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Retraction

Retracted: Analysis of Fuzzy Kuramoto-Sivashinsky Equations under a Generalized Fuzzy Fractional Derivative Operator

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This article has been retracted by Hindawi following an investigation undertaken by the publisher [1]. This investigation has uncovered evidence of one or more of the following indicators of systematic manipulation of the publication process:

(1) Discrepancies in scope
(2) Discrepancies in the description of the research reported
(3) Discrepancies between the availability of data and the research described
(4) Inappropriate citations
(5) Incoherent, meaningless and/or irrelevant content included in the article
(6) Manipulated or compromised peer review

The presence of these indicators undermines our confidence in the integrity of the article's content and we cannot, therefore, vouch for its reliability. Please note that this notice is intended solely to alert readers that the content of this article is unreliable. We have not investigated whether authors were aware of or involved in the systematic manipulation of the publication process.

Wiley and Hindawi regrets that the usual quality checks did not identify these issues before publication and have since put additional measures in place to safeguard research integrity.

We wish to credit our own Research Integrity and Research Publishing teams and anonymous and named external researchers and research integrity experts for contributing to this investigation.

The corresponding author, as the representative of all authors, has been given the opportunity to register their agreement or disagreement to this retraction. We have kept a record of any response received.

References

[1] N. H. Aljahdaly, M. Naeem, and N. Wyal, "Analysis of Fuzzy Kuramoto-Sivashinsky Equations under a Generalized Fuzzy Fractional Derivative Operator," *Journal of Function Spaces*, vol. 2022, Article ID 9517158, 11 pages, 2022.

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Research Article

Analysis of Fuzzy Kuramoto-Sivashinsky Equations under a Generalized Fuzzy Fractional Derivative Operator

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This paper evaluates a semianalytical strategy combined with a novel fuzzy integral transformation and an iterative method inside the fuzziness concept known as the new iterative transform method. Additionally, we apply the abovementioned technique to the fractional fuzzy Kuramoto-Sivashinsky equations with gH-differentiability by employing various initial conditions. Numerous algebraic properties of the fuzzy fractional derivative Atangana-Baleanu operator are illustrated concerning the Shehu transformation to demonstrate their utility. Additionally, a general technique for Atangana-Baleanu fuzzy fractional derivatives is proposed in the sense of Caputo. It is important to note that the purpose of the suggested fuzziness technique is to establish the efficiency and accuracy of analytical solution to nonlinear fuzzy fractional partial differential equations that emerge in complex and physical structures.

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1. Introduction

Physical models of real-world phenomena generally involve considerable uncertainty due to a wide range of variables. Fuzzy models appears to be an effective strategy for modelling the uncertainty introduced by imprecision and ambiguity. We apply it here to disciplines where data is unclear, such as medicine, the environmental, social and economic sciences, and applied sciences. Zadeh contributed fuzziness to the set theory in 1965 to address these concerns. Over the last two centuries, fractional calculus has grown in popularity due to its numerous applications in applied sciences. Hundreds of fuzzy problems rather than stochastic uncertainty in the behavior of the described scheme process can be found. Numerous researchers have been interested in investigating the foundation theoretical fuzzy problems in recent years [1]. Fractional fuzzy differential equations (FFDE) are particularly valuable in scientific modelling and technical examples such as models of population, system of weapon evaluation, civil engineering, and electrohydraulic modelling. Fractional calculus, in conjunction with the theory of fuzzy implementation, is a major tool for resolving uncertainty and dealings with ambiguous or subjective status in Maths and providing more broadly findings. Handled with several real scenarios, for instance, the mean golden [2], medicine [3], gravity and quantum optics [4], systems of practical [5], and engineering phenomenon. Zadeh [6] became familiar with fuzzy sets for the first time. The work on the definition of fuzzy numbers and their applications in fuzzy control [7] and reasoning approximate issues [8, 9] was then completed. It is challenging to accurately depict numerous scenarios in data analysis utilizing real numbers. Furthermore, Dubois and Prade [10, 11], Mizumoto and Tanaka [12, 13], and the elementary principles on noncrisp sets, as well as details on fuzzy differentials and fuzzy differential equations, can be found in [14–16]. The ideas propose a generalization of differential equations. This is a novel idea that has piqued the interest of many researchers. Real-world

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settings involving fractional-order differential equations are extremely important; implementations can be found in chemistry, engineering, physics, and other fields [17–19].

Fuzzy partial differential equations arose from the necessity to replicate many real-world difficulties while accounting for uncertainty data. As we will see later, partially differential equations have essential implications in several domain-applied sciences. Heat transfer is a critical topic of research in mechanic and aeronautical engineering because it affects a wide variety of equipment, mechanical systems, and aerospace engineering domains [20]. Numerous engineering problems, by definition, fall into this category, and scientists must approach them numerically; for more information, see [21, 22].

Furthermore, it is difficult to find a correct solution for the nonlinear equations that illustrate these events but necessary for analyzing the relevant behaviors. In the cited phenomena, in order to describe the behavior of nonlinear systems in real life, analytical and numerical solutions for fractional differential equations are essential; plasma instabilities, chemical reaction-diffusion, flame front propagation, viscous flow issues, and magnetized plasmas are the physical and chemical phenomena described by the Kuramoto-Sivashinsky equation [23, 24]. Several researchers applied many methods to analyze the Kuramoto-Sivashinsky equation like, the homotopy analysis technique [25], finite-difference discretization [26], Lattice Boltzmann technique [27], Chebyshev spectral collocation technique [28], cubic B-spline finite difference-collocation technique [29], and He's variational iteration technique [30, 31].

The structure of the remaining paper is summarized as follows. Section 2 defines some basic definitions related to our present work. The general methodology for solving the fractional fuzzy partial differential equation is provided in Section 3. Section 4 presents the main results, numerical simulations, and graphical representations. The conclusion along with future research directions is drawn in Section 5.

2. Preliminary Concepts

Definition 1. We say that $\mathbb{U} : \mathscr{R} \mapsto [0, 1]$ is a set of fuzzy; then, it is recognized to be a fuzzy set if it continues to hold the subsequent presumptions [32–36]:

(1) $\mathbb{U}$ is normal (for some $\eta_0 \in \mathscr{R}\,; \mathbb{U}(\ell_0) = 1$)

(2) $\mathbb{U}$ is upper semicontinuous

(3) $\mathbb{U}(\ell_1\xi + (1-\xi)\ell_2) \geq (\mathbb{U}(\ell_1) \wedge \mathbb{U}(\ell_2)) \forall \xi \in [0, 1], \ell_1, \ell_2 \in \mathscr{R}$, i.e., $\mathbb{U}$ is convex

(4) $cl\{\ell \in \mathscr{R}, \mathbb{U}(\ell) > 0\}$ is compact

Definition 2. Assume that a fuzzy set $\mathbb{U}$ is the $\mathfrak{P}$ level set discussed as [32–36]

$$[\mathbb{U}]^{\mathfrak{P}} = \{\Psi \in \mathscr{R} : \mathbb{U}(\Psi) \geq 1\}, \tag{1}$$

where $\mathfrak{P} \in [0, 1]$ and $\Psi \in \mathscr{R}$.

