Hindawi
Journal of Function Spaces
Volume 2023, Article ID 9806498, 1 page
https://doi.org/10.1155/2023/9806498

Retraction

Retracted: A Novel MAGDM Approach Based on Cubic *q*-Rung Orthopair Fuzzy Power Generalized Maclaurin Symmetric Mean Operator

Journal of Function Spaces

Received 10 October 2023; Accepted 10 October 2023; Published 11 October 2023

This article has been retracted by Hindawi following an investigation undertaken by the publisher [1]. This investigation has uncovered evidence of one or more of the following indicators of systematic manipulation of the publication process:

(1) Discrepancies in scope

(2) Discrepancies in the description of the research reported

(3) Discrepancies between the availability of data and the research described

(4) Inappropriate citations

(5) Incoherent, meaningless and/or irrelevant content included in the article

(6) Peer-review manipulation

The presence of these indicators undermines our confidence in the integrity of the article's content and we cannot, therefore, vouch for its reliability. Please note that this notice is intended solely to alert readers that the content of this article is unreliable. We have not investigated whether authors were aware of or involved in the systematic manipulation of the publication process.

Wiley and Hindawi regrets that the usual quality checks did not identify these issues before publication and have since put additional measures in place to safeguard research integrity.

We wish to credit our own Research Integrity and Research Publishing teams and anonymous and named external researchers and research integrity experts for contributing to this investigation.

The corresponding author, as the representative of all authors, has been given the opportunity to register their agreement or disagreement to this retraction. We have kept a record of any response received.

References

[1] Q. Khan, H. Garg, H. Khattak, A. A. Al-Babtain, I. Elbatal, and M. Elgarhy, "A Novel MAGDM Approach Based on Cubic *q* -Rung Orthopair Fuzzy Power Generalized Maclaurin Symmetric Mean Operator," *Journal of Function Spaces*, vol. 2022, Article ID 9056605, 33 pages, 2022.

Hindawi
Journal of Function Spaces
Volume 2022, Article ID 9056605, 33 pages
https://doi.org/10.1155/2022/9056605

Research Article

A Novel MAGDM Approach Based on Cubic q-Rung Orthopair Fuzzy Power Generalized Maclaurin Symmetric Mean Operator

Qaisar Khan,[1] **Harish Garg**,[2] **Hizbullah Khattak**,[3] **Abdulhakim A. Al-Babtain**,[4] **I. Elbatal**,[5] **and Mohammed Elgarhy**[6]

[1]*Department of Mathematics and Statistic University of Haripur, Haripur, KPK, Pakistan*
[2]*School of Mathematics, Thapar Institute of Engineering and Technology, Deemed University, Patiala, 147004 Punjab, India*
[3]*Department of Computer Science and Information Technology, Hazara University Mansehra, Pakistan*
[4]*Department of Statistics and Operations Research, King Saud University, Riyadh 11362, Saudi Arabia*
[5]*Department of Mathematics and Statistics-College of Science, Imam Mohammad Ibn Saud Islamic University (IMSIU), Saudi Arabia*
[6]*The Higher Institute of Commercial Sciences, Al Mahalla Al Kubra, Algarbia 31951, Egypt*

Correspondence should be addressed to Mohammed Elgarhy; m_elgarhy85@sva.edu.eg

Received 26 March 2022; Revised 25 May 2022; Accepted 2 July 2022; Published 5 August 2022

Academic Editor: Muhammad Gulzar

The cubic q-rung orthopair fuzzy set (Cq-ROFS) is a basic simplification of several fuzzy notions, including fuzzy set (FS), interval valued FS (INVFS), intuitionistic FS (INTFS), q-rung orthopair FS (q-ROFS), and INV q-rung orthopair FS (INVq-ROFS). By the degrees of INVq-ROFS and q-ROFS, Cq-ROFS exposes fuzzy judgement, and this is one of the advanced mathematical tools to handle more complicated assessment information in multiple attribute group decision-making (MAGDM) problems. In this article, firstly, encouraged from power average (PA) operator, generalized MSM operator, and generalized dual MSM operator, two novel aggregation operators (AOs) such as cubic q-rung orthopair fuzzy power generalized Maclaurin symmetric mean (Cq-ROFPWGMSM) operator and cubic q-rung orthopair fuzzy power generalized dual Maclaurin symmetric mean (Cq-ROFPWGDMSM) operator are initiated by merging PA operator with GMSM and GDMSM operators and some core characteristics of these AOs are investigated. Furthermore, some core cases with respect to different general parameters values are also investigated and found that some aggregation operators are special cases of the initiated AOs. Secondly, the weighted form of these newly initiated aggregation operators is also offered. Thirdly, two MAGDM models are offered based on Cq-OFPGWMSM and Cq-ROFPGWDMSM operators. Lastly, a numerical example about the selection of the best assistant professor of the year is taken as an application to show the effectiveness and capability of the initiated approaches. There is also a comparison with existing decision-making models.

1. Introduction

The fuzzy set (FS) was proposed by Zadeh [1] as a means of expressing and relaying unpredictability and ambiguities. FS has caught the imagination of scholars everywhere around the globe, who might have analyzed its conceptual and technical properties since its conception. Among the most recent academic studies on the theory and applications of FSs include economic and business [2–4], genetic algorithms [5, 6], and supply chain management [7, 8]. Following the development of the concept of FS, a number of FS improvements were projected, including interval-valued FS [9], which revealed membership degree as a subset of $[0, 1]$, and Atanassov's ITFS [10], which clarified membership degree (MED) and nonmembership degree (NOMD) as a single number in $[0, 1]$, with the total of these two degrees having to be less or equal to 1. As a result, ITFS explains uncertainty and unreliability in greater depth than FS. When the MED of such an entity is described as INVFS and FS, the situation becomes more appealing. The traditional ITFS is unable to manage such data in these conditions. Jun et al. [11] proposed the concept of cubic set (CS) to address the above-mentioned problem. The above-mentioned sets are special cases of CS. Mahmood et al. [12] established the concept of CNs and developed a set of weighted aggregation operators (AOs) to solve MADM difficulties. Fahmi et al. [13] intercepted Einstein AOs for CNs and apply these AOs to solve MADM problems under cubic information. Ayub et al. [14] have presented a set of cubic fuzzy Dombi AOs that have been implemented on Dombi t-norm and t-conorm and used to solve MADM issues in cubic fuzzy context. Kaur and Garg [15, 16] extended the idea of CS and introduced the concept of ITF cubic set, anticipated various AOs, and used these AOs to deal with MADM problems under cubic information. The shortcomings of these fuzzy structures are clear, even though research on them has been quite successful. These structures are incapable of dealing with situations in which the total of maximum lower bound membership and maximum upper bound membership and total membership and nonmembership degrees exceeds one. Abbas et al. [17] developed the notion of cubic Pythagorean fuzzy set (CPyTFS), which consists of interval valued Pythagorean FS (IVNPyTFS) [18] and Pythagorean FS (PyTFS), launched certain aggregation operators, and applied these aggregation operators to deal with MADM problems under Pythagorean cubic fuzzy information to overcome this shortcoming of these fuzzy structures. Talukdar and Dutta [19] defined some similarity measures for CPyTFS and apply them to deal with MADM problems under PyTF information. Fahmi et al. [20] defined some CPyTF linguistic Maclaurin symmetric mean (CPyTFLMSM) operators and apply them to solve MADM problems under CPyTF linguistic environment. Naeem et al. [21] redefine CPyTFMSM operators and apply them to solve MADM problems with CPyTFL information. Likewise, CITFSs and CPyTFSs are incapable of dealing with situations in which the total of squares of maximum lower bound membership and maximum upper bound membership and the total of squares of membership and nonmembership degrees exceed one. To deal with such situations, Wang et al. [22] proposed the concept of cubic q-rung orthopair fuzzy set (Cq-ROFS) which consist of q -rung orthopair fuzzy set (q-ROFS) [23] and interval valued q-ROFS [24]. They also proposed some Cq-ROF power AOs and Cq-ROF power Muirhead mean operators and apply them to deal with MAGDM problems under Cq-ROF information. Cq-ROFS is the latest generalization of several fuzzy concepts and can deal with more complex information in MAGDM process. Later, Zhang et al. [25] introduced Cq-ROF Heronian mean operator and apply them to solve MADM problems under Cq-ROF environment. Garg et al. [26] initiated the concept of Cq-ROF linguistic set, developed some basic operational laws, and based on these initiated operational laws proposed some Muirhead mean operator and apply them to deal with MADM problem with Cq-ROF information.

Aggregation operators (AOs) are an important part of the multiattribute group decision-making process. The AOs can have the ability to fuse many real numbers into a single number. The qualities of various AOs varies. Some AOs, such as Yager's PA operator [27], can remove the negative effects of unpleasant data from final ranking results and have been expanded on by many researchers from around the world to figure out how to deal with various situations, such as Xu's [28], who provided the ITF power AOs and implemented it to MAGDM, which can minimise the effects of inaccurate data. Some AOs, such as the BM [29], HM [30], MM [31], and MSM [32], took the link between input arguments into account. BOM and HOM can consider the relationship between two input arguments, but MSM and MM may consider the relationship between any numbers of input arguments. Wei et al. [33] initiated some q-ROF MSM operators and apply these AOs to deal with MADM problems under q-ROF information. Riaz et al. [34] generalized the concept of q-ROFS and initiated the idea of linear Diophantine fuzzy set and give its applications in MADM problems. Liu and Wang [35] proposed some q-ROF aggregation operators and apply them to solve MADM problems under q-ROF information. Liu et al. [36] and Khan et al. [37] initiated power Heronian mean and power Bonferroni mean to deal with 2-tuple and interval neutrosophic information. Wang et al. [38] and Gao et al. [39] initiated MM operators for PFSNs and q-ROFNs and apply them to solve MADM problems. Recently, Khan et al. [40] initiated power Heronian mean operators for T-SPFS and apply them to solve MADM problem under TSPF environment. Riaz et al. [41] generalized the concept of q-ROFS and linear Diophantine fuzzy set and proposed the idea of spherical linear Diophantine fuzzy set and give its applications in MADM problems. Hashmi et al. [41] further generalized the concept of spherical linear Diophantine fuzzy set and introduced spherical linear Diophantine fuzzy soft rough set and give its applications in MADM problems.

It has been observed from above studies that yet no one has attempted to combine PA operator with generalized Maclaurin symmetric mean operator to deal with cubic q-rung orthopair fuzzy data. Therefore, we propose the following:

(1) The PA operator is combined with generalized MSM and generalized dual MSM operators to initiate a hybrid aggregation operators Cq-ROFPGMSM and Cq-ROFPDMSM and is considerably more adaptable and superior to the prior aggregations operators in terms of a general parameter

(2) Fortunately, there are many MAGDM difficulties in which the characteristics are linked, and many existing AOs can only alleviate such scenarios when the attributes are in the form of real integers or other fuzzy structures

(3) There are presently no AOs in place to deal with MAGDM problems arising from cubic q-rung orthopair fuzzy data which have the capacity of removing bad effects of awkward data, can consider interrelationship among any number of input arguments, and can have the general parameter that makes the decision-making process more flexible. In response to this limitation, we combined PA operator with GMSM and GDMSM operators to address MAGDM problems utilizing cubic q-rung orthopair fuzzy information

The following are the priorities and offerings of this effort because of important impacts from earlier studies:

(1) Developing innovative Cq-ROF power GMSM operators, Cq-ROF power GDMSM operators, and their weighted form

(2) Examining the commencing AOs' basic features and exceptional cases

(3) Expecting the deployment of two MAGDM model on these commencing AOs

(4) Assessing selection of the best assistant professor of the year to show the practicality and effectiveness of the anticipated two MAGDM models

(5) Comparison of the launched two MAGDM models with existing MAGDM models

This article is designed in the subsequent way to attain these objectives. Part 2 familiarizes a variability of vital notions such as Cq-ROFSs, score and accuracy functions, PA, and MSM and DMSM operators. In part 3, we look at a few AOs such as Cq-ROFPGMSM and Cq-ROFPGDMSM operator and discussed its core properties and special cases with respect to the general parameter. Part 4 introduces the Cq-ROFPWGMSM and Cq-ROFPWGDMSM operators, as well as their basic properties. In part 5, we initiate two MAGDM model based on these AOs. In part 6, a numerical example about selection of assistant professor of the year is provided to verify the unassailability and compensations of the initiated approaches. Finally, in part 7, a brief conclusion is provided.

2. Preliminaries

For better realizing of this paper, in this part, we will seem back few basic ideas such as definitions, operational rules, distance measure, and score functions of Cq-ROFNs, GMSM, and GDMSM operators.

2.1. The Cq-ROFS

Definition 1 (see [23]). Assume that the domain of universe is represented by Λ and $s \in \Lambda$ be any element. Then, a q-ROFS $\widetilde{QR}$ in the domain of universe is specified by

$$\widetilde{QR} = \left\{ \left\langle s, \widetilde{me}_{\widetilde{QR}}(s), \widetilde{ne}_{\widetilde{QR}}(s) \right\rangle | s \in \Lambda \right\}, \tag{1}$$

where $\widetilde{me}_{\widetilde{QR}}$ and $\widetilde{ne}_{\widetilde{QR}}$, respectively, express the positive grade and the negative grade of s in the q-ROFS $\widetilde{QR}$ such that $0 \le \widetilde{me}_{\widetilde{QR}}(s), \widetilde{ne}_{\widetilde{QR}}(s) \le 1$ with the axiom $0 \le (\widetilde{me}_{\widetilde{QR}}(s))^q + (\widetilde{ne}_{\widetilde{QR}}(s))^q \le 1 (q \ge 1)$.

The hesitancy grade $\widetilde{HS}_{\widetilde{QR}}(s)$ of the element s in the q-ROPFS $\widetilde{QR}$ is delineated by $\widetilde{HS}_{\widetilde{QR}}(s) = (1 - (\widetilde{me}_{\widetilde{QR}}(s))^q - (\widetilde{ne}_{\widetilde{QR}}(s))^q)$, satisfying $0 \le \widetilde{HS}_{\widetilde{QR}}(s) \le 1$ and $s \in \Lambda$.

The pair $\langle \widetilde{me}_{\widetilde{QR}}(s), \widetilde{ne}_{\widetilde{QR}}(s) \rangle$ was named q-ROFN by Liu and Wang [35] and is denoted $\widetilde{qr} = \langle \widetilde{me}, \widetilde{ne} \rangle$ satisfying the condition $0 \le \widetilde{me}, \widetilde{ne} \le 1$ with the axiom $0 \le (\widetilde{me})^q + (\widetilde{ne})^q \le 1 (q \ge 1)$.

Definition 2 (see [24]). Assume that the domain of universe is represented by Λ and $s \in \Lambda$ be any element. Then, an IVq-ROFS $\widetilde{IVQ}$ in the domain of universe is specified by

$$\widetilde{IVQ} = \left\{ \left\langle s, \widetilde{me}_{\widetilde{IVQ}}(s), \widetilde{ne}_{\widetilde{IVQ}}(s) \right\rangle | s \in \Lambda \right\}, \tag{2}$$

where $\widetilde{me}_{\widetilde{IVQ}}, \widetilde{ne}_{\widetilde{IVQ}} \subseteq [0, 1]$ are two interval values, respectively, expressing the positive grade and the negative grade of s in the IVq-ROFS $\widetilde{IVQ}$ such that satisfying the axiom $0 \le \mathrm{Sup}(\widetilde{me}_{\widetilde{IVQ}}(s))^q + \mathrm{Sup}(\widetilde{ne}_{\widetilde{IVQ}}(s))^q \le 1 (q \ge 1)$.

The pair $\kappa = \langle \widetilde{me}_{\widetilde{IVQ}}(s), \widetilde{ne}_{\widetilde{IVQ}}(s) \rangle$ was named IVq-ROFN by Joshi [24] satisfying the condition $0 \le \mathrm{Sup}(\widetilde{me})^q + \mathrm{Sup}(\widetilde{ne})^q \le 1 (q \ge 1)$.

Definition 3 (see [22]). Assume that the domain of universe is represented by Λ and $s \in \Lambda$ be any element. Then, a Cq-ROFS $\widetilde{CQ}$ in the domain of universe is specified by

$$\widetilde{CQ} = \left\{ \left\langle s, \widetilde{IVQ}(s), \widetilde{QR}(s) \right\rangle | s \in \Lambda \right\}, \tag{3}$$

where $\widetilde{IVQ} = \{ \langle s, \widetilde{me}_{\widetilde{IVQ}}(s), \widetilde{ne}_{\widetilde{IVQ}}(s) \rangle | s \in \Lambda \}$ is an IVq-ROFS and $\widetilde{QR} = \{ \langle s, \widetilde{me}_{\widetilde{QR}}(s), \widetilde{ne}_{\widetilde{QR}}(s) \rangle | s \in \Lambda \}$, is a q-ROFS classified on Λ.

For simplicity, the pair $Cq = \langle ([a, b], [c, d]), (e, f) \rangle$ is said to be Cq-ROFN, where $([a, b], [c, d])$ and (e, f) respectively, represent IVq-ROFN and q-ROFN.

Definition 4 (see [22]). Let $Cq_1 = \langle ([a_1, b_1], [c_1, d_1]), (e_1, f_1) \rangle$, $Cq_2 = \langle ([a_2, b_2], [c_2, d_2]), (e_2, f_2) \rangle$ and $Cq_3 = \langle ([a_3, b_3], [c_3, d_3]), (e_3, f_3) \rangle$ be any three Cq-ROFNs. Then, the operational laws are specified as follows:

(1) $Cq_2 \oplus Cq_3 = \langle ([(a_2^q + a_3^q - a_2^q a_3^q)^{1/q}, (b_2^q + b_3^q - b_2^q b_3^q)^{1/q}], [c_2 c_3, d_2 d_3]), ((e_2^q + e_3^q - e_2^q e_3^q)^{1/q}, f_2 f_3) \rangle$

(2) $Cq_2 \otimes Cq_3 = \langle ([a_2 a_3, b_2 b_3], [(c_2^q + c_3^q - c_2^q c_3^q)^{1/q}, (d_2^q + d_3^q - d_2^q d_3^q)^{1/q}]), (e_2 e_3, (f_2^q + f_3^q - f_2^q f_3^q)^{1/q}) \rangle$

(3) $\zeta Cq_1 = \langle ([(1 - (1 - a_1^q)^\zeta)^{1/q}, (1 - (1 - b_1^q)^\zeta)^{1/q}], [c_1^\zeta, d_1^\zeta])((1 - (1 - e_1^q)^\zeta)^{1/q}, f_1^\zeta) \rangle, \zeta > 0$

(4) $Cq_1^\zeta = \langle ([a_1^\zeta, b_1^\zeta], [(1 - (1 - c_1^q)^\zeta)^{1/q}, (1 - (1 - d_1^q)^\zeta)^{1/q}])(e_1^\zeta, (1 - (1 - f_1^q)^\zeta)^{1/q}) \rangle, \zeta > 0$

Definition 5 (see [22]). Let $Cq_1 = \langle ([a_1, b_1], [c_1, d_1]), (e_1, f_1) \rangle$ be a Cq-ROFN. Then, the score and accuracy functions are explained as follows:

$$\widetilde{\mathrm{SCR}}(Cq_1) = \frac{a_1^q + b_1^q - c_1^q - d_1^q}{2} + e_1^q - f_1^q, \tag{4}$$

$$\widetilde{\mathrm{ACR}}(Cq_1) = \frac{a_1^q + b_1^q + c_1^q + d_1^q}{2} + e_1^q + f_1^q. \tag{5}$$

Further, Wang [22] et al. presented the comparison rules based on these scores and accuracy functions, which are stated below.

Let $Cq_1 = \langle([a_1, b_1], [c_1, d_1]), (e_1, f_1)\rangle$ and $Cq_2 = \langle([a_2, b_2], [c_2, d_2]), (e_2, f_2)\rangle$ be two Cq-ROFNs.

(1) If $\widetilde{\mathrm{SCR}}(Cq_1) > \widetilde{\mathrm{SCR}}(Cq_2)$, then $Cq_1 > Cq_2$

(2) If $\widetilde{\mathrm{SCR}}(Cq_1) = \widetilde{\mathrm{SCR}}(Cq_2)$, then

(i) If $\widetilde{\mathrm{ACR}}(Cq_1) > \widetilde{\mathrm{ACR}}(Cq_2)$, then $Cq_1 > Cq_2$

(ii) If $\widetilde{\mathrm{ACR}}(Cq_1) = \widetilde{\mathrm{ACR}}(Cq_2)$, then $Cq_1 = Cq_2$

Now, the definition of distance measure among two Cq-ROFNs presented by Wang et al. [22] is given below.

Definition 6 (see [22]). Let $Cq_1 = \langle([a_1, b_1], [c_1, d_1]), (e_1, f_1)\rangle$ and $Cq_2 = \langle([a_2, b_2], [c_2, d_2]), (e_2, f_2)\rangle$ be two Cq-ROFNs; then, the distance measure among these Cq-ROFNs is specified as follows:

$$\begin{aligned}\widetilde{\mathrm{DE}}(Cq_1, Cq_2) = \frac{1}{6}\Big(&\left|a_1^q - a_2^q\right| + \left|b_1^q - b_2^q\right| + \left|c_1^q - c_2^q\right| \\ &+ \left|d_1^q - d_2^q\right| + \left|e_1^q - e_2^q\right| + \left|f_1^q - f_2^q\right|\Big).\end{aligned} \tag{6}$$

2.2. PA Operator

Definition 7 (see [27]). Let $\mathfrak{J}_l (l = 1, 2, \cdots, p)$ be a set of positive crisp numbers. Then, the PA operator is specified as

$$\mathrm{PA}(\mathfrak{J}_1, \mathfrak{J}_2, \cdots, \mathfrak{J}_p) = \frac{\oplus_{l=1}^{p}(1 + T(\mathfrak{J}_l))\mathfrak{J}_l}{\oplus_{l=1}^{p}(1 + T(\mathfrak{J}_l))}, \tag{7}$$

where $T(\mathfrak{J}_l) = \oplus_{m=1, l \neq m}^{p} \mathrm{Spt}(\mathfrak{J}_l, \mathfrak{J}_m)$, $\mathrm{Spt}(\mathfrak{J}_l, \mathfrak{J}_m)$ indicates the support for $\mathfrak{J}_l$ from $\mathfrak{J}_m$ executing the axioms.

(1) $0 \leq \mathrm{Spt}(\mathfrak{J}_l, \mathfrak{J}_m) \leq 1$, (2) $\mathrm{Spt}(\mathfrak{J}_l, \mathfrak{J}_m) = \mathrm{Spt}(\mathfrak{J}_m, \mathfrak{J}_l)$, and (3) $\mathrm{Spt}(\mathfrak{J}_g, \mathfrak{J}_h) \leq \mathrm{Spt}(\mathfrak{J}_l, \mathfrak{J}_m)$, if $|\mathfrak{J}_l, \mathfrak{J}_m| \geq |\mathfrak{J}_g, \mathfrak{J}_h|$.

2.3. The GMSM and GDMSM Operators

Definition 8 (see [32]). The GMSM operator is described as

$$\mathrm{GMSM}^{(k,\gamma_1,\gamma_2,\cdots,\gamma_k)}\left(\widetilde{gh}_1, \widetilde{gh}_2, \cdots, \widetilde{gh}_s\right) = \left(\frac{\oplus_{1\leq l_1 < l_2 < \cdots < l_k \leq s}\left(\otimes_{m=1}^{k} \widetilde{gh}_{l_m}^{\gamma_m}\right)}{C_s^k}\right)^{1/(\gamma_1+\gamma_2,+\cdots+\gamma_k)}, \tag{8}$$

where $\gamma_1, \gamma_2, \cdots, \gamma_k \geq 0$, k is a parameter and $k = 1, 2, \cdots, s$($l_1, l_2, \cdots, l_k$) traverses all the k-tuple combination of $(1, 2, \cdots, s)$. The denominator C_s^k in Equation (8) expresses the binomial coefficient $s!/k!(s-k)!$ and s is the balancing coefficient.

Certainly, the GMSM operators have the following characteristics [32].

(1) If $\widetilde{gh}_l = \widetilde{gh}(l = 1, 2, \cdots, s)$, $\mathrm{GMSM}^{(k,\gamma_1,\gamma_2,\cdots,\gamma_k)}(\widetilde{gh}, \widetilde{gh}, \cdots, \widetilde{gh}) = \widetilde{gh}$

(2) If $\widetilde{gh}_l \leq gh_l (l = 1, 2, \cdots, s)$, $\mathrm{GMSM}^{(k,\gamma_1,\gamma_2,\cdots,\gamma_k)}(\widetilde{gh}_1, \widetilde{gh}_2, \cdots, \widetilde{gh}_s) \leq \mathrm{GMSM}^{(k,\gamma_1,\gamma_2,\cdots,\gamma_k)}(gh_1, gh_2, \cdots, gh_s)$

(3) If $\min_l\{\widetilde{gh}_l\} \leq \mathrm{GMSM}^{(k,\gamma_1,\gamma_2,\cdots,\gamma_k)}(\widetilde{gh}_1, \widetilde{gh}_2, \cdots, \widetilde{gh}_s) \leq \max_l\{\widetilde{gh}_l\}$

Definition 9 (see [32]). The GDMSM operator is described as

$$\mathrm{GDMSM}^{(k,\gamma_1,\gamma_2,\cdots,\gamma_k)}\left(\widetilde{gh}_1, \widetilde{gh}_2, \cdots, \widetilde{gh}_s\right) = \frac{1}{\gamma_1 + \gamma_2 + \cdots + \gamma_k}\left(\otimes_{1\leq l_1 < l_2 < \cdots < l_k \leq s}\left(\oplus_{m=1}^{k} \gamma_m \widetilde{gh}_{l_m}\right)\right)^{1/C_s^k}, \tag{9}$$

where $\gamma_1, \gamma_2, \cdots, \gamma_k \geq 0$, k is a parameter and $k = 1, 2, \cdots, s$, ($l_1, l_2, \cdots, l_k$) traverses all the k-tuple combination of $(1, 2, \cdots, s)$. The denominator C_s^k in Equation (9) expresses the binomial coefficient $s!/k!(s-k)!$ and s is the balancing coefficient.

Certainly, the GMSM operators have the following characteristics [32].

(1) If $\widetilde{gh}_l = \widetilde{gh}(l = 1, 2, \cdots, s)$, $\mathrm{GDMSM}^{(k,\gamma_1,\gamma_2,\cdots,\gamma_k)}(\widetilde{gh}, \widetilde{gh}, \cdots, \widetilde{gh}) = \widetilde{gh}$

(2) If $\widetilde{gh}_l \leq gh_l (l = 1, 2, \cdots, s)$, $\mathrm{GDMSM}^{(k,\gamma_1,\gamma_2,\cdots,\gamma_k)}(\widetilde{gh}_1, \widetilde{gh}_2, \cdots, \widetilde{gh}_s) \leq \mathrm{GDMSM}^{(k,\gamma_1,\gamma_2,\cdots,\gamma_k)}(gh_1, gh_2, \cdots, gh_s)$

(3) If $\min_l\{\widetilde{gh}_l\} \leq \mathrm{GDMSM}^{(k,\gamma_1,\gamma_2,\cdots,\gamma_k)}(\widetilde{gh}_1, \widetilde{gh}_2, \cdots, \widetilde{gh}_s) \leq \max_l\{\widetilde{gh}_l\}$

3. The Cq-ROFPGMSM and Cq-ROFPGDMSM Operators

In this part, we propose Cq-ROFGMSM and Cq-ROFGDMSM operators by combining PA operator with GMSM and GDMSM operators. Furthermore, some key properties and special cases with respect to generalized parameters of these newly initiated AOs are investigated.

3.1. The Cq-ROFPGMSM Operator. In this subpart, the Cq-ROFGMSM operators are initiated; some key properties and special cases with respect to the parameters are investigated.

