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Research Article

New Subclass of Analytic Function Involving q-Mittag-Leffler Function in Conic Domains

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In this paper, we formulate the q-analogus of differential operator associated with q-Mittag-Leffler function. By using this newly defined operator, we define a new subclass $k-\mathscr{U}\mathscr{S}_{q,\gamma}^{m}(\alpha,\beta)$, of analytic functions in conic domains. We investigate the number of useful properties such that structural formula, coefficient estimates, Fekete–Szego problem and subordination result. We also highlighted some known corollaries of our main results.

1. Introduction Definition

Let $\mathscr{A}$ denote the class of functions $l(z)$ which are analytic in the open unit disk $E=\{z\in\mathbb{C}:|z|<1\}$, satisfying the condition $l(0)=0$ and $l'(0)=1$, and for every $l\in\mathscr{A}$ has the series expansion of the form

$$l(z)=z+\sum_{n=2}^{\infty}a_n z^n. \quad (1)$$

Let $\mathscr{S}\subset\mathscr{A}$ be the class of all functions which are univalent in E (see [1]). Also, $\mathscr{P}$ denotes the well-known Carathéodory class of functions p which are analytic in open unit disk E and has the series expansion of the form

$$p(z)=1+\sum_{n=1}^{\infty}c_n z^n, \quad (2)$$

and satisfying the condition

$$p(0)=1 \text{ and } Rep(z)>0. \quad (3)$$

For the function l given by (1) and the function g defined by $g(z)=z+\sum_{n=2}^{\infty}b_n z^n$, the Hadamard product (convolution) $l*g$ of the functions l and g stated by

$$(l*g)z=z+\sum_{n=2}^{\infty}a_n b_n z^n. \quad (4)$$

For the analytic functions l,g, l is said to be subordinate to g (indicated as $l\prec g$), if there exists a Schwarz function

$$w(z)=\sum_{n=1}^{\infty}c_n z^n, \quad (5)$$

with

$$w(0)=0 \text{ and } |w(z)|<1, \quad (6)$$

such that

$$l(z)=g(w(z)). \quad (7)$$

Furthermore, if g is univalent in E, (see [2]); then, we have

$$l(z)\prec g(z) \text{ if and only if } l(0)=g(0) \text{ and } l(E)\subset g(E), z\in E. \quad (8)$$

The class of starlike functions of order $\alpha(\mathscr{S}^*(\alpha))$ in E and the class of convex functions of order $\alpha(\mathscr{K}(\alpha))$, $0\le\alpha<1$, were defined as follows:

$$\mathcal{S}^*(\alpha)=\left\{l:l\in\mathcal{S}\text{ and }\operatorname{Re}\left(\frac{zl'(z)}{l(z)}\right)>\alpha,(0\le\alpha<1,z\in E)\right\},$$
$$\mathcal{K}(\alpha)=\left\{l:l\in\mathcal{S}\text{ and }\operatorname{Re}\left(\frac{z\left(zl'(z)\right)'}{l'(z)}\right)>\alpha,(0\le\alpha<1,z\in E)\right\}. \tag{9}$$

It should be noted that

$$\mathcal{S}^*(0)=\mathcal{S}^*\text{ and }\mathcal{K}(0)=\mathcal{K}, \tag{10}$$

where $\mathcal{S}^*$ and $\mathcal{K}$ are the well-known function classes of starlike and convex functions, respectively.

In the year of 1991, Goodman [3] introduced the class $\mathcal{UCV}$ of uniformly convex functions which was extensively studied by Ronning [4], and its characterization was given by Ma and Minda [5]. After that, Kanas and Wisniowska [6] defined the class k-uniformly convex functions (k-$\mathcal{UCV}$) and a related class $k-\mathcal{ST}$ was defined by

$$l\in k-\mathcal{UCV}\Longleftrightarrow zl'\in k-\mathcal{ST}$$
$$\Longleftrightarrow l\in\mathcal{A}\text{ and }\operatorname{Re}\left\{\frac{\left(zl'(z)\right)'}{l'(z)}\right\}>\left|\frac{zl''(z)}{l'(z)}\right|,(k\ge0). \tag{11}$$

From different viewpoints, the various subclasses of the normalized analytic function of class $\mathcal{A}$ have been studied in the field of Geometric Function Theory. To investigate various subclasses of $\mathcal{A}$, many authors have been used the q-calculus as well as the fractional q-calculus. In 1910, Jackson [7] was among the one of few researchers who studied q-calculus operator theory on q-definite integrals and also Trjitzinsky in [8] studied about analytic theory of linear q-difference equations. Curmicheal [9] studied general theory of linear q-difference equations and the first use of q-calculus operator theory in Geometric Function Theory in a book chapter by Srivastava (see, for details, [10]). Recently, Hussain et al. discussed the some applications of q-calculus operator theory in [11], while in [12, 13], Ibrahim et al. used the notion of quantum calculus and the Hadamard product to improve an extended Sàlàgean q-differential operator and defined some new subclasses of analytic functions in open unit disk E. Govindaraj and Sivasubramanian [14] as well as Ibrahim et al. [15, 16] employed the quantum calculus and the Hadamard product to defined some new subclasses of analytic functions involving the Sàlàgean q-differential operator and the generalized symmetric Sàlàgean q-differential operator, respectively. Furthermore, Srivastava et al. [17] defined q-Noor integral operator by using q-calculus operator theory and investigated some subclasses of biunivalent functions in open unit disk.

Here, we give some basic definitions and details of the q-calculus and suppose that $0<q<1$.

