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Research Article

On Some w – Interpolative Contractions of Suzuki-Type Mappings in Quasi-Partial b-Metric Space

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In this paper, our focus is to acquaint with the Suzuki-type mappings to establish some fixed point results using the new w-interpolative approach. We present some results for interpolative contraction operators via the w-admissible maps which satisfy the Kannan, Ćirić–Reich–Rus, and Hardy–Rogers contractions in quasi-partial b-metric space. Further, the outcomes so obtained are affirmed with relevant examples.

1. Introduction

In the early 20th century, Fréchet [1], a French mathematician, initiated the concept of metric space, and due to its efficiency and practicable implementations, the idea has been upgraded, improved, and generalized by many authors. In 1922, Banach [2] discovered a remarkable result, that is, Banach contraction principle, which holds a significant position in the field of nonlinear analysis. Later, Karapinar [3] introduced quasi-partial metric spaces which were followed by the discovery of b–metric spaces in 1993, by Czerwik [4]. Gupta and Gautam [5] generalized quasi-partial metric to quasi-partial b–metric space and proved some fixed point results for such spaces. After all these classical results, Suzuki [6] introduced a new type of mappings which generalized the well-known Banach contraction principle.

In 2014, the notion of w-orbital admissible maps was introduced by Popescu [7] which is a refinement of the concept of α-admissible maps of Samet et al. [8].

Suppose S is a self-map defined on G and w: $G \times G \longrightarrow [0, \infty]$ is a mapping where G is nonempty. The mapping S is said to be w-orbital admissible if for all $\eta \in G$, we have

$$w(\eta, S\eta) \geq 1 \longrightarrow w\left(S\eta, S^2\eta\right) \geq 1. \quad (1)$$

If the continuity of the involved contractive mappings is removed, we necessarily need (G, qp_b) to be w-regular, i.e., if $\{\eta_n\}$ is a sequence in (G, qp_b) such that $\{\eta_n\} \longrightarrow t \in G$ as $n \longrightarrow \infty$ and $w\ (\eta_n, \eta_{n+1}) \geq 1$ for each n, then we have $w(\eta_n, t) \geq 1$.

We show that the condition of w-regularity holds in quasi-partial b-metric space by using w-admissibility condition. In our earlier work [9], we have shown that w-admissibility holds in quasi-partial b-metric space, i.e., $w(\eta_n, S\eta_n) \geq 1$; then, as $n \longrightarrow \infty$, we get $\{\eta_n\} \longrightarrow t$, and hence we get the condition for w-regularity.

Throughout the paper, $\mathbf{R}^+$, $\mathbf{N}$, and ϕ stand for the set of positive real numbers, natural numbers, and an empty set, respectively. Let Ψ be the set of all nondecreasing self-mappings ψ on $[0, \infty)$ such that $\sum_{r=1}^{\infty} \psi^r(\eta) < \infty$ for every $\eta > 0$. Notice that for $\Psi \in \psi$, we have $\psi(0) = 0$ and $\psi(\eta) < \eta$ for all $\eta > 0$ (see [10]).

2. Preliminaries

Definition 1 (see [5]). A function qp_b: $G \times G \longrightarrow R^+$ is said to be a quasi-partial b-metric on a nonempty set G if it satisfies the properties

(1) $qp_b(\eta, \eta) = qp_b(\eta, \zeta) = qp_b(\zeta, \zeta)$ implies $\eta = \zeta$;

(2) $qp_b(\eta,\eta) \leq qp_b(\zeta,\eta)$;
(3) $qp_b(\eta,\eta) \leq qp_b(\eta,\zeta)$;
(4) $qp_b(\eta,\zeta) \leq s[qp_b(\eta,\sigma) + qp_b(\sigma,\zeta)] - qp_b(\sigma,\sigma)$,

where s is called the coefficient of (G, qp_b) such that $s \geq 1$ for all $\eta, \zeta, \sigma \in G$.

Definition 2 (see [5]). Suppose (G, qp_b) is a quasi-partial b-metric space. Then,

(1) A sequence $\{\eta_n\}$ is called a Cauchy sequence if and only if $\lim_{n,m\longrightarrow\infty} qp_b(\eta_n, \eta_m)$ and $\lim_{n,m\longrightarrow\infty} qp_b(\eta_m, \eta_n)$ exist finitely.
(2) A sequence $\{\eta_n\} \subset G$ converges to $\eta \in G$ if and only if $qp_b(\eta,\eta) = \lim_{n\longrightarrow\infty} qp_b(\eta,\eta_n) = \lim_{n\longrightarrow\infty} qp_b(\eta_n,\eta)$.
(3) (G, qp_b) is said to be complete if every Cauchy sequence $\{\eta_n\} \subset G$ converges with respect to τ_{qp_b} to a point $\eta \in G$ that holds

$$qp_b(\eta,\eta) = \lim_{n,m\longrightarrow\infty} qp_b(\eta_n,\eta_m) = \lim_{n,m\longrightarrow\infty} qp_b(\eta_m,\eta_n). \tag{2}$$

(4) A mapping $f: G \longrightarrow G$ is said to be continuous at $\eta_0 \in G$ if for every $\varepsilon > 0$, there exists $\delta > 0$ such that $f(B(\eta_0,\delta)) \subset B(f(\eta_0),\varepsilon)$.

Definition 3 (see [9]). Suppose (G, qp_b) is a quasi-partial b-metric space. A self-map $S: G \longrightarrow G$ is known as a w-interpolative Ćirić–Reich–Rus contraction if there exist $\lambda \in \psi$ and a map $w: G \times G \longrightarrow [0,\infty]$ with real numbers $\alpha, \beta > 0$, satisfying $\alpha + \beta < 1$, that holds

$$w(\eta,\zeta) qp_b(S\eta, S\zeta) \leq \lambda [qp_b(\eta,\zeta)]^{\beta} . [qp_b(\eta, S\eta)]^{\alpha} . [qp_b(\zeta, S\zeta)]^{(1-\alpha-\beta)}, \tag{3}$$

for all η, $\zeta \in G$.

Definition 4 (see [11]). Suppose (G, qp_b) is a quasi-partial b-metric. Define a self-mapping $S: G \longrightarrow G$ and a map $w: G \times G \longrightarrow [0,\infty)$ where $\lambda \in \psi$ that holds

$$w(\eta,\zeta) qp_b(S\eta, S\zeta) \leq \lambda \left([qp_b(\eta,\zeta)]^{\alpha} \cdot [qp_b(\eta, S\zeta)]^{\beta} \cdot [qp_b(\zeta, S\zeta)]^{\gamma} \cdot \left[\frac{1}{2s} (qp_b(\eta, S\zeta) + qp_b(\zeta, S\eta) \right]^{1-\alpha-\beta-\gamma}, \tag{4}$$

for all η, $\zeta \in G$ and real numbers $\alpha, \beta, \gamma > 0$ that satisfy the condition $\alpha + \beta + \gamma < 1$. Such a mapping is known as w-interpolative Hardy–Rogers-type contraction.

