rangle,\end{aligned}\tag{65}$$

then

$$\begin{aligned}
\theta\left(\mathbb{N}_{ncg}(v_1)\right) &= \Phi(v_2) = \{([.3, .4], .3), ([.2, .5], .5), ([.6, .8], .7)\},\\
\theta\left(\mathbb{N}_{ncg}(v_2)\right) &= \Phi(v_3) + \Phi(v_4)\\
&= \{([.1, .3], .5), ([.3, .4], .2), ([.5, .7], .9)\}\\
&\quad + \{([.2, .4], .2), ([.4, .5], .3), ([.7, .8], .6)\}\\
&= \{([.3, .7], .7), ([.7, .9], .5), ([1.2, 1.5], 1.5)\},\\
&\quad \cdot \theta\left(\mathbb{N}_{ncg}(v_3)\right) = \Phi(v_2) + \Phi(v_4)\\
&= \{([.3, .4], .3), ([.2, .5], .5), ([.6, .8], .7)\}\\
&\quad + \{([.2, .4], .2), ([.4, .5], .3), ([.7, .8], .6)\}\\
&= \{([.5, .8], .5), ([.6, 1], .8), ([1.3, 1.6], 1.3)\}\,;
\end{aligned} \tag{66}$$

hence,

$$\theta\left(\mathbb{N}_{ncg}(v_1)\right) \neq \theta\left(\mathbb{N}_{ncg}(v_2)\right) \neq \theta\left(\mathbb{N}_{ncg}(v_3)\right) \neq \theta\left(\mathbb{N}_{ncg}(v_4)\right). \tag{67}$$

Hence, $\mathscr{L}_C^G$ is irregular as shown in Figure 5.

Definition 24. A connected $\mathscr{L}_C^G$ is totally irregular, if at least one vertex is adjacent to the vertices having different closed neighborhood degrees.

Example 25. Consider $\mathscr{L}_C^G = (\Gamma, \Lambda)$ for $\mathbf{G} = (A, B)$, with

$$\begin{aligned}
A &= \{v_1, v_2, v_3, v_4, v_5\},\\
B &= \{v_1 v_2, v_2 v_3, v_2 v_4, v_3 v_4, v_1 v_3, v_1 v_4, v_4 v_5\},
\end{aligned} \tag{68}$$

and let

$$\begin{aligned}
\Gamma = \langle &\{v_1, ([.3, .5], .4), ([.2, .3], .1), ([.3, .6], .4)\},\\
&\cdot \{v_2, ([.2, .4], .3), ([.3, .4], .2), ([.6, .7], .5)\},\\
&\cdot \{v_3, ([.2, .6], .5), ([.2, .5], .4), ([.5, .7], .4)\},\\
&\cdot \{v_4, ([.3, .5], .4), ([.3, .6], .5), ([.4, .6], .3)\},\\
&\cdot \{v_5, ([.2, .2], .1), ([.2, .3], .4), ([.3, .5], .2)\}\rangle,
\end{aligned} \tag{69}$$

$$\begin{aligned}
\Lambda = \langle &\{v_1 v_2, ([.2, .4], .3), ([.2, .3], .1), ([.3, .5], .3)\},\\
&\cdot \{v_2 v_3, ([.2, .4], .3), ([.2, .4], .2), ([.5, .6], .2)\},\\
&\cdot \{v_2 v_4, ([.1, .4], .2), ([.3, .4], .2), ([.3, .5], .2)\},\\
&\cdot \{v_3 v_4, ([.2, .5], .3), ([.2, .5], .3), ([.4, .5], .3)\},\\
&\cdot \{v_3 v_1, ([.2, .4], .1), ([.2, .3], .1), ([.3, .6], .4)\},\\
&\cdot \{v_1 v_4, ([.2, .3], .3), ([.2, .3], .1), ([.3, .5], .3)\},\\
&\cdot \{v_4 v_5, ([.1, .2], .1), ([.2, .3], .3), ([.3, .4], .2)\}\rangle.
\end{aligned} \tag{70}$$

Then,

$$\begin{aligned}
\theta\left(\mathbb{N}_{ncg}[v_1]\right) &= (\Phi(v_2) + \Phi(v_3) + \Phi(v_4)) + \Phi(v_1)\\
&= \{\{([.2, .4], .3), ([.3, .4], .2), ([.6, .7], .5)\}\\
&\quad + \{([.2, .6], .5), ([.2, .5], .4), ([.5, .7], .4)\}\\
&\quad + \{([.3, .5], .4), ([.3, .6], .5), ([.4, .6], .3)\}\}\\
&\quad + \{([.3, .5], .4), ([.2, .3], .1), ([.3, .6], .4)\}\\
&= \{([1, 2], 1.6), ([1, 1.8], 1.2), ([1.8, 2.6], 1.6)\},\\
\theta\left(\mathbb{N}_{ncg}[v_2]\right) &= (\Phi(v_1) + \Phi(v_4) + \Phi(v_3)) + \Phi(v_2)\\
&= \{([1, 2], 1.6), ([1, 1.8], 1.2), ([1.8, 2.6], 1.6)\},\\
\theta\left(\mathbb{N}_{ncg}[v_3]\right) &= (\Phi(v_2) + \Phi(v_1) + \Phi(v_4)) + \Phi(v_3)\\
&= \{([1, 2], 1.6), ([1, 1.8], 1.2), ([1.8, 2.6], 1.6)\},\\
\theta\left(\mathbb{N}_{ncg}[v_4]\right) &= (\Phi(v_1) + \Phi(v_2) + \Phi(v_3) + \Phi(v_5)) + \Phi(v_4)\\
&= \{\{([.3, .5], .4), ([.2, .3], .1), ([.3, .6], .4)\}\\
&\quad + \{([.2, .4], .3), ([.3, .4], .2), ([.6, .7], .5)\}\\
&\quad + \{([.2, .6], .5), ([.2, .5], .4), ([.5, .7], .4)\}\\
&\quad + \{([.2, .2], .1), ([.2, .3], .4), ([.3, .5], .2)\}\}\\
&\quad + \{([.3, .5], .4), ([.3, .6], .5), ([.4, .6], .3)\}\\
&= \{([1.2, 2.2], 1.7), ([1.2, 2.1], 1.6),\\
&\quad \cdot ([2.1, 3.1], 1.8)\}, \theta\left(\mathbb{N}_{ncg}[v_5]\right) = \Phi\left(\mathbb{N}_{ncg}[v_4]\right)\\
&= \{([.3, .5], .4), ([.3, .6], .5), ([.4, .6], .3)\}.
\end{aligned} \tag{71}$$

Clearly, $\mathscr{L}_C^G$ is totally irregular as in Figure 6.

3.6. Complement of a Neutrosophic Cubic Graph.

Complement of a neutrosophic cubic graph is a very important concept we discuss here.

Definition 26. The complement of $\mathscr{L}_C^G = (\Gamma, \Lambda)$ is a neutrosophic cubic graph $\overline{\mathscr{L}_C^G} = (\overline{\Gamma}, \bar{\Lambda})$, where

$$\begin{aligned}
\overline{\Gamma} = \bar{\Phi}(g) = \Big\{\Big\langle &g, \left(\left[\bar{\alpha}_T^l, \bar{\alpha}_T^r\right](g), \bar{\alpha}_T(g)\right),\\
&\cdot \left(\left[\bar{\alpha}_I^l, \bar{\alpha}_I^r\right], \bar{\alpha}_I(g)\right), \left(\left[\bar{\alpha}_F^l, \bar{\alpha}_F^r\right](g), \bar{\alpha}_F(g)\right)\rangle \mid g \in A\},\\
\bar{\Lambda} = \bar{\Psi}(g_1 g_2) = \Big\{\Big\langle &g_1 g_2, \left(\left[\bar{\beta}_T^l, \bar{\beta}_T^r\right](g_1 g_2), \bar{\beta}_T(g_1 g_2)\right),\\
&\cdot \left(\left[\bar{\beta}_I^l, \bar{\beta}_I^r\right](g_1 g_2), \bar{\beta}_I(g_1 g_2)\right),\\
&\cdot \left(\left[\bar{\beta}_F^l, \bar{\beta}_F^r\right](g_1 g_2), \bar{\beta}_F(g_1 g_2)\right)\rangle \mid g_1 g_2 \in B\Big\},
\end{aligned} \tag{72}$$