Definition 10. Let $Cq_1, Cq_2, \cdots$ and Cq_s be Cq-ROFNs, where $Cq_l = \langle([a_l, b_l], [c_l, d_l]), (e_l, f_l)\rangle (l = 1, 2, \cdots, s)$ $(q \geq 1)$. The

Cq-ROFPGMSM operator of the Cq-ROFNs $Cq_1, Cq_2, \cdots,$ and Cq_n is describes as

$$\text{Cq-ROFPGMSM}^{(k,\gamma_1,\gamma_2,\cdots,\gamma_k)}(Cq_1, Cq_2, Cq_s) = \left(\frac{\oplus_{1\le l_1<l_2<\cdots<l_k\le s}\left(\otimes_{j=1}^{k}\left(\left(s\left(1+T\left(Cq_{l_j}\right)\right)/\oplus_{z=1}^{s}\left(1+T(Cq_z)\right)\right)Cq_{l_j}\right)^{\gamma_j}\right)}{C_s^k}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}, \tag{10}$$

where $\mathrm{U}_z = (1+T(Cq_z))/\sum_{z=1}^{s}(1+T(Cq_z))$, $\sum_{z=1}^{s}\mathrm{U}_z = 1$, $T(Cq_j) = \prod_{\substack{z=1 \\ z\ne j}}^{s}\operatorname{Spt}(Cq_z, \mathrm{C}q_j)$, is the support degree for Cq_z from Cq_j, which fulfil the following conditions:

(1) $\operatorname{Spt}(Cq_z, \mathrm{C}q_j) \in [0,1]$

(2) $\operatorname{Spt}(Cq_z, \mathrm{C}q_j) = \operatorname{Spt}(Cq_j, \mathrm{C}q_z)$

(3) If $\widetilde{\mathrm{DE}}(Cq_z, \mathrm{C}q_j) \le \widetilde{\mathrm{DE}}(Cq_s, \mathrm{C}q_t)$, then $\operatorname{Spt}(Cq_z, \mathrm{C}q_j) \ge \operatorname{Spt}(Cq_s, \mathrm{C}q_t)$, where $\widetilde{\mathrm{DE}}(Cq_z, \mathrm{C}q_j)$ express the distance measure among two Cq-ROFNs described in Definition 6. $\gamma_1, \gamma_2, \cdots, \gamma_k \ge 0$, k is a parameter and $k = (1, 2, \cdots, s)$, $(l_1, l_2, \cdots, l_k)$ traverses all the k-tuple combination of $(1, 2, \cdots, s)$. The denominator C_s^k in Equation (10) expresses the binomial coefficient $s!/(k!(s-k)!)$ and s is the balancing coefficient

In order to write Equation (10) in an easy way, we can assume

$$\mathrm{o}_z = \frac{(1+T(Cq_z))}{\sum_{z=1}^{s}(1+T(Cq_z))}. \tag{11}$$

Subsequently, we identify $(\mathrm{o}_1, \mathrm{o}_2, \cdots, \mathrm{o}_s)$ as the power weight vector. As a result, Equation (10) can be written in an easy way as follows:

$$\text{Cq-ROFPGMSM}^{(k,\gamma_1,\gamma_2,\cdots,\gamma_k)}(Cq_1, Cq_2,. \cdots, Cq_s) = \left(\frac{\oplus_{1\le l_1<l_2<\cdots<l_k\le s}\left(\otimes_{j=1}^{k}\left(n\mathrm{o}_{l_j}Cq_{l_j}\right)^{\gamma_j}\right)}{C_s^k}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}. \tag{12}$$

Theorem 11. *Let $Cq_1, Cq_2, \cdots,$ and Cq_n be Cq-ROFNs, where $Cq_l = \langle([a_l, b_l], [c_l, d_l]), (e_l, f_l)\rangle (l = 1, 2, \cdots, s)\ (q \ge 1)$. Then, the aggregated value utilizing Equation (12) is still Cq-ROFN, even*

$$\begin{aligned}
&\text{Cq-ROFPGMSM}^{(k,\gamma_1,\gamma_2,\cdots,\gamma_k)}(Cq_1, Cq_2,. \cdots, Cq_s) \\
&= \Bigg\langle \Bigg(\Bigg[\left(\left(\left(1 - \left(\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1 - \prod_{j=1}^{k}\left(1-\left(1-a_{l_j}^{q}\right)^{so_{l_j}}\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}, \\
&\times \left(\left(1 - \left(\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1 - \prod_{j=1}^{k}\left(1-\left(1-b_{l_j}^{q}\right)^{so_{l_j}}\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)} \Bigg], \\
&\times \Bigg[\left(1 - \left(1 - \left(\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1 - \prod_{j=1}^{k}\left(1-\left(c_{l_j}^{so_{l_j}}\right)^{q}\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q}, \\
&\times \left(1 - \left(1 - \left(\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1 - \prod_{j=1}^{k}\left(1-\left(d_{l_j}^{so_{l_j}}\right)^{q}\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q} \Bigg] \Bigg), \\
&\times \left(\left(\left(1 - \left(\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1 - \prod_{j=1}^{k}\left(1-\left(1-e_{l_j}^{q}\right)^{so_{l_j}}\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}, \right. \\
&\times \left. \left(1 - \left(1 - \left(\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1 - \prod_{j=1}^{k}\left(1-\left(f_{l_j}^{so_{l_j}}\right)^{q}\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q} \right) \Bigg\rangle.
\end{aligned} \tag{13}$$

Proof. Based on the operational laws of Cq-ROFNs, we have

$$
\begin{aligned}
\mathrm{so}_{l_j} Cq_{l_j} = \Bigg\langle \Bigg(\Bigg[\left(1-\left(1-a_{l_j}^{q}\right)^{\mathrm{so}_{l_j}}\right)^{1/q}, \left(1-\left(1-b_{l_j}^{q}\right)^{\mathrm{so}_{l_j}}\right)^{1/q} \Bigg], \left[c_{l_j}^{\mathrm{so}_{l_j}}, d_{l_j}^{\mathrm{so}_{l_j}}\right] \Bigg), \\
\cdot \left(\left(1-\left(1-e_{l_j}^{q}\right)^{\mathrm{so}_{l_j}}\right)^{1/q}, f_{l_j}^{\mathrm{so}_{l_j}} \right) \Bigg\rangle,
\end{aligned}
$$

$$
\begin{aligned}
\left(\mathrm{so}_{l_j} Cq_{l_j}\right)^{\gamma_j} = \Bigg\langle \Bigg(\Bigg[\left(\left(1-\left(1-a_{l_j}^{q}\right)^{\mathrm{so}_{l_j}}\right)^{1/q}\right)^{\gamma_j}, \left(\left(1-\left(1-b_{l_j}^{q}\right)^{\mathrm{so}_{l_j}}\right)^{1/q}\right)^{\gamma_j} \Bigg], \\
\cdot \left[\left(1-\left(1-\left(c_{l_j}^{\mathrm{so}_{l_j}}\right)^{q}\right)^{\gamma_j}\right)^{1/q}, \left(1-\left(1-\left(d_{l_j}^{\mathrm{so}_{l_j}}\right)^{q}\right)^{\gamma_j}\right)^{1/q} \right] \Bigg), \\
\cdot \left(\left(\left(1-\left(1-e_{l_j}^{q}\right)^{\mathrm{so}_{l_j}}\right)^{1/q}\right)^{\gamma_j}, \left(1-\left(1-\left(f_{l_j}^{\mathrm{so}_{l_j}}\right)^{q}\right)^{\gamma_j}\right)^{1/q} \right) \Bigg\rangle.
\end{aligned}
\tag{14}
$$

Therefore,

$$
\begin{aligned}
\prod_{j=1}^{k} \left(\mathrm{so}_{l_j} Cq_{l_j}\right)^{\gamma_j} = \Bigg\langle \Bigg(\Bigg[\prod_{j=1}^{k} \left(\left(1-\left(1-a_{l_j}^{q}\right)^{\mathrm{so}_{l_j}}\right)^{1/q}\right)^{\gamma_j}, \prod_{j=1}^{k} \\
\cdot \left(\left(1-\left(1-b_{l_j}^{q}\right)^{\mathrm{so}_{l_j}}\right)^{1/q}\right)^{\gamma_j} \Bigg], \\
\cdot \Bigg[\left(1-\prod_{j=1}^{k}\left(1-\left(c_{l_j}^{\mathrm{so}_{l_j}}\right)^{q}\right)^{\gamma_j}\right)^{1/q}, \\
\cdot \left(1-\prod_{j=1}^{k}\left(1-\left(d_{l_j}^{\mathrm{so}_{l_j}}\right)^{q}\right)^{\gamma_j}\right)^{1/q} \Bigg] \Bigg), \\
\cdot \Bigg(\prod_{j=1}^{k} \left(\left(1-\left(1-e_{l_j}^{q}\right)^{\mathrm{so}_{l_j}}\right)^{1/q}\right)^{\gamma_j}, \\
\cdot \left(1-\prod_{j=1}^{k}\left(1-\left(f_{l_j}^{\mathrm{so}_{l_j}}\right)^{q}\right)^{\gamma_j}\right)^{1/q} \Bigg) \Bigg\rangle.
\end{aligned}
\tag{15}
$$

Further,

$$
\begin{aligned}
&\prod_{1\le l_1<l_2<\cdots<l_k\le s} \left(\prod_{j=1}^{k}\left(so_{l_j} Cq_{l_j}\right)^{\gamma_j}\right) \\
&= \Bigg\langle \Bigg(\Bigg[\left(1-\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(1-a_{l_j}^{q}\right)^{so_{l_j}}\right)^{\gamma_j}\right)\right)^{1/q}, \\
&\times \left(1-\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(1-b_{l_j}^{q}\right)^{so_{l_j}}\right)^{\gamma_j}\right)\right)^{1/q} \Bigg], \\
&\times \Bigg[\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(c_{l_j}^{so_{l_j}}\right)^{q}\right)^{\gamma_j}\right)^{1/q}, \prod_{1\le l_1<l_2<\cdots<l_k\le s} \\
&\times \left(1-\prod_{j=1}^{k}\left(1-\left(d_{l_j}^{so_{l_j}}\right)^{q}\right)^{\gamma_j}\right)^{1/q} \Bigg] \Bigg), \\
&\times \Bigg(\left(1-\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(1-e_{l_j}^{q}\right)^{so_{l_j}}\right)^{\gamma_j}\right)\right)^{1/q}, \prod_{1\le l_1<l_2<\cdots<l_k\le s} \\
&\times \left(1-\prod_{j=1}^{k}\left(1-\left(f_{l_j}^{so_{l_j}}\right)^{q}\right)^{\gamma_j}\right)^{1/q} \Bigg) \Bigg\rangle.
\end{aligned}
\tag{16}
$$

So,

$$
\begin{aligned}
&\frac{\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(\prod_{j=1}^{k}\left(so_{l_j} Cq_{l_j}\right)^{\gamma_j}\right)}{C_s^k} \\
&= \Bigg\langle \Bigg(\Bigg[\left(1-\left(\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(1-a_{l_j}^{q}\right)^{so_{l_j}}\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/q}, \\
&\times \left(1-\left(\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(1-b_{l_j}^{q}\right)^{so_{l_j}}\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/q} \Bigg], \\
&\times \Bigg[\left(\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(c_{l_j}^{so_{i_j}}\right)^{q}\right)^{\gamma_j}\right)^{1/q}\right)^{1/C_s^k}, \\
&\times \left(\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(d_{l_j}^{so_{l_j}}\right)^{q}\right)^{\gamma_j}\right)^{1/q}\right)^{1/C_s^k} \Bigg] \Bigg), \\
&\times \Bigg(\left(1-\left(\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(1-e_{l_j}^{q}\right)^{so_{l_j}}\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/q}, \\
&\times \left(\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(f_{l_j}^{so_{l_j}}\right)^{q}\right)^{\gamma_j}\right)^{1/q}\right)^{1/C_s^k} \Bigg) \Bigg\rangle.
\end{aligned}
\tag{17}
$$

Therefore,

$$
\begin{aligned}
&\left(\frac{\prod_{11\le l_1<l_2<\cdots<l_k\le s}\left(\prod_{j=1}^{k}\left(so_{i_j} Cq_{l_j}\right)^{\gamma_j}\right)}{C_s^k}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)} \\
&= \Bigg\langle \Bigg(\Bigg[\left(\left(1-\left(\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(1-a_{l_j}^{q}\right)^{so_{l_j}}\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}, \\
&\times \left(\left(1-\left(\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(1-b_{l_j}^{q}\right)^{so_{l_j}}\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)} \Bigg], \\
&\times \Bigg[\left(1-\left(1-\left(\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(c_{l_j}^{so_{l_j}}\right)^{q}\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q}, \\
&\times \left(1-\left(1-\left(\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(d_{l_j}^{so_{l_j}}\right)^{q}\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q} \Bigg] \Bigg), \\
&\times \Bigg(\left(\left(1-\left(\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(1-e_{l_j}^{q}\right)^{so_{l_j}}\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}, \\
&\times \left(1-\left(1-\left(\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(f_{l_j}^{so_{l_j}}\right)^{q}\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q} \Bigg) \Bigg\rangle,
\end{aligned}
$$

$$
\begin{aligned}
&\text{Cq-ROFPGMSM}^{(k,\gamma_1,\gamma_2,\cdots,\gamma_k)}(Cq_1, Cq_2, \cdots, Cq_s) \\
&= \left\langle \left(\left[\left(\left(\left(1 - \left(\prod_{1 \le l_1 < l_2 < \cdots < l_k \le s} \left(1 - \prod_{j=1}^{k} \left(1 - \left(1 - a_{l_j}^{q} \right)^{s o_{l_j}} \right)^{\gamma_j} \right) \right) \right)^{1/C_s^k} \right)^{1/q} \right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}, \right. \right. \\
&\times \left. \left(\left(\left(1 - \left(\prod_{1 \le l_1 < l_2 < \cdots < l_k \le s} \left(1 - \prod_{j=1}^{k} \left(1 - \left(1 - b_{l_j}^{q} \right)^{s o_{l_j}} \right)^{\gamma_j} \right) \right) \right)^{1/C_s^k} \right)^{1/q} \right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)} \right], \\
&\times \left[\left(1 - \left(1 - \left(\prod_{1 \le l_1 < l_2 < \cdots < l_k \le s} \left(1 - \prod_{j=1}^{k} \left(1 - \left(c_{l_j}^{s o_{l_j}} \right)^{q} \right)^{\gamma_j} \right) \right)^{1/C_s^k} \right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)} \right)^{1/q}, \right. \\
&\times \left. \left. \left(1 - \left(1 - \left(\prod_{1 \le l_1 < l_2 < \cdots < l_k \le s} \left(1 - \prod_{j=1}^{k} \left(1 - \left(d_{l_j}^{s o_{l_j}} \right)^{q} \right)^{\gamma_j} \right) \right)^{1/C_s^k} \right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)} \right)^{1/q} \right] \right), \\
&\times \left(\left(\left(1 - \left(\prod_{1 \le l_1 < l_2 < \cdots < l_k \le s} \left(1 - \prod_{j=1}^{k} \left(1 - \left(1 - e_{l_j}^{q} \right)^{s o_{l_j}} \right)^{\gamma_j} \right) \right) \right)^{1/C_s^k} \right)^{1/q} \right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}, \\
&\times \left. \left(1 - \left(1 - \left(\prod_{1 \le l_1 < l_2 < \cdots < l_k \le s} \left(1 - \prod_{j=1}^{k} \left(1 - \left(f_{l_j}^{s o_{l_j}} \right)^{q} \right)^{\gamma_j} \right) \right)^{1/C_s^k} \right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)} \right)^{1/q} \right\rangle.
\end{aligned} \tag{18}
$$

□

Example 1. Let $cq_1 = \langle([0.5, 0.6], [0.7, 0.8]), (0.55, 0.75)\rangle$, $cq_2 = \langle([0.4, 0.6], [0.2, 0.4]), (0.5, 0.3)\rangle$, $cq_3 = \langle([0.65, 0.75], [0.3, 0.35]), (0.75, 0.35)\rangle$, and $cq_4 = \langle([0.85, 0.95], [0.1, 0.15]), (0.95, 0.15)\rangle$ be any four Cq-ROFNs. Then, by utilizing Cq-ROFPGMSM$^{(k,\gamma_1,\gamma_2,\cdots,\gamma_k)}$ to get the comprehensive value, the following steps should followed:

Step 1: discover the support degrees $\mathrm{Sup}(Cq_d, Cq_x)(d, x = 1, 2, \cdots, 4)$ by utilizing Equation (6), and then, we can have

$$
\begin{aligned}
\mathrm{Sup}(cq_1, cq_2) &= \mathrm{Sup}(cq_2, cq_1) = 0.7866, \mathrm{Sup}(cq_1, cq_3) \\
&= \mathrm{Sup}(cq_3, cq_1) = 0.7041, \mathrm{Sup}(cq_1, cq_4) \\
&= \mathrm{Sup}(cq_4, cq_1) = 0.4849,
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{Sup}(cq_2, cq_3) &= \mathrm{Sup}(cq_3, cq_2) = 0.8718 \mathrm{Sup}(cq_2, cq_4) \\
&= \mathrm{Sup}(cq_4, cq_2) = 0.6641, \mathrm{Sup}(cq_3, cq_4) \\
&= \mathrm{Sup}(cq_4, cq_3) = 0.7808.
\end{aligned} \tag{19}
$$

Step 2: discover the power weight vector $T(cq_d)(d = 1, 2, 3, 4)$, by utilizing Equation (11). So we can have

$$
\begin{aligned}
T(cq_1) &= \mathrm{Sup}(cq_1, cq_2) + \mathrm{Sup}(cq_1, cq_3) + \mathrm{Sup}(cq_1, cq_4) \\
&= 0.7866 + 0.7041 + 0.4849 = 1.9757,
\end{aligned}
$$

$$
T(cq_2) = 2.3225, T(cq_3) = 2.3567, T(cq_4) = 1.9298,
$$

$$
\begin{aligned}
\mathrm{o}_1 &= \frac{(1 + T(Cq_1))}{((1 + T(Cq_1)) + (1 + T(Cq_2)) + (1 + T(Cq_2)) + (1 + T(Cq_2)))} \\
&= 0.2365,
\end{aligned}
$$

$$
\mathrm{o}_2 = 0.2640, \mathrm{o}_3 = 0.2668, \mathrm{o}_4 = 0.2328. \tag{20}
$$

Step 3: discover the overall Cq-ROFN $cq = \langle([a, b], [c, d]), (e, f)\rangle$ by utilizing Equation (13); we have (assume $q = 3$, $\gamma_1 = 1, \gamma_2 = 2$)

$$
\begin{aligned}
&\text{Cq-ROFPGMSM}^{(2,1,2)}(Cq_1, Cq_2, Cq_3, Cq_4) \\
&= \left\langle \left(\left[\left(\left(\left(1 - \left(\prod_{1 \le l_1 < l_2 \le 4} \left(1 - \prod_{j=1}^{2} \left(1 - \left(1 - a_{l_j}^{3} \right)^{4 o_{l_j}} \right)^{\gamma_j} \right) \right) \right)^{1/6} \right)^{1/3} \right)^{1/3}, \right. \right. \\
&\times \left. \left(\left(\left(1 - \left(\prod_{1 \le l_1 < l_2 \le 4} \left(1 - \prod_{j=1}^{2} \left(1 - \left(1 - b_{l_j}^{3} \right)^{4 o_{l_j}} \right)^{\gamma_j} \right) \right) \right)^{1/6} \right)^{1/3} \right)^{1/3} \right], \\
&\times \left[\left(1 - \left(1 - \left(\prod_{1 \le l_1 < l_2 \le 4} \left(1 - \prod_{j=1}^{2} \left(1 - \left(c_{l_j}^{4 o_{l_j}} \right)^{3} \right)^{\gamma_j} \right) \right)^{1/6} \right)^{1/3} \right)^{1/3}, \right. \\
&\times \left. \left. \left(1 - \left(1 - \left(\prod_{1 \le l_1 < l_2 \le 4} \left(1 - \prod_{j=1}^{2} \left(1 - \left(d_{l_j}^{4 o_{l_j}} \right)^{3} \right)^{\gamma_j} \right) \right)^{1/6} \right)^{1/3} \right)^{1/3} \right] \right), \\
&\times \left(\left(\left(1 - \left(\prod_{1 \le l_1 < l_2 \le 4} \left(1 - \prod_{j=1}^{2} \left(1 - \left(1 - e_{l_j}^{3} \right)^{4 o_{l_j}} \right)^{\gamma_j} \right) \right) \right)^{1/6} \right)^{1/3} \right)^{1/3}, \\
&\times \left. \left(1 - \left(1 - \left(\prod_{1 \le l_1 < l_2 \le 4} \left(1 - \prod_{j=1}^{2} \left(1 - \left(f_{l_j}^{4 o_{l_j}} \right)^{3} \right)^{\gamma_j} \right) \right)^{1/6} \right)^{1/3} \right)^{1/3} \right\rangle.
\end{aligned} \tag{21}
$$

Hence, we can have the overall Cq-ROFN $Cq = \langle([0.7625, 0.8426], [0.3153, 0.4136]), (0.8213, 0.3771)\rangle$.

Theorem 12 (idempotency). *Let $Cq = \langle([a, b], [c, d]), (e, f)\rangle$ and $Cq_l = \langle([a_l, b_l], [c_l, d_l]), (e_l, f_l)\rangle, (l = 1, 2, \cdots, s)$ be two sets of Cq-ROFNs. When $Cq = Cq_i (i = 1, 2, \cdots, n)$, we have*

$$
\textit{Cq-ROFPGMSM}^{(k,\gamma_1,\gamma_2,\cdots,\gamma_k)}(Cq_1, Cq_2, \cdots, Cq_s) = Cq. \tag{22}
$$

Proof. When $Cq = Cq_l (l = 1, 2, \cdots, s)$, we have $s(1 + T(Cq_{l_j}))/\sum_{z=1}^{s}(1 + T(Cq_z)) = s(1 + T(Cq))/\sum_{z=1}^{s}(1 + T(Cq)) = 1$. So, from Theorem 11, we have

$$
\begin{aligned}
&\text{Cq-ROFPGMSM}^{(k,\gamma_1,\gamma_2,\cdots,\gamma_k)}(Cq_1, Cq_2, \cdots, Cq_s) \\
&= \left\langle \left(\left[\left(\left(\left(1 - \left(\prod_{1 \le l_1 < l_2 < \cdots < l_k \le s} \left(1 - \prod_{j=1}^{k} \left(1 - \left(1 - a_{l_j}^{q} \right)^{s o_{l_j}} \right)^{\gamma_j} \right) \right) \right)^{1/C_s^k} \right)^{1/q} \right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}, \right. \right. \\
&\times \left. \left(\left(\left(1 - \left(\prod_{1 \le l_1 < l_2 < \cdots < l_k \le s} \left(1 - \prod_{j=1}^{k} \left(1 - \left(1 - b_{l_j}^{q} \right)^{s o_{l_j}} \right)^{\gamma_j} \right) \right) \right)^{1/C_s^k} \right)^{1/q} \right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)} \right], \\
&\times \left[\left(1 - \left(1 - \left(\prod_{1 \le l_1 < l_2 < \cdots < l_k \le s} \left(1 - \prod_{j=1}^{k} \left(1 - \left(c_{l_j}^{s o_{l_j}} \right)^{q} \right)^{\gamma_j} \right) \right)^{1/C_s^k} \right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)} \right)^{1/q}, \right. \\
&\times \left. \left. \left(1 - \left(1 - \left(\prod_{1 \le l_1 < l_2 < \cdots < l_k \le s} \left(1 - \prod_{j=1}^{k} \left(1 - \left(d_{l_j}^{s o_{l_j}} \right)^{q} \right)^{\gamma_j} \right) \right)^{1/C_s^k} \right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)} \right)^{1/q} \right] \right),
\end{aligned}
$$

$$\times\left(\left(\left(1-\left(\prod_{1\leq l_1<l_2<\cdots<l_k\leq s}\left(1-\prod_{j=1}^{k}\left(1-\left(1-e_{l_j}^{q}\right)^{so_{l_j}}\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)},$$

$$\times\left(1-\left(1-\left(\prod_{1\leq l_1<l_2<\cdots<l_k\leq s}\left(1-\prod_{j=1}^{k}\left(1-\left(f_{l_j}^{so_{l_j}}\right)^{q}\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q}\Bigg)\Bigg)\Bigg\rangle$$

$$=\Bigg\langle\Bigg(\Bigg[\left(\left(\left(1-\left(\prod_{11\leq l_1<l_2<\cdots<l_k\leq s}\left(1-\prod_{j=1}^{k}\left(1-\left(1-a^{q}\right)\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)},$$

$$\times\left(\left(1-\left(\prod_{1\leq l_1<l_2<\cdots<l_k\leq s}\left(1-\prod_{j=1}^{k}\left(1-\left(1-b^{q}\right)\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\Bigg],$$

$$\times\left[\left(1-\left(1-\left(\prod_{1\leq l_1<l_2<\cdots<l_k\leq s}\left(1-\prod_{j=1}^{k}\left(1-(c)^{q}\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q},\right.$$

$$\left.\times\left(1-\left(1-\left(\prod_{1\leq l_1<l_2<\cdots<l_k\leq s}\left(1-\prod_{j=1}^{k}\left(1-(d)^{q}\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q}\right]\Bigg),$$

$$\times\left(\left(\left(1-\left(\prod_{1\leq l_1<l_2<\cdots<l_k\leq s}\left(1-\prod_{j=1}^{k}\left(1-\left(1-e^{q}\right)\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)},\right.$$

$$\left.\times\left(1-\left(1-\left(\prod_{1\leq l_1<l_2<\cdots<l_k\leq s}\left(1-\prod_{j=1}^{k}\left(1-(f)^{q}\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q}\right)\Bigg\rangle$$

$$=\Bigg(\Bigg[\left(\left(\left(1-\left(\prod_{11\leq l_1<l_2<\cdots<l_k\leq s}\left(1-\prod_{j=1}^{k}\left(1-\left(1-a^{q}\right)\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)},$$

$$\times\left(\left(1-\left(\prod_{1\leq l_1<l_2<\cdots<l_k\leq s}\left(1-\prod_{j=1}^{k}\left(1-\left(1-b^{q}\right)\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\Bigg]$$

$$=\Bigg(\Bigg[\left(\left(\left(1-\left(\prod_{11\leq l_1<l_2<\cdots<l_k\leq s}\left(1-\prod_{j=1}^{k}\left(1-\left(1-a^{q}\right)\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)},$$

$$\times\left(\left(1-\left(\prod_{1\leq l_1<l_2<\cdots<l_k\leq s}\left(1-\prod_{j=1}^{k}\left(1-\left(1-b^{q}\right)\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\Bigg]$$

$$=\Bigg(\Bigg[\left(\left(\left(1-\left(\prod_{11\leq l_1<l_2<\cdots<l_k\leq s}\left(1-\prod_{j=1}^{k}\left(1-\left(1-a^{q}\right)\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)},$$

$$\times\left(\left(1-\left(\prod_{1\leq l_1<l_2<\cdots<l_k\leq s}\left(1-\prod_{j=1}^{k}\left(1-\left(1-b^{q}\right)\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\Bigg]$$

$$=\Bigg(\Bigg[\left(\left(\left(1-\left(\prod_{11\leq l_1<l_2<\cdots<l_k\leq s}\left(1-\prod_{j=1}^{k}\left(1-\left(1-a^{q}\right)\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)},$$

$$\times\left(\left(1-\left(\prod_{1\leq l_1<l_2<\cdots<l_k\leq s}\left(1-\prod_{j=1}^{k}\left(1-\left(1-b^{q}\right)\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\Bigg]$$

$$=\Bigg\langle\Bigg(\Bigg[\left(\left(\left(1-\left(\prod_{11\leq l_1<l_2<\cdots<l_k\leq s}\left(1-\prod_{j=1}^{k}\left(1-\left(1-a^{q}\right)\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)},$$

$$\times\left(\left(1-\left(\prod_{1\leq l_1<l_2<\cdots<l_k\leq s}\left(1-\prod_{j=1}^{k}\left(1-\left(1-b^{q}\right)\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\Bigg],$$

$$\times\left[\left(1-\left(1-\left(\prod_{1\leq l_1<l_2<\cdots<l_k\leq s}\left(1-\prod_{j=1}^{k}\left(1-(c)^{q}\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q},\right.$$

$$\left.\times\left(1-\left(1-\left(\prod_{1\leq l_1<l_2<\cdots<l_k\leq s}\left(1-\prod_{j=1}^{k}\left(1-(d)^{q}\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q}\right]\Bigg),$$

$$\times\left(\left(\left(1-\left(\prod_{1\leq l_1<l_2<\cdots<l_k\leq s}\left(1-\prod_{j=1}^{k}\left(1-\left(1-e^{q}\right)\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)},\right.$$

$$\left.\times\left(1-\left(1-\left(\prod_{1\leq l_1<l_2<\cdots<l_k\leq s}\left(1-\prod_{j=1}^{k}\left(1-(f)^{q}\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q}\right)\Bigg\rangle,$$

$$=\Bigg\langle\Bigg(\Bigg[\left(\left(\left(1-\left(\prod_{1\leq l_1<l_2<\cdots<l_k\leq s}\left(1-\prod_{j=1}^{k}a^{q\times\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)},$$

$$\times\left(\left(1-\left(\prod_{1\leq l_1<l_2<\cdots<l_k\leq s}\left(1-\prod_{j=1}^{k}b^{q\times\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\Bigg],$$

$$\times\left[\left(1-\left(1-\left(\prod_{1\leq l_1<l_2<\cdots<l_k\leq s}\left(1-\prod_{j=1}^{k}\left(1-c^{q}\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q},\right.$$

$$\left.\times\left(1-\left(1-\left(\prod_{1\leq l_1<l_2<\cdots<l_k\leq s}\left(1-\prod_{j=1}^{k}\left(1-d^{q}\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q}\right]\Bigg),$$

$$\times\left(\left(\left(1-\left(\prod_{1\leq l_1<l_2<\cdots<l_k\leq s}\left(1-\prod_{j=1}^{k}e^{q\times\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)},\right.$$

$$\left.\times\left(1-\left(1-\left(\prod_{1\leq l_1<l_2<\cdots<l_k\leq s}\left(1-\prod_{j=1}^{k}\left(1-f^{q}\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q}\right)\Bigg\rangle$$

$$=\Bigg\langle\Bigg(\Bigg[\left(\left(\left(1-\left(\prod_{1\leq l_1<l_2<\cdots<l_k\leq s}\left(1-a^{q\times(\gamma_1,\gamma_2,\cdots,\gamma_k)}\right)\right)^{1/C_s^k}\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)},$$

$$\times\left(\left(1-\left(\prod_{1\leq l_1<l_2<\cdots<l_k\leq s}\left(1-b^{q\times(\gamma_1,\gamma_2,\cdots,\gamma_k)}\right)\right)^{1/C_s^k}\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\Bigg],$$

$$\times\left[\left(1-\left(1-\left(\prod_{1\leq l_1<l_2<\cdots<l_k\leq s}\left(1-\left(1-c^{q}\right)^{(\gamma_1,\gamma_2,\cdots,\gamma_k)}\right)\right)^{1/C_s^k}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q},\right.$$

$$\left.\times\left(1-\left(1-\left(\prod_{1\leq l_1<l_2<\cdots<l_k\leq s}\left(1-\left(1-d^{q}\right)^{(\gamma_1,\gamma_2,\cdots,\gamma_k)}\right)\right)^{1/C_s^k}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q}\right]\Bigg),$$

$$\times\left(\left(\left(1-\left(\prod_{1\leq l_1<l_2<\cdots<l_k\leq s}\left(1-e^{q\times(\gamma_1,\gamma_2,\cdots,\gamma_k)}\right)\right)^{1/C_s^k}\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)},\right.$$

$$\left.\times\left(1-\left(1-\left(\prod_{1\leq l_1<l_2<\cdots<l_k\leq s}\left(1-\left(1-f^{q}\right)^{(\gamma_1,\gamma_2,\cdots,\gamma_k)}\right)\right)^{1/C_s^k}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q}\right)\Bigg\rangle$$

$$=\Bigg\langle\Bigg(\Bigg[\left(\left(1-\left(1-a^{q\times(\gamma_1,\gamma_2,\cdots,\gamma_k)}\right)\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)},$$

$$\times\left(\left(1-\left(1-b^{q\times(\gamma_1,\gamma_2,\cdots,\gamma_k)}\right)\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\Bigg],$$

$$\times\left[\left(1-\left(1-\left(1-\left(1-c^{q}\right)^{(\gamma_1,\gamma_2,\cdots,\gamma_k)}\right)\right)\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q},$$

$$\times\left(1-\left(1-\left(1-(1-d^q)^{(\gamma_1,\gamma_2,\cdots,\gamma_k)}\right)\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q}\Bigg]\Bigg),$$
$$\times\left(\left(\left(1-\left(1-e^{q\times(\gamma_1,\gamma_2,\cdots,\gamma_k)}\right)\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)},\right.$$
$$\left.\times\left(1-\left(1-\left(1-(1-f^q)^{(\gamma_1,\gamma_2,\cdots,\gamma_k)}\right)\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q}\right)\Bigg\rangle$$
$$=\Bigg\langle\Bigg(\Bigg[\left(a^{(\gamma_1,\gamma_2,\cdots,\gamma_k)}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)},\left(b^{(\gamma_1,\gamma_2,\cdots,\gamma_k)}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\Bigg],$$
$$\times\Bigg[\left(1-\left((1-c^q)^{(\gamma_1,\gamma_2,\cdots,\gamma_k)}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q},$$
$$\times\left(1-\left((1-d^q)^{(\gamma_1,\gamma_2,\cdots,\gamma_k)}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q}\Bigg]\Bigg),\Bigg(\left(e^{(\gamma_1,\gamma_2,\cdots,\gamma_k)}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)},$$
$$\times\left(1-\left((1-f^q)^{(\gamma_1,\gamma_2,\cdots,\gamma_k)}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q}\Bigg)\Bigg\rangle=\langle([a,b],[c,d]),(e,f)\rangle=Cq. \tag{23}$$

□

Theorem 13 (boundedness). *Let $Cq_l=\langle([a_l,b_l],[c_l,d_l]),(e_l,f_l)\rangle,(l=1,2,\cdots,s)$ be a set of Cq-ROFNs, $\chi=\min(Cq_1,Cq_2,\cdots,Cq_s)$ and $\alpha=\max(Cq_1,Cq_2,\cdots,Cq_s)$, then*

$$\chi\le \text{Cq-ROFPGMSM}^{(k,\gamma_1,\gamma_2,\cdots,\gamma_k)}(Cq_1,Cq_2,\cdots,Cq_s)\le\alpha. \tag{24}$$

Proof. Since, $\chi\le Cq_l\le\alpha$, then

$$\begin{aligned}&\left(\frac{\sum_{1\le l_1<l_2<\cdots<l_k\le s}\left(\prod_{j=1}^{k}\left(sU_{l_j}\chi_{l_j}\right)^{\gamma_j}\right)}{C_s^k}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\\&\le\left(\frac{\sum_{1\le l_1<l_2<\cdots<l_k\le s}\left(\prod_{j=1}^{k}\left(\mathrm{so}_{l_j}Cq_{l_j}\right)^{\gamma_j}\right)}{C_s^k}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\\&\le\left(\frac{\sum_{1\le l_1<l_2<\cdots<l_k\le s}\left(\prod_{j=1}^{k}\left(\mathrm{so}_{l_j}\alpha_{l_j}\right)^{\gamma_j}\right)}{C_s^k}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}.\end{aligned} \tag{25}$$

Therefore,

$$\begin{aligned}&\text{Cq-ROFPGMSM}^{(k,\gamma_1,\gamma_2,\cdots,\gamma_k)}(\chi_1,\chi_2,\cdots,\chi_s)\\&\le\text{Cq-ROFPGMSM}^{(k,\gamma_1,\gamma_2,\cdots,\gamma_k)}(Cq_1,Cq_2,\cdots,Cq_s)\\&\le\text{Cq-ROFPGMSM}^{(k,\gamma_1,\gamma_2,\cdots,\gamma_k)}(\alpha_1,\alpha_2,\cdots,\alpha_s).\end{aligned} \tag{26}$$

Hence,

$$\chi\le\text{Cq-ROFPGHAM}^{(k,\gamma_1,\gamma_2,\cdots,\gamma_k)}(Cq_1,Cq_2,\cdots,Cq_s)\le\alpha. \tag{27}$$

The Cq-ROFPGMSM operator does not satisfy the property of monotonicity.

