Hindawi
Journal of Function Spaces
Volume 2024, Article ID 9837418, 1 page
https://doi.org/10.1155/2024/9837418

Retraction

Retracted: Applications of Karry-Kalim-Adnan Transformation (KKAT) to Mechanics and Electrical Circuits

Journal of Function Spaces

Received 23 January 2024; Accepted 23 January 2024; Published 24 January 2024

This article has been retracted by Hindawi following an investigation undertaken by the publisher [1]. This investigation has uncovered evidence of one or more of the following indicators of systematic manipulation of the publication process:

(1) Discrepancies in scope
(2) Discrepancies in the description of the research reported
(3) Discrepancies between the availability of data and the research described
(4) Inappropriate citations
(5) Incoherent, meaningless and/or irrelevant content included in the article
(6) Manipulated or compromised peer review

The presence of these indicators undermines our confidence in the integrity of the article's content and we cannot, therefore, vouch for its reliability. Please note that this notice is intended solely to alert readers that the content of this article is unreliable. We have not investigated whether authors were aware of or involved in the systematic manipulation of the publication process.

Wiley and Hindawi regrets that the usual quality checks did not identify these issues before publication and have since put additional measures in place to safeguard research integrity.

We wish to credit our own Research Integrity and Research Publishing teams and anonymous and named external researchers and research integrity experts for contributing to this investigation.

The corresponding author, as the representative of all authors, has been given the opportunity to register their agreement or disagreement to this retraction. We have kept a record of any response received.

References

[1] K. Iqbal, M. Kalim, and A. Khan, "Applications of Karry-Kalim-Adnan Transformation (KKAT) to Mechanics and Electrical Circuits," *Journal of Function Spaces*, vol. 2022, Article ID 8722534, 11 pages, 2022.

Hindawi
Journal of Function Spaces
Volume 2022, Article ID 8722534, 11 pages
https://doi.org/10.1155/2022/8722534

Research Article

Applications of Karry-Kalim-Adnan Transformation (KKAT) to Mechanics and Electrical Circuits

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Received 9 March 2022; Revised 12 May 2022; Accepted 15 June 2022; Published 4 July 2022

Academic Editor: Muhammad Gulzar

In this paper, we have used the new integral transformation known as Karry-Kalim-Adnan transformation (KKAT) to solve the ordinary linear differential equations as well as partial differential equations. We have used KKAT to solve the problems of engineering and sciences specially electrical and mechanical problems. Also, we have established the comparison between KKAT and existing transformations. We have determined the KKAT of error functions and complementary error function. The fundamental purpose of this research paper is to transform the given problem into the easier form to get its solution.

1. Introduction

Integral transformation is a very important tool to solve the differential equations for engineers and scientists. Integral transforms are widely used to determine the solution of differential equations with the initial values. Integral transforms have played a great role in engineering, chemistry, biology, astronomy, social sciences, radio physics, and nuclear science. KKAT is closely related to Laplace transformation [1, 2], Fourier transformation [3], Sumudu transformation [4], Elzaki transformation [5], Aboodh transformation [6], Mahgoub transformation [7], etc. Mohand and Laplace transformations [8] were studied and used by Aggarwal and Chaudhary to solve the system of advanced differential equations. Sedeeg has used the Kamal transformation [9], to solve the linear ordinary differential equations having constant coefficients. Aggarwal et al. [10] determined the Elzaki transformation of error function.

In general integral transformation, we take a set of functions of $\{f(x)\}$ in the region $-\infty < a \le x \le b < \infty$, and then, we select a field function $\kappa(x, \beta)$. So the integral transformation [4] is defined by

$$T\{f(x)\} = F(\beta) = \int_a^b f(x)\kappa(x, \beta)dx, \quad (1)$$

where the function $\kappa(x, \beta)$ is known as the kernel of the transformation T and β is a parameter. There are different kinds of transformations depending upon the kernel and the limits a and b.

This field of study is very important for the researchers. In classical analysis, some special types of the integral transformations as mentioned above have been extensively studied and used in solving several problems of engineering and mathematical sciences. Gupta [11] has used Rohit transform to analyze the boundary value problems of science and engineering. All transformations correspond to different types of kernel $\kappa(x, \beta)$. The limits of the integral are also different in each case.

Historically, Pierre Simon de Laplace (1749-1827) [12] initiated the integral transformations. Joseph Fourier (1768-1830) used the modern mathematical theory on heat conduction, Fourier series, integral transforms, and inverse Fourier transformation.

1.1. Laplace Transformation. The Laplace transformation [1] for $f(x)$, $x \ge 0$, is given by

$$L(f(x)) = \int_0^\infty e^{-\beta x} f(x)dx = A(\beta). \quad (2)$$

The parameter β belongs to real or complex plane.

1.2. Elzaki Transformation. The Elzaki transformation [1, 5] for the function $f(x)$, $x \geq 0$, is given by

$$E(f(x)) = \beta \int_0^\infty e^{-x/\beta} f(x) dx = C(\beta), \beta \in [-\beta_1 \beta_2], \quad \beta_1, \beta_2 > 0. \tag{3}$$

1.3. Sumudu Transformation. The Sumudu transformation [4] for the function $f(x)$, $x \geq 0$, is given by

$$S(f(x)) = \int_0^\infty e^{-x} f(\beta x) dx = D(\beta), \quad 0 < \beta_1 \leq \beta \leq \beta_2. \tag{4}$$

1.4. Rohit Transformation. The Rohit transformation [11] for the function $f(x)$, $x \geq 0$, is given by

$$R(f(x)) = \beta^3 \int_0^\infty e^{-\beta x} f(x) dx = I(\beta). \tag{5}$$

The parameter β belongs to real or complex plane.

The Kernels of well-known transformations are given in Table 1.

2. Definition of KKAT

A transformation defined for function of exponential order from set S (see [4, 5])

$$S = \left\{ f(x): \exists P\,; a_1, a_2 > 0, |f(x)| < P e^{|x|/a_i} \right\}, x \in (=1)^i \times [0, \infty), \tag{6}$$

where constant P is a finite number and a_1, a_2 may be finite or infinite.

KKAT is represented by operator $K(.)$ and is defined by

$$K(f(x)) = \frac{1}{\beta} \int_0^\infty f(\alpha x) e^{-\beta x} dx, x \geq 0\,; \alpha, \beta \in [a_1\, a_2], \tag{7}$$

where α and β are constants and $\alpha, \beta \neq 0$.