Definition 5 (see [10]). A mapping S is said to satisfy C-condition on (G, qp_b), if it satisfies

$$\frac{1}{2} [qp_b(\eta, S\eta)] \leq qp_b(\eta,\zeta) \Rightarrow qp_b(S\eta, S\zeta) \leq qp_b(\eta,\zeta), \tag{5}$$

for all η, $\zeta \in G$.

Throughout the paper, qp_b and Cqp_b denote the quasi-partial b-metric space and complete quasi-partial b-metric space, respectively. One can see for more related point results in [12–15] and the references therein.

3. Main Results

We now define the main results for Suzuki-type mappings using the notion of w-interpolation (see 16, 17) and the fact that the condition of w-regularity holds in quasi-partial b–metric space (see 18–20]).

Definition 6. Let (G, qp_b) be a quasi-partial b-metric space and there exists a self-map $w: G \times G \longrightarrow [0,\infty)$ with a real number $\alpha \in [0, 1)$. A self-map $S: G \longrightarrow G$ is said to be a w-ψ-interpolative Kannan contraction of Suzuki type if there exist $\psi \in \Psi$ that satisfies

$$\frac{1}{2} [qp_b(\eta, S\eta)] \leq qp_b(\eta,\zeta) \Rightarrow w(\eta,\zeta) qp_b(S\eta, S\zeta) \leq \psi\left([qp_b(\eta, S\eta)]^{\alpha} [qp_b(\zeta, S\zeta)]^{1-\alpha} \right), \tag{6}$$

for all η, $\zeta \in G$.

Theorem 1. *Suppose (G, qp_b) is a Cqp_b and $S: G \longrightarrow G$ is a w-ψ-interpolative Kannan contraction of Suzuki type. Let S be a w-orbital admissible map and $w(\eta_0, S\eta_0) \geq 1$ for some $\eta_0 \in G$. Then, S possesses a fixed point in G if any of the following conditions hold:*

(1) (G, qp_b) is w-regular.
(2) S is continuous.
(3) ψS^2 is continuous and $w(\eta, S\eta) \geq 1$ when $\eta \in Fix(S^2)$.

Proof. Let $\eta_0 \in G$ with the condition $w(\eta_0, S\eta_0) \geq 1$ and $\{\eta_n\}$ be the sequence such that $S^n(\eta_0) = \eta_n$ for each positive integer n. For some $\eta_0 \in \mathbf{N}$, we have $\eta_{n_0} = \eta_{n_{0+1}}$. Hence, we get $\eta_{n_0} = S\eta_{n_0}$, so η_{n_0} is a fixed point of S. Hence, the proof is complete.

On the contrary, take $\eta_n \neq \eta_{n+1}$ for every positive integer n. As S is w-orbital admissible, we have the condition $w(\eta_0, S\eta_0) = w(\eta_0, \eta_1) \geq 1$ which implies that $w(\eta_1, S\eta_1) = w(\eta_1, \eta_2) \geq 1$. Proceeding in a similar way, we get

$$w(\eta_n, \eta_{n+1}) \geq 1. \tag{7}$$

Hence, choosing $\eta = \eta_{n-1}$ and $\zeta = S\eta_{n-1}$ in (6) gives

$$\begin{aligned}\frac{1}{2} qp_b(\eta, \zeta) &= \frac{1}{2} qp_b(\eta_{n-1}, S\eta_{n-1}) \\ &= \frac{1}{2} qp_b(\eta_{n-1}, \eta_n) \leq qp_b(\eta_{n-1}, \eta_n),\end{aligned} \tag{8}$$

which implies

$$\begin{aligned} qp_b(\eta_n, \eta_{n+1}) &\leq w(\eta_{n-1}, \eta_n) qp_b(S\eta_{n-1}, S\eta_n) \\ &\leq \psi\left([qp_b(\eta_{n-1}, S\eta_{n-1})]^{\alpha} \cdot [qp_b(\eta_n, S\eta_n)]^{1-\alpha} \right) \\ &= \psi\left([qp_b(\eta_{n-1}, \eta_n)]^{\alpha} \cdot [qp_b(\eta_n, \eta_{n+1})]^{1-\alpha} \right) \\ &< [qp_b(\eta_{n-1}, \eta_n)]^{\alpha} \cdot [qp_b(\eta_n, \eta_{n+1})]^{1-\alpha}. \end{aligned} \tag{9}$$

Hence, we have

$$[qp_b(\eta_n, \eta_{n+1})]^{\alpha} < [qp_b(\eta_{n-1}, \eta_n)]^{\alpha}, \tag{10}$$

which equivalently can be written as

$$qp_b(\eta_n, \eta_{n+1}) < qp_b(\eta_{n-1}, \eta_n). \tag{11}$$

Thus, we get that $\{qp_b(\eta_{n-1}, \eta_n)\}$ is a nonincreasing sequence of positive terms, so there exists $l \geq 0$ such that $\lim_{n\longrightarrow\infty} qp_b(\eta_{n-1}, \eta_n) = l$. On the other hand, from the above equations and the nondecreasing nature of function ψ, we obtain

$$\begin{aligned} qp_b(\eta_n, \eta_{n+1}) &\leq \psi(qp_b(\eta_{n-1}, \eta_n)) \leq \psi^2(qp_b(\eta_{n-2}, \eta_{n-1})) \\ &\leq \ldots \leq \psi^n(qp_b(\eta_0, \eta_1)). \end{aligned} \tag{12}$$

By triangular inequality, for all $j \geq 1$, we get

$$\begin{aligned} qp_b(\eta_n, \eta_{n+j}) &\leq \left[s qp_b(\eta_n, \eta_{n+1}) + s^2 qp_b(\eta_{n+1}, \eta_{n+2}) + \cdots + s^j qp_b(\eta_{n+j-1}, \eta_{n+j}) \right] \\ &\leq \left[s\psi^n qp_b(\eta_0, \eta_1) + s^2 \psi^{n+1} qp_b(\eta_0, \eta_1) + \cdots + s^j \psi^{n+j-1} qp_b(\eta_0, \eta_1) \right] \\ &= \sum_{m=n, r=1}^{n+j-1} s^r \psi^m (qp_b(\eta_0, \eta_1)) \\ &= P_{n+j-1} - P_{n-1}, \end{aligned} \tag{13}$$

where $P_k = s^k \sum_{m=0}^{k} \psi^m (qp_b(\eta_0, \eta_1))$. But, the series $\sum_{m=0}^{\infty} \psi^m (qp_b(\eta_0, \eta_1))$ is convergent, so there exists a positive real number p such that $\lim_{k\longrightarrow\infty} P_k = p$. Letting n and $j \longrightarrow \infty$ in the above inequality, we get

$$qp_b(\eta_n, \eta_{n+j}) \longrightarrow 0. \tag{14}$$

Hence, $\{\eta_n\}$ is a Cauchy sequence and using the completeness property of qp_b space, it shows that there exists $t \in G$ such that

$$\lim_{n\longrightarrow\infty} \eta_n = t. \tag{15}$$

Also, we claim that S possesses a fixed point as t.