In addition, we can acquire some novel aggregation operators by giving distinct values for the parameters as follows:

(1) When $k=2$ and $\gamma_1=\gamma_2=p$, the developed Cq-ROFPGMSM operator degenerates into the cubic q-rung orthopair fuzzy power Bonferroni mean (Cq-ROFPBM) operator (when $p=r$), as shown below

$$\text{Cq}-\text{ROFPGMSM}^{(2,p,p)}(Cq_1,Cq_2,\cdots,Cq_s)$$
$$=\left(\frac{\sum_{1\le l_1<l_2\le s}\left(\prod_{j=1}^{2}\left(\mathrm{so}_{l_j}Cq_{l_j}\right)^{p}\right)}{C_s^2}\right)^{1/2p}=\left(\frac{\sum_{1\le l_1<l_2\le s}\left(\left(\mathrm{so}_{l_1}Cq_{l_1}\right)^{p}\otimes\left(\mathrm{so}_{l_2}Cq_{l_2}\right)^{p}\right)}{s(s-1)/2}\right)^{1/2p}$$
$$\times\left(\frac{2\sum_{1\le l_1<l_2\le n}\left((\mathrm{so}_lCq_l)^p\otimes\left(\mathrm{so}_jCq_j\right)^p\right)}{s(s-1)}\right)^{1/2p}$$
$$=\left(\frac{1}{s(s-1)}\sum_{\substack{l,j=1\\l\ne j}}^{s}\left((\mathrm{so}_lCq_l)^p\otimes\left(\mathrm{so}_jCq_j\right)^p\right)\right)^{1/2p}=\text{Cq}-\text{ROFPBM}^{(p,p)}(Cq_1,Cq_2,\cdots,Cq_s)$$
$$=\Bigg\langle\Bigg(\Bigg[\left(\left(1-\left(\prod_{\substack{l=j=1\\l\ne j}}^{s}\left(1-\left(1-(1-a_l^q)^{s\mathrm{o}_l}\right)^p\left(1-\left(1-a_j^q\right)^{s\mathrm{o}_j}\right)^p\right)\right)^{1/(s(s-1))}\right)^{1/q}\right)^{1/2p},$$
$$\times\left(\left(1-\left(\prod_{\substack{l=j=1\\l\ne j}}^{s}\left(1-\left(1-(1-b_l^q)^{s\mathrm{o}_l}\right)^p\left(1-\left(1-b_j^q\right)^{s\mathrm{o}_j}\right)^p\right)\right)^{1/(s(s-1))}\right)^{1/q}\right)^{1/2p}\Bigg],$$
$$\times\Bigg[\left(1-\left(1-\left(\prod_{\substack{l=j=1\\l\ne j}}^{s}\left(1-\left(1-(c_l^q)^{s\mathrm{o}_l}\right)^p\left(1-\left(c_j^q\right)^{s\mathrm{o}_l}\right)^p\right)\right)^{1/(s(s-1))}\right)^{1/2p}\right)^{1/q},$$
$$\times\left(1-\left(1-\left(\prod_{\substack{l=j=1\\l\ne j}}^{s}\left(1-\left(1-(d_l^q)^{s\mathrm{o}_l}\right)^p\left(1-\left(d_j^q\right)^{s\mathrm{o}_l}\right)^p\right)\right)^{1/(s(s-1))}\right)^{1/2p}\right)^{1/q}\Bigg]\Bigg),$$
$$\times\Bigg(\left(\left(1-\left(\prod_{\substack{l=j=1\\l\ne j}}^{s}\left(1-\left(1-(1-e_l^q)^{s\mathrm{o}_l}\right)^p\left(1-\left(1-e_j^q\right)^{s\mathrm{o}_j}\right)^p\right)\right)^{1/(s(s-1))}\right)^{1/q}\right)^{1/2p},$$
$$\times\left(1-\left(1-\left(\prod_{\substack{l=j=1\\l\ne j}}^{s}\left(1-\left(1-(f_l^q)^{s\mathrm{o}_i}\right)^p\left(1-\left(f_j^q\right)^{s\mathrm{o}_j}\right)^p\right)\right)^{1/(s(s-1))}\right)^{1/2p}\right)^{1/q}\Bigg)\Bigg\rangle=$$
$$\times\Bigg\langle\Bigg(\Bigg[\left(\left(1-\left(\prod_{\substack{l=j=1\\l\ne j}}^{s}\left(1-\left(1-(1-a_l^q)^{s\mathrm{o}_l}\right)^p\left(1-\left(1-a_j^q\right)^{s\mathrm{o}_j}\right)^p\right)\right)^{1/(s(s-1))}\right)^{1/q}\right)^{1/2p},$$
$$\times\left(\left(1-\left(\prod_{\substack{l=j=1\\l\ne j}}^{s}\left(1-\left(1-(1-b_l^q)^{s\mathrm{o}_l}\right)^p\left(1-\left(1-b_j^q\right)^{s\mathrm{o}_j}\right)^p\right)\right)^{1/(s(s-1))}\right)^{1/q}\right)^{1/2p}\Bigg],$$
$$\times\Bigg[\left(1-\left(1-\left(\prod_{\substack{l=j=1\\l\ne j}}^{s}\left(1-\left(1-(c_l^q)^{s\mathrm{o}_l}\right)^p\left(1-\left(c_j^q\right)^{s\mathrm{o}_l}\right)^p\right)\right)^{1/(s(s-1))}\right)^{1/2p}\right)^{1/q},$$

$$
\begin{aligned}
&\times\left(1-\left(1-\left(\prod_{\substack{l=j=1\\ l\neq j}}^{s}\left(1-\left(1-\left(d_l^q\right)^{so_l}\right)^p\left(1-\left(d_j^q\right)^{so_j}\right)^p\right)\right)^{1/(s(s-1))}\right)^{1/2p}\right)^{1/q}\Bigg]\Bigg),\\
&\Bigg[\times\left(1-\left(1-\left(\prod_{\substack{l=j=1\\ l\neq j}}^{s}\left(1-\left(1-\left(c_l^q\right)^{so_l}\right)^p\left(1-\left(c_j^q\right)^{so_j}\right)^p\right)\right)^{1/(s(s-1))}\right)^{1/2p}\right)^{1/q},\\
&\times\left(1-\left(1-\left(\prod_{\substack{l=j=1\\ l\neq j}}^{s}\left(1-\left(1-\left(d_l^q\right)^{so_l}\right)^p\left(1-\left(d_j^q\right)^{so_j}\right)^p\right)\right)^{1/(s(s-1))}\right)^{1/2p}\right)^{1/q}\Bigg]\Bigg),\\
&\times\left(\left(\left(1-\left(\prod_{\substack{l=j=1\\ l\neq j}}^{s}\left(1-\left(1-\left(1-e_l^q\right)^{so_l}\right)^p\left(1-\left(1-e_j^q\right)^{so_j}\right)^p\right)\right)^{1/(s(s-1))}\right)^{1/q}\right)^{1/2p},\\
&\times\left(1-\left(1-\left(\prod_{\substack{l=j=1\\ l\neq j}}^{s}\left(1-\left(1-\left(f_l^q\right)^{so_l}\right)^p\left(1-\left(f_j^q\right)^{so_j}\right)^p\right)\right)^{1/(s(s-1))}\right)^{1/2p}\right)^{1/q}\Bigg\rangle.
\end{aligned}
\tag{28}
$$

(2) When $k=3$ and $\gamma_1=\gamma_2=\gamma_3=p$, the developed Cq-ROFPGMSM operator degenerates into the cubic q-rung orthopair fuzzy power generalized Bonferroni mean (Cq-ROFPGBM) operator (when $p=r=s$), as shown below

$$
\begin{aligned}
&\mathrm{Cq-ROFPGMSM}^{(2,p,p)}(Cq_1,Cq_2,\cdots,Cq_s)=\left(\frac{\sum_{1\le l_1<l_2<l_3\le s}\left(\prod_{j=1}^{3}\left(so_{l_j}Cq_{l_j}\right)^p\right)}{C_s^3}\right)^{1/3p}\\
&=\left(\frac{(1/3!)\sum_{\substack{_1,l_2,l_3=1\\ l_1\neq l_2\neq l_3}}^{s}\left(\left(so_{l_1}Cq_{l_1}\right)^p\otimes\left(so_{l_2}Cq_{l_2}\right)^p\otimes\left(so_{l_3}Cq_{l_3}\right)^p\right)}{(s(s-1)(s-2))/3!}\right)^{1/3p}\\
&=\left(\frac{\sum_{\substack{i,j,l=1\\ i\neq j\neq l}}^{s}\left(\left(so_iCq_i\right)^p\otimes\left(so_jCq_j\right)^p\otimes\left(so_lCq_l\right)^p\right)}{s(s-1)(s-2)}\right)^{1/2p}\\
&=\left(\frac{1}{s(s-1)(s-2)}\sum_{\substack{i,j,l=1\\ i\neq j\neq l}}^{s}\left(\left(so_iCq_i\right)^p\otimes\left(so_jCq_j\right)^p\otimes\left(so_lCq_l\right)^p\right)\right)^{1/2p}\\
&=\mathrm{Cq-ROFPGBM}^{(p,p,p)}(Cq_1,Cq_2,\cdots,Cq_s)\\
&=\Bigg\langle\Bigg(\Bigg[\Bigg(\Bigg(1-\left(\prod_{\substack{=j=\\ l=1\\ i\neq j\neq l}}^{s}\left(1-\left(1-\left(1-a_i^q\right)^{so_i}\right)^p\right.\right.\\
&\times\left(1-\left(1-a_j^q\right)^{so_j}\right)^p\left(1-\left(1-a_l^q\right)^{so_l}\right)^p\Big)\Big)^{1/(s(s-1)(s-2))}\Big)^{1/q}\Big)^{1/3p},\\
&\times\left(\left(\left(1-\left(\prod_{\substack{=j=\\ l=1\\ i\neq j\neq l}}^{s}\left(1-\left(1-\left(1-b_i^q\right)^{so_i}\right)^p\right.\right.\right.\right.\\
&\times\left(1-\left(1-b_j^q\right)^{so_j}\right)^p\left(1-\left(1-b_l^q\right)^{so_l}\right)^p\Big)\Big)^{1/(s(s-1)(s-2))}\Big)^{1/q}\Big)^{1/3p}\Bigg],\\
&\Bigg[\times\left(\left(1-\left(1-\left(\prod_{\substack{=j=\\ l=1\\ i\neq j\neq l}}^{s}\left(1-\left(1-\left(c_i^q\right)^{so_i}\right)^p\right.\right.\right.\right.\\
&\times\left(1-\left(c_j^q\right)^{so_j}\right)^p\left(1-\left(c_l^q\right)^{so_l}\right)^p\Big)\Big)^{1/(s(s-1)(s-2))}\Big)^{1/3p}\Big)^{1/q},\\
&\times\left(1-\left(1-\left(\prod_{\substack{=j=\\ l=1\\ i\neq j\neq l}}^{s}\left(1-\left(1-\left(d_i^q\right)^{so_i}\right)^p\right.\right.\right.\\
&\times\left(1-\left(d_j^q\right)^{so_j}\right)^p\left(1-\left(d_l^q\right)^{so_l}\right)^p\Big)\Big)^{1/(s(s-1)(s-2))}\Big)^{1/3p}\Big)^{1/q}\Bigg]\Bigg),\\
&\times\left(\left(\left(1-\left(\prod_{\substack{=j=\\ l=1\\ i\neq j\neq l}}^{s}\left(1-\left(1-\left(1-e_i^q\right)^{so_i}\right)^p\right.\right.\right.\right.\\
&\times\left(1-\left(1-e_j^q\right)^{so_j}\right)^p\left(1-\left(1-e_l^q\right)^{so_j}\right)^p\Big)\Big)^{1/(s(s-1)(s-2))}\Big)^{1/q}\Big)^{1/3p},\\
&\times\left(1-\left(1-\left(\prod_{\substack{=j=\\ l=1\\ i\neq j\neq l}}^{s}\left(1-\left(1-\left(f_i^q\right)^{so_i}\right)^p\right.\right.\right.\\
&\times\left(1-\left(f_j^q\right)^{so_i}\right)^p\left(1-\left(f_l^q\right)^{so_i}\right)^p\Big)\Big)^{1/(s(s-1)(s-2))}\Big)^{1/3p}\Big)^{1/q}\Bigg)\Bigg\rangle.
\end{aligned}
\tag{29}
$$

(3) When $\gamma_1=\gamma_2=\cdots=\gamma_n=1$, the developed Cq-ROFPGMSM operator degenerates into the cubic q-rung orthopair fuzzy power MSM (Cq-ROFPMSM) operator, as shown below.

$$
\begin{aligned}
&\mathrm{Cq-ROFPGMSM}^{(k,1,,\cdots,1)}(Cq_1, Cq_2, \cdots, Cq_s)\\
&= \left(\frac{\sum_{1\le l_1<l_2<l_3\le s}\left(\prod_{j=1}^{k}\left(so_{l_j}Cq_{l_j}\right)^{1}\right)}{C_s^k}\right)^{1/(1+1+\cdots+1)}\\
&= \left(\frac{\sum_{1\le l_1<l_2<l_3\le s}\left(\prod_{j=1}^{k}\left(so_{l_j}Cq_{l_j}\right)\right)}{C_s^3}\right)^{1/k} = \mathrm{Cq-ROFPMSM}^{(k)}(Cq_1, Cq_2, \cdots, Cq_s)\\
&= \left\langle \left(\left[\left(\left(1-\left(\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{s}\left(1-\left(1-a_{l_j}^{q}\right)^{so_{l_j}}\right)\right)\right)^{1/C_s^k}\right)^{1/q}\right)^{1/k},\right.\right.\\
&\quad \left.\times\left(\left(1-\left(\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{s}\left(1-\left(1-b_{l_j}^{q}\right)^{so_{l_j}}\right)\right)\right)^{1/C_s^k}\right)^{1/q}\right)^{1/k}\right],\\
&\quad \times\left[\left(1-\left(1-\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(c_{l_j}^{so_{l_j}}\right)^{q}\right)\right)\right)^{1/C_s^k}\right)^{1/k}\right)^{1/q},\\
&\quad \left.\times\left(1-\left(1-\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(d_{l_j}^{so_{l_j}}\right)^{q}\right)\right)\right)^{1/C_s^k}\right)^{1/k}\right)^{1/q}\right]\Bigg),\\
&\quad \times\left(\left(\left(1-\left(\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{s}\left(1-\left(1-e_{l_j}^{q}\right)^{so_{l_j}}\right)\right)\right)^{1/C_s^k}\right)^{1/q}\right)^{1/k},\\
&\quad \left.\times\left(1-\left(1-\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(f_{l_j}^{so_{l_j}}\right)^{q}\right)\right)\right)^{1/C_s^k}\right)^{1/k}\right)^{1/q}\right)\right\rangle.
\end{aligned}
\tag{30}
$$

(4) When $\gamma_1 = \gamma_2 = \cdots = \gamma_n = 1/k$, the developed Cq-ROFPGMSM operator degenerates into the cubic q-rung orthopair fuzzy power HAM (Cq-ROFP-HAM) operator, as shown below.

$$
\begin{aligned}
&\mathrm{Cq-ROFPGMSM}^{(k,1/k,1/k,\cdots,1/k)}(Cq_1, Cq_2, \cdots, Cq_s)\\
&= \left(\frac{\oplus_{1\le l_1<l_2<\cdots<l_k\le s}\left(\otimes_{j=1}^{k}\left(so_{l_j}Cq_{l_j}\right)^{1/k}\right)}{C_s^k}\right)^{1/(1/k+1/k+\cdots+1/k)}\\
&= \left(\frac{\oplus_{1\le l_1<l_2<\cdots<l_k\le s}\left(\otimes_{j=1}^{k}\left(so_{l_j}Cq_{l_j}\right)^{1/k}\right)}{C_s^3}\right)^{1} = \left(\frac{\oplus_{1\le l_1<l_2<\cdots<l_k\le s}\left(\otimes_{j=1}^{k}\left(so_{l_j}Cq_{l_j}\right)\right)^{1/k}}{C_s^3}\right)\\
&= \mathrm{Cq-ROFPHAM}^{(k)}(Cq_1, Cq_2, \cdots, Cq_s)\\
&= \left\langle \left(\left[\left(1-\left(\sum_{11\le l_1<l_2<\cdots<l_k\le s}\left(1-\left(\prod_{j=1}^{k}\left(1-\left(1-a_{l_j}^{q}\right)^{s\mathrm{U}_{l_j}}\right)\right)^{1/k}\right)\right)^{1/C_s^k}\right)^{1/q},\right.\right.\\
&\quad \left.\times\left(1-\left(\sum_{11\le l_1<l_2<\cdots<l_k\le s}\left(1-\left(\prod_{j=1}^{k}\left(1-\left(1-b_{l_j}^{q}\right)^{so_{l_j}}\right)\right)^{1/k}\right)\right)^{1/C_s^k}\right)^{1/q}\right],\\
&\quad \times\left[\left(\sum_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\left(\prod_{j=1}^{k}\left(1-\left(c_{l_j}^{so_{l_j}}\right)^{q}\right)\right)^{1/k}\right)^{1/q}\right)^{1/C_s^k},\right.\\
&\quad \left.\times\left(\sum_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\left(\prod_{j=1}^{k}\left(1-\left(d_{l_j}^{so_{l_j}}\right)^{q}\right)\right)^{1/k}\right)^{1/q}\right)^{1/C_s^k}\right]\Bigg),\\
&\quad \times\left(\left(1-\left(\sum_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\left(\prod_{j=1}^{k}\left(1-\left(1-e_{l_j}^{q}\right)^{so_{l_j}}\right)\right)^{1/k}\right)\right)^{1/C_s^k}\right)^{1/q},\right.\\
&\quad \left.\left.\times\left(\sum_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\left(\prod_{j=1}^{k}\left(1-\left(f_{l_j}^{so_{l_j}}\right)^{q}\right)\right)^{1/k}\right)^{1/q}\right)^{1/C_s^k}\right)\right\rangle.
\end{aligned}
\tag{31}
$$

In the next subpart, we present a dual form of the Cq-ROFPGMSM operators based on the Cq-ROFNs and dual MSM operator. □

3.2. The Cq-ROFPGDMSM Operator

Definition 14. Let $Cq_1, Cq_2, \cdots$, and Cq_s be Cq-ROFNs, where $Cq_l = \langle([a_l, b_l], [c_l, d_l]), (e_l, f_l)\rangle, (l = 1, 2, \cdots, s), (q \ge 1)$. Then, the dual form of the Cq-ROFPDMSM operator is described as follows:

$$
\mathrm{Cq\text{-}ROFPGDMSM}^{(k,\gamma_1,\gamma_2,\cdots,\gamma_k)}(Cq_1, Cq_2, \cdots, Cq_s) = \frac{1}{\gamma_1+\gamma_2+\cdots+\gamma_k}\left(\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(\sum_{j=1}^{k}\gamma_j\left(Cq_{l_j}\right)^{s\left(1+T\left(Cq_{l_j}\right)\right)/\sum_{z=1}^{s}\left(1+T\left(Cq_z\right)\right)}\right)\right)^{1/C_s^k},
\tag{32}
$$

where $Ł_z = (1 + T(Cq_z))/\sum_{z=1}^{s}(1 + T(Cq_z)), \sum_{z=1}^{s} Ł_z = 1$, $T(Cq_j) = \prod_{z=1, z\ne j}^{s}\mathrm{Spt}(Cq_z, Cq_j)$ is the support degree for Cq_z from Cq_j, which fulfil the following conditions:

(1) $\mathrm{Spt}(Cq_z, Cq_j) \in [0, 1]$

(2) $\mathrm{Spt}(Cq_z, Cq_j) = \mathrm{Spt}(Cq_j, Cq_z)$

(3) If $\widetilde{\mathrm{DE}}(Cq_z, Cq_j) \le \widetilde{\mathrm{DE}}(Cq_s, Cq_t)$, then $\mathrm{Spt}(Cq_z, Cq_j) \ge \mathrm{Spt}(Cq_s, Cq_t)$, where $\widetilde{\mathrm{DE}}(Cq_z, Cq_j)$express the distance measure among two Cq-ROFNs described in Definition 6. $\gamma_1, \gamma_2, \cdots, \gamma_k \ge 0$, k is a parameter and $k = (1, 2, \cdots, n)$, $(l_1, l_2, \cdots, l_k)$ traverses all the k-tuple combination of $(1, 2, \cdots, n)$. The denominator C_n^k in Equation (32) expresses the binomial coefficient $s!/(k!(s-k)!)$ and s is the balancing coefficient

In order to write Equation (32) in an easy way, we can assume

$$
Ł_z = \frac{(1 + T(Cq_z))}{\sum_{z=1}^{s}(1 + T(Cq_z))}.
\tag{33}
$$

Subsequently, we identify $(\tilde{\mathrm{O}}_1, \tilde{\mathrm{O}}_2, \cdots, \tilde{\mathrm{O}}_n)$ as the power weight vector. As a result, Equation (32) can be written in an easy way as follows:

$$\text{Cq-ROFPGDMSM}^{(k,\gamma_1,\gamma_2,\cdots,\gamma_k)}(Cq_1, Cq_2, \cdots, Cq_s) = \frac{1}{\gamma_1+\gamma_2+\cdots+\gamma_k}\left(\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(\sum_{j=1}^{k}\gamma_j\left(Cq_{l_j}\right)^{s\text{Ł}_{l_j}}\right)\right)^{1/C_s^k}. \tag{34}$$

Theorem 15. *Let $Cq_1, Cq_2, \cdots$, and Cq_s be Cq-ROFNs, where $Cq_l = \langle([a_l, b_l], [c_l, d_l]), (e_l, f_l)\rangle (l = 1, 2, \cdots, s)$ $(q \ge 1)$. Then, the aggregated value utilizing Equation (34) is still Cq-ROFN, even*

$$\begin{aligned}
&Cq-ROFPGDMSM^{(k,\gamma_1,\gamma_2,\cdots,\gamma_k)}(Cq_1, Cq_2, \cdots, Cq_s)\\
&= \left\langle\left(\left[\left(1-\left(1-\left(\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(a_{l_j}^{s\text{Ł}_{l_j}}\right)^q\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q},\right.\right.\right.\\
&\times\left.\left(1-\left(1-\left(\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(b_{l_j}^{s\text{Ł}_{l_j}}\right)^q\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q}\right],\\
&\times\left[\left(\left(1-\left(\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(1-c_{l_j}^q\right)^{s\text{Ł}_{l_j}}\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)},\right.\\
&\times\left.\left.\left(\left(1-\left(\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(1-d_{l_j}^q\right)^{s\text{Ł}_{l_j}}\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right]\right),\\
&\times\left(\left(1-\left(1-\left(\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(e_{l_j}^{s\text{Ł}_{l_j}}\right)^q\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q},\right.\\
&\times\left.\left.\left(\left(1-\left(\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(1-f_{l_j}^q\right)^{s\text{Ł}_{l_j}}\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)\right\rangle.
\end{aligned} \tag{35}$$

Proof. Since

$$\begin{aligned}
\left(Cq_{l_j}\right)^{s\text{Ł}_{l_j}} &= \left\langle\left(\left[a_{l_j}^{s\text{Ł}_{l_j}}, b_{l_j}^{s\text{Ł}_{l_j}}\right], \left[\left(1-\left(1-c_{l_j}^q\right)^{s\text{Ł}_{l_j}}\right)^{1/q}, \left(1-\left(1-d_{l_j}^q\right)^{s\text{Ł}_{l_j}}\right)^{1/q}\right]\right),\right.\\
&\quad\cdot\left.\left(e_{l_j}^{s\text{Ł}_{l_j}}, \left(1-\left(1-f_{l_j}^q\right)^{s\text{Ł}_{l_j}}\right)^{1/q}\right)\right\rangle,
\end{aligned}$$

$$\begin{aligned}
\gamma_j\left(Cq_{l_j}\right)^{s\text{Ł}_{l_j}} &= \left\langle\left(\left[\left(1-\left(1-\left(a_{l_j}^{s\text{Ł}_{l_j}}\right)^q\right)^{\gamma_j}\right)^{1/q}, \left(1-\left(1-\left(b_{l_j}^{s\text{Ł}_{l_j}}\right)^q\right)^{\gamma_j}\right)^{1/q}\right],\right.\right.\\
&\quad\cdot\left.\left[\left(\left(1-\left(1-c_{l_j}^q\right)^{s\text{Ł}_{l_j}}\right)^{1/q}\right)^{\gamma_j}, \left(\left(1-\left(1-d_{l_j}^q\right)^{s\text{Ł}_{l_j}}\right)^{1/q}\right)^{\gamma_j}\right]\right),\\
&\quad\cdot\left.\left(\left(1-\left(1-\left(e_{l_j}^{s\text{Ł}_{l_j}}\right)^q\right)^{\gamma_j}\right)^{1/q}, \left(\left(1-\left(1-f_{l_j}^q\right)^{s\text{Ł}_{l_j}}\right)^{1/q}\right)^{\gamma_j}\right)\right\rangle.
\end{aligned} \tag{36}$$

Therefore,

$$\begin{aligned}
\sum_{j=1}^{k}\gamma_j\left(Cq_{l_j}\right)^{s\text{Ł}_{l_j}} &= \left\langle\left(\left[\left(1-\prod_{j=1}^{k}\left(1-\left(a_{l_j}^{s\text{Ł}_{l_j}}\right)^q\right)^{\gamma_j}\right)^{1/q},\right.\right.\right.\\
&\quad\cdot\left.\left(1-\prod_{j=1}^{k}\left(1-\left(b_{l_j}^{s\text{Ł}_{l_j}}\right)^q\right)^{\gamma_j}\right)^{1/q}\right],\\
&\quad\cdot\left[\prod_{j=1}^{k}\left(\left(1-\left(1-c_{l_j}^q\right)^{s\text{Ł}_{l_j}}\right)^{1/q}\right)^{\gamma_j}, \prod_{j=1}^{k}\right.\\
&\quad\cdot\left.\left.\left(\left(1-\left(1-d_{l_j}^q\right)^{s\text{Ł}_{l_j}}\right)^{1/q}\right)^{\gamma_j}\right]\right),\\
&\quad\cdot\left(\left(1-\prod_{j=1}^{k}\left(1-\left(e_{l_j}^{s\text{Ł}_{l_j}}\right)^q\right)^{\gamma_j}\right)^{1/q}, \prod_{j=1}^{k}\right.\\
&\quad\cdot\left.\left.\left(\left(1-\left(1-f_{l_j}^q\right)^{s\text{Ł}_{l_j}}\right)^{1/q}\right)^{\gamma_j}\right)\right\rangle.
\end{aligned} \tag{37}$$

Further,

$$\begin{aligned}
&\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(\sum_{j=1}^{k}\gamma_j\left(Cq_{l_j}\right)^{s\text{Ł}_{l_j}}\right)\\
&= \left\langle\left(\left[\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(a_{l_j}^{s\text{Ł}_{l_j}}\right)^q\right)^{\gamma_j}\right)^{1/q}, \prod_{1\le l_1<l_2<\cdots<l_k\le s}\right.\right.\right.\\
&\quad\times\left.\left(1-\prod_{j=1}^{k}\left(1-\left(b_{l_j}^{s\text{Ł}_{l_j}}\right)^q\right)^{\gamma_j}\right)^{1/q}\right],\\
&\quad\times\left[\left(1-\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(1-c_{l_j}^q\right)^{s\text{Ł}_{l_j}}\right)^{\gamma_j}\right)\right)^{1/q},\right.\\
&\quad\times\left.\left.\left(1-\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(1-d_{l_j}^q\right)^{s\text{Ł}_{l_j}}\right)^{\gamma_j}\right)\right)^{1/q}\right]\right),\\
&\quad\times\left(\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(e_{l_j}^{s\text{Ł}_{l_j}}\right)^q\right)^{\gamma_j}\right)^{1/q},\right.\\
&\quad\times\left.\left.\left(1-\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(1-f_{l_j}^q\right)^{s\text{Ł}_{l_j}}\right)^{\gamma_j}\right)\right)^{1/q}\right)\right\rangle.
\end{aligned} \tag{38}$$

So,

$$\begin{aligned}&\left(\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(\sum_{j=1}^{k}\gamma_j\left(Cq_{l_j}\right)^{s\text{Ł}_{l_j}}\right)\right)^{1/C_s^k}\\
&\quad=\left\langle\left(\left[\left(\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(a_{l_j}^{s\text{Ł}_{l_j}}\right)^{q}\right)^{\gamma_j}\right)^{1/q}\right)^{1/C_s^k},\right.\right.\right.\\
&\qquad\left.\left(\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(b_{l_j}^{s\text{Ł}_{l_j}}\right)^{q}\right)^{\gamma_j}\right)^{1/q}\right)^{1/C_s^k}\right],\\
&\qquad\left[\left(1-\left(\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(1-c_{l_j}^{q}\right)^{s\text{Ł}_{l_j}}\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/q},\right.\\
&\qquad\left.\left.\left(1-\left(\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(1-d_{l_j}^{q}\right)^{s\text{Ł}_{l_j}}\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/q}\right]\right),\\
&\qquad\left(\left(\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(e_{l_j}^{s\text{Ł}_{l_j}}\right)^{q}\right)^{\gamma_j}\right)^{1/q}\right)^{1/C_s^k},\right.\\
&\qquad\left.\left.\left(1-\left(\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(1-f_{i_j}^{q}\right)^{n\text{Ł}_{l_j}}\right)^{\gamma_j}\right)\right)^{1/C_n^k}\right)^{1/q}\right)\right\rangle.\end{aligned}\tag{39}$$

Therefore,

$$\begin{aligned}&\frac{1}{\gamma_1+\gamma_2+\cdots+\gamma_k}\left(\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(\prod_{j=1}^{k}\gamma_j\left(Cq_{l_j}\right)^{s\text{Ł}_{l_j}}\right)\right)^{1/C_s^k}\\
&\quad=\left\langle\left(\left[\left(1-\left(1-\left(\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(a_{l_j}^{s\text{Ł}_{l_j}}\right)^{q}\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q},\right.\right.\right.\\
&\qquad\left.\left(1-\left(1-\left(\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(b_{l_j}^{s\text{Ł}_{l_j}}\right)^{q}\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q}\right],\\
&\qquad\left[\left(\left(1-\left(\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(1-c_{l_j}^{q}\right)^{s\text{Ł}_{l_j}}\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)},\right.\\
&\qquad\left.\left.\left(\left(1-\left(\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(1-d_{l_j}^{q}\right)^{s\text{Ł}_{l_j}}\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right]\right),\\
&\qquad\left(\left(1-\left(1-\left(\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(e_{l_j}^{s\text{Ł}_{l_j}}\right)^{q}\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q},\right.\\
&\qquad\left.\left.\left(\left(1-\left(\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(1-f_{l_j}^{q}\right)^{s\text{Ł}_{l_j}}\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)\right\rangle,\end{aligned}\tag{40}$$

$$\begin{aligned}&Cq-\mathrm{ROFPGDMSM}^{(k,\gamma_1,\gamma_2,\cdots,\gamma_k)}(Cq_1,Cq_2,\cdots,Cq_s)\\
&\quad=\left\langle\left(\left[\left(1-\left(1-\left(\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(a_{l_j}^{s\text{Ł}_{l_j}}\right)^{q}\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q},\right.\right.\right.\\
&\qquad\left.\left(1-\left(1-\left(\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(b_{l_j}^{s\text{Ł}_{l_j}}\right)^{q}\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q}\right],\\
&\qquad\left[\left(\left(1-\left(\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(1-c_{l_j}^{q}\right)^{s\text{Ł}_{l_j}}\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)},\right.\\
&\qquad\left.\left.\left(\left(1-\left(\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(1-d_{l_j}^{q}\right)^{s\text{Ł}_{l_j}}\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right]\right),\\
&\qquad\left(\left(1-\left(1-\left(\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(e_{l_j}^{s\text{Ł}_{l_j}}\right)^{q}\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q},\right.\\
&\qquad\left.\left.\left(\left(1-\left(\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(1-f_{l_j}^{q}\right)^{s\text{Ł}_{l_j}}\right)^{\gamma_j}\right)\right)^{1/C_s^k}\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)\right\rangle.\end{aligned}\tag{41}$$

□

Example 2. Consider the four Cq-ROFNs given in Example 1. Then, by utilizing Cq-ROFPGDMSM$^{(k,\gamma_1,\gamma_2,\cdots,\gamma_k)}$ to get the comprehensive value. The following steps should be followed.

