Hindawi
Journal of Function Spaces
Volume 2024, Article ID 9768762, 1 page
https://doi.org/10.1155/2024/9768762

Retraction

Retracted: Study of Neighborhood Degree-Based Topological Indices via Direct and NM-Polynomial of Starphene Graph

Journal of Function Spaces

Received 23 January 2024; Accepted 23 January 2024; Published 24 January 2024

This article has been retracted by Hindawi following an investigation undertaken by the publisher [1]. This investigation has uncovered evidence of one or more of the following indicators of systematic manipulation of the publication process:

(1) Discrepancies in scope

(2) Discrepancies in the description of the research reported

(3) Discrepancies between the availability of data and the research described

(4) Inappropriate citations

(5) Incoherent, meaningless and/or irrelevant content included in the article

(6) Manipulated or compromised peer review

The presence of these indicators undermines our confidence in the integrity of the article's content and we cannot, therefore, vouch for its reliability. Please note that this notice is intended solely to alert readers that the content of this article is unreliable. We have not investigated whether authors were aware of or involved in the systematic manipulation of the publication process.

Wiley and Hindawi regrets that the usual quality checks did not identify these issues before publication and have since put additional measures in place to safeguard research integrity.

We wish to credit our own Research Integrity and Research Publishing teams and anonymous and named external researchers and research integrity experts for contributing to this investigation.

The corresponding author, as the representative of all authors, has been given the opportunity to register their agreement or disagreement to this retraction. We have kept a record of any response received.

References

[1] D. Afzal, S. Hameed, U. Ashraf, A. Mehmood, F. Chaudhry, and D. K. Thapa, "Study of Neighborhood Degree-Based Topological Indices via Direct and NM-Polynomial of Starphene Graph," *Journal of Function Spaces*, vol. 2022, Article ID 8661489, 16 pages, 2022.

Hindawi
Journal of Function Spaces
Volume 2022, Article ID 8661489, 16 pages
https://doi.org/10.1155/2022/8661489

Research Article

Study of Neighborhood Degree-Based Topological Indices via Direct and NM-Polynomial of Starphene Graph

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Received 10 March 2022; Revised 16 April 2022; Accepted 13 May 2022; Published 15 June 2022

Academic Editor: Ganesh Ghorai

Our objective is to compute the neighborhood degree-based topological indices via NM-polynomial for starphene. In the neighborhood degree-based topological indices, we compute the third version of the Zagreb index; neighborhood second Zagreb index; neighborhood forgotten topological index; neighborhood second modified Zagreb index; neighborhood general Randic index; neighborhood harmonic index; neighborhood inverse sum index; first, second, third, fourth, and fifth NDe indices; fourth atom bond connective index; fifth geometric arithmetic index; fifth arithmetic-geometric index; fifth hyper-first and second Zagreb index; general first neighborhood index; and Sanskruti index. These neighborhood topological indices are computed both direct and via the NM-polynomial approach.

1. Introduction

Graph theory deals with the study of lines and points. It is the division of mathematics which is concerned with graphs (structures that include points and lines and which frequently pictorially denote mathematical facts). Graph theory is the study of the relationship between edges and vertices. Formally, a graph is a pair (V, E) where $V(G)$ and $E(G)$ are the vertex set and edge set of a graph G, respectively. A simple graph is a graph that has no loops and multiple edges.

The total number of edges incident to $x \in V(G)$ is known as the degree of x and is denoted by d_u. Here, δ_x denotes the degree sum of neighbors of x in G. By neighbors of a vertex, we mean the vertices adjacent to that vertex.

Various physical properties, chemical reactivity, and biological activities of a chemical molecule are strongly connected to its graphical structure, and this fact is the main topic of interest in chemical graph theory. The topological index plays a key role in predicting such a connection without involving a wet lab. A topological index is a function from the collection of graphs to the set of real numbers that describe the topology of the graph and are used in QSPR/QSAR analysis. It remains unchanged for isomorphic graphs. Thousands of indices are developed in the literature on chemical graph theory.

A molecular graph is a graph such that its vertices represent the atoms and the edges of the bonds. Chemical graph theory is a branch of mathematical chemistry whose focus of interest is on finding topological indices of a molecular graph that correlate well with the chemical properties of the chemical molecules. Several topological indices have been considered in theoretical chemistry and have found some applications, especially in QSPR/QSAR.

Topological indices are numerical parameters associated with a graph that characterize its topology. These indices are usually graphed invariant. The topology of chemical structures is described by these indices [1]. To make the computation of neighborhood degree-based topological indices easier, [2] introduced neighborhood M-polynomial whose role for neighborhood degree-based indices is parallel to the role of the M-polynomial for degree-based indices. NM-polynomial is another correct method to prove the neighborhood degree-based topological indices.

A starphene $St_{l,m,n}$ can be considered as a structure obtained by fusing three linear polyacenes of length l, m, and n, respectively. Starphenes are widely used in many electronic devices and played a key role in the revolution of miniaturization of electronic devices. I want to explore the structure of starphene by neighborhood degree-based topological indices by direct and NM-polynomial [3].

For a graph G, neighborhood degree-based topological invariants are defined as

$$\mathrm{NI}(G) = \sum_{xy\in E_G} f\left(\delta_x, \delta_y\right). \tag{1}$$

By counting edges that have the same end degrees in the chemical graph, then we can rewrite equation (1) as

$$\mathrm{NI}(G) = \sum_{j\le k} m_{jk} f(j, k), \tag{2}$$

where the relation $\{\delta_x, \delta_y\} = \{j, k\}$ is satisfied and m_{jk} is the total count of edges xy of the graph G.

In 2021, Mondal et al. introduced the ND indices [4]:

$$\begin{aligned}
&\text{First ND index} = \mathrm{ND}_1(G) = \sum_{xy\in E_G} \sqrt{(\delta_x)(\delta_y)},\\
&\text{Second ND index} = \mathrm{ND}_2(G) = \sum_{xy\in E_G} \frac{1}{\sqrt{\delta_x + \delta_y}},\\
&\text{Third ND index} = \mathrm{ND}_3(G) = \sum_{xy\in E_G} \delta_x\delta_y\left(\delta_x + \delta_y\right),\\
&\text{Fourth ND index} = \mathrm{ND}_4(G) = \sum_{xy\in E_G} \frac{1}{\sqrt{\delta_x \cdot \delta_y}},\\
&\text{Fifth ND index} = \mathrm{ND}_5(G) = \sum_{xy\in E_G} \left[\frac{\delta_x}{\delta_y} + \frac{\delta_y}{\delta_x}\right].
\end{aligned} \tag{3}$$

Ghorbani and Hosseinzadeh defined the third version of the Zagreb index in 2013 [5].

$$M_1'(G) = \sum_{xy\in E_G} \left(\delta_x + \delta_y\right). \tag{4}$$

Mondal et al. introduced the neighborhood second Zagreb index in 2019 [6].

$$M_2^{\ddagger}(G) = \sum_{xy\in E_G} \delta_x\delta_y. \tag{5}$$

Verma and Mondal defined the neighborhood second modified Zagreb index in 2019 [7].

$$M_2^{nm}(G) = \sum_{xy\in E_G} \frac{1}{\delta_x\delta_y} \tag{6}$$

Mondal et al. introduced the neighborhood forgotten topological index in 2019 [6].

$$F_N^{\ddagger}(G) = \sum_{xy\in E_G} \left(\delta_x^2 + \delta_y^2\right). \tag{7}$$

Verma and Mondal defined the neighborhood general Randic index in 2019 [7].

$$\mathrm{NR}_\alpha(G) = \sum_{xy\in E_G} \left(\delta_x\delta_y\right)^\alpha. \tag{8}$$

Verma and Mondal defined the neighborhood harmonic index in 2019 [7].

$$\mathrm{NH}(G) = \sum_{xy\in E_G} \frac{2}{\delta_x + \delta_y}. \tag{9}$$

Verma and Mondal defined the neighborhood inverse sum index in 2019 [7].

$$\mathrm{NI}(G) = \sum_{xy\in E_G} \frac{\delta_x\delta_y}{\delta_x + \delta_y}. \tag{10}$$

Ghorbani and Hosseinzadeh present in 2010 the fourth atom bond connectivity index as [8]

$$ABC_4(G) = \sum_{xy\in E_G} \sqrt{\frac{\delta_x + \delta_y - 2}{\delta_x \cdot \delta_y}}. \tag{11}$$

The fifth geometric arithmetic index was proposed by Grovac et al. in 2011 and defined as [9]

$$\mathrm{GA}_5(G) = \sum_{xy\in E_G} \frac{2\sqrt{\delta_x \cdot \delta_y}}{\delta_x + \delta_y}. \tag{12}$$

Kulli introduced the fifth arithmetic geometric index in 2017 and defined it as [10]

$$\mathrm{AG}_5(G) = \sum_{xy\in E_G} \frac{\delta_x + \delta_y}{2\sqrt{\delta_x \cdot \delta_y}}. \tag{13}$$

Kulli [11] proposed the fifth hyper-first and second Zagreb index in 2017 and defined it as

$$\begin{aligned}
\mathrm{HM}_1G_5(G) &= \sum_{xy\in E_G} \left(\delta_x + \delta_y\right)^2,\\
\mathrm{HM}_2G_5(G) &= \sum_{xy\in E_G} \left(\delta_x \cdot \delta_y\right)^2,
\end{aligned} \tag{14}$$

Hosamani proposed the Sanskruti index in 2020 [12].

$$S(G)=\sum_{xy\in E_G}\left(\frac{\delta_x\delta_y}{\delta_x+\delta_y-2}\right)^3. \quad (15)$$

2. NM-Polynomial

Verma and Mondal defined the neighborhood M-polynomial in 2019 [7, 11].

$$\mathrm{NM}_G(u,v)=\sum_{\psi\le j\le k\le\psi} m_{jk}u^j v^k. \quad (16)$$

Here, $\psi=\min\{d_x|x\in V_G\}$ and $\Psi=\max\{d_x|x\in V_G\}$.

3. Induced Neighborhood Degree-Based Topological Indices via NM-Polynomial

Some operators which are used in the above table are defined as

$$\begin{aligned}
D_u\mathrm{NM}[G:u,v]&=u\frac{\partial}{\partial u}\mathrm{NM}[G:u,v],\\
D_v\mathrm{NM}[G:u,v]&=v\frac{\partial}{\partial v}\mathrm{NM}[G:u,v],\\
D_u^{1/2}\mathrm{NM}[G:u,v]&=\sqrt{u\frac{\partial}{\partial \mathrm{u}}\mathrm{NM}[G:u,v]}\bullet\sqrt{\mathrm{NM}[G:u,v]},\\
D_v^{1/2}\mathrm{NM}[G:u,v]&=\sqrt{v\frac{\partial}{\partial v}\mathrm{NM}[G:u,v]}\cdot\sqrt{\mathrm{NM}[G:u,v]},\\
S_u^{1/2}\mathrm{NM}[G:u,v]&=\sqrt{\int_0^u\frac{\mathrm{NM}[G:t,v]}{t}dt}\cdot\sqrt{\mathrm{NM}[G:u,v]},\\
S_v^{1/2}\mathrm{NM}[G:u,v]&=\sqrt{\int_0^v\frac{\mathrm{NM}[G:u,t]}{t}dt}\cdot\sqrt{\mathrm{NM}[G:u,v]},\\
J\mathrm{NM}[G:u,v]&=\mathrm{NM}[G:u,v],\\
Q_{u(\alpha)}\mathrm{NM}[G:u,v]&=u^\alpha\mathrm{NM}[G:u,v],\\
Q_{v(\alpha)}\mathrm{NM}[G:u,v]&=v^\alpha\mathrm{NM}[G:u,v].
\end{aligned} \quad (17)$$

In Table 1, we have computed Neighbourhood degree dependent topological indices via NM-polynomial.