For any nonnegative integer n, the q-integer number $[n]_q$ is defined by

$$[n]_q=\frac{1-q^n}{1-q},[0]_q=0, \tag{12}$$

and for any nonnegative integer n, the q-number shift factorial is defined by

$$[n]_q!=[1]_q[2]_q[3]_q\cdots[n]_q,\left([0]_q!=1\right). \tag{13}$$

We note that when $q\longrightarrow1-$, then $[n]!=n$.

The q-difference operator was introduced by Jackson (see in [7]). For $l\in\mathcal{A}$, the q-derivative operator or q-difference operator is defined as

$$\partial_q l(z)=\frac{l(qz)-l(z)}{z(q-1)},z\in E,z\ne0,q\ne1. \tag{14}$$

It is readily deduced from (1) and (14) that

$$\partial_q z^n=[n]_q z^{n-1},\partial_q l(z)=1+\sum_{n=2}^{\infty}[n]_q a_n z^{n-1}. \tag{15}$$

We can observe that

$$\lim_{q\longrightarrow1-}\partial_q l(z)=l'(z). \tag{16}$$

The familiar Mittag-Leffler function $\mathcal{H}_\alpha(z)$ introduced by Mittag-Leffler [18] and its generalization $\mathcal{H}_{\alpha,\beta}(z)$ introduced by Wiman [19] which are defined by

$$\mathcal{H}_\alpha(z)=\sum_{n=0}^{\infty}\frac{1}{\Gamma(\alpha n+1)}z^n,(\alpha\in\mathbb{C},\operatorname{Re}(\alpha))>0,$$
$$\mathcal{H}_{\alpha,\beta}(z)=\sum_{n=0}^{\infty}\frac{1}{\Gamma(\alpha n+\beta)}z^n,(\alpha,\beta\in\mathbb{C},\operatorname{Re}(\alpha),\operatorname{Re}(\beta))>0. \tag{17}$$

Recently, Attiya [20] investigated some applications of Mittag-Leffler functions and generalized k-Mittag-Leffler studied by Rehman et al. in [21]. Moreover, Srivastava et al. [22, 23] introduced the generalization of Mittag-Leffler functions.

The q-Mittag-Leffler function was defined by (see [24]):

$$\mathcal{H}_{\alpha,\beta}(z,q)=\sum_{n=0}^{\infty}\frac{1}{\Gamma_q(\alpha n+\beta)}z^n,(\alpha,\beta\in\mathbb{C},\operatorname{Re}(\alpha),\operatorname{Re}(\beta))>0. \tag{18}$$

The q-Mittag-Leffler function has also been investigated in [25, 26]. Since the q-Mittag-Leffler function $\mathcal{H}_{\alpha,\beta}(z,q)$ defined by (18) does not belong to the normalized analytic function class $\mathcal{A}$. Hence, we define the normalization of q-Mittag-Leffler function as

$$\mathscr{M}_{\alpha,\beta}(z,q)=z\Gamma_q(\beta)\mathscr{H}_{\alpha,\beta}(z)=z+\sum_{n=2}^{\infty}\frac{\Gamma_q(\beta)}{\Gamma_q(\alpha(n-1)+\beta)}z^n, \tag{19}$$

where $z\in E, Re\alpha>0$, $\beta\in\mathbb{C}\setminus\{0,-1,-2,-3\cdots\}$). Corresponding to $\mathscr{M}_{\alpha,\beta}(z,q)$ and for $l\in\mathscr{A}$, we define the following q-analogous of differential operator $\mathscr{D}_q^m(\alpha,\beta)$: $\mathscr{A}\longrightarrow\mathscr{A}$ by

$$\begin{aligned}\mathscr{D}_q^0(\alpha,\beta)l(z)&=l(z)*\mathscr{M}_{\alpha,\beta}(z,q),\\ \mathscr{D}_q^1(\alpha,\beta)l(z)&=z\partial_q\left(l(z)*\mathscr{M}_{\alpha,\beta}(z,q)\right),\\ \mathscr{D}_q^2(\alpha,\beta)l(z)&=\mathscr{D}\left(\mathscr{D}^1(\alpha,\beta)l(z)\right),\\ \mathscr{D}_q^m(\alpha,\beta)l(z)&=\mathscr{D}\left(\mathscr{D}^{m-1}(\alpha,\beta)l(z)\right).\end{aligned} \tag{20}$$

We note that

$$\mathscr{D}_q^m(\alpha,\beta)l(z)=z+\sum_{n=2}^{\infty}[n]_q^m\mathscr{T}_n(\alpha,q)a_nz^n, \tag{21}$$

where

$$\mathscr{T}_n(\alpha,q)=\frac{\Gamma_q(\beta)}{\Gamma_q(\alpha(n-1)+\beta)}. \tag{22}$$

Note that

(i) For $\alpha=0$ and $\beta=1$, we get Salagean q-differential operator [14]

(ii) For $q\longrightarrow 1-$, $\alpha=0$, and $\beta=1$, we get Salagean differential operator [27]

(iii) For $m=0$, we get $E_{\alpha,\beta}(z,q)$ (see [24])

(iv) For $m=0$, we get $E_{\alpha,\beta}(z)$ (see [22])

Definition 1. Let $l(z)\in\mathscr{A}$, then $l(z)$ is in the class $k-\mathscr{US}_{q,\gamma}^m(\alpha,\beta)$, $\gamma\in\mathbb{C}\setminus\{0\}$, if it satisfies the condition

$$\begin{aligned}&\operatorname{Re}\left\{1+\frac{1}{\gamma}\left(\frac{z\partial_q\mathscr{D}_q^m(\alpha,\beta)l(z)}{\mathscr{D}_q^m(\alpha,\beta)l(z)}-1\right)\right\}\\ &\quad>k\left|\frac{1}{\gamma}\left(\frac{z\partial_q\mathscr{D}_q^m(\alpha,\beta)l(z)}{\mathscr{D}_q^m(\alpha,\beta)l(z)}-1\right)\right|, z\in E.\end{aligned} \tag{23}$$

Remark 2.