since

$$\bar{\Psi}(xy) = \min\{\Phi(x), \Phi(y)\} - \Psi(xy), \tag{73}$$

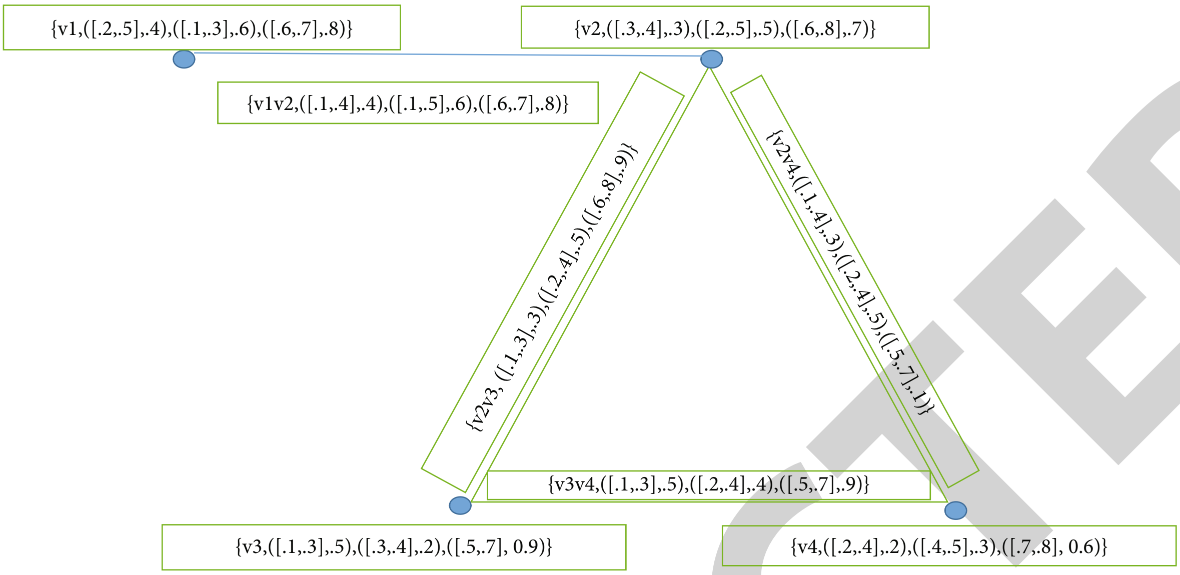

Figure 5: Represents irregular neutrosophic cubic graph.

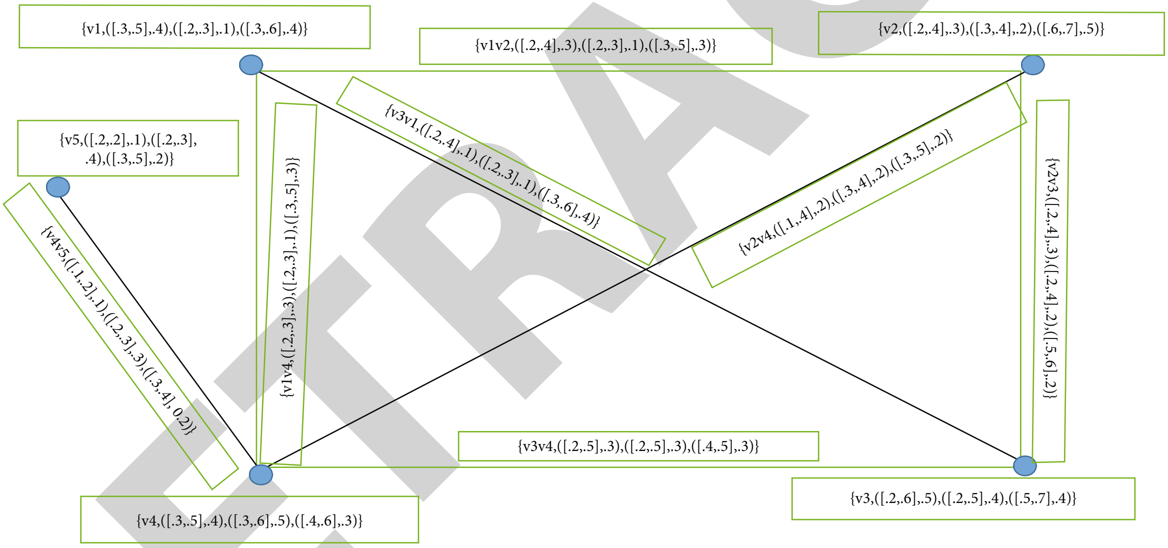

Figure 6: Represents totally irregular neutrosophic cubic graph.

or for truth membership functions, we have

$$\begin{aligned}
\bar{\beta}^l_T(xy) &= \min\left\{\alpha^l_T(x), \alpha^l_T(y)\right\} - \beta^l_T(xy),\\
\bar{\beta}_T(xy) &= \min\left\{\alpha_T(x), \alpha_T(y)\right\} - \beta_T(xy),\\
\bar{\beta}^r_T(xy) &= \min\left\{\alpha^r_T(x), \alpha^r_T(y)\right\} - \beta^r_T(xy)\,;
\end{aligned} \tag{74}$$

similarly, for indeterminate membership functions, we have

$$\begin{aligned}
\bar{\beta}^l_I(xy) &= \min\left\{\alpha^l_I(x), \alpha^l_I(y)\right\} - \beta^l_I(xy),\\
\bar{\beta}^r_I(xy) &= \min\left\{\alpha^r_I(x), \alpha^r_I(y)\right\} - \beta^r_I(xy),\\
\bar{\beta}_I(xy) &= \min\left\{\alpha_I(x), \alpha_I(y)\right\} - \beta_I(xy)\,;
\end{aligned} \tag{75}$$

also, for falsity membership functions, we have similar results.

Proposition 27. *For self-complementary* $\mathscr{L}^G_C = (\Gamma, \Lambda)$, *we have*

$$\wp\left(\mathscr{L}^G_C\right) = \{([1, 1], 1), ([1, 1], 1), ([1, 1], 1)\}. \tag{76}$$

Proof. Given $\mathscr{L}^G_C$ is self-complementary, so $\bar{\Psi}(xy) = \Psi(xy)$; also, by definition of a self-complementary neutrosophic cubic graph, we have

$$\bar{\Psi}(xy) = \min\left\{\Phi(x), \Phi(y)\right\} - \Psi(xy). \tag{77}$$

Dividing both sides of equation (77) by min $\{\Phi(x), \Phi($

$y)\}$,

$$\frac{\Psi(xy)}{\min\{\Phi(x),\Phi(y)\}} = \{([1,1],1),([1,1],1),([1,1],1)\} - \frac{\Psi(xy)}{\min\{\Phi(x),\Phi(y)\}}, \tag{78}$$

we get

$$\sum_{x,y\in A}\frac{\Psi(xy)}{\min\{\Phi(x),\Phi(y)\}} = \{([1,1],1),([1,1],1),([1,1],1)\} - \sum_{x,y\in A}\frac{\Psi(xy)}{\min\{\Phi(x),\Phi(y)\}}. \tag{79}$$

Hence,

$$2\sum_{x,y\in A}\frac{\Psi(xy)}{\min\{\Phi(x),\Phi(y)\}} = \{([1,1],1),([1,1],1),([1,1],1)\}, \tag{80}$$

so

$$\wp\left(\mathscr{L}_C^S\right) = \{([1,1],1),([1,1],1),([1,1],1)\}. \tag{81}$$

□

Proposition 28. *Let $\mathscr{L}_C^G = (\Gamma, \Lambda)$ be strictly balanced, let $\overline{\mathscr{L}_C^G}$ be its complement, then $\wp(\mathscr{L}_C^G) + \wp(\overline{\mathscr{L}_C^G}) = \{([2,2],2),([2,2],2),([2,2],2)\}$.*

Proof. Let $\mathscr{L}_C^G$ be strictly balanced; let $\overline{\mathscr{L}_C^G}$ be its complement. Let H be a nonempty subgraph of $\mathscr{L}_C^G$. Since $\mathscr{L}_C^G$ is strictly balanced, $\wp(\mathscr{L}_C^G) = \wp(H)$ for every subset $H \subseteq \mathscr{L}_C^G$ and for any $x, y \in A$. In $\overline{\mathscr{L}_C^G}$, we have $\bar{\Psi}(xy) = \min\{\Phi(x), \Phi(y)\} - \Psi(xy)$, and so, for truth membership functions, we have

$$\bar{\beta}_T^l(xy) = \min\left\{\alpha_T^l(x), \alpha_T^l(y)\right\} - \beta_T^l(xy), \text{for all } x, y \in A. \tag{82}$$

Dividing equation (82) by $\min\{\alpha_T^l(x), \alpha_T^l(y)\}$, we get

$$\frac{\bar{\beta}_T^l(xy)}{\min\left\{\alpha_T^l(x), \alpha_T^l(y)\right\}} = 1 - \frac{\beta_T^l(xy)}{\min\left\{\alpha_T^l(x), \alpha_T^l(y)\right\}}. \tag{83}$$

Hence,

$$\sum_{x,y\in A}\frac{\bar{\beta}_T^l(xy)}{\min\left\{\alpha_T^l(x), \alpha_T^l(y)\right\}} = 1 - \sum_{x,y\in E}\frac{\beta_T^l(xy)}{\min\left\{\alpha_T^l(x), \alpha_T^l(y)\right\}}. \tag{84}$$

Multiplying both sides by 2,we get

$$2\sum_{x,y\in A}\frac{\bar{\beta}_T^l(xy)}{\min\left\{\alpha_T^l(x), \alpha_T^l(y)\right\}} = 2 - 2\sum_{x,y\in E}\frac{\beta_T^l(xy)}{\min\left\{\alpha_T^l(x), \alpha_T^l(y)\right\}}. \tag{85}$$

Hence, $\wp_T^l(\overline{\mathscr{L}_C^G}) = 2 - \wp_T^l(\mathscr{L}_C^G)$. Similarly for right end point of interval in truth valued membership functions, we have $\wp_T^r(\overline{\mathscr{L}_C^G}) = 2 - \wp_T^r(\mathscr{L}_C^G)$. Similar results hold for the rest of membership functions. Hence,

$$\begin{aligned} \wp\left(\overline{\ell_C^G}\right) &= \{([2,2],2),([2,2],2),([2,2],2)\} - \wp\left(\ell_C^G\right), \\ \wp\left(\overline{\ell_C^G}\right) + \wp\left(\ell_C^G\right) &= \{([2,2],2),([2,2],2),([2,2],2)\}. \end{aligned} \tag{86}$$

This completes the proof. □

3.7. Neighborly Irregular and Neighborly Totally Irregular Neutrosophic Cubic Graphs. In this subsection, we use the neighborhood degrees to discuss the idea of neighborly irregular and neighborly totally irregular neutrosophic cubic graphs.

