Research Article

Unique Common Fixed Points for Occasionally Weakly Biased Maps of Type \((\mathcal{A})\) in \(b\)-Metric-Like Spaces

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We start this work by demonstrating the existence of unique common fixed points for two pairs of occasionally weakly biased maps of type \((\mathcal{A})\) in a \(b\)-metric-like space, and we end it by producing two illustrative examples in order to support and show that our results are meaningful.

1. Introduction

With no doubt, the theory of fixed point is an active domain in mathematics. Moreover, Banach’s theorem is considered a guarantee of the existence and uniqueness of fixed points. Several mathematicians made some changes to this distinguished result, in order to improve, extend, and generalize this important theorem. On the one hand, they increased the number of maps to investigate common fixed points. On the other hand, they concentrated on the complete metric space. For this end, many authors generalized the metric space to other spaces such as symmetric, fuzzy, generalized, partial, probabilistic, dislocated, Menger, and ultrametric spaces. Particularly in 1985, in his dissertation \([1]\), Matthews suggested the class of metric domains. According to him, those domains permit a normal distinction to be formed in the middle of complete and partial topics. Again, Matthews stated that metric domain has been introduced so as to boost the concept of completeness in domain theory, and he pointed out that there exists a bijection between the category of metric domains and the category of metric spaces. In 1992, in his paper \([2]\), the same author provided another generalization of metric spaces under the name of partial metric spaces in which he keeps the symmetry axiom. In 2012, in his paper \([3]\), Amini-Harandi initiated a novel generalization of partial metrics under the name of metric-like spaces. Then he gave some fixed-point theorems in these new spaces which generalize and ameliorate several recognized outcomes in both metric-like and partial metric spaces. In fact, the concepts of metric domains, metric-like spaces, and dislocated metric spaces are exactly the same, also named \(d\)-metric spaces. Now, according to \([4–6, 7]\), and others, in 1989, the \(b\)-metric’s definition was established by Bakhtin \([8]\) as a generalization of the metric, while \([9–12]\) and others wrote that the pioneer of the \(b\)-metric idea is Czerwik, who presented a few generalizations of the renowned Banach’s theorem in these modern spaces. The year 2014 witnessed the birth of the so-called partial \(b\)-metric spaces by Shukla \([13]\) as a generalization of partial metric and \(b\)-metric spaces. Let us return to 2013. This year, Alghamdi et al. \([14]\) suggested a recent generalization of metric-like, partial metric, and \(b\)-metric spaces named a \(b\)-metric-like space. Thereafter, they provided several results in these current spaces. According to them, their theorems
generalize and ameliorate many prominent results in both metric-like and partial metric spaces. Several authors proved the existence and uniqueness of fixed points in \( b \)-metric and \( b \)-metric-like spaces (see, for instance, [15–18] and [19]). In addition, fixed points can exist under several different conditions. For this end, in a few months we introduced a new type of compatibility called occasionally weakly biased maps of type \( (\mathcal{A}) \) for generalizing weakly compatible [20], weakly biased [21], occasionally weakly compatible [22], and weakly biased of type \( (\mathcal{A}) \) maps [23]. In this article, we will especially improve and expand the principal results of [9] and some similar results in metric, partial metric, metric-like, \( b \)-metric, partial \( b \)-metric, and \( b \)-metric-like spaces. Our motivation for working in \( b \)-metric-like spaces is that we can benefit from the condition \( d(x,x) \) may be positive for some \( x \). Again, since \( b \)-metric-like is a combination of metric-like spaces used in computer sciences (see [1, 25]) and \( b \)-metric spaces, working in this space allows us to benefit from practical applications and can provide us with more applicable fields. We do not deny that the work in \( b \)-metric spaces is useful; however, such spaces lose us benefit of the first condition.

2. Preliminary Notes

In this part, we only give the below concepts:

**Definition 1** ([1]). A metric domain is a pair \( \langle \mathcal{D}, \mathfrak{D} \rangle \) where \( \mathcal{D} \) is a nonempty set and \( \mathfrak{D} \) is a function from \( \mathcal{D} \times \mathcal{D} \) to \( \mathbb{R} \) such that

1. For all \( x, y \in \mathcal{D} \), \( \mathfrak{D}(x, y) = 0 \iff x = y \)
2. For all \( x, y \in \mathcal{D} \), \( \mathfrak{D}(x, y) = \mathfrak{D}(y, x) \)
3. For all \( x, y, z \in \mathcal{D} \), \( \mathfrak{D}(x, y) \leq \mathfrak{D}(x, z) + \mathfrak{D}(z, y) \).