In the case when assumption (1) holds true, we have $w(\eta_n, t) \geq 1$ and we claim that

$$\begin{aligned} \frac{1}{2} qp_b(\eta_n, S\eta_n) \leq qp_b(\eta_n, t) \text{ or } &\frac{1}{2} qp_b(S\eta_n, S(S\eta_n)) \\ &\leq qp_b(S\eta_n, t), \end{aligned} \tag{16}$$

for every $n \in \mathbf{N}$. On the contrary, if the above condition is not true, then by triangular inequality in qp_b space, we have

$$\begin{aligned} qp_b(\eta_n, \eta_{n+1}) &= qp_b(\eta_n, S\eta_n) + qp_b(t,t) \le qp_b(\eta_n, t) + qp_b(t, S\eta_n) \\ &< \frac{1}{2} qp_b(\eta_n, S\eta_n) + \frac{1}{2} qp_b(S\eta_n, S(S\eta_n)) \\ &= \frac{1}{2} qp_b(\eta_n, \eta_{n+1}) + \frac{1}{2} qp_b(\eta_{n+1}, \eta_{n+2}) \\ &\le \frac{1}{2} qp_b(\eta_n, \eta_{n+1}) + \frac{1}{2} qp_b(\eta_n, \eta_{n+1}) = qp_b(\eta_n, \eta_{n+1}), \end{aligned} \tag{17}$$

which is a contradiction, and hence our claim is proved. If the first condition holds, we obtain

$$\begin{aligned} qp_b(\eta_{n+1}, St) &\le w(\eta_n, t) qp_b(S\eta_n, St) \\ &\le \psi[qp_b(\eta_n, S\eta_n)]^\alpha . [qp_b(t, St)]^{1-\alpha} \\ &= \psi[qp_b(\eta_n, S\eta_{n+1})]^\alpha . [qp_b(t, St)]^{1-\alpha} \\ &< [qp_b(\eta_n, S\eta_{n+1})]^\alpha . [qp_b(t, St)]^{1-\alpha}. \end{aligned} \tag{18}$$

If the second condition holds, we get

$$\begin{aligned} qp_b(\eta_{n+2}, St) &\le w(\eta_{n+1}, t) qp_b(S^2\eta_n, St) \\ &\le \psi[qp_b(S\eta_n, S^2\eta_n)]^\alpha . [qp_b(t, St)]^{1-\alpha} \\ &= \psi[qp_b(\eta_{n+1}, \eta_{n+2})]^\alpha . [qp_b(t, St)]^{1-\alpha} \\ &< [qp_b(\eta_{n+1}, \eta_{n+2})]^\alpha . [qp_b(t, St)]^{1-\alpha}. \end{aligned} \tag{19}$$

Therefore, letting $n \longrightarrow \infty$, we get $qp_b(t, St) = 0$, that is, $t = St$.

In the case when assumption (2) holds, we have that the mapping S is continuous, so we get

$$St = \lim_{n \longrightarrow \infty} S\eta_n = \lim_{n \longrightarrow \infty} S\eta_{n+1} = t. \tag{20}$$

In the case when assumption (3) holds, we have $\psi S^2 t = \psi \lim_{n \longrightarrow \infty} S^2 \eta_n = \psi \lim_{n \longrightarrow \infty} \eta_{n+2} = \psi t$ and we prove that $St = t$. On the contrary, take $St \ne t$; then,

$$\frac{1}{2} qp_b(St, \psi S^2 t) = \frac{1}{2} qp_b(St, \psi t) \le qp_b(St, \psi t). \tag{21}$$

By (6), we get

$$\begin{aligned} qp_b(t, St) &\le w(St, t) qp_b(S^2 t, St) \\ &\le \psi[qp_b(St, S^2 t)]^\alpha . [qp_b(t, St)]^{1-\alpha} \\ &= \psi[qp_b(\eta_{n+1}, \eta_{n+2})]^\alpha . [qp_b(t, St)]^{1-\alpha} \\ &< [qp_b(St, t)]^\alpha . [qp_b(t, St)]^{1-\alpha} \\ &= qp_b(t, St). \end{aligned} \tag{22}$$

Hence, it is a contradiction. Thus, $t = St$, that is, t is a fixed point of the mapping S. □

Example 1. Let $G = [0, \pi/4]$ and define qp_b: $G \times G \longrightarrow [0, \infty)$ such that

$$qp_b(\eta, \zeta) = \text{Sin}\eta + \text{Sin}\zeta. \tag{23}$$

Define a self-mapping S: $G \longrightarrow G$ as

$$S\eta = \begin{cases} \pi/9, & \eta \in [0, \pi/8), \\ \eta, & \eta \in [\pi/8, \pi/4]. \end{cases} \tag{24}$$

Also, define w: $G \times G \longrightarrow [0, \infty)$ such that

$$w(\eta, \zeta) = \begin{cases} 2, & \eta = \pi/9 \text{ and } \zeta = \pi/5, \\ 0, & \text{otherwise}. \end{cases} \tag{25}$$

Choose $\alpha = 1/3$ and define the function $\psi \in \Psi$ as $\psi(\eta) = 2\eta/3$. The only case we need to verify is when $\eta = \pi/9$ and $\zeta = \pi/5$; as for the remaining cases, we have $w(\eta, \zeta) = 0$, which clearly implies that inequality (6) holds. So, when $\eta = \pi/9$ and $\zeta = \pi/5$, we get

$$\begin{aligned} \frac{1}{2}\left[qp_b\left(\frac{\pi}{9}, \frac{S\pi}{9}\right)\right] &= \frac{1}{2}\left[qp_b\left(\frac{\pi}{9}, \frac{\pi}{9}\right)\right] = 0.342 \le 0.929 \\ &= qp_b\left(\frac{\pi}{9}, \frac{\pi}{5}\right), \end{aligned} \tag{26}$$

which implies

$$w\left(\frac{\pi}{9}, \frac{\pi}{5}\right) qp_b\left(\frac{\pi}{9}, \frac{\pi}{5}\right) \le \psi\left(\left[qp_b\left(\frac{\pi}{9}, \frac{\pi}{9}\right)\right]^{1/3} \left[qp_b\left(\frac{\pi}{5}, \frac{\pi}{5}\right)\right]^{2/3}\right). \tag{27}$$

Hence, all the assumptions of Theorem 1 are satisfied, which follows that the mapping S owns a fixed point, that is, $\eta = \pi/9$, as shown in Figure 1.

Corollary 1. *Let (G, qp_b) be a $C\, qp_b$ and S be a self-map on G, satisfying*

$$\begin{aligned} &\frac{1}{2}[qp_b(\eta, S\eta)] \le qp_b(\eta, \zeta) \\ &\Rightarrow qp_b(S\eta, S\zeta) \le \psi\left([qp_b(\eta, S\eta)]^\alpha [qp_b(\zeta, S\zeta)]^{1-\alpha}\right), \end{aligned} \tag{28}$$

for all η, $\zeta \in G$, where $\alpha \in\alpha \in [0, 1)$. Then, S owns a fixed point in G.