Table 1: Neighborhood degree dependent topological indices via NM-polynomial.

$\mathrm{ND}_1[G]=D_u^{1/2}D_v^{1/2}\mathrm{NM}[G:u,v]\|_{u=v=1}$
$\mathrm{ND}_2[G]=S_u^{1/2}J\mathrm{NM}[G:u,v]\|_{u=1}$
$\mathrm{ND}_3[G]=D_uD_v(D_u+D_v)\mathrm{NM}[G:u,v]\|_{u=v=1}$
$\mathrm{ND}_4[G]=S_u^{1/2}S_v^{1/2}\mathrm{NM}_G(u,v)\|_{u=v=1}$
$\mathrm{ND}_5[G]=(D_uS_v+S_uD_v)\mathrm{NM}[G:u,v]\|_{u=v=1}$
$M_1'[G]=(D_u+D_v)\mathrm{NM}[G:u,v]\|_{u=v=1}$
$M_2^{\ddagger}[G]=D_uD_v\mathrm{NM}[G:u,v]\|_{u=v=1}$
$M_2^{nm}[G]=S_uS_v\mathrm{NM}[G:u,v]\|_{u=v=1}$
$F_N^{\ddagger}[G]=(D_u^2+D_v^2)\mathrm{NM}[G:u,v]\|_{u=v=1}$
$\mathrm{NR}_\alpha[G]=D_u^\alpha D_v^\alpha\mathrm{NM}[G:u,v]\|_{u=v=1}$
$\mathrm{NH}[G]=2S_uJ\mathrm{NM}[G:u,v]\|_{u=1}$
$\mathrm{NI}[G]=S_uJD_uD_v\mathrm{NM}[G:u,v]\|_{u=1}$
$ABC_4[G]=D_u^{1/2}Q_{u(-2)}JS_u^{1/2}S_v^{1/2}\mathrm{NM}[G:u,v]\Big\|_{u=1}$
$\mathrm{GA}_5[G]=2S_uJD_u^{1/2}D_v^{1/2}\mathrm{NM}[G:u,v]\|_{u=1}$
$\mathrm{AG}_5[G]=1/2S_u^{1/2}S_v^{1/2}(D_u+D_v)\mathrm{NM}[G:u,v]\|_{u=v=1}$
$\mathrm{HM}_1G_5[G]=D_u^2J\mathrm{NM}[G:u,v]\|_{u=1}$
$\mathrm{HM}_2G_5[G]=D_u^2D_v^2\mathrm{NM}[G:u,v]\|_{u=v=1}$
$S[G]=S_u^3Q_{u(-2)}JD_u^3D_v^3\mathrm{NM}[G:u,v]\Big\|_{u=1}$

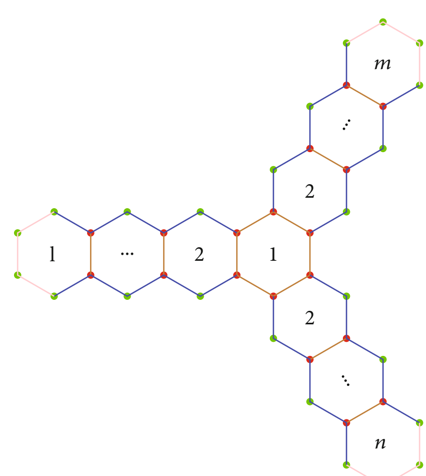


Figure 1: Starphene.

4. Starphene Graph

Starphenes are polycyclic aromatic hydrocarbons and build by three different acene arms. Starphenes are the basic building blocks for the miniaturization of different especially organic electronic devices. It also played an important role in different logical gates. A starphene $\mathrm{St}_{l,m,n}$ shown in Figure 1 can be considered as a structure obtained by fusing three linear polyacenes of length l, m, and n, respectively. The edge partition of $\mathrm{St}_{l,m,n}$ is shown in 2. A starphene S $\mathrm{t}_{l,m,n}$, shown in Figure 1 can be considered as a structure obtained by fusing three linear polyacenes of length l, m, and n, respectively. The edge partition of $\mathrm{St}_{l,m,n}$ is shown Table 2.

To compute the edge partitions, first, we count the number of vertices and edges of the molecular graph and find the degree and neighborhood degree of the vertices and then, we count the edges that have a specific neighborhood vertex

TABLE 2: Edge partition of starphene $St_{l,m,n}$.

(d_x, d_y)	Number of edges
(4,4)	3
(4,5)	6
(5,7)	6
(6,7)	$4(l+m+n)-30$
(7,7)	$(l+m+n)-6$
(6,8)	6
(8,8)	6
Total edges	$5(l+m+n)-9$

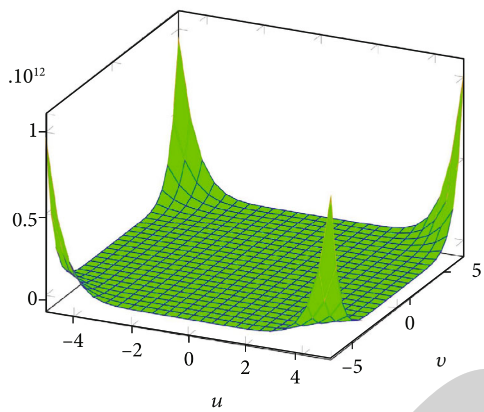


FIGURE 2: 3D plot of NM-polynomial of starphene $(St_{l,m,n})$ for $l+m+n=12$.

degree. First, we compute the smallest dimension and then generalize.

5. NM-Polynomial of Starphene Graph

In this section, we compute the NM-polynomial of $St_{l,m,n}$.

Theorem 1. *If starphene is denoted by $St_{l,m,n}$, then for l,m,n ≥3, NM-polynomial of $St_{l,m,n}$ is $NM[St_n : u, v] = 3u^4v^4 + 6u^4v^5 + 6u^5v^7 + [4(l+m+n)-30]u^6v^7 + [(l+m+n)-6]u^7v^7 + 6u^6v^8 + 6u^8v^8$.*

Proof. Let $St_{l,m,n}$ represent the starphene; then, by using Figure 1 and Table 2, we have the following edge partition of $St_{l,m,n}$ which is

$$E_{4,4}(St_{l,m,n}) = \{e = xy \in E(St_{l,m,n}) : Nd_x = 4, Nd_y = 4\} \Rightarrow |E_{4,4}St_{l,m,n}| = 3,$$

$$E_{4,5}(St_{l,m,n}) = \{e = xy \in E(St_{l,m,n}) : Nd_x = 4, Nd_y = 5\} \Rightarrow |E_{4,5}St_{l,m,n}| = 6,$$

$$E_{5,7}(St_{l,m,n}) = \{e = xy \in E(St_{l,m,n}) : Nd_x = 5, Nd_y = 7\} \Rightarrow |E_{5,7}St_{l,m,n}| = 6,$$

$$E_{6,7}(St_{l,m,n}) = \{e = xy \in E(St_{l,m,n}) : Nd_x = 6, Nd_y = 7\} \Rightarrow |E_{6,7}St_{l,m,n}| = 4(l+m+n)-30,$$

$$E_{7,7}(St_{l,m,n}) = \{e = xy \in E(St_{l,m,n}) : Nd_x = 7, Nd_y = 7\} \Rightarrow |E_{7,7}St_{l,m,n}| = (l+m+n)-6,$$

$$E_{6,8}(St_{l,m,n}) = \{e = xy \in E(St_{l,m,n}) : Nd_x = 6, Nd_y = 8\}. \quad (18)$$

The following result was obtained by using the definition of NM-polynomial:

$$NM[St_n : u, v] = \sum_{\delta\le i\le j\le \Delta} m_{i,j}(St_{l,m,n})u^i v^j,$$
$$NM[St_n : u, v] = \sum_{4\le i\le j\le 8} m_{i,j}(St_{l,m,n})u^i v^j,$$
$$NM[St_n : u, v] = \sum_{4\le 4} m_{4,4}(St_{l,m,n})u^4v^4 + \sum_{4\le 5} m_{4,5}(St_{l,m,n})u^4v^5 + \sum_{5\le 7} m_{5,7}(St_{l,m,n})u^5v^7 + \sum_{6\le 7} m_{6,7}(St_{l,m,n})u^6v^7 + \sum_{7\le 7} m_{7,7}(St_{l,m,n})u^7v^7 + \sum_{6\le 8} m_{6,8}(St_{l,m,n})u^6v^8 + \sum_{8\le 8} m_{8,8}(St_{l,m,n})u^8v^8,$$
$$NM[St_n : u, v] = |E_{4,4}|u^4v^4 + |E_{4,5}|u^4v^5 + |E_{5,7}|u^5v^7 + |E_{6,7}|u^6v^7 + |E_{7,7}|u^7v^7 + |E_{6,8}|u^6v^8 + |E_{8,8}|u^8v^8,$$
$$NM[St_n : u, v] = 3u^4v^4 + 6u^4v^5 + 6u^5v^7 + [4(l+m+n)-30]u^6v^7 + [(l+m+n)-6]u^7v^7 + 6u^6v^8 + 6u^8v^8. \quad (19)$$

□

The plot of NM-polynomial of $St_{l,m,n}$ is shown in Figure 2.

6. Topological Indices of Starphene

In this section, we calculate a few topological indices by direct formulae for $St_{l,m,n}$.

Theorem 2. *Let $St_{l,m,n}$ be a starphene, and then,*

(1) $ND_1(St_{l,m,n}) = (7 + 4\sqrt{42})(l+m+n) + 18 + 24\sqrt{3} + 12\sqrt{5} + 6\sqrt{35} - 30\sqrt{42}$

(2) $ND_2(St_{l,m,n}) = 1/182(56\sqrt{13} + 13\sqrt{14})(l+m+n) + 1/52(182 + 29\sqrt{2} + 52\sqrt{3} - 120\sqrt{13})$

(3) $ND_3(St_{l,m,n}) = 2870(l+m+n) - 6336$

(4) $ND_4(St_{l,m,n}) = 1/21(2 + 2\sqrt{42})(l+m+n) + 1/210($

$135+126\sqrt{5}+105\sqrt{6}+36\sqrt{35}-150\sqrt{42})$

(5) $ND_5(St_{l,m,n}) = 212/21(l+m+n) - 603/35$

(6) $M_1'(St_{l,m,n}) = 66(l+m+n) - 144$

(7) $M_2^{\ddagger}(St_{l,m,n}) = 217(l+m+n) - 504$

(8) $M_2^{nm}(St_{l,m,n}) = 17/47(l+m+n) + 321/840$

(9) $F_N^{\ddagger}(St_{l,m,n}) = 438(l+m+n) - 984$

(10) $NR_\alpha(St_{l,m,n}) = (4\cdot 42^\alpha + 49^\alpha)(l+m+n) + (3\cdot 16^\alpha + 6\cdot 20^\alpha + 6\cdot 35^\alpha - 30\cdot 42^\alpha - 6\cdot 48^\alpha + 6\cdot 48^\alpha + 6\cdot 64^\alpha)$

(11) $NH(St_{l,m,n}) = 69/91(l+m+n) - 61/78$

(12) $NI(St_{l,m,n}) = 427/26(l+m+n) - 19939/546$

(13) $ABC_4(St_{l,m,n}) = 1/21(6\sqrt{3}+\sqrt{462})(l+m+n) + 1/140(420 - 240\sqrt{3} + 105\sqrt{6} + 225\sqrt{14} + 84\sqrt{35} - 100\sqrt{462})$

(14) $GA_5(St_{l,m,n}) = 1/13(13+8\sqrt{42})(l+m+n) + 1/273(819 + 936\sqrt{3} + 728\sqrt{5} + 273\sqrt{35} - 1260\sqrt{42})$

(15) $AG_5(St_{l,m,n}) = 1/21(21+13\sqrt{42})(l+m+n) + 1/210(630 + 735\sqrt{3} + 567\sqrt{5} + 216\sqrt{35} - 975\sqrt{42})$

(16) $HM_1G_5(St_{l,m,n}) = 872(l+m+n) - 1992$

(17) $HM_2G_5(St_{l,m,n}) = 9457(l+m+n) - 28408$

(18) $S(St_{l,m,n}) = 668687075/2299968(l+m+n) - 555184266893/832716192$

Proof.