Definition 3. The parameter version of a fuzzy set is signified as $[\underline{\mathbb{U}}(\mathfrak{P}), \bar{\mathbb{U}}(\mathfrak{P})]$ so that $\mathfrak{P} \in [0, 1]$ fulfils the successive presumptions [32–36]:

(1) $\bar{\mathbb{U}}(\mathfrak{P})$ is right continuous, nonincreasing, bounded over $(0, 1]$, and right continuous at 0

(2) $\mathbb{U}(\mathfrak{P})$ is left continuous, nondecreasing, bounded over $(0, 1]$, and left continuous at 0

(3) $\mathbb{U}(\mathfrak{P}) \leq \bar{\mathbb{U}}(\mathfrak{P})$

Definition 4. For $\mathfrak{P} \in [0, 1]$ and Y to be scalar, suppose that there are two fuzzy sets $\tilde{\alpha}_1 = (\underline{\alpha_1}, \bar{\alpha_1}), \tilde{\alpha}_2 = (\underline{\alpha_2}, \bar{\alpha_2})$; then the addition, subtraction, and scalar multiplication are stated as [32–35]

(1) $\tilde{\alpha}_1 \oplus \tilde{\alpha}_2 = (\underline{\alpha_1}(\mathfrak{P}) + \underline{\alpha_2}(\mathfrak{P}), \bar{\alpha_1}(\mathfrak{P}) + \bar{\alpha_2}(\mathfrak{P}))$

(2) $\tilde{\alpha}_1 ! \tilde{\alpha}_2 = (\underline{\alpha_1}(\mathfrak{P}) - \underline{\alpha_2}(\mathfrak{P}), \bar{\alpha_1}(\mathfrak{P}) - \bar{\alpha_2}(\mathfrak{P}))$

(3) $Ye\tilde{\alpha}_1 = \{(Y\underline{\alpha_1}, Y\bar{\alpha_1})\,\mathrm{Y} \geq 0, (Y\bar{\alpha_1}, Y\underline{\alpha_1})\mathrm{Y} < 0$

Definition 5. Supposing that a fuzzy modelling $\Theta : \tilde{E} \times \tilde{E} \mapsto \mathscr{R}$ having fuzzy two sets $\tilde{\alpha}1 = (\underline{\alpha 1}, \bar{\alpha 1}), \tilde{\alpha}2 = (\underline{\alpha 2}, \bar{\alpha 2})$, then, the Θ distance between $\tilde{\alpha}1$ and $\tilde{\alpha}2$ is defined as [32–36]

$$\Theta(\tilde{\alpha}_1, \tilde{\alpha}_2) = \sup_{\mathfrak{P} \in [0,1]} \left[\max\left\{\left|\underline{\alpha_1}(\mathfrak{P}) - \underline{\alpha_2}(\mathfrak{P})\right|, |\bar{\alpha_1}(\mathfrak{P}) - \bar{\alpha_2}(\mathfrak{P})|\right\}\right]. \tag{2}$$

Definition 6. Recognize a fuzzy modelling $\Xi : \mathscr{R} \mapsto \tilde{E}$; if for any $\varepsilon > 0$, there exists $\delta > 0$ and a fixed value of $\mu 0 \in [a1, a2]$, we have [32–36]:

$$\Theta(\Xi(\mu), \Xi(\mu_0)) < \varepsilon\,; \quad \text{whenever } |\mu - \mu_0| < \delta. \tag{3}$$

Then, Ξ is known to be continuous.

Definition 7. Suppose that $\Xi : (b_1, b_2) \mapsto \tilde{E}$ and $\tau_0 \in (b_1, b_2)$. Then, Ξ is said to be strongly generalized differentiable at τ_0 if $\Xi'(\tau_0) \in \tilde{E}$ exists such that [32–35]

(i) $\Xi'(\tau_0) = \lim_{\hbar \mapsto 0}((\Xi(\tau_0 + \hbar) ! g\mathscr{H}\Xi(\tau_0))/h) = \lim_{\hbar \mapsto 0}((\Xi(\tau_0) !^{g\mathscr{H}} \Xi(\tau_0 - \hbar))/h)$

(ii) $\Xi'(\tau_0) = \lim_{\hbar \mapsto 0}((\Xi(\tau_0) ! g^{\mathscr{H}} \Xi(\tau_0 + \hbar))/-\hbar) = \lim_{\hbar \mapsto 0}((\Xi(\tau_0 - \hbar) !^{g\mathscr{H}} \Xi(\tau_0))/-\hbar)$

Theorem 8. *Consider a fuzzy value term $\Xi : \mathscr{R} \mapsto \bar{E}$ such that $\Xi(\tau_0\,; \mathfrak{P}) = [\Xi(\tau_0\,; \mathfrak{P}), \bar{\bar{\Xi}}(\tau_0\,; \mathfrak{P})]$ and $\mathfrak{P} \in [0, 1]$. Then,* *(I) $\Xi(\tau_0;\mathfrak{P})$ and $\Xi(\tau_0;\mathfrak{P})$ are differentiable, if Ξ is (1)-differentiable, and*

$$\left[\Xi'(\tau_0)\right]^{\mathfrak{P}} = \left[\underline{\Xi}'(\tau_0;\mathfrak{P}), \bar{\Xi}'(\tau_0;\mathfrak{P})\right] \tag{4}$$

(II) $\Xi(\tau_0;\mathfrak{P})$ and $\bar{\Xi}(\tau_0;\mathfrak{P})$ are differentiable, if Ξ is (2)-differentiable, and

$$\left[\Xi'(\tau_0)\right]^{\mathfrak{P}} = \left[\bar{\Xi}'(\tau_0;\mathfrak{P}), \Xi'(\tau_0;\mathfrak{P})\right] \tag{5}$$

Definition 9. Assume that a fuzzy mapping $\Psi^{(r)}_{g\mathscr{H}} = \Psi(r) \in \mathbb{C}^F[0,s] \cap \mathbb{L}^F[0,s]$. Then, the fuzzy $g\mathscr{H}$ -fractional Caputo differentiability of fuzzy valued mapping Ψ is defined as

$$\begin{aligned}&\left({}^{c}_{g\mathscr{H}}\mathscr{D}^{\theta}\Psi\right)(y)\\ &= \mathscr{I}^{r-\theta}_{a_1}\odot\left(\Psi^{(r)}\right)(\tau)\\ &= \frac{1}{\Gamma(r-\vartheta)}\odot\int_{a_1}^{y}(y-\ell)^{r-\theta-1}\odot\Psi(r)(\ell)d\ell, \quad \theta\in(r-1,r], r\in\mathbb{N}, y>a_1.\end{aligned} \tag{6}$$

Therefore, the parameterized versions of $\Psi = [\underline{\Psi}_{\mathfrak{P}}(y), \bar{\Psi}_{\mathfrak{P}}(y)]$, $\mathfrak{P}\in[0,1]$ and $y_0\in(0,s)$ and CFD in a fuzzy sense is stated as

$$\begin{aligned}&\left[\mathscr{D}^{\vartheta}_{(i)-g\mathscr{H}}\Psi(y_0)\right]_{\mathfrak{P}}\\ &= \left[\mathscr{D}^{\vartheta}_{(i)-g\mathscr{H}}\Psi(y_0), \mathscr{D}^{\vartheta}_{(i)-g\mathscr{H}}\bar{\Psi}(y_0)\right], \quad \mathfrak{P}\in[0,1],\end{aligned} \tag{7}$$

where $r=[\mathfrak{P}]$:

$$\begin{aligned}&\left[\mathscr{D}^{\vartheta}_{(i)-g\mathscr{H}}\underline{\Psi}(y_0)\right]\\ &= \frac{1}{\Gamma(r-\vartheta)}\left[\int_0^y (y-\mathbf{x})^{r-\vartheta-1}\frac{d^r}{d\mathbf{x}^r}\underline{\Psi}_{(i)-g\mathscr{H}}(\mathbf{x})d\mathbf{x}\right]'_{y=y_0},\\ &\left[\mathscr{D}^{\vartheta}_{(i)-g\mathscr{H}}\bar{\Psi}(y_0)\right]\\ &= \frac{1}{\Gamma(r-\theta)}\left[\int_0^y (y-\mathbf{x})^{r-\theta-1}\frac{d^r}{d\mathbf{x}^r}\bar{\Psi}_{(i)-g\mathscr{H}}(\mathbf{x})d\mathbf{x}\right]_{y=y_0}.\end{aligned} \tag{8}$$

Definition 10. Assume that a fuzzy mapping $\tilde{\Psi}(y)\in\widetilde{\mathbb{H}}^1(0,T)$ and $\vartheta\in[0,1]$; then, the fuzzy $g\mathscr{H}$-fractional Atangana-Baleanu differentiability of fuzzy-valued mapping is defined as

$$\left(g\mathscr{H}^{\vartheta}\Psi\right)(y) = \frac{\mathbb{B}(\vartheta)}{1-\vartheta}\odot\left[\int_0^y \underline{\Psi}'(\mathbf{x})\odot E_\theta\left[\frac{-\vartheta(y-\mathbf{x})^\theta}{1-\vartheta}\right]d\mathbf{x}\right]. \tag{9}$$