Definition 29. A connected $\mathscr{L}_C^G$ is neighborly irregular if every two adjacent vertices in $\mathscr{L}_C^G$ have different closed neighborhood degrees.

Example 30. Consider $\mathscr{L}_C^G = (\Gamma, \Lambda)$ for any $\mathbf{G} = (A, B)$ with $A = \{a, b, c, d\}$ and $B = \{ab, bc, cd, da\}$ and let neutrosophic cubic membership functions for vertices and edges be represented in Tables 6 and 7, respectively.

Then, clearly $\mathscr{L}_C^G$ is neighborly irregular, as shown in Figure 7.

Definition 31. $\mathscr{L}_C^G$ is said to be neighborly totally irregular if every two adjacent vertices of $\mathscr{L}_C^G$ have different closed neighborhood degrees.

Example 32. Consider $\mathscr{L}_C^G = (\Gamma, \Lambda)$ for any $\mathbf{G} = (A, B)$ with $A = \{v_1, v_2, v_3, v_4\}$ and $B = \{v_1v_2, v_2v_3, v_3v_4, v_1v_4\}$. Also, let

$$\begin{aligned} \Gamma = \langle &\{v_1, ([.2,.6],.4), ([.2,.4],.5), ([.4,.6],.5)\}, \\ &\cdot\{v_2, ([.3,.5],.3), ([.4,.5],.5), ([.5,.7],.4)\}, \\ &\cdot\{v_3, ([.4,.7],.5), ([.3,.5],.2), ([.3,.6],.1)\}, \\ &\cdot\{v_4, ([.5,.6],.2), ([.2,.5],.3), ([.6,.8],.2)\}\rangle, \end{aligned} \tag{87}$$

$$\begin{aligned} \Lambda = \langle &\{v_1v_2, ([.2,.5],.4), ([.2,.4],.4), ([.5,.7],.4)\}, \\ &\cdot\{v_2v_3, ([.3,.5],.5), ([.3,.4],.4), ([.5,.6],.1)\}, \\ &\cdot\{v_3v_4, ([.2,.6],.5), ([.2,.5],.3), ([.5,.7],.1)\}, \\ &\cdot\{v_1v_4, ([.1,.4],.3), ([.2,.4],.5), ([.5,.7],.2)\}\rangle. \end{aligned} \tag{88}$$

Then, clearly $\mathscr{L}_C^G$ is neighborly totally irregular, as shown in Figure 8.

3.8. Highly Irregular Neutrosophic Cubic Graphs. In this subsection, we use the neighborhood degrees to discuss the idea of highly irregular neutrosophic cubic graphs.

Definition 33. Consider a connected $\mathscr{L}_C^G$; then, $\mathscr{L}_C^G$ is highly irregular if every vertex in $\mathscr{L}_C^G$ is adjacent to vertices with different neighborhood degrees.

Example 34. Consider $\mathscr{L}_C^G = (\Gamma, \Lambda)$ for a graph $\mathbf{G} = (A, B)$ with $A = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ and $B = \{v_1 v_2, v_2 v_3, v_2 v_6, v_3 v_4, v_3 v_5, v_4 v_5, v_1 v_5\}$ and let

$$\begin{aligned}\Gamma = \langle &\{v_1, ([.2, .5], .3), ([.1, .3], .4), ([.4, .5], .3)\},\\ &\cdot \{v_2, ([.1, .4], .2), ([.2, .4], .2), ([.3, .6], .4)\},\\ &\cdot \{v_3, ([.3, .5], .6), ([.2, .5], .3), ([.5, .7], .4)\},\\ &\cdot \{v_4, ([.4, .6], .3), ([.3, .6], .5), ([.4, .6], .3)\},\\ &\cdot \{v_5, ([.2, .4], .5), ([.2, .3], .4), ([.2, .5], .2)\},\\ &\cdot \{v_6, ([.1, .4], .3), ([.2, .4], .2), ([.3, .5], .2)\}\rangle.\end{aligned} \tag{89}$$

By routine computations, we have

$$\begin{aligned}\theta(\mathbb{N}_{ncg}(v_1)) &= \Phi(v_2) + \Phi(v_5)\\ &= \{([.1, .4], .2), ([.2, .4], .2), ([.3, .6], .4)\}\\ &\quad + \{([.2, .4], .5), ([.2, .3], .4), ([.2, .5], .2)\}\\ &= \{([.3, .8], .7), ([.4, .7], .6), ([.5, 1.1], .6)\},\\ \theta(\mathbb{N}_{ncg}(v_2)) &= \Phi(v_1) + \Phi(v_6) + \Phi(v_3)\\ &= \{([.2, .5], .3), ([.1, .3], .4), ([.4, .5], .3)\}\\ &\quad + \{([.1, .4], .3), ([.2, .4], .2), ([.3, .5], .2)\}\\ &\quad + \{([.3, .5], .6), ([.2, .5], .3), ([.5, .7], .4)\}\\ &= \{([.6, 1.4], 1.2), ([.5, 1.2\%], .9), ([1.2, 1.7], .9)\},\\ \theta(\mathbb{N}_{ncg}(v_4)) &= \Phi(v_3) + \Phi(v_5)\\ &= \{([.3, .5], .6), ([.2, .5], .3), ([.5, .7], .4)\}\\ &\quad + \{([.2, .4], .5), ([.2, .3], .4), ([.2, .5], .2)\}\\ &= \{([.5, .9], 1.1), ([.4, .8], .7), ([.7, 1.2], .6)\},\\ \theta(\mathbb{N}_{ncg}(v_5)) &= \Phi(v_3) + \Phi(v_4) + \Phi(v_1)\\ &= \{([.3, .5], .6), ([.2, .5], .3), ([.5, .7], .4)\}\\ &\quad + \{([.4, .6], .3), ([.3, .6], .5), ([.4, .6], .3)\}\\ &\quad + \{([.2, .5], .3), ([.1, .3], .4), ([.4, .5], .3)\}\\ &= \{([.9, 1.6], 1.2), ([.6, 1.4\%], 1.2), ([1.3, 1.8], 1)\},\\ \theta(\mathbb{N}_{ncg}(v_6)) &= \Phi(v_2)\{([.1, .4], .2), ([.2, .4], .2), ([.3, .6], .4)\}.\end{aligned} \tag{90}$$

Clearly, $\mathscr{L}_C^G$ as shown in Figure 9 is highly irregular.

Theorem 35. *$\mathscr{L}_C^G$ is highly irregular and neighborly irregular if and only if open neighborhood degrees for all vertices of $\mathscr{L}_C^G$ are different.*

Table 6: Neutrosophic membership functions for vertices.

	A	$[\alpha_T^l, \alpha_T^r]$	α_T	$[\alpha_I^l, \alpha_I^r]$	α_I	$[\alpha_F^l, \alpha_F^r]$	α_F
$\Gamma =$	a	[.2, .6]	.3	[.1, .3]	.2	[.5, .7]	.5
	b	[.3, .7]	.2	[.2, .4]	.3	[.4, .5]	.6
	c	[.4, .8]	.3	[.3, .5]	.4	[.6, .7]	.4
	d	[.5, .7]	.4	[.1, .2]	.3	[.7, .8]	.7

Table 7: Neutrosophic membership functions for edges.

	B	$[\beta_T^l, \beta_T^r]$	β_T	$[\beta_I^l, \beta_I^r]$	β_I	$[\beta_F^l, \beta_F^r]$	β_F
$\Lambda =$	ab	[.1, .4]	.2	[.1, .2]	.1	[.3, .5]	.5
	bc	[.2, .5]	.1	[.2, .3]	.2	[.4, .5]	.3
	cd	[.3, .6]	.3	[.1, .2]	.3	[.5, .6]	.4
	da	[.2, .5]	.2	[.1, .2]	.2	[.4, .5]	.5

Proof. Suppose $\mathscr{L}_C^G$ has n vertices $v_1, v_2, \cdots, v_n$. Also, let $\mathscr{L}_C^G$ be highly irregular and neighborly irregular.