$K\{f(x)\}$ can also be written in the form

$$K\{f(x)\} = \frac{1}{\alpha\beta} \int_0^\infty f(x) e^{-(\beta/\alpha)x} dx = F\left(\frac{\beta}{\alpha}\right). \tag{8}$$

Also

$$f(x) = K^{-1}\left\{ F\left(\frac{\beta}{\alpha}\right) \right\}. \tag{9}$$

2.1. Some Useful Results of KKAT

Result 1. If

$$f(x) = 1, \tag{10}$$

Table 1: Kernel of well-known transformations.

Sr. no.	Transformation	Kernel
1	Laplace transformation [1]	$\kappa(x, \beta) = e^{-\beta x}$
2	Sumudu transformation [4]	$\kappa(x, \beta) = (1/\beta) e^{-(1/\beta)x}$
3	Elzaki transformation [1, 5]	$\kappa(x, \beta) = \beta e^{-(1/\beta)x}$
4	Rohit transformation [11]	$\kappa(x, \beta) = \beta^3 e^{-\beta x}$
5	Karry Kalim Adnan Transformation	$\kappa(x, (\beta/\alpha)) = (1/\alpha\beta) e^{-(\beta/\alpha)x}$

then

$$K\{1\} = \frac{1}{\alpha\beta} \int_0^\infty e^{-(\beta/\alpha)x} dx. \tag{11}$$

By evaluating integral, we get

$$K\{1\} = \frac{1}{\beta^2}. \tag{12}$$

Result 2. If

$$f(x) = x^n e^{\lambda x}, \tag{13}$$

then

$$K\left\{x^n e^{\lambda x}\right\} = \frac{1}{\alpha\beta} \int_0^\infty x^n e^{-((\beta/\alpha)-\lambda)x} dx. \tag{14}$$

We assume that

$$\left(\frac{\beta}{\alpha} - \lambda\right) x = t \Rightarrow dx = \frac{dt}{((\beta/\alpha) - \lambda)}. \tag{15}$$

Thus, we get

$$K\left\{x^n e^{\lambda x}\right\} = \frac{1}{\alpha\beta((\beta/\alpha) - \lambda)^{n+1}} \int_0^\infty t^n e^{-t} dt. \tag{16}$$

Hence,

$$K\left\{x^n e^{\lambda x}\right\} = \frac{\Gamma(n+1)\alpha^n}{\beta(\beta - \lambda\alpha)^{n+1}}, \text{for } n > -1. \tag{17}$$

Result 3. If

$$f(x) = \sin \lambda x, \tag{18}$$

then

$$K\{\sin \lambda x\} = \frac{1}{\alpha\beta} \int_0^\infty e^{-(\beta/\alpha)x} \sin \lambda x dx. \tag{19}$$

Table 2: KKAT of some important functions.

Sr. no.	$f(x)$	$F(\beta/\alpha)$
1	1	$1/\beta^2$
2	$x^n, n \in N$	$n!\alpha^n/\beta^{n+2}$
3	$x^n e^{\lambda x}, n \in N$	$n!\alpha^n/\beta(\beta-\lambda\alpha)^{n+1}$
4	$x^n, n > -1$	$\Gamma(n+1)\alpha^n/\beta^{n+2}$
5	$x^n e^{\lambda x}, n > -1$	$\Gamma(n+1)\alpha^n/\beta(\beta-\lambda\alpha)^{n+1}$
6	$e^{\lambda x}$	$1/\beta(\beta-\lambda\alpha)$
7	$\sin \lambda x$	$\lambda\alpha/\beta\left(\beta^2+\lambda^2\alpha^2\right)$
8	$\cos \lambda x$	$1/\beta^2+\lambda^2\alpha^2$
9	$\sinh x$	$\alpha/\beta\left(\beta^2-\alpha^2\right)$
10	$\cosh \lambda x$	$1/\beta^2-\lambda^2\alpha^2$
11	$e^{\lambda x}\cosh x$	$(\beta-\lambda\alpha)/\beta\left((\beta-\lambda\alpha)^2-\alpha^2\right)$
12	$e^{\lambda x}\sinh x$	$\alpha/\beta\left((\beta-\lambda\alpha)^2-\alpha^2\right)$
13	$\sin \lambda x + \cos \lambda x$	$\lambda\alpha+\beta/\beta\left(\beta^2+\lambda^2\alpha^2\right)$
14	$f'(x)$	$(\beta/\alpha)K\{f(x)\}-f(0)/\alpha\beta$
15	$f''(x)$	$\left(\beta^2/\alpha^2\right)K\{f(x)\}-f'(0)/\alpha\beta-f(0)/\alpha^2$
16	$e^{\lambda x}f(x)$	$(\beta-\lambda\alpha/\beta)F(\alpha,\beta-\lambda\alpha)$
17	$xf(x)$	$-\alpha(d/d\beta)F(\alpha,\beta)-(\alpha/\beta)K\{f(x)\}$
18	$xf'(x)$	$-\alpha(d/d\beta)((\beta/\alpha)K\{f(x)\}-f(0)/\alpha\beta)-K\{f(x)\}+\left(f(0)/\beta^2\right)$

By using

$$\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}\{a\sin bx - b\cos bx\},$$

$$\frac{1}{\alpha\beta}\int_0^\infty e^{-(\beta/\alpha)x}\sin\lambda x\,dx = \frac{1}{\alpha\beta}\left[\frac{e^{-(\beta/\alpha)x}}{(-\beta/\alpha)^2+\lambda^2}\left\{\left(-\frac{\beta}{\alpha}\right)\sin\lambda x - \lambda\cos\lambda x\right\}\Bigg|_0^\infty\right]. \tag{20}$$

Hence, we get

$$K\{\sin\lambda x\} = \frac{\lambda\alpha}{\beta\left(\beta^2+\lambda^2\alpha^2\right)}. \tag{21}$$

Thus, the KKAT of some important functions is given in Table 2 as follows.

3. Linearity Property of KKAT

If $G_1(\beta/\alpha), G_2(\beta/\alpha)$ are KKAT of $g_1(x), g_2(x) \in A$, then KKAT of $ag_1(x) + b\,g_2(x)$ is $aG_1(\beta/\alpha) + bG_2(\beta/\alpha)$, where a, b are any constant.