Remark 11. In (33), Ψ satisfy the presumption of the decreasing diameter Ψ and increasing diameter Ψ of a fuzzy modelling Ψ. If $\nu=1$, then, the fuzzy Shehu transformation is simplified to fuzzy Laplace transformation. Applying the evidence of Salahshour et al. [30], we get

$$\begin{aligned}&\int_0^{+\infty}\exp\left(\frac{-\zeta}{\nu}y\right)\odot\bar{\Psi}(y)dy\\ &= \left(\int_0^{+\infty}\exp\left(\frac{-\xi}{\nu}y\right)\Psi(y;\mathfrak{P})dy, \int_0^{+\infty}\exp\left(\frac{-\xi}{\nu}y\right)\bar{\Psi}(y;\mathfrak{P})dy\right).\end{aligned} \tag{10}$$

Moreover, when we consider the classic Shehu transformation, we get

$$\mathbb{S}[\underline{\Psi}(y;\mathfrak{P})] = \int_0^{+\infty}\exp\left(\frac{-\xi}{\nu}y\right)\Psi(y;\mathfrak{P})dy, \tag{11}$$

and

$$\mathbb{S}\left[\bar{\Psi}(y;\mathfrak{P})\right] = \int_0^{+\infty}\exp\left(\frac{-\zeta}{\nu}y\right)\bar{\Psi}(y;\mathfrak{P})dy. \tag{12}$$

The aforementioned ideas can then be expressed as

$$\begin{aligned}S\left[\bar{\Psi}(y)\right] &= \left(\mathbb{S}[\underline{\Psi}(y;\mathfrak{P})], \mathbb{S}\left[\bar{\Psi}(y;\mathfrak{P})\right]\right)\\ &= \left(\underline{\mathcal{S}}\left(\bar{\zeta},\nu\right), \bar{\mathcal{S}}\left(\bar{\zeta},\nu\right)\right).\end{aligned} \tag{13}$$

The fuzzy Shehu transformation of the Caputo generalized Hukuhara derivative will then be defined ${}^{c}_{g}\mathscr{H}^{\theta}\mathscr{D}^{\theta}_{y}\Psi(y)$.

Definition 12. Suppose that there is an integrable fuzzy-valued mapping ${}^{c}_{g\mathscr{H}}\mathscr{D}^{\theta}_{\mathrm{y}}\bar{\Psi}(\mathrm{y})$ and $\Psi(\mathrm{y})$ is the primitive of ${}^{c}_{g}\mathscr{H}^{\theta}_{\mathrm{y}}\bar{\Psi}(\mathrm{y})$ on $[0,+\infty)$; then, the CFD of order θ is defined as

$$\begin{aligned}&S\left[{}^{c}_{g}\mathscr{H}\mathscr{D}^{\theta}_{y}\bar{\Psi}(y)\right]\\ &= \left(\frac{\bar{\zeta}}{V}\right)^{\theta}\odot S\left[\bar{\Psi}(y)\right]!\sum_{x=0}^{r-1}\left(\frac{\bar{\xi}}{V}\right)^{\theta-\kappa-1}\odot\bar{\Psi}^{(\kappa)}(0), \quad \theta\in(r-1,r].\end{aligned} \tag{14}$$

Again, using the fact of Maitama et al. [35], we have

$$\begin{aligned}&\left(\frac{\bar{\zeta}}{V}\right)^{\theta}\odot\mathcal{S}\left[\bar{\Psi}(y)\right]!\sum_{k=0}^{r-1}\left(\frac{\xi}{V}\right)^{\theta-\kappa-1}\odot\bar{f}^{(\kappa)}(0)\\ &= \left(\left(\frac{\xi}{V}\right)^{\theta}\mathcal{S}[\underline{\Psi}(y;\mathfrak{P})] - \sum_{k=0}^{r-1}\left(\frac{\xi}{V}\right)^{\theta-\kappa-1}\right.\\ &\odot\underline{\Psi}^{(\kappa)}(0;\mathfrak{P}), \left(\frac{\xi}{V}\right)^{\theta}\mathcal{S}\left[\bar{\Psi}(y;\mathfrak{P})\right]\\ &\left.-\sum_{\kappa=0}^{r-1}\left(\frac{\xi}{V}\right)^{\theta-\kappa-1}\bar{\Psi}(\kappa)(0;\mathfrak{P})\right).\end{aligned} \tag{15}$$

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The fractional derivative of the ABC operator was defined in the sense of Shehu by Bokhari et al. Furthermore, in the context of a fuzzy Shehu transform, we extend the concept of the fuzzy ABC fractional derivative as follows:

Definition 13. Consider $\Psi \in \mathbb{C}^F[0, s] \cap \mathbb{L}^F[0, s]$ such that $\tilde{\Psi}(y) = [\underline{\Psi}(\mathfrak{y}, \mathfrak{P}), \bar{\Psi}(\mathfrak{y}, \mathfrak{P})]$, $\mathfrak{P} \in [0, 1]$; then, the Shehu transformation of the fuzzy ABC of order $\theta \in [0, 1]$ is describe as follows:

$$\mathcal{S}\left[{}_{g\mathscr{H}}\mathscr{D}_y^{\theta}\bar{\Psi}(y)\right] = \frac{\mathbb{B}(\vartheta)}{1-\theta+\theta(\nu/\xi)^{\theta}} \odot \left(\bar{\mathbf{V}}(\nu, \xi)! \frac{\nu}{\xi}\bar{\Psi}(0)\right). \tag{16}$$

Moreover, utilizing the evidence of Porshokouhi and Ghanbari [30], we get

$$\begin{aligned} &\frac{\mathbb{B}(\theta)}{1-\theta+\theta(\nu/\xi)^{\theta}} \odot \left(\bar{\mathbf{V}}\left(\nu, \bar{\xi}\right)! \frac{\xi}{\nu}\tilde{\Psi}(0)\right) \\ &= \left(\frac{\mathbb{B}(\theta)}{1-\theta+\vartheta(\nu/\xi)}\left(\underline{\mathbf{V}}\left(\nu, \bar{\xi}; \mathfrak{P}\right) - \frac{\nu}{\xi}\underline{\Psi}(0; \mathfrak{P})\right), \frac{\mathbb{B}(\vartheta)}{1-\theta+\theta(\nu/\xi)^{\theta}} \right. \\ &\left. \cdot \left(\nabla\left(\nu, \bar{\xi}; \mathfrak{P}\right) - \frac{\nu}{\xi}\Psi(0; \mathfrak{P})\right)\right). \end{aligned} \tag{17}$$

3. General Implementation of the Proposed Method

In this section, we investigated the proposed method for an analytical solution. For this, we implemented the Shehu transform of the fractional Atangana-Baleanu operator along with the iterative transformation technique as

$$\mathcal{S}\left[{}_{\mathrm{ABC}}D_{\wp}^{\varsigma}\tilde{\mathbb{U}}(\psi,\wp)\right] = \mathcal{S}\left[D_{\psi}^{2}\tilde{\mathbb{U}}(\psi,\wp) + D_{\psi}^{3}\tilde{\mathbb{U}}(\psi,\wp) + \tilde{k}(r)\mathscr{F}(\psi,\wp)\right], \tag{18}$$

where $\varsigma \in (0, 1]$; therefore, the Shehu transformation of (18) is

$$\begin{aligned} &\frac{\mathbb{B}(\varsigma)}{1-\varsigma+\varsigma(\sigma/\omega)^{\varsigma}}\mathcal{S}\left[\tilde{\mathbb{U}}(\psi,\wp)\right] - \frac{\mathbb{B}(\varsigma)}{1-\varsigma+\varsigma(\sigma/\omega)^{\varsigma}}\left(\frac{\nu}{\omega}\right)\tilde{\mathbb{U}}(\psi, 0) \\ &= \mathcal{S}\left[D_{\psi}^{2}\tilde{\mathbb{U}}(\psi,\wp) + D_{\psi}^{3}\tilde{\mathbb{U}}(\psi,\wp) + \tilde{k}(r)\mathscr{F}(\psi,\wp)\right], \end{aligned} \tag{19}$$

using the initial condition, we obtain as

$$\begin{aligned} \mathcal{S}\left[\tilde{\mathbb{U}}(\psi,\wp)\right] &= \frac{g(\psi)}{\omega} + \frac{1-\varsigma+\varsigma(\sigma/\omega)^{\varsigma}}{\mathbb{B}(\varsigma)}\mathcal{S} \\ &\cdot \left[D_{\psi}^{2}\tilde{\mathbb{U}}(\psi,\wp) + D_{\psi}^{3}\tilde{\mathbb{U}}(\psi,\wp) + \tilde{k}(r)\mathscr{F}(\psi,\wp)\right]. \end{aligned} \tag{20}$$