(i) For $\alpha=0$ and $\beta=1$, the class $k-\mathscr{US}_{q,\gamma}^m(\alpha,\beta)=k-\mathscr{US}(q,\gamma,m)$ studied in [11]

(ii) For $m=0,\alpha=0,\beta=1,q\longrightarrow 1-$, and $\gamma=1/(1-\eta)$, $\eta\in\mathbb{C}\setminus\{1\}$, the class $k-\mathscr{US}_{q,\gamma}^m(\alpha,\beta)=\mathscr{SD}(k,\eta)$ studied in [28]

(iii) For $m=0,\alpha=0,\beta=1,q\longrightarrow 1-$, and $\gamma=2/(1-\eta)$, $\eta\in\mathbb{C}\setminus\{1\}$, the class $k-\mathscr{US}_{q,\gamma}^m(\alpha,\beta)=\mathscr{KD}(k,\eta)$, studied in [29]

(iv) For $k=1,m=0,\alpha=0,\beta=1,q\longrightarrow 1-$, and $\gamma=(1-\eta)$, $\eta\in\mathbb{C}\setminus\{1\}$, the class $k-\mathscr{US}_{q,\gamma}^m(\alpha,\beta)=\mathscr{S}_p(\eta)$ studied in [30]

(v) For $k=1,m=0,\alpha=0,\beta=1,q\longrightarrow 1-$, and $\gamma=2/(1-\eta)$, $\eta\in\mathbb{C}\setminus\{1\}$, $k-\mathscr{US}_{q,\gamma}^m(\alpha,\beta)=\mathscr{K}_p(\eta)$, studied in [30]

2. Geometric Interpretation

A function $l(z)\in\mathscr{A}$, belongs to $k-\mathscr{US}_{q,\gamma}^m(\alpha,\beta)$ if and only if $z\partial_q\mathscr{D}_q^m(\alpha,\beta)l(z)/\mathscr{D}_q^m(\alpha,\beta)l(z)$ takes all the values in the conic domain $\Omega_{k,\gamma}=p_{k,\gamma}(E)$, such that

$$\Omega_{k,\gamma}=\gamma\Omega_k+(1-\gamma), 0\le\gamma<1, k\ge 0, \tag{24}$$

where

$$\Omega_k=u+iv: u^2>k^2\left((u-1)^2+v^2\right). \tag{25}$$

The domain $\Omega_{k,\gamma}$ is not always well defined because in general $(1,0)\notin\Omega_{k,\gamma}$ (for example, in particular $(1,0)\notin\Omega_{2,0.5}$). We see that in [31], the conic domain $\Omega_k(0,b)$ concides with $\Omega_{k,b}$ only when b is chosen according to

(i) For $k=0$, we take $b=0$

(ii) For $k\in(0,1/\sqrt{2})$, we take $b\in[1/2k^2-1,1)$

(iii) For $k\in[1/\sqrt{2}1/\sqrt{2},1]$, we take $b\in(-\infty,1)$

(iv) For $k\in(1,\infty)$, we take $b\in(-\infty,1/2k^2-1]$

This means that for $\Omega_{k,\gamma}$ to contain the point $(1,0)$,γ must be chosen according as follows:

$$\gamma\in\begin{cases}(0,1) & \text{if } 0\le k\le 1,\\ \left[0,1-\dfrac{\sqrt{k^2-1}}{k}\right] & \text{if } k\ge 0.\end{cases} \tag{26}$$

Since $p_{k,\gamma}(z)$ is convex univalent, the above definition can be written as

$$\frac{z\partial_q\mathscr{D}_q^m(\alpha,\beta)l(z)}{\mathscr{D}_q^m(\alpha,\beta)l(z)}\prec p_{k,\gamma}(z), \tag{27}$$

where

$$p_{k,\gamma}(z)=\begin{cases}\dfrac{1+z}{1-z}, & \text{for } k=0,\\ U_1(\gamma,k), & \text{for } k=1,\\ U_2(\gamma,k), & \text{for } 0<k<1,\\ U_3(\gamma,k), & \text{for } k>1,\end{cases} \tag{28}$$

$$U_1(\gamma,k)=1+\frac{2\gamma}{\pi^2}\left(\log\frac{1+\sqrt{z}}{1-\sqrt{z}}\right)^{2'}, \tag{29}$$

$$U_2(\gamma,k)=1+\frac{2\gamma}{1-k^2}\sinh^2\left\{\left(\frac{2}{\pi}\arccos k\right)\arctan h\sqrt{z}\right\}, \tag{30}$$

$$U_3(\gamma,k)=1+\frac{\gamma}{k^2-1}\sin\left(\frac{\pi}{2R(t)}\int_0^{\frac{u(z)}{\sqrt{t}}}\frac{1}{\sqrt{1-x^2}\sqrt{1-(tx)^2}}dx\right)+\frac{\gamma}{1-k^2}. \tag{31}$$

For more detail (see [32, 33]).