Proof. By definition of KKAT from equation (8), we get

$$K\{g(x)\} = G\left(\frac{\beta}{\alpha}\right) = \frac{1}{\alpha\beta}\int_0^\infty e^{-(\beta/\alpha)x}g(x)dx,$$

$$\begin{aligned} K(ag_1(x)+bg_2(x)) &= \frac{1}{\alpha\beta}\int_0^\infty e^{-(\beta/\alpha)x}(ag_1(x)+bg_2(x))dx \\ &= a\frac{1}{\alpha\beta}\int_0^\infty e^{-(\beta/\alpha)x}g_1(x)dx \\ &\quad + b\frac{1}{\alpha\beta}\int_0^\infty e^{-(\beta/\alpha)x}g_2(x)dx. \end{aligned} \tag{22}$$

Hence, we obtain

$$K(ag_1(x)+b\,g_2(x)) = aG_1\left(\frac{\beta}{\alpha}\right) + bG_2\left(\frac{\beta}{\alpha}\right). \tag{23}$$

□

4. Inverse of KKAT

If $F(\beta/\alpha)$ be the KKAT of $f(x)$, then $f(x)$ is called the inverse KKAT of $F(\beta/\alpha)$. The inverse KKAT is expressed in equation (9)

$$K^{-1}\left\{F\left(\frac{\beta}{\alpha}\right)\right\} = f(x). \tag{24}$$

4.1. KKAT Laplace Duality and KKAT Inversion Integral. If $F(s)$ and $F(\beta/\alpha)$ are Laplace transform and KKAT of function $f(x)$, respectively, then $\alpha\beta F(\beta/\alpha) = F(s)$ by assuming $\beta/\alpha = s$.

If $f(x)$ be the inverse KKAT of $F(\beta/\alpha)$ and all the singularities of $F(\beta/\alpha)$ in the complex plane $s = \beta/\alpha = x + iy$ lie to the left of the line $x = x_\circ$, then

$$\begin{aligned} f(x) &= \frac{1}{2\pi i} \lim_{x_\circ \longrightarrow \infty} \int_{x_\circ - iR}^{x_\circ + iR} \beta e^{(\beta/\alpha)x} F\left(\frac{\beta}{\alpha}\right) d\beta, \\ f(x) &= \sum_j R_j, \end{aligned} \tag{25}$$

where R_j is residue of $\beta e^{(\beta/\alpha)x} F(\beta/\alpha)$ at the poles $\beta = \beta_j$.

4.2. Verification of Inverse Function. To verify the inverse of KKAT, the following examples are given.

Example 1. If $K(1) = F(\beta/\alpha) = 1/\beta^2$, then $K^{-1}(1/\beta^2) = 1$, where $f(x) = 1$ and $F(\beta/\alpha)$ is defined in equation (8).

Proof. If

$$F\left(\frac{\beta}{\alpha}\right) = \frac{1}{\beta^2}, \tag{26}$$

then the function $\beta e^{(\beta/\alpha)x} F(\beta/\alpha) = e^{(\beta/\alpha)x}/\beta$ has simple pole at $\beta = 0$.

Thus, we have

$$f(t) = K^{-1}\left(\frac{1}{\beta^2}\right) = R_0, \tag{27}$$

where

$$R_0 = \text{residue of } \beta e^{(\beta/\alpha)x} F\left(\frac{\beta}{\alpha}\right) \text{ at } \beta = 0, \tag{28}$$

also

$$R_0 = \lim_{\beta \longrightarrow 0} \frac{\beta^2 e^{(\beta/\alpha)x}}{\beta^2}, \tag{29}$$

thus

$$R_0 = 1, \tag{30}$$

therefore

$$f(x) = K^{-1}\left(\frac{1}{\beta^2}\right) = 1. \tag{31}$$

□

Example 2. If

$$K\left(e^{\lambda x}\right) = F\left(\frac{\beta}{\alpha}\right) = \frac{1}{\beta(\beta - \lambda\alpha)}, \tag{32}$$

then

$$K^{-1}\left(\frac{1}{\beta(\beta - \lambda\alpha)}\right) = e^{\lambda x}. \tag{33}$$

Proof. If

$$F\left(\frac{\beta}{\alpha}\right) = \frac{1}{\beta(\beta - \lambda\alpha)}, \tag{34}$$

then the function $\beta e^{(\beta/\alpha)x} F(\beta/\alpha) = e^{(\beta/\alpha)x}/(\beta - \lambda\alpha)$ has simple pole at $\beta = \lambda\alpha$,thus

$$f(t) = K^{-1}\left(F\left(\frac{\beta}{\alpha}\right)\right) = R_0, \tag{35}$$

where

$$R_0 = \text{residue of } \beta e^{(\beta/\alpha)x} F\left(\frac{\beta}{\alpha}\right), \text{ at } \beta = \lambda\alpha, \tag{36}$$

also

$$R_0 = \lim_{\beta \longrightarrow \lambda\alpha} \frac{(\beta - \lambda\alpha) e^{(\beta/\alpha)x}}{(\beta - \lambda\alpha)}, \tag{37}$$

hence

$$R_0 = e^{\lambda x}, \tag{38}$$

therefore

$$f(x) = e^{\lambda x}. \tag{39}$$

□

Example 3. If $K(\sin \lambda x) = F(\beta/\alpha) = \lambda\alpha/(\beta(\beta^2 + \lambda^2\alpha^2))$, then $K^{-1}(\lambda\alpha/(\beta(\beta^2 + \lambda^2\alpha^2))) = \sin \lambda x$.

Proof. Since $F(\beta/\alpha) = \lambda\alpha/(\beta(\beta^2 + \lambda^2\alpha^2))$, then the function $\beta e^{(\beta/\alpha)x} F(\beta/\alpha) = \lambda\alpha e^{(\beta/\alpha)x}/(\beta^2 + \lambda^2\alpha^2)$ has simple pole at $\beta = \pm\lambda\alpha i$; thus,

$$f(t) = K^{-1}\left(F\left(\frac{\beta}{\alpha}\right)\right) = R_0 + R_1, \tag{40}$$

where

$$R_0 = \text{residue of } \beta e^{(\beta/\alpha)x} F\left(\frac{\beta}{\alpha}\right) \text{ at } \beta = \lambda\alpha i, \tag{41}$$

also

$$R_0 = \lim_{\beta \longrightarrow \lambda\alpha i} \frac{(\beta - \lambda\alpha i)\lambda\alpha e^{(\beta/\alpha)x}}{(\beta - \lambda\alpha i)(\beta + \lambda\alpha i)}, \tag{42}$$

thus

$$R_0 = \frac{e^{i\lambda x}}{2i}. \tag{43}$$

And

$$R_1 = \text{residue of } \beta e^{(\beta/\alpha)x} F\left(\frac{\beta}{\alpha}\right) \text{ at } \beta = -\lambda\alpha i, \tag{44}$$

thus

$$R_1 = -\frac{e^{-i\lambda x}}{2i}, \tag{45}$$

hence

$$f(t) = R_0 + R_1 = \frac{e^{i\lambda x}}{2i} - \frac{e^{-i\lambda x}}{2i} = \sin \lambda x, \tag{46}$$

therefore

$$f(t) = \sin \lambda x. \tag{47}$$

□

The inverse of KKAT is given in Table 3.