(1) First NDe index

$$\begin{aligned}
\mathrm{ND}_1(\mathrm{St}_{l,m,n}) &= \sum_{xy\in E_{\mathrm{St}_{l,m,n}}} \sqrt{(\delta_x)(\delta_y)} = \sqrt{4\cdot 4}(3) + \sqrt{4\cdot 5}(6) \\
&+ \sqrt{5\cdot 7}(6) + \sqrt{6\cdot 7}[4(l+m+n)-30] \\
&+ \sqrt{7\cdot 7}[(l+m+n)-6] + \sqrt{6\cdot 8}(6) \\
&+ \sqrt{8\cdot 8}(6) = 12 + 12\sqrt{5} + 6\sqrt{35} \\
&+ \sqrt{42}[4(l+m+n)-30] + 7[(l+m+n)-6] \\
&+ 24\sqrt{3} + 48 = \left(7 + 4\sqrt{42}\right)(l+m+n) \\
&+ 18 + 24\sqrt{3} + 12\sqrt{5} + 6\sqrt{35} - 30\sqrt{42}.
\end{aligned} \tag{20}$$

(2) Second NDe index

$$\begin{aligned}
\mathrm{ND}_2[\mathrm{St}_{l,m,n}] &= \sum_{xy\in E_G} \frac{1}{\sqrt{\delta_x+\delta_y}} = \frac{3}{\sqrt{4+4}} + \frac{6}{\sqrt{4+5}} \\
&+ \frac{6}{\sqrt{5+7}} + \frac{1}{\sqrt{6+7}}[4(l+m+n)-30] \\
&+ \frac{1}{\sqrt{7+7}}[(l+m+n)-6] + \frac{6}{\sqrt{6+8}} + \frac{6}{\sqrt{8+8}} \\
&= \frac{3\sqrt{2}}{4} + \frac{6\sqrt{11}}{11} + \frac{6\sqrt{13}}{13} + \frac{13}{13}[4(l+m+n)-30] \\
&+ \frac{\sqrt{14}}{14}[(l+m+n)-6] + \frac{6\sqrt{14}}{14} + \frac{3}{2} \\
&= \frac{1}{182}\left(56\sqrt{13} + 13\sqrt{14}\right)(l+m+n) \\
&+ \frac{1}{52}\left(182 + 29\sqrt{2} + 52\sqrt{3} - 120\sqrt{13}\right).
\end{aligned} \tag{21}$$

(3) Third NDe index

$$\begin{aligned}
\mathrm{ND}_3[\mathrm{St}_{l,m,n}] &= \sum_{xy\in E_G} \delta_x\delta_y(\delta_x+\delta_y) = (4\cdot 4)(4+4)(3) \\
&+ (4\cdot 5)(4+5)(6) + (5\cdot 7)(5+7)(6) \\
&+ (6\cdot 7)(6+7)[4(l+m+n)-30] \\
&+ (7\cdot 7)(7+7)\,[(l+m+n)-6] \\
&+ (6\cdot 8)(6+8)(6) + (8\cdot 8)(8+8)(6) \\
&= 384 + 1080 + 2520 + 564[4(l+m+n)-30] \\
&+ 686[(l+m+n)-6] + 4032 + 6144 \\
&= 2870(l+m+n) - 6336.
\end{aligned} \tag{22}$$

(4) Fourth NDe index

$$\begin{aligned}
\mathrm{ND}_4[\mathrm{St}_{l,m,n}] &= \sum_{xy\in E_G} \frac{1}{\sqrt{\delta_x\cdot\delta_y}} = \frac{3}{\sqrt{4\cdot 4}} + \frac{6}{\sqrt{4\cdot 5}} \\
&+ \frac{6}{\sqrt{5\cdot 7}} + \frac{1}{\sqrt{6\cdot 7}}[4(l+m+n)-30] \\
&+ \frac{1}{\sqrt{7\cdot 7}}[(l+m+n)-6] + \frac{6}{\sqrt{6\cdot 8}} + \frac{6}{\sqrt{8\cdot 8}} \\
&= \frac{3}{4} + \frac{3}{\sqrt{5}} + \frac{6}{\sqrt{35}} + \frac{1}{\sqrt{42}}[4(l+m+n)-30] \\
&+ \frac{1}{7}[(l+m+n)-6] + \frac{\sqrt{3}}{2} + \frac{3}{4} \\
&= \frac{1}{21}\left(2 + 2\sqrt{42}\right)(l+m+n) + \frac{1}{210}\Big(135 \\
&+ 126\sqrt{5} + 105\sqrt{6} + 36\sqrt{35} - 150\sqrt{42}\Big).
\end{aligned} \tag{23}$$

(5) Fifth NDe index

$$\mathrm{ND}_5[\mathrm{St}_{l,m,n}] = \sum_{xy\in E_G}\left[\frac{\delta_x}{\delta_y} + \frac{\delta_y}{\delta_x}\right] = \left(\frac{4}{4} + \frac{4}{4}\right)(3) + \left(\frac{4}{5} + \frac{5}{4}\right)(6)$$

$$+\left(\frac{5}{7}+\frac{7}{5}\right)(6)+\left(\frac{6}{7}+\frac{7}{6}\right)[4(l+m+n)-30] +\left(\frac{7}{7}+\frac{7}{7}\right)[(l+m+n)-6]+\left(\frac{6}{8}+\frac{8}{6}\right)(6) +\left(\frac{8}{8}+\frac{8}{8}\right)(6)=6+\frac{123}{10}+\frac{444}{35} +\frac{85}{42}[4(l+m+n)-30]+2[(l+m+n)-6] +\frac{25}{2}+12=\frac{212}{21}(l+m+n)-\frac{603}{35}. \tag{24}$$

(6) Third version of Zagreb index

$$M_1'[\mathrm{St}_{l,m,n}]=\sum_{xy\in E_G}(\delta_x+\delta_y)=(4+4)(3)+(4+5)(6)+(5+7)\cdot(6)+(6+7)[4(l+m+n)-30]+(7+7)\cdot[(l+m+n)-6]+(6+8)(6)+(8+8)(6) =24+54+72+13[4(l+m+n)-30]+14[(l+m+n)-6]+84+96 =66(l+m+n)-144. \tag{25}$$

(7) Neighborhood second Zagreb index

$$M_2^{\ast}[\mathrm{St}_{l,m,n}]=\sum_{xy\in E_G}\delta_x\delta_y=(4\cdot4)(3)+(4\cdot5)(6)+(5\cdot7)(6)+(6\cdot7)[4(l+m+n)-30]+(7\cdot7)\cdot[(l+m+n)-6]+(6\cdot8)(6)+(8\cdot8)(6) =48+120+210+42[4(l+m+n)-30]+49[(l+m+n)-6]+288+384 =217(l+m+n)-504. \tag{26}$$

(8) Neighborhood second modified Zagreb index

$$M_2^{nm}[\mathrm{St}_{l,m,n}]=\sum_{xy\in E_G}\frac{1}{\delta_x\delta_y}=\frac{1}{(4\cdot4)}(3)+\frac{1}{(4\cdot5)}(6)+\frac{1}{(5\cdot7)}(6)+\frac{1}{(6\cdot7)}[4(l+m+n)-30]+\frac{1}{(7\cdot7)}\cdot[(l+m+n)-6]+\frac{1}{(6\cdot8)}(6)+\frac{1}{(8\cdot8)}(6) =\frac{3}{16}+\frac{3}{10}+\frac{6}{35}+\frac{1}{42}[4(l+m+n)-30]+\frac{1}{49}[(l+m+n)-6]+\frac{1}{8}+\frac{3}{32} =\frac{17}{47}(l+m+n)+\frac{321}{840}. \tag{27}$$

(9) Neighborhood forgotten topological index

$$F_N^{\ast}[\mathrm{St}_{l,m,n}]=\sum_{xy\in E_G}\left(\delta_x^2+\delta_y^2\right)=\left(4^2+4^2\right)(3)+\left(4^2+5^2\right)(6)+\left(5^2+7^2\right)(6)+\left(6^2+7^2\right)[4(l+m+n)-30]+\left(7^2+7^2\right)[(l+m+n)-6]+\left(6^2+8^2\right)(6)+\left(8^2+8^2\right)(6)=96+246+444+34(p+s+t)-2550+98(p+s+t)-588+600+768 =438(l+m+n)-984. \tag{28}$$

(10) Neighborhood general Randic index

$$\mathrm{NR}_\alpha[\mathrm{St}_{l,m,n}]=\sum_{xy\in E_G}(\delta_x\delta_y)^\alpha=(4\cdot4)^\alpha(3)+(4\cdot5)^\alpha(6)+(5\cdot7)^\alpha(6)+(6\cdot7)^\alpha[4(l+m+n)-30]+(7\cdot7)^\alpha[(l+m+n)-6]+(6\cdot8)^\alpha(6)+(8\cdot8)^\alpha(6)=3\cdot16^\alpha+20^\alpha(6)+6\cdot35^\alpha+42^\alpha[4(l+m+n)-30]+49^\alpha[(l+m+n)-6]+6\cdot48^\alpha+6\cdot64^\alpha=(4\cdot42^\alpha+49^\alpha)(l+m+n)+(3\cdot16^\alpha+6\cdot20^\alpha+6\cdot35^\alpha-30\cdot42^\alpha-6\cdot48^\alpha+6\cdot48^\alpha+6\cdot64^\alpha). \tag{29}$$

(11) Neighborhood harmonic index

$$\mathrm{NH}[\mathrm{St}_{l,m,n}]=\sum_{xy\in E_G}\frac{2}{\delta_x+\delta_y}=\frac{2}{4+4}(3)+\frac{2}{4+5}(6)+\frac{2}{5+7}(6)+\frac{2}{6+7}[4(l+m+n)-30]+\frac{2}{7+7}[(l+m+n)-6]+\frac{2}{6+8}(6)+\frac{2}{8+8}(6) =\frac{3}{4}+\frac{4}{3}+1+\frac{2}{13}[4(l+m+n)-30]+\frac{1}{7}[(l+m+n)-6]+\frac{6}{7}+\frac{3}{4} =\frac{69}{91}(l+m+n)-\frac{61}{78}. \tag{30}$$

(12) Neighborhood inverse sum index

$$\mathrm{NI}[\mathrm{St}_{l,m,n}]=\sum_{xy\in E_G}\frac{\delta_x\delta_y}{\delta_x+\delta_y}=\frac{4\cdot4}{4+4}(3)+\frac{4\cdot5}{4+5}(6)+\frac{5\cdot7}{5+7}(6)+\frac{6\cdot7}{6+7}[4(l+m+n)-30]+\frac{7\cdot7}{7+7}[(l+m+n)-6]+\frac{6\cdot8}{6+8}(6)+\frac{8\cdot8}{8+8}(6)$$

$$= 6 + \frac{40}{3} + \frac{35}{2} + \frac{42}{13}[4(l+m+n) - 30] + \frac{7}{2}[(l+m+n) - 6] + \frac{144}{7} + 24 = \frac{427}{26}(l+m+n) - \frac{19939}{546}. \quad (31)$$