**Definition 2** ([2]). A partial metric (\( p \)-metric) is a function \( p : \mathcal{A} \times \mathcal{A} \to \mathbb{R} \), such that

1. For all \( x, y \in \mathcal{A} \), \( x = y \implies p(x, x) = p(x, y) = p(y, y) \)
2. For all \( x, y \in \mathcal{A} \), \( p(x, x) \leq p(x, y) \)
3. For all \( x, y \in \mathcal{A} \), \( p(x, x) = p(y, y) \)
4. For all \( x, y \in \mathcal{A} \), \( p(x, z) \leq p(x, y) + p(y, z) - p(y, y) \).

**Definition 3** ([3]). A map \( \varsigma : X \times X \to \mathbb{R} \), where \( X \) is a nonempty set, is said to be metric-like on \( X \) if, for any \( l, m, n \in X \), the following three conditions hold true:

1. \( \varsigma(l, m) = 0 \iff l = m \)
2. \( \varsigma(l, m) = \varsigma(m, l) \)
3. \( \varsigma(l, m) \leq \varsigma(l, n) + \varsigma(n, m) \).

The pair \( (\mathcal{X}, \varsigma) \) is then called a metric-like space. Then a metric-like on \( \mathcal{X} \) satisfies all of the conditions of a metric except that \( \varsigma(l, l) \) may be positive for \( l \in \mathcal{X} \).

**Definition 4** ([26]). Let \( \mathcal{X} \) be a space, and let \( \mathbb{R}_+ \) denotes the set of all nonnegative numbers. A function \( D : \mathcal{X} \times \mathcal{X} \to \mathbb{R}_+ \) is said to be a \( b \)-metric if and only if for all \( x, y, z \in \mathcal{X} \) and all \( \mathcal{R} > 0 \), the following conditions are satisfied:

1. \( D(x, x) = 0 \) if and only if \( x = y \),
2. \( D(x, y) = D(y, x) \),
3. \( D(x, y) < D, D(x, y) < D \) imply \( D(y, y) < 2D \).

The pair \( (\mathcal{X}, D) \) is called a \( b \)-metric space.

**Lemma 5** ([26]). The condition (3) is equivalent to the following one:

\[
D(x, y) \leq 2D(x, z) + 2D(z, y).
\]

**Definition 6**. A \( b \)-metric on a nonempty set \( X \) is a function \( d' : X \times X \to [0, +\infty) \) such that for all \( f, g, h \in X \) and a constant \( \xi \geq 1 \), the following three conditions hold true:

1. \( d'(f, g) = 0 \iff f = g \)
2. \( d'(f, g) = d'(g, f) \)
3. \( d'(f, g) \leq \xi[d'(f, h) + d'(h, g)] \).

The pair \( (X, d') \) is called a \( b \)-metric space.

**Definition 7** ([13]). Given a nonempty set \( X \) and \( s \geq 1 \). If \( p_b : X \times X \to [0, +\infty) \) such that

1. \( (pb1)q = \delta \) if and only if \( p_b(q, q) = p_b(q, \delta) = p_b(\delta, \delta) \)
2. \( (pb2)p_b(q, q) \leq p_b(q, \delta) \)
3. \( (pb3)p_b(q, \delta) = p_b(\delta, q) \)
4. \( (pb4)p_b(q, \delta) \leq s[p_b(q, \zeta) + p_b(\zeta, \delta)] - p_b(\zeta, \zeta) \).

For all \( q, \delta, \zeta \in X \), then \( p_b \) is a partial \( b \)-metric with a coefficient \( s \geq 1 \).

**Definition 8** ([14]). A \( b \)-metric-like on a nonempty set \( X \) is a function \( \mathfrak{D} : X \times X \to [0, +\infty) \) such that for all \( r, s, t \in X \)
and a constant $\mathcal{K} \geq 1$ the following three conditions hold true:

1. $\mathcal{D}(r, s) = 0 \implies r = s$
2. $\mathcal{D}(r, s) = \mathcal{D}(s, r)$
3. $\mathcal{D}(r, s) \leq \mathcal{K}[\mathcal{D}(r, t) + \mathcal{D}(t, s)]$

The pair $(\mathcal{X}, \mathcal{D})$ is called a $b$-metric-like space.