3. Set of Lemmas

Lemma 3. (see [34]). *Let $p(z)=\sum_{n=1}^{\infty}p_n z^n \prec F(z)=\sum_{n=1}^{\infty}d_n z^n$ in E. If $F(z)$ is convex univalent in E, then*

$$|p_n|\le|d_1|, n\ge 1. \tag{32}$$

Lemma 4. (see [35]). *Let $k\in[0,\infty)$ be fixed and let $p_{k,\gamma}(z)$ of the form (28). If*

$$p_{k,\gamma}(z)=1+Q_1 z+Q_2 z^2+\cdots, \tag{33}$$

where

$$Q_1=\begin{cases}\dfrac{2\gamma A^2}{1-k^2}, & 0\le k<1,\\ \dfrac{8\gamma}{\pi^2}, & k=1,\\ \dfrac{\pi^2\gamma}{4(1+t)\sqrt{t}K^2(t)\left(k^2-1\right)}, & k>1,\end{cases} \tag{34}$$

$$Q_2=\begin{cases}\dfrac{A^2+2}{3}Q_1, & 0\le k<1,\\ \dfrac{2}{3}Q_1, & k=1,\\ \dfrac{4K^2(t)\left(t^2+6t+1\right)-\pi^2}{24K^2(t)(1+t)\sqrt{t}}Q_1, & k>1.\end{cases} \tag{35}$$

Lemma 5. (see [36]). *Let $p(z)=1+\sum_{n=1}^{\infty}c_n z^n\in\mathcal{P}$ and let $p(z)$ be analytic in E and satisfy $\operatorname{Re}(p(z))>0$ for z in E, then*

$$\left|c_2-\mu c_1^2\right|\le 2\max\{1,|2\mu-1|\},\forall\mu\in\mathbb{C}. \tag{36}$$

4. Main Results

Theorem 6. *Let $l(z)\in k-\mathcal{US}_{q,\gamma}^{m}(\alpha,\beta)$. Then,*

$$\mathcal{D}_q^m(\alpha,\beta)l(z))\prec z\exp\int_0^z\frac{p_{k,\gamma}(w(\xi))-1}{\zeta}d\xi, \tag{37}$$

where $w(z)$ is a Schwarz function given in (5). Moreover, for $|z|=\rho$, we have

$$\exp\left(\int_0^1\frac{p_{k,\gamma}(-\rho)-1}{\rho}d\rho\right)\le\left|\frac{\mathcal{D}_q^m(\alpha,\beta)l(z)}{z}\right|\le\exp\left(\int_0^1\frac{p_{k,\gamma}(\rho)-1}{\rho}d\rho\right), \tag{38}$$

where $p_{k,\gamma}(z)$ is given by (28).

Proof. If $l(z)\in k-\mathcal{US}_{q,\gamma}^{m}(\alpha,\beta)$, then by using (27), we obtain

$$\frac{\partial_q\mathcal{D}_q^m(\alpha,\beta)l(z)}{\mathcal{D}_q^m(\alpha,\beta)l(z)}-\frac{1}{z}=\frac{p_{k,\gamma}(w(z))-1}{z}. \tag{39}$$

Integrating (39) and after some simplification, we have

$$\mathcal{D}_q^m(\alpha,\beta)l(z)\prec z\exp\int_0^z\frac{p_{k,\gamma}(w(\xi))-1}{\zeta}d\xi. \tag{40}$$

This proves (37). We know that

$$p_{k,\gamma}(-\rho|z|)\le\operatorname{Re}\left\{p_{k,\gamma}(w(\rho z)\right\}\le p_{k,\gamma}(\rho|z|)\ (0<\rho\le 1, z\in E). \tag{41}$$

Using (40) and (41), we have

$$\int_0^1\frac{p_{k,\gamma}(-\rho|z|)-1}{\rho}d\rho\le\operatorname{Re}\int_0^1\frac{p_{k,\gamma}(w(\rho(z))-1}{\rho}d\rho\le\int_0^1\frac{p_{k,\gamma}(\rho|z|)-1}{\rho}d\rho, \tag{42}$$

for $z\in E$. From (40), we have

$$\begin{aligned}&\frac{\mathcal{D}_q^m(\alpha,\beta)l(z)}{z}\prec\exp\int_0^z\frac{p_{k,\gamma}(w(\xi))-1}{\zeta}d\xi,\\ &\int_0^1\frac{p_{k,\gamma}(-\rho|z|)-1}{\rho}d\rho\le\log\left|\frac{\mathcal{D}_q^m(\alpha,\beta)l(z)}{z}\right|\le\int_0^1\frac{p_{k,\gamma}(\rho|z|)-1}{\rho}d\rho,\\ &\exp\int_0^1\frac{p_{k,\gamma}(-\rho|z|)-1}{\rho}d\rho\le\exp\left(\log\left|\frac{\mathcal{D}_q^m(\alpha,\beta)l(z)}{z}\right|\right)\le\exp\int_0^1\frac{p_{k,\gamma}(\rho|z|)-1}{\rho}d\rho,\end{aligned} \tag{43}$$

which implies that

$$\exp \int_0^1 \frac{p_{k,\gamma}(-\rho)-1}{\rho} d\rho \le \left| \frac{\mathcal{D}_q^m(\alpha,\beta)l(z)}{z} \right| \le \exp \int_0^1 \frac{p_{k,\gamma}(\rho)-1}{\rho} d\rho. \quad (44)$$

□

Corollary 7. (see [11]). *Let $l(z) \in k-\mathcal{US}_{q,\gamma}^m(0,1)$. Then,*

$$\mathcal{D}_q^m l(z)\Big) \prec z \exp \int_0^z \frac{p_{k,\gamma}(w(\xi))-1}{\zeta} d\xi, \quad (45)$$

where $w(z)$ is a Schwarz function given in (5). Moreover, for $|z|=\rho$, we have

$$\exp\left(\int_0^1 \frac{p_{k,\gamma}(-\rho)-1}{\rho} d\rho \right) \le \left| \frac{\mathcal{D}_q^m l(z)}{z} \right| \le \exp\left(\int_0^1 \frac{p_{k,\gamma}(\rho)-1}{\rho} d\rho \right). \quad (46)$$