5. Application of KKAT on the Integral Function

If $g(x) = \int_0^x f(z)dz$, then $K\{\int_0^x f(z)dz\} = (\alpha/\beta)F(\beta/\alpha)$.

Proof. It is given that

$$g(x) = \int_0^x f(z)dz. \tag{48}$$

By using fundamental theorem of calculus,

$$g'(x) = f(x). \tag{49}$$

We know that

$$K\left\{g'(x)\right\} = \frac{\beta}{\alpha}K\{g(x)\} - \frac{g(0)}{\alpha\beta}. \tag{50}$$

But $g(0) = 0$ and $g'(x) = f(x)$,thus

$$K\{f(x)\} = \frac{\beta}{\alpha}K\{g(x)\}, \tag{51}$$

Table 3: Inverse KKAT.

Sr. no.	$K^{-1}\{F(\beta/\alpha)\}$	$f(x)$
1	$K^{-1}\{1/\beta^2\}$	1
2	$K^{-1}\{n!\alpha^n/\beta^{n+2}\}$	x^n
3	$K^{-1}\{1/\beta(\beta-\lambda\alpha)\}$	$e^{\lambda x}$
4	$K^{-1}\{\lambda\alpha/\beta(\beta^2+\lambda^2\alpha^2)\}$	$\sin \lambda x$
5	$K^{-1}\{1/\beta^2+\lambda^2\alpha^2\}$	$\cos \lambda x$
6	$K^{-1}\{\alpha/\beta(\beta^2-\alpha^2)\}$	$\sinh x$
7	$K^{-1}\{1/\beta^2-\alpha^2\}$	$\cosh x$

hence

$$K\left\{\int_0^x f(z)dz\right\} = \frac{\alpha}{\beta}F\left(\frac{\beta}{\alpha}\right). \tag{52}$$

□

6. Application of KKAT on Derivatives

Let $K\{f(x)\}$ be KKAT of $f(x)$; then,

$$K\left[f'(x)\right] = \frac{\beta}{\alpha}K[f(x)] - \frac{f(0)}{\alpha\beta}, \tag{53}$$

$K[f''(x)] = (\beta/\alpha)K[f'(x)] - f'(0)/\alpha\beta$,
$K[f^{(n)}(x)] = (\beta/\alpha)K(f^{(n-1)}(x)) - f^{(n-1)}(0)/\alpha\beta$.

7. Applications of KKAT to Mechanics

7.1. Application No. 1. Let a body A of mass 1 gram moves on x-axis. It is attracted towards the origin O with a force equals to $4x$. Also, assume that initially it is at rest when $x = 5$; then, determine its position by considering

(i) no any other forces acting on it

(ii) damping force or in other words resistance to the particle is equal to the 8 times the velocity at any instant [12]

Solution: see Figure 1.

From Figure 1, for $x > 0$, the net force towards left is given by $-4x$, while for $x < 0$, the net force towards right is given by $-4x$. Thus, for both cases, the net force is equal to $-4x$.

By Newton's second law of motion,

$$\begin{gathered} \text{mass} \times \text{acceleration} = \text{net force}, \\ \frac{d^2x}{dt^2} = -4x, \\ \frac{d^2x}{dt^2} + 4x = 0. \end{gathered} \tag{54}$$

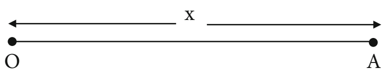

Figure 1

The initial conditions are

$$\begin{aligned} x(0) &= 5, \\ x'(0) &= 0. \end{aligned} \tag{55}$$

Applying KKAT and using initial conditions on equation (54), we get

$$\begin{aligned} K\left\{x''\right\} + 4K\{x\} &= 0, \\ \left[\frac{\beta^2}{\alpha^2} + 4\right] K\{x\} - \frac{5}{\alpha^2} &= 0. \end{aligned} \tag{56}$$

Now, by using inverse KKAT on equation (56), we obtain

$$X(t) = 5 \cos 2t. \tag{57}$$

7.2. Application No. 2. The relationship between resistive force of air and the velocity of a freely falling body is given by $dv/dt = g - \lambda v$, where $v(t)$ is the velocity at any time t. Initially, the body is at rest [11].

Solution: see Figure 2.

The equation for the motion of body moving under constant gravitational acceleration (as shown in Figure 2) is

$$\frac{dv}{dt} = g - \lambda v. \tag{58}$$

Applying KKAT on equation (58), we get

$$\begin{aligned} K\left\{\frac{dv}{dt}\right\} &= K\{g - \lambda v\}, \\ \frac{\beta}{\alpha} K\{v(t)\} - \frac{v(0)}{\alpha\beta} &= \frac{g}{\beta^2} - \lambda K\{v\}, \\ \frac{\beta}{\alpha} K\{v(t)\} &= \frac{g}{\beta^2} - \lambda K\{v\}, \\ \left\{\frac{\beta}{\alpha} + \lambda\right\} K\{v\} &= \frac{g}{\beta^2}, \\ K\{v\} = \frac{g}{\beta^2}\left\{\frac{1}{\lambda} - \frac{\beta/\lambda}{\beta + \lambda\alpha}\right\} &= \frac{g}{\beta^2}\left\{\frac{1}{\lambda} - \frac{\beta}{\lambda(\beta + \lambda\alpha)}\right\}, \\ K\{v\} &= \frac{g}{\lambda}\left\{\frac{1}{\beta^2} - \frac{1}{\beta(\beta + \lambda\alpha)}\right\}. \end{aligned} \tag{59}$$

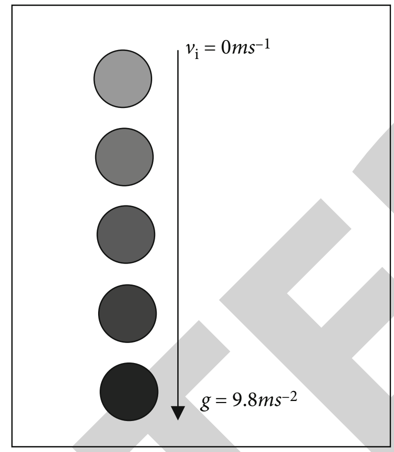

Figure 2

Applying inverse KKAT both sides on equation (59), we get

$$v(t) = \frac{g}{\lambda}\left\{1 - e^{-\lambda t}\right\}. \tag{60}$$

7.3. Application No. 3

7.3.1. KKAT and Simple Harmonic Motion (SHM). Let a body of mass m executing SHM (as shown in Figure 3) and z be the displacement from the mean position at any time. The equation for this motion is given by $d^2z/dx^2 = + \lambda^2 z = 0$, where $\lambda = k/m$ and k is constant of proportionality. For $t = 0$, $z(0) = 0, z'(0) = 1$. We use KKAT to find the displacement at any time t [11].

