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Research Article

Fixed-Point Theorems for $\omega - \psi$-Interpolative Hardy-Rogers-Suzuki-Type Contraction in a Compact Quasipartial b-Metric Space

Santosh Kumar and Jonasi Chilongola

Department of Mathematics, College of Natural and Applied Sciences, University of Dar es Salaam, Tanzania

Correspondence should be addressed to Santosh Kumar; drsengar2002@gmail.com

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This paper is aimed at proving the existence and uniqueness of a common fixed point for a pair of $\omega - \psi$-interpolative Hardy-Rogers-Suzuki-type contractions in the context of quasipartial b-metric space. Thus, several results in literature such as Hardy and Rogers, Suzuki, and others have been generalized in this work. We also offer a demonstrative example and an application of fractional differential equations to validate the findings.

1. Introduction and Preliminaries

Fixed-point theory is one of the fascinating research areas in pure mathematics, which has many applications in both pure and applied mathematics. Picard presented an iterative procedure for the solution of a functional equation first time in his research paper. This notion was later developed into an abstract framework by the Polish mathematician Stephan Banach [1] who presented a powerful tool known as the Banach contraction principle to determine the fixed point of mapping in complete metric space. It states as follows:

Theorem 1 (see [1]). *Let (M, d) be a complete metric space and let $f : M \longrightarrow M$ be a contraction; that is, there exists a number $k \in [0, 1)$ such that for all $u, v \in M$,*

$$d(fu, fv) \leq kd(u, v). \tag{1}$$

Then, f has a unique fixed point w in M.

By altering the contraction conditions, maps, and other conditions, several researchers have generalized the Banach contraction principle.

The Banach contraction principle needs continuity of the map involved in the contraction condition. In 1968, Kannan [2] relaxed the continuity condition and introduced a new fixed-point theorem with a new contraction condition as follows:

Theorem 2. *Let (M, d) be a complete metric space. A mapping $T : M \longrightarrow M$ is said to be a Kannan contraction if there exists $\lambda \in [0, 1/2)$ such that*

$$d(Tx, Ty) \leq \lambda[d(x, Tx) + d(y, Ty)], \tag{2}$$

for all $x, y \in X \setminus Fix(T)$. Then, T possesses a unique fixed point.

In 2018, Karapinar first established the interpolative Kannan-type contraction in his paper [3] as follows:

Definition 3. Let (M, d) be a metric space. We say that the self-mapping $T : M \longrightarrow M$ is an interpolative Kannan-type contraction, if there exists a constant $\lambda \in [0, 1)$ and $\alpha \in (0, 1)$

such that

$$d(Tx, Ty) \leq \lambda[d(x, Tx)]^{\alpha}[\mathrm{d}(y, Ty)]^{1-\alpha}, \tag{3}$$

for all $x, y \in X$ with $x \neq Tx$.

Karapinar et al. [4] proved some results in the setting of $(\alpha, \beta, \psi, \phi)$ -interpolative contractions. Again in 2021, Khan et al. [5] proved some fixed-point results on the interpolative (ϕ, ψ)-type Z-contraction. For more results on interpolative-type contractions, one can see [6–8] and the references therein.

Following the results due to Karapinar et al. [9], Gaba and Karapinar [10] introduced a new approach to the interpolative contraction as follows:

Definition 4 (see [10]). Let (M, d) be a metric space and $f : M \longrightarrow M$ be a self-map. We shall call T a relaxed (λ, α, β) -interpolative Kannan contraction, if there exists $0 \leq \lambda, \alpha, \beta$ such that

$$d(fu, fv) \leq \lambda d(u, fu)^{\alpha} d(v, fv).^{\beta}. \tag{4}$$

Gaba and Karapinar [10] introduced the following definition of optimal interpolative triplet as follows:

Definition 5 (see [10]). Let (M, d) be a metric space and $f : M \longrightarrow M$ be a relaxed (λ, α, β)-interpolative Kannan contraction. The triplet (λ, α, β) will be called an "optimal interpolative triplet" if for any $\epsilon > 0$, the inequality (4) fails for at least one of the triplet $(\lambda - \epsilon, \alpha, \beta), (\lambda, \alpha - \epsilon, \beta)$, and $(\lambda, \alpha, \beta - \epsilon)$.

In view of the above definitions, Gaba and Karapinar [10] proved the following theorem:

Theorem 6 (see [10]). *Let (M, d) be a complete metric space, and $f : M \longrightarrow M$ be a (λ, α, β)-interpolative Kannan contraction with $\lambda \in [0, 1), \alpha, \beta \in (0, 1)$ so that $\alpha + \beta < 1$. Then, f has a fixed point in M.*

In 1973, Hardy and Rogers [11] introduced a natural modification of the Banach contraction principle.

Theorem 7. *Let (M, d) be a complete metric space. The mapping $f : M \longrightarrow M$ is called an interpolative Hardy-Rogers type of contraction if there exist positive real numbers $\alpha, \beta, \gamma, \delta > 0$, with $\beta + \alpha + \gamma + \delta < 1$ such that*

$$d(fu, fv) \leq [\alpha d(u, v) + \beta d(u, fu) + \gamma d(v, fv)] + \delta\left[\frac{1}{2}(d(u, fv) + d(v, fu))\right], \tag{5}$$

for each $u, v \in M \setminus Fix(f)$. Then, a mapping f has a unique fixed point in M.

Later in 2018, Karapinar et al. [12] introduced the following notion of interpolative Hardy-Rogers-type contraction.

Theorem 8 (see [12]). *Let (M, d) be a complete metric space. The mapping $f : M \longrightarrow M$ is called an interpolative Hardy-Rogers type of contraction if there exist $\lambda \in (0, 1)$ and positive reals $\alpha, \beta, \gamma > 0$, with $\beta + \alpha + \gamma < 1$ such that*

$$d(fu, fv) \leq \lambda\left([d(u, v)]^{\beta}.[d(u, fu)]^{\alpha}.[d(v, fv)]^{\gamma}.\left[\frac{1}{2}(d(u, fv) + d(v, fu))\right]^{1-\alpha-\beta-\gamma}\right), \tag{6}$$

for each $u, v \in M \setminus Fix(f)$. Then, a mapping f has a unique fixed point in M.