Theorem 8. *If $l(z) \in k-\mathcal{US}_{q,\gamma}^m(\alpha,\beta)$. Then,*

$$|a_2| \le \frac{\delta}{[2]_q^m \left\{ [2]_q - 1 \right\} \mathcal{T}_2(\alpha,q)}, \quad (47)$$

$$|a_n| \le \frac{\delta}{[n]_q^m \left\{ [n]_q - 1 \right\} \mathcal{T}_n(\alpha,q)} \prod_{j=1}^{n-2} \left(1 + \frac{\delta}{[j+1]_q - 1} \right), \quad for n \ge 3, \quad (48)$$

where $\delta = |Q_1|$ with Q_1 and $\mathcal{T}_n(\alpha,q)$ are given by (34) and (22).

Proof. Let

$$\frac{z\partial_q \mathcal{D}_q^m(\alpha,\beta)l(z)}{\mathcal{D}_q^m(\alpha,\beta)l(z)} = p(z), \quad (49)$$

where $p(z)$ is the analytic in E and $p(0)=1$. Let $p(z) = 1 + \sum_{n=1}^{\infty} c_n z^n$ and $\mathcal{D}_q^m(\alpha,\beta)l(z)$ is given by (21). Then, (49) implies that

$$\begin{aligned} z + \sum_{n=2}^{\infty} [n]_q^{m+1} \mathcal{T}_n(\alpha,q) a_n z^n &= \left(\sum_{n=0}^{\infty} c_n z^n \right) \left(z + \sum_{n=2}^{\infty} [n]_q^m \mathcal{T}_n(\alpha,q) a_n z^n \right), \\ &= \sum_{n=0}^{\infty} c_n z^{n+1} + \left(\sum_{n=0}^{\infty} c_n z^n \right) \left(\sum_{n=2}^{\infty} [n]_q^m \mathcal{T}_n(\alpha,q) a_n z^n \right). \end{aligned} \quad (50)$$

Now comparing the coefficients of z^n, we obtain

$$\begin{aligned} [n]_q^{m+1} \mathcal{T}_n(\alpha,q) a_n &= [n]_q^m \mathcal{T}_n(\alpha,q) a_n + \sum_{j=1}^{n-1} [j]_q^m \frac{\Gamma(\beta)}{\Gamma_q(\alpha(j-1)+\beta)} a_j c_{n-j}, \\ [n]_q^{m+1} \mathcal{T}_n(\alpha,q) a_n - [n]_q^m \mathcal{T}_n(\alpha,q) a_n &= \sum_{j=1}^{n-1} [j]_q^m \frac{\Gamma(\beta)}{\Gamma_q(\alpha(j-1)+\beta)} a_j c_{n-j}, \\ [n]_q^m \left\{ [n]_q - 1 \right\} \mathcal{T}_n(\alpha,q) a_n &= \sum_{j=1}^{n-1} [j]_q^m \frac{\Gamma(\beta)}{\Gamma_q(\alpha(j-1)+\beta)} a_j c_{n-j}, \end{aligned} \quad (51)$$

which implies

$$a_n = \frac{1}{[n]_q^m \left\{ [n]_q - 1 \right\} \mathcal{T}_n(\alpha,q)} \sum_{j=1}^{n-1} [j]_q^m \frac{\Gamma(\beta)}{\Gamma_q(\alpha(j-1)+\beta)} a_j c_{n-j}. \quad (52)$$

Using the results that $|c_n| \le |Q_1|$ given in ([33]), we have

$$|a_n| \le \frac{|Q_1|}{[n]_q^m \left\{ [n]_q - 1 \right\} \mathcal{T}_n(\alpha,q)} \sum_{j=1}^{n-1} [j]_q^m \frac{\Gamma(\beta)}{\Gamma_q(\alpha(j-1)+\beta)} |a_j|. \quad (53)$$

Let us take $\delta = |Q_1|$. Then, we have

$$|a_n| \le \frac{\delta}{[n]_q^m \left\{ [n]_q - 1 \right\} \mathcal{T}_n(\alpha,q)} \sum_{j=1}^{n-1} [j]_q^m \frac{\Gamma(\beta)}{\Gamma_q(\alpha(j-1)+\beta)} |a_j|. \quad (54)$$

For $n=2$ in (54), we have

$$\begin{aligned} |a_2| &\le \frac{\delta}{[2]_q^m \left\{ [2]_q - 1 \right\} \mathcal{T}_2(\alpha,q)} \sum_{j=1}^{1} [j]_q^m \frac{\Gamma(\beta)}{\Gamma_q(\alpha(j-1)+\beta)} |a_j| \\ &= \frac{\delta}{[2]_q^m \left\{ [2]_q - 1 \right\} \mathcal{T}_2(\alpha,q)}, \end{aligned} \quad (55)$$

Hence, for $n=2$ the inequality (48) holds. To prove (48), we use mathematical induction, for $n=3$

$$|a_3| \le \frac{\delta}{[3]_q^m \left\{ [3]_q - 1 \right\} \mathcal{T}_3(\alpha,q)} \left\{ 1 + [2]_q^m \mathcal{T}_2(\alpha,q) |a_2| \right\}. \quad (56)$$

By using (55), we have

$$|a_3| \le \frac{\delta}{[3]_q^m \left\{ [3]_q - 1 \right\} \mathcal{T}_3(\alpha,q)} \left\{ 1 + [2]_q^m \mathcal{T}_2(\alpha,q) \left(\frac{\delta}{[2]_q^m \left\{ [2]_q - 1 \right\} \mathcal{T}_2(\alpha,q)} \right) \right\}. \quad (57)$$