Solution: see Figure 3.

The equation for SHM is given by

$$\frac{d^2z}{dx^2} + \lambda^2 z = 0. \tag{61}$$

Applying KKAT and initial conditions on equation (61), we get

$$\begin{aligned} K\left\{\frac{d^2z}{dx^2} + \lambda^2 z\right\} &= 0, \\ K\left\{\frac{d^2z}{dx^2}\right\} + \lambda^2 K\{z\} &= 0, \\ \frac{\beta^2}{\alpha^2} K\{z\} - \frac{z'(0)}{\alpha\beta} - \frac{z(0)}{\alpha^2} + \lambda^2 K\{z\} &= 0, \\ \left\{\frac{\beta^2}{\alpha^2} + \lambda^2\right\} K\{z\} - \frac{1}{\alpha\beta} &= 0, \\ K\{z\} &= \frac{1}{\beta}\left\{\frac{\alpha}{\beta^2 + \lambda^2\alpha^2}\right\}. \end{aligned} \tag{62}$$

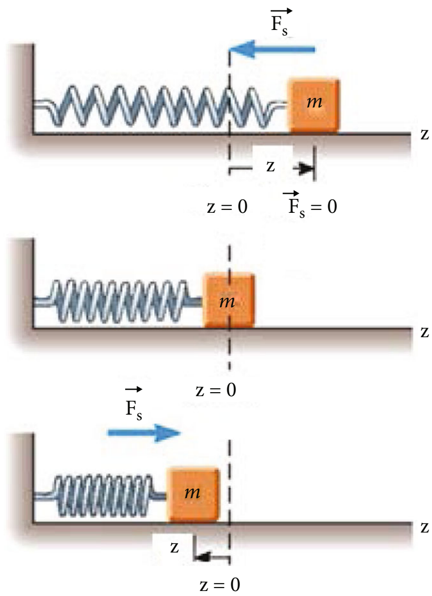

Figure 3

Applying inverse KKAT both sides on equation (62), we get

$$z = \frac{1}{\lambda} K^{-1}\left(\frac{\lambda\alpha}{\beta\left(\beta^2 + (\lambda\alpha)^2\right)}\right),$$
$$z(t) = \frac{1}{\lambda} \sin \lambda t. \tag{63}$$

By using the value of λ, we get

$$z(t) = \sqrt{\frac{m}{k}} \sin \sqrt{\frac{k}{m}} t. \tag{64}$$

8. Applications of KKAT to Electrical Circuits

8.1. Application No. 4. A capacitor of capacitance 0.04 Farad, inductor of inductance 1 Henry, and resistor of 8 ohms are connected in series with battery of emf $E = 150$ V (as shown in Figure 4) at $t = 0$ current and charge are zero. Determine charge and current at any time [12].

Solution: see Figure 4.

The voltage $V_R(t)$ across the resistor R when current $I(t)$ is flowing is given by

$$V_R(t) = RI. \tag{65}$$

The voltage $V_L(t)$ across the inductor L when current $I(t)$ is flowing given by the relation

$$V_L(t) = L\frac{dI}{dt}. \tag{66}$$

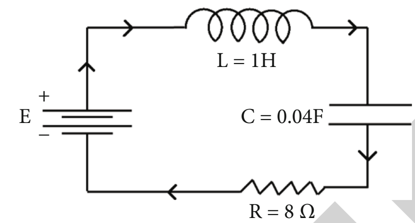

Figure 4: RLC circuit.

The voltage $V_C(t)$ across the capacitor C is flowing given by

$$V_C(t) = \frac{Q}{C}. \tag{67}$$

By Kirchhoff second law,

$$\begin{aligned}
V_R(t) + V_L(t) + V_C(t) &= E,\\
L\frac{dI}{dt} + RI + \frac{Q}{C} &= E,\\
L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} &= E,\\
\frac{d^2Q}{dt^2} + 8\frac{dQ}{dt} + \frac{Q}{.04} &= 150,\\
\frac{d^2Q}{dt^2} + 8\frac{dI}{dt} + 25Q &= 150.
\end{aligned} \tag{68}$$

Using KKAT and initial conditions on equation (68), we get

$$K\left\{\frac{d^2Q}{dt^2}\right\} + 8K\left\{\frac{dQ}{dt}\right\} + 25K\{Q\} = 150K\{1\},$$

$$\frac{\beta^2}{\alpha^2}K\{Q\} - \frac{Q'(0)}{\alpha\beta} - \frac{Q(0)}{\alpha^2} + 8\left[\frac{\beta}{\alpha}K\{Q\} - \frac{Q(0)}{\alpha\beta}\right] + 25K\{Q\} = \frac{150}{\beta^2},$$

$$\frac{\beta^2}{\alpha^2}K\{Q\} + 8\frac{\beta}{\alpha}K\{Q\} + 25K\{Q\} = \frac{150}{\beta^2},$$

$$\left(\frac{\beta^2}{\alpha^2} + 8\frac{\beta}{\alpha} + 25\right)K\{Q\} = \frac{150}{\beta^2},$$

$$\left(\frac{\beta^2 + 8\alpha\beta + 25\alpha^2}{\alpha^2}\right)K\{Q\} = \frac{150}{\beta^2},$$

$$K\{Q\} = \frac{150}{\beta^2} \cdot \frac{\alpha^2}{\beta^2 + 8\alpha\beta + 25\alpha^2},$$

$$K\{Q\} = \frac{1}{\beta^2}\left\{6 - \frac{(6\beta^2 + 48\alpha\beta)}{\beta^2 + 8\alpha\beta + 25\alpha^2}\right\},$$

$$K\{Q\} = \frac{1}{\beta^2}\left\{6 - \frac{(6\beta^2 + 24\alpha\beta)}{\beta^2 + 8\alpha\beta + 25\alpha^2} - \frac{24\alpha\beta}{\beta^2 + 8\alpha\beta + 25\alpha^2}\right\},$$

$$K\{Q\} = \frac{1}{\beta^2}\left\{6 - \frac{(6\beta^2 + 24\alpha\beta)}{(\beta+4\alpha)^2 + 9\alpha^2} - \frac{24\alpha\beta}{(\beta+4\alpha)^2 + 9\alpha^2}\right\},$$

$$K\{Q\} = \frac{6}{\beta^2} - \frac{(6\beta^2 + 24\alpha\beta)}{\beta^2\left((\beta+4\alpha)^2 + 9\alpha^2\right)} - \frac{24\alpha\beta}{\beta^2\left((\beta+4\alpha)^2 + 9\alpha^2\right)},$$

$$K\{Q\} = \frac{6}{\beta^2} - \frac{(6\beta + 24\alpha)}{\beta\left((\beta+4\alpha)^2 + 9\alpha^2\right)} - \frac{24\alpha}{\beta\left((\beta+4\alpha)^2 + 9\alpha^2\right)},$$

$$K\{Q\} = \frac{6}{\beta^2} - \frac{(6\beta + 24\alpha)}{\beta\left((\beta+4\alpha)^2 + 9\alpha^2\right)} - \frac{(\beta+4\alpha)8(3\alpha)}{\beta(\beta+4\alpha)\left((\beta+4\alpha)^2 + 9\alpha^2\right)}. \tag{69}$$