Claim 1. The open neighborhood degrees for all vertices in $\mathscr{L}_C^G$ are different. Let

$$\theta(\mathbb{N}_{ncg}(vi)) = \left\{ \left[\lambda_{iT}^l, \lambda_{iT}^r\right], \lambda_{iT}, \left[\lambda_{iI}^l, \lambda_{iI}^r\right], \lambda_{iI}, \left[\lambda_{\mathrm{iF}}^l, \lambda_{\mathrm{iF}}^r\right], \lambda_{\mathrm{iF}} \right\} \tag{91}$$

for all $i = 1, 2, 3, \cdots, n$. Let the adjacent vertices of v_1 be $v_2, v_3, \cdots, v_n$ with open neighborhood degrees:

$$\theta(\mathbb{N}_{ncg}(vi)) = \left\{ \left[\lambda_{iT}^l, \lambda_{iT}^r\right], \lambda_{iT}, \left[\lambda_{iI}^l, \lambda_{iI}^r\right], \lambda_{iI}, \left[\lambda_{\mathrm{iF}}^l, \lambda_{\mathrm{iF}}^r\right], \lambda_{\mathrm{iF}} \right\} \tag{92}$$

for all $i = 2, 3, \cdots, n$, respectively. Then, as $\mathscr{L}_C^G$ is highly irregular, we have

$$\begin{aligned}&\lambda_{2T}^l \neq \lambda_{3T}^l \neq \cdots \neq \lambda_{nT}^l,\\ &\lambda_{2T}^r \neq \lambda_{3T}^r \neq \cdots \neq \lambda_{nT}^r,\\ &\lambda_{2T} \neq \lambda_{3T} \neq \cdots \neq \lambda_{nT}\end{aligned} \tag{93}$$

for all $i = 2, 3, \cdots, n$. Similar holds for indeterminacy and falsity membership functions. Also, $\mathscr{L}_C^G$ is neighborly irregular, so we have

$$\begin{aligned}&\lambda_{1T}^l \neq \lambda_{2T}^l \neq, \cdots, \neq \lambda_{nT}^l,\\ &\lambda_{1T}^r \neq \lambda_{2T}^r \neq \cdots \neq \lambda_{nT}^r,\\ &\lambda_{1T} \neq \lambda_{2T} \neq \cdots \neq \lambda_{nT}.\end{aligned} \tag{94}$$

Similar holds for indeterminacy and falsity membership functions. Hence, open neighborhood degrees of all vertices of $\mathscr{L}_C^G$ are different.

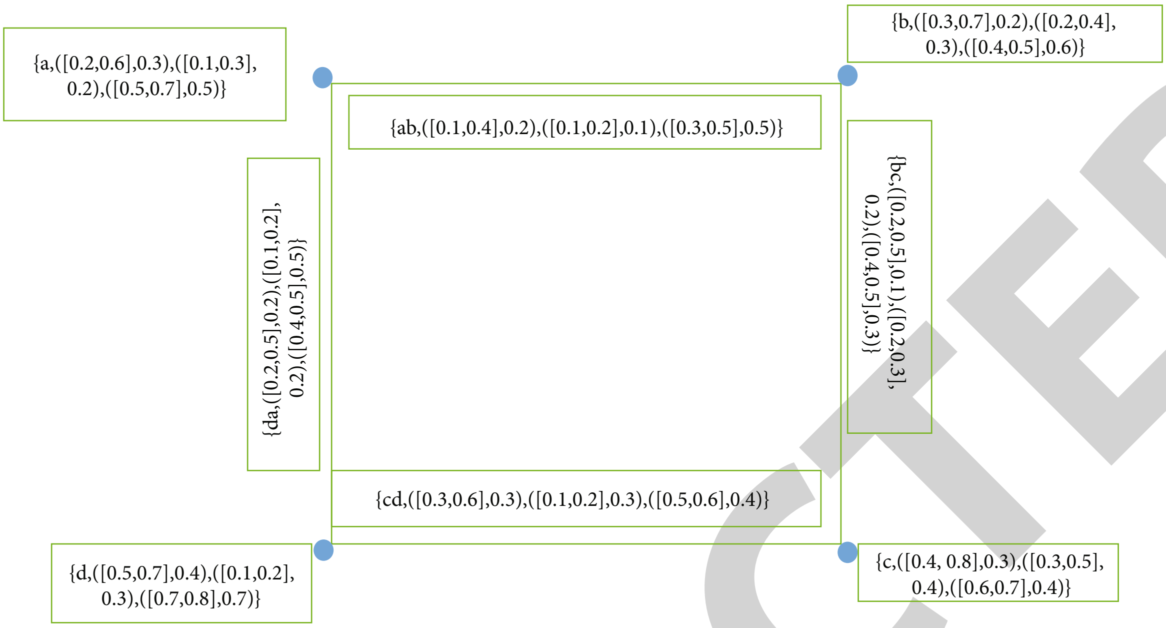

Figure 7: Represents neighborly irregular neutrosophic cubic graph.

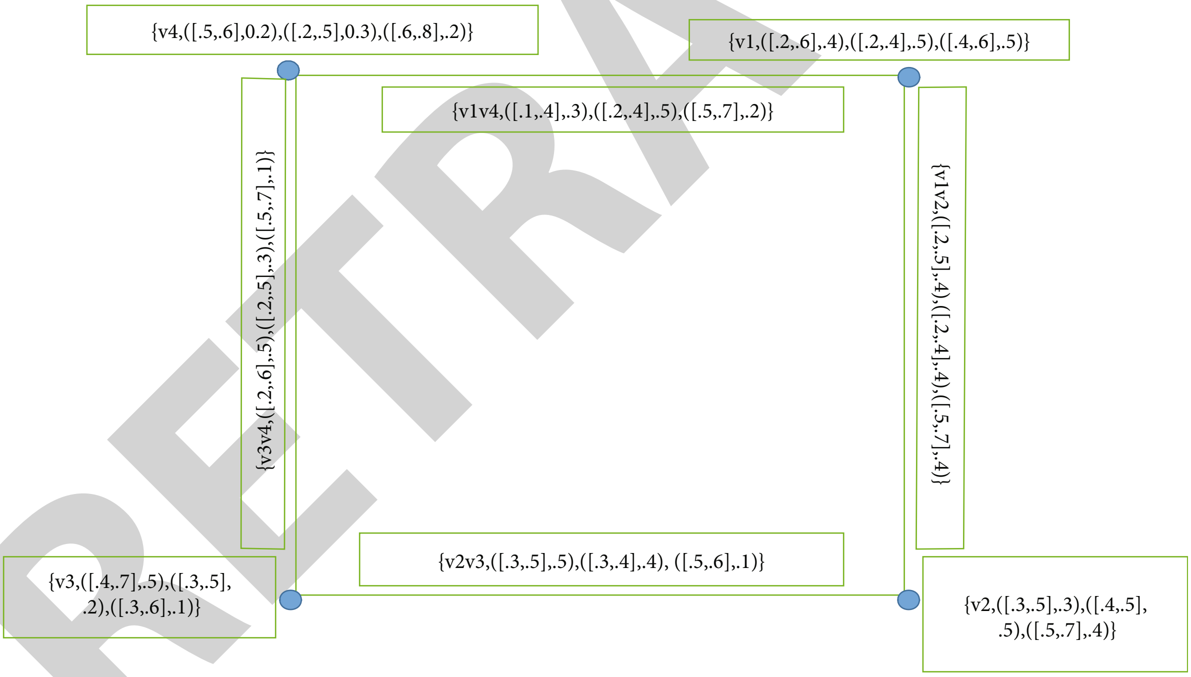

Figure 8: Represents neighborly totally irregular neutrosophic cubic graph.

Claim 2. $\mathscr{L}_C^G$ is highly irregular and neighborly irregular. Let

$$\theta\left(\mathbb{N}_{ncg}(vi)\right)=\left\{\left[\lambda_{iT}^l,\lambda_{iT}^r\right],\lambda_{iT},\left[\lambda_{iI}^l,\lambda_{iI}^r\right],\lambda_{iI},\left[\lambda_{iF}^l,\lambda_{iF}^r\right],\lambda_{iF}\right\} \tag{95}$$

for all $i=1,2,3,\cdots,n$ are degrees for all vertices of $\mathscr{L}_C^G$. Given open neighborhood degrees of all vertices of $\mathscr{L}_C^G$ are different, so

$$\begin{gathered}\theta\left(\mathbb{N}_{ncg}(v_1)\right)\neq\theta\left(\mathbb{N}_{ncg}(v_2)\right)\neq\cdots\neq\theta\left(\mathbb{N}_{ncg}(v_n)\right),\\ \lambda_{1T}^l\neq\lambda_{2T}^l\neq\cdots\neq\lambda_{nT}^l,\\ \lambda_{1T}^r\neq\lambda_{2T}^r\neq\cdots\neq\lambda_{nT}^r,\\ \lambda_{1T}\neq\lambda_{2T}\neq\cdots\neq\lambda_{nT}\end{gathered} \tag{96}$$