Therefore,

$$|a_3| \le \frac{\delta}{[3]_q^m \left\{[3]_q - 1\right\} \mathcal{T}_3(\alpha, q)} \left\{1 + \frac{\delta}{[2]_q - 1}\right\}. \quad (58)$$

Hence, (48) holds for $n = 3$. Now, we suppose that (48) is true for $n = t + 1$, that is

$$|a_t| \le \frac{\delta}{[t]_q^m \left\{[t]_q - 1\right\} \mathcal{T}_t(\alpha, q)} \prod_{j=1}^{t-2} \left(1 + \frac{\delta}{[j+1]_q - 1}\right), \quad n \ge 3. \quad (59)$$

Consider

$$\begin{aligned} |a_{t+1}| \le & \frac{\delta}{[t+1]_q^m \left\{[t+1]_q - 1\right\} \mathcal{T}_{t+1}(\alpha, q)} \\ & \times \left\{1 + [2]_q^m \mathcal{T}_2(\alpha, q)|a_2| + [3]_q^m \mathcal{T}_3(\alpha, q)|a_3| \right. \\ & \left. + [4]_q^m \mathcal{T}_4(\alpha, q)|a_4| + \cdots + [t]_q^m \mathcal{T}_t(\alpha, q)|a_t|\right\} \\ \le & \frac{\delta}{[t+1]_q^m \left\{[t+1]_q - 1\right\} \mathcal{T}_{t+1}(\alpha, q)} \\ & \cdot \left\{1 + \frac{\delta}{[2]_q - 1} + \frac{\delta}{[3]_q - 1}\left(1 + \frac{\delta}{[2]_q - 1}\right) \right. \\ & \left. + \cdots + \frac{\delta}{[t]_q - 1} \prod_{j=1}^{t-2}\left(1 + \frac{\delta}{[j+1]_q - 1}\right)\right\} \\ = & \frac{\delta}{[t+1]_q^m \left\{[t+1]_q - 1\right\} \mathcal{T}_{t+1}(\alpha, q)} \prod_{j=1}^{t-1}\left(1 + \frac{\delta}{[j+1]_q - 1}\right). \end{aligned} \quad (60)$$

Hence, (48) holds for $n = t + 1$. Hence, proof is complete. □

Corollary 9. (see [11]). *f* $l(z) \in k - \mathcal{U}\mathcal{S}_{q,\gamma}^m(0, 1)$. *Then,*

$$|a_2| \le \frac{\delta}{[2]_q^m \left\{[2]_q - 1\right\}}$$

$$|a_n| \le \frac{\delta}{[n]_q^m \left\{[n]_q - 1\right\}} \prod_{j=1}^{n-2}\left(1 + \frac{\delta}{[j+1]_q - 1}\right), \quad \text{for } n \ge 3. \quad (61)$$

Theorem 10. *Let* $0 \le k < \infty$ *be fixed and let* $l(z) \in k - \mathcal{U}\mathcal{S}_{q,\gamma}^m(\alpha, \beta)$ *with the form (1), then, for* $\mu \in \mathbb{C}$

$$\left|a_3 - \mu a_2^2\right| \le \frac{Q_1}{2[3]_q^m \mathcal{T}_3(\alpha, q)\left\{[3]_q - 1\right\}} \max\left[1, |2\nu - 1|\right], \quad (62)$$

where Q_1 *and* Q_2 *are given by (34) and (35).*

Proof. Let $l(z) \in k - \mathcal{U}\mathcal{S}_{q,\gamma}^m(\alpha, \beta)$, then there exists a Schwarz function $w(z)$ given by (5), such that

$$\frac{z\partial_q \mathcal{D}_q^m(\alpha, \beta) l(z)}{\mathcal{D}_q^m(\alpha, \beta) l(z)} \prec p_{k,\gamma}(z), z \in E \quad (63)$$

$$\frac{z\partial_q \mathcal{D}_q^m(\alpha, \beta) l(z)}{\mathcal{D}_q^m(\alpha, \beta) l(z)} = p_{k,\gamma}(w(z)), z \in E. \quad (64)$$

Let $p(z) \in \mathcal{P}$ be defined as

$$p(z) = 1 + c_1 z + c_2 z^2 + \cdots. \quad (65)$$

This gives

$$w(z) = \frac{c_1}{2} z + \frac{1}{2}\left(c_2 - \frac{c_1^2}{2}\right) z^2 + \cdots \quad (66)$$

$$p_{k,\gamma}(w(z)) = 1 + \frac{Q_1 c_1}{2} z + \left\{\frac{Q_2 c_1^2}{4} + \frac{1}{2}\left(c_2 - \frac{c_1^2}{2}\right) Q_1\right\} z^2 + \cdots. \quad (67)$$

$$\begin{aligned} \frac{z\partial_q \mathcal{D}_q^m(\alpha, \beta) l(z)}{\mathcal{D}_q^m(\alpha, \beta) l(z)} = & 1 + [2]_q^m \mathcal{T}_2(\alpha, q)\left\{[2]_q - 1\right\} a_2 z \\ & + \left\{[3]_q^m \mathcal{T}_3(\alpha, q)\left\{[3]_q - 1\right\} a_3 \right. \\ & \left. - \left([2]_q^m \mathcal{T}_2(\alpha, q)\right)^2 \left\{[2]_q - 1\right\} a_2^2\right\} z^2. \end{aligned} \quad (68)$$