Several other versions of this type of results were proven by researchers. Some of them can be seen in [9, 13–15].

In 2008, Suzuki [16] introduced a generalization of the Banach contraction principle and characterizes the metric completeness of the underlying space. The generalized result is as follows:

Theorem 9 (see [16]). *Let (M, d) be a complete metric space and let $f : M \longrightarrow M$ be a mapping such that for all $u, v \in M$,*

$$\Phi(k) d(u, fu) \leq d(u, v) \Rightarrow d(fu, fv) \leqslant k d(u, v), \tag{7}$$

where $\Phi : [0, 1) \longrightarrow (1/2, 1)$ is a nonincreasing function defined by

$$\Phi(k) = \begin{cases} 1 & \text{if } 0 \leq k \leq \dfrac{\left(\sqrt{5} - 1\right)}{2}, \\ (1-k)k^{-2} & \text{if } \dfrac{\left(\sqrt{5} - 1\right)}{2} \leq k \leq 2^{-1/2}, \\ (1+k)^{-1} & \text{if } 2^{-1/2} \leq k < 1. \end{cases} \tag{8}$$

Then, there exists a unique fixed-point $w \in M$. A mapping f satisfying (7) is called as the Suzuki contraction.

Example 10 (see [16]). Let $M = \{(1, 1), (4, 1), (1, 4), (4, 5), (5, 4)\}$ with a metric d be defined by

$$d((u_1, u_2), (v_1, v_2)) = |u_1 - v_1| + |u_2 - v_2|. \tag{9}$$

Define a mapping

$$f(u_1, u_2) = \begin{cases} (u_1, 1) & \text{if } u_1 \geq u_2, \\ (1, u_2) & \text{if } u_1 > u_2. \end{cases} \quad (10)$$

Then, the map f satisfies all the hypotheses of Theorem 9, and $(1, 1)$ is the unique fixed point of f. However, for $u = (4, 5)$ and $v = (5, 4)$, $d(fu, fv) = 6 > 2 = d(u, v)$. Thus, f does not satisfy the assumptions in Theorem 9 for any $k \in [0, 1)$.

In 2021, Yeşilkaya [17] generalized the Banach contraction principle to (λ, α, β)-interpolative Kannan contraction as follows:

Definition 11 (see [17]). Let (M, d) be a metric space. The mapping $f : M \longrightarrow M$ is called an $\omega - \phi$ interpolative Hardy-Rogers contraction of the Suzuki type. If there exist $\psi \in \Psi$, $\omega : M \times M \longrightarrow [0,\infty)$, and positive reals $\alpha, \beta, \gamma > 0$, with $\alpha + \beta + \gamma < 1$, such that

$$\frac{1}{2} d(u, fu) \leq d(u, v) \Rightarrow \omega(u, v) d(fu, fv) \leq \psi\left\{ [d(u, v)]^{\beta}.[d(u, fu)]^{\alpha}.[d(v, fv)]^{\gamma}.\left[\frac{1}{2}(d(u, fv) + d(v, fu))\right]^{1-\alpha-\beta-\gamma} \right\}, \quad (11)$$

where Ψ is the set of all nondecreasing self-mappings ψ on $[0, \infty)$ such that $\sum_{n=1}^{\infty} \psi^n(t) < \infty$ for all $t > 0$.

Similar results can be seen in [6, 7] and the references therein.

In 2012, Wardowski [18] generalized the Banach contraction principle into F-contraction mapping principle as follows:

Definition 12 (see [18]). Let (M, d) be a metric space. A mapping $f : M \longrightarrow M$ is called an F-contraction if there exist $\tau > 0$ and $F \in \mathcal{F}$ such that

$$\tau + F(d(fu, fv)) \leqslant F(d(u, v)), \quad (12)$$

holds for any $u, v \in M$ with $d(fu, fv) > 0$, where F is the set of all functions $F : R^+ \longrightarrow R$ satisfying the following conditions:

(F_1) F is strictly increasing: $u < v \Rightarrow F(u) < F(v)$,

(F_2) For each sequence $\{\alpha_n\}_{n\in N}$ in R^+, $\lim_{n\longrightarrow\infty} F(\alpha_n) = -\infty$,

(F_3) There exists $k \in (0, 1)$ such that $\lim_{\alpha\longrightarrow\infty} \alpha^k F(\alpha) = 0$.

We denote by $\mathcal{F}$ the set of all functions satisfying the conditions (F_1) and (F_2).

Example 13 (see [18]). The following $F : (0,+\infty)$ are the elements of $\mathcal{F}$

(1) $F\theta = \theta$,

(2) $F\theta = ln\theta + \theta$,

(3) $F\theta = -1/\sqrt{\theta}$,

(4) $F\theta = \ln(\theta^2 + \theta)$.

In 2013, Salimi et al. [19] and Hussain et al. [20] modified the notions of $\alpha - \phi$-contractive and α-admissible mappings and established certain fixed-point theorem as given below:

Definition 14 (see [19]). Let f be a self-mapping on M and $\alpha, \eta : M \times M \longrightarrow [0,+\infty)$ be two functions. We say that T is an α-admissible mapping with respect to η if $u, v \in M$,

$$\alpha(u, v) \geq \eta(u, v) \Rightarrow \alpha(fu, fv) \geq \eta(fu, fv). \quad (13)$$

Remark 15. It should be noted that Definition 14 reduces to α-admissible mapping definition due to Samet et al. [21] if we assume that $\alpha(u, v) = 1$. Furthermore, if we suppose that $\eta(u, v) = 1$, we may argue that f is an admissible η-sub admissible mapping.