Steps 1 and 2 are the same as for Example 1.

Step 3: discover the overall Cq-ROFN $cq=\langle([a,b],[c,d]),(e,f)\rangle$ by utilizing Equation (35); we have (assume $q=3$, $\gamma_1=1,\gamma_2=2$)

$$\begin{aligned}&Cq-\mathrm{ROFPGDMSM}^{(2,1,2)}(Cq_1,Cq_2,Cq_3,Cq_4)\\
&\quad=\left\langle\left(\left[\left(1-\left(1-\left(\prod_{1\le l_1<l_2\le 4}\left(1-\prod_{j=1}^{2}\left(1-\left(a_{l_j}^{4\text{Ł}_{l_j}}\right)^{3}\right)^{\gamma_j}\right)\right)^{1/6}\right)^{1/3}\right)^{1/3},\right.\right.\right.\\
&\qquad\left.\left(1-\left(1-\left(\prod_{1\le l_1<l_2\le 4}\left(1-\prod_{j=1}^{2}\left(1-\left(b_{l_j}^{4\text{Ł}_{l_j}}\right)^{3}\right)^{\gamma_j}\right)\right)^{1/6}\right)^{1/3}\right)^{1/3}\right],\\
&\qquad\left[\left(\left(1-\left(\prod_{1\le l_1<l_2\le 4}\left(1-\prod_{j=1}^{2}\left(1-\left(1-c_{l_j}^{3}\right)^{4\text{Ł}_{l_j}}\right)^{\gamma_j}\right)\right)^{1/6}\right)^{1/3}\right)^{1/3},\right.\\
&\qquad\left.\left.\left(\left(1-\left(\prod_{1\le l_1<l_2\le 4}\left(1-\prod_{j=1}^{2}\left(1-\left(1-d_{l_j}^{3}\right)^{4\text{Ł}_{l_j}}\right)^{\gamma_j}\right)\right)^{1/6}\right)^{1/3}\right)^{1/3}\right]\right),\\
&\qquad\left(\left(1-\left(1-\left(\prod_{1\le l_1<l_2\le 4}\left(1-\prod_{j=1}^{2}\left(1-\left(e_{l_j}^{4\text{Ł}_{l_j}}\right)^{3}\right)^{\gamma_j}\right)\right)^{1/6}\right)^{1/3}\right)^{1/3},\right.\\
&\qquad\left.\left.\left(\left(1-\left(\prod_{1\le l_1<l_2\le 4}\left(1-\prod_{j=1}^{2}\left(1-\left(1-f_{l_j}^{3}\right)^{4\text{Ł}_{l_j}}\right)^{\gamma_j}\right)\right)^{1/6}\right)^{1/3}\right)^{1/3}\right)\right\rangle.\end{aligned}\tag{42}$$

Hence, we can have the overall Cq-ROFN $Cq=\langle([0.6326,0.7426],[0.5449,0.6475]),(0.7119,0.6048)\rangle$.

Now, we will discuss some basic properties of the Cq-ROFPGDMSM operators.

Theorem 16 (idempotency). *Let $Cq=\langle([a,b],[c,d]),(e,f)\rangle$ and $Cq_l=\langle([a_l,b_l],[c_l,d_l]),(e_l,f_l)\rangle,(l=1,2,\cdots,s)$ be two sets of Cq-ROFNs. When $Cq=Cq_l(l=1,2,\cdots,s)$, we have*

$$Cq\text{-}ROFPGDMSM^{(k,\gamma_1,\gamma_2,\cdots,\gamma_k)}(Cq_1,Cq_2,\cdots,Cq_s)=Cq.\tag{43}$$

Table 1: Evaluation values provided by experts E_1.

Alternatives/attributes	$\overline{\overline{\text{Cri}}}_1$	$\overline{\overline{\text{Cri}}}_2$	$\overline{\overline{\text{Cri}}}_3$	$\overline{\overline{\text{Cri}}}_4$
$\overline{\overline{\text{ale}}}_1$	$\langle[0.75, 0.85], [0.2, 0.25], (0.85, 0.25)\rangle$	$\langle[0.65, 0.75], [0.3, 0.35], (0.75, 0.35)\rangle$	$\langle[0.75, 0.85], [0.2, 0.25], (0.85, 0.25)\rangle$	$\langle[0.65, 0.75], [0.3, 0.35], (0.75, 0.35)\rangle$
$\overline{\overline{\text{ale}}}_2$	$\langle[0.85, 0.95], [0.1, 0.15], (0.95, 0.15)\rangle$	$\langle[0.75, 0.85], [0.2, 0.25], (0.85, 0.25)\rangle$	$\langle[0.65, 0.75], [0.3, 0.35], (0.75, 0.35)\rangle$	$[0.85, 0.95], [0.1, 0.15], (0.95, 0.15)\rangle$
$\overline{\overline{\text{ale}}}_2$	$\langle[0.85, 0.95], [0.1, 0.15], (0.95, 0.15)\rangle$	$\langle[0.85, 0.95], [0.1, 0.15], (0.95, 0.15)\rangle$	$\langle[0.45, 0.55], [0.45, 0.55], (0.55, 0.55)\rangle$	$\langle[0.45, 0.55], [0.45, 0.55], (0.55, 0.55)\rangle$
$\overline{\overline{\text{ale}}}_4$	$\langle[0.85, 0.95], [0.1, 0.15], (0.95, 0.15)\rangle$	$\langle[0.75, 0.85], [0.2, 0.25], (0.85, 0.25)$	$\langle[0.85, 0.95], [0.1, 0.15], (0.95, 0.15)\rangle$	$\langle[0.75, 0.85], [0.2, 0.25], (0.85, 0.25)\rangle$
$\overline{\overline{\text{ale}}}_5$	$\langle[0.75, 0.85], [0.2, 0.25], (0.85, 0.25)\rangle$	$\langle[0.65, 0.75], [0.3, 0.35], (0.75, 0.35)\rangle$	$\langle[0.65, 0.75], [0.3, 0.35], (0.75, 0.35)\rangle$	$\langle[0.65, 0.75], [0.3, 0.35], (0.75, 0.35)\rangle$

Table 2: Evaluation values provided by experts E_2.

Alternatives/attributes	$\overline{\overline{\text{Cri}}}_1$	$\overline{\overline{\text{Cri}}}_2$	$\overline{\overline{\text{Cri}}}_3$	$\overline{\overline{\text{Cri}}}_4$
$\overline{\overline{\text{ale}}}_1$	$\langle[0.65, 0.75], [0.3, 0.35], (0.75, 0.35)\rangle$	$\langle[0.75, 0.85], [0.2, 0.25], (0.85, 0.25)\rangle$	$\langle[0.65, 0.75], [0.3, 0.35], (0.75, 0.35)\rangle$	$\langle[0.85, 0.95], [0.1, 0.15], (0.95, 0.15)\rangle$
$\overline{\overline{\text{ale}}}_2$	$\langle[0.75, 0.85], [0.2, 0.25], (0.85, 0.25)\rangle$	$\langle[0.85, 0.95], [0.1, 0.15], (0.95, 0.15)\rangle$	$\langle[0.85, 0.95], [0.1, 0.15], (0.95, 0.15)\rangle$	$\langle[0.45, 0.55], [0.45, 0.55], (0.55, 0.55)\rangle$
$\overline{\overline{\text{ale}}}_3$	$\langle[0.65, 0.75], [0.3, 0.35], (0.75, 0.35)\rangle$	$\langle[0.65, 0.75], [0.2, 0.25], (0.75, 0.25)\rangle$	$\langle[0.75, 0.85], [0.2, 0.25], (0.85, 0.25)\rangle$	$\langle[0.65, 0.75], [0.3, 0.35], (0.75, 0.35)\rangle$
$\overline{\overline{\text{ale}}}_4$	$\langle[0.45, 0.55], [0.45, 0.55], (0.55, 0.55)\rangle$	$\langle[0.75, 0.85], [0.2, 0.25], (0.85, 0.25)\rangle$	$\langle[0.65, 0.75], [0.3, 0.35], (0.75, 0.35)\rangle$	$\langle[0.45, 0.55], [0.45, 0.55], (0.55, 0.55)\rangle$
$\overline{\overline{\text{ale}}}_5$	$[0.3, 0.35], [0.65, 0.75], (0.35, 0.75)\rangle$	$\langle[0.75, 0.85], [0.2, 0.25], (0.85, 0.25)\rangle$	$\langle[0.45, 0.55], [0.45, 0.55], (0.55, 0.55)\rangle$	$\langle[0.45, 0.55], [0.45, 0.55], (0.55, 0.55)\rangle$

Table 3: Evaluation values provided by experts E_3.

Alternatives/attributes	$\overline{\overline{\text{Cri}}}_1$	$\overline{\overline{\text{Cri}}}_2$	$\overline{\overline{\text{Cri}}}_3$	$\overline{\overline{\text{Cri}}}_4$
$\overline{\overline{\text{ale}}}_1$	$\langle[0.85, 0.95], [0.1, 0.15], (0.95, 0.15)\rangle$	$\langle[0.65, 0.75], [0.3, 0.35], (0.75, 0.35)\rangle$	$\langle[0.45, 0.55], [0.45, 0.55], (0.55, 0.55)\rangle$	$\langle[0.75, 0.85], [0.2, 0.25], (0.85, 0.25)\rangle$
$\overline{\overline{\text{ale}}}_2$	$\langle[0.65, 0.75], [0.3, 0.35], (0.75, 0.35)\rangle$	$\langle[0.65, 0.75], [0.3, 0.35], (0.75, 0.35)\rangle$	$\langle[0.75, 0.85], [0.2, 0.25], (0.85, 0.25)\rangle$	$\langle[0.75, 0.85], [0.2, 0.25], (0.85, 0.25)\rangle$
$\overline{\overline{\text{ale}}}_3$	$\langle[0.75, 0.85], [0.2, 0.25], (0.85, 0.25)\rangle$	$\langle[0.75, 0.85], [0.2, 0.25], (0.85, 0.25)\rangle$	$\langle[0.85, 0.95], [0.1, 0.15], (0.95, 0.15)\rangle$	$\langle[0.75, 0.85], [0.2, 0.25], (0.85, 0.25)\rangle$
$\overline{\overline{\text{ale}}}_4$	$\langle[0.2, 0.25], [0.75, 0.85], (0.25, 0.85)\rangle$	$\langle[0.85, 0.95], [0.1, 0.15], (0.95, 0.15)\rangle$	$\langle[0.75, 0.85], [0.2, 0.25], (0.85, 0.25)\rangle$	$\langle[0.65, 0.75], [0.3, 0.35], (0.75, 0.35)\rangle$
$\overline{\overline{\text{ale}}}_5$	$\langle[0.85, 0.95], [0.1, 0.15], (0.95, 0.15)\rangle$	$\langle[0.45, 0.55], [0.45, 0.55], (0.55, 0.55)\rangle$	$\langle[0.75, 0.85], [0.2, 0.25], (0.85, 0.25)\rangle$	$\langle[0.85, 0.95], [0.10.15], (0.95, 0.15)\rangle$

Proof. Proof of Theorem 16 is the same as that of Theorem 12. □

Theorem 17 (boundedness). *Let* $Cq_l = \langle([a_l, b_l], [c_l, d_l]), (e_l, f_l)\rangle, (l = 1, 2, \cdots, s)$ *be a set of Cq-ROFNs,* $\chi = \min(Cq_1, Cq_2, \cdots, Cq_s)$ *and* $\alpha = \max(Cq_1, Cq_2, \cdots, Cq_s)$*, then*

$$\chi \leq Cq\text{-}ROFPGDMSM^{(k, \gamma_1, \gamma_2, \cdots, \gamma_k)}(Cq_1, Cq_2, \cdots, Cq_s) \leq \alpha. \tag{44}$$

Table 4: Collective decision matrix $\widetilde{DMT}$ employing Cq-ROFPGWMSM operator.

Alternative/attributes	$\overline{\overline{Cri}}_1$	$\overline{\overline{Cri}}_2$	$\overline{\overline{Cri}}_3$	$\overline{\overline{Cri}}_5$
$\overline{\overline{ale}}_1$	$\langle [0.5916, 0.6899], [0.4974, 0.5405], (0.6914, 0.5405) \rangle$	$\langle [0.5275, 0.6159], [0.5551, 0.5935], (0.6159, 0.5935) \rangle$	$\langle [0.4889, 0.5750], [0.5891, 0.6368], (0.5750, 0.6368) \rangle$	$\langle [0.5791, 0.6761], [0.5128, 0.5538], (0.6761, 0.553811) \rangle$
$\overline{\overline{ale}}_2$	$\langle [0.5975, 0.6980], [0.4935, 0.5372], (0.6980, 0.5365) \rangle$	$\langle [0.5916, 0.6914], [0.4975, 0.5410], (0.6914, 0.5405) \rangle$	$\langle [0.5791, 0.6761], [0.5128, 0.5538], (0.6761, 0.5538) \rangle$	$\langle [0.5568, 0.6555], [0.5367, 0.5884], (0.6555, 0.5884) \rangle$
$\overline{\overline{ale}}_3$	$\langle [0.5975, 0.6980], [0.4935, 0.5365], (0.6980, 0.5365) \rangle$	$\langle [0.5974, 0.6979], [0.4595, 0.5067], (0.6979, 0.5067) \rangle$	$\langle [0.5136, 0.6057], [0.5772, 0.6305], (0.6068, 0.6305) \rangle$	$\langle [0.4606, 0.5448], [0.6111, 0.6619], (0.5448, 0.6619) \rangle$
$\overline{\overline{ale}}_4$	$\langle [0.4023, 0.4804], [0.6852, 0.7441], (0.4804, 0.7441) \rangle$	$\langle [0.6159, 0.7191], [0.4665, 0.5123], (0.7191, 0.5132) \rangle$	$\langle [0.5975, 0.6980], [0.4935, 0.5373], (0.6980, 0.5365) \rangle$	$\langle [0.4889, 0.5750], [0.5891, 0.6368], (0.5750, 0.6368) \rangle$
$\overline{\overline{ale}}_5$	$\langle [0.5453, 0.6398], [0.5676, 0.6179], (0.6398, 0.6179) \rangle$	$\langle [0.4851, 0.5709], [0.5922, 0.6394], (0.57087, 0.6394) \rangle$	$\langle [0.4851, 0.5709], [0.5922, 0.6394], (0.5709, 0.6260) \rangle$	$\langle [0.5140, 0.6052], [0.5744, 0.6221], (0.6052, 0.6221) \rangle$

Table 5: Collective decision matrix $\widetilde{DM}$employing Cq-ROFPGWDMSM operator.

Alternatives/ attributes	$\overline{\overline{\mathrm{Cri}}}_1$	$\overline{\overline{\mathrm{Cri}}}_2$	$\overline{\overline{\mathrm{Cri}}}_3$	$\overline{\overline{\mathrm{Cri}}}_4$
$\overline{\overline{\mathrm{ale}}}_1$	$[0.8622, 0.9180], [0.1565, 0.1937], (0.9198, 0.1937)\rangle$	$[0.8207, 0.8774]\rangle, [0.2066, 0.2445], (0.8774, 0.2445)\rangle$	$\langle[0.8010, 0.8599], [0.2294, 0.2787], (0.8599, 0.2787)\rangle$	$\langle[0.8594, 0.9181], [0.1585, 0.1965], (0.9181, 0.1965)\rangle$
$\overline{\overline{\mathrm{ale}}}_2$	$\langle[0.8712, 0.9306], [0.1427, 0.1807], (0.9306, 0.1807)\rangle$	$\langle[0.8622, 0.9198], [0.1565, 0.1936], (0.9198, 0.1936)\rangle$	$\langle[0.8594, 0.9181], [0.1585, 0.1965], (0.9181, 0.1965)\rangle$	$\langle[0.8503, 0.9140], [0.1707, 0.2197], (0.9140, 0.2197)\rangle$
$\overline{\overline{\mathrm{ale}}}_3$	$\langle[0.7940, 0.9306], [0.1427, 0.1807], (0.9306, 0.1807)\rangle$	$\langle[0.8712, 0.9306], [0.1210, 0.1589], (0.9306, 0.1589)\rangle$	$\langle[0.8319, 0.8955], [0.1919, 0.2430], (0.8955, 0.2430)\rangle$	$\langle[0.7851, 0.8445], [0.24738, 0.2992], (0.8445, 0.2992)\rangle$
$\overline{\overline{\mathrm{ale}}}_4$	$\langle[0.7738, 0.8333], [0.3134, 0.3787], (0.8333, 0.3787)\rangle$	$\langle[0.8774, 0.9342], [0.1313, 0.1688], (0.9342, 0.1688)\rangle$	$\langle[0.8712, 0.9306], [0.1427, 0.1807], (0.9306, 0.1807)\rangle$	$\langle[0.8010, 0.8599], [0.2294, 0.2787], (0.8599, 0.2787)\rangle$
$\overline{\overline{\mathrm{ale}}}_5$	$\langle[0.8179, 0.8800], [0.2303, 0.2815], (0.8800, 0.2815)\rangle$	$\langle[0.7909, 0.8494], [0.2427, 0.2925], (0.8494, 0.2925)\rangle$	$\langle[0.7909, 0.8494], [0.2427, 0.2925], (0.8494, 0.2925)\rangle$	$\langle[0.8111, 0.8705], [0.2294, 0.278015], (0.8705, 0.278015)\rangle$

Proof. Proof of Theorem 17 is the same as that of Theorem 13.

In addition, we can acquire some novel aggregation operators by giving distinct values for the parameters as follows:

(1) When $k=2$ and $\gamma_1=\gamma_2=p$, the developed Cq-ROFPGDMSM operator degenerates into the cubic q-rung orthopair fuzzy power geometric Bonferroni mean (Cq-ROFPGBM) operator (when $p=r$), as shown below

$$\begin{aligned}
&\mathrm{Cq-ROFPGDMSM}^{(2,p,p)}(Cq_1, Cq_2, \cdots, Cq_n) = \frac{1}{2p}\left(\prod_{1\le l_1<l_2\le s}\left(\sum_{j=1}^{k} p\left(Cq_{l_j}\right)^{s\text{Ł}_{l_j}}\right)\right)^{1/C_s^2} \\
&= \frac{1}{2p}\left(\prod_{1\le l_1<l_2\le s}\left(p\left(Cq_{l_1}\right)^{s\text{Ł}_{l_1}} \oplus p\left(Cq_{l_2}\right)^{s\text{Ł}_{l_2}}\right)\right)^{1/(s(s-1))} \\
&= \frac{1}{2p}\left(\prod_{\substack{l,j=1\\ l\neq j}}^{s}\left(p(Cq_l)^{s\text{Ł}_l} \oplus p\left(Cq_j\right)^{s\text{Ł}_j}\right)\right)^{1/(s(s-1))} = \mathrm{Cq-ROFPGBM}^{(p,p)}(Cq_1, Cq_2, \cdots, Cq_s) \\
&= \Bigg\langle \Bigg(\Bigg[\Bigg(1-\Bigg(1-\Bigg(\prod_{\substack{l=j=1\\ l\neq j}}^{s}\left(1-\left(1-(a_l^q)^{s\text{Ł}_l}\right)^p\left(1-\left(a_j^q\right)^{s\text{Ł}_j}\right)^p\right)\Bigg)^{1/(s(s-1))}\Bigg)^{1/2p}\Bigg)^{1/q}, \\
&\times \Bigg(1-\Bigg(1-\Bigg(\prod_{\substack{l=j=1\\ l\neq j}}^{s}\left(1-\left(1-(b_l^q)^{s\text{Ł}_l}\right)^p\left(1-\left(b_j^q\right)^{s\text{Ł}_j}\right)^p\right)\Bigg)^{1/(s(s-1))}\Bigg)^{1/2p}\Bigg)^{1/q}\Bigg], \\
&\times \Bigg[\Bigg(\Bigg(1-\Bigg(\prod_{\substack{l=j=1\\ l\neq j}}^{s}\left(1-\left(1-(1-c_l^q)^{s\text{Ł}_l}\right)^p\left(1-\left(1-c_j^q\right)^{s\text{Ł}_j}\right)^p\right)\Bigg)^{1/(s(s-1))}\Bigg)^{1/q}\Bigg)^{1/2p}, \\
&\times \Bigg(\Bigg(1-\Bigg(\prod_{\substack{l=j=1\\ l\neq j}}^{s}\left(1-\left(1-(1-d_l^q)^{s\text{Ł}_l}\right)^p\left(1-\left(1-d_j^q\right)^{s\text{Ł}_j}\right)^p\right)\Bigg)^{1/(s(s-1))}\Bigg)^{1/q}\Bigg)^{1/2p}\Bigg]\Bigg), \\
&\times \Bigg(\Bigg(1-\Bigg(1-\Bigg(\prod_{\substack{l=j=1\\ l\neq j}}^{s}\left(1-\left(1-(e_l^q)^{s\text{Ł}_l}\right)^p\left(1-\left(e_j^q\right)^{s\text{Ł}_j}\right)^p\right)\Bigg)^{1/(s(s-1))}\Bigg)^{1/2p}\Bigg)^{1/q}, \\
&\times \Bigg(\Bigg(1-\Bigg(\prod_{\substack{l=j=1\\ l\neq j}}^{s}\left(1-\left(1-(1-f_l^q)^{s\text{Ł}_l}\right)^p\left(1-\left(1-f_j^q\right)^{s\text{Ł}_j}\right)^p\right)\Bigg)^{1/(s(s-1))}\Bigg)^{1/q}\Bigg)^{1/2p}\Bigg)\Bigg\rangle.
\end{aligned} \tag{45}$$

(2) When $k=3$ and $\gamma_1=\gamma_2=\gamma_3=p$, the developed Cq-ROFPGDMSM operator degenerates into the cubic q-rung orthopair fuzzy power generalized geometric Bonferroni mean (Cq-ROFPGGBM) operator when $(p=r=s)$, as shown below

$$\begin{aligned}
&\mathrm{Cq-ROFPGDMSM}^{(3,p,p,p)}(Cq_1, Cq_2, \cdots, Cq_s) = \frac{1}{3p}\left(\prod_{1\le l_1<l_2<l_3\le s}\left(\sum_{j=1}^{3} p\left(Cq_{l_j}\right)^{s\text{Ł}_{l_j}}\right)\right)^{1/C_n^3} \\
&= \frac{1}{3p}\left(\left(\bigotimes_{\substack{l_1,l_2,l_3=1\\ l_1\neq l_2\neq l_3}}^{s}\left(p\left(Cq_{l_1}\right)^{s\text{Ł}_{l_1}} \oplus p\left(Cq_{l_2}\right)^{s\text{Ł}_{l_2}} \oplus \left(Cq_{l_3}\right)^{s\text{Ł}_{l_3}}\right)\right)^{1/3!}\right)^{3!/(s(s-1)(s-2))} \\
&= \frac{1}{3p}\left(\bigotimes_{\substack{l,m,n=1\\ l\neq m\neq n}}^{n}\left(p(Cq_l)^{s\text{Ł}_l} \otimes p(Cq_m)^{s\text{Ł}_j} \otimes p(Cq_n)^{s\text{Ł}_n}\right)\right)^{1/(s(s-1)(s-2))} \\
&= \mathrm{Cq-ROFPGGBM}^{(p,p,p)}(Cq_1, Cq_2, \cdots, Cq_s) \\
&= \Bigg\langle \Bigg(\Bigg[\Bigg(1-\Bigg(1-\Bigg(\prod_{\substack{l=m=\\ n=1,\\ l\neq m\neq n}}^{s}\left(1-\left(1-(a_l^q)^{s\text{Ł}_l}\right)^p\right. \\
&\times \left.\left(1-(a_m^q)^{s\text{Ł}_m}\right)^p\left(1-(a_n^q)^{s\text{Ł}_n}\right)^p\right)\Bigg)^{1/(s(s-1)(s-2))}\Bigg)^{1/3p}\Bigg)^{1/q}, \\
&\times \Bigg(1-\Bigg(1-\Bigg(\prod_{\substack{l=m=\\ n=1,\\ l\neq m\neq n}}^{s}\left(1-\left(1-(a_l^q)^{s\tilde{O}_l}\right)^p\right. \\
&\times \left.\left(1-(b_m^q)^{s\text{Ł}_m}\right)^p\left(1-(b_n^q)^{s\text{Ł}_n}\right)^p\right)\Bigg)^{1/(s(s-1)(s-2))}\Bigg)^{1/3p}\Bigg)^{1/q}\Bigg], \\
&\times \Bigg[\Bigg(\Bigg(1-\Bigg(\prod_{\substack{l=m=\\ n=1,\\ l\neq m\neq n}}^{s}\left(1-\left(1-(1-c_l^q)^{s\text{Ł}_l}\right)^p\right. \\
&\times \left.\left(1-(1-c_m^q)^{m\text{Ł}_j}\right)^p\left(1-(1-c_n^q)^{s\text{Ł}_l}\right)^p\right)\Bigg)^{1/(s(s-1)(s-2))}\Bigg)^{1/q}\Bigg)^{1/3p}, \\
&\times \Bigg(\Bigg(1-\Bigg(\prod_{\substack{l=m=\\ n=1,\\ l\neq m\neq n}}^{s}\left(1-\left(1-(1-d_l^q)^{s\text{Ł}_l}\right)^p\right. \\
&\times \left.\left(1-(1-d_m^q)^{m\text{Ł}_j}\right)^p\left(1-(1-d_n^q)^{s\text{Ł}_l}\right)^p\right)\Bigg)^{1/(s(s-1)(s-2))}\Bigg)^{1/q}\Bigg)^{1/3p}\Bigg], \\
&\times \Bigg(\Bigg(1-\Bigg(1-\Bigg(\prod_{\substack{l=m=\\ n=1,\\ l\neq m\neq n}}^{s}\left(1-\left(1-(e_l^q)^{s\text{Ł}_l}\right)^p\right.
\end{aligned}$$

$$\times\left(1-\left(e_m^q\right)^{sŁ_m}\right)^p\left(1-\left(e_n^q\right)^{sŁ_n}\right)^p\Big)\Big)^{1/(s(s-1)(s-2))}\Bigg)^{1/3p}\Bigg)^{1/q},$$

$$\times\left(\left(\left(1-\left(\prod_{\substack{l=m=\\ n=1,\\ l\neq m\neq n}}^{s}\left(1-\left(1-\left(1-f_l^q\right)^{sŁ_i}\right)\right.\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\left.\times^{p}\left(1-\left(1-f_m^q\right)^{mŁ_j}\right)^p\left(1-\left(1-f_n^q\right)^{sŁ_l}\right)^p\right)\right)^{1/(s(s-1)(s-2))}\right)^{1/q}\right)^{1/3p}\right)\right\rangle. \tag{46}$$

(3) When $\gamma_1=\gamma_2=\cdots=\gamma_n=1$, the developed Cq-ROFPGMSM operator degenerates into the cubic q-rung orthopair fuzzy power MSM (Cq-ROFPDMSM) operator, as shown below.

$$\begin{aligned}
&\mathrm{Cq-ROFPGDMSM}^{(k,1,,\cdots,1)}(Cq_1,Cq_2,\cdots,Cq_s)\\
&=\frac{1}{1+1+\cdots+1}\left(\prod_{1\le l_1<l_2<l_3\le s}\left(\sum_{j=1}^{k}1\left(Cq_{l_j}\right)^{sŁ_{l_j}}\right)\right)^{1/C_s^k}\\
&=\frac{1}{k}\left(\prod_{1\le l_1<l_2<l_3\le s}\left(\sum_{j=1}^{k}\left(Cq_{l_j}\right)^{sŁ_{l_j}}\right)\right)^{1/C_s^3}\\
&=Cq-\mathrm{ROFPDMSM}^{(k)}(Cq_1,Cq_2,\cdots,Cq_s)\\
&=\Bigg\langle\Bigg(\Bigg[\Bigg(1-\Bigg(1-\prod_{1\le l_1<l_2<\cdots<l_k\le s}\Bigg(1-\prod_{j=1}^{k}\Bigg(1-\left(a_{l_j}^{sŁ_{l_j}}\right)^q\Bigg)\Bigg)^{1/C_s^k}\Bigg)^{1/k}\Bigg)^{1/q},\\
&\times\Bigg(1-\Bigg(1-\prod_{1\le l_1<l_2<\cdots<l_k\le s}\Bigg(1-\prod_{j=1}^{k}\Bigg(1-\left(b_{l_j}^{sŁ_{l_j}}\right)^q\Bigg)\Bigg)^{1/C_s^k}\Bigg)^{1/k}\Bigg)^{1/q}\Bigg],\\
&\times\Bigg[\Bigg(\Bigg(\Bigg(1-\Bigg(\prod_{1\le l_1<l_2<\cdots<l_k\le s}\Bigg(1-\prod_{j=1}^{k}\Bigg(1-\left(1-c_{l_j}^q\right)^{sŁ_l}\Bigg)\Bigg)\Bigg)^{1/C_s^k}\Bigg)^{1/q}\Bigg)^{1/k},\\
&\times\Bigg(\Bigg(1-\Bigg(\prod_{1\le l_1<l_2<\cdots<l_k\le s}\Bigg(1-\prod_{j=1}^{k}\Bigg(1-\left(1-d_{l_j}^q\right)^{sŁ_{l_j}}\Bigg)\Bigg)\Bigg)^{1/C_s^k}\Bigg)^{1/q}\Bigg)^{1/k}\Bigg]\Bigg),\\
&\times\Bigg(\Bigg(1-\Bigg(1-\prod_{1\le l_1<l_2<\cdots<l_k\le s}\Bigg(1-\prod_{j=1}^{k}\Bigg(1-\left(e_{l_j}^{sŁ_{l_j}}\right)^q\Bigg)\Bigg)^{1/C_s^k}\Bigg)^{1/k}\Bigg)^{1/q},\\
&\times\Bigg(\Bigg(1-\Bigg(\prod_{1\le l_1<l_2<\cdots<l_k\le s}\Bigg(1-\prod_{j=1}^{k}\Bigg(1-\left(1-f_{l_j}^q\right)^{sŁ_{l_j}}\Bigg)\Bigg)\Bigg)^{1/C_s^k}\Bigg)^{1/q}\Bigg)^{1/k}\Bigg)\Bigg\rangle.
\end{aligned}\tag{47}$$

(4) When $\gamma_1=\gamma_2=\cdots=\gamma_n=1/k$, the developed Cq-ROFPGDMSM operator degenerates into the cubic q-rung orthopair fuzzy power HAM (Cq-ROFPD-HAM) operator, as shown below.

$$\begin{aligned}
&\mathrm{Cq-ROFPGDMSM}^{(k,1/k,1/k,\cdots,1/k)}(Cq_1,Cq_2,\cdots,Cq_n)\\
&=\frac{1}{1/k+1/k+\cdots+1/k}\left(\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(\sum_{j=1}^{k}\frac{1}{k}\left(Cq_{l_j}\right)^{sŁ_{l_j}}\right)\right)^{1/C_s^k}\\
&=\left(\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(\sum_{j=1}^{k}\frac{1}{k}\left(Cq_{l_j}\right)^{sŁ_{l_j}}\right)\right)^{1/C_s^k}\\
&=Cq-\mathrm{ROFPDHAM}^{(k)}(Cq_1,Cq_2,\cdots,Cq_s)\\
&=\Bigg\langle\Bigg(\Bigg[\Bigg(\sum_{1\le l_1<l_2<\cdots<l_k\le s}\Bigg(1-\Bigg(\prod_{j=1}^{k}\Bigg(1-\left(a_{l_j}^{sŁ_{l_j}}\right)^q\Bigg)\Bigg)^{1/k}\Bigg)^{1/q}\Bigg)^{1/C_s^k},\\
&\times\Bigg(\sum_{1\le l_1<l_2<\cdots<l_k\le s}\Bigg(1-\Bigg(\prod_{j=1}^{k}\Bigg(1-\left(b_{l_j}^{sŁ_{l_j}}\right)^q\Bigg)\Bigg)^{1/k}\Bigg)^{1/q}\Bigg)^{1/C_s^k}\Bigg],\\
&\times\Bigg[\Bigg(1-\Bigg(\sum_{1\le l_1<l_2<\cdots<l_k\le s}\Bigg(1-\Bigg(\prod_{j=1}^{k}\Bigg(1-\left(1-c_{l_j}^q\right)^{sŁ_{l_j}}\Bigg)\Bigg)^{1/k}\Bigg)\Bigg)^{1/C_s^k}\Bigg)^{1/q},\\
&\times\Bigg(1-\Bigg(\sum_{1\le l_1<l_2<\cdots<l_k\le s}\Bigg(1-\Bigg(\prod_{j=1}^{k}\Bigg(1-\left(1-d_{l_j}^q\right)^{sŁ_{l_j}}\Bigg)\Bigg)^{1/k}\Bigg)\Bigg)^{1/C_s^k}\Bigg)^{1/q}\Bigg]\Bigg),\\
&\times\Bigg(\Bigg(\sum_{1\le l_1<l_2<\cdots<l_k\le s}\Bigg(1-\Bigg(\prod_{j=1}^{k}\Bigg(1-\left(e_{l_j}^{sŁ_{l_j}}\right)^q\Bigg)\Bigg)^{1/k}\Bigg)^{1/q}\Bigg)^{1/C_s^k},\\
&\times\Bigg(1-\Bigg(\sum_{1\le l_1<l_2<\cdots<l_k\le s}\Bigg(1-\Bigg(\prod_{j=1}^{k}\Bigg(1-\left(1-f_{l_j}^q\right)^{sŁ_{l_j}}\Bigg)\Bigg)^{1/k}\Bigg)\Bigg)^{1/C_s^k}\Bigg)^{1/q}\Bigg)\Bigg\rangle.
\end{aligned}\tag{48}$$

□

4. The Cq-ROFPWGMSM Operator and Cq-ROFPWGDMSM Operator

In Definitions 10 and 14, they can deal with the situation when the importance degrees of all the input arguments are the same. But in real decision-making, sometimes attributes have their own importance, so to deal with such situation, we propose the weighted form of the proposed aggregation operators given in Definitions 10 and 14.