**Remark 9.** According to Mirko et al. [27] and Fabiano et al. [28], we have

1. (R1): metric space $\rightarrow$ partial metric space $\rightarrow$ metric-like space $\rightarrow b$-metric-like space
2. (R2): metric space $\rightarrow b$-metric space $\rightarrow$ partial $b$-metric space $\rightarrow b$-metric-like space
3. (R3): partial metric space $\rightarrow$ partial $b$-metric space $\rightarrow b$-metric-like space.

**Definition 10.** Let $\mathcal{S}$ and $\mathcal{T}$ be maps from a nonempty set $\mathcal{X}$ into itself. $\mathcal{S}$ and $\mathcal{T}$ are occasionally weakly $\mathcal{S}$-biased of type $(\mathcal{A})$ and occasionally weakly $\mathcal{T}$-biased of type $(\mathcal{A})$, respectively, if, there is an element $i \in \mathcal{X}$ such that $\mathcal{S}i = \mathcal{T}i$ and

$$d(\mathcal{S}i, \mathcal{T}i) \leq d(\mathcal{S}i, \mathcal{S}i),$$
$$d(\mathcal{T}i, \mathcal{S}i) \leq d(\mathcal{T}i, \mathcal{T}i),$$

respectively.

### 3. The Head of the Paper

In this subdivision, we will prove the existence and uniqueness of common fixed points for two couples of occasionally weakly biased maps of type $(\mathcal{A})$, under a contractive condition, and in $b$-metric-like spaces. The two outcomes are supported by two examples.

**Theorem 11.** Suppose that $\mathcal{S}$, $\mathcal{R}$, $\mathcal{Y}$, and $\mathcal{X}$ are maps from a $b$-metric-like space $(\mathcal{X}, d)$ into itself satisfying

$$d(\mathcal{S}x, \mathcal{Y}y) \leq \frac{1}{k^4} \rho(d(\mathcal{Y}x, \mathcal{Y}y)) \max\left\{d(\mathcal{Y}x, \mathcal{Y}y), \frac{1}{2} d(\mathcal{S}x, \mathcal{Y}x), \frac{1}{2} d(\mathcal{Y}x, \mathcal{S}x), \frac{1}{2} d(\mathcal{Y}x, \mathcal{Y}x), \frac{1}{2} d(\mathcal{Y}x, \mathcal{Y}x), \right\},$$

for all $x, y \in \mathcal{X}$, where $k \geq 1$ is a real number and $\rho: [0, +\infty) \rightarrow [0, 1]$ is a nondecreasing function. If $\mathcal{S}$ and $\mathcal{Y}$ are occasionally weakly $\mathcal{Y}$-biased of type $(\mathcal{A})$ and $\mathcal{R}$ and $\mathcal{X}$ are occasionally weakly $\mathcal{X}$-biased of type $(\mathcal{A})$, then, there exists a unique point $p$ such that $\mathcal{S}p = \mathcal{R}p = \mathcal{Y}p = \mathcal{X}p = p$.

**Proof.** Since pairs of maps $(\mathcal{S}, \mathcal{Y})$ and $(\mathcal{R}, \mathcal{X})$ are occasionally weakly $\mathcal{Y}$-biased and $\mathcal{X}$-biased of type $(\mathcal{A})$, then, there exist two elements $s$ and $t$ such that

$$d(\mathcal{S}s, \mathcal{Y}s) \leq d(\mathcal{S}s, \mathcal{Y}s),$$
$$d(\mathcal{R}t, \mathcal{X}t) \leq d(\mathcal{R}t, \mathcal{X}t),$$

to achieve our goals, we will use four steps.

First step: we claim that $d(\mathcal{S}s, \mathcal{R}t) > 1$, using condition (7) we obtain

$$d(\mathcal{S}s, \mathcal{R}t) \leq \frac{1}{k^4} \rho(d(\mathcal{Y}s, \mathcal{X}t)) \max\left\{d(\mathcal{Y}s, \mathcal{X}t), \frac{1}{2} d(\mathcal{S}s, \mathcal{Y}s), \frac{1}{2} d(\mathcal{R}t, \mathcal{X}t), \frac{1}{2} d(\mathcal{S}s, \mathcal{Y}s), \right\},$$

respectively.

Then, $\mathcal{S}s = \mathcal{R}t$.