Decompose the result as $\tilde{\mathbb{U}}(\psi,\wp) = \sum_{n=0}^{\infty}\tilde{\mathbb{U}}_n(\psi,\wp)$; then, (20) apply that

$$\begin{aligned} &\mathcal{S}\sum_{n=0}^{\infty}\tilde{\mathbb{U}}_n(\psi,\wp) \\ &= \frac{g(\psi)}{\omega} + \frac{1-\varsigma+\varsigma(\sigma/\omega)^{\varsigma}}{\mathbb{B}(\varsigma)}\mathcal{S} \\ &\cdot \left[D_{\psi}^{2}\sum_{n=0}^{\infty}\tilde{\mathbb{U}}_n(\psi,\wp) + D_{\psi}^{3}\sum_{n=0}^{\infty}\tilde{\mathbb{U}}_n(\psi,\wp) + \tilde{k}(r)\mathscr{F}(\psi,\wp)\right]. \end{aligned} \tag{21}$$

Aspects of the outcome may be seen as comparisons

$$\begin{aligned} \mathcal{S}\left[\tilde{\mathbb{U}}_0(\psi,\wp)\right] &= \frac{g(\psi)}{\omega} + \frac{1-\varsigma+\varsigma(\sigma/\omega)^{\varsigma}}{\mathbb{B}(\varsigma)}\mathcal{S}\left[\tilde{k}(r)\mathscr{F}(\psi,\wp)\right], \\ \mathcal{S}\left[\tilde{\mathbb{U}}_1(\psi,\wp)\right] &= \frac{1-\varsigma+\varsigma(\sigma/\omega)^{\varsigma}}{\mathbb{B}(\varsigma)}\mathcal{S}\left[D_{\psi}^{2}\tilde{\mathbb{U}}_0(\psi,\wp) + D_{\psi}^{3}\tilde{\mathbb{U}}_0(\psi,\wp)\right], \\ \mathcal{S}\left[\tilde{\mathbb{U}}_2(\psi,\wp)\right] &= \frac{1-\varsigma+\varsigma(\sigma/\omega)^{\varsigma}}{\mathbb{B}(\varsigma)}\mathcal{S}\left[D_{\psi}^{2}\tilde{\mathbb{U}}_1(\psi,\wp) + D_{\psi}^{3}\tilde{\mathbb{U}}_1(\psi,\wp)\right], \\ &\vdots \\ \mathcal{S}\left[\tilde{\mathbb{U}}_{n+1}(\psi,\wp)\right] &= \frac{1-\varsigma+\varsigma(\sigma/\omega)^{\varsigma}}{\mathbb{B}(\varsigma)}\mathcal{S}\left[D_{\psi}^{2}\tilde{\mathbb{U}}_n(\psi,\wp) + D_{\psi}^{3}\tilde{\mathbb{U}}_n(\psi,\wp)\right]. \end{aligned} \tag{22}$$

Applying the Shehu inverse transformation, we achieve

$$\begin{aligned} \underline{\mathbb{U}}_0(\psi,\wp) &= g(\psi) + \mathcal{S}^{-1}\left[\frac{1-\varsigma+\varsigma(\sigma/\omega)^{\varsigma}}{\mathbb{B}(\varsigma)}\mathcal{S}\left[\underline{k}(r)\mathscr{F}(\psi,\wp)\right]\right], \\ \bar{\mathbb{U}}_0(\psi,\wp) &= g(\psi) + \mathcal{S}^{-1}\left[\frac{1-\varsigma+\varsigma(\sigma/\omega)^{\varsigma}}{\mathbb{B}(\varsigma)}\mathcal{S}\left[\bar{k}(r)\mathscr{F}(\psi,\wp)\right]\right], \\ \underline{\mathbb{U}}_1(\psi,\wp) &= \mathcal{S}^{-1}\left[\frac{1-\varsigma+\varsigma(\sigma/\omega)^{\varsigma}}{\mathbb{B}(\varsigma)}\mathcal{S}\left[D_{\psi}^{2}\underline{\mathbb{U}}_0(\psi,\wp) + D_{\psi}^{3}\underline{\mathbb{U}}_0(\psi,\wp)\right]\right], \\ \bar{\mathbb{U}}_1(\psi,\wp) &= \mathcal{S}^{-1}\left[\frac{1-\varsigma+\varsigma(\sigma/\omega)^{\varsigma}}{\mathbb{B}(\varsigma)}\mathcal{S}\left[D_{\psi}^{2}\bar{\mathbb{U}}_0(\psi,\wp) + D_{\psi}^{3}\bar{\mathbb{U}}_0(\psi,\wp)\right]\right], \\ \underline{\mathbb{U}}_2(\psi,\wp) &= \mathcal{S}^{-1}\left[\frac{1-\varsigma+\varsigma(\sigma/\omega)^{\varsigma}}{\mathbb{B}(\varsigma)}\mathcal{S}\left[D_{\psi}^{2}\underline{\mathbb{U}}_1(\psi,\wp) + D_{\psi}^{3}\underline{\mathbb{U}}_1(\psi,\wp)\right]\right], \\ \bar{\mathbb{U}}_2(\psi,\wp) &= \mathcal{S}^{-1}\left[\frac{1-\varsigma+\varsigma(\sigma/\omega)^{\varsigma}}{\mathbb{B}(\varsigma)}\mathcal{S}\left[D_{\psi}^{2}\bar{\mathbb{U}}_1(\psi,\wp) + D_{\psi}^{3}\bar{\mathbb{U}}_1(\psi,\wp)\right]\right], \\ &\vdots \\ \underline{\mathbb{U}}_{n+1}(\psi,\wp) &= \mathcal{S}^{-1}\left[\frac{1-\varsigma+\varsigma(\sigma/\omega)^{\varsigma}}{\mathbb{B}(\varsigma)}\mathcal{S}\left[D_{\psi}^{2}\underline{\mathbb{U}}_n(\psi,\wp) + D_{\psi}^{3}\underline{\mathbb{U}}_n(\psi,\wp)\right]\right], \\ \bar{\mathbb{U}}_{n+1}(\psi,\wp) &= \mathcal{S}^{-1}\left[\frac{1-\varsigma+\varsigma(\sigma/\omega)^{\varsigma}}{\mathbb{B}(\varsigma)}\mathcal{S}\left[D_{\psi}^{2}\bar{\mathbb{U}}_n(\psi,\wp) + D_{\psi}^{3}\bar{\mathbb{U}}_n(\psi,\wp)\right]\right]. \end{aligned} \tag{23}$$

Thus, the result becomes

$$\begin{aligned}\underline{\mathbb{U}}(\psi,\wp) &= \underline{\mathbb{U}}_0(\psi,\wp) + \underline{\mathbb{U}}_1(\psi,\wp) + \underline{\mathbb{U}}_2(\psi,\wp) + \cdots,\\ \bar{\mathbb{U}}(\psi,\wp) &= \bar{\mathbb{U}}_0(\psi,\wp) + \bar{\mathbb{U}}_1(\psi,\wp) + \bar{\mathbb{U}}_2(\psi,\wp) + \cdots.\end{aligned} \tag{24}$$

The solution to equation (24) in the series form is obtained as follows.