Proof. For the proof, take $w(\eta, \zeta) = 1$ in Theorem 1. □

Corollary 2. *Let (G, qp_b) be a $C\, qp_b$ and S be a self-map on G, satisfying*

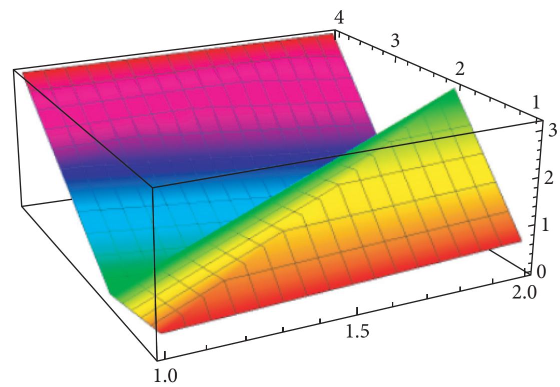

Figure 1: The graphical surface represents a 3-D view of the function, $qp_b(\eta,\zeta) = \text{Sin}\eta + \text{Sin}\zeta$. Clearly, the fixed point of the map S is $\pi/9$.

$$\begin{aligned}&\frac{1}{2}[qp_b(\eta,S\eta)] \le qp_b(\eta,\zeta)\\ &\quad\Rightarrow qp_b(S\eta,S\zeta) \le g\left([qp_b(\eta,S\eta)]^{\alpha}[qp_b(\zeta,S\zeta)]^{1-\alpha}\right),\end{aligned} \tag{29}$$

for all η, $\zeta \in G$, where $\alpha \in [0, 1)$. Then, S owns a fixed point in G.

Proof. For the proof, take $\psi(\eta) = \eta g$, with $g \in [0, 1)$ and $\eta > 0$ in Corollary 1. □

Definition 7. Suppose (G, qp_b) is a quasi-partial b-metric space. Define a self-mapping $S: G \longrightarrow G$ such that there exist $\psi \in \Psi$ satisfying

$$\begin{aligned}&\frac{1}{2}[qp_b(\eta,S\eta)] \le qp_b(\eta,\zeta),\\ &w(\eta,\zeta)qp_b(S\eta,S\zeta) \le \psi\left([qp_b(\eta,\zeta)]^{\beta}[qp_b(\eta,S\eta)]^{\alpha}.[qp_b(\zeta,S\zeta)]^{1-\alpha-\beta}\right),\end{aligned} \tag{30}$$

for all η, $\zeta \in G$ and real numbers α, $\beta > 0$ that satisfy $\alpha+\beta < 1$. Such a mapping is called w-ψ-interpolative Ćirić–Reich–Rus contraction of Suzuki type (see 21–23).

Theorem 2. *Suppose (G, qp_b) is a $C\,qp_b$ and $S: G \longrightarrow G$ is a w-ψ-interpolative Ćirić–Reich–Rus contraction of Suzuki type. Let S be a w-orbital admissible map and $w(\eta_0, S\eta_0) \ge 1$ for some $\eta_0 \in G$. Then, S possesses a fixed point in G if any of the conditions hold:*

(1) (G, qp_b) is w-regular.

(2) S is continuous.

(3) ψS^2 is continuous and $w(S\eta,\eta) \ge 1$ when $\eta \in Fix(S^2)$.

Proof. Let $\eta_0 \in G$ with the condition $w(\eta_0, S\eta_0) \ge 1$ and $\{\eta_n\}$ be the sequence such that $S^n(\eta_0) = \eta_n$ for each positive integer n. Assume that for some $\eta_0 \in \mathbf{N}$, we have the condition $\eta_{n_0} = \eta_{n_{0+1}}$. Hence, we get $\eta_{n_0} = S\,\eta_{n_0}$, which implies η_{n_0} is the fixed point of S. So, the proof is complete.

On the contrary, take $\eta_n \ne \eta_{n+1}$ for each positive integer n. Since S is w-orbital admissible, we have $w(\eta_0, S\,\eta_0) = w(\eta_0, \eta_1) \ge 1$, which implies that $w(\eta_1, S\,\eta_1) = w(\eta_1, \eta_2) \ge 1$. Proceeding similarly,

$$w(\eta_n,\eta_{n+1}) \ge 1. \tag{31}$$

Hence, choosing $\eta = \eta_{n-1}$ and $\zeta = S\,\eta_{n-1}$ in (30), we get

$$\begin{aligned}\frac{1}{2}qp_b(\eta,\zeta) &= \frac{1}{2}qp_b(\eta_{n-1},S\eta_{n-1})\\ &= \frac{1}{2}qp_b(\eta_{n-1},\eta_n) \le qp_b(\eta_{n-1},\eta_n),\end{aligned} \tag{32}$$

which implies

$$\begin{aligned}qp_b(\eta_n,\eta_{n+1}) &\le w(\eta_{n-1},\eta_n)qp_b(S\eta_{n-1},S\eta_n)\\ &\le \psi\left([qp_b(\eta_{n-1},\eta_n)]^{\beta}.[qp_b(\eta_{n-1},S\eta_{n-1})]^{\alpha}.[qp_b(\eta_n,S\eta_n)]^{1-\alpha-\beta}\right)\\ &= \psi\left([qp_b(\eta_{n-1},\eta_n)]^{\beta}.[qp_b(\eta_{n-1},\eta_n)]^{\alpha}.[qp_b(\eta_n,\eta_{n+1})]^{1-\alpha-\beta}\right).\end{aligned} \tag{33}$$

Then, using $\psi(\eta) < \eta$ for every $\eta > 0$, we get

$$qp_b(\eta_n, \eta_{n+1}) \leq [qp_b(\eta_{n-1}, \eta_n)]^{\beta+\alpha} . [qp_b(\eta_n, \eta_{n+1})]^{1-\beta-\alpha}, \tag{34}$$

which equivalently can be written as

$$[qp_b(\eta_n, \eta_{n+1})]^{\alpha+\beta} < [qp_b(\eta_{n-1}, \eta_n)]^{\alpha+\beta}. \tag{35}$$

So, we get

$$qp_b(\eta_n, \eta_{n+1}) < qp_b(\eta_{n-1}, \eta_n). \tag{36}$$

Hence, it shows that $\{qp_b(\eta_{n-1}, \eta_n)\}$ is a decreasing sequence. Eventually, we have

$$qp_b(\eta_n, \eta_{n+1}) \leq \psi(qp_b(\eta_{n-1}, \eta_n)). \tag{37}$$

In a similar way, we get

$$qp_b(\eta_n, \eta_{n+1}) \leq \psi^n(qp_b(\eta_0, \eta_1)). \tag{38}$$

Since $\{\eta_n\}$ is a fundamental sequence, applying triangular inequality, we get

$$\begin{aligned} qp_b(\eta_n, \eta_{n+l}) &\leq \left[s qp_b(\eta_n, \eta_{n+1}) + s^2 qp_b(\eta_{n+1}, \eta_{n+2}) + \cdots + s^l qp_b(\eta_{n+l-1}, \eta_{n+l})\right] \\ &\leq \left[s\psi^n qp_b(\eta_0, \eta_1) + s^2 \psi^{n+1} qp_b(\eta_0, \eta_1) + \cdots + s^l \psi^{n+l-1} qp_b(\eta_0, \eta_1)\right] \\ &= \sum_{k=n, r=1}^{\infty} s^r \psi^k (qp_b(\eta_0, \eta_1)). \end{aligned} \tag{39}$$