for all $i=1,2,3,\cdots,n$. Similar holds for indeterminacy and

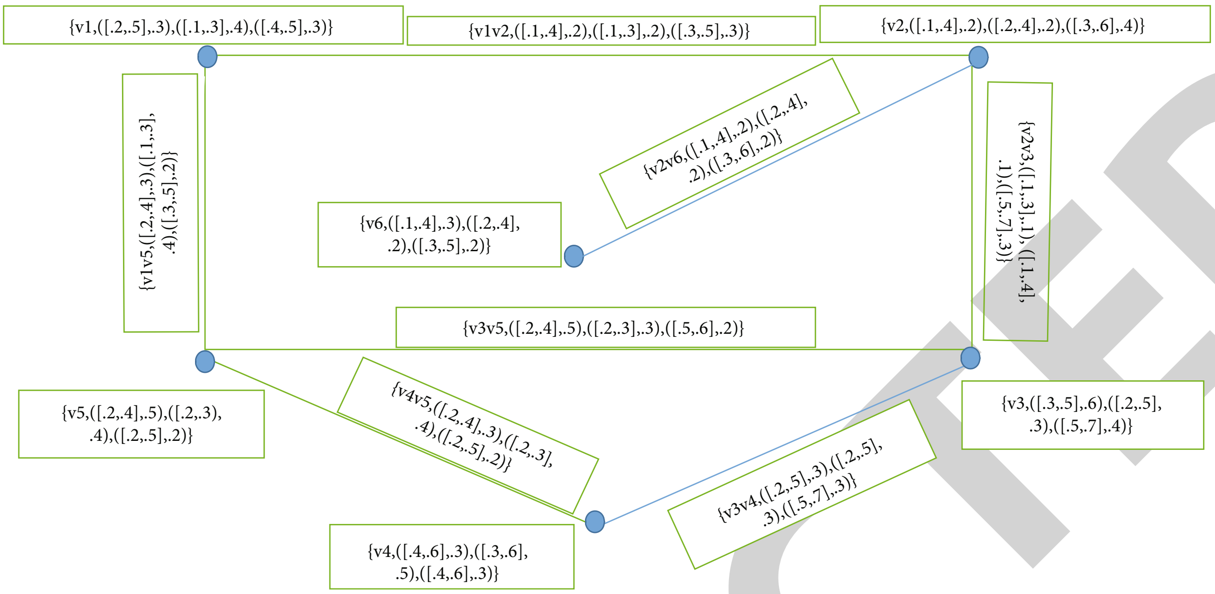

FIGURE 9: Represents highly irregular neutrosophic cubic graph.

falsity membership functions. So, any two vertices in $\mathscr{L}_C^G$ have different open neighborhood degrees, and for every vertex, adjacent vertices have different open neighborhood degrees, which proves the result. □

Remark 36. A complete $\mathscr{L}_C^G$ may not be neighborly irregular.

Example 37. Let $\mathscr{L}_C^G = (\Gamma, \Lambda)$ for any $\mathbf{G} = (A, B)$, with $A = \{v_1, v_2, v_3\}$ and $B = \{v_1v_2, v_2v_3, v_1v_3\}$ such that

$$\begin{aligned}\Gamma = \langle \{v_1, ([.4,.8],.4), ([.3,.5\%],.4), ([.6,.7],.3)\},\\ \cdot \{v_2, ([.2,.4],.2), ([.4,.7\%],.5), ([.5,.8],.2)\},\\ \cdot \{v_3, ([.2,.4],.2), ([.4,.7\%],.5), ([.5,.8],.2)\}\rangle,\end{aligned} \tag{97}$$

$$\begin{aligned}\Lambda = \langle \{v_1v_2, ([.2,.4],.2), ([.3,.5\%],.4), ([.5,.7],.2)\},\\ \cdot \{v_2v_3, ([.2,.4],.2), ([.4,.7\%],.5), ([.5,.8],.2)\},\\ \cdot \{v_1v_3, ([.2,.4],.2), ([.3,.5\%],.4), ([.5,.7],.2)\}\rangle.\end{aligned} \tag{98}$$

By simple computation, we get

$$\begin{aligned}\theta(\mathbb{N}_{ncg}(v_1)) = \{([.4,.8],.4), ([.8,1.4],1), ([1,1.6],.4)\},\\ \theta(\mathbb{N}_{ncg}(v_2)) = \{([.6,1.2],.6), ([.7,1.2],.9), ([1.1,1.5],.5)\},\\ \theta(\mathbb{N}_{ncg}(v_3)) = \{([.6,1.2],.6), ([.7,1.2],.9), ([1.1,1.5],.5)\}.\end{aligned} \tag{99}$$

Here, $\theta(\mathbb{N}_{ncg}(v_2)) = \theta(\mathbb{N}_{ncg}(v_3))$, so the neighborhood degree is not different. Hence, $\mathscr{L}_C^G$ is not neighborly irregular; also, we have similar holds for all vertices and edges. So, $\mathscr{L}_C^G$ is complete. Hence, a complete $\mathscr{L}_C^G$ may not be neighborly irregular as shown in Figure 10.

Theorem 38. *If $\mathscr{L}_C^G$ is neighborly irregular and*

$$\begin{aligned}\Gamma = \Phi(x) = \Big\{\Big(\big[\alpha_T^l, \alpha_T^r\big](x), \alpha_T(x)\Big), \Big(\big[\alpha_I^l, \alpha_I^r\big](x), \alpha_I(x)\Big),\\ \cdot \Big(\big[\alpha_F^l, \alpha_F^r\big](x), \alpha_F(x)\Big)\Big\}\end{aligned} \tag{100}$$

for all $x \in A$ is a constant function, then it is neighborly totally irregular.

Proof. Assume that $\mathscr{L}_C^G$ is a neighborly irregular. Then, open neighborhood degrees of every two adjacent vertices are different. Let $v_i, v_j \in A$ be adjacent vertices with different open neighborhood degrees. Then, $\theta(\mathbb{N}_{ncg}(v_i)) \neq \theta(\mathbb{N}_{ncg}(v_j))$ for all $i \neq j$; let $\theta(\mathbb{N}_{ncg}(v_i)) = d_1 \& \theta(\mathbb{N}_{ncg}(v_j)) = d_2$ then $d_1 \neq d_2$. Also, as

$$\begin{aligned}\Gamma = \Phi(x) = \Big\{\Big(\big[\alpha_T^l, \alpha_T^r\big](x), \alpha_T(x)\Big), \Big(\big[\alpha_I^l, \alpha_I^r\big](x), \alpha_I(x)\Big),\\ \cdot \Big(\big[\alpha_F^l, \alpha_F^r\big](x), \alpha_F(x)\Big)\Big\}\end{aligned} \tag{101}$$

is constant for all $x \in A$. Hence, $\Phi(v_i) = \Phi(v_j) = k$; suppose that $\mathscr{L}_C^G$ is not neighborly totally irregular, then

$$\theta(\mathbb{N}_{ncg}[v_i]) = \theta(\mathbb{N}_{ncg}[v_j]), \tag{102}$$

for some $i \neq j$ but $\theta(\mathbb{N}_{ncg}[v_i]) = \theta(\mathbb{N}_{ncg}(v_i)) + \Phi(v_i)$ and $\theta(\mathbb{N}_{ncg}[v_j]) = \theta(\mathbb{N}_{ncg}(v_j)) + \Phi(v_j)$ using these values in

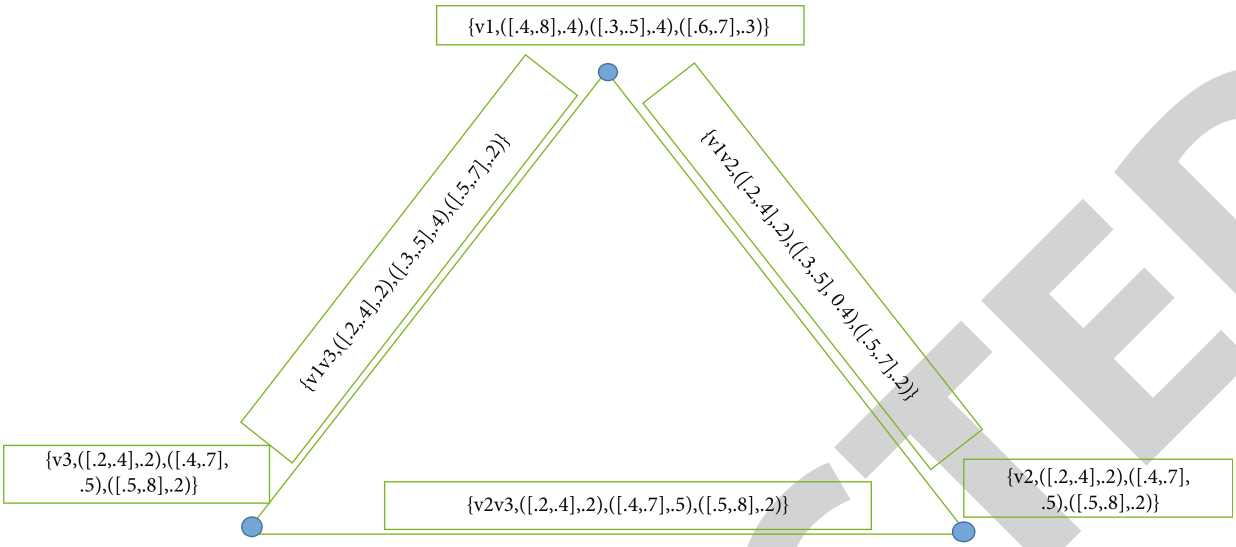

Figure 10: Represents a complete neutrosophic cubic graph, but not neighborly irregular.

equation (102), we get

$$\theta(\mathbb{N}_{ncg}(v_i)) + \Phi(v_i) = \theta(\mathbb{N}_{ncg}(v_j)) + \Phi(v_j) \Rightarrow d_1 + k = d_2 + k \Rightarrow d_1 = d_2, \quad (103)$$

as cancellation law holds in $[0, 1]$, which contradicts, as

$$d_1 \neq d_2. \quad (104)$$

Hence,

$$\theta(\mathbb{N}_{ncg}[v_i]) \neq \theta(\mathbb{N}_{ncg}[v_j]), \quad (105)$$

so $\mathscr{L}_C^G$ is neighborly totally irregular. This proves the result. □

Theorem 39. *If $\mathscr{L}_C^G$ is neighborly totally irregular and*

$$\Gamma = \Phi(x) = \left\{ \left(\left[\alpha_T^l, \alpha_T^r\right](x), \alpha_T(x) \right), \left(\left[\alpha_I^l, \alpha_I^r\right](x), \alpha_I(x) \right), \cdot \left(\left[\alpha_F^l, \alpha_F^r\right](x), \alpha_F(x) \right) \right\}, \quad x \in A, \quad (106)$$

is a constant function, then it is neighborly irregular.