Using (67) in (64) and comparing with (68), we obtain

$$a_2 = \frac{Q_1 c_1}{2[2]_q^m \mathcal{T}_2(\alpha, q)\left\{[2]_q - 1\right\}},$$

$$a_3 = \frac{1}{[3]_q^m \mathcal{T}_3(\alpha, q)\left\{[3]_q - 1\right\}}\left\{\frac{Q_1 c_2}{2} + \frac{c_1^2}{4}\left(Q_2 - Q_1 + \frac{Q_1^2}{\left\{[2]_q - 1\right\}}\right)\right\}.$$

$$\begin{aligned} a_3 - \mu a_2^2 = & \frac{1}{[3]_q^m \mathcal{T}_3(\alpha, q)\left\{[3]_q - 1\right\}}\left\{\frac{Q_1 c_2}{2} + \frac{c_1^2}{4}\left(Q_2 - Q_1 + \frac{Q_1^2}{\left\{[2]_q - 1\right\}}\right)\right\} \\ & - \mu\left(\frac{Q_1 c_1}{2[2]_q^m \mathcal{T}_2(\alpha, q)\left\{[2]_q - 1\right\}}\right)^2. \end{aligned} \quad (69)$$

For any complex number μ and after some calculation we have

$$a_3-\mu a_2^2=\frac{Q_1}{2[3]_q^m\mathcal{T}_3(\alpha,q)\left\{[3]_q-1\right\}}\left\{c_2-vc_1^2\right\}, \quad (70)$$

where

$$v=\frac{1}{2}\left\{1-\frac{Q_2}{Q_1}-Q_1\left(\frac{1}{\left\{[2]_q-1\right\}}-\mu\frac{[3]_q^m\left\{[3]_q-1\right\}}{2\mathcal{T}_2(\alpha,q)\left([2]_q^m\left\{[2]_q-1\right\}\right)^2}\right)\right\}. \quad (71)$$

Using a lemma (36) on (70), we have the required result. □

Corollary 11. (see [11]). *Let $0\leq k<\infty$ be fixed and let $l(z)\in k-\mathcal{US}_{q,\gamma}^m(0,1)$ with the form (1.1), then, for $\mu\in\mathbb{C}$*

$$\left|a_3-\mu a_2^2\right|\leq\frac{Q_1}{2[3]_q^m\left\{[3]_q-1\right\}}\max\left[1,|2v-1|\right], \quad (72)$$

where

$$v=\frac{1}{2}\left\{1-\frac{Q_2}{Q_1}-Q_1\left(\frac{1}{\left\{[2]_q-1\right\}}-\mu\frac{[3]_q^m\left\{[3]_q-1\right\}}{2\left([2]_q^m\left\{[2]_q-1\right\}\right)^2}\right)\right\}. \quad (73)$$

Theorem 12. *Let $l(z)\in\mathcal{A}$ of the form (1) and satisfy the condition*

$$\sum_{n=2}^{\infty}\left\{\left\{[n]_q-1\right\}(k+1)+|\gamma|\right\}|\mathcal{T}_n(\alpha,q)|\left|[n]_q^m\right||a_n|\leq|\gamma|, \quad (74)$$

then, $l(z)\in k-\mathcal{US}_{q,\gamma}^m(\alpha,\beta)$.

Proof. Let we note that

$$\left|\frac{z\partial_q\mathcal{D}_q^m(\alpha,\beta)l(z)}{\mathcal{D}_q^m(\alpha,\beta)l(z)}-1\right|=\left|\frac{z\partial_q\mathcal{D}_q^m(\alpha,\beta)l(z)-\mathcal{D}_q^m(\alpha,\beta)l(z)}{\mathcal{D}_q^m(\alpha,\beta)l(z)}\right|$$
$$=\left|\frac{\sum_{n=2}^{\infty}[n]_q^m\mathcal{T}_n(\alpha,q)\left\{[n]_q-1\right\}a_nz^n}{z+\sum_{n=2}^{\infty}[n]_q^m\mathcal{T}_n(\alpha,q)a_nz^n}\right|$$
$$\leq\frac{\sum_{n=2}^{\infty}\left|[n]_q^m\mathcal{T}_n(\alpha,q)\left\{[n]_q-1\right\}\right||a_n|}{1-\sum_{n=2}^{\infty}\left|[n]_q^m\right||\mathcal{T}_n(\alpha,q)||a_n|}. \quad (75)$$

From (74), we get

$$1-\sum_{n=2}^{\infty}\left|[n]_q^m\right||\mathcal{T}_n(\alpha,q)||a_n|>0. \quad (76)$$

To show that $l(z)\in k-\mathcal{US}_{q,\gamma}^m(\alpha,\beta)$, it suffices that

$$\left|\frac{k}{\gamma}\left(\frac{z\partial_q\mathcal{D}_q^m(\alpha,\beta)l(z)}{\mathcal{D}_q^m(\alpha,\beta)l(z)}-1\right)\right|-\mathrm{Re}\left\{\frac{1}{\gamma}\left(\frac{z\partial_q\mathcal{D}_q^m(\alpha,\beta)l(z)}{\mathcal{D}_q^m(\alpha,\beta)l(z)}-1\right)\right\}\leq1. \quad (77)$$