Taking $n \longrightarrow \infty$, we deduce that $\{\eta_n\}$ is a fundamental sequence in qp_b and by the completeness property of qp_b, there exists $t \in G$ satisfying

$$\lim_{n\longrightarrow\infty} qp_b(\eta_n, t) = 0. \tag{40}$$

If assumption (1) holds, then we have $w\ (\eta_n, t) \geq 1$ and we claim that

$$\begin{aligned} &\frac{1}{2} qp_b(\eta_n, S\eta_n) \leq qp_b(\eta_n, t) \text{ or } \frac{1}{2} qp_b(S\eta_n, S(S\eta_n)) \\ &\leq qp_b(S\eta_n, t), \end{aligned} \tag{41}$$

for every $n \in \mathbf{N}$. On the contrary, suppose the above inequality does not hold; then, by triangular inequality in qp_b, we have

$$\begin{aligned} qp_b(\eta_n, \eta_{n+1}) = qp_b(\eta_n, S\eta_n) + qp_b(t, t) &\leq qp_b(\eta_n, t) + qp_b(t, S\eta_n) \\ &< \frac{1}{2} qp_b(\eta_n, S\eta_n) + \frac{1}{2} qp_b(S\eta_n, S(S\eta_n)) \\ &= \frac{1}{2} qp_b(\eta_n, \eta_{n+1}) + \frac{1}{2} qp_b(\eta_{n+1}, \eta_{n+2}) \\ &\leq \frac{1}{2} qp_b(\eta_n, \eta_{n+1}) + \frac{1}{2} qp_b(\eta_n, \eta_{n+1}) = qp_b(\eta_n, \eta_{n+1}). \end{aligned} \tag{42}$$

Hence, the contradiction occurs. Therefore, for every $n \in \mathbf{N}$, our claim holds. If the first condition holds, we obtain

$$\begin{aligned} qp_b(\eta_{n+1}, St) &\leq w(\eta_n, t) qp_b(S\eta_n, St) \\ &\leq \psi[qp_b(\eta_n, t)]^{\beta} . [qp_b(\eta_n, S\eta_n)]^{\alpha} . [qp_b(t, St)]^{1-\alpha-\beta} \\ &= \psi[qp_b(\eta_n, t)]^{\beta} . [qp_b(\eta_n, S\eta_{n+1})]^{\alpha} . [qp_b(t, St)]^{1-\alpha-\beta} \\ &< [qp_b(\eta_n, t)]^{\beta} . [qp_b(\eta_n, S\eta_{n+1})]^{\alpha} . [qp_b(t, St)]^{1-\alpha-\beta}. \end{aligned} \tag{43}$$

If the second condition holds true, clearly t is the fixed point of S in a similar manner. Furthermore, if the w-regular condition is removed and S is a continuous map, we get a fixed point in G because

$$t = \lim_{n\longrightarrow\infty} \eta_{n+1} = \lim_{n\longrightarrow\infty} S\eta_n = S\left(\lim_{n\longrightarrow\infty} \eta_n\right) = St. \quad (44)$$

Finally, if the last condition holds, i.e., ψS^2 is continuous, we easily obtain $\psi S^2 = \psi t$. Suppose on the contrary that $St \neq t$, since $w(S\eta, \eta) \leq 1$ for any $\eta \in \text{Fix}\,(S^2)$ and

$$\frac{1}{2} qp_b\left(St, \psi S^2 t\right) = \frac{1}{2} qp_b(St, \psi t) \leq qp_b(St, \psi t). \quad (45)$$

We have

$$\begin{aligned} qp_b(t, St) &= qp_b\left(S^2 t, St\right) \leq w(St, t) qp_b\left(S^2 t, St\right) \\ &\leq \psi\left[qp_b(St, t)\right]^{\alpha} \cdot \left[qp_b\left(St, S^2 t\right)\right]^{\beta} \cdot \left[qp_b(t, St)\right]^{1-\alpha-\beta} \\ &< \left[qp_b(St, t)\right]^{\alpha} \cdot \left[qp_b(St, t)\right]^{\beta} \cdot \left[qp_b(t, St)\right]^{1-\alpha-\beta} \\ &= qp_b(t, St), \end{aligned} \quad (46)$$

which is a contradiction. So, $t = St$, that is, the mapping S owns a fixed point t. □

Example 2. Suppose $G = \{0, 1/4, 1/3, 1/2, 1\}$ and define qp_b: $G \times G \longrightarrow [0, \infty)$ such that

$$qp_b(\eta, \zeta) = \eta + \zeta. \quad (47)$$

Let the transformation S: $G \longrightarrow G$ maps as follows:

$$\begin{aligned} &S(0) = S\left(\frac{1}{4}\right) = S(1) \\ &S\left(\frac{1}{3}\right) = \frac{1}{3}, \\ &S\left(\frac{1}{2}\right) = 0. \end{aligned} \quad (48)$$

Also, define w: $G \times G \longrightarrow [0, \infty)$ such that

$$w(\eta, \zeta) = \begin{cases} 0.2, & (\eta, \zeta) = \left\{(0, 1), \left(0, \frac{1}{3}\right), (1, 0)\right\}, \\ 0, & \text{otherwise.} \end{cases} \quad (49)$$

Choose $\alpha = \beta = 1/3$ and define the function $\psi \in \Psi$ as $\psi(\eta) = \eta/2$. We need to check when $w(\eta, \zeta) = 1$. So, the following cases occur.

Case 1. When $(\eta, \zeta) = (0, 1)$,

$$\frac{1}{2}\left[qp_b(0, S0)\right] = \frac{1}{2}\left[qp_b\left(0, \frac{1}{2}\right)\right] = \frac{1}{4} \leq 1 = qp_b(0, 1), \quad (50)$$

which implies

$$\begin{aligned} w(0, 1) qp_b\left(\frac{1}{2}, \frac{1}{2}\right) \leq \psi\Bigg(&\left[qp_b(0, 1)\right]^{1/3} \left[qp_b\left(0, \frac{1}{2}\right)\right]^{1/3} \\ &\cdot \left[qp_b\left(1, \frac{1}{2}\right)\right]^{1/3}\Bigg). \end{aligned} \quad (51)$$

Case 2. When $(\eta, \zeta) = (0, 1/3)$,

$$\frac{1}{2}\left[qp_b(0, S0)\right] = \frac{1}{2}\left[qp_b\left(0, \frac{1}{2}\right)\right] = \frac{1}{4} \leq \frac{1}{3} = qp_b\left(0, \frac{1}{3}\right), \quad (52)$$

which implies

$$\begin{aligned} w\left(0, \frac{1}{3}\right) qp_b\left(\frac{1}{2}, \frac{1}{3}\right) \leq \psi\Bigg(&\left[qp_b\left(0, \frac{1}{3}\right)\right]^{1/3} \left(qp_b\left(0, \frac{1}{2}\right)\right)^{1/3} \\ &\cdot \left(qp_b\left(\frac{1}{3}, \frac{1}{3}\right)\right)^{1/3}\Bigg). \end{aligned} \quad (53)$$