Note that a self-map f can be ω-orbital admissible as stated in the definition below:

Definition 16 (see [11]). Let f be a self-map defined on M, and $\omega : M \times M \longrightarrow [0,\infty)$ be a function. f is said to be an ω-orbital admissible if for all $u \in M$, we have

$$\omega(u, fu) \geq 1 \Rightarrow \omega\left(u, f^2 u\right) \geq 1. \quad (14)$$

Gopal et al. [22] established the idea of α-type F-contractions and α-type F-weak contractions by combining the concepts of α-admissible mappings with F-contractions and F-weak contractions:

Definition 17 (see [22]). Let (M, d) be a metric space and $g : M \longrightarrow M$ be a mapping. Suppose $\alpha : M \times M \longrightarrow \{-\infty\} \cup (0,\infty)$ be a function. The function g is said to be an α-type F-contraction if there exists $\tau > 0$ such that for all $u, v \in M$,

$$d(fu, fv) > 0 \Rightarrow \tau + \alpha(u, v) F(d(gu, gv)) \leq F(d(u, v)). \quad (15)$$

In 2019, Dey et al. [23] introduced the notion of generalized α-F-contraction mapping as follows:

Theorem 18 (see [23]). *Let (M, d) be a metric space and $g : M \longrightarrow M$ be a mapping. Let $\alpha : M \times M \longrightarrow [0,\infty)$ be a function and $F \in \mathscr{F}$. The function g is said to be a modified generalized $\alpha - F$-contraction mapping if there exists $\tau > 0$ such that for all $u, v \in M$,*

$$d(gu, gv) > 0 \Rightarrow \tau + \alpha(u, v)F(d(gu, gv)) \leq F\left(N_{g(u,v)}\right), \tag{16}$$

where

$$N_{g(u,v)} = \max\left\{d(u, v), \frac{d(u, gv) + d(v, gu)}{2}, \frac{d\left(g^2u, u\right) + d\left(g^2u, gv\right)}{2}, d\left(g^2u, gu\right), d\left(g^2u, v\right), d(gu, v) + d(v, gv), d\left(g^2u, gv\right) + d(u, gu)\right\}. \tag{17}$$

Later, Wangwe and Kumar [24] proved results for $\alpha - F$-type contractions. One can see more results in [25–28] and the references therein.

$F-$contraction mapping of Hardy-Rogers type was introduced by Cosentino and Vetro [29] as follows:

Definition 19 (see [29]). Let (M, d) be a metric space. A self-mapping f on M is called an F-contraction of Hardy-Rogers type if there exists $F \in \mathscr{F}$ and $\tau \in S$ such that

$$\begin{aligned}\tau(d(u, v) + F(d(fu, fv))) \leq F[\alpha d(u, v) + \beta d(u, fu) \\ + \gamma d(v, fv) + \delta d(u, fv) + Ld(v, fu)],\end{aligned} \tag{18}$$

for all $u, v \in M$ with $fu \neq fv$ where $\alpha, \beta, \gamma, \delta, L \in [0,+\infty)$,

$$\alpha + \beta + \gamma + 2\delta = 1. \tag{19}$$

Moreover, f is said to be a F-contraction of Suzuki-Hardy-Rogers type [30] if contraction Condition (18) holds for all $u, v \in M$ with $fu \neq fv$ and $d(u, fu)/2 < d(u, v)$.

Many researchers generalized the concept of metric space. The concept of b-metric space was first introduced by Bakhtin in 1989. By adding a variable $s \geq 1$ to the definition of metric space, the triangle inequality in this concept was relaxed as follows:

Definition 20 (see [31]). A b-metric on a nonempty set M is a function $d : M \times M \longrightarrow [0,\infty)$, such that for all $u, v, w \in M$ and for some real number $s \geq 1$, it satisfies the following:

(i) if $d(u, v) = 0$, then $u = v$,

(ii) $d(u, v) = d(v, u)$,

(iii) $d(u, v) \leqslant s[d(u, w) + d(w, v)]$,

then, a pair (M, d) is called b-metric space.

In 2021, Pauline and Kumar [32] presented an extension of the fixed-point theorem for T-Hardy-Rodgers contraction mappings in b-metric space. Czerwick [33] proved the existence of fixed point in b-metric space as follows:

Theorem 21 (see [33]). *Let v be a topological space and let (M, d) be a complete b-metric space. Let $f : M \longrightarrow M$ be continuous and satisfy for each $w \in v$*

$$d[f(u, w), f(v, w)] \leq \alpha d(u, v), \tag{20}$$

for all $u, v \in M$, where $0 < \alpha < 1$. Then for each $w \in v$, there exists a unique fixed-point $u(w)$ of f, i.e., $f[u(w), w] = u(w)$ and the function $w \longrightarrow u(w)$ is continuous on v.

In 1994, Matthews [34] introduced partial metric space as a result of the failure of metric functions in computer science as follows:

Definition 22 (see [34]). Let $M \neq \varnothing$. A partial metric is a function $p : M \times M \longrightarrow R^+$ satisfying

(i) $p(u, v) = p(v, u)$,

(ii) If $0 \leqslant p(u, u) = p(u, v) = p(v, v)$, then $u = v$,

(iii) $p(u, v) + p(w, w) \leqslant p(u, w) + p(w, v)$ for all $u, v, w \in M$.

Then, a pair (M, p) is called partial metric space. It is clear that if $p(u, v) = 0$, then $u = v$; however, if $u = v$, then $p(u, v)$ may not be zero.

Remark 23 (see [34]). As partial metrics have a wider range of topological features and may easily support partial ordering, partial metrics are more versatile than metric spaces.

Künzi et al. [35] proposed the idea of partial quasimetric by eliminating symmetry condition from the notion of partial metric space.