(2) Proof of this part is easy, so it is omitted here

Definition 18. Let $Cq_l=\langle([a_l,b_l],[c_l,d_l]),(e_l,f_l)\rangle$ be a set of Cq-ROFNs, $w=(w_1,w_2,\cdots,w_s)^T$ be the importance degree of $Cq_l, w_l\in[0,1]$, such that $\oplus_s^s w_l=1$; then, the Cq-ROFPWGMSM operator can be revealed as follows:

$$\mathrm{Cq\text{-}ROFPWGMSM}^{(k,\gamma_1,\gamma_2,\cdots,\gamma_k)}(Cq_1,Cq_2,\cdots,Cq_s)=\begin{cases}\left(\dfrac{\sum_{1\le l_1<l_2<\cdots<l_k\le s}\left(\left(1-\oplus_{j=1}^{k}w_{l_j}\right)\left(\sum_{j=1}^{k}\left(Ł_{l_j}Cq_{l_j}\right)^{\gamma_j}\right)\right)}{C_{s-1}^{k}}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}, & \text{when } 1\le k<s,\\[2ex] \left(\bigotimes_{i=1}^{k}\left(Ł_iCq_i\right)^{\gamma_i+((1-w_i)/(n-1))}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}, & \text{when } (k=s),\end{cases}\tag{49}$$

where $Ł_z=(1+T(Cq_z))/\oplus_{z=1}^{s}(1+T(Cq_z)), \sum_{z=1}^{s}Ł_z=1$,

$T(Cq_j) = \prod_{a=1, z \neq j}^{s} \mathrm{Spt}(Cq_z, Cq_j)$, is the support degree for Cq_z from Cq_j, which fulfil the following conditions:

(1) $\mathrm{Spt}(Cq_z, Cq_j) \in [0, 1]$

(2) $\mathrm{Spt}(Cq_z, Cq_j) = \mathrm{Spt}(Cq_j, Cq_z)$

(3) If $\widetilde{\mathrm{DE}}(Cq_z, Cq_j) \leq \widetilde{\mathrm{DE}}(Cq_s, Cq_t)$, then $\mathrm{Spt}(Cq_z, Cq_j) \geq \mathrm{Spt}(Cq_s, Cq_t)$, where $\widetilde{\mathrm{DE}}(Cq_z, Cq_j)$ express the distance measure among two Cq-ROFNs described in Definition 6. $\gamma_1, \gamma_2, \cdots, \gamma_k \geq 0, k$, is a parameter and $k = (1, 2, \cdots, \mathrm{n})$, $(l_1, l_2, \cdots, l_k)$ traverses all the k-tuple combination of $(1, 2, \cdots, s)$. The denominator C_s^k in Equation (49) expresses the binomial coefficient $s!/(k!(s-k)!)$ and n is the balancing coefficient

Theorem 19. *Let $Cq_1, Cq_2, \cdots,$ and Cq_s be Cq-ROFNs, where $Cq_l = \langle([a_l, b_l], [c_l, d_l]), (e_l, f_l)\rangle (l = 1, 2, \cdots, s)\ (q \geq 1)$. Then, the aggregated value utilizing Equation (49) is still Cq-ROFN, even*

$$\begin{aligned}
&Cq-ROFPWGMSM^{(k,\gamma_1,\gamma_2,\cdots,\gamma_k)}(Cq_1, Cq_2, \cdots, Cq_s)\\
&= \Bigg\langle\Bigg(\Bigg[\Bigg(\Bigg(\Bigg(1 - \prod_{1\leq l_1<l_2<\cdots<l_k\leq s}\Bigg(1 - \prod_{j=1}^{k}\left(1-\left(1-a_{l_j}^{q}\right)^{s\text{Ł}_{l_j}}\right)^{\gamma_j}\Bigg)^{\left(1-\oplus_{j=1}^{k}\mathrm{w}_{l_j}\right)/C_{s-1}^{k}}\Bigg)^{1/q}\Bigg)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)},\\
&\times\Bigg(\Bigg(1 - \prod_{1\leq l_1<l_2<\cdots<l_k\leq s}\Bigg(1 - \prod_{j=1}^{k}\left(1-\left(1-b_{l_j}^{q}\right)^{s\text{Ł}_{l_j}}\right)^{\gamma_j}\Bigg)^{\left(1-\oplus_{j=1}^{k}\mathrm{w}_{l_j}\right)/C_{s-1}^{k}}\Bigg)^{1/q}\Bigg)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\Bigg],\\
&\times\Bigg[\Bigg(1-\Bigg(1 - \prod_{1\leq l_1<l_2<\cdots<l_k\leq s}\Bigg(1 - \prod_{j=1}^{k}\left(1-\left(c_{l_j}^{s\text{Ł}_{l_j}}\right)^{q}\right)^{\gamma_j}\Bigg)^{\left(1-\oplus_{j=1}^{k}\mathrm{w}_{l_j}\right)/C_{s-1}^{k}}\Bigg)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\Bigg)^{1/q},\\
&\times\Bigg(1-\Bigg(1 - \prod_{1\leq l_1<l_2<\cdots<l_k\leq s}\Bigg(1 - \prod_{j=1}^{k}\left(1-\left(d_{l_j}^{s\text{Ł}_{l_j}}\right)^{q}\right)^{\gamma_j}\Bigg)^{\left(1-\oplus_{j=1}^{k}\mathrm{w}_{l_j}\right)/C_{s-1}^{k}}\Bigg)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\Bigg)^{1/q}\Bigg]\Bigg\rangle,\\
&\times\Bigg(\Bigg(\Bigg(1 - \prod_{1\leq l_1<l_2<\cdots<l_k\leq s}\Bigg(1 - \prod_{j=1}^{k}\left(1-\left(1-e_{l_j}^{q}\right)^{s\text{Ł}_{l_j}}\right)^{\gamma_j}\Bigg)^{\left(1-\oplus_{j=1}^{k}\mathrm{w}_{l_j}\right)/C_{s-1}^{k}}\Bigg)^{1/q}\Bigg)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)},\\
&\times\Bigg(1-\Bigg(1 - \prod_{1\leq l_1<l_2<\cdots<l_k\leq s}\Bigg(1 - \prod_{j=1}^{k}\left(1-\left(f_{l_j}^{s\text{Ł}_{l_j}}\right)^{q}\right)^{\gamma_j}\Bigg)^{\left(1-\oplus_{j=1}^{k}\mathrm{w}_{l_j}\right)/C_{s-1}^{k}}\Bigg)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\Bigg)^{1/q}\Bigg)\Bigg\rangle,
\end{aligned} \tag{50}$$

or

$$\begin{aligned}
&Cq-ROFPWGMSM^{(k,\gamma_1,\gamma_2,\cdots,\gamma_k)}(Cq_1, Cq_2, \cdots, Cq_s)\\
&= \Bigg\langle\Bigg(\Bigg[\Bigg(\prod_{j=1}^{k}\left(\left(1-\left(1-a_l^{q}\right)^{s\text{Ł}_l}\right)^{1/q}\right)^{\gamma_l+((1-s\mathrm{w}_l)/(s-1))}\Bigg)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)},\\
&\times\Bigg(\prod_{j=1}^{k}\left(\left(1-\left(1-b_l^{q}\right)^{s\text{Ł}_l}\right)^{1/q}\right)^{\gamma_l+((1-s\mathrm{w}_l)/(s-1))}\Bigg)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\Bigg],\\
&\times\Bigg[\Bigg(1-\Bigg(\prod_{j=1}^{k}\left(1-\left(c_l^{s\text{Ł}_l}\right)^{q}\right)^{\gamma_l+((1-s\mathrm{w}_l)/(s-1))}\Bigg)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\Bigg)^{1/q},\\
&\times\Bigg(1-\Bigg(\prod_{j=1}^{k}\left(1-\left(d_l^{s\text{Ł}_l}\right)^{q}\right)^{\gamma_l+((1-s\mathrm{w}_l)/(s-1))}\Bigg)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\Bigg)^{1/q}\Bigg]\Bigg),\\
&\times\Bigg(\Bigg(\prod_{j=1}^{k}\left(\left(1-\left(1-e_l^{q}\right)^{s\text{Ł}_l}\right)^{1/q}\right)^{\gamma_l+((1-s\mathrm{w}_l)/(s-1))}\Bigg)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)},\\
&\times\Bigg(1-\Bigg(\prod_{j=1}^{k}\left(1-\left(f_l^{s\text{Ł}_l}\right)^{q}\right)^{\gamma_l+((1-s\mathrm{w}_l)/(s-1))}\Bigg)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\Bigg)^{1/q}\Bigg)\Bigg\rangle,\\
&(\mathit{for}\ k = s).
\end{aligned} \tag{51}$$

Proof. Based on the operational laws of Cq-ROFNs, we can prove Theorem 19.

(1) For $1 \leq k < n$, we can have

$$\begin{aligned}
s\text{Ł}_{l_j}Cq_{l_j} &= \Bigg\langle\Bigg(\Bigg[\left(1-\left(1-a_{l_j}^{q}\right)^{s\text{Ł}_{l_j}}\right)^{1/q}, \left(1-\left(1-b_{l_j}^{q}\right)^{s\text{Ł}_{l_j}}\right)^{1/q}\Bigg],\\
&\times\left[c_{l_j}^{s\text{Ł}_{l_j}}, d_{l_j}^{s\text{Ł}_{l_j}}\right]\Bigg), \Bigg(\left(1-\left(1-e_{l_j}^{q}\right)^{s\text{Ł}_{l_j}}\right)^{1/q}, f_{l_j}^{s\text{Ł}_{l_j}}\Bigg)\Bigg\rangle,\\
&\times\left(s\text{Ł}_{l_j}Cq_{l_j}\right)^{\gamma_j} = \Bigg\langle\Bigg(\Bigg[\Bigg(\left(1-\left(1-a_{l_j}^{q}\right)^{s\text{Ł}_{l_j}}\right)^{1/q}\Bigg)^{\gamma_j},\\
&\times\Bigg(\left(1-\left(1-b_{l_j}^{q}\right)^{s\text{Ł}_{l_j}}\right)^{1/q}\Bigg)^{\gamma_j}\Bigg], \Bigg[\left(1-\left(1-\left(c_{l_j}^{s\text{Ł}_{l_j}}\right)^{q}\right)^{\gamma_j}\right)^{1/q},\\
&\times\left(1-\left(1-\left(d_{l_j}^{s\text{Ł}_{l_j}}\right)^{q}\right)^{\gamma_j}\right)^{1/q}\Bigg]\Bigg), \Bigg(\Bigg(\left(1-\left(1-e_{l_j}^{q}\right)^{s\text{Ł}_{l_j}}\right)^{1/q}\Bigg)^{\gamma_j},\\
&\times\left(1-\left(1-\left(f_{l_j}^{s\text{Ł}_{l_j}}\right)^{q}\right)^{\gamma_j}\right)^{1/q}\Bigg)\Bigg\rangle,
\end{aligned}$$

$$\begin{aligned}
\prod_{j=1}^{k}\left(s\text{Ł}_{l_j}Cq_{l_j}\right)^{\gamma_j} &= \Bigg\langle\Bigg(\Bigg[\prod_{j=1}^{k}\Bigg(\left(1-\left(1-a_{l_j}^{q}\right)^{s\text{Ł}_{l_j}}\right)^{1/q}\Bigg)^{\gamma_j}, \prod_{j=1}^{k}\\
&\times\Bigg(\left(1-\left(1-b_{l_j}^{q}\right)^{s\text{Ł}_{l_j}}\right)^{1/q}\Bigg)^{\gamma_j}\Bigg],\\
&\times\Bigg[\left(1-\prod_{j=1}^{k}\left(1-\left(c_{l_j}^{s\text{Ł}_{l_j}}\right)^{q}\right)^{\gamma_j}\right)^{1/q},\\
&\times\left(1-\prod_{j=1}^{k}\left(1-\left(d_{l_j}^{s\text{Ł}_{l_j}}\right)^{q}\right)^{\gamma_j}\right)^{1/q}\Bigg]\Bigg),\\
&\times\Bigg(\prod_{j=1}^{k}\Bigg(\left(1-\left(1-e_{l_j}^{q}\right)^{s\text{Ł}_{l_j}}\right)^{1/q}\Bigg)^{\gamma_j},\\
&\times\left(1-\prod_{j=1}^{k}\left(1-\left(f_{l_j}^{s\text{Ł}_{l_j}}\right)^{q}\right)^{\gamma_j}\right)^{1/q}\Bigg)\Bigg\rangle.
\end{aligned} \tag{52}$$

So,

$$
\begin{aligned}
&\left(1-\bigoplus_{j=1}^{k}\mathrm{w}_{l_j}\right)\left(\prod_{j=1}^{k}\left(s\L_{l_j}Cq_{l_j}\right)^{\gamma_j}\right)\\
&=\Bigg\langle\Bigg(\Bigg[\left(1-\left(1-\prod_{j=1}^{k}\left(1-\left(1-a_{l_j}^{q}\right)^{s\L_{l_j}}\right)^{\gamma_j}\right)^{\left(1-\oplus_{j=1}^{k}\mathrm{w}_{l_j}\right)}\right)^{1/q},\\
&\times\left(1-\left(1-\prod_{j=1}^{k}\left(1-\left(1-b_{l_j}^{q}\right)^{s\L_{l_j}}\right)^{\gamma_j}\right)^{\left(1-\oplus_{j=1}^{k}\mathrm{w}_{l_j}\right)}\right)^{1/q}\Bigg],\\
&\times\Bigg[\left(\left(1-\prod_{j=1}^{k}\left(1-\left(c_{l_j}^{s\L_{l_j}}\right)^{q}\right)^{\gamma_j}\right)^{1/q}\right)^{\left(1-\oplus_{j=1}^{k}\mathrm{w}_{l_j}\right)},\\
&\times\left(\left(1-\prod_{j=1}^{k}\left(1-\left(d_{l_j}^{s\L_{l_j}}\right)^{q}\right)^{\gamma_j}\right)^{1/q}\right)^{\left(1-\oplus_{j=1}^{k}\mathrm{w}_{l_j}\right)}\Bigg]\Bigg),\\
&\times\Bigg(\left(1-\left(1-\prod_{j=1}^{k}\left(1-\left(1-e_{l_j}^{q}\right)^{s\L_{l_j}}\right)^{\gamma_j}\right)^{\left(1-\oplus_{j=1}^{k}\mathrm{w}_{l_j}\right)}\right)^{1/q},\\
&\times\left(\left(1-\prod_{j=1}^{k}\left(1-\left(f_{l_j}^{s\L_{l_j}}\right)^{q}\right)^{\gamma_j}\right)^{1/q}\right)^{\left(1-\oplus_{j=1}^{k}\mathrm{w}_{l_j}\right)}\Bigg)\Bigg\rangle.
\end{aligned}
\tag{53}
$$

Further,

$$
\begin{aligned}
&\sum_{1\le l_1<l_2<\cdots<l_k\le s}\left(\left(1-\bigoplus_{j=1}^{k}\mathrm{w}_{l_j}\right)\left(\prod_{j=1}^{k}\left(s\L_{l_j}Cq_{l_j}\right)^{\gamma_j}\right)\right)\\
&=\Bigg\langle\Bigg(\Bigg[\left(1-\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(1-a_{l_j}^{q}\right)^{s\L_{l_j}}\right)^{\gamma_j}\right)^{\left(1-\oplus_{j=1}^{k}\mathrm{w}_{l_j}\right)}\right)^{1/q},\\
&\times\left(1-\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(1-b_{l_j}^{q}\right)^{s\L_{l_j}}\right)^{\gamma_j}\right)^{\left(1-\oplus_{j=1}^{k}\mathrm{w}_{l_j}\right)}\right)^{1/q}\Bigg],\\
&\times\Bigg[\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(\left(1-\prod_{j=1}^{k}\left(1-\left(c_{l_j}^{s\L_{l_j}}\right)^{q}\right)^{\gamma_j}\right)^{1/q}\right)^{\left(1-\oplus_{j=1}^{k}\mathrm{w}_{l_j}\right)},\prod_{1\le l_1<l_2<\cdots<l_k\le s}\\
&\times\left(\left(1-\prod_{j=1}^{k}\left(1-\left(d_{l_j}^{s\L_{l_j}}\right)^{q}\right)^{\gamma_j}\right)^{1/q}\right)^{\left(1-\oplus_{j=1}^{k}\mathrm{w}_{l_j}\right)}\Bigg]\Bigg),\\
&\times\Bigg(\left(1-\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(1-e_{l_j}^{q}\right)^{s\L_{l_j}}\right)^{\gamma_j}\right)^{\left(1-\oplus_{j=1}^{k}\mathrm{w}_{l_j}\right)}\right)^{1/q},\prod_{1\le l_1<l_2<\cdots<l_k\le s}\\
&\times\left(\left(1-\prod_{j=1}^{k}\left(1-\left(f_{l_j}^{s\L_{l_j}}\right)^{q}\right)^{\gamma_j}\right)^{1/q}\right)^{\left(1-\oplus_{j=1}^{k}\mathrm{w}_{l_j}\right)}\Bigg)\Bigg\rangle.
\end{aligned}
\tag{54}
$$

Therefore,

$$
\begin{aligned}
&\frac{\sum_{1\le l_1<l_2<\cdots<l_k\le s}\left(\left(1-\oplus_{j=1}^{k}\mathrm{w}_{l_j}\right)\left(\prod_{j=1}^{k}\left(s\L_{l_j}Cq_{l_j}\right)^{\gamma_j}\right)\right)}{C_{s-1}^{k}}\\
&=\Bigg\langle\Bigg(\Bigg[\left(1-\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(1-a_{l_j}^{q}\right)^{s\L_{l_j}}\right)^{\gamma_j}\right)^{\left(1-\oplus_{j=1}^{k}\mathrm{w}_{l_j}\right)/C_{s-1}^{k}}\right)^{1/q},\\
&\times\left(1-\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(1-b_{l_j}^{q}\right)^{s\L_{l_j}}\right)^{\gamma_j}\right)^{\left(1-\oplus_{j=1}^{k}\mathrm{w}_{l_j}\right)/C_{s-1}^{k}}\right)^{1/q}\Bigg],\\
&\times\Bigg[\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(\left(1-\prod_{j=1}^{k}\left(1-\left(c_{l_j}^{s\L_{l_j}}\right)^{q}\right)^{\gamma_j}\right)^{1/q}\right)^{\left(1-\oplus_{j=1}^{k}\mathrm{w}_{l_j}\right)/C_{s-1}^{k}},\prod_{1\le l_1<l_2<\cdots<l_k\le s}\\
&\times\left(\left(1-\prod_{j=1}^{k}\left(1-\left(d_{l_j}^{s\L_{l_j}}\right)^{q}\right)^{\gamma_j}\right)^{1/q}\right)^{\left(1-\oplus_{j=1}^{k}\mathrm{w}_{l_j}\right)/C_{s-1}^{k}}\Bigg]\Bigg),\\
&\times\Bigg(\left(1-\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(1-e_{l_j}^{q}\right)^{s\L_{l_j}}\right)^{\gamma_j}\right)^{\left(1-\oplus_{j=1}^{k}\mathrm{w}_{l_j}\right)/C_{s-1}^{k}}\right)^{1/q},\prod_{1\le l_1<l_2<\cdots<l_k\le s}\\
&\times\left(\left(1-\prod_{j=1}^{k}\left(1-\left(f_{l_j}^{s\L_{l_j}}\right)^{q}\right)^{\gamma_j}\right)^{1/q}\right)^{\left(1-\oplus_{j=1}^{k}\mathrm{w}_{l_j}\right)/C_{s-1}^{k}}\Bigg)\Bigg\rangle.
\end{aligned}
\tag{55}
$$

Hence,

$$
\begin{aligned}
&\left(\frac{\sum_{1\le l_1<l_2<\cdots<l_k\le s}\left(\left(1-\oplus_{j=1}^{k}\mathrm{w}_{l_j}\right)\left(\prod_{j=1}^{k}\left(s\L_{l_j}Cq_{l_j}\right)^{\gamma_j}\right)\right)}{C_{s}^{k}}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\\
&=\Bigg\langle\Bigg(\Bigg[\left(\left(1-\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(1-a_{l_j}^{q}\right)^{s\L_{l_j}}\right)^{\gamma_j}\right)^{\left(1-\oplus_{j=1}^{k}\mathrm{w}_{l_j}\right)/C_{s-1}^{k}}\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)},\\
&\cdot\left(\left(1-\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(1-b_{l_j}^{q}\right)^{s\L_{l_j}}\right)^{\gamma_j}\right)^{\left(1-\oplus_{j=1}^{k}\mathrm{w}_{l_j}\right)/C_{s-1}^{k}}\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\Bigg],\\
&\cdot\Bigg[\left(1-\left(1-\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(c_{l_j}^{s\L_{l_j}}\right)^{q}\right)^{\gamma_j}\right)^{\left(1-\oplus_{j=1}^{k}\mathrm{w}_{l_j}\right)/C_{s-1}^{k}}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q},\\
&\cdot\left(1-\left(1-\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(d_{l_j}^{s\L_{l_j}}\right)^{q}\right)^{\gamma_j}\right)^{\left(1-\oplus_{j=1}^{k}\mathrm{w}_{l_j}\right)/C_{s-1}^{k}}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q}\Bigg]\Bigg),\\
&\cdot\Bigg(\left(\left(1-\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(1-e_{l_j}^{q}\right)^{s\L_{l_j}}\right)^{\gamma_j}\right)^{\left(1-\oplus_{j=1}^{k}\mathrm{w}_{l_j}\right)/C_{s-1}^{k}}\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)},\\
&\cdot\left(1-\left(1-\prod_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\prod_{j=1}^{k}\left(1-\left(f_{l_j}^{s\L_{l_j}}\right)^{q}\right)^{\gamma_j}\right)^{\left(1-\oplus_{j=1}^{k}\mathrm{w}_{l_j}\right)/C_{s-1}^{k}}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q}\Bigg)\Bigg\rangle.
\end{aligned}
\tag{56}
$$

(2) For $k=s$, we can have

$$
\begin{aligned}
sŁ_l Cq_l &= \left\langle \left(\left[\left(1-\left(1-a_l^q\right)^{sŁ_l}\right)^{1/q}, \left(1-\left(1-b_l^q\right)^{sŁ_l}\right)^{1/q} \right], \left[c_l^{sŁ_l}, d_l^{sŁ_l} \right] \right), \right. \\
&\quad \cdot \left. \left(\left(1-\left(1-e_l^q\right)^{sŁ_l}\right)^{1/q}, f_l^{sŁ_l} \right) \right\rangle, \left(sŁ_l Cq_l\right)^{\gamma_l+((1-s\mathrm{w}_l)/(s-1))} \\
&= \left\langle \left(\left[\left(\left(1-\left(1-a_l^q\right)^{sŁ_l}\right)^{1/q} \right)^{\gamma_l+((1-s\mathrm{w}_l)/(s-1))}, \right. \right. \right. \\
&\quad \cdot \left. \left(\left(1-\left(1-b_l^q\right)^{sŁ_l}\right)^{1/q} \right)^{\gamma_l+((1-s\mathrm{w}_l)/(s-1))} \right], \\
&\quad \cdot \left[\left(1-\left(1-\left(c_l^{sŁ_l}\right)^q\right)^{\gamma_l+((1-s\mathrm{w}_l)/(s-1))}\right)^{1/q}, \right. \\
&\quad \cdot \left. \left. \left(1-\left(1-\left(d_l^{sŁ_l}\right)^q\right)^{\gamma_l+((1-s\mathrm{w}_l)/(s-1))}\right)^{1/q} \right] \right), \\
&\quad \cdot \left(\left(\left(1-\left(1-e_l^q\right)^{sŁ_l}\right)^{1/q} \right)^{\gamma_l+((1-s\mathrm{w}_l)/(s-1))}, \right. \\
&\quad \cdot \left. \left. \left(1-\left(1-\left(f_l^{sŁ_l}\right)^q\right)^{\gamma_l+((1-s\mathrm{w}_l)/(s-1))}\right)^{1/q} \right) \right\rangle,
\end{aligned}
$$

$$
\begin{aligned}
\prod_{j=1}^{k} \left(sŁ_l Cq_l\right)^{\gamma_l+((1-s\mathrm{w}_l)/(s-1))} &= \left\langle \left(\left[\prod_{j=1}^{k} \left(\left(1-\left(1-a_l^q\right)^{sŁ_l}\right)^{1/q} \right)^{\gamma_l+((1-s\mathrm{w}_l)/(s-1))}, \prod_{j=1}^{k} \right. \right. \right. \\
&\quad \cdot \left. \left(\left(1-\left(1-b_l^q\right)^{sŁ_l}\right)^{1/q} \right)^{\gamma_l+((1-s\mathrm{w}_l)/(s-1))} \right], \\
&\quad \cdot \left[\left(1-\prod_{j=1}^{k}\left(1-\left(c_l^{sŁ_l}\right)^q\right)^{\gamma_l+((1-s\mathrm{w}_l)/(s-1))}\right)^{1/q}, \right. \\
&\quad \cdot \left. \left. \left(1-\prod_{j=1}^{k}\left(1-\left(d_l^{sŁ_l}\right)^q\right)^{\gamma_l+((1-s\mathrm{w}_l)/(s-1))}\right)^{1/q} \right] \right), \\
&\quad \cdot \left(\prod_{j=1}^{k} \left(\left(1-\left(1-e_l^q\right)^{sŁ_l}\right)^{1/q} \right)^{\gamma_l+((1-s\mathrm{w}_l)/(s-1))}, \right. \\
&\quad \cdot \left. \left. \left(1-\prod_{j=1}^{k}\left(1-\left(f_l^{sŁ_l}\right)^q\right)^{\gamma_l+((1-s\mathrm{w}_l)/(s-1))}\right)^{1/q} \right) \right\rangle.
\end{aligned} \tag{57}
$$

So,

$$
\begin{aligned}
&\left(\prod_{j=1}^{k} \left(sŁ_l Cq_l\right)^{\gamma_l+((1-s\mathrm{w}_l)/(s-1))} \right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)} \\
&= \left\langle \left(\left[\left(\prod_{j=1}^{k} \left(\left(1-\left(1-a_l^q\right)^{sŁ_l}\right)^{1/q} \right)^{\gamma_l+((1-s\mathrm{w}_l)/(s-1))} \right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}, \right. \right. \right. \\
&\quad \times \left. \left(\prod_{j=1}^{k} \left(\left(1-\left(1-b_l^q\right)^{sŁ_l}\right)^{1/q} \right)^{\gamma_l+((1-s\mathrm{w}_l)/(s-1))} \right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)} \right], \\
&\quad \times \left[\left(1-\left(\prod_{j=1}^{k} \left(1-\left(c_l^{sŁ_l}\right)^q\right)^{\gamma_l+((1-s\mathrm{w}_l)/(s-1))} \right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q}, \right. \\
&\quad \times \left. \left. \left(1-\left(\prod_{j=1}^{k} \left(1-\left(d_l^{sŁ_l}\right)^q\right)^{\gamma_l+((1-s\mathrm{w}_l)/(s-1))} \right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q} \right] \right), \\
&\quad \times \left(\left(\prod_{j=1}^{k} \left(\left(1-\left(1-e_l^q\right)^{sŁ_l}\right)^{1/q} \right)^{\gamma_l+((1-s\mathrm{w}_l)/(s-1))} \right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}, \right. \\
&\quad \times \left. \left. \left(1-\left(\prod_{j=1}^{k} \left(1-\left(f_l^{sŁ_l}\right)^q\right)^{\gamma_l+((1-s\mathrm{w}_l)/(s-1))} \right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q} \right) \right\rangle.
\end{aligned} \tag{58}
$$

This is the required proof. □

Theorem 20 (idempotency). *Let $Cq = \langle([a, b], [c, d]), (e, f)\rangle$ and $Cq_l = \langle([a_l, b_l], [c_l, d_l]), (e_l, f_l)\rangle$, $(l = 1, 2, \cdots, s)$ be two sets of Cq-ROFNs. When $Cq = Cq_l (l = 1, 2, \cdots, s)$, we have*

$$
Cq\text{-}ROFPGWMSM^{(k,\gamma_1,\gamma_2,\cdots,\gamma_k)}\left(Cq_1, Cq_2, \cdots, Cq_s\right) = Cq. \tag{59}
$$

Proof. When $Cq = Cq_l (l = 1, 2, \cdots, s)$, we have $s(1 + T(Cq_{l_j}))/\sum_{z=1}^{s}(1 + T(Cq_z)) = s(1 + T(Cq))/\sum_{z=1}^{s}(1 + T(Cq)) = 1$. So, from Theorem 19, we have the following.