Case 3. When $(\eta, \zeta) = (1, 0)$,

$$\frac{1}{2}\left[qp_b(1, S1)\right] = \frac{1}{2}\left[qp_b\left(1, \frac{1}{2}\right)\right] = \frac{3}{4} \leq 1 = qp_b(1, 0), \quad (54)$$

which implies

$$\begin{aligned} w(1, 0) qp_b\left(\frac{1}{2}, \frac{1}{2}\right) \leq \psi\Bigg(&\left[qp_b(1, 0)\right]^{1/3} \left[qp_b\left(1, \frac{1}{3}\right)\right]^{1/2} \\ &\cdot \left[qp_b\left(0, \frac{1}{2}\right)\right]^{1/3}\Bigg). \end{aligned} \quad (55)$$

Hence, all the assumptions of Theorem 2 are satisfied, and it follows that the mapping S owns a fixed point, that is, $\eta = 1/3$, as shown in Figure 2.

Definition 8. Let (G, qp_b) be a quasi-partial b-metric space. Define a self-mapping S: $G \longrightarrow G$ with $\psi \in \Psi$ satisfying

$$\begin{aligned} &\frac{1}{2}\left[qp_b(\eta, S\eta)\right] \leq qp_b(\eta, \zeta), \\ &qp_b(S\eta, S\zeta) \leq \psi\left(\left[qp_b(\eta, \zeta)\right]^{\beta} \text{middot; } \left[qp_b(\eta, S\eta)\right]^{\alpha} \right. \\ &\qquad \left. \cdot \left[qp_b(\zeta, S\zeta)\right]^{1-\alpha-\beta}\right), \end{aligned} \quad (56)$$

for all $\eta, \zeta \in G$ and $\alpha, \beta > 0$ with the condition $\alpha + \beta < 1$. Such a mapping is called a ψ-interpolative Ćirić–Reich–Rus contraction of Suzuki type.

Corollary 3. *Suppose (G, qp_b) is a $C\ qp_b$ and S is a ψ-interpolative Ćirić–Reich–Rus contraction of Suzuki type. Then, S owns a fixed point in G.*

Proof. For the proof, take $w(\eta, \zeta) = 1$ in Theorem 2. □

Corollary 4. *Suppose (G, qp_b) is a $C\ qp_b$ and S is an interpolative Ćirić–Reich–Rus contraction of Suzuki type if there exist $g \in [0, 1)$ and positive reals $\alpha, \beta > 0$, with $\alpha + \beta < 1$ such that*

$$\begin{aligned} &\frac{1}{2}\left[qp_b(\eta, S\eta)\right] \leq qp_b(\eta, \zeta) \Rightarrow qp_b(S\eta, S\zeta) \\ &\qquad \leq g\left(\left[qp_b(\eta, \zeta)\right]^{\beta} \left[qp_b(\eta, S\eta)\right]^{\alpha} \left[qp_b(\zeta, S\zeta)\right]^{1-\alpha-\beta}\right), \end{aligned} \quad (57)$$

for all $\eta, \zeta \in G$. Then, S possesses a fixed point in G.

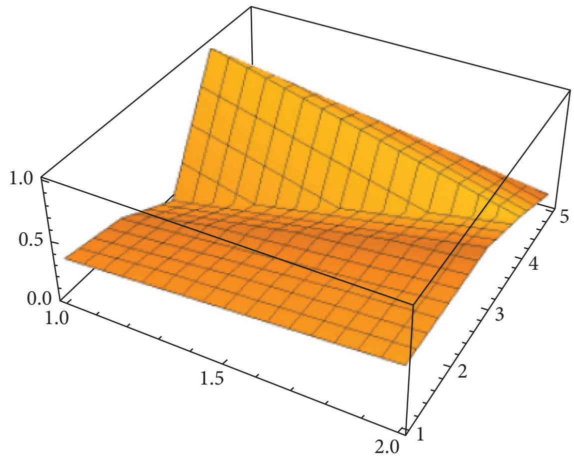

Figure 2: The 3-D plane in yellow represents the quasi-partial b-metric space defined by the function, $qp_b(\eta,\zeta)=\eta+\zeta$. Clearly, the fixed point of S is 1/3.

Proof. In Theorem 2, it is sufficient to put $\psi(\eta)=g\eta$, for all $\eta>0$ and $g\in[0,1)$, for the proof. □

Definition 9. Let (G,qp_b) be a qp_b space and define a map $w\colon G\times G\longrightarrow[0,\ \infty)$. A mapping $S\colon G\longrightarrow G$ is w-ψ-interpolative Hardy–Rogers contraction of Suzuki type if there exists $\psi\in\Psi$ with real numbers $\alpha,\beta,\gamma>0$, holding $\alpha+\beta+\gamma<1$ such that

$$\begin{aligned}&\frac{1}{2}\left[qp_b(\eta,S\eta)\right]\le qp_b(\eta,\zeta),\\ &w(\eta,\zeta)qp_b(S\eta,S\zeta)\le\psi\left(\left[qp_b(\eta,\zeta)\right]^{\alpha}\text{middot; }\left[qp_b(\eta,S\eta)\right]^{\beta}.\left[qp_b(\zeta,S\zeta)\right]^{\gamma}.\left[\frac{1}{2\nu}\left[qp_b(\zeta,S\eta)+qp_b(\eta,S\zeta)\right]\right]^{1-\alpha-\beta-\gamma}\right),\end{aligned}\tag{58}$$

for all $\eta,\ \zeta\in G$ and $\nu\ge1$ (see [24]).

Theorem 3. *Suppose (G,qp_b) is a $C\,qp_b$ and $S\colon G\longrightarrow G$ is a $w-\psi$-interpolative Hardy–Rogers contraction of Suzuki type. Let S be a w-orbital admissible mapping and $w(\eta_0,\ S\,\eta_0)\ge1$ for some $\eta_0\in G$. Then, S possesses a fixed point in G if any of the conditions hold:*

(1) $(G,\ qp_b)$ is w-regular.

(2) S is continuous.

(3) ψS^2 is continuous and $w(S\,\eta,\eta)\ge1$ when $\eta\in Fix\ (S^2)$.

Proof. Let $\eta_0\in G$ with the condition $w(\eta_0,\ S\,\eta_0)\ge1$ and $\{\eta_n\}$ be the sequence such that $S^n(\eta_0)=\eta_n$ for each positive integer n. Assume that for some $\eta_0\in\mathbf{N}$, we have the condition $\eta_{n_0}=\eta_{n_{0+1}}$. Hence, we get $\eta_{n_0}=S\,\eta_{n_0}$ which implies η_{n_0} is a unique fixed point of S. Hence, the proof is complete.