Proof. Assume that $\mathscr{L}_C^G$ is a neighborly totally irregular. Then, closed neighborhood degrees of every two adjacent vertices are distinct. Let $v_i, v_j \in A$ be adjacent vertices with distinct closed neighborhood degrees. Then, for all $i \neq j$,

$$\theta(\mathbb{N}_{ncg}[v_i]) \neq \theta(\mathbb{N}_{ncg}[v_j]), \quad (107)$$

let

$$\theta(\mathbb{N}_{ncg}(v_i)) = f_1 \& \theta(\mathbb{N}_{ncg}(v_j)) = f_2, \quad (108)$$

then $f_1 \neq f_2$. Also, as

$$\Gamma = \Phi(x) = \left\{ \left(\left[\alpha_T^l, \alpha_T^r\right](x), \alpha_T(x) \right), \cdot \left(\left[\alpha_I^l, \alpha_I^r\right](x), \alpha_I(x) \right), \left(\left[\alpha_F^l, \alpha_F^r\right](x), \alpha_F(x) \right) \right\}; \quad (109)$$

suppose that $\mathscr{L}_C^G$ is not neighborly irregular, then

$$\theta(\mathbb{N}_{ncg}(v_i)) = \theta(\mathbb{N}_{ncg}(v_j)) = w; \quad (110)$$

say, for some $i \neq j$ but

$$\theta(\mathbb{N}_{ncg}[v_i]) = \theta(\mathbb{N}_{ncg}(v_i)) + \Phi(v_i), \quad (111)$$

$$\theta(\mathbb{N}_{ncg}[v_j]) = \theta(\mathbb{N}_{ncg}(v_j)) + \Phi(v_j), \quad (112)$$

using these values in equation (112), we get

$$\theta(\mathbb{N}_{ncg}(v_i)) + \Phi(v_i) = \theta(\mathbb{N}_{ncg}(v_j)) + \Phi(v_j) = w + r, \quad (113)$$

so

$$\theta(\mathbb{N}_{ncg}[v_i]) = \theta(\mathbb{N}_{ncg}[v_j]), \quad (114)$$

for some $i \neq j$ which is a contradiction to the fact that $\mathscr{L}_C^G$ is a neighborly totally irregular neutrosophic cubic graph. Hence,

$$\theta(\mathbb{N}_{ncg}(v_i)) \neq \theta(\mathbb{N}_{ncg}(v_j)), \quad (115)$$

so $\mathscr{L}_C^G$ is neighborly irregular. This proves the result. □

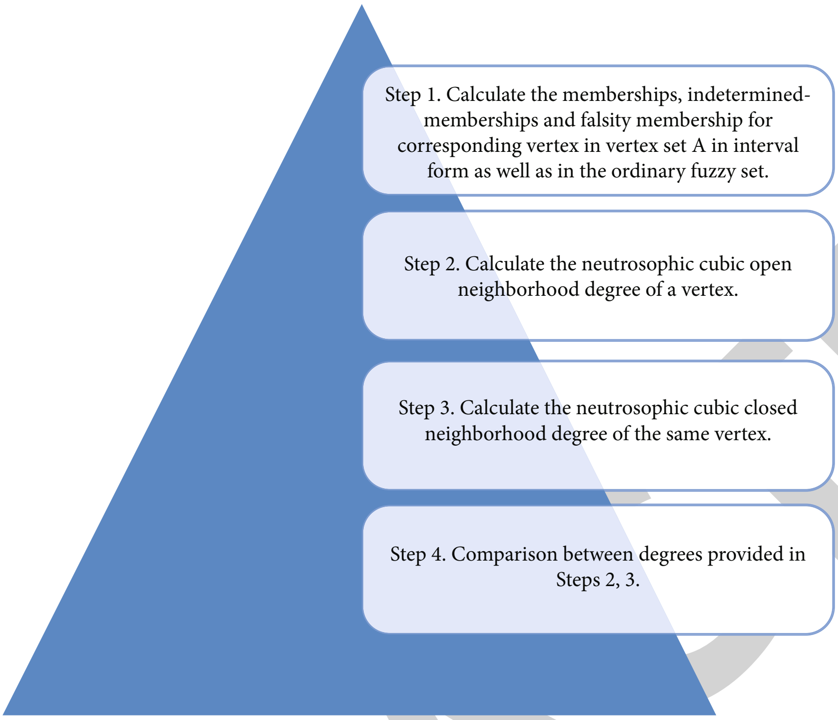

Figure 11: Frame diagram of the proposed method.

Proposition 40. *If $\mathscr{L}_C^G$ is neighborly irregular as well as neighborly totally irregular, then*

$$\Gamma = \Phi(x) = \left\{\left(\left[\alpha_T^l, \alpha_T^r\right](x), \alpha_T(x)\right), \left(\left[\alpha_I^l, \alpha_I^r\right](x), \alpha_I(x)\right), \cdot \left(\left[\alpha_F^l, \alpha_F^r\right](x), \alpha_F(x)\right)\right\} \tag{116}$$

need not be a constant function.

Remark 41. If $\mathscr{L}_C^G$ is neighborly irregular, then a neutrosophic cubic subgraph H of $\mathscr{L}_C^G$ may not be neighborly irregular.

Remark 42. If $\mathscr{L}_C^G$ is neighborly totally irregular, then a neutrosophic cubic subgraph H of $\mathscr{L}_C^G$ may not be neighborly totally irregular.

4. Applications

As neutrosophic cubic graph theory is a developing field of modern mathematics, it has many applications in different fields. In this section, we discuss applications of neutrosophic cubic graphs in finding the effects of different factors in the neighboring countries of Pakistan. Further, we used our proposed model in decision-making while selecting a house in a certain locality.

We will use the following proposed algorithm in the following real-life problems.

Step 1. Calculate the memberships, indetermined-memberships and falsity membership for corresponding vertex in vertex set A in interval form as well as in the ordinary fuzzy set.

Step 2. Calculate the neutrosophic cubic open neighborhood degree of a vertex.

Step 3. Calculate the neutrosophic cubic closed neighborhood degree of the same vertex.

Step 4. Comparison between degrees provided in Steps 2 and 3.

The frame diagram to clarify the organization of the proposed method is given in Figure 11.

4.1. Effects of Different Factors on the Neighboring Countries of Pakistan.

Suppose we are interested to check the effects (e.g., time/durations/situations) on different factors in the neighboring countries of Pakistan. These factors may be the population, literacy, health conditions, etc., of these countries. So, we take Pakistan and its neighboring countries as a set of vertices and link between these countries through roads as our edge set. Hence, graph $G = (A, B)$ has set of vertices $A = \{\text{Pak}, \text{Ir}, \text{In}, \text{Ch}, \text{Af}\}$, where Pak stands for Pakistan, Ir for Iran, In for India, Ch for China, and Af for Afghanistan. Let the set of edges be $E = A$ network of roads between these countries, so we can define membership function for each vertex $v \in A$ to denote strength or degree of these vertices as $\Phi(v) = \{\text{Pop}, \text{PCI}, \text{LR}, \text{WTRU}, \text{HE}, \text{PSI}\}$;

Table 8: A neutrosophic cubic representation for vertex set A.

A	Pop [past, future]	PCI (present)	LR [past, future]	WTRU (present)	HE [past, future]	PSI (present)
Af	[.026, .026]	.06	[.32, .38]	.01	[.92, .82]	.13
Ch	[1, 1]	1	[.92, .96]	1	[.49, .55]	.67
In	[.94, .95]	.212	[.69, .72]	.21	[.43, .47]	1
Ir	[.057, .058]	.57	[.86, .95]	.123	[.81, .69]	.2
Pak	[.148, .151]	.162	[.55, .57]	.06	[.31, .26]	.33

here, we have three different categories/situations/time/duration say past, future, and present. Also, here, interval membership represents past and future for truth and indeterminate membership, respectively, and present time represents falsity memberships.