4. Numerical Problems

Example 14. Consider the fuzzy fractional Kuramoto-Sivashinsky equation defined as $\tau = \sigma = 1$ and $\omega = 4$:

$$ {}^{ABC}D_{\wp}^{\varsigma}\tilde{\mathbb{U}}(\psi,\wp) + \tilde{\mathbb{U}}\frac{\partial \tilde{\mathbb{U}}}{\partial \psi} + \tau\frac{\partial^2 \tilde{\mathbb{U}}}{\partial \psi^2} + \sigma\frac{\partial^3 \tilde{\mathbb{U}}}{\partial \psi^3} + \omega\frac{\partial^4 \tilde{\mathbb{U}}}{\partial \psi^4} = 0, \tag{25}$$

with the initial condition

$$\tilde{\mathbb{U}}_0(\psi,0) = \tilde{k}\left\{11 + 15\tan h\left(\frac{-1}{2}\psi\right) - 15\tan h^2\left(\frac{-1}{2}\psi\right) - 15\tan h^3\left(\frac{-1}{2}\psi\right)\right\}. \tag{26}$$

Using the scheme of equation (23), we obtain

$$\begin{aligned}
\underline{\mathbb{U}}_0(\psi,\wp) &= \underline{k}(r)\left\{11 + 15\tan h\left(\frac{-1}{2}\psi\right) - 15\tan h^2\left(\frac{-1}{2}\psi\right) - 15\tan h^3\left(\frac{-1}{2}\psi\right)\right\},\\
\bar{\mathbb{U}}_0(\psi,\wp) &= \bar{k}(r)\left\{11 + 15\tan h\left(\frac{-1}{2}\psi\right) - 15\tan h^2\left(\frac{-1}{2}\psi\right) - 15\tan h^3\left(\frac{-1}{2}\psi\right)\right\},\\
\underline{\mathbb{U}}_1(\psi,\wp) &= -\underline{k}(r)\frac{60(-2 + \cos h(\psi) - \sin h(\psi))}{\cos h^2(\psi) + 2\cos h(\psi) + 1}\frac{1}{\mathbb{B}(\varsigma)}\left\{\frac{\varsigma\wp^{\varsigma}}{\Gamma(\varsigma+1)} + (1-\varsigma)\right\},\\
\bar{\mathbb{U}}_1(\psi,\wp) &= -\bar{k}(r)\frac{60(-2 + \cos h(\psi) - \sin h(\psi))}{\cos h^2(\psi) + 2\cos h(\psi) + 1}\frac{1}{\mathbb{B}(\varsigma)}\left\{\frac{\varsigma\wp^{\varsigma}}{\Gamma(\varsigma+1)} + (1-\varsigma)\right\},\\
\underline{\mathbb{U}}_2(\psi,\wp) &= \underline{k}(r)\frac{60\left(\cos h^2(\psi) - \cos h(\psi) - \sin h(\psi)\cos h(\psi) - 2 + 5\sin h(\psi)\right)}{\cos h^3(\psi) + 3\cos h^2(\psi) + 3\cos h(\psi) + 1}\frac{1}{\mathbb{B}^2(\varsigma)}\\
&\quad\cdot\left\{\frac{\varsigma^2\wp^{2\varsigma}}{\Gamma(2\varsigma+1)} + 2\varsigma(1-\varsigma)\frac{\wp^{\varsigma}}{\Gamma(\varsigma+1)} + (1-\varsigma)^2\right\},\\
\bar{\mathbb{U}}_2(\psi,\wp) &= \bar{k}(r)\frac{60\left(\cos h^2(\psi) - \cos h(\psi) - \sin h(\psi)\cos h(\psi) - 2 + 5\sin h(\psi)\right)}{\cos h^3(\psi) + 3\cos h^2(\psi) + 3\cos h(\psi) + 1}\frac{1}{\mathbb{B}^2(\varsigma)}\\
&\quad\cdot\left\{\frac{\varsigma^2\wp^{2\varsigma}}{\Gamma(2\varsigma+1)} + 2\varsigma(1-\varsigma)\frac{\wp^{\varsigma}}{\Gamma(\varsigma+1)} + (1-\varsigma)^2\right\},\\
\underline{\mathbb{U}}_3(\psi,\wp) &= \underline{k}(r)\frac{\cos h^2(\psi) - 13\cos h(\psi) - \sin h(\psi)\cos h(\psi) + 16 + 5\sin h(\psi)}{\cos h^3(\psi) + 3\cos h^2(\psi) + 3\cos h(\psi) + 1}\frac{1}{\mathbb{B}^3(\varsigma)}\\
&\quad\cdot\left\{\frac{\varsigma^3\wp^{3\theta}}{\Gamma(3\varsigma+1)} + 3\varsigma^2(1-\varsigma)\frac{\wp^{2\varsigma}}{\Gamma(2\varsigma+1)} + 3\varsigma(1-\varsigma)^2\frac{\wp^{\theta}}{\Gamma(\varsigma+1)}\right\},\\
\bar{\mathbb{U}}_3(\psi,\wp) &= \bar{k}(r)\frac{\cos h^2(\psi) - 13\cos h(\psi) - \sin h(\psi)\cos h(\psi) + 16 + 5\sin h(\psi)}{\cos h^3(\psi) + 3\cos h^2(\psi) + 3\cos h(\psi) + 1}\frac{1}{\mathbb{B}^3(\varsigma)}\\
&\quad\cdot\left\{\frac{\varsigma^3\wp^{3\theta}}{\Gamma(3\varsigma+1)} + 3\varsigma^2(1-\varsigma)\frac{\wp^{2\varsigma}}{\Gamma(2\varsigma+1)} + 3\varsigma(1-\varsigma)^2\frac{\wp^{\theta}}{\Gamma(\varsigma+1)}\right\}.
\end{aligned} \tag{27}$$

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The series solution is obtained using equation (24); therefore, we write

$$\tilde{\mathbb{U}}(\psi,\wp) = \tilde{\mathbb{U}}_0(\psi,\wp) + \tilde{\mathbb{U}}_1(\psi,\wp) + \tilde{\mathbb{U}}_2(\psi,\wp) + \tilde{\mathbb{U}}_3(\psi,\wp) + \tilde{\mathbb{U}}_4(\psi,\wp) + \cdots. \quad (28)$$

The upper and bottom portion forms can be written as

$$\begin{aligned}
&\underline{\mathbb{U}}(\psi,\wp) = \underline{\mathbb{U}}_0(\psi,\wp) + \underline{\mathbb{U}}_1(\psi,\wp) + \underline{\mathbb{U}}_2(\psi,\wp) + \underline{\mathbb{U}}_3(\psi,\wp) + \underline{\mathbb{U}}_4(\psi,\wp) + \cdots,\\
&\bar{\mathbb{U}}(\psi,\wp) = \bar{\mathbb{U}}_0(\psi,\wp) + \bar{\mathbb{U}}_1(\psi,\wp) + \bar{\mathbb{U}}_2(\psi,\wp) + \bar{\mathbb{U}}_3(\psi,\wp) + \bar{\mathbb{U}}_4(\psi,\wp) + \cdots.\\
&\bar{\mathbb{U}}(\psi,\wp) = \bar{k}(r)\left\{11 + 15 \tan h\left(\frac{-1}{2}\psi\right) - 15 \tan h^2\left(\frac{-1}{2}\psi\right) - 15 \tan h^3\left(\frac{-1}{2}\psi\right)\right\} - \bar{k}(r)\frac{60(-2 + \cos h(\psi) - \sin h(\psi))}{\cos h^2(\psi) + 2 \cos h(\psi) + 1}\frac{1}{\mathbb{B}(\varsigma)}\\
&\quad \cdot \left\{\frac{\varsigma\wp^\varsigma}{\Gamma(\varsigma+1)} + (1-\varsigma)\right\} + \bar{k}(r)\frac{60\left(\cos h^2(\psi) - \cos h(\psi) - \sin h(\psi) \cos h(\psi) - 2 + 5 \sin h(\psi)\right)}{\cos h^3(\psi) + 3 \cos h^2(\psi) + 3 \cos h(\psi) + 1}\frac{1}{\mathbb{B}(\varsigma)}\\
&\quad \cdot \left\{\frac{\varsigma\wp^\varsigma}{\Gamma(\varsigma+1)} + (1-\varsigma)\right\} + \underline{k}(r)\frac{60\left(\cos h^2(\psi) - \cos h(\psi) - \sin h(\psi) \cos h(\psi) - 2 + 5 \sin h(\psi)\right)}{\cos h^3(\psi) + 3 \cos h^2(\psi) + 3 \cos h(\psi) + 1}\frac{1}{\mathbb{B}^2(\varsigma)}\\
&\quad \cdot \left\{\frac{\varsigma^2\wp^{2\varsigma}}{\Gamma(2\varsigma+1)} + 2\varsigma(1-\varsigma)\frac{\wp^\varsigma}{\Gamma(\varsigma+1)} + (1-\varsigma)^2\right\} + \underline{k}(r)\frac{\cosh^2(\psi) - 13 \cosh(\psi) - \sinh(\psi) \cosh(\psi) + 16 + 5 \sinh(\psi)}{\cos \mathrm{h}^3(\psi) + 3 \cosh^2(\psi) + 3 \cosh(\psi) + 1}\\
&\quad \cdot \frac{1}{\mathbb{B}^3(\varsigma)}\left\{\frac{\varsigma^3\wp^{3\theta}}{\Gamma(3\varsigma+1)} + 3\varsigma^2(1-\varsigma)\frac{\wp^{2\varsigma}}{\Gamma(2\varsigma+1)} + 3\varsigma(1-\varsigma)^2\frac{\wp^\theta}{\Gamma(\varsigma+1)}\right\} + \cdots,\\
&\bar{\mathbb{U}}(\psi,\wp) = \bar{k}(r)\left\{11 + 15 \tan h\left(\frac{-1}{2}\psi\right) - 15 \tan h^2\left(\frac{-1}{2}\psi\right) - 15 \tan h^3\left(\frac{-1}{2}\psi\right)\right\} - \bar{k}(r)\frac{60(-2 + \cos h(\psi) - \sin h(\psi))}{\cos h^2(\psi) + 2 \cos h(\psi) + 1}\frac{1}{\mathbb{B}(\varsigma)}\\
&\quad \cdot \left\{\frac{\varsigma\wp^\varsigma}{\Gamma(\varsigma+1)} + (1-\varsigma)\right\} + \bar{k}(r)\frac{60\left(\cos h^2(\psi) - \cos h(\psi) - \sin h(\psi) \cos h(\psi) - 2 + 5 \sin h(\psi)\right)}{\cos h^3(\psi) + 3 \cos h^2(\psi) + 3 \cos h(\psi) + 1}\frac{1}{\mathbb{B}^2(\varsigma)}\\
&\quad \cdot \left\{\frac{\varsigma^2\wp^{2\varsigma}}{\Gamma(2\varsigma+1)} + 2\varsigma(1-\varsigma)\frac{\wp^\varsigma}{\Gamma(\varsigma+1)} + (1-\varsigma)^2\right\} + \bar{k}(r)\frac{\cos h^2(\psi) - 13 \cos h(\psi) - \sin h(\psi) \cos h(\psi) + 16 + 5 \sin h(\psi)}{\cos h^3(\psi) + 3 \cos h^2(\psi) + 3 \cos h(\psi) + 1}\\
&\quad \cdot \frac{1}{\mathbb{B}^3(\varsigma)}\left\{\frac{\varsigma^3\wp^{3\theta}}{\Gamma(3\varsigma+1)} + 3\varsigma^2(1-\varsigma)\frac{\wp^{2\varsigma}}{\Gamma(2\varsigma+1)} + 3\varsigma(1-\varsigma)^2\frac{\wp^\varsigma}{\Gamma(\varsigma+1)}\right\} + \cdots.
\end{aligned} \quad (29)$$