Applying inverse KKAT both sides on equation (69), we obtain

$$\begin{aligned} Q &= K^{-1}\left\{\frac{6}{\beta^2} - \frac{(6\beta + 24\alpha)}{\beta\left((\beta+4\alpha)^2 + 9\alpha^2\right)} - \frac{(\beta+4\alpha)8(3\alpha)}{\beta(\beta+4\alpha)\left((\beta+4\alpha)^2 + 9\alpha^2\right)}\right\} \\ &= K^{-1}\left\{\frac{6}{\beta^2}\right\} - K^{-1}\left\{\frac{(6\beta + 24\alpha)}{\beta\left((\beta+4\alpha)^2 + 9\alpha^2\right)}\right\} \\ &\quad - K^{-1}\left\{\frac{(\beta+4\alpha)8(3\alpha)}{\beta(\beta+4\alpha)\left((\beta+4\alpha)^2 + 9\alpha^2\right)}\right\} \\ &= K^{-1}\left\{\frac{6}{\beta^2}\right\} - K^{-1}\left\{\frac{6(\beta + 4\alpha)}{\beta\left((\beta+4\alpha)^2 + 9\alpha^2\right)}\right\} \\ &\quad - K^{-1}\left\{\frac{(\beta+4\alpha)8(3\alpha)}{\beta(\beta+4\alpha)\left((\beta+4\alpha)^2 + 9\alpha^2\right)}\right\}, \\ Q(t) &= 6 - 6e^{-4t}\cos 3t - 8e^{-4t}\sin 3t. \end{aligned} \tag{70}$$

Now, by differentiating equation (70), we get

$$\begin{aligned} I &= \frac{dQ(t)}{dt} = \frac{d}{dt}\left\{6 - 6e^{-4t}\cos 3t - 8e^{-4t}\sin 3t\right\}, \\ I &= 50e^{-4t}\sin 3t. \end{aligned} \tag{71}$$

9. Connection between KKAT and Some Useful Transforms

In this paper, we have established the connection between KKAT and some useful integral transforms [1, 4, 5, 11] (Laplace, Elzaki, Sumudu, and Rohit).

9.1. Connection between KKAT and Laplace Transform. If KKAT of function $f(x)$ is $K(f(x)) = F(\beta/\alpha)$ and Laplace transform of $f(x)$ is

$L(f(x)) = A(\beta)$ defined in equation (2), then $F(\beta/\alpha) = (1/\alpha\beta)A(\beta/\alpha)$.

Proof. From equation (8)

$$\begin{aligned} K\{f(x)\} &= \frac{1}{\alpha\beta}\int_0^\infty f(x)e^{(-\beta/\alpha)x}dx = F\left(\frac{\beta}{\alpha}\right) \\ &= \frac{1}{\alpha\beta}\left(\int_0^\infty f(x)e^{(-\beta/\alpha)x}dx\right), \\ F\left(\frac{\beta}{\alpha}\right) &= \frac{1}{\alpha\beta}A\left(\frac{\beta}{\alpha}\right). \end{aligned} \tag{72}$$

□

9.2. Connection between KKAT and Elzaki Transform. If KKAT of function $f(x)$ is $K(f(x)) = F(\beta/\alpha)$ and $E(f(x)) = C(\beta)$ defined in equation (3), then

$$F\left(\frac{\beta}{\alpha}\right) = \frac{1}{\alpha^2}C\left(\frac{\alpha}{\beta}\right). \tag{73}$$

Proof. From equation (8)

$$\begin{aligned} K\{f(x)\} &= \frac{1}{\alpha\beta}\int_0^\infty f(x)e^{-(\beta/\alpha)x}dx = F\left(\frac{\beta}{\alpha}\right) \\ &= \frac{1}{\alpha^2}\left(\frac{\alpha}{\beta}\int_0^\infty f(x)e^{-x/(\alpha/\beta)}dx\right), \\ F\left(\frac{\beta}{\alpha}\right) &= \frac{1}{\alpha^2}C\left(\frac{\alpha}{\beta}\right). \end{aligned} \tag{74}$$

□

9.3. Connection between KKAT and Sumudu Transform. If KKAT of function $f(x)$ is $K(f(x)) = F(\beta/\alpha)$ and $S(f(x)) = D(\beta)$ defined in equation (4), then

$$F\left(\frac{\beta}{\alpha}\right) = \frac{1}{\beta^2}D\left(\frac{\alpha}{\beta}\right). \tag{75}$$

9.4. Connection between KKAT and Rohit Transform. If KKAT of function $f(x)$ is $K(f(x)) = F(\beta/\alpha)$ from equation (6) and $k(f(x)) = I(\beta)$ defined in equation (5), then

$$F\left(\frac{\beta}{\alpha}\right) = \frac{\alpha^2}{\beta^4}I(\beta). \tag{76}$$

10. Comparison with Laplace, Sumudu, Elzaki, and Rohit Transformations

We have presented KKAT and integral transforms (Laplace, Elzaki, Sumudu, and Rohit) of some functions in Table 4.

Table 4: Some important functions with KKAT, Laplace, Sumudu, Elzaki, and Rohit Transformations.