Definition 24 (see [35]). A quasipartial metric on a nonempty set M is a function $qp : M \times M \longrightarrow [0,\infty)$ such that

(1) $qp(u, u) \leqslant qp(u, v)$ whenever $u, v \in M$,

(2) $qp(u, u) \leqslant qp(v, u)$ whenever $u, v \in M$,

(3) $qp(u, w) + qp(v, v) \leqslant (qp(u, v) + qp(v, w))$, whenever $u, v, w \in M$,

(4) $u = v$ if and only if $qp(u, u) = qp(u, v) = qp(v, v)$ whenever $u, v \in M$.

A pair (M, qp) is called a quasipartial metric space.

In 2015, Gupta and Gautam [36] introduced the notion of quasipartial b-metric space as follows:

Definition 25 (see [36]). A quasipartial b-metric on a nonempty set M is a function $qp_b : M \times M \longrightarrow [0,\infty)$ such that for some real number $s \geq 1$, it satisfies the following:

(i) if $qp_b(u, u) = qp_b(u, v) = qp_b(v, v)$, then $u = v$ (indistancy implies equality),

(ii) $qp_b(u, u) \leqslant qp_b(u, v)$ (small self-distances),

(iii) $qp_b(u, u) \leqslant qp_b(v, u)$ (small self-distances)

(iv) $qp_b(u, v) + qp_b(w, w) \leqslant s[qp_b(u, w) + qp_b(w, v)]$ (triangularity), for all $u, v \in M$.

Then, the pair (M, qp_b) is quasipartial b-metric on space M.

Example 26 (see [36]). Let $M = \mathbb{R}$ be the set of all real numbers. Define $qp_b : \mathbb{R} \times \mathbb{R} \longrightarrow \mathbb{R}^+$ by

$$qp_b(u, v) = |u - v| + |u|. \tag{21}$$

Then, it is a quasipartial b-metric on M.

Gautam et al. [37, 38] extended several results in quasipartial b-metric spaces.

In this article, we establish the existence and uniqueness of fixed-point theorems for $\omega - \psi$- interpolative Hardy-Rogers-Suzuki-type contraction in a compact quasipartial b-metric spaces with an application to fractional differential equations. An example is given to use the results that have been proven. The outcomes of this study will generalize several results obtained in [11, 12, 16–18, 25, 39, 40] and the references therein.

2. Main Results

To establish our first main results, we will begin by generalizing Definition 11 and extend it to a compact quasipartial b-metric space.

Definition 27. Let (M, qp_b) be a compact quasipartial b-metric space. A map $f : M \longrightarrow M$ is called $\omega - \psi$-interpolative Hardy-Rogers contraction of Suzuki type, if there exist $\psi \in \Psi$, where Ψ is the set of all nondecreasing self-mappings ψ on $[0, \infty)$ such that $\sum_{n=1}^{\infty} \psi^n(t) < \infty$ for all $t > 0$ and $\alpha, \beta, \gamma > 0$, with $\alpha + \beta + \gamma < 1$,

$$\frac{1}{2} qp_b(u, fu) < qp_b(u, v) \Rightarrow \omega(u, v) qp_b(fu, fv) < \psi\left\{ [qp_b(u, v)]^{\beta} [qp_b(u, fu)]^{\alpha} [qp_b(v, fv)]^{\gamma} \left[\frac{1}{2}(qp_b(u, fv) + qp_b(v, fu))\right]^{1-\alpha-\beta-\gamma} \right\}, \quad \forall u, v \in M \setminus \mathrm{Fix}(f). \tag{22}$$

We now present our main theorem as follows:

Theorem 28. *Let (M, qp_b) be a compact quasipartial b-metric space and $f : M \longrightarrow M$ be $\omega - \psi$-interpolative Hardy-Rogers contraction of Suzuki type. If f is ω-orbital admissible mappings such that*

$$\omega(u_0, fu_0) \geq 1, \tag{23}$$

for some $u_0 \in M$. Then, a mapping f has a fixed point in M if at least one of the following properties holds

(i) (M, qp_b) is ω-regular

(ii) f is a continuous map

(iii) f^2 is continuous, $\omega(u, fu) \geq 1$ where $u \in Fix(f^2)$.

Proof. Let $u_0 \in M$ satisfies

$$\omega(u_0, fu_0) \geq 1. \tag{24}$$

We construct a sequence $\{u_n\}_{n=1}^{\infty}$ as shown below

$$u_1 = fu_0, u_2 = fu_1, \cdots, u_n = fu_{n-1}. \tag{25}$$

Assume that

$$u_{n_0} = u_{n_0+1} \tag{26}$$

for some $n_0 \in \mathbb{N}$, so that u_{n_0} is a fixed point of f. Thus on contrary, we can suppose that

$$u_n \neq u_{n+1}, \tag{27}$$

for each $n \in \mathbb{N} \cup \{0\}$. As f is ω-orbital admissible

$$\omega(u_0, fu_0) = \omega(u_0, u_1) \geq 1, \tag{28}$$

implies that

$$\omega(u_1, fu_1) = \omega(u_1, u_2) \geq 1. \tag{29}$$

Similarly, continuing this process, we get a sequence,

$$\omega(u_{n-1}, u_n) \geq 1. \tag{30}$$

By substituting $u = u_{n-1}$ and $v = fu_{n-1} = u_n$ in Definition 27, we obtain

$$\begin{aligned}
&\frac{1}{2} qp_b(u_{n-1}, fu_{n-1}) = \frac{1}{2} qp_b(u_{n-1}, u_n) < qp_b(u_{n-1}, u_n) \\
&\quad \Rightarrow \omega(u_{n-1}, u_n) qp_b(fu_{n-1}, fu_n) \\
&\quad < \psi\Big([qp_b(u_{n-1}, u_n)]^\beta [qp_b(u_{n-1}, fu_{n-1})]^\alpha [qp_b(u_n, fu_n)]^\gamma \\
&\qquad \times \left[\frac{1}{2}(qp_b(u_{n-1}, fu_n) + qp_b(u_n, fu_{n-1}))\right]^{1-\alpha-\beta-\gamma}\Big) \\
&\quad = \psi\Big([qp_b(u_{n-1}, u_n)]^\beta [qp_b(u_{n-1}, u_n)]^\alpha [qp_b(u_n, u_{n+1})]^\gamma \\
&\qquad \times \left[\frac{1}{2}(qp_b(u_{n-1}, u_{n+1}) + qp_b(u_n, u_n))\right]^{1-\alpha-\beta-\gamma}\Big).
\end{aligned} \tag{31}$$