From (Proof), we have

$$\left|\frac{k}{\gamma}\left(\frac{z\partial_q\mathcal{D}_q^m(\alpha,\beta)l(z)}{\mathcal{D}_q^m(\alpha,\beta)l(z)}-1\right)\right|-\mathrm{Re}\left\{\frac{1}{\gamma}\left(\frac{z\partial_q\mathcal{D}_q^m(\alpha,\beta)l(z)}{\mathcal{D}_q^m(\alpha,\beta)l(z)}-1\right)\right\}$$
$$\leq\frac{k}{|\gamma|}\left|\frac{z\partial_q\mathcal{D}_q^m(\alpha,\beta)l(z)}{\mathcal{D}_q^m(\alpha,\beta)l(z)}-1\right|+\frac{1}{|\gamma|}\left|\frac{z\partial_q\mathcal{D}_q^m(\alpha,\beta)l(z)}{\mathcal{D}_q^m(\alpha,\beta)l(z)}-1\right|$$
$$\leq\frac{(k+1)}{|\gamma|}\left|\frac{z\partial_q\mathcal{D}_q^m(\alpha,\beta)l(z)}{\mathcal{D}_q^m(\alpha,\beta)l(z)}-1\right|$$
$$=\frac{(k+1)}{|\gamma|}\left|\frac{z\partial_q\mathcal{D}_q^m(\alpha,\beta)l(z)-\mathcal{D}_q^m(\alpha,\beta)l(z)}{\mathcal{D}_q^m(\alpha,\beta)l(z)}\right|$$
$$\leq\frac{(k+1)}{|\gamma|}\left(\frac{\left|\sum_{n=2}^{\infty}[n]_q^m\mathcal{T}_n(\alpha,q)\left\{[n]_q-1\right\}\right||a_n|}{1-\sum_{n=2}^{\infty}\left|[n]_q^m\right||\mathcal{T}_n(\alpha,q)||a_n|}\right)\leq1. \quad (78)$$

Because of (74). □

Corollary 13. (see [11]). *If a function $l(z)\in\mathcal{A}$ of the form (1) and satisfy the condition*

$$\sum_{n=2}^{\infty}[n]_q^m\left\{\left\{[n]_q-1\right\}(k+1)+|\gamma|\right\}|a_n|\leq\gamma, \quad (79)$$

then, $l(z)\in k-\mathcal{US}_{q,\gamma}^m(0,1)$.

Corollary 14. (see [28]). *A function $l\in\mathcal{A}$ of the form (1) belongs to $k-\mathcal{US}(1-2\eta)$, if*

$$\sum_{n=2}^{\infty}\left\{n(k+1)-(k+\eta)\right\}|a_n|\leq1-\eta, \quad (80)$$

where $0\leq\eta<1$ and $k\geq0$. Then, $l(z)\in k-\mathcal{US}_{q\longrightarrow1-,1-\eta}^0(0,1)$.

When $q\longrightarrow1-$, then, $m=0,\alpha=0,\beta=1,\gamma=1-\eta$, with $0\leq\eta<1$ and $k=0$.

Corollary 15. (see [37]). *A function $l\in\mathcal{A}$ of the form (1) is in the class $0-\mathcal{US}(1-\eta)$, if*

$$\sum_{n=2}^{\infty}(n-\eta)|a_n|\leq1-\delta,0\leq\eta<1. \quad (81)$$

Theorem 16. *Let $l(z) \in k - \mathcal{US}_{q,\gamma}^{m}(\alpha, \beta)$. Then, $l(E)$ includes an open disk of radius*

$$\frac{[2]^{m}\mathcal{T}_{2}(\alpha, q)\left\{[2]_{q} - 1\right\}}{2[2]_{q}^{m}\left\{[2]_{q} - 1\right\}\mathcal{T}_{2}(\alpha, q) + \delta}, \qquad (82)$$

where Q_1 is given by (34).

Proof. Let a nonzero complex number w_0, such that $l(z) \neq w_0$ for $z \in E$. Then,

$$l_1(z) = \frac{w_0 l(z)}{w_0 - l(z)} = z + \left(a_2 + \frac{1}{w_0}\right)z^2 + \cdots. \qquad (83)$$

Since $l_1(z)$ is univalent, therefore

$$\left|a_2 + \frac{1}{w_0}\right| \leq 2. \qquad (84)$$

Now using (47), we have

$$\left|\frac{1}{w_0}\right| \leq 2 + \frac{\delta}{[2]_{q}^{m}\left\{[2]_{q} - 1\right\}\mathcal{T}_{2}(\alpha, q)} = \frac{\left(2[2]_{q}^{m}\left\{[2]_{q} - 1\right\}\mathcal{T}_{2}(\alpha, q)\right) + \delta}{[2]_{q}^{m}\left\{[2]_{q} - 1\right\}\mathcal{T}_{2}(\alpha, q)}. \qquad (85)$$

Hence we have

$$|w_0| \geq \frac{[2]_{q}^{m}\left\{[2]_{q} - 1\right\}\mathcal{T}_{2}(\alpha, q)}{\left(2[2]_{q}^{m}\left\{[2]_{q} - 1\right\}\mathcal{T}_{2}(\alpha, q)\right) + \delta}. \qquad (86)$$

When $\alpha = 0$ and $\beta = 1$, then we have known result [11]. □

Corollary 17. *Let $l(z) \in k - \mathcal{US}_{q,\gamma}^{m}(0, 1)$. Then, $l(E)$ includes an open disk of radius*

$$\frac{[2]^{m}\left\{[2]_{q} - 1\right\}}{2[2]_{q}^{m}\left\{[2]_{q} - 1\right\} + \delta}. \qquad (87)$$

5. Conclusion

In this paper, we formulate the q-analogous of differential operator associated with q-Mittag-Leffler function. By applying newly defined operator, we defined and investigated a new subclass $k - \mathcal{US}_{q,\gamma}^{m}(\alpha, \beta)$, of analytic functions in conic domains. We investigated the number of useful properties such that structural formula, coefficient estimates, Fekete–Szego problem, and subordination results. We also highlighted some known consequences of our main result. For future work, one can employ the q-analogous of differential operator (21) in different classes of analytic functions such as the meromorphic and multivalent functions (see [38–42].

Data Availability

All data are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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