(i) Pop represents interval membership for the population of a country in the duration (1st July 2018, 1st July 2019)/max population of the corresponding country in the same duration. Here, interval represents past and future to represent truth membership and indetermined-membership for members of the vertex set A

(ii) PCI is for per capita income of a country which represents falsity membership for the corresponding vertex in vertex set A

(iii) LR represents interval membership for literacy rate of a country in the duration [2011, 2014]. Here again, we have interval to represent past and future for truth membership and indetermined-memberships for members of vertex set A

(iv) WTRU is the position of the corresponding country in the world's top-ranking universities/max number of universities in these countries

(v) HE represents interval membership for %age of health expenditure of a country in the duration [2010, 2015] where interval shows past and future to represent truth membership and indetermined-membership for members of vertex set A

(vi) PSI is the number of popular sports interest/max number of sports played in the corresponding country as health depends on sports

By data collection for different time intervals, we have neutrosophic cubic membership for vertex set A represented in Table 8.

The neutrosophic cubic open neighborhood degree of a vertex (say Pak) is

$$\begin{aligned} N(\mathrm{Pak}) &= \Phi(\mathrm{Af}) + \Phi(\mathrm{Ch}) + \Phi(\mathrm{In}) + \Phi(\mathrm{Ir}) \\ &= \{([2.0303, 2.0373], 1.841), ([2.785, 3.015], 1.343), \\ &\quad \cdot ([2.639, 2.531], 2)\}; \end{aligned} \tag{117}$$

also, the neutrosophic cubic closed neighborhood degree for the same vertex is defined as

$$\begin{aligned} N[\mathrm{Pak}] &= \Phi(\mathrm{Pak}) + \Phi(\mathrm{Af}) + \Phi(\mathrm{Ch}) + \Phi(\mathrm{In}) + \Phi(\mathrm{Ir}) \\ &= \{([2.1789, 2.1883], 2.003), ([3.332, 3.585], 1.403), \\ &\quad \cdot ([2.941, 2.792], 2.33)\}. \end{aligned} \tag{118}$$

The neutrosophic cubic open neighborhood degree of a vertex (say Pak) is less than the neutrosophic cubic closed neighborhood degree of a vertex (say, Pak). Thus, we may conclude that the vertex (say, Pak) has more closed neighborhoods than the open neighborhoods that can change their loyalties according to time as shown in Figure 12. Similarly, we may check other countries.

4.2. Decision-Making while Selecting a House. Suppose we are interested to purchase a house in a housing society. Then, we have to consider certain features before making our final decision like availability of mosque, workplace, school, college, university, clinic/hospital, market, park, and gym, width/condition of roads, and the distance of the house and all these facilities. We also keep in view past, future, and present situations of all these attributes, or we keep in view trends and demands and check effects of duration on these areas. So, we take a survey of different areas in a locality and take a set of different houses with different features as our set of vertices and link or distance between these as our edge set. Let h_1, h_2, h_3, h_4 be different choices of houses, and we define neutrosophic cubic membership function of a house $h \in V$ as

$$\begin{aligned} M = \Phi(h) = \{&[\text{school, university}], \text{mosque}), \\ &\cdot ([\text{workplace, hospital}], \text{gym}), \\ &\cdot ([\text{shops, market}], \text{park})\}; \end{aligned} \tag{119}$$

here, interval membership represents past and future for truth and indeterminate membership, respectively, and present time represents falsity memberships, and

$$N = \Psi(h_1 h_2) = \text{distance between these houses.} \tag{120}$$

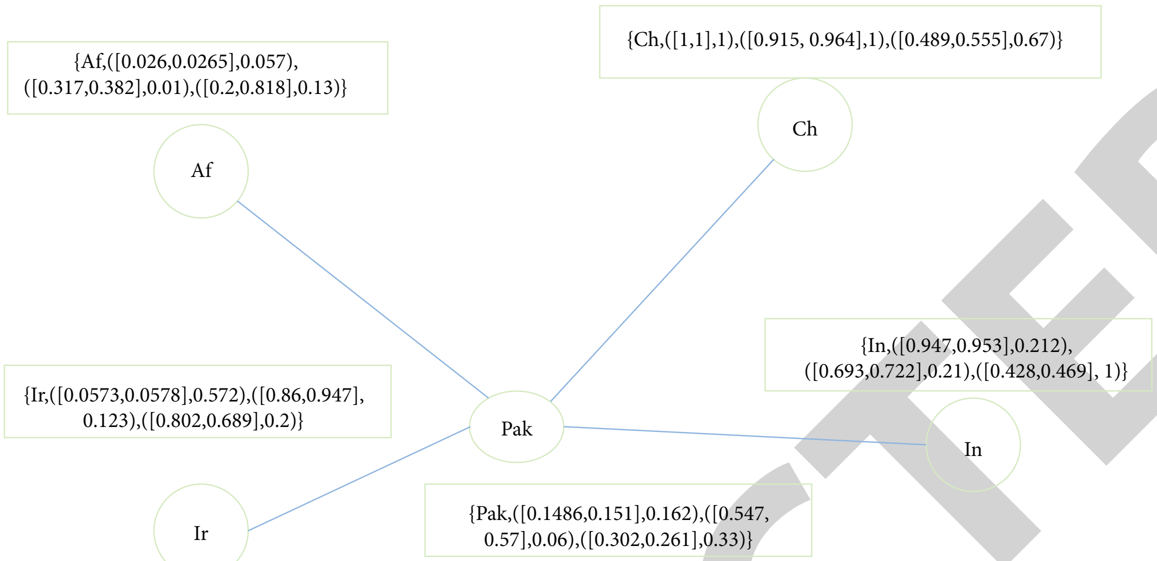

Figure 12: Represents neighborhood of Pakistan.

Let

$$\begin{aligned}\Phi(h_1) &= \{[.6, .7], 1), ([.7, .2], .3), ([.8, 1], 1)\},\\ \Phi(h_2) &= \{[.2, .3],.2), ([.5, .6], .3), ([.4, .7], .9)\},\\ \Phi(h_3) &= \{[.5, .6],.5), ([.3, .9], .4), ([0, .2], .7)\},\\ \Phi(h_4) &= \{[.1, .5],.4), ([.6, .8], .1), ([.6, .5], .4)\}.\end{aligned} \tag{121}$$

Then, the neutrosophic cubic open neighborhood degree of each vertex (house) is given for house h_1; we have

$$\begin{aligned}\theta(\mathbb{N}_{ncg}(h_1)) &= \Phi(h_2) + \Phi(h_3) + \Phi(h_4)\\ &= \{[.2, .3],.2), ([.5, .6], .3), ([.4, .7], .9)\}\\ &\quad + \{[.5, .6],.5), ([.3, .9], .4), ([, .2], .7)\}\\ &\quad + \{[.1, .5], 0.4), ([.6, .8], .1), ([.6, .5], .4)\}\\ &= \{[.8, 1.4], 1.1), ([1.5, 2.3], .8), ([1.0, 1.4], 2.0)\}\,;\end{aligned} \tag{122}$$

similarly, for house h_2, we have

$$\begin{aligned}\theta(\mathbb{N}_{ncg}(h_2)) &= \Phi(h_1) + \Phi(h_4)\\ &= \{[.6, .7], 1), ([.7, .2], .3), ([.8, 1], 1)\}\\ &\quad + \{[.1, .5], .4), ([.6, .8], .1), ([.6, .5], .4)\}\\ &= \{[.7, 1.2], 1.4), ([1.3, 1.0], .4), ([1.4, 1.5], 1.4)\},\end{aligned} \tag{123}$$

and for house h_3, we have

$$\begin{aligned}\theta(\mathbb{N}_{ncg}(h_3)) &= \Phi(h_1) + \Phi(h_4)\\ &= \{[.6, .7], 1), ([.7, .2], .3), ([.8, 1], 1)\}\\ &\quad + \{[.1, .5], .4), ([.6, .8], .1), ([.6, .5], .4)\}\\ &= \{[.7, 1.2], 1.4), ([1.3, 1.0], .4), ([1.4, 1.5], 1.4)\}\,;\end{aligned} \tag{124}$$

also, for house h_4, we have

$$\begin{aligned}\theta(\mathbb{N}_{ncg}(h_4)) &= \Phi(h_3) + \Phi(h_1) + \Phi(h_2)\\ &= \{[.5, .6], .5), ([.3, .9], .4), ([0, .2], .7)\}\\ &\quad + \{[.6, .7], 1), ([.7, .2], .3), ([.8, 1], 1)\}\\ &\quad + \{[.2, .3], .2), ([.5, .6], .3), ([.4, .7], .9)\}\\ &= \{[1.3, 1.6], 1.7), ([1.5, 1.7], 1.0), ([1.4, 1.9], 2.6)\}.\end{aligned} \tag{125}$$