Second step: assume that $d(\mathcal{S}s, \mathcal{R}t) > 0$, then, inequality (7) yields

$$d(\mathcal{S}s, \mathcal{S}s) = d(\mathcal{S}s, \mathcal{R}t) \leq \frac{1}{k^4} \rho(d(\mathcal{Y}s, \mathcal{X}t)) \max\left\{d(\mathcal{Y}s, \mathcal{X}t), \frac{1}{2} d(\mathcal{S}s, \mathcal{Y}s), \frac{1}{2} d(\mathcal{R}t, \mathcal{X}t), \frac{1}{2} d(\mathcal{S}s, \mathcal{Y}s), \right\},$$

for all $x, y \in \mathcal{X}$, where $k \geq 1$ is a real number and $\rho: [0, +\infty) \rightarrow [0, 1]$ is a nondecreasing function. If $\mathcal{S}$ and $\mathcal{Y}$ are occasionally weakly $\mathcal{Y}$-biased of type $(\mathcal{A})$ and $\mathcal{R}$ and $\mathcal{X}$ are occasionally weakly $\mathcal{X}$-biased of type $(\mathcal{A})$, then, there exists a unique point $p$ such that $\mathcal{S}p = \mathcal{R}p = \mathcal{Y}p = \mathcal{X}p = p$. 
Since pair of maps \((\mathcal{O}, \mathcal{Y})\) is occasionally weakly \(\mathcal{Y}\)-biased of type \((\mathcal{A})\), one acquire

\[
d(\mathcal{O}s, \mathcal{Q}s) \leq \frac{1}{\kappa^3} \varphi(d(\mathcal{O}Ys, \mathcal{Y}s)) \max \left\{ d(\mathcal{O}Ys, \mathcal{Y}s), \right. \\
\frac{1}{2} \kappa(d(\mathcal{O}s, \mathcal{Q}s) + d(\mathcal{Y}s, \mathcal{O}Ys)), \kappa d(\mathcal{O}s, \mathcal{Q}s), \\
\left. d(\mathcal{O}Ys, \mathcal{Y}s), d(\mathcal{O}s, \mathcal{Q}s) \right\} \\
= \frac{1}{\kappa^3} \varphi(d(\mathcal{O}s, \mathcal{Q}s)) \max \left\{ d(\mathcal{O}s, \mathcal{Q}s), \kappa d(\mathcal{O}s, \mathcal{Q}s), \\
\kappa d(\mathcal{O}s, \mathcal{Q}s), d(\mathcal{O}s, \mathcal{Q}s), d(\mathcal{O}s, \mathcal{Q}s) \right\} \\
= \frac{1}{\kappa^3} \varphi(d(\mathcal{O}s, \mathcal{Q}s)) d(\mathcal{O}s, \mathcal{Q}s) < d(\mathcal{O}s, \mathcal{Q}s),
\]

(11)
a contradiction; hence, \(\mathcal{Q}s = \mathcal{Q}s\), consequently, \(\mathcal{Y}s = \mathcal{Q}s\).

Third step: we claim that \(\mathcal{R}\mathcal{R}t = \mathcal{R}t\). If not, then, the utilisation of inequality (7) offers

\[
d(\mathcal{Q}s, \mathcal{R}t) \leq \frac{1}{\kappa^3} \varphi(d(\mathcal{Y}s, \mathcal{L}t)) \max \left\{ d(\mathcal{Y}s, \mathcal{L}t), \frac{1}{2} d(\mathcal{Q}s, \mathcal{Y}s), \\
\frac{1}{2} d(\mathcal{R}t, \mathcal{R}t), d(\mathcal{Y}s, \mathcal{R}t), d(\mathcal{Q}s, \mathcal{L}t) \right\},
\]

(12)
i.e.,

\[
d(\mathcal{R}t, \mathcal{R}t) \leq \frac{1}{\kappa^3} \varphi(d(\mathcal{R}t, \mathcal{L}t)) \max \left\{ d(\mathcal{R}t, \mathcal{L}t), \frac{1}{2} d(\mathcal{R}t, \mathcal{R}t), \\
\frac{1}{2} d(\mathcal{R}t, \mathcal{L}t), d(\mathcal{R}t, \mathcal{R}t), d(\mathcal{R}t, \mathcal{L}t) \right\} \\
\leq \frac{1}{\kappa^3} \varphi(d(\mathcal{R}t, \mathcal{L}t)) \max \left\{ d(\mathcal{R}t, \mathcal{L}t), \\
\frac{1}{2} \kappa d(\mathcal{R}t, \mathcal{R}t) + d(\mathcal{R}t, \mathcal{L}t), \\
\frac{1}{2} \kappa d(\mathcal{R}t, \mathcal{R}t) + d(\mathcal{R}t, \mathcal{L}t), \\
d(\mathcal{R}t, \mathcal{L}t), d(\mathcal{R}t, \mathcal{L}t) \right\}.
\]

(13)

This contradiction confirms that \(\mathcal{R}Rt = \mathcal{R}t\) and consequently \(\mathcal{L}t = \mathcal{R}t\).