(1) For $1 \le k < s$, we have

$$
\begin{aligned}
&\mathrm{Cq}-\mathrm{ROFPWGMSM}^{(k,\gamma_1,\gamma_2,\cdots,\gamma_k)}\left(Cq_1, Cq_2, \cdots, Cq_s\right) \\
&= \left\langle \left(\left[\left(\left(1 - \prod_{1\le l_1<l_2<\cdots<l_k\le s} \left(1-\prod_{j=1}^{k}\left(1-\left(1-a^q\right)\right)^{\gamma_j}\right)^{\left(1-\oplus_{j=1}^{k}\mathrm{w}_{l_j}\right)/C_{s-1}^{k}} \right)^{1/q} \right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}, \right. \right. \right. \\
&\quad \times \left. \left(\left(1 - \prod_{1\le l_1<l_2<\cdots<l_k\le s} \left(1-\prod_{j=1}^{k}\left(1-\left(1-b^q\right)\right)^{\gamma_j}\right)^{\left(1-\oplus_{j=1}^{k}\mathrm{w}_{l_j}\right)/C_{s-1}^{k}} \right)^{1/q} \right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)} \right], \\
&\quad \times \left[\left(1 - \left(1 - \prod_{1\le l_1<l_2<\cdots<l_k\le s} \left(1-\prod_{j=1}^{k}\left(1-c^q\right)^{\gamma_j}\right)^{\left(1-\oplus_{j=1}^{k}\mathrm{w}_{l_j}\right)/C_{s-1}^{k}} \right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)} \right)^{1/q}, \right. \\
&\quad \times \left. \left. \left(1 - \left(1 - \prod_{1\le l_1<l_2<\cdots<l_k\le s} \left(1-\prod_{J=1}^{K}\left(1-d^q\right)^{\gamma_j}\right)^{\left(1-\oplus_{j=1}^{k}\mathrm{w}_{l_j}\right)/C_{s-1}^{k}} \right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)} \right)^{1/q} \right] \right), \\
&\quad \times \left(\left(\left(1 - \prod_{1\le l_1<l_2<\cdots<l_k\le s} \left(1-\prod_{j=1}^{k}\left(1-\left(1-e^q\right)\right)^{\gamma_j}\right)^{\left(1-\oplus_{j=1}^{k}\mathrm{w}_{l_j}\right)/C_{s-1}^{k}} \right)^{1/q} \right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}, \right. \\
&\quad \times \left. \left. \left(1 - \left(1 - \prod_{1\le l_1<l_2<\cdots<l_k\le s} \left(1-\prod_{j=1}^{k}\left(1-f^q\right)^{\gamma_j}\right)^{\left(1-\oplus_{j=1}^{k}\mathrm{w}_{l_j}\right)/C_{s-1}^{k}} \right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)} \right)^{1/q} \right) \right\rangle, \\
&= \left\langle \left(\left[\left(\left(1 - \prod_{1\le l_1<l_2<\cdots<l_k\le s} \left(1-a^{q\times(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{\left(1-\oplus_{j=1}^{k}\mathrm{w}_{l_j}\right)/C_{s-1}^{k}} \right)^{1/q} \right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}, \right. \right. \right. \\
&\quad \times \left. \left(\left(1 - \prod_{1\le l_1<l_2<\cdots<l_k\le s} \left(1-b^{q\times(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{\left(1-\oplus_{j=1}^{k}\mathrm{w}_{l_j}\right)/C_{s-1}^{k}} \right)^{1/q} \right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)} \right], \\
&\quad \times \left[\left(1 - \left(1 - \prod_{1\le l_1<l_2<\cdots<l_k\le s} \left(1-\left(1-c^q\right)^{(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{\left(1-\oplus_{j=1}^{k}\mathrm{w}_{l_j}\right)/C_{s-1}^{k}} \right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)} \right)^{1/q}, \right. \\
&\quad \times \left. \left. \left(1 - \left(1 - \prod_{1\le l_1<l_2<\cdots<l_k\le s} \left(1-\left(1-d^q\right)^{(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{\left(1-\oplus_{j=1}^{k}\mathrm{w}_{l_j}\right)/C_{s-1}^{k}} \right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)} \right)^{1/q} \right] \right), \\
&\quad \times \left(\left(\left(1 - \prod_{1\le l_1<l_2<\cdots<l_k\le s} \left(1-e^{q\times(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{\left(1-\oplus_{j=1}^{k}\mathrm{w}_{l_j}\right)/C_{s-1}^{k}} \right)^{1/q} \right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}, \right. \\
&\quad \times \left. \left. \left(1 - \left(1 - \prod_{1\le l_1<l_2<\cdots<l_k\le s} \left(1-\left(1-f^q\right)^{(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{\left(1-\oplus_{j=1}^{k}\mathrm{w}_{l_j}\right)/C_{s-1}^{k}} \right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)} \right)^{1/q} \right) \right\rangle, \\
&= \left\langle \left(\left[\left(\left(1 - \left(1-a^{q\times(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/C_{s-1}^{k} \oplus_{1\le l_1<l_2<\cdots<l_k\le s} \left(1-\oplus_{j=1}^{k}\mathrm{w}_{l_j}\right)} \right)^{1/q} \right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}, \right. \right. \right.
\end{aligned}
$$

$$
\begin{aligned}
&\times\left(\left(\left(1-\left(1-b^{q\times(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/C_{s-1}^{k}\bigoplus_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\bigoplus_{j=1}^{k}\mathrm{w}_{l_j}\right)}\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right],\\
&\times\left[\left(1-\left(1-\left(1-(1-c^q)^{(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/C_{s-1}^{k}\bigoplus_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\bigoplus_{j=1}^{k}\mathrm{w}_{l_j}\right)}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q},\right.\\
&\left.\times\left(1-\left(1-\left(1-(1-d^q)^{(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/C_{s-1}^{k}\bigoplus_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\bigoplus_{j=1}^{k}\mathrm{w}_{l_j}\right)}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q}\right]\Bigg),\\
&\times\left(\left(\left(1-\left(1-e^{q\times(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/C_{s-1}^{k}\bigoplus_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\bigoplus_{j=1}^{k}\mathrm{w}_{l_j}\right)}\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)},\right.\\
&\left.\times\left(1-\left(1-\left(1-(1-f^q)^{(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/C_{s-1}^{k}\bigoplus_{1\le l_1<l_2<\cdots<l_k\le s}\left(1-\bigoplus_{j=1}^{k}\mathrm{w}_{l_j}\right)}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q}\right)\Bigg\rangle.\\
&=\Bigg\langle\left(\left[\left(\left(1-\left(1-a^{q\times(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/C_{s-1}^{k}\left(C_s^k-\bigoplus_{1\le l_1<l_2<\cdots<l_k\le s}\left(\bigoplus_{j=1}^{k}\mathrm{w}_{l_j}\right)\right)}\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)},\right.\right.\\
&\times\left.\left(\left(1-\left(1-b^{q\times(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/C_{s-1}^{k}\left(C_s^k-\bigoplus_{1\le l_1<l_2<\cdots<l_k\le s}\left(\bigoplus_{j=1}^{k}\mathrm{w}_{l_j}\right)\right)}\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right],\\
&\times\left[\left(1-\left(1-\left(1-(1-c^q)^{(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/C_{s-1}^{k}\left(C_s^k-\bigoplus_{1\le l_1<l_2<\cdots<l_k\le s}\left(\bigoplus_{j=1}^{k}\mathrm{w}_{l_j}\right)\right)}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q},\right.\\
&\left.\left.\times\left(1-\left(1-\left(1-(1-d^q)^{(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/C_{s-1}^{k}\left(C_s^k-\bigoplus_{1\le l_1<l_2<\cdots<l_k\le s}\left(\bigoplus_{j=1}^{k}\mathrm{w}_{l_j}\right)\right)}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q}\right]\right),\\
&\times\left(\left(\left(1-\left(1-e^{q\times(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/C_{s-1}^{k}\left(C_s^k-\bigoplus_{1\le l_1<l_2<\cdots<l_k\le s}\left(\bigoplus_{j=1}^{k}\mathrm{w}_{l_j}\right)\right)}\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)},\right.\\
&\left.\times\left(1-\left(1-\left(1-(1-f^q)^{(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/C_{s-1}^{k}\left(C_s^k-\bigoplus_{1\le l_1<l_2<\cdots<l_k\le s}\left(\bigoplus_{j=1}^{k}\mathrm{w}_{l_j}\right)\right)}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q}\right)\Bigg\rangle.\\
&=\Bigg\langle\left(\left[\left(\left(1-\left(1-a^{q\times(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/C_{s-1}^{k}\left(C_s^k-\bigoplus_{i_j=1}^{k}C_{s-1}^{k-1}\mathrm{w}_{l_j}\right)}\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)},\right.\right.\\
&\times\left.\left(\left(1-\left(1-b^{q\times(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/C_{s-1}^{k}\left(C_n^k-\bigoplus_{l_j=1}^{k}C_{s-1}^{k-1}\mathrm{w}_{l_j}\right)}\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right],\\
&\times\left[\left(1-\left(1-\left(1-(1-c^q)^{(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/C_{s-1}^{k}\left(C_s^k-\bigoplus_{l_j=1}^{k}C_{s-1}^{k-1}\mathrm{w}_{i_j}\right)}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q},\right.\\
&\left.\left.\times\left(1-\left(1-\left(1-(1-d^q)^{(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/C_{s-1}^{k}\left(C_s^k-\bigoplus_{l_j=1}^{k}C_{s-1}^{k-1}\mathrm{w}_{l_j}\right)}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q}\right]\right),\\
&\times\left(\left(\left(1-\left(1-e^{q\times(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/C_{s-1}^{k}\left(C_s^k-\bigoplus_{l_j=1}^{k}C_{s-1}^{k-1}\mathrm{w}_{l_j}\right)}\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)},\right.\\
&\left.\times\left(1-\left(1-\left(1-(1-f^q)^{(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/C_{s-1}^{k}\left(C_s^k-\bigoplus_{l_j=1}^{k}C_{s-1}^{k-1}\mathrm{w}_{l_j}\right)}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q}\right)\Bigg\rangle\\
&=\Bigg\langle\left(\left[\left(\left(1-\left(1-a^{q\times(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/C_{s-1}^{k}\left(C_s^k-C_{s-1}^{k-1}\right)}\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)},\right.\right.\\
&\times\left.\left(\left(1-\left(1-b^{q\times(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/C_{s-1}^{k}\left(C_s^k-C_{s-1}^{k-1}\right)}\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right],\\
&\times\left[\left(1-\left(1-\left(1-(1-c^q)^{(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/C_{s-1}^{k}\left(C_s^k-C_{s-1}^{k-1}\right)}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q},\right.\\
&\left.\left.\times\left(1-\left(1-\left(1-(1-d^q)^{(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/C_{s-1}^{k}\left(C_s^k-C_{s-1}^{k-1}\right)}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q}\right]\right),\\
&\times\left(\left(\left(1-\left(1-e^{q\times(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/C_{s-1}^{k}\left(C_s^k-C_{s-1}^{k-1}\right)}\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)},\right.\\
&\left.\times\left(1-\left(1-\left(1-(1-f^q)^{(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/C_{s-1}^{k}\left(C_s^k-C_{s-1}^{k-1}\right)}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q}\right)\Bigg\rangle;\\
&=\Bigg\langle\left(\left[\left(\left(1-\left(1-a^{q\times(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)},\right.\right.\\
&\times\left.\left(\left(1-\left(1-b^{q\times(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right],\\
&\times\left[\left(1-\left(1-\left(1-(1-c^q)^{(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q},\right.\\
&\left.\left.\times\left(1-\left(1-\left(1-(1-d^q)^{(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q}\right]\right)\\
&\times\left(\left(\left(1-\left(1-e^{q\times(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)},\right.\\
&\left.\times\left(1-\left(1-\left(1-(1-f^q)^{(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q}\right)\Bigg\rangle\\
&=\Bigg\langle\left(\left[\left(\left(1-\left(1-a^{q\times(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)},\right.\right.\\
&\times\left.\left(\left(1-\left(1-b^{q\times(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right],\\
&\times\left[\left(1-\left(1-\left(1-(1-c^q)^{(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q},\right.\\
&\left.\left.\times\left(1-\left(1-\left(1-(1-d^q)^{(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q}\right]\right)\\
&\times\left(\left(\left(1-\left(1-e^{q\times(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)},\right.\\
&\left.\times\left(1-\left(1-\left(1-(1-f^q)^{(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q}\right)\Bigg\rangle\\
&=\Bigg\langle\left(\left[\left(\left(a^{q\times(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)},\left(\left(b^{q\times(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right],\right.\\
&\times\left[\left(1-\left((1-c^q)^{(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q},\right.\\
&\left.\left.\times\left(1-\left((1-d^q)^{(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q}\right]\right)\\
&\times\left(\left(\left(e^{q\times(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)},\left(1-\left((1-f^q)^{(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q}\right)\Bigg\rangle\\
&=\langle([a,b],[c,d]),(e,f)\rangle=Cq.
\end{aligned}
\tag{60}
$$

□

Theorem 21 (boundedness). *Let* $Cq_l=\langle([a_l,b_l],[c_l,d_l]),(e_l,f_l)\rangle,(l=1,2,\cdots,s)$ *be a set of Cq-ROFNs,* $\chi=\min(Cq_1,$

$Cq_2, \cdots, Cq_n)$ *and* $\alpha = \max(Cq_1, Cq_2, \cdots, Cq_n)$, *then*

$$\chi \leq Cq\text{-}ROFPGWMSM^{(k, \gamma_1, \gamma_2, \cdots, \gamma_k)}(Cq_1, Cq_2, \cdots, Cq_s) \leq \alpha. \tag{61}$$

Proof. Proof of Theorem 21 is the same as that of Theorem 13. □

Definition 22. Let $Cq_l = \langle([a_l, b_l], [c_l, d_l]), (e_l, f_l)\rangle$ be a set of Cq-ROFNs, $\mathrm{w} = (\mathrm{w}_1, \mathrm{w}_2, \cdots, \mathrm{w}_s)^T$ be the importance degree of Cq_i, $\mathrm{w}_l \in [0, 1]$ such that $\oplus_{l=1}^{s} \mathrm{w}_l = 1$; then, the Cq-ROFPWGDMSM operator can be revealed as follows:

$$\begin{aligned}&\text{Cq-ROFPWGDMSM}^{(k, \gamma_1, \gamma_2, \cdots, \gamma_k)}(Cq_1, Cq_2, \cdots, Cq_s)\\ &= \begin{cases} \dfrac{1}{\gamma_1 + \gamma_2 + \cdots + \gamma_k}\left(\prod\limits_{1 \leq l_1 < l_2 < \cdots < l_k \leq s}\left(\left(\prod\limits_{j=1}^{k} \gamma_j \left(Cq_{l_j}\right)^{sŁ_{l_j}}\right)^{\left(1-\oplus_{j=1}^{k}\mathrm{w}_{l_j}\right)}\right)^{1/C_{s-1}^{k}}\right), & \text{when } 1 \leq k < s, \\ \dfrac{1}{\gamma_1 + \gamma_2 + \cdots + \gamma_k}\left(\sum\limits_{j=1}^{k}\left(\gamma_l + \dfrac{1-\mathrm{w}_l}{s-1}\right)(Cq_l)^{s\tilde{O}_l}\right), & \text{when}(k = s), \end{cases}\end{aligned} \tag{62}$$

where $Ł_z = (1 + T(Cq_z)) / \oplus_{z=1}^{s}(1 + T(Cq_z))$, $\oplus_{z=1}^{s} Ł_z = 1$, $T(Cq_j) = \oplus_{\substack{z=1 \\ z \neq j}}^{s} \mathrm{Spt}(Cq_z, Cq_j)$, is the support degree for Cq_z from Cq_j, which fulfil the following conditions:

(1) $\mathrm{Spt}(Cq_z, Cq_j) \in [0, 1]$

(2) $\mathrm{Spt}(Cq_z, Cq_j) = \mathrm{Spt}(Cq_j, Cq_z)$

(3) If $\widetilde{\mathrm{DE}}(Cq_z, Cq_j) \leq \widetilde{\mathrm{DE}}(Cq_s, Cq_t)$, then $\mathrm{Spt}(Cq_z, Cq_j) \geq \mathrm{Spt}(Cq_s, Cq_t)$, where $\widetilde{\mathrm{DE}}(Cq_z, Cq_j)$ express the distance measure among two Cq-ROFNs described in Definition 6. $\gamma_1, \gamma_2, \cdots, \gamma_k \geq 0$, k is a parameter and $k = (1, 2, \cdots, s)$, $(l_1, l_2, \cdots, l_k)$ traverses all the k-tuple combination of $(1, 2, \cdots, s)$. The denominator C_s^k in Equation (62) expresses the binomial coefficient $s!/(k!(s-k)!)$ and s is the balancing coefficient

Theorem 23. *Let* $Cq_1, Cq_2, \cdots,$ *and* Cq_n *be Cq-ROFNs, where* $Cq_l = \langle([a_l, b_l], [c_l, d_l]), (e_l, f_l)\rangle (l = 1, 2, \cdots, s)$ $(q \geq 1)$. *Then, the aggregated value utilizing Equation (62) is still Cq-ROFN, even*

$$\begin{aligned}&Cq-ROFPWGDMSM^{(k, \gamma_1, \gamma_2, \cdots, \gamma_k)}(Cq_1, Cq_2, \cdots, Cq_s)\\
&= \left\langle \left(\left[\left(1 - \left(1 - \prod_{1 \leq l_1 < l_2 < \cdots < l_k \leq s}\left(\left(1 - \prod_{j=1}^{k}\left(1 - \left(a_{l_j}^{sŁ_{l_j}}\right)^q\right)^{\gamma_j}\right)^{1/q}\right)^{\left(1-\oplus_{j=1}^{k}\mathrm{w}_{l_j}\right)/C_s^k}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q},\right.\right.\right.\\
&\cdot \left.\left(1 - \left(1 - \prod_{1 \leq l_1 < l_2 < \cdots < l_k \leq s}\left(\left(1 - \prod_{j=1}^{k}\left(1 - \left(b_{l_j}^{sŁ_{l_j}}\right)^q\right)^{\gamma_j}\right)^{1/q}\right)^{\left(1-\oplus_{j=1}^{k}\mathrm{w}_{l_j}\right)/C_s^k}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q}\right],\\
&\cdot \left[\left(\left(1 - \prod_{1 \leq l_1 < l_2 < \cdots < l_k \leq s}\left(1 - \prod_{j=1}^{k}\left(1 - \left(1 - c_{l_j}^q\right)^{sŁ_{l_j}}\right)^{\gamma_j}\right)^{\left(1-\oplus_{j=1}^{k}\mathrm{w}_{l_j}\right)/C_s^k}\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)},\right.\\
&\cdot \left.\left.\left(\left(1 - \prod_{1 \leq l_1 < l_2 < \cdots < l_k \leq s}\left(1 - \prod_{j=1}^{k}\left(1 - \left(1 - d_{l_j}^q\right)^{sŁ_{l_j}}\right)^{\gamma_j}\right)^{\left(1-\oplus_{j=1}^{k}\mathrm{w}_{l_j}\right)/C_s^k}\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right]\right),\\
&\cdot \left(\left(1 - \left(1 - \prod_{1 \leq l_1 < l_2 < \cdots < l_k \leq s}\left(\left(1 - \prod_{j=1}^{k}\left(1 - \left(e_{l_j}^{sŁ_{l_j}}\right)^q\right)^{\gamma_j}\right)^{1/q}\right)^{\left(1-\oplus_{j=1}^{k}\mathrm{w}_{l_j}\right)/C_s^k}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q},\\
&\cdot \left.\left(\left(1 - \prod_{1 \leq l_1 < l_2 < \cdots < l_k \leq s}\left(1 - \prod_{j=1}^{k}\left(1 - \left(1 - f_{l_j}^q\right)^{sŁ_{l_j}}\right)^{\gamma_j}\right)^{\left(1-\oplus_{j=1}^{k}\mathrm{w}_{l_j}\right)/C_s^k}\right)^{1/q}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)\right\rangle,\\
&\cdot (for\ 1 \leq k < s),\end{aligned} \tag{63}$$

or

$$\begin{aligned}&Cq-ROFPWGMSM^{(k, \gamma_1, \gamma_2, \cdots, \gamma_k)}(Cq_1, Cq_2, \cdots, Cq_s)\\
&= \left\langle\left(\left[\left(1 - \left(\prod_{j=1}^{k}\left(1 - \left(a_l^{sŁ_l}\right)^q\right)^{\gamma_l + ((1-s\mathrm{w}_i)/(s-1))}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q},\right.\right.\right.\\
&\times \left.\left(1 - \left(\prod_{j=1}^{k}\left(1 - \left(b_l^{sŁ_l}\right)^q\right)^{\gamma_l + ((1-s\mathrm{w}_i)/(s-1))}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q}\right],\\
&\times \left[\left(\prod_{j=1}^{k}\left(\left(1 - \left(1 - c_l^q\right)^{sŁ_l}\right)^{1/q}\right)^{\gamma_l + ((1-s\mathrm{w}_i)/(s-1))}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)},\right.\\
&\times \left.\left.\left(\prod_{j=1}^{k}\left(\left(1 - \left(1 - d_l^q\right)^{sŁ_l}\right)^{1/q}\right)^{\gamma_l + ((1-s\mathrm{w}_i)/(s-1))}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right],\right.\\
&\times \left(\left(1 - \left(\prod_{j=1}^{k}\left(1 - \left(e_l^{sŁ_l}\right)^q\right)^{\gamma_l + ((1-s\mathrm{w}_i)/(s-1))}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)^{1/q},\right.\\
&\times \left.\left.\left(\prod_{j=1}^{k}\left(\left(1 - \left(1 - f_l^q\right)^{sŁ_l}\right)^{1/q}\right)^{\gamma_l + ((1-s\mathrm{w}_i)/(s-1))}\right)^{1/(\gamma_1+\gamma_2+\cdots+\gamma_k)}\right)\right\rangle, (for\ k = s).\end{aligned} \tag{64}$$

Proof. Proof of Theorem 23 is the same as that of Theorem 19. So, it is omitted here.

Similar to Cq-ROFPGWMSM operator, Cq-ROFPWGDMSM has the properties of idempotency and boundedness. □

5. A Novel Approach to MAGDM Established on the Initiated Operators

In this section, we pertain the afore-conferred Cq-ROFPGWMSM and Cq-ROFGWDMM operators to ascertain productive model for MAGDM under Cq-ROF environments. Let $\overline{\overline{\mathrm{ALE}}} = \{\overline{\overline{\mathrm{ale}}}_1, \overline{\overline{\mathrm{ale}}}_2, \cdots, \overline{\overline{\mathrm{ale}}}_m\}$ be the group of alternatives, the group of attributes is intimated by $\overline{\overline{\mathrm{Cri}}} = \{\overline{\overline{\mathrm{Cri}}}_1, \overline{\overline{\mathrm{Cri}}}_2, \cdots, \overline{\overline{\mathrm{Cri}}}_n\}$, the importance degree of the attributes is intimated by $\overline{\overline{\mathrm{Wet}}} = (\overline{\overline{\mathrm{wet}}}_1, \overline{\overline{\mathrm{wet}}}_2, \cdots, \overline{\overline{\mathrm{wet}}}_n)^T$ such that $\overline{\overline{\mathrm{wet}}}_e \in [0, 1]$, $\sum_{e=1}^{n} \overline{\overline{\mathrm{wet}}}_e = 1$, and $\overline{\overline{\mathrm{DMR}}} = (\overline{\overline{\mathrm{dmr}}}_1, \overline{\overline{\mathrm{dmr}}}_2, \cdots, \overline{\overline{\mathrm{dmr}}}_a)$ refer to the group of a decision-makers (DMR), with importance degree intimated by $\overline{\overline{\mathrm{WGT}}} = (\overline{\overline{\mathrm{wgt}}}_1, \overline{\overline{\mathrm{wgt}}}_2, \cdots, \overline{\overline{\mathrm{wgt}}}_a)^T$ such that $\overline{\overline{\mathrm{wgt}}}_b \in [0, 1]$, $\sum_{b=1}^{a} \overline{\overline{\mathrm{wgt}}} = 1$. Assume that $\overline{\overline{\mathrm{DMT}}}^b = (Cq_{ce}^b)_{m \times n}$ is the decision matrix (DMT), where $Cq_{ce}^b = \langle ([a_{ce}^b, b_{ce}^b], [c_{ce}^b, d_{ce}^b]), (e_{ce}^b, f_{ce}^b) \rangle$ takes the structure of Cq-ROFN, specified by DMR $\overline{\overline{\mathrm{dmr}}}_b$ for alternative $\overline{\overline{\mathrm{ale}}}_c$ with respect to the attribute $\overline{\overline{\mathrm{Cri}}}_e$.

Then, turning on practical decision circumstances where the importance degrees of both DMRs and attributes are entirely identified in advance. As a result, in the following, we ascertain a MAGDM approach hinge on the anticipated Cq-ROFPGWMSM and Cq-ROFPGDWMSM operators. To do accordingly, now pursue the steps below.

5.1. Decision-Making Model Utilizing Cq-ROFPGWMSM

Step 1. Firstly, the specified DMTs $\overline{\overline{\mathrm{DMT}}}^b = (Cq_{ce}^b)_{m \times n}$ should be distorted into uniform DMT $\overline{\overline{\mathrm{DMT}}}^b = (Cq_{ce}^b)_{m \times n}$. We modify the cost-type attribute into benefit-type attribute utilizing the following formula.

$$Cq_{ce}^b = \begin{cases} Cq_{ce}^b = \left\langle \left(\left[a_{ce}^b, b_{ce}^b\right], \left[c_{ce}^b, d_{ce}^b\right] \right), e_{ce}^b, f_{ce}^b \right\rangle \text{for benefit-type attribute } \overline{\overline{\mathrm{Cri}}}_{ec=1,2,\cdots,m,e=1,2,\cdots,n}, \\ \left(Cq_{ce}^b\right)^c = \left\langle \left(\left[c_{ce}^b, d_{ce}^b\right], \left[a_{ce}^b, b_{ce}^b\right] \right), f_{ce}^b, e_{ce}^b \right\rangle \text{ for benefit-type attribute } \overline{\overline{\mathrm{Cri}}}_e. \end{cases} \tag{65}$$

Step 2. Ascertain the support degrees by the following formula:

$$\mathrm{Sup}\left(Cq_{ce}^b, Cq_{ce}^l\right) = 1 - \widetilde{\mathrm{DE}}_H\left(Cq_{ce}^b, Cq_{ce}^l\right), (b, l = 1, 2, \cdots, a\,; c = 1, 2, \cdots, m\,; e = 1, 2, \cdots, n)., \tag{66}$$

which gratify the conditions for support function and $\bar{\bar{\mathrm{D}}}\mathrm{E}(Cq_{ce}^b, Cq_{ce}^l)$ is the distance measure stipulated in Definition 6.

Step 3. Ascertain the support degree $T(Cq_{ce}^b)$ that Cq-ROFN Cq_{ce}^b accepts from other Cq-ROFNs $Cq_{ce}^l (l = 1, 2, \cdots, a\,; l \neq b)$, where

$$T\left(Cq_{ce}^b\right) = \sum_{l=1, l \neq b}^{a} \mathrm{sup}\left(Cq_{ce}^b, Cq_{ce}^l\right). \tag{67}$$

Step 4. Discover the weights o_{de}^c associated with the Cq-OFN Cq_{de}^c,

$$o_{de}^c = \frac{a(1 + T(Cq_{de}^c))}{\sum_{c=1}^{a}(1 + T(Cq_{de}^c))}, (c = 1, 2, \cdots, a), \tag{68}$$

where $o_{de}^c \geq 0$ and $\sum_{c=1}^{a} o_{de}^c = 1$..

Step 5. Aggregate all the individual decision matrices $\overline{\overline{\mathrm{DMT}}}^b = (Cq_{ce}^b)_{m \times n} (b = 1, 2, \cdots, a)$ into group decision matrix $\widetilde{\mathrm{DMT}} = (Cq_{ce})_{m \times n}$ by utilizing Cq-ROFPGWMSM operators, where

$$Cq_{ce}^b = \text{Cq-ROFPGMSM}\left(Cq_{ce}^1, Cq_{ce}^2, \cdots, Cq_{ce}^a\right). \tag{69}$$

Step 6. Determine support degrees $\mathrm{Sup}(Cq_{ce}, Cq_{cx})$ by the following formula:

$$\mathrm{Sup}(Cq_{ce}, Cq_{cx}) = 1 - \widetilde{\mathrm{DE}}(Cq_{ce}, Cq_{cx})\,; (c = 1, 2, \cdots, m, e = 1, 2, \cdots, n, e \neq x), \tag{70}$$

where $\widetilde{\mathrm{DE}}(Cq_{ce}, Cq_{cx})$ is distance measure given in Definition 6.

Step 7. Ascertain the support degree $T(Cq_{ce})$ that Cq-ROFNs Cq_{ce} collects from other Cq-ROFNs $Cq_{cx}(x = 1, 2, \cdots, n\,; e \neq x)$, where

$$T(Cq_{ce}) = \sum_{x=1, x \neq e}^{n} \mathrm{Sup}(Cq_{ce}, Cq_{cx}). \tag{71}$$

Step 8. Ascertain weighting vector $\tilde{\mathrm{O}}_{ce}(c = 1, 2, . \cdots, m, e = 1$

Table 6: Effect of parameter q on the final ranking results utilizing Cq-ROFPGWMSM and Cq-ROFPGWDMSM operator.