Thus, using $\psi(t) < t$ for $t > 0$, we have

$$\begin{aligned}
qp_b(u_n, u_{n+1}) &< \psi\left([qp_b(u_{n-1}, u_n)]^\beta [qp_b(u_{n-1}, u_n)]^\alpha [qp_b(u_n, u_{n+1})]^\gamma \left[\frac{1}{2}(qp_b(u_{n-1}, u_{n+1}) + qp_b(u_n, u_n))\right]^{1-\alpha-\beta-\gamma}\right) \\
&< [qp_b(u_{n-1}, u_n)]^\beta [qp_b(u_{n-1}, u_n)]^\alpha [qp_b(u_n, u_{n+1})]^\gamma \left[\frac{1}{2}(qp_b(u_{n-1}, u_n) + qp_b(u_n, u_{n+1}))\right]^{1-\alpha-\beta-\gamma}.
\end{aligned} \tag{32}$$

Assuming that,

$$qp_b(u_{n-1}, u_n) < qp_b(u_n, u_{n+1}), \tag{33}$$

for all $n \in \mathbb{N}$, then

$$\frac{1}{2}(qp_b(u_{n-1}, u_n) + qp_b(u_n, u_{n+1})) \leq qp_b(u_n, u_{n+1}), \tag{34}$$

Thus,

$$[qp_b(u_n, u_{n+1})]^{\alpha+\beta} < [qp_b(u_{n-1}, u_n)]^{\alpha+\beta}, \tag{35}$$

which is a contradiction. Hence, we get $\forall n \in \mathbb{N}$,

$$qp_b(u_n, u_{n+1}) \leq qp_b(u_{n-1}, u_n). \tag{36}$$

Then, the positive sequence $\{qp_b(u_{n-1}, u_n)\}$ is a nonincreasing sequence with positive terms, so we attain that there exists $a \geq 0$ such that

$$\lim_{n \longrightarrow \infty} qp_b(u_{n-1}, u_n) = a. \tag{37}$$

Accordingly, we get

$$\frac{1}{2}(qp_b(u_{n-1}, u_n) + qp_b(u_n, u_{n+1})) < qp_b(u_n, u_{n+1}). \tag{38}$$

Furthermore, using Equation (32),

$$[qp_b(u_n, u_{n+1})]^{1-\gamma} < \psi[qp_b(u_{n-1}, u_n)], \tag{39}$$

or equivalent

$$qp_b(u_n, u_{n-1}) < \psi(qp_b(u_{n-1}, u_n)). \tag{40}$$

Hence, by repeating this condition, we can write

$$\begin{aligned}
qp_b(u_n, u_{n+1}) &< qp_b(u_{n-1}, u_n) < \psi^2 qp_b(qp_b(u_{n-2}, u_{n-1})) \\
&< \cdots < \psi^n qp_b(u_0, u_1).
\end{aligned} \tag{41}$$

Now, we claim that $\{x_n\}$ is a Cauchy sequence in (X, qp_b). Then, we shall use the triangle inequality with Equation (41) for $s \geq 1$ and find that

$$\begin{aligned}
qp_b(u_n, u_{n+l}) &\leq s(qp_b(u_n, u_{n+1}) + qp_b(u_{n+1}, u_{n+2}) \\
&\quad + \cdots + qp_b(u_{n+l-1}, u_{n+l}) - qp_b(u_{n+l-1}, u_{n+l-1})), \\
&< \psi^n\big(qp_b(u_0, u_1) + \psi^{n+1} qp_b(u_0, u_1) \\
&\quad + .. + \psi^{n+l-1} qp_b(u_0, u_1)\big), < \sum_{k=1}^{\infty} \psi^k (qp_b(u_0, u_1)).
\end{aligned} \tag{42}$$

Letting $n \longrightarrow \infty$ in Equation (42), we find that $\{u_n\}$ is a Cauchy sequence in (M, qp_b). Regarding that (M, qp_b) is complete, there exists $t \in M$ such that

$$\lim_{n \longrightarrow \infty} qp_b(u_n, t) = 0. \tag{43}$$

We will show that the point t is a fixed point of f. If Equation (32) holds, that is, (M, qp_b) is ω-regular, then $\{u_n\}$ verify Equation (32), that

$$\omega(u_n, u_{n+1}) \geq 1. \tag{44}$$

and $\forall n \in \mathbb{N}$, we get

$$\omega(u_n, t) \geq 1. \tag{45}$$

We assert that

$$\frac{1}{2} qp_b(u_n, fu_n) \leq qp_b(u_n, t), \tag{46}$$

or

$$\frac{1}{2} qp_b(fu_n, f(fu_n)) \leq qp_b(fu_n, t), \tag{47}$$

$\forall n \in \mathbb{N}$. Assuming on the contrary that

$$\frac{1}{2} qp_b(u_n, fu_n) > qp_b(u_n, t), \tag{48}$$

and

$$\frac{1}{2} qp_b(fu_n, f(fu_n)) > qp_b(fu_n, t). \tag{49}$$

Using triangle inequality for $s \geq 1$, we obtain

$$\begin{aligned} qp_b(u_n, u_{n+1}) = qp_b(u_n, fu_n) &\leq s(qp_b(u_n, t) + qp_b(t, fu_n) - qp_b(t, t)) \\ &< \frac{1}{2} qp_b(u_n, u_{n+1}) + \frac{1}{2} qp_b(u_n, u_{n+2}) = qp_b(u_n, u_{n+1}), \end{aligned} \tag{50}$$