(13) Fourth atom bond connectivity index

$$ABC_4[\mathrm{St}_{l,m,n}] = \sum_{xy\in E_G} \sqrt{\frac{\delta_x + \delta_y - 2}{\delta_x \cdot \delta_y}} = \sqrt{\frac{4+4-2}{4\cdot 4}}(3) + \sqrt{\frac{4+5-2}{4\cdot 5}}(6) + \sqrt{\frac{5+7-2}{5\cdot 7}}(6) + \sqrt{\frac{6+7-2}{6\cdot 7}}[4(l+m+n) - 30] + \sqrt{\frac{7+7-2}{7\cdot 7}}[(l+m+n) - 6] + \sqrt{\frac{6+8-2}{6\cdot 8}}(6) + \sqrt{\frac{8+8-2}{8\cdot 8}}(6) = \frac{3\sqrt{6}}{4} + \frac{3\sqrt{35}}{5} ++ \frac{6\sqrt{14}}{7} + \frac{\sqrt{462}}{42}[4(l+m+n) - 30] + \frac{2\sqrt{3}}{7}[(l+m+n) - 6] + 3 + \frac{3\sqrt{14}}{4} = \frac{1}{21}\left(6\sqrt{3} + \sqrt{462}\right)(l+m+n) + \frac{1}{140}\left(420 - 240\sqrt{3} + 105\sqrt{6} + 225\sqrt{14} + 84\sqrt{35} - 100\sqrt{462}\right). \quad (32)$$

(14) Fifth geometric arithmetic index

$$\mathrm{GA}_5[\mathrm{St}_{l,m,n}] = \sum_{xy\in E_G} \frac{2\sqrt{\delta_x \cdot \delta_y}}{\delta_x + \delta_y} = 2\frac{2\sqrt{4\cdot 4}}{4+4}(3) + \frac{2\sqrt{4\cdot 5}}{4+5}(6) + \frac{2\sqrt{5\cdot 7}}{5+7}(6) + \frac{2\sqrt{6\cdot 7}}{6+7}[4(l+m+n) - 30] + \frac{2\sqrt{7\cdot 7}}{7+7}[(l+m+n) - 6] + \frac{2\sqrt{6\cdot 8}}{6+8}(6) + \frac{2\sqrt{8\cdot 8}}{8+8}(6) = 3 + \frac{8\sqrt{5}}{3} + \sqrt{35} + \frac{2\sqrt{42}}{13} \cdot [4(l+m+n) - 30] + [(l+m+n) - 6]\frac{24\sqrt{3}}{7} + 6 = \frac{1}{13}\left(13 + 8\sqrt{42}\right)(l+m+n) + \frac{1}{273}\left(819 + 936\sqrt{3} + 728\sqrt{5} + 273\sqrt{35} - 1260\sqrt{42}\right). \quad (33)$$

(15) Fifth arithmetic geometric index

$$\mathrm{AG}_5[\mathrm{St}_{l,m,n}] = \sum_{xy\in E_G} \frac{\delta_x + \delta_y}{2\sqrt{\delta_x \cdot \delta_y}} = \frac{4+4}{2\sqrt{4\cdot 4}}(3) + \frac{4+5}{2\sqrt{4\cdot 5}}(6) + \frac{5+}{2\sqrt{5\cdot 7}}(6) + \frac{6+7}{2\sqrt{6\cdot 7}}[4(l+m+n) - 30] + \frac{7+7}{2\sqrt{7\cdot 7}}[(l+m+n) - 6] + \frac{6+8}{2\sqrt{6\cdot 8}}(6) + \frac{8+8}{2\sqrt{8\cdot 8}}(6) = 3 + \frac{27\sqrt{5}}{10} + \frac{36\sqrt{35}}{35} + \frac{13\sqrt{42}}{84}[4(l+m+n) - 30] + [(l+m+n) - 6]\frac{7\sqrt{3}}{2} + 6 = \frac{1}{21}\left(21 + 13\sqrt{42}\right)(l+m+n) + \frac{1}{210}\left(630 + 735\sqrt{3} + 567\sqrt{5} + 216\sqrt{35} - 975\sqrt{42}\right). \quad (34)$$

(16) Fifth hyper-first Zagreb index

$$\mathrm{HM}_1G_5[\mathrm{St}_{l,m,n}] = \sum_{xy\in E_G} \left(\delta_x + \delta_y\right)^2 = (4+4)^2(3) + (4+5)^2(6) + (5+7)^2(6) + (6+7)^2[4(l+m+n) - 30] + (7+7)^2[(l+m+n) - 6] + (6+8)^2(6) + (8+8)^2(6) = 192 + 486 + 864 + 169[4(l+m+n) - 30] + 196[(l+m+n) - 6] + 1176 + 1536 = 872(l+m+n) - 1992. \quad (35)$$

(17) Fifth hyper-second Zagreb index

$$\mathrm{HM}_2G_5[\mathrm{St}_{l,m,n}] = \sum_{xy\in E_G} \left(\delta_x \cdot \delta_y\right)^2 = (4\cdot 4)^2(3) + (4\cdot 5)^2(6) + (5\cdot 7)^2(6) + (6\cdot 7)^2[4(l+m+n) - 30] + (7\cdot 7)^2[(l+m+n) - 6] + (6\cdot 8)^2(6) + (8\cdot 8)^2(6) = 768 + 2400 + 7350 + 1764[4(l+m+n) - 30] + 2401[(l+m+n) - 6] + 13824 + 24576 = 9457(l+m+n) - 28408. \quad (36)$$

(18) Sanskruti index

$$S[\mathrm{St}_{l,m,n}] = \sum_{xy\in E_G} \left(\frac{\delta_x \delta_y}{\delta_x + \delta_y - 2}\right)^3 = \left(\frac{4\cdot 4}{4+4-2}\right)^3(3)$$

$$+\left(\frac{4\cdot5}{4+5-2}\right)^3(6)+\left(\frac{5\cdot7}{5+7-2}\right)^3$$
$$+\left(\frac{6\cdot7}{6+7-2}\right)^3[4(l+m+n)-30]$$
$$+\left(\frac{7\cdot7}{7+7-2}\right)^3[(l+m+n)-6]$$
$$+\left(\frac{6\cdot8}{6+8-2}\right)^3(6)+\left(\frac{8\cdot8}{8+8-2}\right)^3(6)$$
$$=\frac{512}{9}+\frac{48000}{343}+\frac{1029}{4}+\frac{74088}{1331}[4(l+m+n)-30]+\frac{117649}{1728}[(l+m+n)-6]+384+\frac{196608}{343}$$
$$=\frac{668687075}{2299968}(l+m+n)-\frac{555184266893}{832716192}. \tag{37}$$

□

7. Topological Indices of Starphene via NM-Polynomial

In this section, we calculate some topological indices via NM-polynomial, computed in Section 5, of $St_{l,m,n}$.

Theorem 3. *Let $St_{l,m,n}$ be a starphene and*

$$NM[St_n:u,v]=(2n+4)u^6v^6+(4n-1)u^6v^7+(4n-1)u^7v^{10}+2(n-1)u^8v^{10}+(n-1)u^{10}v^{10}, \tag{38}$$

then

$$ND_1(St_{l,m,n})=\left(7+4\sqrt{42}\right)(l+m+n)+18+24\sqrt{3}+12\sqrt{5}+6\sqrt{35}-30\sqrt{42},$$

$$ND_2(St_{l,m,n})=\frac{1}{182}\left(56\sqrt{13}+13\sqrt{14}\right)(l+m+n)+\frac{1}{52}\left(182+29\sqrt{2}+52\sqrt{3}-120\sqrt{13}\right),$$

$$ND_3(St_{l,m,n})=2870(l+m+n)-6336,$$

$$ND_4(St_{l,m,n})=\frac{1}{21}\left(2+2\sqrt{42}\right)(l+m+n)+\frac{1}{210}\left(135+126\sqrt{5}+105\sqrt{6}+36\sqrt{35}-150\sqrt{42}\right),$$

$$ND_5(St_{l,m,n})=\frac{212}{21}(l+m+n)-\frac{603}{35},$$

$$M_1'(St_{l,m,n})=66(l+m+n)-144,$$

$$M_2^{\mp}(St_{l,m,n})=217(l+m+n)-504,$$

$$M_2^{nm}(St_{l,m,n})=\frac{17}{47}(l+m+n)+\frac{321}{840},$$

$$F_N^{\mp}(St_{l,m,n})=438(l+m+n)-984,$$

$$NR_\alpha(St_{l,m,n})=(4\cdot42^\alpha+49^\alpha)(l+m+n)+(3\cdot16^\alpha+6\cdot20^\alpha+6\cdot35^\alpha-30\cdot42^\alpha-6\cdot48^\alpha+6\cdot48^\alpha+6\cdot64^\alpha),$$

$$NH(St_{l,m,n})=\frac{69}{91}(l+m+n)-\frac{61}{78},$$

$$NI(St_{l,m,n})=\frac{427}{26}(l+m+n)-\frac{19939}{546},$$

$$ABC_4(St_{l,m,n})=\frac{1}{21}\left(6\sqrt{3}+\sqrt{462}\right)(l+m+n)+\frac{1}{140}\left(420-240\sqrt{3}+105\sqrt{6}+225\sqrt{14}+84\sqrt{35}-100\sqrt{462}\right),$$

$$GA_5(St_{l,m,n})=\frac{1}{13}\left(13+8\sqrt{42}\right)(l+m+n)+\frac{1}{273}\left(819+936\sqrt{3}+728\sqrt{5}+273\sqrt{35}-1260\sqrt{42}\right),$$

$$AG_5(St_{l,m,n})=\frac{1}{21}\left(21+13\sqrt{42}\right)(l+m+n)+\frac{1}{210}\left(630+735\sqrt{3}+567\sqrt{5}+216\sqrt{35}-975\sqrt{42}\right),$$

$$HM_1G_5(St_{l,m,n})=872(l+m+n)-1992,$$

$$HM_2G_5(St_{l,m,n})=9457(l+m+n)-28408,$$

$$S(St_{l,m,n})=\frac{668687075}{2299968}(l+m+n)-\frac{555184266893}{832716192}. \tag{39}$$

Proof. Let $NM[St_n:u,v]=3u^4v^4+6u^4v^5+6u^5v^7+[4(l+m+n)-30]u^6v^7+[(l+m+n)-6]u^7v^7+6u^6v^8+6u^8v^8$.