Parameter	Score value Cq-ROFPGWMSM	Score value Cq-ROFPGWDMSM	Ranking order Cq-ROFPGWMSM and Cq-ROFPGWDMSM
$q=3$	$\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_1)=-0.31159$, $\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_2)=-0.20941$, $\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_3)=-0.3605$, $\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_4)=-0.4766$, $\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_5)=-0.5062$.	$\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_1)=1.5550$, $\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_2)=1.6360$, $\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_3)=1.5475$, $\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_4)=1.5113$, $\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_5)=1.4323$.	$\overline{\overline{\mathrm{ale}}}_2>\overline{\overline{\mathrm{ale}}}_1>\overline{\overline{\mathrm{ale}}}_3>\overline{\overline{\mathrm{ale}}}_4>\overline{\overline{\mathrm{ale}}}_5$. and $\overline{\overline{\mathrm{ale}}}_2>\overline{\overline{\mathrm{ale}}}_1>\overline{\overline{\mathrm{ale}}}_3>\overline{\overline{\mathrm{ale}}}_4>\overline{\overline{\mathrm{ale}}}_5$.
$q=5$	$\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_1)=-0.1986$, $\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_2)=-0.1021$, $\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_3)=-0.2459$, $\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_4)=-0.3669$, $\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_5)=-0.3831$.	$\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_1)=1.3895$, $\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_2)=1.4957$, $\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_3)=1.3902$, $\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_4)=1.3677$, $\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_5)=1.2791$.	$\overline{\overline{\mathrm{ale}}}_2>\overline{\overline{\mathrm{ale}}}_1>\overline{\overline{\mathrm{ale}}}_3>\overline{\overline{\mathrm{ale}}}_4>\overline{\overline{\mathrm{ale}}}_5$. and $\overline{\overline{\mathrm{ale}}}_2>\overline{\overline{\mathrm{ale}}}_1>\overline{\overline{\mathrm{ale}}}_3>\overline{\overline{\mathrm{ale}}}_4>\overline{\overline{\mathrm{ale}}}_5$.
$q=10$	$\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_1)=-0.0725$, $\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_2)=-0.0022$, $\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_3)=-0.1045$, $\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_4)=-0.2100$, $\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_5)=-0.1980$.	$\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_1)=1.0841$, $\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_2)=1.2357$, $\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_3)=1.1104$, $\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_4)=1.0901$, $\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_5)=0.9928$.	$\overline{\overline{\mathrm{ale}}}_2>\overline{\overline{\mathrm{ale}}}_1>\overline{\overline{\mathrm{ale}}}_3>\overline{\overline{\mathrm{ale}}}_4>\overline{\overline{\mathrm{ale}}}_5$. and $\overline{\overline{\mathrm{ale}}}_2>\overline{\overline{\mathrm{ale}}}_3>\overline{\overline{\mathrm{ale}}}_4>\overline{\overline{\mathrm{ale}}}_1>\overline{\overline{\mathrm{ale}}}_5$.
$q=18$	$\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_1)=-0.0129$, $\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_2)=0.0205$, $\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_3)=-0.0283$, $\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_4)=-0.0924$, $\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_5)=-0.0709$.	$\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_1)=0.7950$, $\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_2)=0.9774$, $\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_3)=0.8473$, $\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_4)=0.8116$, $\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_5)=0.7171$.	$\overline{\overline{\mathrm{ale}}}_2>\overline{\overline{\mathrm{ale}}}_1>\overline{\overline{\mathrm{ale}}}_3>\overline{\overline{\mathrm{ale}}}_4>\overline{\overline{\mathrm{ale}}}_5$. and $\overline{\overline{\mathrm{ale}}}_2>\overline{\overline{\mathrm{ale}}}_3>\overline{\overline{\mathrm{ale}}}_4>\overline{\overline{\mathrm{ale}}}_1>\overline{\overline{\mathrm{ale}}}_5$.
$q=25$	$\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_1)=0.0008$, $\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_2)=0.0164$, $\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_3)=-0.0052$, $\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_4)=-0.0379$, $\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_5)=-0.0282$.	$\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_1)=0.6339$, $\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_2)=0.8236$, $\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_3)=0.6978$, $\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_4)=0.6506$, $\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_5)=0.5604$.	$\overline{\overline{\mathrm{ale}}}_2>\overline{\overline{\mathrm{ale}}}_1>\overline{\overline{\mathrm{ale}}}_3>\overline{\overline{\mathrm{ale}}}_5>\overline{\overline{\mathrm{ale}}}_4$. and $\overline{\overline{\mathrm{ale}}}_2>\overline{\overline{\mathrm{ale}}}_3>\overline{\overline{\mathrm{ale}}}_4>\overline{\overline{\mathrm{ale}}}_1>\overline{\overline{\mathrm{ale}}}_5$.
$q=50$	$\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_1)=0.0008$, $\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_2)=0.0017$, $\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_3)=0.0011$, $\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_4)=0.0006$, $\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_5)=0.0002$.	$\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_1)=0.3264$, $\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_2)=0.4961$, $\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_3)=0.4019$, $\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_4)=0.3325$, $\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_5)=0.2641$.	$\overline{\overline{\mathrm{ale}}}_2>\overline{\overline{\mathrm{ale}}}_3>\overline{\overline{\mathrm{ale}}}_1>\overline{\overline{\mathrm{ale}}}_4>\overline{\overline{\mathrm{ale}}}_5$. and $\overline{\overline{\mathrm{ale}}}_2>\overline{\overline{\mathrm{ale}}}_3>\overline{\overline{\mathrm{ale}}}_4>\overline{\overline{\mathrm{ale}}}_1>\overline{\overline{\mathrm{ale}}}_5$.

$, 2, \cdots, n)$ associated with Cq_{ce},

$$\mathrm{o}_{ce}=\frac{n(1+T(Cq_{ce}))}{\sum_{e=1}^{n}(1+T(Cq_{ce}))}. \tag{72}$$

Step 9. Utilize Cq-ROFPGWMSM operator to aggregate all assessment values $Cq_{ce}(c=1,2,.\cdots,m,e=1,2,\cdots,n)$ into overall assessment value $Cq_c(c=1,2,.\cdots,m)$ corresponding to the alternatives $\overline{\overline{\mathrm{ale}}}_c\ (c=1,2,\cdots,m)$:

$$Cq_c=\text{Cq-ROFPWMSM}(Cq_{c1},Cq_{c2},\cdots,Cq_{cn}). \tag{73}$$

Step 10. Ascertain the scores $\widetilde{\mathrm{SCR}}(Cq_d)$ for the overall Cq-RFN of the alternatives $\overline{\overline{\mathrm{ale}}}_d(d=1,2,\cdots,g)$ by utilizing Definition 6.

Step 11. Rank all alternatives $\overline{\overline{\mathrm{ale}}}_d(d=1,2,\cdots,g)$ and select the optimal one (s) with the ranking order $Cq_d(d=1,2,\cdots,g)$.

5.2. Decision-Making Model Utilizing Cq-ROFPGWDMSM Operator. Steps 1–4. These are the same as for Cq-ROFPGWMSM operator.

Step 5. Aggregate all the individual decision matrices $\overline{\overline{\mathrm{DMT}}}^b=(Cq_{ce}^b)_{m\times n}(b=1,2,\cdots,a)$ into group decision matrix $\overline{\mathrm{DMT}}=(Cq_{ce})_{m\times n}$ by utilizing Cq-ROFPGWDMSM operators, where

$$Cq_{ce}^b=\text{Cq-ROFPGWDMSM}\left(Cq_{ce}^1,Cq_{ce}^2,\cdots,Cq_{ce}^a\right). \tag{74}$$

Step 6. Determine support degrees $\mathrm{Sup}(Cq_{ce},Cq_{cx})$ by the following formula:

$$\text{Sup}(Cq_{ce}, Cq_{cx}) = 1 - \widetilde{\text{DE}}(Cq_{ce}, Cq_{cx})\,;\,(c = 1, 2, \cdots, m, e = 1, 2, \cdots, n, e \neq x), \tag{75}$$

where $\widetilde{\text{DE}}(Cq_{ce}, Cq_{cx})$ is distance measure given in Definition 5.

Step 7. Ascertain the support degree $T(Cq_{ce})$ that Cq-ROFNs Cq_{ce} collects from other Cq-ROFNs $Cq_{cx}(x = 1, 2, \cdots, n\,; e \neq x)$, where

$$T(Cq_{ce}) = \sum_{x=1, x\neq e}^{n} \text{Sup}(Cq_{ce}, Cq_{cx}). \tag{76}$$

Step 8. Ascertain weighting vector $\tilde{\text{O}}_{ce}(c = 1, 2,. \cdots, m, e = 1, 2, \cdots, n)$ associated with Cq_{ce},

$$Ł_{ce} = \frac{n(1 + T(Cq_{ce}))}{\sum_{e=1}^{n}(1 + T(Cq_{ce}))}. \tag{77}$$

Step 9. Utilize Cq-ROFPGWDMSM operator to aggregate all assessment values $Cq_{ce}(c = 1, 2,. \cdots, m, e = 1, 2, \cdots, n)$ into overall assessment value $Cq_c(c = 1, 2,. \cdots, m)$ corresponding to the alternatives $\overline{\overline{\text{ale}}}_c$ $(c = 1, 2, \cdots, m)$:

$$Cq_c = \text{Cq-ROFPWDMSM}(Cq_{c1}, Cq_{c2}, \cdots, Cq_{cn}). \tag{78}$$

Step 10. Ascertain the scores $\widetilde{\text{SCR}}(Cq_d)$ for the overall Cq-RFN of the alternatives $\overline{\overline{\text{ale}}}_d(d = 1, 2, \cdots, g)$ by utilizing Definition 6.

Step 11. Rank all alternatives $\overline{\overline{\text{ale}}}_d(d = 1, 2, \cdots, g)$ and select the optimal one (s) with the ranking order $Cq_d(d = 1, 2, \cdots, g)$.

6. Numerical Examples

In this part, we apply the intended approach based on the intended AOs in the case of a university in Pakistan, that is, International Islamic University Islamabad Pakistan. The Dean of the faculty of basic and applied sciences is mystified on how to choose the most excellent assistant professor at the end of the year. The intended approach based on intended AOs was pertained to the procedure of assistant professor (AP) selection of performance assessment of this university to help it evaluating their APs and investigate the efficiency of suggested approach based on these newly suggested AOs.

The input data of the suggested approach were assembled by accomplishing semi-structured interviews with the head of departments (HODs) of each department of basic and applied sciences of the university. After initial screening, fixed APs Dr. Syed Salman Hussian $A\overline{\overline{\text{ale}}}_1$, Dr. Tahir Mahmood $\overline{\overline{\text{ale}}}_2$, Dr. M. Imran Shabbir $\overline{\overline{\text{ale}}}_3$, Dr. Zafeer Saqib $\overline{\overline{\text{ale}}}_4$, and Dr. Usman Nasir $\overline{\overline{\text{ale}}}_5$ are picked for further assessment. A team of three experts E_1, E_2, and E_3 is liable for the assessment of these five selected APs and the picking of the most excellent APs. Four attributes are taken into account consisting of teaching method As_1, total number of publications As_2, student evaluation As_3, and personality As_4. The experts provide their evaluation exploiting C-qROFNs and are shown in Tables 1–3.

6.1. Decision-Making Steps Utilizing Cq-ROFPGWDMSM Operator

Step 1. Since all the attributes are of the same type, so there is no need to normalize it.

Step 2. Discover the support degrees $\text{Sup}(Cq_{de}^{b}, Cq_{de}^{l})$, $(d = 1, 2, \cdots, 5, e = 1, 2, 3, c, l = 1, 2, \cdots, 4)$ by the consuming Formula (66). For easiness, we shall write $S_{de}^{c}(d = 1, 2, \cdots, 5, e = 1, 2, 3, c = 1, 2, \cdots, 4)$ in place of $\text{Sup}(Cq_{de}^{c}, Cq_{de}^{l})$. We have

$$\begin{aligned}
&S^1_{11,12} = S^1_{12,11} = 0.8991, S^1_{11,13} = S^1_{13,11} = 0.8791,\\
&S^1_{12,13} = S^1_{13,13} = 0.7808, S^1_{21,22} = S^1_{22,21} = 0.8816,\\
&S^1_{21,23} = S^1_{23,21} = 0.7808,\\
&S^1_{22,22} = S^1_{23,22} = 0.8991, S^1_{31,32} = S^1_{32,31} = 0.7808,\\
&S^1_{31,33} = S^1_{33,31} = 0.8816, S^1_{32,33} = S^1_{33,32} = 0.8991,\\
&S^1_{41,42} = S^1_{42,41} = 0.6131,\\
&S^1_{41,43} = S^1_{43,41} = 0.3447, S^1_{42,43} = S^1_{43,42} = 0.7315,\\
&S^1_{51,52} = S^1_{52,51} = 0.5639, S^1_{51,53} = S^1_{53,51} = 0.8816,\\
&S^1_{52,53} = S^1_{53,52} = 0.4455,\\
&S^3_{11,12} = S^3_{12,11} = 0.8991, S^3_{11,13} = S^3_{13,11} = 0.7315,\\
&S^3_{12,13} = S^3_{13,13} = 0.8324, S^3_{21,22} = S^3_{22,21} = 0.7808,\\
&S^3_{21,23} = S^3_{23,21} = 0.8991,\\
&S^3_{22,22} = S^3_{23,22} = 0.8816, S^3_{31,32} = S^3_{32,31} = 0.7315,\\
&S^3_{31,33} = S^3_{33,31} = 0.6131, S^3_{32,33} = S^3_{33,32} = 0.8816,\\
&S^3_{41,42} = S^3_{42,41} = 0.7808,\\
&S^3_{41,43} = S^3_{43,41} = 0.8816, S^3_{42,43} = S^3_{43,42} = 0.8991,\\
&S^3_{51,52} = S^3_{52,51} = 0.8324, S^3_{51,53} = S^3_{53,51} = 0.8991,\\
&S^3_{52,53} = S^3_{53,52} = 0.7315,\\
&S^4_{11,12} = S^4_{12,11} = 0.7808, S^4_{11,13} = S^4_{13,11} = 0.8991,\\
&S^4_{12,13} = S^4_{13,13} = 0.8816, S^4_{21,22} = S^4_{22,21} = 0.6131,\\
&S^4_{21,23} = S^4_{23,21} = 0.8816,\\
&S^4_{22,22} = S^4_{23,22} = 0.7315, S^4_{31,32} = S^4_{32,31} = 0.8324,\\
&S^4_{31,33} = S^4_{33,31} = 0.7315, S^4_{32,33} = S^4_{33,32} = 0.8991,\\
&S^4_{41,42} = S^4_{42,41} = 0.7315,\\
&S^4_{41,43} = S^4_{43,41} = 0.8991, S^4_{42,43} = S^4_{43,42} = 0.8324,\\
&S^4_{51,52} = S^4_{52,51} = 0.8324, S^4_{51,53} = S^4_{53,51} = 0.7808,\\
&S^4_{52,53} = S^4_{53,52} = 0.6131.
\end{aligned} \tag{79}$$

Table 7: Influence of the generalized parameters γ_1, γ_2 on final ranking results.

Parameter	Score value Cq-ROFPGWMSM	Score value Cq-ROFPGWDMSM	Ranking order
$\gamma_1 = 2, \gamma_2 = 2$	$\widetilde{SCR}(\overline{\overline{ale}}_1) = 0.6819, \widetilde{SCR}(\overline{\overline{ale}}_2) = 0.7760, \widetilde{SCR}(\overline{\overline{ale}}_3) = 0.6228, \widetilde{SCR}(\overline{\overline{ale}}_4) = 0.5136, \widetilde{SCR}(\overline{\overline{ale}}_5) = 0.4817.$	$\widetilde{SCR}(\overline{\overline{ale}}_1) = 1.0694, \widetilde{SCR}(\overline{\overline{ale}}_2) = 1.2103, \widetilde{SCR}(\overline{\overline{ale}}_3) = 1.0667, \widetilde{SCR}(\overline{\overline{ale}}_4) = 0.9187, \widetilde{SCR}(\overline{\overline{ale}}_5) = 0.8065.$	$\overline{\overline{ale}}_2 > \overline{\overline{ale}}_1 > \overline{\overline{ale}}_3 > \overline{\overline{ale}}_4 > \overline{\overline{ale}}_5.$ and $\overline{\overline{ale}}_2 > \overline{\overline{ale}}_1 > \overline{\overline{ale}}_3 > \overline{\overline{ale}}_4 > \overline{\overline{ale}}_5.$
$\gamma_1 = 4, \gamma_2 = 5$	$\widetilde{SCR}(\overline{\overline{ale}}_1) = 1.4374, \widetilde{SCR}(\overline{\overline{ale}}_2) = 1.4913, \widetilde{SCR}(\overline{\overline{ale}}_3) = 1.3867, \widetilde{SCR}(\overline{\overline{ale}}_4) = 1.3291, \widetilde{SCR}(\overline{\overline{ale}}_5) = 1.3019.$	$\widetilde{SCR}(\overline{\overline{ale}}_1) = 0.1293, \widetilde{SCR}(\overline{\overline{ale}}_2) = 0.3069, \widetilde{SCR}(\overline{\overline{ale}}_3) = 0.1218, \widetilde{SCR}(\overline{\overline{ale}}_4) = -0.0897, \widetilde{SCR}(\overline{\overline{ale}}_5) = -0.2203.$	$\overline{\overline{ale}}_2 > \overline{\overline{ale}}_1 > \overline{\overline{ale}}_3 > \overline{\overline{ale}}_4 > \overline{\overline{ale}}_5.$ and $\overline{\overline{ale}}_2 > \overline{\overline{ale}}_1 > \overline{\overline{ale}}_3 > \overline{\overline{ale}}_4 > \overline{\overline{ale}}_5.$
$\gamma_1 = 10, \gamma_2 = 8$	$\widetilde{SCR}(\overline{\overline{ale}}_1) = 1.7306, \widetilde{SCR}(\overline{\overline{ale}}_2) = 1.7588, \widetilde{SCR}(\overline{\overline{ale}}_3) = 1.7090, \widetilde{SCR}(\overline{\overline{ale}}_4) = 1.6619, \widetilde{SCR}(\overline{\overline{ale}}_5) = 1.6642.$	$\widetilde{SCR}(\overline{\overline{ale}}_1) = -0.5304, \widetilde{SCR}(\overline{\overline{ale}}_2) = -0.3679, \widetilde{SCR}(\overline{\overline{ale}}_3) = -0.5529, \widetilde{SCR}(\overline{\overline{ale}}_4) = -0.8495, \widetilde{SCR}(\overline{\overline{ale}}_5) = -0.8023.$	$\overline{\overline{ale}}_2 > \overline{\overline{ale}}_1 > \overline{\overline{ale}}_3 > \overline{\overline{ale}}_4 > \overline{\overline{ale}}_5.$ and $\overline{\overline{ale}}_2 > \overline{\overline{ale}}_1 > \overline{\overline{ale}}_3 > \overline{\overline{ale}}_5 > \overline{\overline{ale}}_4.$
$\gamma_1 = 15, \gamma_2 = 25$	$\widetilde{SCR}(\overline{\overline{ale}}_1) = 1.8655, \widetilde{SCR}(\overline{\overline{ale}}_2) = 1.8823, \widetilde{SCR}(\overline{\overline{ale}}_3) = 1.8496, \widetilde{SCR}(\overline{\overline{ale}}_4) = 1.8312, \widetilde{SCR}(\overline{\overline{ale}}_5) = 1.8355.$	$\widetilde{SCR}(\overline{\overline{ale}}_1) = -1.0354, \widetilde{SCR}(\overline{\overline{ale}}_2) = -0.7914, \widetilde{SCR}(\overline{\overline{ale}}_3) = -0.9676, \widetilde{SCR}(\overline{\overline{ale}}_4) = -1.2396, \widetilde{SCR}(\overline{\overline{ale}}_5) = -1.2463.$	$\overline{\overline{ale}}_2 > \overline{\overline{ale}}_1 > \overline{\overline{ale}}_3 > \overline{\overline{ale}}_5 > \overline{\overline{ale}}_4.$ and $\overline{\overline{ale}}_2 > \overline{\overline{ale}}_3 > \overline{\overline{ale}}_1 > \overline{\overline{ale}}_4 > \overline{\overline{ale}}_5.$
$\gamma_1 = 30, \gamma_2 = 35$	$\widetilde{SCR}(\overline{\overline{ale}}_1) = 1.9032, \widetilde{SCR}(\overline{\overline{ale}}_2) = 1.9186, \widetilde{SCR}(\overline{\overline{ale}}_3) = 1.9010, \widetilde{SCR}(\overline{\overline{ale}}_4) = 1.8843, \widetilde{SCR}(\overline{\overline{ale}}_5) = 1.8893.$	$\widetilde{SCR}(\overline{\overline{ale}}_1) = -1.1533, \widetilde{SCR}(\overline{\overline{ale}}_2) = -0.2095, \widetilde{SCR}(\overline{\overline{ale}}_3) = -0.2537, \widetilde{SCR}(\overline{\overline{ale}}_4) = -1.4230, \widetilde{SCR}(\overline{\overline{ale}}_5) = -1.3193.$	$\overline{\overline{ale}}_2 > \overline{\overline{ale}}_1 > \overline{\overline{ale}}_3 > \overline{\overline{ale}}_5 > \overline{\overline{ale}}_4.$ and $\overline{\overline{ale}}_2 > \overline{\overline{ale}}_3 > \overline{\overline{ale}}_1 > \overline{\overline{ale}}_5 > \overline{\overline{ale}}_4.$
$\gamma_1 = 50, \gamma_2 = 50$	$\widetilde{SCR}(\overline{\overline{ale}}_1) = 1.9245, \widetilde{SCR}(\overline{\overline{ale}}_2) = 1.9389, \widetilde{SCR}(\overline{\overline{ale}}_3) = 1.9288, \widetilde{SCR}(\overline{\overline{ale}}_4) = 1.9163, \widetilde{SCR}(\overline{\overline{ale}}_5) = 1.9175.$	$\widetilde{SCR}(\overline{\overline{ale}}_1) = -0.3141, \widetilde{SCR}(\overline{\overline{ale}}_2) = 0.0656, \widetilde{SCR}(\overline{\overline{ale}}_3) = -0.3119, \widetilde{SCR}(\overline{\overline{ale}}_4) = -0.3697, \widetilde{SCR}(\overline{\overline{ale}}_5) = -0.3531.$	$\overline{\overline{ale}}_2 > \overline{\overline{ale}}_1 > \overline{\overline{ale}}_3 > \overline{\overline{ale}}_5 > \overline{\overline{ale}}_4.$ and $\overline{\overline{ale}}_2 > \overline{\overline{ale}}_3 > \overline{\overline{ale}}_1 > \overline{\overline{ale}}_5 > \overline{\overline{ale}}_4.$

Table 8: Effect of parameter k on the final ranking results.

Parameter	Score value Cq-ROFPGWMSM	Score value Cq-ROFPGWDMSM	Ranking orders
$k = 2$	$\widetilde{SCR}(\overline{\overline{ale}}_1) = -0.4042, \widetilde{SCR}(\overline{\overline{ale}}_2) = -0.3031, \widetilde{SCR}(\overline{\overline{ale}}_3) = -0.4487, \widetilde{SCR}(\overline{\overline{ale}}_4) = -0.5560, \widetilde{SCR}(\overline{\overline{ale}}_5) = -0.5895.$	$\widetilde{SCR}(\overline{\overline{ale}}_1) = 1.6246, \widetilde{SCR}(\overline{\overline{ale}}_2) = 1.6950, \widetilde{SCR}(\overline{\overline{ale}}_3) = 1.6168, \widetilde{SCR}(\overline{\overline{ale}}_4) = 1.5718, \widetilde{SCR}(\overline{\overline{ale}}_5) = 1.4965.$	$\overline{\overline{ale}}_2 > \overline{\overline{ale}}_1 > \overline{\overline{ale}}_3 > \overline{\overline{ale}}_4 > \overline{\overline{ale}}_5.$ and $\overline{\overline{ale}}_2 > \overline{\overline{ale}}_1 > \overline{\overline{ale}}_3 > \overline{\overline{ale}}_4 > \overline{\overline{ale}}_5.$
$k = 3$	$\widetilde{SCR}(\overline{\overline{ale}}_1) = 0.3027, \widetilde{SCR}(\overline{\overline{ale}}_2) = 0.3552, \widetilde{SCR}(\overline{\overline{ale}}_3) = 0.1937, \widetilde{SCR}(\overline{\overline{ale}}_4) = -0.0768, \widetilde{SCR}(\overline{\overline{ale}}_5) = 0.0381.$	$\widetilde{SCR}(\overline{\overline{ale}}_1) = 1.4176, \widetilde{SCR}(\overline{\overline{ale}}_2) = 1.4887, \widetilde{SCR}(\overline{\overline{ale}}_3) = 1.4236, \widetilde{SCR}(\overline{\overline{ale}}_4) = 1.3206, \widetilde{SCR}(\overline{\overline{ale}}_5) = 1.3970.$	$\overline{\overline{ale}}_2 > \overline{\overline{ale}}_1 > \overline{\overline{ale}}_3 > \overline{\overline{ale}}_5 > \overline{\overline{ale}}_4.$ and $\overline{\overline{ale}}_2 > \overline{\overline{ale}}_3 > \overline{\overline{ale}}_1 > \overline{\overline{ale}}_5 > \overline{\overline{ale}}_4.$

Table 9: Comparison with existing approaches.

Approach	Score values	Ranking order
Cq-PWA operator [22]	$\widetilde{\mathrm{SCR}}\left(\overline{\overline{\mathrm{ale}}}_1\right)=0.9298, \widetilde{\mathrm{SCR}}\left(\overline{\overline{\mathrm{ale}}}_2\right)=1.2256,$ $\widetilde{\mathrm{SCR}}\left(\overline{\overline{\mathrm{ale}}}_3\right)=1.1570, \widetilde{\mathrm{SCR}}\left(\overline{\overline{\mathrm{ale}}}_4\right)=1.0640,$ $\widetilde{\mathrm{SCR}}\left(\overline{\overline{\mathrm{ale}}}_5\right)=0.8413.$	$\overline{\overline{\mathrm{ale}}}_2>\overline{\overline{\mathrm{ale}}}_3>\overline{\overline{\mathrm{ale}}}_4>\overline{\overline{\mathrm{ale}}}_1>\overline{\overline{\mathrm{ale}}}_5.$
Cq-HM operator [25]	$\widetilde{\mathrm{SCR}}\left(\overline{\overline{\mathrm{ale}}}_1\right)=-1.3039, \widetilde{\mathrm{SCR}}\left(\overline{\overline{\mathrm{ale}}}\ 2\right)=-1.0502,$ $\widetilde{\mathrm{SCR}}\left(\overline{\overline{\mathrm{ale}}}\ 3\right)=-1.0627, \widetilde{\mathrm{SCR}}\left(\overline{\overline{\mathrm{ale}}}_4\right)=-1.1320,$ $\widetilde{\mathrm{SCR}}\left(\overline{\overline{\mathrm{ale}}}\ 5\right)=-1.2982.$	$\overline{\overline{\mathrm{ale}}}_2>\overline{\overline{\mathrm{ale}}}_3>\overline{\overline{\mathrm{ale}}}_4>\overline{\overline{\mathrm{ale}}}_1>\overline{\overline{\mathrm{ale}}}_5.$
PhyWA [42]	$\widetilde{\mathrm{SCR}}\left(\overline{\overline{\mathrm{ale}}}\ 1\right)=1.0372, \widetilde{\mathrm{SCR}}\left(\overline{\overline{\mathrm{ale}}}_2\right)=1.3125,$ $\widetilde{\mathrm{SCR}}\left(\overline{\overline{\mathrm{ale}}}_3\right)=1.2453, \widetilde{\mathrm{SCR}}\left(\overline{\overline{\mathrm{ale}}}_4\right)=1.1450,$ $\widetilde{\mathrm{SCR}}\left(\overline{\overline{\mathrm{ale}}}_5\right)=1.0721.$	$\overline{\overline{\mathrm{ale}}}_2>\overline{\overline{\mathrm{ale}}}_3>\overline{\overline{\mathrm{ale}}}_4>\overline{\overline{\mathrm{ale}}}_5>\overline{\overline{\mathrm{ale}}}_1.$
In this article	$\widetilde{\mathrm{SCR}}\left(\overline{\overline{\mathrm{ale}}}_1\right)=-0.4042, \widetilde{\mathrm{SCR}}\left(\overline{\overline{\mathrm{ale}}}_2\right)=-0.3031,$ $\widetilde{\mathrm{SCR}}\left(\overline{\overline{\mathrm{ale}}}_3\right)=-0.4487, \widetilde{\mathrm{SCR}}\left(\overline{\overline{\mathrm{ale}}}_4\right)=-0.5560,$ $\widetilde{\mathrm{SCR}}\left(\overline{\overline{\mathrm{ale}}}_5\right)=-0.5895.$	$\overline{\overline{\mathrm{ale}}}_2>\overline{\overline{\mathrm{ale}}}_1>\overline{\overline{\mathrm{ale}}}_3>\overline{\overline{\mathrm{ale}}}_4>\overline{\overline{\mathrm{ale}}}_5.$
In this article	$\widetilde{\mathrm{SCR}}\left(\overline{\overline{\mathrm{ale}}}_1\right)=1.6246, \widetilde{\mathrm{SCR}}\left(\overline{\overline{\mathrm{ale}}}_2\right)=1.6950,$ $\widetilde{\mathrm{SCR}}\left(\overline{\overline{\mathrm{ale}}}_3\right)=1.6168, \widetilde{\mathrm{SCR}}\left(\overline{\overline{\mathrm{ale}}}_4\right)=1.5718,$ $\widetilde{\mathrm{SCR}}\left(\overline{\overline{\mathrm{ale}}}_5\right)=1.4965.$	$\overline{\overline{\mathrm{ale}}}_2>\overline{\overline{\mathrm{ale}}}_1>\overline{\overline{\mathrm{ale}}}_3>\overline{\overline{\mathrm{ale}}}_4>\overline{\overline{\mathrm{ale}}}_5.$

Step 3. Discover the support degree $T(Cq^c_{de})$ by utilizing Formula (67). For easiness, we shall write $T^c_{de} (c=1,2,..,4, d=1,2,.\cdots,5, e=1,2,3)$ in place of $T(Cq^c_{de})$. We have

$$
\begin{aligned}
&T^1_{11}=1.7783, T^1_{12}=1.6799, T^1_{13}=1.6599, T^1_{21}=1.6624,\\
&T^1_{22}=1.7808, T^1_{23}=1.6799, T^1_{31}=1.6624, T^1_{32}=1.6799,\\
&T^1_{33}=1.7808, T^1_{41}=0.9578, T^1_{42}=1.3447,\\
&T^1_{43}=1.0762, T^1_{51}=1.4455, T^1_{52}=1.0095, T^1_{53}=1.3272,\\
&T^2_{11}=1.8991, T^2_{12}=1.7983, T^2_{13}=1.8991, T^2_{21}=1.7808,\\
&T^2_{22}=1.6624, T^2_{23}=1.6799, T^2_{31}=1.6746, T^2_{32}=1.7044,\\
&T^2_{33}=1.7930, T^2_{41}=1.8816, T^2_{42}=1.8816,\\
&T^2_{43}=1.7633, T^2_{51}=1.7315, T^2_{52}=1.6306, T^2_{53}=1.5639,\\
&T^3_{11}=1.6306, T^3_{12}=1.7315, T^3_{13}=1.5639,\\
&T^3_{21}=1.6799, T^3_{22}=1.6624, T^3_{23}=1.7808,\\
&T^3_{31}=1.3447, T^3_{32}=1.6131, T^3_{33}=1.4948,\\
&T^3_{41}=1.6624, T^3_{42}=1.6799, T^3_{43}=1.7808, T^3_{51}=1.7315,\\
&T^3_{52}=1.5639, T^3_{53}=1.6306, T^4_{11}=1.6799, T^4_{12}=1.6624,\\
&T^4_{13}=1.7808, T^4_{21}=1.4948, T^4_{22}=1.3447, T^4_{23}=1.6131,\\
&T^4_{31}=1.5639, T^4_{32}=1.7315, T^4_{33}=1.6306, T^4_{41}=1.6306,\\
&T^4_{42}=1.5639, T^4_{43}=1.7315, T^4_{51}=1.6131, T^4_{52}=1.4455,\\
&T^4_{53}=1.3939.
\end{aligned}
\tag{80}
$$

Step 4. Employ Formula (68) to discover the weights o^c_{de}; we have

$$
\begin{aligned}
&\mathrm{o}^1_{11}=1.0253, \mathrm{o}^1_{12}=0.9879, \mathrm{o}^1_{13}=0.9867, \mathrm{o}^1_{21}=0.9871,\\
&\mathrm{o}^1_{22}=1.0257, \mathrm{o}^1_{23}=0.9871, \mathrm{o}^1_{31}=0.9871,\\
&\mathrm{o}^1_{32}=0.9871, \mathrm{o}^1_{33}=1.0257, \mathrm{o}^1_{41}=0.9156, \mathrm{o}^1_{42}=1.1174,\\
&\mathrm{o}^1_{43}=0.9670, \mathrm{o}^1_{51}=1.1659, \mathrm{o}^1_{52}=0.9171, \mathrm{o}^1_{53}=1.0457,\\
&\mathrm{o}^2_{11}=1.0122, \mathrm{o}^2_{12}=0.9755, \mathrm{o}^2_{13}=1.0122, \mathrm{o}^2_{21}=1.0257,\\
&\mathrm{o}^2_{22}=0.9871, \mathrm{o}^2_{23}=0.9871, \mathrm{o}^2_{31}=0.9839,\\
&\mathrm{o}^2_{32}=0.9942, \mathrm{o}^2_{33}=1.0220, \mathrm{o}^2_{41}=1.0122, \mathrm{o}^2_{42}=1.0122,\\
&\mathrm{o}^2_{43}=0.9755, \mathrm{o}^2_{51}=1.0395, \mathrm{o}^2_{52}=0.9990, \mathrm{o}^2_{53}=0.9615,\\
&\mathrm{o}^3_{11}=0.9990, \mathrm{o}^3_{12}=1.0395, \mathrm{o}^3_{13}=0.9615, \mathrm{o}^3_{21}=0.9871,\\
&\mathrm{o}^3_{22}=0.9871, \mathrm{o}^3_{23}=1.0257, \mathrm{o}^3_{31}=0.9307,\\
&\mathrm{o}^3_{32}=1.0561, \mathrm{o}^3_{33}=1.0132, \mathrm{o}^3_{41}=0.9871, \mathrm{o}^3_{42}=0.9871,\\
&\mathrm{o}^3_{43}=1.0257, \mathrm{o}^3_{51}=1.0395, \mathrm{o}^3_{52}=0.9615, \mathrm{o}^3_{53}=0.9990,\\
&\mathrm{o}^4_{11}=0.9871, \mathrm{o}^4_{12}=0.9871, \mathrm{o}^4_{13}=1.0257, \mathrm{o}^4_{21}=1.0132,\\
&\mathrm{o}^4_{22}=0.9307, \mathrm{o}^4_{23}=1.0561, \mathrm{o}^4_{31}=0.9615, \mathrm{o}^4_{32}=1.0395,\\
&\mathrm{o}^4_{33}=0.9990, \mathrm{o}^4_{41}=0.9990, \mathrm{o}^4_{42}=0.9615, \mathrm{o}^4_{43}=1.0395,\\
&\mathrm{o}^4_{51}=1.0561, \mathrm{o}^4_{52}=0.9736, \mathrm{o}^4_{53}=0.9704.
\end{aligned}
\tag{81}
$$

Step 5. Employ Formula (69) to aggregate all the individual decision matrices $\overline{\overline{\mathrm{DMT}}}^{c} = (Cq^{c}_{de})_{m\times n} (c = 1, 2, \cdots, 4)$ into group decision matrix $\widetilde{\mathrm{DMT}} = (Cq_{dc})_{m\times n}$, which is given in Table 4 $(q = 2, \gamma_1 = \gamma_2 = 1)$.