Fourth and last step: taking \(\mathcal{Q}s = \mathcal{Y}s = \mathcal{R}t = \mathcal{L}t = p\), and let \(p^\ast\) be a second distinct fixed point, then, by condition (7) one find

\[
d(\mathcal{Q}p, \mathcal{R}p) \leq \frac{1}{\kappa^3} \varphi(d(\mathcal{Y}p, \mathcal{L}p^\ast)) \max \left\{ d(\mathcal{Y}p, \mathcal{L}p^\ast), \\
\frac{1}{2} d(\mathcal{Q}p, \mathcal{Y}p), \frac{1}{2} d(\mathcal{R}p^\ast, \mathcal{L}p^\ast), d(\mathcal{Y}p, \mathcal{R}p), \\
d(\mathcal{Q}p, \mathcal{L}p^\ast) \right\},
\]

(15)
i.e.,

\[
d(p, p^\ast) \leq \frac{1}{\kappa^3} \varphi(d(p, p^\ast)) \max \left\{ d(p, p^\ast), \frac{1}{2} d(p, p^\ast), \\
\frac{1}{2} d(p^\ast, p^\ast), d(p, p^\ast), d(p, p^\ast) \right\} \\
\leq \frac{1}{\kappa^3} \varphi(d(p, p^\ast)) \max \left\{ d(p, p^\ast), \frac{1}{2} \kappa d(p, p^\ast) + d(p^\ast, p^\ast), \\
\frac{1}{2} \kappa d(p^\ast, p) + d(p, p^\ast), d(p, p^\ast), d(p, p^\ast) \right\} \\
= \frac{1}{\kappa^3} \varphi(d(p, p^\ast)) d(p, p^\ast) < d(p, p^\ast).
\]

(16)

Therefore, \(p^\ast = p\), so, all objectives are reached.

Below, one furnish an example to boost the above theorem

**Example 1.** Equip \(\mathcal{X} = (-\pi/2, \pi/2)\) with the \(b\)-metric-like
\(d(x, y) = (\max \{ |x|, |y| \})^2\), where \(\kappa = 2\). Taking \(\varphi = 3/4|\sin (t)|\) and define
\( \Omega(x) = \begin{cases} \frac{x + \pi}{200} & \text{if } x \in \left( -\frac{\pi}{2}, 0 \right), \\ 0 & \text{if } x \in \left[ 0, \frac{\pi}{2} \right), \\ \frac{\pi}{3} & \text{if } x \in \left( -\frac{\pi}{3}, 0 \right). \\ \end{cases} \)

\( \mathcal{R}(x) = \begin{cases} \frac{x + \pi}{300} & \text{if } x \in \left( -\frac{\pi}{2}, 0 \right), \\ 0 & \text{if } x \in \left[ 0, \frac{\pi}{2} \right), \\ \frac{\pi}{4} & \text{if } x \in \left( -\frac{\pi}{4}, 0 \right). \\ \end{cases} \)

\( \mathcal{Y}(x) = \begin{cases} \frac{\pi}{4} & \text{if } x \in \left( -\frac{\pi}{2}, 0 \right), \\ x & \text{if } x \in \left[ 0, \frac{\pi}{2} \right), \\ \frac{\pi}{3} & \text{if } x \in \left( -\frac{\pi}{3}, 0 \right). \\ \end{cases} \)

\( \mathcal{I}(x) = \begin{cases} \frac{\pi}{3} & \text{if } x \in \left( -\frac{\pi}{3}, 0 \right), \\ x & \text{if } x \in \left[ 0, \frac{\pi}{2} \right). \\ \end{cases} \)

Before starting, note that \( \omega \) and \( \mathcal{Y} \) as well as \( \mathcal{R} \) and \( \mathcal{I} \) are occasionally weakly \( \mathcal{Y} \)-biased and \( \mathcal{I} \)-biased of type \( \omega \). We have