(1) First NDe index

$$D_v^{1/2}NM[St_n:u,v]=6u^4v^4+6\sqrt{5}u^4v^5+6\sqrt{7}u^5v^7+\sqrt{7}[4(l+m+n)-30]u^6v^7+\sqrt{7}[(l+m+n)-6]u^7v^7+12\sqrt{2}u^6v^8+12\sqrt{2}u^8v^8,$$

$$D_u^{1/2}D_v^{1/2}NM[St_n:u,v]=12u^4v^4+12\sqrt{5}u^4v^5+6\sqrt{35}u^5v^7+\sqrt{42}[4(l+mn)30]u^6v^7+7[(l+m+n)-6]u^7v^7+24\sqrt{3}u^6v^8+48u^8v^8,$$

$$\begin{aligned}ND_1(St_{l,m,n}) &= D_u^{1/2}D_v^{1/2}NM[St_n : u, v]|u = v = 1\\ &= \left(7 + 4\sqrt{42}\right)(l + m + n) + 18 + 24\sqrt{3}\\ &\quad + 12\sqrt{5} + 6\sqrt{35} - 30\sqrt{42}.\end{aligned} \tag{40}$$

(2) Second NDe index

$$\begin{aligned}&S_u^{1/2}JNM[St_n : u, v]\\ &= \frac{3}{2\sqrt{2}}u^8 + 2u^9 + \sqrt{3}u^{12} + \frac{1}{\sqrt{13}}[4(l + m + n) - 30]u^{13}\\ &\quad + \frac{1}{\sqrt{13}}[(l + m + n) - 6]u^{14} + \frac{6}{\sqrt{14}}u^{14} + \frac{3}{2}u^{16},\\ &ND_2[St_{l,m,n}] = S_u^{1/2}JNM[St_n : u, v]|u = 1\\ &= \frac{1}{182}\left(56\sqrt{13} + 13\sqrt{14}\right)(l + m + n)\\ &\quad + \frac{1}{52}\left(182 + 29\sqrt{2} + 52\sqrt{3} - 120\sqrt{13}\right).\end{aligned} \tag{41}$$

(3) Third NDe index

$$\begin{aligned}D_vNM[St_n : u, v] &= 12u^4v^4 + 30u^4v^5 + 42u^5v^7 + 7[4(l + m + n)\\ &\quad - 30]u^6v^7 + 7[(l + m + n) - 6]u^7v^7\\ &\quad + 48u^6v^8 + 48u^8v^8,\\ D_uNM[St_n : u, v] &= 12u^4v^4 + 24u^4v^5 + 30u^5v^7 + 6[4(l + m + n)\\ &\quad - 30]u^6v^7 + 7[(l + m + n) - 6]u^7v^7\\ &\quad + 36u^6v^8 + 48u^8v^8,\\ &(D_u + D_v)NM[St_n : u, v]\\ &= 24u^4v^4 + 54u^4v^5 + 72u^5v^7 + 13[4(l + m + n) - 30]u^6v^7\\ &\quad + 14[(l + m + n) - 6]u^7v^7 846u^6v^8 + 96u^8v^8,\\ &D_v(D_u + D_v)NM[St_n : u, v]\\ &= 96u^4v^4 + 270u^4v^5 + 504u^5v^7 + 91[4(l + m + n) - 30]u^6v^7\\ &\quad + 98[(l + m + n) - 6]u^7v^7 + 672u^6v^8 + 768u^8v^8,\\ &D_uD_v(D_u + D_v)NM[St_n : u, v]\\ &= 384u^4v^4 + 1080u^4v^5 + 2520u^5v^7 + 546[4(l + m + n)\\ &\quad - 30]u^6v^7 + 686[(l + m + n) - 6]u^7v^7 + 4032u^6v^8\\ &\quad + 6144u^8v^8,\\ ND_3[St_{l,m,n}] &= D_uD_v(D_u + D_v)NM[St_n : u, v]\\ &\quad \cdot |u = v = 1 = 2870(l + m + n) - 6336.\end{aligned} \tag{42}$$

(4) Fourth NDe index

$$\begin{aligned}S_v^{1/2}NM[St_n : u, v] &= \frac{3}{2}u^4v^4 + \frac{6}{\sqrt{5}}u^4v^5 + \frac{6}{\sqrt{7}}u^5v^7\\ &\quad + \frac{1}{\sqrt{7}}[4(l + m + n) - 30]u^6v^7\\ &\quad + \frac{1}{\sqrt{7}}[(l + m + n) - 6]u^7v^7\\ &\quad + \frac{3}{\sqrt{2}}u^6v^8 + \frac{3}{\sqrt{2}}u^8v^8,\\ S_u^{1/2}S_v^{1/2}NM[St_n : u, v] &= \frac{3}{4}u^4v^4 + \frac{3}{\sqrt{5}}u^4v^5 + \frac{6}{\sqrt{35}}u^5v^7\\ &\quad + \frac{1}{\sqrt{42}}[4(l + m + n) - 30]u^6v^7\\ &\quad + \frac{1}{7}[(l + m + n) - 6]u^7v^7\\ &\quad + \frac{\sqrt{3}}{2}u^6v^8 + \frac{3}{4}u^8v^8,\\ ND_4[St_{l,m,n}] &= S_u^{1/2}S_v^{1/2}NM[St_n : u, v]|u = v = 1\\ &= \frac{1}{21}\left(2 + 2\sqrt{42}\right)(l + m + n) + \frac{1}{210}\left(135\right.\\ &\quad \left. + 126\sqrt{5} + 105\sqrt{6} + 36\sqrt{35} - 150\sqrt{42}\right).\end{aligned} \tag{43}$$

(5) Fifth NDe index

$$\begin{aligned}&D_vNM[St_n : u, v]\\ &= 12u^4v^4 + 30u^4v^5 + 42u^5v^7 + 7[4(l + m + n) - 30]u^6v^7\\ &\quad + 7[(l + m + n) - 6]u^7v^7 + 48u^6v^8 + 48u^8v^8,\end{aligned}$$

$$\begin{aligned}&S_uD_vNM[St_n : u, v]\\ &= 3u^4v^4 + \frac{15}{2}u^4v^5 + \frac{42}{5}u^5v^7 + \frac{7}{6}[4(l + m + n) - 30]u^6v^7\\ &\quad + [(l + m + n) - 6]u^7v^7 + 8u^6v^8 + 6u^8v^8,\end{aligned}$$

$$\begin{aligned}&S_vNM[St_n : u, v]\\ &= \frac{3}{4}u^4v^4 + \frac{6}{5}u^4v^5 + \frac{6}{7}u^5v^7 + \frac{1}{7}[4(l + m + n) - 30]u^6v^7\\ &\quad + \frac{1}{7}[(l + m + n) - 6]u^7v^7 + \frac{3}{4}u^6v^8 + \frac{3}{4}u^8v^8,\end{aligned}$$

$$\begin{aligned}&D_uS_vNM[St_n : u, v]\\ &= 3u^4v^4 + \frac{24}{5}u^4v^5 + \frac{30}{7}u^5v^7 + \frac{6}{7}[4(l + m + n) - 30]u^6v^7\\ &\quad + [(l + m + n) - 6]u^7v^7 + \frac{9}{4}u^6v^8 + 6u^8v^8,\end{aligned}$$

$$\begin{aligned}(D_uS_v + S_uD_v)\mathrm{NM}[\mathrm{St}_n : u, v] \\ = 6u^4v^4 + \frac{123}{10}u^4v^5 + \frac{444}{35}u^5v^7 + \frac{85}{42}[4(l+m+n) \\ - 30]u^6v^7 + 2[(l+m+n) - 6]u^7v^7 + \frac{25}{2}u^6v^8 + 12u^8v^8,\end{aligned}$$

$$\begin{aligned}\mathrm{ND}_5[\mathrm{St}_{l,m,n}] &= (D_uS_v + S_uD_v)\mathrm{NM}[\mathrm{St}_n : u, v] \\ &\cdot |u = v = 1 = \frac{212}{21}(l+m+n) - \frac{603}{35}.\end{aligned} \tag{44}$$

(6) Third version of Zagreb index

$$\begin{aligned}D_v\mathrm{NM}[\mathrm{St}_n : u, v] &= 12u^4v^4 + 30u^4v^5 + 42u^5v^7 \\ &+ 7[4(l+m+n) - 30]u^6v^7 \\ &+ 7[(l+m+n) - 6]u^7v^7 \\ &+ 48u^6v^8 + 48u^8v^8, \\ D_u\mathrm{NM}[\mathrm{St}_n : u, v] &= 12u^4v^4 + 24u^4v^5 + 30u^5v^7 \\ &+ 6[4(l+m+n) - 30]u^6v^7 \\ &+ 7[(l+m+n) - 6]u^7v^7 \\ &+ 36u^6v^8 + 48u^8v^8, \\ (D_u + D_v)\mathrm{NM}[\mathrm{St}_n : u, v] &= 24u^4v^4 + 54u^4v^5 + 72u^5v^7 \\ &+ 13[4(l+m+n) - 30] - 30u^6v^7 \\ &+ 14[(l+m+n) - 6]u^7v^7 846u^6v^8 \\ &+ 96u^8v^8, \\ M_1'[\mathrm{St}_{l,m,n}] &= (D_u + D_v)\mathrm{NM}[\mathrm{St}_n : u, v]|u = v = 1 \\ &= 66(l+m+n) - 144.\end{aligned} \tag{45}$$

(7) Neighborhood second Zagreb index

$$\begin{aligned}D_v\mathrm{NM}[\mathrm{St}_n : u, v] &= 12u^4v^4 + 30u^4v^5 + 42u^5v^7 \\ &+ 7[4(l+m+n) - 30]u^6v^7 \\ &+ 7[(l+m+n) - 6]u^7v^7 \\ &+ 48u^6v^8 + 48u^8v^8, \\ D_uD_v\mathrm{NM}[\mathrm{St}_n : u, v] &= 48u^4v^4 + 120u^4v^5 + 210u^5v^7 \\ &+ 42[4(l+m+n) - 30]u^6v^7 \\ &+ 49[(l+m+n) - 6]u^7v^7 \\ &+ 288u^6v^8 + 384u^8v^8, \\ M_2^{\mp}[\mathrm{St}_{l,m,n}] &= D_uD_v\mathrm{NM}[\mathrm{St}_n : u, v]|u = v = 1 \\ &= 217(l+m+n) - 504.\end{aligned} \tag{46}$$

(8) Neighborhood second modified Zagreb index

$$\begin{aligned}S_v\mathrm{NM}[\mathrm{St}_n : u, v] &= \frac{3}{4}u^4v^4 + \frac{6}{5}u^4v^5 + \frac{6}{7}u^5v^7 \\ &+ \frac{1}{7}[4(l+m+n) - 30]u^6v^7 \\ &+ \frac{1}{7}[(l+m+n) - 6]u^7v^7 \\ &+ \frac{3}{4}u^6v^8 + \frac{3}{4}u^8v^8, \\ S_uS_v\mathrm{NM}[\mathrm{St}_n : u, v] &= \frac{3}{16}u^4v^4 + \frac{3}{10}u^4v^5 + \frac{6}{35}u^5v^7 \\ &+ \frac{1}{42}[4(l+m+n) - 30]u^6v^7 \\ &+ \frac{1}{49}[(l+m+n) - 6]u^7v^7 \\ &+ \frac{1}{8}u^6v^8 + \frac{3}{32}u^8v^8, \\ M_2^{nm}[\mathrm{St}_{l,m,n}] &= S_uS_v\mathrm{NM}[\mathrm{St}_n : u, v]|u = v = 1 \\ &= \frac{17}{47}(l+m+n) + \frac{321}{840}.\end{aligned} \tag{47}$$