Also, the neutrosophic cubic closed neighborhood degree of each vertex (house) is given for house h_1; we have

$$\begin{aligned}\theta(\mathbb{N}_{ncg}[h_1]) &= \Phi(h_1) + \Phi(h_2) + \Phi(h_3) + \Phi(h_4)\\ &= \{[.6, .7], 1), ([.7, .2], .3), ([.8, 1], 1)\}\\ &\quad + \{[.2, .3], .2), ([.5, .6], .3), ([.4, .7], .9)\}\\ &\quad + \{[.5, .6], .5), ([.3, .9], .4), ([0, .2], .7)\}\\ &\quad + \{[.1, .5], .4), ([.6, .8], .1), ([.6, 0.5], .4)\}\\ &= \{[1.4, 2.1], 2.1), ([2.1, 2.5], 1.1), ([1.8, 2.4], 3.0)\},\end{aligned} \tag{126}$$

and for house h_2, we have

$$\begin{aligned}\theta(\mathbb{N}_{ncg}[h_2]) &= \Phi(h_2) + \Phi(h_1) + \Phi(h_4)\\ &= \{[.2, .3],.2), ([.5, .6], .3), ([.4, .7], .9)\}\\ &\quad + \{[.6, .7], 1), ([.7, .2], .3), ([.8, 1], 1)\}\\ &\quad + \{[.1, .5], .4), ([.6, .8], .1), ([.6, .5], .4)\}\\ &= \{[.9, 1.5], 1.6), ([1.8, 1.6], .7), ([1.8, 2.2], 2.3)\},\end{aligned} \tag{127}$$

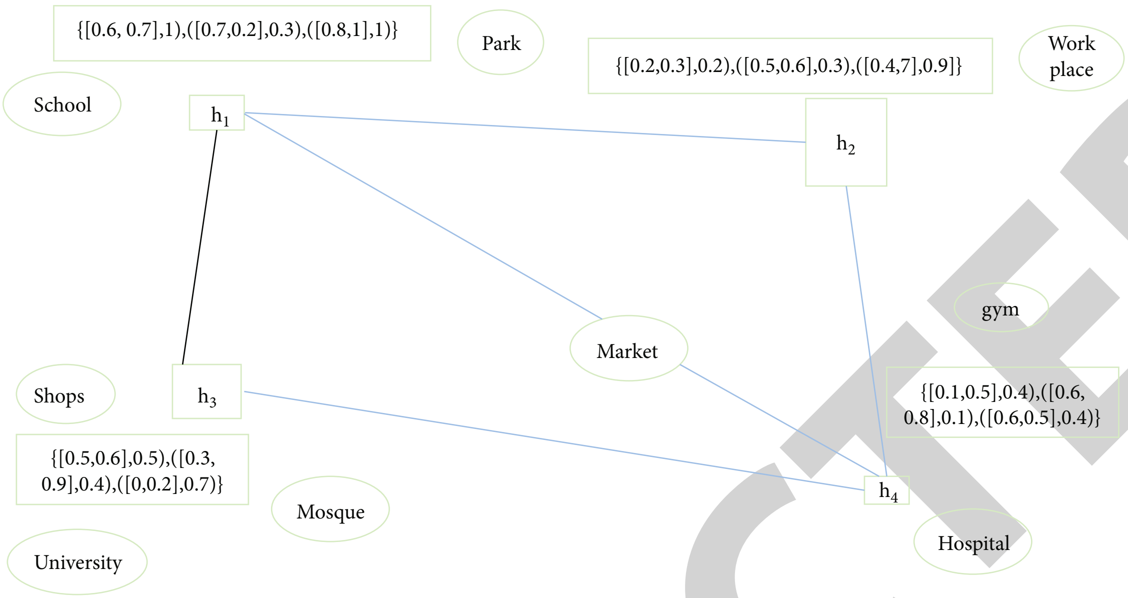

Figure 13: Represents position of four houses with different facilities.

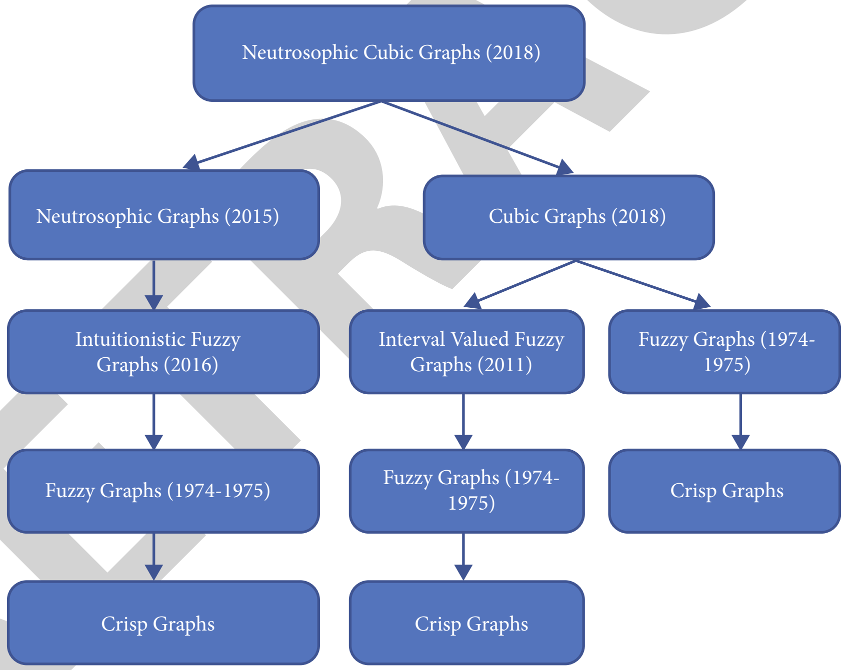

Figure 14: Flow chart of comparison analysis.

and for house h_3, we have

$$\begin{aligned}\theta\left(\mathbb{N}_{ncg}[h_3]\right) &= \Phi(h_1)+\Phi(h_3)+\Phi(h_4)\\ &= \{[.6,.7],1),([.7,.2],.3),\\ &\quad\cdot([.8,1],1)\}+\{[.5,.6],.5),\\ &\quad\cdot([.3,.9],.4),([0,.2],.7)\}\\ &\quad+\{[.1,.5],.4),([.6,.8],.1),([.6,.5],.4)\}\\ &= \{[1.2,1.8],1.9),([1.6,1.9],.8),\\ &\quad\cdot([1.4,1.7],2.1)\}.\end{aligned} \tag{128}$$

Also, for house h_4, we have

$$\begin{aligned}\theta\left(\mathbb{N}_{ncg}[h_4]\right) &= \Phi(h_1)+\Phi(h_2)+\Phi(h_3)+\Phi(h_4)\\ &= \{[.6,.7],1),([.7,.2],.3),([.8,1],1)\}\\ &\quad+\{[.2,.3],.2),([.5,.6],.3),([.4,.7],.9)\}\\ &\quad+\{[.5,.6],.5),([.3,.9],.4),([0,.2],.7)\}\\ &\quad+\{[.1,.5],0.4),([.6,.8],.1),([.6,.5],.4)\}\\ &= \{[1.4,2.1],2.1),([2.1,2.5],1.1),([1.8,2.4],3.0)\}.\end{aligned} \tag{129}$$

We compare neutrosophic cubic open neighborhood and observe that neutrosophic cubic open neighborhood of h_2 and h_3 is the same, but comparison of neutrosophic cubic open neighborhood of h_1 and h_4 shows that h_1 and h_4 are more effective than h_2 and h_3. Also, we observe that neutrosophic cubic open neighborhood of h_4 is effective than h_1. Moreover, one can observe that the neutrosophic cubic closed neighborhood degree of houses h_1 and h_4 is the same, but in view of neutrosophic cubic open neighborhood, h_4 is the best choice in all respects as compared to other houses h_2 and h_3. So, choice of house h_4 is the best choice for our selection of an ideal house. Position of four houses with different facilities is shown in Figure 13.

5. Comparison Analysis

In this paper, our focus is to introduce some different types of neutrosophic cubic graphs. These include balanced, strictly balanced, complete, regular, totally regular, and irregular neutrosophic cubic graphs. In this regard, we explained the open and closed neighborhood of a vertex of the neutrosophic cubic graph and its role in the art of decision-making. Many of these graphs have already been discussed from a different perspective by the other researchers, for example, Poulik et al. [29–32], Akram [44, 45], and Gulistan et al. [65]. We have tried to discuss them concerning the neutrosophic cubic graphs. The neutrosophic cubic graphs are the generalization of different versions of the fuzzy graph which is extended to the neutrosophic cubic graph. The idea is summarized in the form of a flow chart (Figure 14).

This flow chart shows under certain conditions neutrosophic cubic graphs reduced to crisp graphs. So, under certain conditions, all the different types described are reduced for neutrosophic graphs, cubic graphs, intuitionistic graphs, fuzzy graphs, and crisp graphs.

6. Conclusion and Future Work

In this article, we provided different types of neutrosophic cubic graphs with examples and give many results which correlate with these neutrosophic cubic graphs. We used the idea of the neutrosophic cubic open neighborhood degree and neutrosophic cubic closed neighborhood degree of the same vertex in two real-life problems. We concluded the following: (1) As the neutrosophic cubic open neighborhood degree of a vertex (say Pak) is less than the neutrosophic cubic closed neighborhood degree of a vertex (say, Pak), the vertex (say, Pak) has more closed neighborhoods than the open neighborhoods. (2) Also, we observe that house h_4 is the best choice for our selection of an ideal house using the idea of neutrosophic cubic open neighborhood degree and neutrosophic cubic closed neighborhood degree of the same vertex. The limitation of the presented method is the data collection which is not an easy task. In the future, we aim to make more different types of graphs such as line, planer, and directed neutrosophic cubic graphs. We are also aiming to have more real-life applications of neutrosophic cubic graphs.

Data Availability

There is no data related to this article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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