Sr. no.	$f(x)$	KKAT $K\{f(x)\} = F(\beta/\alpha)$	LAPLACE $L(f(x)) = A(\beta)$	SUMUDU $S(f(x)) = D(\beta)$	ELZAKI $E(f(x)) = C(\beta)$	ROHIT $k(f(x)) = I(\beta)$
1	1	$1/\beta^2$	$1/\beta$	1	β^2	β^2
2	$x^n, n \in N$	$n!\alpha^n/\beta^{n+2}$	$n!/\beta^{n+1}$	$n!\beta^n$	$n!\beta^{n+2}$	$n!/\beta^{n-2}$
3	$e^{\lambda x}$	$1/\beta(\beta - \lambda\alpha)$	$1/(\beta - \lambda)$	$1/(1 - \lambda\beta)$	$\beta^2/(1 - \lambda\beta)$	$\beta^3/(\beta - \lambda)$
4	$\sin \lambda x$	$\lambda\alpha/\beta(\beta^2 + \lambda^2\alpha^2)$	$\lambda/(\beta^2 + \lambda^2)$	$\lambda\beta/(1 + \lambda^2\beta^2)$	$\lambda\beta^3/(1 + \lambda^2\beta^2)$	$\lambda\beta^3/(\beta^2 + \lambda^2)$
5	$\cos \lambda x$	$1/\beta^2 + \lambda^2\alpha^2$	$\beta/\beta^2 + \lambda^2$	$1/(1 + \lambda^2\beta^2)$	$\beta^2/1 + \lambda^2\beta^2$	$\beta^4/\beta^2 + \lambda^2$
6	$\sinh x$	$\alpha/\beta(\beta^2 - \alpha^2)$	$1/(\beta^2 - 1)$	$\beta/(1 - \beta^2)$	$\beta^3/(1 + \beta^2)$	$\beta^3/(\beta^2 - 1)$
7	$\cosh x$	$1/(\beta^2 - \alpha^2)$	$\beta/(\beta^2 - 1)$	$1/(1 - \beta^2)$	$\beta^2/(1 + \beta^2)$	$\beta^4/(\beta^2 - 1)$

11. Error Function and Complementary Error Function

Error function and complementary error function [2, 10] are the special functions in mathematics, science, and engineering and it is defined as

$$\operatorname{erf}(t) = \frac{2}{\sqrt{\pi}} \int_0^t e^{-x^2} dx, \tag{77}$$

$$\operatorname{erf} c(t) = \frac{2}{\sqrt{\pi}} \int_t^\infty e^{-x^2} dx. \tag{78}$$

The sum of error function and complementary error function is equal to unity.

$$\operatorname{erf}(t) + \operatorname{erf} c(t) = 1. \tag{79}$$

12. KKAT of Error Function

By using equation (77),

$$\begin{aligned}
\operatorname{erf}\left(\sqrt{t}\right) &= \frac{2}{\sqrt{\pi}} \int_0^{\sqrt{t}} e^{-x^2} dx = \frac{2}{\sqrt{\pi}} \int_0^{\sqrt{t}} \left(1 - \frac{x^2}{1!} + \frac{x^4}{2!} - \frac{x^6}{3!} + \cdots\right) dx \\
&= \frac{2}{\sqrt{\pi}} \left[x - \frac{x^3}{1!.3} + \frac{x^5}{2!.5} - \frac{x^7}{3!.7} + \cdots\right]_0^{\sqrt{t}} \\
&= \frac{2}{\sqrt{\pi}} \left(t^{1/2} - \frac{t^{3/2}}{1!.3} + \frac{t^{5/2}}{2!.5} - \frac{t^{7/2}}{3!.7} + \cdots\right),
\end{aligned}$$

$$\begin{aligned}
K\left[\operatorname{erf}\left(\sqrt{t}\right)\right] &= K\left\{\frac{2}{\sqrt{\pi}} \left(t^{1/2} - \frac{t^{3/2}}{1!.3} + \frac{t^{5/2}}{2!.5} - \frac{t^{7/2}}{3!.7} + \cdots\right)\right\} \\
&= \frac{2}{\sqrt{\pi}} \left(\frac{\Gamma(3/2)\alpha^{1/2}}{\beta^{5/2}} - \frac{\Gamma(5/2)\alpha^{3/2}}{\beta^{7/2} 1!.3} + \frac{\Gamma(7/2)\alpha^{5/2}}{\beta^{9/2} 2!.5} - \frac{\Gamma(9/2)\alpha^{7/2}}{\beta^{11/2} 3!.7} + \cdots\right) \\
&= \frac{2\Gamma(3/2)}{\sqrt{\pi}} \left(\frac{\alpha^{1/2}}{\beta^{5/2}} - \frac{3\alpha^{3/2}}{\beta^{7/2} 1!.3.2} + \frac{5.3\alpha^{5/2}}{\beta^{9/2} 2!.5.4} - \frac{7.5.3\alpha^{7/2}}{\beta^{11/2} 3!.7.8} + \cdots\right) \\
&= \frac{2}{\sqrt{\pi}} \cdot \frac{\sqrt{\pi}}{2} \cdot \frac{\alpha^{1/2}}{\beta^{5/2}} \left(1 - \frac{3\alpha}{1!.3.2\beta} + \frac{5.3\alpha^2}{2!.5.4\beta^2} - \frac{7.5.3\alpha^3}{3!.7.8\beta^3} + \cdots\right),
\end{aligned}$$

$$K\left[\operatorname{erf}\left(\sqrt{t}\right)\right] = \frac{\alpha^{1/2}}{\beta^{5/2}} \left(1 + \frac{\alpha}{\beta}\right)^{-1/2},$$

$$K\left[\operatorname{erf}\left(\sqrt{t}\right)\right] = \frac{1}{\beta^2} \left(\frac{\alpha}{\alpha + \beta}\right)^{1/2}. \tag{80}$$

13. KKAT of Complementary Error Function

By using equation (79), we get

$$\begin{aligned}
\operatorname{erf}\left(\sqrt{t}\right) + \operatorname{erf} c\left(\sqrt{t}\right) &= 1, \\
\operatorname{erf} c\left(\sqrt{t}\right) &= 1 - \operatorname{erf}\left(\sqrt{t}\right).
\end{aligned} \tag{81}$$

Applying KKAT both sides on equations (81), we get

$$K\left[\operatorname{erf} c\left(\sqrt{t}\right)\right] = K\left[1 - \operatorname{erf}\left(\sqrt{t}\right)\right]. \tag{82}$$

Using linear property of KKAT on equation (82), we get

$$K\left[\operatorname{erf} c\left(\sqrt{t}\right)\right] = K[1] - K\left[\operatorname{erf}\left(\sqrt{t}\right)\right] = \frac{1}{\beta^2} - \frac{\alpha^{1/2}}{\beta^{5/2}} \left(1 + \frac{\alpha}{\beta}\right)^{-1/2},$$

$$K\left[\operatorname{erf} c\left(\sqrt{t}\right)\right] = \frac{1}{\beta^2} - \frac{1}{\beta^2}\left(\frac{\alpha}{\alpha+\beta}\right)^{1/2},$$
$$K\left[\operatorname{erf} c\left(\sqrt{t}\right)\right] = \frac{1}{\beta^2}\left(1 - \left(\frac{\alpha}{\alpha+\beta}\right)^{1/2}\right). \tag{83}$$