Step 6. Employ Formula (70) to discover the support degrees $\mathrm{Sup}(Cq_{dc}, Cq_{dx}) (d = 1, 2, \cdots, 5, c, x = 1, 2, \cdots, 4)$. For easiness, we shall designate S_{dc} in place of $\mathrm{Sup}(Cq_{dc}, Cq_{dx})$. We have

$$\begin{aligned}
&S_{12} = S_{21} = 0.9254, S_{13} = S_{31} = 0.8783,\\
&S_{14} = S_{41} = 0.9835, S_{23} = S_{32} = 0.9530,\\
&S_{24} = S_{42} = 8514, S_{34} = S_{43} = 0.8044,\\
&S_{12} = S_{21} = 0.9937, S_{13} = S_{31} = 0.9769,\\
&S_{14} = S_{41} = 9462, S_{23} = S_{32} = 0.9832,\\
&S_{24} = S_{42} = 0.8736, S_{34} = S_{43} = 0.8834,\\
&S_{12} = S_{21} = 0.9842, S_{13} = S_{31} = 0.8931,\\
&S_{14} = S_{41} = 0.8407, S_{23} = S_{32} = 0.8774,\\
&S_{24} = S_{42} = 0.7712, S_{34} = S_{43} = 0.8537,\\
&S_{12} = S_{21} = 0.7294, S_{13} = S_{31} = 0.7559,\\
&S_{14} = S_{41} = 0.8841, S_{23} = S_{32} = 0.9735,\\
&S_{24} = S_{42} = 0.7960, S_{34} = S_{43} = 0.8124,\\
&S_{12} = S_{21} = 0.9481, S_{13} = S_{31} = 0.9509,\\
&S_{14} = S_{41} = 0.9770, S_{23} = S_{32} = 0.9972,\\
&S_{24} = S_{42} = 0.8654, S_{34} = S_{43} = 0.8682.
\end{aligned} \tag{82}$$

Step 7. Employ Formula (71), to discover the support degree $T(Cq_{dc})$. For easiness, we shall write T_{dc} in place of $T(Cq_{dc})$. We have

$$\begin{aligned}
&T_{11} = 2.7872, T_{12} = 2.7297, T_{13} = 2.6357,\\
&T_{14} = 2.6392, T_{21} = 2.9169, T_{22} = 2.8505,\\
&T_{23} = 2.8435, T_{24} = 2.7032,\\
&T_{31} = 2.7180, T_{32} = 2.6328, T_{33} = 2.6242,\\
&T_{34} = 2.4656, T_{41} = 2.3693, T_{42} = 2.4989,\\
&T_{43} = 2.5418, T_{44} = 2.4924,\\
&T_{51} = 2.8760, T_{52} = 2.8106, T_{53} = 2.8163,\\
&T_{54} = 2.7106.
\end{aligned} \tag{83}$$

Step 8. By employing Formula (72), to discover the weighting vector $o_{dc} (d = 1, 2, . \cdots, 5, c = 1, 2, \cdots, 4)$, we have

$$\begin{aligned}
&o_{11} = 1.0241, o_{12} = 1.0086, o_{13} = 0.9832,\\
&o_{14} = 0.9841, o_{21} = 1.0231, o_{22} = 1.0057,\\
&o_{23} = 1.0039, o_{24} = 0.9673,\\
&o_{31} = 1.0299, o_{32} = 1.0063, o_{33} = 1.0039,\\
&o_{34} = 0.9600, o_{41} = 0.9694, o_{42} = 1.0067,\\
&o_{43} = 1.0191, o_{44} = 1.0048,\\
&o_{51} = 1.0191, o_{52} = 1.0019, o_{53} = 1.0034,\\
&o_{54} = 0.9756.
\end{aligned} \tag{84}$$

Step 9. Employ Formula (73) to aggregate all assessment values $\overline{\overline{\mathrm{ale}}}_{dc} (d = 1, 2, . \cdots, 5, c = 1, 2, \cdots, 4)$ into overall assessment value $Cq_d (d = 1, 2, . \cdots, 5)$ corresponding to the alternatives $\overline{\overline{\mathrm{ale}}}_d$ $(d = 1, 2, \cdots, 5)$:

$$\begin{aligned}
&(q = 2, \gamma_1 = \gamma_2 = 1)\overline{\overline{\mathrm{ale}}}_1 = \langle [0.4681, 0.5503],\\
&\quad \cdot [0.6724, 0.7035], (0.5506, 0.7035) \rangle,\\
&\overline{\overline{\mathrm{ale}}}_2 = \langle [0.4914, 0.5794], [0.6558, 0.6895], (0.5799, 0.6893) \rangle,\\
&\overline{\overline{\mathrm{ale}}}_3 = \langle [0.4575, 0.5402], [0.6772, 0.7127], (0.5404, 0.7127) \rangle,\\
&\overline{\overline{\mathrm{ale}}}_4 = \langle [0.4399, 0.5199], [0.7000, 0.7363], (0.5199, 0.7363) \rangle,\\
&\overline{\overline{\mathrm{ale}}}_5 = \langle [0.4306, 0.5085], [0.7063, 0.7410], (0.5085, 0.7390) \rangle.
\end{aligned} \tag{85}$$

Step 10. By employing Definition 6 to discover the scores $\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_d)$, we have

$$\begin{aligned}
&\widetilde{\mathrm{SCR}}\left(\overline{\overline{\mathrm{ale}}}_1\right) = -0.4042, \widetilde{\mathrm{SCR}}\left(\overline{\overline{\mathrm{ale}}}_2\right) = -0.3031,\\
&\widetilde{\mathrm{SCR}}\left(\overline{\overline{\mathrm{ale}}}_3\right) = -0.4487,\\
&\widetilde{\mathrm{SCR}}\left(\overline{\overline{\mathrm{ale}}}_4\right) = -0.5560, \widetilde{\mathrm{SCR}}\left(\overline{\overline{\mathrm{ale}}}_5\right) = -0.5895.
\end{aligned}$$

Step 11. According to score values, the ranking order of the alternatives $\overline{\overline{\mathrm{ale}}}_d$ $(d = 1, 2, \cdots, 5)$ is $\overline{\overline{\mathrm{ale}}}_2 > \overline{\overline{\mathrm{ale}}}_1 > \overline{\overline{\mathrm{ale}}}_3 > \overline{\overline{\mathrm{ale}}}_4 > \overline{\overline{\mathrm{ale}}}_5$. According to the ranking order, the best alternative is $\overline{\overline{\mathrm{ale}}}_2$ (i.e., Dr. Tahir Mahmood), while the last one is $\overline{\overline{\mathrm{ale}}}_5$ (i.e., Dr. Usman Nasir).

6.2. Decision-Making Steps Utilizing Cq-ROFPGWDMSM Operator. Steps 1–4. These are the same as those for decision-making model utilizing Cq-ROFPGWMSM operator.

Step 5. Employ Formula (74) to aggregate all the individual decision matrices $\overline{\overline{\mathrm{DMT}}}^{c} = (Cq^{c}_{de})_{m\times n} (c = 1, 2, \cdots, 4)$ into group decision matrix $\widetilde{\mathrm{DMT}} = (Cq_{dc})_{m\times n}$, which is given in Table 5. $(q = 2, \gamma_1 = \gamma_2 = 1)$.

Step 6. Employ Formula (75) to discover the support degrees $\mathrm{Sup}(Cq_{dc}, Cq_{dx})(d = 1, 2, \cdots, 5, c, x = 1, 2, \cdots, 4)$. For easiness, we shall designate S_{dc} in place of $\mathrm{Sup}(Cq_{dc}, Cq_{dx})$. We have

$$\begin{aligned} &S_{12} = S_{21} = 0.9525, S_{13} = S_{31} = 0.9295, \\ &S_{14} = S_{41} = 0.9977, S_{23} = S_{32} = 0.9769, \\ &S_{24} = S_{42} = 0.9286, S_{34} = S_{43} = 0.9056, \\ &S_{12} = S_{21} = 0.9884, S_{13} = S_{31} = 0.9861, \\ &S_{14} = S_{41} = 0.9771, S_{23} = S_{32} = 0.9977, \\ &S_{24} = S_{42} = 0.9648, S_{34} = S_{43} = 0.9661, \\ &S_{12} = S_{21} = 0.9751, S_{13} = S_{31} = 0.9568, \\ &S_{14} = S_{41} = 0.9209, S_{23} = S_{32} = 0.9525, \\ &S_{24} = S_{42} = 0.8992, S_{34} = S_{43} = 0.9254, \\ &S_{12} = S_{21} = 0.8602, S_{13} = S_{31} = 0.8662, \\ &S_{14} = S_{41} = 0.9483, S_{23} = S_{32} = 0.9940, \\ &S_{24} = S_{42} = 0.9129, S_{34} = S_{43} = 0.9166, \\ &S_{12} = S_{21} = 0.9721, S_{13} = S_{31} = 0.9728, \\ &S_{14} = S_{41} = 0.9919, S_{23} = S_{32} = 1.0000, \\ &S_{24} = S_{42} = 0.9383, S_{34} = S_{43} = 0.9383. \end{aligned} \tag{86}$$

Step 7. Employ Formula (76), to discover the support degree $T(Cq_{dc})$. For easiness, we shall write T_{dc} in place of $T(Cq_{dc})$. We have

$$\begin{aligned} &T_{11} = 2.8797, T_{12} = 2.8581, T_{13} = 2.8120, \\ &T_{14} = 2.8319, T_{21} = 2.9516, T_{22} = 2.9509, \\ &T_{23} = 2.9498, T_{24} = 2.9080, \\ &T_{31} = 2.8529, T_{32} = 2.8269, T_{33} = 2.8347, \\ &T_{34} = 2.7456, T_{41} = 2.6747, T_{42} = 2.7672, \\ &T_{43} = 2.7769, T_{44} = 2.7779, \\ &T_{51} = 2.9368, T_{52} = 2.9104, T_{53} = 2.9111, \\ &T_{54} = 2.8685. \end{aligned} \tag{87}$$

Step 8. By employing Formula (77), to discover the weighting vector $\tilde{\mathrm{O}}_{dc}(d = 1, 2, \cdots, 5, c = 1, 2, \cdots, 4)$, we have

$$\begin{aligned} &Ł_{11} = 1.0089, Ł_{12} = 1.0033, Ł_{13} = 0.9913, \\ &Ł_{14} = 0.9965, Ł_{21} = 1.0029, Ł_{22} = 1.0027, \\ &Ł_{23} = 1.0025, Ł_{24} = 0.9919, \\ &Ł_{31} = 1.0099, Ł_{32} = 1.0031, Ł_{33} = 1.0052, \\ &Ł_{34} = 0.9818, Ł_{41} = 0.9801, Ł_{42} = 1.0048, \\ &Ł_{43} = 1.0074, Ł_{44} = 1.0077, \\ &Ł_{51} = 1.0077, Ł_{52} = 1.0009, Ł_{53} = 1.0011, \\ &Ł_{54} = 0.9902. \end{aligned} \tag{88}$$

Step 9. Employ Formula (73) to aggregate all assessment values $\overline{\overline{\mathrm{ale}}}_{dc}(d = 1, 2, \cdots, 5, c = 1, 2, \cdots, 4)$ into overall assessment value $Cq_d(d = 1, 2, \cdots, 5)$ corresponding to the alternatives $\overline{\overline{\mathrm{ale}}}_d\ (d = 1, 2, \cdots, 5)$, $(q = 2, \gamma_1 = \gamma_2 = 1)$

$$\overline{\overline{\mathrm{ale}}}_1 = \langle [0.8887, 0.9292], [0.1548, 0.1885], (0.9292, 0.1885) \rangle, \tag{89}$$

$\overline{\overline{\mathrm{ale}}}_2 = \langle [0.9028, 0.9445], [0.1324, 0.1670], (0.9445, 0.1670) \rangle$,
$\overline{\overline{\mathrm{ale}}}_3 = \langle [0.8738, 0.9310], [0.1511, 0.1889], (0.9310, 0.1889) \rangle$,
$\overline{\overline{\mathrm{ale}}}_4 = \langle [0.8805, 0.9217], [0.1808, 0.2218], (0.9217, 0.2218) \rangle$,
$\overline{\overline{\mathrm{ale}}}_5 = \langle [0.8639, 0.9053], [0.1980, 0.2399], (0.9053, 0.2399) \rangle$.

Step 10. By employing Definition 6 to discover the scores $\widetilde{\mathrm{SCR}}(\overline{\overline{\mathrm{ale}}}_d)$, we have

$$\begin{aligned} &\widetilde{\mathrm{SCR}}\left(\overline{\overline{\mathrm{ale}}}_1\right) = 1.6246, \widetilde{\mathrm{SCR}}\left(\overline{\overline{\mathrm{ale}}}_2\right) = 1.6950, \\ &\widetilde{\mathrm{SCR}}\left(\overline{\overline{\mathrm{ale}}}_3\right) = 1.6168, \\ &\widetilde{\mathrm{SCR}}\left(\overline{\overline{\mathrm{ale}}}_4\right) = 1.5718, \widetilde{\mathrm{SCR}}\left(\overline{\overline{\mathrm{ale}}}_5\right) = 1.4965. \end{aligned} \tag{90}$$

Step 11. According to score values, the ranking order of the alternatives $\overline{\overline{\mathrm{ale}}}_d\ (d = 1, 2, \cdots, 5)$ is $\overline{\overline{\mathrm{ale}}}_2 > \overline{\overline{\mathrm{ale}}}_1 > \overline{\overline{\mathrm{ale}}}_3 > \overline{\overline{\mathrm{ale}}}_4 > \overline{\overline{\mathrm{ale}}}_5$. According to the ranking order, the best alternative is (*i.e.*, Dr. Tahir Mahmood), while the last one is $\overline{\overline{\mathrm{ale}}}_5$ (*i.e.*, Dr. Usman Nasir).

6.3. Effect of Parameter q on Final Ranking Result. In this subpart, the influence of the parameter q is discussed on the final ranking results while utilizing Cq-ROFPGWMSM and Cq-ROFPGWDMSM operators. The score values and ranking orders are given in Table 6. From Table 6, one can observe distinct values of the parameter q, while the values of generalized parameters $\gamma_1 = \gamma_2 = 1$ and $k = 2$ are fixed, the obtained ranking orders are different. When the value of parameter $q = 25$, while utilizing Cq-ROFPGWMSM operator the best alternative remains the same, but the worst alternative is $\overline{\overline{\mathrm{ale}}}_4$. One can also observe from Table 6, when the value of parameter q increases, the score values increase utilizing Cq-ROFPGWMSM operator. Similarly, when the value of parameter q increases, the score values decrease utilizing Cq-ROFPGWDMSM operator. So, the decision-makers choose the value of parameter q according to the actual need of the situation.

6.4. Effect of the Generalized Parameter. In this subpart, the influence of the generalized parameters γ_1, γ_2 are discussed on the final ranking results while utilizing Cq-ROFPGWMSM and Cq-ROFPGWDMSM operators. The values of the parameter $q = 2, k = 2$ are fixed. The score values and ranking orders are given in Table 7. From Table 7, one can observe that for distinct values of the parameter γ_1, γ_2, the obtained ranking orders are different. When the value of the parameter $\gamma_1 = 15, \gamma_2 = 25$, while utilizing Cq-ROFPGWMSM operator, the best alternative remains the same, but the worst alternative is $\overline{\overline{\text{ale}}}_4$. One can also observe from Table 7, when the value of the parameters γ_1, γ_2 increases, the score values increases utilizing Cq-ROFPGWMSM operator. Similarly, when the value of the parameters γ_1, γ_2 increases, the score values decrease utilizing Cq-ROFPGWDMSM operator. So, the decision-makers choose the value of the parameters γ_1, γ_2 according to the actual need of the situation.

6.5. Effect of Parameter k on Final Ranking Results. In this subpart, the effect of parameter k is discussed on the final ranking results, where the values of the generalized parameters $\gamma_1 = \gamma_2 = 1$ and $q = 2$ are fixed. From Table 8, one can observe that for different values of parameter k, the obtained ranking orders are different. When the value of the parameter $k = 2$, that is, when considering interrelationship among input arguments, the best alternatives is $\overline{\overline{\text{ale}}}_2$.

6.6. Comparison with Existing Approaches. In this subpart, we compare our initiated approach with some of the existing approaches such Wang et al. [22], Zhang et al. [25], and Abbas et al. [42]. From Table 9, one can notice that the ranking orders obtained from the existing approaches and the developed approach in this article are slightly different, although the best alternatives obtained from the existing approaches and the approaches initiated in this paper are the same. This shows that the proposed approaches are valid. The developed approaches, which are based on these newly AOs, have certain advantages over the existing developed approaches. The developed approaches based on these newly AOs can remove the effect of awkward data, can take interrelationship among any number of input arguments, at the same time, and have a general parameter, while the existing approaches can only have the capability of removing the effect of awkward data or can consider interrelationship between any two input arguments at a time. Due to the above characteristics, these newly initiated AOs can make the decision-making process more flexible. So, the proposed AOs are more practical and effective while dealing with Cq-ROF information.

7. Conclusion

The cubic q-rung orthopair fuzzy set (Cq-ROFS) is a fundamental simplification of numerous fuzzy concepts and one of the latest generalizations of q-ROFSs. Cq-ROFS exposes fuzzy judgement to the degrees of INVq-ROFS and q-ROFS, and it is one of the sophisticated mathematical tools for dealing with more intricate assessment information in multiple attribute group decision-making (MAGDM) situations. The article mainly introduced two types of hybrid AOs such as cubic q-rung orthopair fuzzy power generalized Maclaurin symmetric mean operator and cubic q-rung orthopair fuzzy power generalized dual Maclaurin symmetric mean operator. Then, a few of its core characteristics and some of the existing AOs are obtained from the proposed AOs by assigning different values to general parameters. Furthermore, weighted forms of the initiated AOs are also proposed. After that, two MAGDM models are offered based on Cq-OFPGWMSM and Cq-ROFPGWDMSM operators. Lastly, a numerical example about the selection of the best assistant professor of the year is taken as an application to show the effectiveness and capability of the initiated approaches. There is also a comparison with some existing decision-making models and discussed advantages of the proposed two MAGDM models over the existing MAGDM models for Cq-ROFS.

In the future, we will explore these AOs for other generalizations of linear Diophantine fuzzy set [34], T-spherical fuzzy sets [40], spherical linear Diophantine fuzzy set [41], and spherical linear Diophantine fuzzy soft rough set [43] on different t-norm and t-conorm and apply these AOs to solve MADM and MAGDM problems in different fields.

Data Availability

Data sharing does not apply to this article as no data set was generated or analyzed during the current study.

Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

Acknowledgments

This project is supported by Researchers Supporting Project number RSP-2021/156, King Saud University, Riyadh, Saudi Arabia.

References

[1] L. A. Zadeh, "Fuzzy sets," *Information and Control*, vol. 8, no. 3, pp. 338–353, 1965.

[2] Q. Pan, G. Chhipi-Shrestha, D. Zhou, K. Zhang, K. Hewage, and R. Sadiq, "Evaluating water reuse applications under uncertainty: generalized intuitionistic fuzzy-based approach," *Stochastic Environmental Research and Risk Assessment*, vol. 32, no. 4, pp. 1099–1111, 2018.

[3] P. Phochanikorn and C. Tan, "A new extension to a multi-criteria decision-making model for sustainable supplier selection under an intuitionistic fuzzy environment," *Sustainability*, vol. 11, no. 19, p. 5413, 2019.

[4] M. L. Tseng, P. A. Tan, S. Y. Jeng, C. W. R. Lin, Y. T. Negash, and S. N. A. C. Darsono, "Sustainable investment: interrelated among corporate governance, economic performance and market risks using investor preference approach," *Sustainability*, vol. 11, no. 7, p. 2108, 2019.

[5] R. Wang, G. Nan, L. Chen, and M. Li, "Channel integration strategies for competing dual-channel retailers: effect of cross-selling," *IEEE Transactions on Engineering Management*, vol. 16, no. 5, pp. 2260–2274, 2019.

[6] S. H. Mousavi-Avval, S. Rafiee, and A. Mohammadi, “Development and evaluation of combined adaptive neuro-fuzzy inference system and multi-objective genetic algorithm in energy, economic and environmental life cycle assessments of oilseed production,” *Sustainability*, vol. 13, no. 1, p. 290, 2021.

[7] R. Stekelorum, I. Laguir, S. Gupta, and S. Kumar, “Green supply chain management practices and third-party logistics providers’ performances: a fuzzy-set approach,” *International Journal of Production Economics*, vol. 235, p. 108093, 2021.

[8] L. Coppolino, L. Romano, A. Scaletti, and L. Sgaglione, “Fuzzy set theory-based comparative evaluation of cloud service offerings: an agro-food supply chain case study,” *Technology Analysis & Strategic Management*, vol. 33, no. 8, pp. 900–913, 2021.

[9] I. B. Turksen, “Interval valued fuzzy sets based on normal forms,” *Fuzzy Sets and Systems*, vol. 20, no. 2, pp. 191–210, 1986.

[10] K. Atanassov, “Intuitionistic fuzzy sets,” *Fuzzy sets and systems*, vol. 20, no. 1, pp. 87–96, 1986.

[11] Y. B. Jun, C. S. Kim, and K. O. Yang, “Cubic sets,” *Annals of Fuzzy Mathematics and Informatics*, vol. 4, no. 1, pp. 83–98, 2012.

[12] T. Mahmood, S. Abdullah, Saeed-ur-rashid, and M. Bilal, “Multicriteria decision making based on cubic set,” *Journal of New Theory*, vol. 16, pp. 1–9, 2017.

[13] S. Ayub, S. Abdullah, F. Ghani, M. Qiyas, and M. Y. Khan, “Cubic fuzzy Heronian mean Dombi aggregation operators and their application on multi-attribute decision-making problem,” *Soft Computing*, vol. 25, no. 6, pp. 4175–4189, 2021.

[14] A. Fahmi, F. Amin, S. Abdullah, and A. Ali, “Cubic fuzzy Einstein aggregation operators and its application to decision-making,” *International Journal of Systems Science*, vol. 49, no. 11, pp. 2385–2397, 2018.

[15] G. Kaur and H. Garg, “Multi-attribute decision-making based on Bonferroni mean operators under cubic intuitionistic fuzzy set environment,” *Entropy*, vol. 20, no. 1, p. 65, 2018.

[16] H. Garg and G. Kaur, “Cubic intuitionistic fuzzy sets and its fundamental properties,” *Journal of Multiple-Valued Logic & Soft Computing*, vol. 33, no. 6, 2019.

[17] Z. Yang and J. Chang, “Interval-valued Pythagorean normal fuzzy information aggregation operators for multi-attribute decision making,” *IEEE Access*, vol. 8, pp. 51295–51314, 2020.

[18] R. R. Yager, “Pythagorean fuzzy subsets,” in *2013 joint IFSA world congress and NAFIPS annual meeting (IFSA/NAFIPS)*, pp. 57–61, Edmonton, AB, Canada, 2013.

[19] P. Talukdar and P. Dutta, “Distance measures for cubic Pythagorean fuzzy sets and its applications to multicriteria decision making,” *Granular Computing*, vol. 6, no. 2, pp. 267–284, 2021.

[20] A. Fahmi, N. Yaqoob, and W. Chammam, “Maclaurin symmetric mean aggregation operators based on cubic Pythagorean linguistic fuzzy number,” *Journal of Ambient Intelligence and Humanized Computing*, vol. 12, no. 2, pp. 1925–1942, 2021.

[21] M. Naeem, S. Ashraf, S. Abdullah, and F. M. al-Harbi, “Redefined “Maclaurin symmetric mean aggregation operators based on cubic Pythagorean linguistic fuzzy numbers”,” *Mathematical Problems in Engineering*, vol. 2021, 19 pages, 2021.

[22] J. Wang, X. Shang, K. Bai, and Y. Xu, “A new approach to cubic q-rung orthopair fuzzy multiple attribute group decision-making based on power Muirhead mean,” *Neural Computing and Applications*, vol. 32, no. 17, pp. 14087–14112, 2020.

[23] R. R. Yager, “Generalized orthopair fuzzy sets,” *IEEE Transactions on Fuzzy Systems*, vol. 25, no. 5, pp. 1222–1230, 2017.

[24] B. P. Joshi, A. Singh, P. K. Bhatt, and K. S. Vaisla, “Interval valued q-rung orthopair fuzzy sets and their properties,” *Journal of Intelligent & Fuzzy Systems*, vol. 35, no. 5, pp. 5225–5230, 2018.

[25] B. Zhang, T. Mahmood, J. Ahmmad, Q. Khan, Z. Ali, and S. Zeng, “Cubic q-rung orthopair fuzzy Heronian mean operators and their applications to multi-attribute group decision making,” *Mathematics*, vol. 8, no. 7, p. 1125, 2020.

[26] H. Garg, T. Mahmood, J. Ahmmad, Q. Khan, and Z. Ali, “Cubic q-rung orthopair fuzzy linguistic set and their application to multiattribute decision-making with Muirhead mean operator,” *Journal of Artificial Intelligence and Technology*, vol. 1, no. 1, pp. 37–50, 2021.

[27] R. R. Yager, “The power average operator,” *IEEE Transactions on Systems, Man, and Cybernetics - Part A: Systems and Humans*, vol. 31, no. 6, pp. 724–731, 2001.

[28] Z. Xu, “Approaches to multiple attribute group decision making based on intuitionistic fuzzy power aggregation operators,” *Knowledge-Based Systems*, vol. 24, no. 6, pp. 749–760, 2011.

[29] C. Bonferroni, “Sulle medie multiple di potenze,” *Bollettino dell'Unione Matematica Italiana*, vol. 5, pp. 267–270, 1950.

[30] S. Sykora, “Mathematical means and averages: generalized Heronian means,” *Stan’s Library: Castano Primo, Italy*, vol. 3, 2009.

[31] R. F. Muirhead, “Some methods applicable to identities and inequalities of symmetric algebraic functions of n letters,” *Proceedings of the Edinburgh Mathematical Society*, vol. 21, pp. 144–162, 1902.

[32] C. Maclaurin, “A second letter to Martin Folkes, Esq.; concerning the roots of equations, with the demonstration of other rules of algebra,” *Philosophical Transactions*, vol. 36, pp. 59–96, 1729.

[33] G. Wei, C. Wei, J. Wang, H. Gao, and Y. Wei, “Some q-rung orthopair fuzzy maclaurin symmetric mean operators and their applications to potential evaluation of emerging technology commercialization,” *International Journal of Intelligent Systems*, vol. 34, no. 1, pp. 50–81, 2019.

[34] M. Riaz and M. R. Hashmi, “Linear Diophantine fuzzy set and its applications towards multi-attribute decision-making problems,” *Journal of Intelligent & Fuzzy Systems*, vol. 37, no. 4, pp. 5417–5439, 2019.

[35] P. Liu and P. Wang, “Some q-rung orthopair fuzzy aggregation operators and their applications to multiple-attribute decision making,” *International Journal of Intelligent Systems*, vol. 33, no. 2, pp. 259–280, 2018.

[36] P. Liu, Q. Khan, T. Mahmood, F. Smarandache, and Y. Li, “Multiple attribute group decision making based on 2-tuple linguistic neutrosophic Dombi power Heronian mean operators,” *IEEE Access*, vol. 7, pp. 100205–100230, 2019.

[37] Q. Khan, P. Liu, T. Mahmood, F. Smarandache, and K. Ullah, “Some interval neutrosophic dombi power bonferroni mean operators and their application in multi–attribute decision–making,” *Symmetry*, vol. 10, no. 10, p. 459, 2018.

[38] R. Wang, J. Wang, H. Gao, and G. Wei, “Methods for MADM with picture fuzzy Muirhead mean operators and their application for evaluating the financial investment risk,” *Symmetry*, vol. 11, no. 1, p. 6, 2019.

[39] H. Gao, Y. Ju, W. Zhang, and D. Ju, “Multi-attribute decision-making method based on interval-valued q -rung orthopair fuzzy Archimedean Muirhead mean operators,” *IEEE Access*, vol. 7, pp. 74300–74315, 2019.

[40] Q. Khan, J. Gwak, M. Shahzad, and M. K. Alam, “A novel approached based on T-spherical fuzzy Schweizer-Sklar power Heronian mean operator for evaluating water reuse applications under uncertainty,” *Sustainability*, vol. 13, no. 13, p. 7108, 2021.

[41] M. Riaz, M. R. Hashmi, D. Pamucar, and Y. M. Chu, “Spherical linear Diophantine fuzzy sets with modeling uncertainties in MCDM,” *Computer Modeling in Engineering & Sciences*, vol. 126, no. 3, pp. 1125–1164, 2021.

[42] S. Z. Abbas, M. S. Ali Khan, S. Abdullah, H. Sun, and F. Hussain, “Cubic Pythagorean fuzzy sets and their application to multi-attribute decision making with unknown weight information,” *Journal of Intelligent & Fuzzy Systems*, vol. 37, no. 1, pp. 1529–1544, 2019.

[43] M. R. Hashmi, S. T. Tehrim, M. Riaz, D. Pamucar, and G. Cirovic, “Spherical linear Diophantine fuzzy soft rough sets with multi-criteria decision making,” *Axioms*, vol. 10, no. 3, p. 185, 2021.