Now, consider $\eta_n\ne\eta_{n+1}$ for each positive integer n. As S is w-orbital admissible, we have the condition $w(\eta_0,\ S\ \eta_0)=w(\eta_0,\ \eta_1)\ge1$ which implies that $w(\eta_1,\ S\ \eta_1)=w(\eta_1,\ \eta_2)\ge1$. Proceeding in a similar way, we get

$$w(\eta_n,\eta_{n+1})\ge1.\tag{59}$$

Choosing $\eta=\ \eta_{n-1}$ and $\zeta=S\ \eta_{n-1}$ in (58), we get

$$\begin{aligned}\frac{1}{2}qp_b(\eta,\zeta)&=\frac{1}{2}qp_b(\eta_{n-1},S\eta_{n-1})\\ &=\frac{1}{2}qp_b(\eta_{n-1},\eta_n)\le qp_b(\eta_{n-1},\eta_n),\end{aligned}\tag{60}$$

which implies

$$\begin{aligned}qp_b(\eta_n,\eta_{n+1})&\le w(\eta_{n-1},\eta_n)qp_b(S\eta_{n-1},S\eta_n)\\ &\le\psi\left(\left[qp_b(\eta_{n-1},\eta_n)\right]^{\alpha}.\left[qp_b(\eta_{n-1},S\eta_{n-1})\right]^{\beta}.\left[qp_b(\eta_n,S\eta_n)\right]^{\gamma}.\left[\frac{1}{2\nu}\left[qp_b(\eta_n,S\eta_{n-1})+qp_b(\eta_{n-1},S\eta_n)\right]\right]^{1-\alpha-\beta-\gamma}\right)\\ &=\psi\left(\left[qp_b(\eta_{n-1},\eta_n)\right]^{\alpha}.\left[qp_b(\eta_{n-1},\eta_n)\right]^{\beta}.\left[qp_b(\eta_n,\eta_{n+1})\right]^{\gamma}.\left[\frac{1}{2\nu}\left[qp_b(\eta_n,\eta_n)+qp_b(\eta_{n-1},\eta_{n+1})\right]\right]^{1-\alpha-\beta-\gamma}\right).\end{aligned}\tag{61}$$

Then, using $\psi(\eta) < \eta$ for every $\eta > 0$, we get

$$qp_b[(\eta_n, \eta_{n+1})]^{1-\gamma} \leq [qp_b(\eta_{n-1}, \eta_n)]^{\beta+\alpha} \cdot \left[\frac{1}{2v}[qp_b(\eta_n, \eta_n) + qp_b(\eta_{n-1}, \eta_{n+1})]\right]^{1-\alpha-\beta-\gamma}\Bigg), \tag{62}$$

which is equivalent to

$$qp_b(\eta_n, \eta_{n+1}) < qp_b(\eta_{n-1}, \eta_n). \tag{63}$$

Hence, $\{qp_b(\eta_{n-1}, \eta_n)\}$ is a decreasing sequence. Eventually, we have

$$qp_b(\eta_n, \eta_{n+1}) \leq \psi(qp_b(\eta_{n-1}, \eta_n)). \tag{64}$$

In a similar way, we get

$$qp_b(\eta_n, \eta_{n+1}) \leq \psi^n(qp_b(\eta_0, \eta_1)). \tag{65}$$

Since $\{\eta_n\}$ is a fundamental sequence, applying triangular inequality,

$$\begin{aligned} qp_b(\eta_n, \eta_{n+l}) &\leq \left[s qp_b(\eta_n, \eta_{n+1}) + s^2 qp_b(\eta_{n+1}, \eta_{n+2}) + \cdots + s^l qp_b(\eta_{n+l-1}, \eta_{n+l})\right] \\ &\leq \left[s\psi^n qp_b(\eta_0, \eta_1) + s^2\psi^{n+1} qp_b(\eta_0, \eta_1) + \cdots + s^l\psi^{n+l-1} qp_b(\eta_0, \eta_1)\right] \\ &= \sum_{k=n,r=1}^{\infty} s^r \psi^k(qp_b(\eta_0, \eta_1)). \end{aligned} \tag{66}$$

Taking $n \longrightarrow \infty$, we deduce that $\{\eta_n\}$ is a fundamental sequence in qp_b space and by the completeness property of qp_b, there exists $t \in G$ satisfying

$$\lim_{n\longrightarrow\infty} qp_b(\eta_n, t) = 0. \tag{67}$$

Now we show that t is the fixed point of S. If assumption (1) holds true, then we have $w(\eta_n, t) \geq 1$ and we claim that

$$\begin{aligned} \frac{1}{2} qp_b(\eta_n, S\eta_n) \leq qp_b(\eta_n, t) \text{ or } \frac{1}{2} qp_b(S\eta_n, S(S\eta_n)) \\ \leq qp_b(S\eta_n, t), \end{aligned} \tag{68}$$

for every $n \in \mathbf{N}$. Suppose the above condition does not hold; then, by triangular inequality in qp_b, we have

$$\begin{aligned} qp_b(\eta_n, \eta_{n+1}) = qp_b(\eta_n, S\eta_n) + qp_b(t, t) &\leq qp_b(\eta_n, t) + qp_b(t, S\eta_n) \\ &< \frac{1}{2} qp_b(\eta_n, S\eta_n) + \frac{1}{2} qp_b(S\eta_n, S(S\eta_n)) \\ &= \frac{1}{2} qp_b(\eta_n, \eta_{n+1}) + \frac{1}{2} qp_b(\eta_{n+1}, \eta_{n+2}) \\ &\leq \frac{1}{2} qp_b(\eta_n, \eta_{n+1}) + \frac{1}{2} qp_b(\eta_n, \eta_{n+1}) = qp_b(\eta_n, \eta_{n+1}), \end{aligned} \tag{69}$$

which is a contradiction. Therefore, for every $n \in \mathbf{N}$, either

$$\frac{1}{2} qp_b(\eta_n, S\eta_n) \leq qp_b(\eta_n, t) \text{ or } \frac{1}{2} qp_b(S\eta_n, S(S\eta_n)) \leq qp_b(S\eta_n, t) \tag{70}$$

holds. If the first condition holds, we obtain

$$
\begin{aligned}
qp_b(\eta_{n+1}, St) &\le w(\eta_n, t) qp_b(S\eta_n, St) \\
&\le \psi[qp_b(\eta_n, t)]^{\alpha}.[qp_b(\eta_n, S\eta_n)]^{\beta}.[qp_b(t, St)]^{\gamma}.\left[\frac{1}{2\nu}[qp_b(\eta_n, \eta_n) + qp_b(t, St)]\right]^{1-\alpha-\beta-\gamma}\Bigg) \\
&= \psi[qp_b(\eta_n, t)]^{\alpha}.[qp_b(\eta_n, S\eta_n)]^{\beta}.[qp_b(t, St)]^{\gamma}.\left[\frac{1}{2\nu}[qp_b(\eta_n, \eta_n) + qp_b(t, St)]\right]^{1-\alpha-\beta-\gamma}\Bigg) \\
&< [qp_b(\eta_n, t)]^{\alpha}.[qp_b(\eta_n, \eta_{n+1})]^{\beta}.[qp_b(t, St)]^{\gamma}.\left[\frac{1}{2\nu}[qp_b(t, \eta_{n+1}) + qp_b(\eta_n, St)]\right]^{1-\alpha-\beta-\gamma}\Bigg).
\end{aligned} \tag{71}
$$