which is a contradiction. Therefore, $\forall n \in \mathbb{N}$, either

$$\frac{1}{2} qp_b(u_n, fu_n) \leq qp_b(u_n, t), \tag{51}$$

or

$$\frac{1}{2} qp_b(fu_n, f(fu_n)) \leq qp_b(fu_n, t), \tag{52}$$

holds. In case that inequality (46) holds, we get

$$\begin{aligned} qp_b(u_{n+1}, \mathrm{ft}) < \omega(u_n, t).qp_b(fu_n, \mathrm{ft}) &< \psi\left([(qp_b(u_n, t)]^{\beta} [qp_b(u_n, fu_n)]^{\alpha} [qp_b(t, \mathrm{ft})]^{\gamma} \left[\frac{1}{2}(qp_b(u_n, \mathrm{ft}) + qp_b(t, u_{n+1}))\right]^{1-\alpha-\beta-\gamma} \right). \\ &< [(qp_b(u_n, t)]^{\beta} [qp_b(u_n, u_{n+1})]^{\alpha} [qp_b(t, \mathrm{ft})]^{\gamma} \left[\frac{1}{2}(qp_b(u_n, \mathrm{ft}) + qp_b(t, u_{n+1}))\right]^{1-\alpha-\beta-\gamma}. \end{aligned} \tag{53}$$

If Equation (47) holds, we have

$$qp_b(u_{n+2}, \mathrm{ft}) < \omega(u_{n+1}, t) qp_b(f(fu_n), \mathrm{ft})) < \psi\left([qp_b(fu_n, t)]^{\beta} [qp_b(fu_n, f(fu_n))]^{\alpha} [qp_b(t, \mathrm{ft})]^{\gamma} \left[\frac{1}{2}(qp_b(fu_n, \mathrm{ft}) + qp_b(t, f(fu_n)))\right]^{1-\alpha-\beta-\gamma} \right), \tag{54}$$

$$= \psi\left([qp_b(u_{n+1}, t)]^{\beta} [qp_b(u_{n+1}, u_{n+2})]^{\alpha} [qp_b(t, \mathrm{ft})]^{\gamma} \left[\frac{1}{2}(qp_b(u_{n+1}, \mathrm{ft}) + qp_b(t, fu_{n+2}))\right]^{1-\alpha-\beta-\gamma} \right). \tag{55}$$

Therefore, letting $n \longrightarrow \infty$ in Equations (54) and (55), we get $qp_b(t, t) = 0$, that is,

$$\mathrm{ft} = t. \tag{56}$$

In case that assumption (47) is true, that is the mapping f is continuous,

$$t = \mathrm{ft} = \lim_{n \longrightarrow \infty} fu_n = \lim_{n \longrightarrow \infty} u_{n+1}, \tag{57}$$

and we want to show that also

$$\text{ft} = t. \tag{58}$$

Assuming on the contrary that

$$t \neq \text{ft}. \tag{59}$$

Since,

$$\frac{1}{2} qp_b\left(\text{ft}, f^2(t)\right) = \frac{1}{2} qp_b(\text{ft}, t) < qp_b(\text{ft}, t), \tag{60}$$

by Equation (47), we get

$$\begin{aligned} qp_b(t, \text{ft}) &< \omega(t, \text{ft}).qp_b\left(f^2 t, \text{ft}\right) < \psi\left([qp_b(\text{ft}, t)]^\beta \left[qp_b\left(\text{ft}, f^2 t\right)\right]^\alpha [qp_b(t, \text{ft})]^\gamma \left[\frac{1}{2}\left(qp_b(\text{ft}, \text{ft}) + qp_b\left(\text{ft}, f^2 t\right)\right)\right]^{1-\alpha-\beta-\gamma}\right) \\ &< [qp_b(\text{ft}, t)]^\beta [qp_b(\text{ft}, t)]^\alpha [qp_b(t, \text{ft})]^\gamma \left[\frac{1}{2} qp_b(\text{ft}, t)\right]^{1-\alpha-\beta-\gamma} < qp_b(t, \text{ft}), \end{aligned} \tag{61}$$

which is a contradiction. Consequently,

$$t = \text{ft}, \tag{62}$$

that is, t is a fixed point of f. □

The following corollary is obtained by substituting $\omega = 1$ in Theorem 28.

Corollary 29. *Let (M, qp_b) be a complete and compact metric space and f be self-mapping on M, such that*

$$\frac{1}{2} qp_b(u, fu) < qp_b(u, v), \tag{63}$$

implies

$$qp_b(fu, fv) < \psi\left([qp_b(u, v)]^\beta [qp_b(u, fu)]^\alpha [qp_b(v, fv)]^\gamma \left[\frac{1}{2}\left(qp_b(u, fv) + qp_b(v, fu)\right)\right]^{1-\alpha-\beta-\gamma}\right), \tag{64}$$

for each $u, v \in M \setminus Fix(f)$, where $\psi \in \Psi$ and positive real β, $\alpha, \gamma > 0$, with $\alpha + \beta + \gamma < 1$. Then, f has a fixed point in M.

Proof. In Theorem 28, it is sufficient to get

$$\omega(u, v) = 1, \tag{65}$$

for proof. □

Further, taking $\psi(p) = p\lambda$, with $\lambda \in [0, 1)$ in Corollary 29, we obtain the following Corollary.

Corollary 30. *Let (M, qp_b) be a compact quasipartial b-metric space and f be a self-mapping on space M such that*

$$\frac{1}{2} qp_b(u, fu) < qp_b(u, v), \tag{66}$$

implies that

$$\begin{aligned} qp_b(fu, fv) &< \lambda [qp_b(u, v)]^\beta . [qp_b(u, fu)]^\alpha [qp_b(v, fv)]^\gamma \\ &\cdot \left[\frac{1}{2}\left(qp_b(u, fv) + qp_b(v, fu)\right)\right]^{1-\alpha-\beta-\gamma}, \end{aligned} \tag{67}$$

for each $u, v \in M \setminus Fix(f)$, where positive reals $\alpha, \beta, \gamma > 0$, with $\alpha + \beta + \gamma < 1$. Then, f has a fixed point in M.