14. Applications of KKAT to Partial Differential Equations

Suppose the partial differential equation contains unknown variable $v(y,x)$. Then, we will take KKAT of $v(y,x)$ w.r.t. the variable x.

$$K[v(y,x)] = V\left(y, \frac{\beta}{\alpha}\right). \tag{84}$$

For example,

$$\begin{aligned} K\left\{e^{-\lambda x}\sin \pi y\right\} &= \frac{\sin \pi y}{\beta(\beta+\lambda\alpha)},\\ K\{\sin(x+y)\} &= K\{\sin x\cos y + \cos x \sin y\},\\ K\{\sin(x+y)\} &= \frac{\alpha\cos y}{\beta\left(\beta^2+\alpha^2\right)} + \frac{\sin y}{\beta^2+\alpha^2}. \end{aligned} \tag{85}$$

Similarly,

$$\begin{aligned} K\left\{\frac{\partial v}{\partial y}\right\} &= \frac{1}{\alpha\beta}\int_0^\infty e^{-(\beta/\alpha)x}\frac{\partial v}{\partial y}dx = \frac{1}{\alpha\beta}\cdot\frac{\partial}{\partial y}\int_0^\infty e^{-(\beta/\alpha)x}v(y,x)dx\\ &= \frac{\partial}{\partial y}\left\{\frac{1}{\alpha\beta}\int_0^\infty e^{-(\beta/\alpha)x}v(y,x)dx\right\},\\ K\left\{\frac{\partial v}{\partial y}\right\} &= \frac{\partial}{\partial y}V\left(y,\frac{\beta}{\alpha}\right). \end{aligned} \tag{86}$$

And

$$K\left\{\frac{\partial v}{\partial x}\right\} = \frac{\beta}{\alpha}K(v(y,x)) - \frac{v(y,0)}{\alpha\beta}. \tag{87}$$

15. Example

Use KKAT to solve

$$\frac{\partial^2 v}{\partial y^2} = \frac{\partial v}{\partial x}, 0 < y < a, 0 \le x < \infty, \tag{88}$$

$$v(0,x) = 1, v(1,x) = 1, x > 0, \tag{89}$$

$$v(y,0) = 1 + \sin \pi y, < 0y < 1. \tag{90}$$

Solution: the above example is one-dimensional heat problem. It describes the conduction of heat through a rod of unit length. The end points of rod are kept at zero temperature and initial temperature conditions are given.

Taking KKAT both sides on equation (88), we get

$$K\left\{\frac{\partial^2 v}{\partial y^2}\right\} = K\left\{\frac{\partial v}{\partial x}\right\}. \tag{91}$$

Using equation (84) and equation (86), we obtain

$$\begin{aligned} \frac{\partial^2}{\partial y^2}V\left(y,\frac{\beta}{\alpha}\right) &= \frac{\beta}{\alpha}V\left(y,\frac{\beta}{\alpha}\right) - \frac{v(y,0)}{\alpha\beta},\\ \frac{\partial^2}{\partial y^2}V\left(y,\frac{\beta}{\alpha}\right) - \frac{\beta}{\alpha}V\left(y,\frac{\beta}{\alpha}\right) &= -\frac{(1+\sin\pi y)}{\alpha\beta}. \end{aligned} \tag{92}$$

Equation (92) is a nonhomogenous linear second-order differential equation, and its solution is given by

$$V\left(y,\frac{\beta}{\alpha}\right) = V_c + V_p, \tag{93}$$

where

$$\begin{aligned} V_c &= ce^{\sqrt{(\beta/\alpha)}\,x} + de^{-\sqrt{(\beta/\alpha)}\,x},\\ V_p &= \frac{-1}{\left(D^2 - (\beta/\alpha)\right)}\frac{(1+\sin\pi y)}{\alpha\beta},\\ V_p &= \frac{1}{\beta^2} + \frac{\sin \pi y}{(\pi^2\alpha+\beta)\beta}. \end{aligned} \tag{94}$$

Thus, equation (93) becomes

$$V\left(y,\frac{\beta}{\alpha}\right) = ce^{\sqrt{(\beta/\alpha)}\,x} + de^{-\sqrt{(\beta/\alpha)}\,x} + \frac{1}{\beta^2} + \frac{\sin\pi y}{(\pi^2\alpha+\beta)\beta}. \tag{95}$$

Applying KKAT on conditions of equation (89), we get

$$v(0,x) = 1 \Rightarrow V\left(0,\frac{\beta}{\alpha}\right) = \frac{1}{\beta^2}, \tag{96}$$

$$v(1,x) = 1 \Rightarrow V\left(1,\frac{\beta}{\alpha}\right) = \frac{1}{\beta^2}. \tag{97}$$

Using equation (96) and equation (97) in equation (87), we get

$$\begin{aligned} c &= 0,\\ d &= 0. \end{aligned} \tag{98}$$

Thus, equation (95) becomes

$$V\left(y,\frac{\beta}{\alpha}\right) = \frac{1}{\beta^2} + \frac{\sin\pi y}{(\pi^2\alpha+\beta)\beta}. \tag{99}$$

Applying inverse KKAT both sides on equation (99), we get

$$K^{-1}\left\{V\left(y,\frac{\beta}{\alpha}\right)\right\}=K^{-1}\left\{\frac{1}{\beta^2}\right\}+K^{-1}\left\{\frac{\sin\pi y}{(\pi^2\alpha+\beta)\beta}\right\},$$
$$v(y,x)=1+e^{-\pi^2 x}\sin\pi y. \tag{100}$$

16. Conclusion

In this paper, we have introduced the new integral transformation KKAT and reached at the conclusion that this transformation is easier to apply for difficult problems. Furthermore, it is compared with other transformations and found that KKAT is easier to apply and get the better results. This transformation is more effective and convenient to find the solution of a linear system of ordinary differential equations and partial differential equations. In future work, it will be useful for solving the problems where the other transformations fail and get the analytic problems in complex analysis where the Laplace transformation or other transformations fail to get exact results.

Data Availability

Data sharing is not applicable to this article as no datasets were generated or analyzed during current study.

Conflicts of Interest

The authors declare that they have no competing interest.

Authors' Contributions

The author confirms sole responsibility for this contribution. All authors read and approved the final manuscript.

Acknowledgments

The author would like to thank Professor Dr. Muhammad Kalim and Dr. Adnan Khan of NCBA&E for fruitful discussions while performing this research. This research is self-supported.

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