If assumption (2) holds, we get that t is the fixed point of S in a similar manner. Furthermore, if the w-regular condition is removed and S is continuous, then we get that S owns a unique fixed point in G because

$$
t = \lim_{n\longrightarrow\infty} \eta_{n+1} = \lim_{n\longrightarrow\infty} S\eta_n = S\left(\lim_{n\longrightarrow\infty} \eta_n\right) = St. \tag{72}
$$

Finally, if the last condition holds, i.e., ψS^2 is continuous, we easily obtain $\psi S^2 = \psi t$. Suppose on the contrary that $St \neq t$, since $w(S\eta, \eta) \le 1$ for any $\eta \in \mathrm{Fix}\ (S^2)$ and

$$
\frac{1}{2} qp_b\left(St, \psi S^2 t\right) = \frac{1}{2} qp_b(St, \psi t) \le qp_b(St, \psi t). \tag{73}
$$

We have

$$
\begin{aligned}
qp_b(t, St) &= qp_b\left(S^2 t, St\right) \le w(St, t) qp_b\left(S^2 t, St\right) \\
&\le \psi[qp_b(St, t)]^{\alpha}.\left[qp_b\left(St, S^2 t\right)\right]^{\beta}.[qp_b(t, St)]^{\gamma}.\left[\frac{1}{2\nu}[qp_b(t, St) + qp_b(t, St)]\right]^{1-\alpha-\beta-\gamma}\Bigg) \\
&< [qp_b(St, t)]^{\alpha}.[qp_b(St, t)]^{\beta}.[qp_b(t, St)]^{\gamma}.\left[\frac{1}{2}[qp_b(t, St) + qp_b(t, St)]\right]^{1-\alpha-\beta-\gamma}\Bigg) \\
&= qp_b(t, St),
\end{aligned} \tag{74}
$$

which is a contradiction. So, $t = St$, which implies that t is a fixed point of the map S. □

Definition 10. Let (G, qp_b) be a quasi-partial b-metric space. A mapping $S: G \longrightarrow G$ is said to be a ψ-interpolative Hardy–Rogers contraction of Suzuki type if there exist ψ and α, β, $\gamma > 0$, with the condition $\alpha + \beta + \gamma < 1$ such that

$$
\begin{aligned}
&\frac{1}{2}[qp_b(\eta, S\eta)] \le qp_b(\eta, \zeta), \\
&qp_b(S\eta, S\zeta) \le \psi\Bigg([qp_b(\eta, \zeta)]^{\alpha}[qp_b(\eta, S\eta)]^{\beta}.[qp_b(\zeta, S\zeta)]^{\gamma}.\left[\frac{1}{2\nu}[qp_b(\zeta, S\eta) + qp_b(\eta, S\zeta)]\right]^{1-\alpha-\beta-\gamma}\Bigg),
\end{aligned} \tag{75}
$$

for all η, $\zeta \in G$.

Corollary 5. *Let (G, qp_b) be a C qp_b and S be a ψ-interpolative Hardy–Rogers contraction of Suzuki type. Then, the mapping S possesses a fixed point in G.*

Proof. For the proof, take $w(\eta, \zeta) = 1$ in Theorem 3. □

Example 3. Let $G = [0, 3]$ and define $qp_b: G \times G \longrightarrow [0, \infty)$ such that

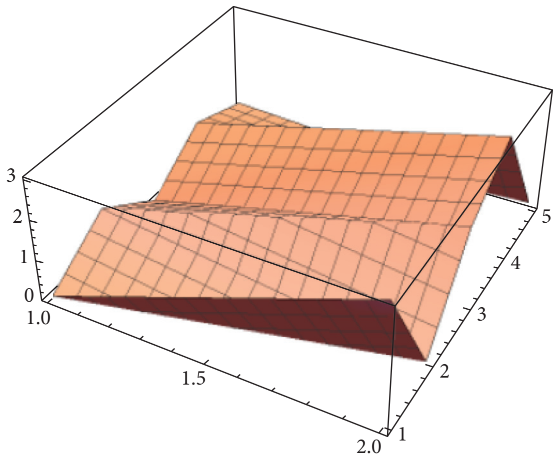

FIGURE 3: The graphical surface clearly shows that 2 is the fixed point of the map S.

$$qp_b(\eta,\zeta) = |\eta + \zeta| + \eta. \tag{76}$$

Let the mapping $S: G \longrightarrow G$ be defined as

$$S\eta = \begin{cases} 2, & \eta \in [0,1) \\ \frac{2}{3}, & \eta \in [1,2), \\ \eta, & \eta \in [2,3]. \end{cases} \tag{77}$$

Also, define $w: G \times G \longrightarrow [0,\infty)$ such that

$$w(\eta,\zeta) = \begin{cases} 0.1, & \eta = 0 \text{ and } \zeta = 3, \\ 0, & \text{otherwise.} \end{cases} \tag{78}$$

Choose $\alpha = \beta = \gamma = 1/4$ and define the function $\psi \in \Psi$ as $\psi(\eta) = \eta/3$. For $\eta,\ \zeta \in [0,\ 2)$, we have $w(\eta,\zeta) = 0$, which clearly implies that inequality (58) holds. As per the definition of function w, the only case left is when we have $\eta = 0$ and $\zeta = 3$ as $w(\eta,\zeta) = 1$, so

$$\frac{1}{2}\left[qp_b(0,S0)\right] = \frac{1}{2}\left[qp_b(0,2)\right] = 1 \leq 3 = qp_b(0,3), \tag{79}$$

which implies

$$w(0,3)qp_b(2,3) \leq \psi\left(\left[qp_b(0,3)\right]^{1/4}\left[qp_b(0,2)\right]^{1/4}\left[qp_b(3,3)\right]^{1/4}\left[\frac{1}{2\nu}\left[qp_b(3,2) + qp_b(0,3)\right]\right]^{1/4}\right), \tag{80}$$

where we assume $\nu = 1$. Hence, the assumptions of Theorem 3 are satisfied, so the mapping S owns a fixed point, that is, $\eta = 2$, as shown in Figure 3.

4. Conclusion and Future Aspects

The paper propounds the idea of using interpolation in noteworthy Suzuki-type mappings in the quasi-partial b-metric space. The incentive behind the paper was to introduce new concepts on completeness of w-ψ-interpolative Kannan, Ćirić–Reich–Rus, and Hardy–Rogers contractions of Suzuki-type mappings in quasi-partial b-metric space. Further, some fixed point results are obtained and are validated by illustrative examples. Interpolation is a noble concept which can be utilized to obtain different interpolative contraction of Suzuki-type mappings in other well-known spaces in the future. We are certain that the paper is a significant improvement of the known results in the existing fixed point literature.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

All authors contributed equally and significantly in writing this article. All authors read and approved the final manuscript.

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