The exact solution of equation (25) is given as

$$\tilde{\mathbb{U}}(\psi,\wp) = \tilde{k}\left\{11 + 15 \tan h\left(-\frac{1}{2}\psi+\wp\right) - 15 \tan h^2\left(-\frac{1}{2}\psi+\wp\right) - 15 \tan h^3\left(-\frac{1}{2}\psi+\wp\right)\right\}. \quad (30)$$

Figure 1 represents the three-dimensional upper and lower fuzzy branch plots of the approximate series result and the different fractional order of ς. Figure 2 represents the two-dimensional upper and lower fuzzy branch plots of the approximate series result and the different fractional order of ς.

Example 15. Consider the fuzzy fractional Kuramoto-Sivashinsky equation defined as $\tau = 2$, $\sigma = 1$, and $\omega = 0$:

$$^{ABC}D_\wp^\varsigma \tilde{\mathbb{U}}(\psi,\wp) + \tilde{\mathbb{U}}\frac{\partial\tilde{\mathbb{U}}}{\partial\psi} + \tau\frac{\partial^2\tilde{\mathbb{U}}}{\partial\psi^2} + \sigma\frac{\partial^3\tilde{\mathbb{U}}}{\partial\psi^3} + \omega\frac{\partial^4\tilde{\mathbb{U}}}{\partial\psi^4} = 0, \quad (31)$$

with the initial condition

$$\tilde{\mathbb{U}}_0(\psi,0) = \tilde{k}\left\{-\frac{\sqrt{418}}{11} - \frac{270}{361}\sqrt{418}\tan h\left(\frac{\sqrt{418}}{38}\psi\right) + \frac{330}{361}\sqrt{418}\tan h^3\left(\frac{\sqrt{418}}{38}\psi\right)\right\}. \quad (32)$$

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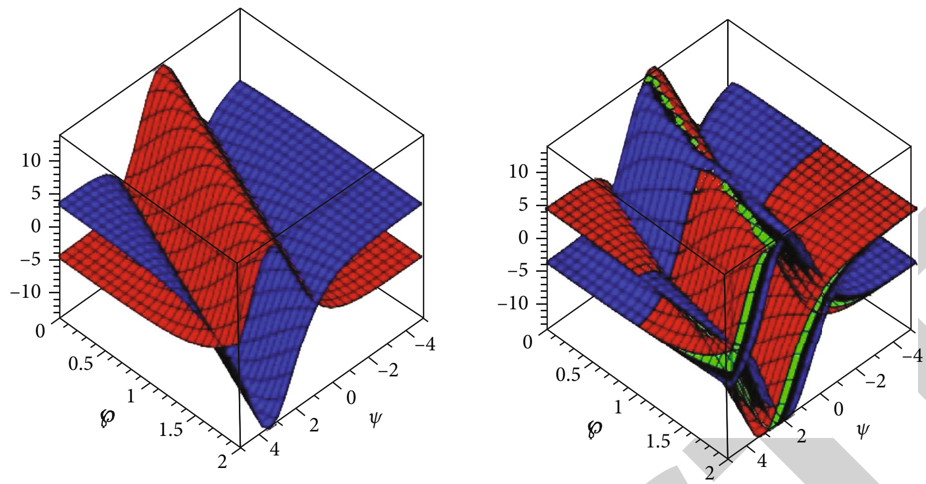

FIGURE 1: (a) 3D upper and lower fuzzy branch plots of the approximate series result and (b) the different fractional order of ς.

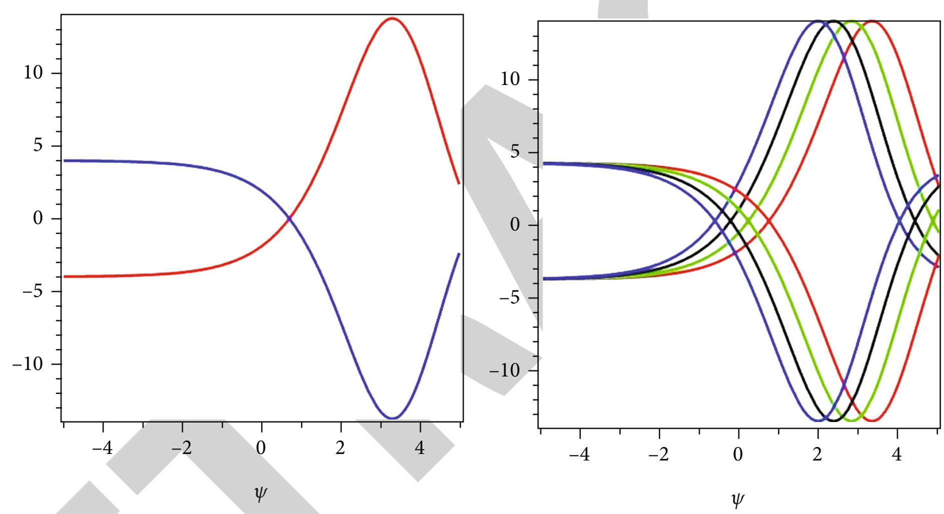

FIGURE 2: (a) 2D upper and lower fuzzy branch plots of the approximate series result and (b) the different fractional order of ς.