(9) Neighborhood forgotten topological index

$$\begin{aligned}D_u\mathrm{NM}[\mathrm{St}_n : u, v] &= 12u^4v^4 + 24u^4v^5 + 30u^5v^7 \\ &+ 6[4(l+m+n) - 30]u^6v^7 \\ &+ 7[(l+m+n) - 6]u^7v^7 \\ &+ 36u^6v^8 + 48u^8v^8, \\ D_u^2\mathrm{NM}[\mathrm{St}_n : u, v] &= 48u^4v^4 + 96u^4v^5 + 150u^5v^7 \\ &+ 36[4(l+m+n) - 30]u^6v^7 \\ &+ 49[(l+m+n) - 6]u^7v^7 \\ &+ 216u^6v^8 + 384u^8v^8, \\ D_v\mathrm{NM}[\mathrm{St}_n : u, v] &= 12u^4v^4 + 30u^4v^5 + 42u^5v^7 \\ &+ 7[4(l+m+n) - 30]u^6v^7 \\ &+ 7[(l+m+n) - 6]u^7v^7 \\ &+ 48u^6v^8 + 48u^8v^8, \\ D_v^2\mathrm{NM}[\mathrm{St}_n : u, v] &= 48u^4v^4 + 150u^4v^5 + 294u^5v^7 \\ &+ 49[4(l+m+n) - 30]u^6v^7 \\ &+ 49[(l+m+n) - 6]u^7v^7 \\ &+ 384u^6v^8 + 384u^8v^8,\end{aligned}$$

$$\begin{aligned}&\left(D_u^2 + D_v^2\right)\mathrm{NM}[\mathrm{St}_n : u, v] \\ &= 96u^4v^4 + 246u^4v^5 + 444u^5v^7 + 85[4(l+m+n) - 30]u^6v^7 \\ &+ 98[(l+m+n) - 6]u^7v^7 + 600u^6v^8 + 768u^8v^8, \\ &F_N^{\mp}[\mathrm{St}_{l,m,n}] = \left(D_u^2 + D_v^2\right)\mathrm{NM}[\mathrm{St}_n : u, v]|u = v = 1 \\ &= 438(l+m+n) - 984.\end{aligned} \tag{48}$$

(10) Neighborhood general Randic index

$$\begin{aligned} D_v^{\alpha}\mathrm{NM}[\mathrm{St}_n : u, v] &= 3\cdot 4^{\alpha}u^4v^4 + 6\cdot 5^{\alpha}u^4v^5 + 6\cdot 7^{\alpha}u^5v^7 + 7^{\alpha}[4(l+m+n) \\ &\quad - 30]u^6v^7 + 7^{\alpha}[(l+m+n)-6]u^7v^7 + 6\cdot 8^{\alpha}u^6v^8 \\ &\quad + 6\cdot 8^{\alpha}u^8v^8, \\ D_u^{\alpha}D_v^{\alpha}\mathrm{NM}[\mathrm{St}_n : u, v] &= 3\cdot 16^{\alpha}u^4v^4 + 6\cdot 20^{\alpha}u^4v^5 + 6\cdot 35^{\alpha}u^5v^7 + 42^{\alpha}[4(l+m+n) \\ &\quad - 30]u^6v^7 + 49^{\alpha}[(l+m+n)-6]u^7v^7 + 6\cdot 48^{\alpha}u^6v^8 \\ &\quad + 6\cdot 64^{\alpha}u^8v^8, \\ \mathrm{NR}_{\alpha}[\mathrm{St}_{l,m,n}] &= D_u^{\alpha}D_v^{\alpha}\mathrm{NM}[\mathrm{St}_n : u, v]|u=v=1 \\ &= (4\cdot 42^{\alpha} + 49^{\alpha})(l+m+n) + (3\cdot 16^{\alpha} + 6\cdot 20^{\alpha} \\ &\quad + 6\cdot 35^{\alpha} - 30\cdot 42^{\alpha} - 6\cdot 48^{\alpha} + 6\cdot 48^{\alpha} + 6\cdot 64^{\alpha}). \end{aligned} \tag{49}$$

(11) Neighborhood harmonic index

$$\begin{aligned} J\mathrm{NM}[\mathrm{St}_n : u, v] &= 3u^8 + 6u^9 + 6u^{12} + [4(l+m+n)-30]u^{13} \\ &\quad + [(l+m+n)-6]u^{14} + 6u^{14} + 6u^{16}, \\ S_uJ\mathrm{NM}[\mathrm{St}_n : u, v] &= \frac{3}{8}u^8 + \frac{2}{3}u^9 + \frac{1}{2}u^{12} + \frac{1}{13}[4(l+m+n)-30]u^{13} \\ &\quad + \frac{1}{14}[(l+m+n)-6]u^{14} + \frac{3}{7}u^{14} + \frac{3}{8}u^{16}, \\ 2S_uJ\mathrm{NM}[\mathrm{St}_n : u, v] &= \frac{3}{4}u^8 + \frac{4}{3}u^9 + u^{12} + \frac{2}{13}[4(l+m+n)-30]u^{13} \\ &\quad + \frac{1}{7}[(l+m+n)-6]u^{14} + \frac{6}{7}u^{14} + \frac{3}{4}u^{16}, \\ \mathrm{NH}[\mathrm{St}_{l,m,n}] &= 2S_uJ\mathrm{NM}[\mathrm{St}_n : u, v]|u=1 \\ &= \frac{69}{91}(l+m+n) - \frac{61}{78}. \end{aligned} \tag{50}$$

(12) Neighborhood inverse sum index

$$\begin{aligned} D_v\mathrm{NM}[\mathrm{St}_n : u, v] &= 12u^4v^4 + 30u^4v^5 + 42u^5v^7 \\ &\quad + 7[4(l+m+n)-30]u^6v^7 \\ &\quad + 7[(l+m+n)-6]u^7v^7 \\ &\quad + 48u^6v^8 + 48u^8v^8, \\ D_uD_v\mathrm{NM}[\mathrm{St}_n : u, v] &= 48u^4v^4 + 120u^4v^5 + 210u^5v^7 \\ &\quad + 42[4(l+m+n)-30]u^6v^7 \\ &\quad + 49[(l+m+n)-6]u^7v^7 \\ &\quad + 288u^6v^8 + 384u^8v^8, \\ JD_uD_v\mathrm{NM}[\mathrm{St}_n : u, v] &= 48u^8 + 120u^9 + 210u^{210} \\ &\quad + 42[4(l+m+n)-30]u^{13} \\ &\quad + 49[(l+m+n)-6]u^{14} \\ &\quad + 288u^{14} + 384u^{16}, \\ S_uJD_uD_v\mathrm{NM}[\mathrm{St}_n : u, v] &= 6u^8 + \frac{120}{9}u^9 + \frac{35}{2}u^{12} \\ &\quad + \frac{42}{13}[4(l+m+n)-30]u^{13} \\ &\quad + \frac{7}{2}[(l+m+n)-6]u^{14} \\ &\quad + \frac{144}{7}u^{14} + 24u^{16}, \\ \mathrm{NI}[\mathrm{St}_{l,m,n}] &= S_uJD_uD_v\mathrm{NM}[\mathrm{St}_n : u, v]|u=1 \\ &= \frac{427}{26}(l+m+n) - \frac{19939}{546}. \end{aligned} \tag{51}$$

(13) Fourth atom bond connectivity index

$$\begin{aligned} S_u^{1/2}S_v^{1/2}\mathrm{NM}[\mathrm{St}_n : u, v] &= \frac{3}{4}u^4v^4 + \frac{3}{\sqrt{5}}u^4v^5 + \frac{6}{\sqrt{35}}u^5v^7 + \frac{1}{\sqrt{42}}[4(l+m+n) \\ &\quad - 30]u^6v^7 + \frac{1}{7}[(l+m+n)-6]u^7v^7 + \frac{\sqrt{3}}{2}u^6v^8 + \frac{3}{4}u^8v^8, \\ JS_u^{1/2}S_v^{1/2}\mathrm{NM}[\mathrm{St}_n : u, v] &= \frac{3}{4}u^8 + \frac{3}{\sqrt{5}}u^9 + \frac{6}{\sqrt{35}}u^{12} + \frac{1}{\sqrt{42}}[4(l+m+n) \\ &\quad - 30]u^{13} + \frac{1}{7}[(l+m+n)-6]u^{14} + \frac{\sqrt{3}}{2}u^{14} + \frac{3}{4}u^{16}, \\ Q_{u(-2)}JS_u^{1/2}S_v^{1/2}\mathrm{NM}[\mathrm{St}_n : u, v] &= \frac{3}{4}u^6 + \frac{3}{\sqrt{5}}u^7 + \frac{6}{\sqrt{35}}u^{10} + \frac{1}{\sqrt{42}}[4(l+m+n) \\ &\quad - 30]u^{11} + \frac{1}{7}[(l+m+n)-6]u^{12} + \frac{\sqrt{3}}{2}u^{12} + \frac{3}{4}u^{14}, \\ D_u^{1/2}Q_{u(-2)}JS_u^{1/2}S_v^{1/2}\mathrm{NM}[\mathrm{St}_n : u, v] &= \frac{3\sqrt{6}}{4}u^6 + \frac{3\sqrt{7}}{\sqrt{5}}u^7 + \frac{6\sqrt{2}}{\sqrt{7}}u^{10} + \frac{\sqrt{11}}{\sqrt{42}}[4(l+m+n) \\ &\quad - 30]u^{11} + \frac{2\sqrt{3}}{7}[(l+m+n)-6]u^{12} + 3u^{12} + \frac{3\sqrt{14}}{4}u^{14}, \\ ABC_4[\mathrm{St}_{l,m,n}] &= D_u^{1/2}Q_{u(-2)}JS_u^{1/2}S_v^{1/2}\mathrm{NM}[\mathrm{St}_n : u, v]|u=1 \\ &= \frac{1}{21}\left(6\sqrt{3} + \sqrt{462}\right)(l+m+n) + \frac{1}{140} \\ &\quad \cdot \left(420 - 240\sqrt{3} + 105\sqrt{6} + 225\sqrt{14} \right. \\ &\quad \left. + 84\sqrt{35} - 100\sqrt{462}\right). \end{aligned} \tag{52}$$

(14) Fifth geometric arithmetic index

$$\begin{aligned}
D_v^{1/2}\mathrm{NM}[\mathrm{St}_n : u, v] &= 6u^4v^4 + 6\sqrt{5}u^4v^5 + 6\sqrt{7}u^5v^7 + \sqrt{7}[4(l+m+n) \\
&\quad - 30]u^6v^7 + \sqrt{7}[(l+m+n) - 6]u^7v^7 \\
&\quad + 12\sqrt{2}u^6v^8 + 6u^8v^8,
\end{aligned}$$

$$\begin{aligned}
D_u^{1/2}D_v^{1/2}\mathrm{NM}[\mathrm{St}_n : u, v] &= 12u^4v^4 + 12\sqrt{5}u^4v^5 + 6\sqrt{35}u^5v^7 + \sqrt{42}[4(l+m+n) \\
&\quad - 30]u^6v^7 + 7[(l+m+n) - 6]u^7v^7 + 24\sqrt{3}u^6v^8 + 48u^8v^8,
\end{aligned}$$

$$\begin{aligned}
JD_u^{1/2}D_v^{1/2}\mathrm{NM}[\mathrm{St}_n : u, v] &= 12u^8 + 12\sqrt{5}u^9 + 6\sqrt{35}u^{12} + \sqrt{42}[4(l+m+n) \\
&\quad - 30]u^{13} + 7[(l+m+n) - 6]u^{14} + 24\sqrt{3}u^{14} + 48u^{16},
\end{aligned}$$

$$\begin{aligned}
S_uJD_u^{1/2}D_v^{1/2}\mathrm{NM}[\mathrm{St}_n : u, v] &= \frac{3}{2}u^8 + \frac{4\sqrt{5}}{3}\sqrt{5}u^9 + \frac{\sqrt{35}}{2}u^{12} + \frac{\sqrt{42}}{13}[4(l+m+n) \\
&\quad - 30]u^{13} + \frac{1}{2}[(l+m+n) - 6]u^{14} + \frac{12\sqrt{3}}{7}u^{14} + 3u^{16},
\end{aligned}$$

$$\begin{aligned}
2S_uJD_u^{1/2}D_v^{1/2}\mathrm{NM}[\mathrm{St}_n : u, v] &= 3u^8 + \frac{8\sqrt{5}}{3}u^9 + \sqrt{35}u^{12} + \frac{2\sqrt{42}}{13}[4(l+m+n) - 30]u^{13} \\
&\quad + [(l+m+n) - 6]u^{14} + \frac{24\sqrt{3}}{7}u^{14} + 6u^{16},
\end{aligned}$$

$$\begin{aligned}
\mathrm{GA}_5[\mathrm{St}_{l,m,n}] &= 2S_uJD_u^{1/2}D_v^{1/2}\mathrm{NM}[\mathrm{St}_n : u, v]|u = 1 \\
&= \frac{1}{13}\left(13 + 8\sqrt{42}\right)(l+m+n) + \frac{1}{273}\Big(819 \\
&\quad + 936\sqrt{3} + 728\sqrt{5} + 273\sqrt{35} - 1260\sqrt{42}\Big).
\end{aligned} \tag{53}$$