Remark 31. If we replace the quasipartial b-metric space by the metric space in Theorem 28, then we get the result due to Yeşilkaya [17] as a corollary.

Kumar [27] discussed the concept of orbital continuity. Using this concept, we formulate the following example which validates the result proved in Theorem 28.

Example 32. Let $M = [0, 2]$ and

$$qp_b = |u - v| + |u|. \tag{68}$$

Here, (M, qp_b) is a complete and compact quasipartial b-metric space defined by

$$f(u) = \begin{cases} \dfrac{1}{3} & \text{if}, 0 \le u \le 1, \\ \dfrac{u}{5} & \text{if}, 1 < u \le 2, \end{cases} \tag{69}$$

and further, let

$$\omega(u, v) = \begin{cases} 3, & \text{if}, 0 \le u \le 1, \\ 1, & \text{if}, u = 0, \text{and } v = 2, \\ 0, & \text{otherwise}. \end{cases} \quad (70)$$

The mapping f is not continuous but since

$$f^2 = \frac{1}{3}, \quad (71)$$

we have f^2 is continuous mapping. Let a function $\psi \in \Psi$ defined as $\psi = t/6$ and we choose $\beta = 1/2, \alpha = 1/3, \gamma = 1/7$, and $t = 1$. Then, we have to check if Theorem 28 holds. We have to consider the following cases:

(i) For $u, v \in [0, 1]$, we have

$$\frac{1}{2} qp_b(u, fu) < qp_b(u, v), \quad (72)$$

implies

$$\omega(u, v) qp_b(fu, fv) < \psi\left([qp_b(u, v)]^\beta [qp_b(u, fu)]^\alpha [qp_b(v, fv)]^\gamma \left[\frac{1}{2}(qp_b(u, fv) + qp_b(v, fu))\right]^{1-\alpha-\beta-\gamma}, \right. \quad (73)$$
$$\forall u, v \in M$$

(ii) For $u = 0$ and $v = 2$, we have

$$\frac{1}{2} qp_b(0, f0) = \frac{1}{3} < qp_b(0, 2) = 2, \quad (74)$$

implies

$$\omega(0, 2) qp_b(f0, f2) = \frac{2}{5}, < \frac{1}{6}\left([2]^{1/2}\right)\left(\left[\frac{2}{5}\right]^{1/3}\right)\left(\left[\frac{18}{5}\right]^{1/7}\right)\left(\frac{1}{2}\left[\frac{2}{5} + \frac{18}{5}\right]\right)^{1/42}. \quad (75)$$

For all other cases, Theorem 28 holds, since

$$\omega(u, v) = 0. \quad (76)$$

As a result, the assumptions of Theorem 28 are satisfied, also the mappings f has a fixed point $u = 1/3$.

3. An Application to Fractional Differential Equations

Several authors gave solutions of fractional differential equations using fixed-point theorems. Some of them are worth noting in this direction [41–45]. In this section, Theorem 28 is used to establish the existence and uniqueness of the solution of the fractional order differential equation. Here, we consider the following initial valued problem (IVP) of the form

$$D^\alpha u(t) = f(t, u_t), \forall t \in \gamma = [0, b], \alpha \in (0, 1), \quad (77)$$

$$u(t) = \phi(t), t \in (-\infty, 0), \quad (78)$$

where D^α is the standard Riemann-Liouville fractional derivative

$$f : \gamma \times A \longrightarrow \mathbb{R}, \phi \in A, \phi(0) = 0, \quad (79)$$

and A is called a phase, space, or state space. Consider a quasipartial b-metric qp_b on X given by

$$qp_b(u, v) = |u - v| + |u|, \quad (80)$$

$\forall u, v \in M$ then, it is obvious that (M, qp_b) is a compact quasipartial b-metric space. If $u : (-\infty, b] \longrightarrow \mathbb{R}$, and $u_0 \in \gamma$, then for every $t \in [0, b] u_t$ is a γ-valued continuous function on $[0, b]$. The space γ is complete by a solution of problems (77) and (78); we mean a space $\Omega = \{u : (-\infty, b] \longrightarrow \mathbb{R} : u|_{(-\infty,0)\in B} \text{ and } u|_{[0,b]}\}$. Therefore, a function $u \in \Omega$ is called a solution of Equations (77) and (78) if it satisfies the equation $D^s u(t) = f(t, u_t)$ on γ and condition $u(t) = \phi(t)$ on $(-\infty, 0]$.

Lemma 33 (see [41]). *Let $0 < \beta < 1$ and $h : (0, b] \longrightarrow \mathbb{R}$ be continuous and*

$$\lim_{t \longrightarrow 0^+} v(t) = v(0^+) \in \mathbb{R}. \quad (81)$$

Then, u is a solution of the fractional integral equation

$$u(t) = \frac{1}{\Gamma(\beta)} \int_0^t (t - s)^{\beta - 1} v(s) ds, \quad (82)$$

if and only if u is a solution of the initial value problem for the fractional differential equation

$$D^\beta u(t) = v(t), t \in (0, b], u(0) = 0. \quad (83)$$

Theorem 34. *Let $f:\gamma \times A \longrightarrow \mathbb{R}$. Assume that there exists $q>0$ such that*

$$|f(t,u)-f(t,v)|+|f(t,u)| \le q(|u-v|+|u|), \tag{84}$$

for $t \in \gamma$ and $\forall u, v \in A$. If $b^{\beta}k_b q/\Gamma(\beta+1)=k_1\lambda<1$ where $0 \le k_1<1/7$ and

$$k_b=\sup\{|k(t)|: t\in[0,b]\}, \tag{85}$$

then, there exists a unique solution for (IVP) (77) and (78) on the interval $(-\infty, b]$.