Using the scheme of equation (23), we obtain

$$\underline{\mathbb{U}}_0(\psi,\wp)=\underline{k}(r)\left\{-\frac{\sqrt{418}}{11}-\frac{270}{361}\sqrt{418}\tan h\left(\frac{\sqrt{418}}{38}\psi\right)+\frac{330}{361}\sqrt{418}\tan h^3\left(\frac{\sqrt{418}}{38}\psi\right)\right\},$$

$$\overline{\mathbb{U}}_0(\psi,\wp)=\overline{k}(r)\left\{-\frac{\sqrt{418}}{11}-\frac{270}{361}\sqrt{418}\tan h\left(\frac{\sqrt{418}}{38}\psi\right)+\frac{330}{361}\sqrt{418}\tan h^3\left(\frac{\sqrt{418}}{38}\psi\right)\right\},$$

$$\underline{\mathbb{U}}_1(\psi,\wp)=\underline{k}(r)\left[-\left\{90\sqrt{418}\left(-152\cos h^3\left(\frac{\sqrt{418}}{38}\psi\right)+209\cos h\left(\frac{\sqrt{418}}{38}\psi\right)+88\sin h\left(\frac{\sqrt{418}}{38}\psi\right)\cdot\cos h^2\left(\frac{\sqrt{418}}{38}\psi\right)-242\sin h\left(\frac{\sqrt{418}}{38}\psi\right)\right)\right\}\right]\cdot\frac{1}{6859\left(\cos h^5\left(\sqrt{418}/38\psi\right)\right)}\frac{1}{\mathbb{B}(\varsigma)}\cdot\left\{\frac{\varsigma\wp^{\varsigma}}{\Gamma(\varsigma+1)}+(1-\varsigma)\right\},$$

$$\overline{\mathbb{U}}_1(\psi,\wp)=\overline{k}(r)\left[-\left\{90\sqrt{418}\left(-152\cos h^3\left(\frac{\sqrt{418}}{38}\psi\right)+209\cos h\left(\frac{\sqrt{418}}{38}\psi\right)+88\sin h\left(\frac{\sqrt{418}}{38}\psi\right)\cdot\cos h^2\left(\frac{\sqrt{418}}{38}\psi\right)-242\sinh\left(\frac{\sqrt{418}}{38}\psi\right)\right)\right\}\right]$$

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$$\cdot \frac{1}{6859\left(\cos h^5\left(\sqrt{418}/38\psi\right)\right)}\frac{1}{\mathbb{B}(\varsigma)} \cdot \left\{\frac{\varsigma\wp^{\varsigma}}{\Gamma(\varsigma+1)} + (1-\varsigma)\right\},$$

$$\underline{\mathbb{U}}_2(\psi,\wp) = \underline{k}(r)\left[\left[180\sqrt{418}\left\{288574 \sin h\left(\frac{\sqrt{418}}{38}\psi\right) \cdot \cos h^4\left(\frac{\sqrt{418}}{38}\psi\right) - 2495625 \sin h\left(\frac{\sqrt{418}}{38}\psi\right) \cdot \cos h^2\left(\frac{\sqrt{418}}{38}\psi\right) + 2635380 \sin h\left(\frac{\sqrt{418}}{38}\psi\right) + 444752 \cos h^5\left(\frac{\sqrt{418}}{38}\psi\right) - 436810 \cos h^3\left(\frac{\sqrt{418}}{38}\psi\right) - 63536 \cos h^7\left(\frac{\sqrt{418}}{38}\right)\psi + 73264 \sin h\left(\frac{\sqrt{418}}{38}\psi\right) \cosh^6\left(\frac{\sqrt{418}}{38}\psi\right)\right\}\right] \cdot \frac{1}{2476099 \cos h^9\left(\sqrt{418}/38\right)\psi}\right]\frac{1}{\mathbb{B}^2(\varsigma)} \cdot \left\{\frac{\varsigma^2\wp^{2\varsigma}}{\Gamma(2\varsigma+1)} + 2\varsigma(1-\varsigma)\frac{\wp^{\varsigma}}{\Gamma(\varsigma+1)} + (1-\varsigma)^2\right\},$$

$$\bar{\mathbb{U}}_2(\psi,\wp) = \bar{k}(r)\left[\left[180\sqrt{418}\left\{288574 \sin h\left(\frac{\sqrt{418}}{38}\psi\right) \cdot \cos h^4\left(\frac{\sqrt{418}}{38}\psi\right) - 2495625 \sin h\left(\frac{\sqrt{418}}{38}\psi\right) \cdot \cos h^2\left(\frac{\sqrt{418}}{38}\psi\right) + 2635380 \sin h\left(\frac{\sqrt{418}}{38}\psi\right) + 444752 \cos h^5\left(\frac{\sqrt{418}}{38}\psi\right) - 436810 \cos h^3\left(\frac{\sqrt{418}}{38}\psi\right) - 63536 \cos h^7\left(\frac{\sqrt{418}}{38}\right)\psi + 73264 \sin h\left(\frac{\sqrt{418}}{38}\psi\right) \cos h^6\left(\frac{\sqrt{418}}{38}\psi\right)\right\}\right] \cdot \frac{1}{2476099 \cos h^9\left(\sqrt{418}/38\right)\psi}\right]\frac{1}{\mathbb{B}^2(\varsigma)} \cdot \left\{\frac{\varsigma^2\wp^{2\varsigma}}{\Gamma(2\varsigma+1)} + 2\varsigma(1-\varsigma)\frac{\wp^{\varsigma}}{\Gamma(\varsigma+1)} + (1-\varsigma)^2\right\}. \quad (33)$$

The series solution is obtained using equation (24); therefore, we write

$$\tilde{\mathbb{U}}(\psi,\wp) = \tilde{\mathbb{U}}_0(\psi,\wp) + \tilde{\mathbb{U}}_1(\psi,\wp) + \tilde{\mathbb{U}}_2(\psi,\wp) + \tilde{\mathbb{U}}_3(\psi,\wp) + \tilde{\mathbb{U}}_4(\psi,\wp) + \cdots. \quad (34)$$

The upper and bottom portion forms can be written as

$$\underline{\mathbb{U}}(\psi,\wp) = \underline{\mathbb{U}}_0(\psi,\wp) + \underline{\mathbb{U}}_1(\psi,\wp) + \underline{\mathbb{U}}_2(\psi,\wp) + \underline{\mathbb{U}}_3(\psi,\wp) + \underline{\mathbb{U}}_4(\psi,\wp) + \cdots,$$

$$\bar{\mathbb{U}}(\psi,\wp) = \bar{\mathbb{U}}_0(\psi,\wp) + \bar{\mathbb{U}}_1(\psi,\wp) + \bar{\mathbb{U}}_2(\psi,\wp) + \bar{\mathbb{U}}_3(\psi,\wp) + \bar{\mathbb{U}}_4(\psi,\wp) + \cdots.$$

$$\underline{\mathbb{U}}(\psi,\wp) = \underline{k}(r)\left\{-\frac{\sqrt{418}}{11} - \frac{270}{361}\sqrt{418} \tan h\left(\frac{\sqrt{418}}{38}\psi\right) + \frac{330}{361}\sqrt{418} \tan h^3\left(\frac{\sqrt{418}}{38}\psi\right)\right\} + \underline{k}(r)\left[-\left\{90\sqrt{418}\left(-152 \cos h^3\left(\frac{\sqrt{418}}{38}\psi\right) + 209 \cos h\left(\frac{\sqrt{418}}{38}\psi\right) + 88 \sin h\left(\frac{\sqrt{418}}{38}\psi\right) \cdot \cos h^2\left(\frac{\sqrt{418}}{38}\psi\right) - 242 \sin h\left(\frac{\sqrt{418}}{38}\psi\right)\right)\right\}\right] \cdot \frac{1}{6859\left(\cos h^5\left(\left(\sqrt{418}/38\right)\psi\right)\right)}\frac{1}{\mathbb{B}(\varsigma)} \cdot \left\{\frac{\varsigma\wp^{\varsigma}}{\Gamma(\varsigma+1)} + (1-\varsigma)\right\} + \underline{k}(r)\left[\left[180\sqrt{418}\left\{288574 \sin h\left(\frac{\sqrt{418}}{38}\psi\right) \cdot \cos h^4\left(\frac{\sqrt{418}}{38}\psi\right) - 2495625 \sin h\left(\frac{\sqrt{418}}{38}\psi\right) \cdot \cos h^2\left(\frac{\sqrt{418}}{38}\psi\right) + 2635380 \sin h\left(\frac{\sqrt{418}}{38}\psi\right) + 444752 \cos h^5\left(\frac{\sqrt{418}}{38}\psi\right) - 436810 \cos h^3\left(\frac{\sqrt{418}}{38}\psi\right) - 63536 \cos h^7\left(\frac{\sqrt{418}}{38}\right)\psi\right.\right.\right.$$