(15) Fifth arithmetic geometric index

$$\begin{aligned}
D_v\mathrm{NM}[\mathrm{St}_n : u, v] &= 12u^4v^4 + 30u^4v^5 + 42u^5v^7 \\
&\quad + 7[4(l+m+n) - 30]u^6v^7 \\
&\quad + 7[(l+m+n) - 6]u^7v^7 \\
&\quad + 48u^6v^8 + 48u^8v^8,
\end{aligned}$$

$$\begin{aligned}
D_u\mathrm{NM}[\mathrm{St}_n : u, v] &= 12u^4v^4 + 24u^4v^5 + 30u^5v^7 \\
&\quad + 6[4(l+m+n) - 30]u^6v^7 \\
&\quad + 7[(l+m+n) - 6]u^7v^7 \\
&\quad + 36u^6v^8 + 48u^8v^8,
\end{aligned}$$

$$\begin{aligned}
(D_u + D_v)\mathrm{NM}[\mathrm{St}_n : u, v] &= 24u^4v^4 + 54u^4v^5 + 72u^5v^7 \\
&\quad + 13[4(l+m+n) - 30]u^6v^7 \\
&\quad + 14[(l+m+n) - 6]u^7v^7 \\
&\quad + 84u^6v^8 + 96u^8v^8,
\end{aligned}$$

$$\begin{aligned}
S_v^{1/2}(D_u + D_v)\mathrm{NM}[\mathrm{St}_n : u, v] &= 12u^4v^4 + \frac{54}{\sqrt{5}}u^4v^5 + \frac{72}{\sqrt{7}}u^5v^7 \\
&\quad + \frac{13}{7}[4\,(l+m+n) - 30]u^6v^7 \\
&\quad + \frac{14}{\sqrt{7}}[(l+m+n) - 6]u^7v^7 \\
&\quad + \frac{42}{\sqrt{2}}u^6v^8 + \frac{48}{\sqrt{2}}u^8v^8,
\end{aligned}$$

$$\begin{aligned}
S_u^{1/2}S_v^{1/2}(D_u + D_v)\mathrm{NM}[\mathrm{St}_n : u, v] &= 6u^4v^4 + \frac{27}{\sqrt{5}}u^4v^5 + \frac{72}{\sqrt{35}}u^5v^7 \\
&\quad + \frac{13}{\sqrt{42}}[4\,(l+m+n) - 30]u^6v^7 \\
&\quad + 2[(l+m+n) - 6]u^7v^7 + \frac{21}{\sqrt{3}}u^6v^8 + 12u^8v^8,
\end{aligned}$$

$$\begin{aligned}
\frac{1}{2}S_u^{1/2}S_v^{1/2}(D_u + D_v)\mathrm{NM}[\mathrm{St}_n : u, v] &= 3u^4v^4 + \frac{27}{2\sqrt{5}}u^4v^5 + \frac{36}{\sqrt{35}}u^5v^7 \\
&\quad + \frac{13}{2\sqrt{42}}[4(l+m+n) - 30]u^6v^7 \\
&\quad + [(l+m+n) - 6]u^7v^7 + \frac{21}{2\sqrt{3}}u^6v^8 + 6u^8v^8,
\end{aligned}$$

$$\begin{aligned}
\mathrm{AG}_5[\mathrm{St}_{l,m,n}] &= \frac{1}{2}S_u^{1/2}S_v^{1/2}(D_u + D_v)\mathrm{NM}[\mathrm{St}_n : u, v] \\
&\quad \cdot |u = v = 1 = \frac{1}{21}\left(21 + 13\sqrt{42}\right)(l+m+n) \\
&\quad + \frac{1}{210}\Big(630 + 735\sqrt{3} + 567\sqrt{5} + 216\sqrt{35} \\
&\quad - 975\sqrt{42}\Big).
\end{aligned} \tag{54}$$

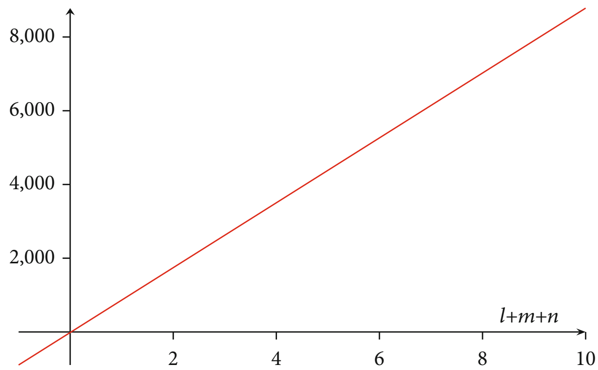
(16) Fifth hyper-first Zagreb index

$$\begin{aligned}
J\mathrm{NM}[\mathrm{St}_n : u, v] &= 3u^8 + 6u^9 + 6u^{12} + [4(l+m+n) - 30]u^{13} \\
&\quad + [(l+m+n) - 6]u^{14} + 6u^{14} + 6u^{16}
\end{aligned}$$

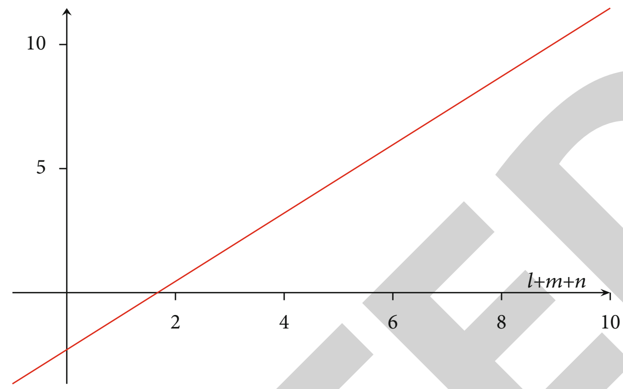
$$\begin{aligned}
D_uJ\mathrm{NM}[G : u, v] &= 24u^8 + 54u^9 + 72u^{12} + 13[4(l+m+n) \\
&\quad - 30]u^{13} + 14[(l+m+n) - 6]u^{14} \\
&\quad + 84u^{14} + 96u^{16},
\end{aligned}$$

$$\begin{aligned}
D_u^2J\mathrm{NM}[G : u, v] &= 192u^8 + 486u^9 + 864u^{12} \\
&\quad + 169[4(l+m+n) - 30]u^{13} \\
&\quad + 196[(l+m+n) - 6]u^{14} \\
&\quad + 1176u^{14} + 1536u^{16},
\end{aligned}$$

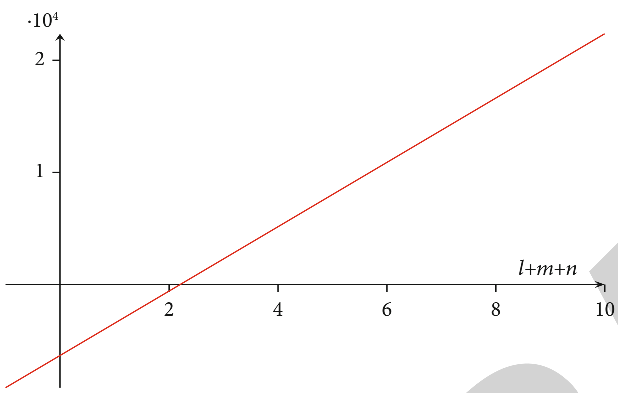
$$\begin{aligned}
\mathrm{HM}_1G_5[\mathrm{St}_{l,m,n}] &= D_u^2J\mathrm{NM}[G : u, v]\big|_{u=1} \\
&= 872(l+m+n) - 1992.
\end{aligned} \tag{55}$$