Proof. We first transform the given initial value problem into a fixed point problem. For this, we consider an operator $N:\Omega \longrightarrow \Omega$ defined by

$$N(u)(t)=\begin{cases}\phi(t) & \text{if}, t\in(-\infty,0],\\ \dfrac{1}{\Gamma(\beta)}\displaystyle\int_1^0 (t-s)^{\beta-1}f(s,y_s) & \text{if}, t\in[0,b].\end{cases} \tag{86}$$

Let $\rho(\cdot)$: $(-\infty,b] \longrightarrow \mathbb{R}$ be a function defined by

$$\rho(t)=\begin{cases}\phi(t) & \text{if}, t\in(-\infty,0],\\ 0 & \text{if}, t\in(0,b).\end{cases} \tag{87}$$

Then, $\xi_0=\phi$. For each $\eta \in C([0,b],\mathbb{R})$ with $\eta(0)=0$, we denote by $\bar{\eta}$ the function defined by

$$\bar{\eta}(t)=\begin{cases}0 & \text{if}, t\in(-\infty,0],\\ \eta(t) & \text{if}, t\in(0,b).\end{cases} \tag{88}$$

If

$$u(t)=\frac{1}{\Gamma(\beta)}\int_0^t (t-s)^{\beta-1}f(s,u_s)ds, \tag{89}$$

for every $0\le t\le b$ and the function $\eta(\cdot)$ satisfies

$$\eta(t)=\frac{1}{\Gamma(\beta)}\int_0^t (t-s)^{\beta-1}f(s,\bar{\eta}_s+\rho_s)ds. \tag{90}$$

Set

$$C_0=\{\eta\in C([0,b],\mathbb{R}): \eta_0=0\}. \tag{91}$$

Now, let $f:C_0 \longrightarrow C_0$ be $\omega-\psi$ Hardy-Rogers-Suzuki operator be defined by

$$f\eta(t)=\frac{1}{\Gamma(\beta)}\int_0^t (t-s)^{\beta-1}f(s,\bar{\eta}_s+u_s). \tag{92}$$

The operator N has a fixed-point equivalent to f; hence, we have to prove that f has a fixed point. Indeed, if we consider that $\eta,\eta^*\in C_0$, then for all $t\in[0,b]$, we have

$$\begin{aligned}
&|f\eta(t)-f\eta^*(t)|+|f\eta(t)|\\
&\quad=\left|\frac{1}{\Gamma(\beta)}\int_0^t (t-s)^{\beta-1}f(s,\bar{\eta}_s+u_s)ds\right.\\
&\qquad\left.-\frac{1}{\Gamma(\beta)}\int_0^t (t-s)^{\beta-1}f\left(s,\bar{\eta^*}_s+u_s\right)ds\right|\\
&\qquad+\left|\frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}f(s,\bar{\eta}_s+u_s)ds\right|\\
&\quad<\frac{1}{\Gamma(\beta)}\int_0^t (t-s)^{\beta-1}\left|f(s,\bar{\eta}_s+u_s)-f\left(s,\bar{\eta^*}+\rho_s\right)\right|\\
&\qquad+\frac{1}{\Gamma(\beta)}\int_0^t (t-s)^{\beta-1}|f(s,\bar{\eta}_s+\rho_s)|\\
&\quad<\frac{1}{\Gamma(\beta)}\int_0^t (t-s)^{\beta-1}\left(\left|f(s,\bar{\rho}_s+\rho_s)-f\left(s,\bar{\eta^*}+\rho_s\right)\right|\right.\\
&\qquad\left.+|f(s,\bar{\eta}_s+u_s)|\right)ds\\
&\quad<\frac{1}{\Gamma(\beta)}\int_0^t (t-s)^{\beta-1}\left(q\left|\bar{\eta}_s-\bar{\eta^*}\right|+q|\bar{\eta}_s|\right)ds\\
&\quad<\frac{1}{\Gamma(\beta)}\int_0^t (t-s)^{\beta-1}qk_b\sup\left(|\eta(s)-\eta^*(s)|+|\eta(s)|\right)\\
&\quad<\frac{k_b}{\Gamma(\beta)}\int_0^t (t-s)^{\beta-1}qds|\eta-\eta^*|+|\eta|.
\end{aligned} \tag{93}$$

Therefore,

$$\begin{aligned}
|f(\eta)-f(\eta^*)|+|f(\eta)| &< \frac{qb^{\beta}k_b}{\Gamma(\beta+1)}|\eta-\eta^*|_b\\
&\quad+|\eta|qp_b(f(\eta),f(\eta^*))<\lambda k_1 qp_b(\eta,\eta^*).
\end{aligned} \tag{94}$$

Suppose $\psi\in\Psi$ and $\alpha,\beta,\gamma\ge0$ with $\alpha+\beta+\gamma<1$ such that

$$\frac{1}{2}qp_b(\eta,f\eta)<qp_b(\eta,\eta^*) \tag{95}$$

implies that

$$\begin{aligned}
\omega(\eta,\eta^*)qp_b(f\eta,f\eta^*)&<\psi\left([qp_b(\eta,\eta^*)]^{\beta}.[qp_b(\eta,f\eta)]^{\beta}[qp_b(\eta^*,f\eta^*)]^{\gamma}\right)\\
&\quad\cdot\left(\frac{1}{2}(qp_b(\eta,f\eta)^*)+qp_b(\eta^*,f\eta)\right).
\end{aligned} \tag{96}$$

Thus, we deduce that the operator f satisfy all the hypothesis of Theorem 28. Therefore, f has a unique fixed point. ☐

Data Availability

There is no data required in this research.

Conflicts of Interest

The authors declare that they have no competing interests.

Authors' Contributions

Both authors contributed equally and significantly in writing this article. Both authors read and approved the final manuscript.

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