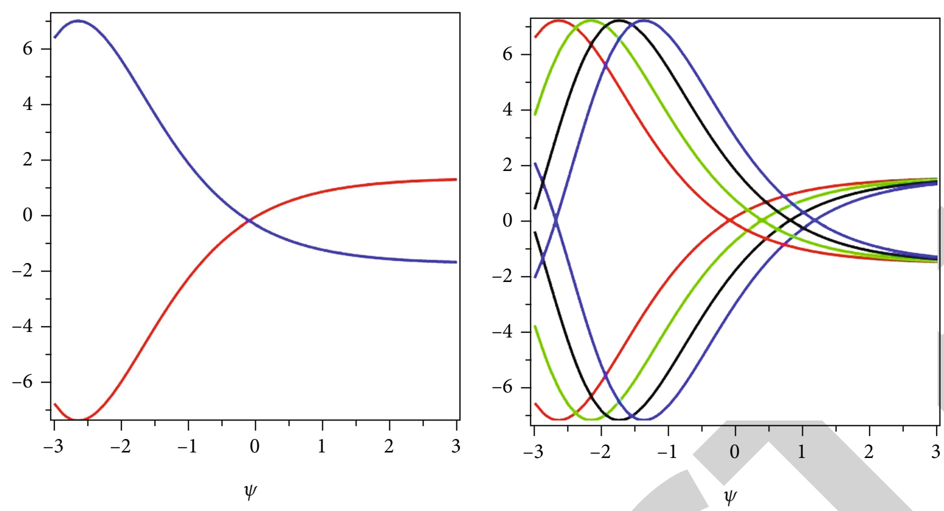

FIGURE 3: (a) 2D upper and lower fuzzy branch plots of the approximate series result and (b) the different fractional order of ς.

$$+73264 \sin h\left(\frac{\sqrt{418}}{38}\psi\right) \cos h^{6}\left(\frac{\sqrt{418}}{38}\psi\right)\Bigg\}\Bigg] \cdot \frac{1}{2476099 \cos h^{9}\left(\sqrt{418}/38\right)\psi}\Bigg] \cdot \frac{1}{\mathbb{B}^{2}(\varsigma)}\left\{\frac{\varsigma^{2}\wp^{2\varsigma}}{\Gamma(2\varsigma+1)}+2\varsigma(1-\varsigma)\frac{\wp^{\varsigma}}{\Gamma(\varsigma+1)} + (1-\varsigma)^{2}\right\}+\cdots,$$

$$\bar{\mathbb{U}}(\psi,\wp)=\bar{k}(r)\left\{-\frac{\sqrt{418}}{11}-\frac{270}{361}\sqrt{418}\tan h\left(\frac{\sqrt{418}}{38}\psi\right) + \frac{330}{361}\sqrt{418}\tan h^{3}\left(\frac{\sqrt{418}}{38}\psi\right)\right\} + \bar{k}(r)\Bigg[-\Bigg\{90\sqrt{418}\Bigg(-152 \cos h^{3}\left(\frac{\sqrt{418}}{38}\psi\right) + 209 \cos h\left(\frac{\sqrt{418}}{38}\psi\right) + 88 \sin h\left(\frac{\sqrt{418}}{38}\psi\right) \cdot \cos h^{2}\left(\frac{\sqrt{418}}{38}\psi\right) - 242 \sin h\left(\frac{\sqrt{418}}{38}\psi\right)\Bigg)\Bigg\}\Bigg] \cdot \frac{1}{6859\left(\cos h^{5}\left(\left(\sqrt{418}/38\right)\psi\right)\right)}\frac{1}{\mathbb{B}(\varsigma)} \cdot \left\{\frac{\varsigma\wp^{\varsigma}}{\Gamma(\varsigma+1)}+(1-\varsigma)\right\} + \bar{k}(r)\Bigg[\Bigg[180\sqrt{418}\Bigg\{288574 \sin h\left(\frac{\sqrt{418}}{38}\psi\right) \cdot \cos h^{4}\left(\frac{\sqrt{418}}{38}\psi\right) - 2495625 \sin h\left(\frac{\sqrt{418}}{38}\psi\right) \cdot \cos h^{2}\left(\frac{\sqrt{418}}{38}\psi\right) + 2635380 \sin h\left(\frac{\sqrt{418}}{38}\psi\right) + 444752 \cos h^{5}\left(\frac{\sqrt{418}}{38}\psi\right) - 436810 \cos h^{3}\left(\frac{\sqrt{418}}{38}\psi\right) - 63536 \cos h^{7}\left(\frac{\sqrt{418}}{38}\right)\psi + 73264 \sin h\left(\frac{\sqrt{418}}{38}\psi\right) \cos h^{6}\left(\frac{\sqrt{418}}{38}\psi\right)\Bigg\}\Bigg] \cdot \frac{1}{2476099 \cos h^{9}\left(\sqrt{418}/38\right)\psi}\Bigg]\frac{1}{\mathbb{B}^{2}(\varsigma)} \cdot \left\{\frac{\varsigma^{2}\wp^{2\varsigma}}{\Gamma(2\varsigma+1)}+2\varsigma(1-\varsigma)\frac{\wp^{\varsigma}}{\Gamma(\varsigma+1)}+(1-\varsigma)^{2}\right\}+\cdots. \quad (35)$$

The exact result of equation (31) is given as

$$\tilde{\mathbb{U}}(\psi,\wp)=\tilde{k}\left\{-\frac{1}{\rho}+\frac{60}{19}\rho\left(-38\theta\rho^{2}+\wp\right)\tan h(\mu) + 120\theta\rho^{3}\tan h^{3}(\mu)\right\}. \quad (36)$$

Then $\mu=\rho\psi+\wp$ and $\rho=0.5\sqrt{22/19}$, on the interval $[-1, 1]$.

Figure 3 represents the two-dimensional upper and lower fuzzy branch plots of the approximate series result and different fractional order of ς. Figure 4 represents the two-dimensional upper and lower fuzzy branch plots with respect to time.

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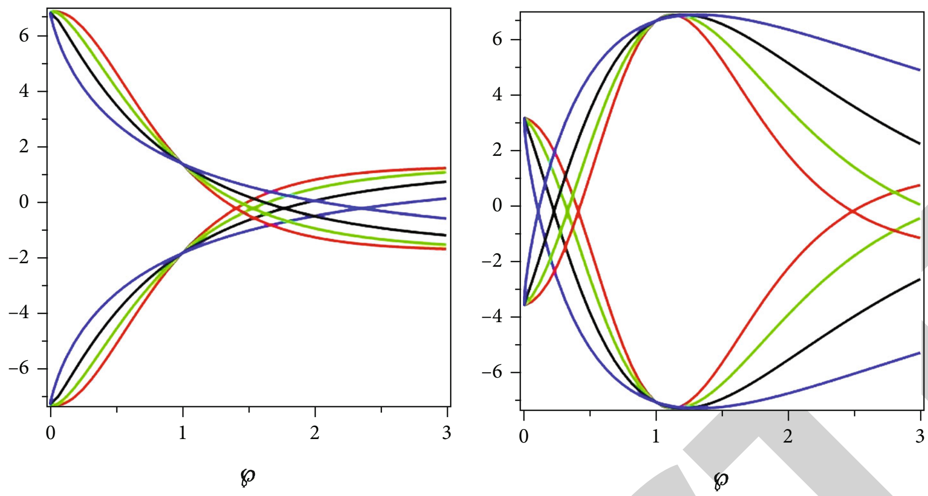

Figure 4: (a) 2D upper and lower fuzzy branch plots with respect to time and (b) different fractional-order of ς.

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5. Conclusion

Finally, we suggest that an efficient method for computing semianalytical or approximate solutions to two-dimensional fuzzy fractional-order Kuramoto-Sivashinsky equations as external terms has been implemented. The techniques of the novel iterative transform approach were used to analyze a semianalytical result in the series form solution. These techniques are applicable to a variety of fractional-order issues. Additionally, we validated our generic solution strategies using three specific examples. All two models were studied using the aforementioned procedures, with the fuzzy number multiplied at two different fractional orders to account for the uncertainty in the beginning values. The reported findings for both scenarios were simulated using two distinct graph formats: two dimensions and three dimensions. The observed solutions were also achieved in the uncertainty form, using fuzzy solutions with upper and lower branches. Additionally, we can take alternative fractional-order values and compare them to the integer order. We can converge to the values of integer order 1 by raising the order of q. In the future, this approach may be applied to a variety of partial differential equations with series-type solutions in both integer and noninteger orders. Additionally, the adopted methodologies can be employed as a robust tool for studying nonlinear and linear dynamical systems of fractional order.

Data Availability

The numerical data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

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