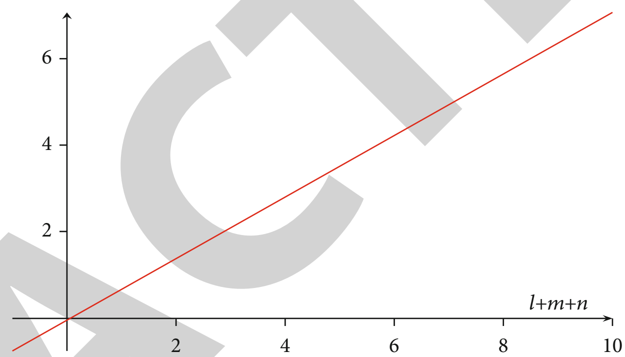
(a) First ND index



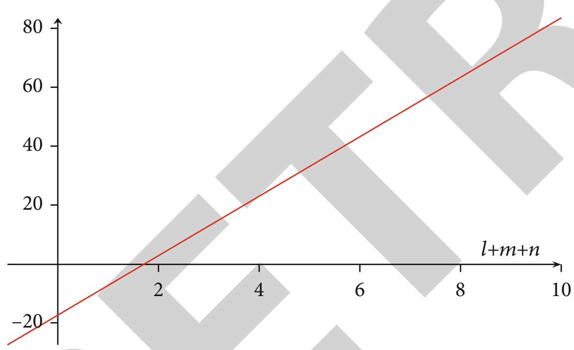
(b) Second ND index



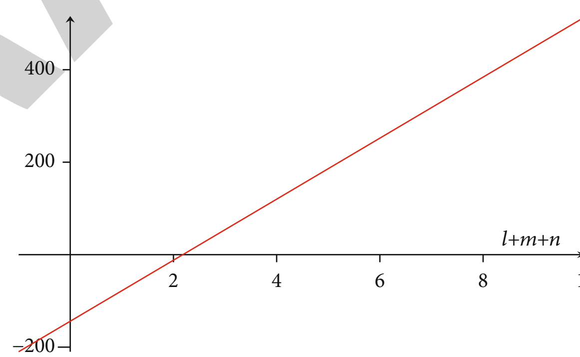
(c) Third ND index



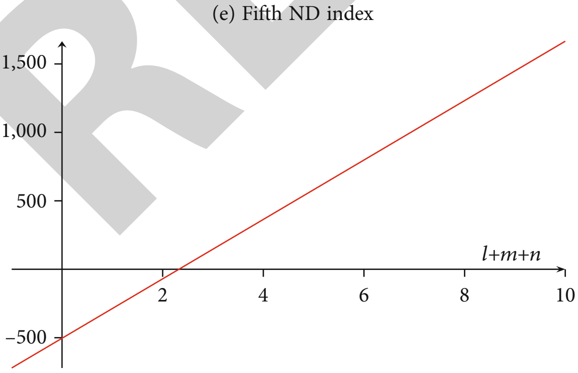
(d) Fourth ND index



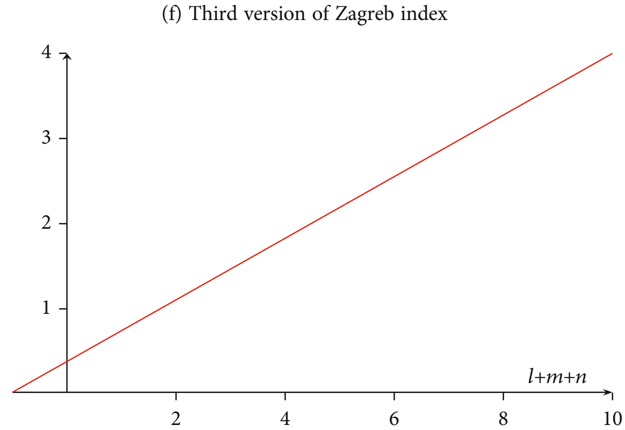
(e) Fifth ND index



(f) Third version of Zagreb index

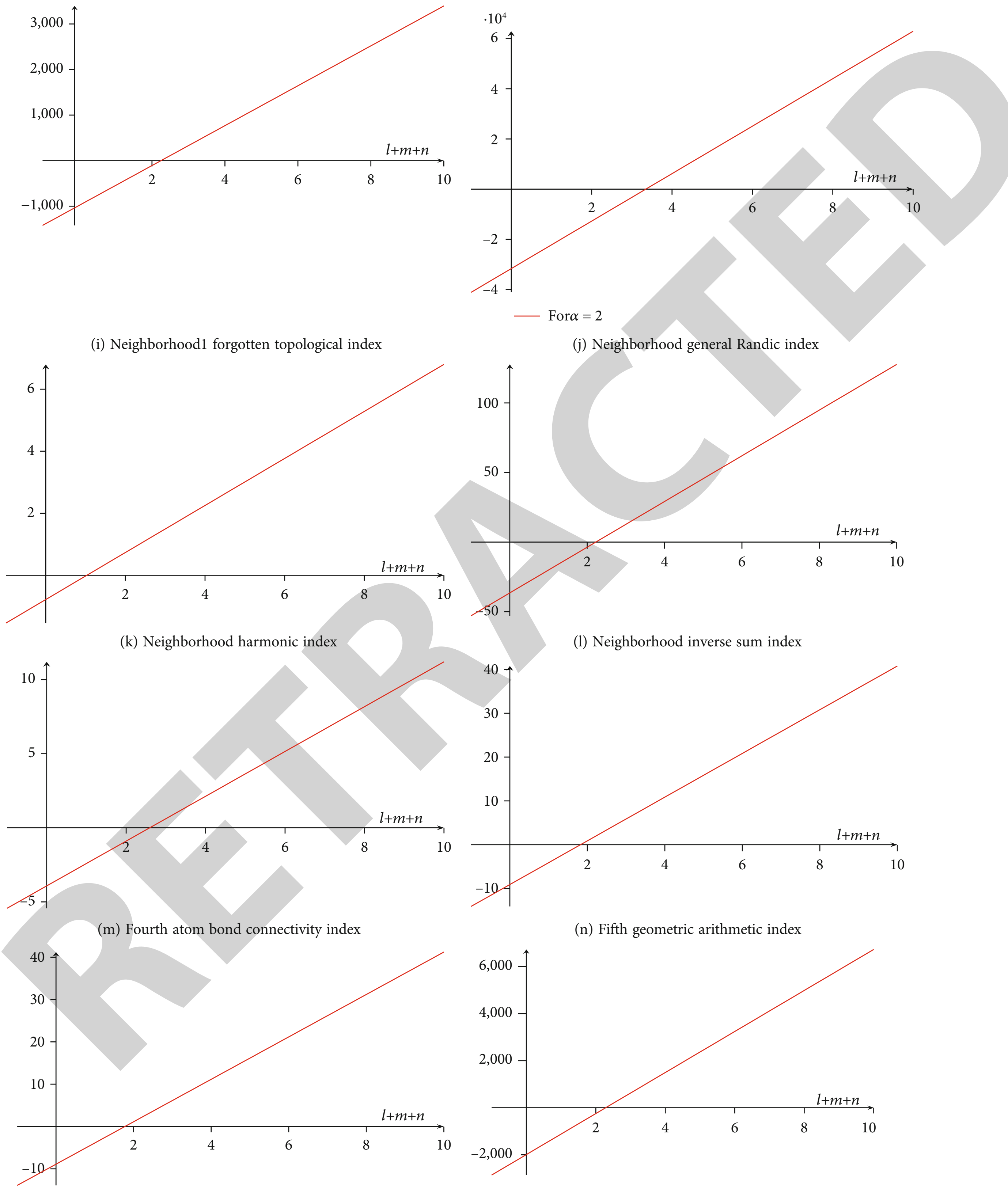


(g) Neighborhood second Zagreb index



(h) Neighborhood1 second modified Zagreb index

Figure 3: Continued.



(i) Neighborhood1 forgotten topological index

(j) Neighborhood general Randic index

(k) Neighborhood harmonic index

(l) Neighborhood inverse sum index

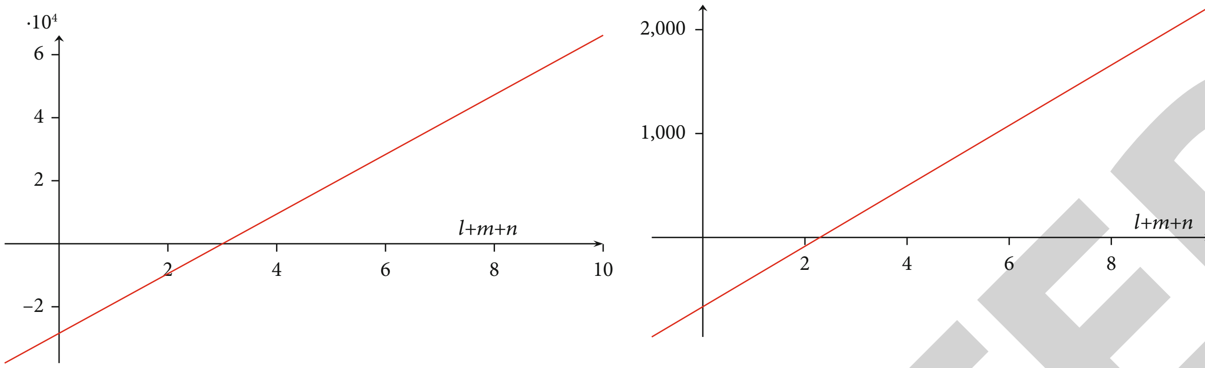
(m) Fourth atom bond connectivity index

(n) Fifth geometric arithmetic index

(o) Fifth arithmetic geometric index

(p) Fifth hyper-first Zagreb index

Figure 3: Continued.



(q) Fifth hyper-second Zagreb index

(r) Sanskruti index

Figure 3: Neighborhood topological indices of $St_{l,m,n}$.

(17) Fifth hyper-second Zagreb index

$$\begin{aligned}
D_u D_v^2 NM[St_n : u, v] &= 192u^4v^4 + 600u^4v^5 + 1470u^5v^7 \\
&\quad + 294[4(l+m+n) - 30]u^6v^7 \\
&\quad + 343[(l+m+n) - 6]u^7v^7 \\
&\quad + 2304u^6v^8 + 3072u^8v^8, \\
D_u^2 D_v^2 NM[G : u, v] &= 768u^4v^4 + 2400u^4v^5 + 7350u^5v^7 \\
&\quad + 1764[4(l+m+n) - 30]u^6v^7 \\
&\quad + 2401[(l+m+n) - 6]u^7v^7 \\
&\quad + 1382u^6v^8 + 24576u^8v^8, \\
HM_2G_5[St_{l,m,n}] &= D_u^2 D_v^2 NM[G : u, v]\big|_{u=v=1} \\
&= 9457(l+m+n) - 28408.
\end{aligned} \tag{56}$$

(18) Sanskruti Index

$$\begin{aligned}
D_v^3 NM[G : u, v] &= 192u^4v^4 + 750u^4v^5 + 2058u^5v^7 \\
&\quad + 343[4(l+m+n) - 30]u^6v^7 \\
&\quad + 343[(l+m+n) - 6]u^7v^7 \\
&\quad + 3072u^6v^8 + 3072u^8v^8,
\end{aligned}$$

$$\begin{aligned}
D_u^3 D_v^3 NM[G : u, v] &= 12288u^4v^4 + 48000u^4v^5 + 257250u^5v^7 \\
&\quad + 74088[4(l+m+n) - 30]u^6v^7 \\
&\quad + 117649[(l+m+n) - 6]u^7v^7 \\
&\quad + 663552u^6v^8 + 1572864u^8v^8,
\end{aligned}$$

$$\begin{aligned}
JD_u^3 D_v^3 NM[G : u, v] &= 12288u^8 + 48000u^9 + 257250u^{12} \\
&\quad + 74088[4(l+m+n) - 30]u^{13} \\
&\quad + 117649[(l+m+n) - 6]u^{14} \\
&\quad + 663552u^{14} + 1572864u^{16},
\end{aligned}$$

$$\begin{aligned}
Q_{u(-2)} JD_u^3 D_v^3 NM[G : u, v] &= 12288u^6 + 48000u^7 + 257250u^{10} \\
&\quad + 74088[4(l+m+n) - 30]u^{11} \\
&\quad + 117649[(l+m+n) - 6]u^{12} \\
&\quad + 663552u^{12} + 1572864u^{14},
\end{aligned}$$

$$\begin{aligned}
S_u^3 Q_{u(-2)} JD_u^3 D_v^3 NM[G : u, v] &= \frac{512}{9}u^6 + \frac{48000u^7}{343} + \frac{257250}{1000}u^{10} \\
&\quad + \frac{74088}{1331}[4(l+m+n) - 30]u^{11} \\
&\quad + \frac{117649}{1728}[(l+m+n) - 6]u^{12} \\
&\quad + \frac{663552}{1728}u^{12} + \frac{1572864}{2744}u^{14},
\end{aligned}$$

$$\begin{aligned}
S[St_{l,m,n}] &= S_u^3 Q_{u(-2)} JD_u^3 D_v^3 NM[G : u, v]\big|_{u=1} \\
&= \frac{668687075}{2299968}(l+m+n) - \frac{555184266893}{832716192}.
\end{aligned} \tag{57}$$

□

In Section 6, I calculate a few neighborhood degree-based topological indices by direct formula, and in Section 7, I calculate these topological indices via the NM-polynomial. I see that the results of these neighborhood degree-based topological indices are the same and NM-polynomial is another method of finding degree-based topological indices.

Figure 3 shows a graphical representation of topological indices of $St_{l,m,n}$. From the graphs, we see the behavior of the topological indices along with different parameters. Even though the graphs are looking to be identical, they have distinct gradients.

8. Conclusion

In this paper, we consider starphene. We derived the edge partitions of the molecular graph with respect to the neighborhood degree of the vertex, then computed the different molecular descriptors based on the neighborhood degree sum of nodes via NM-polynomial. All types of neighborhood degree sum-based indices available in the literature till now are considered in this paper. Each of them has a significant ability to predict different physiochemical properties and biological activities. The isomer discrimination ability of the indices is also remarkable as compared to other indices. Considered topological indices are therefore useful molecular descriptors in the area of chemical graph theory to establish structure-property/structure-activity relationship. Thus, the findings capture several information about different properties and activities of the considered structures through mathematical formulations.

Data Availability

There is no data associated with this paper.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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