(1) For \( x, y \in (-\pi/2, 0) \), \( \Omega(x) = -(x + \pi)/200 \), \( \Omega(y) = -(y + \pi)/300 \), \( \mathcal{R}(x) = -\pi/4 \), \( \mathcal{R}(y) = -\pi/3 \), and

\[
\begin{align*}
    d(\Omega(x), \Omega(y)) &= \frac{1}{\kappa^2} \rho(d(\mathcal{Y}(x), \mathcal{Y}(y))) \max \left\{ d(\mathcal{Y}(x), \mathcal{Y}(y)), \frac{1}{2} d(\Omega(x), \mathcal{Y}(y)), \frac{1}{2} d(\Omega(x), \mathcal{R}(y)), d(\Omega(x), \mathcal{I}(y)) \right\}.
\end{align*}
\]

(2) For \( x, y \in [0, \pi/2) \), \( \Omega(x) = 0 \), \( \Omega(y) = 0 \), \( \mathcal{R}(x) = x \), \( \mathcal{R}(y) = y \), and

\[
\begin{align*}
    d(\Omega(x), \Omega(y)) &= \frac{1}{\kappa^2} \rho(d(\mathcal{Y}(x), \mathcal{Y}(y))) \max \left\{ d(\mathcal{Y}(x), \mathcal{Y}(y)), \frac{1}{2} d(\Omega(x), \mathcal{Y}(y)), \frac{1}{2} d(\Omega(x), \mathcal{R}(y)), d(\Omega(x), \mathcal{I}(y)) \right\}.
\end{align*}
\]

(3) For \( x \in (-\pi/2, 0) \) and \( y \in [0, \pi/2) \), \( \Omega(x) = -(x + \pi)/200 \), \( \Omega(y) = 0 \), \( \mathcal{R}(x) = -\pi/4 \), \( \mathcal{R}(y) = y \), and

\[
\begin{align*}
    d(\Omega(x), \Omega(y)) &= \frac{1}{\kappa^2} \rho(d(\mathcal{Y}(x), \mathcal{Y}(y))) \max \left\{ d(\mathcal{Y}(x), \mathcal{Y}(y)), \frac{1}{2} d(\Omega(x), \mathcal{Y}(y)), \frac{1}{2} d(\Omega(x), \mathcal{R}(y)), d(\Omega(x), \mathcal{I}(y)) \right\}.
\end{align*}
\]

Thereby, all hypotheses of Theorem 11 are fulfilled; the four maps admit 0 as the only fixed point.

**Remark 12.** Note that \( \mathcal{X} = (-\pi/200, -\pi/400) \cup \{0\} \not\subseteq \mathcal{I} \mathcal{X} = [0, \pi/2) \cup \{-\pi/3\} \),

\[
\mathcal{R} \mathcal{X} = \left( -\frac{\pi}{300}, -\frac{\pi}{600} \right) \cup \{0\} \not\subseteq \mathcal{Y} \mathcal{X} = \left[ 0, \frac{\pi}{2} \right) \cup \left\{ -\frac{\pi}{4} \right\}.
\]

(22)

**Remark 13.** Mention that Theorem 11 remains valid if we replace inequality (7) by

\[
\begin{align*}
    d(\Omega(x), \Omega(y)) &\leq \frac{1}{\kappa^2} \rho(d(\mathcal{Y}(x), \mathcal{Y}(y))) \max \left\{ d(\mathcal{Y}(x), \mathcal{Y}(y)), d(\Omega(x), \mathcal{Y}(y)), \frac{1}{2} d(\Omega(x), \mathcal{R}(y)), d(\Omega(x), \mathcal{I}(y)) \right\}.
\end{align*}
\]

(23)
with \( \varphi : [0, +\infty) \rightarrow [0, 1/2) \) is a nondecreasing function.

\[
d\left(d(x, y) \leq \frac{1}{k^1} [\varphi(d(y, x))] \cdot d(x, y) + \frac{1}{4} (d(2x, y) + d(2y, x)), d(y, 2x) + d(2y, x),] \right)
\]

with \( \varphi : [0, +\infty) \rightarrow [0, 1) \) is a nondecreasing function.

\[
d\left(d(x, y) \leq \frac{1}{k^1} [\varphi(d(y, x))] \cdot d(x, y) + \frac{1}{2} (d(2x, y) + d(2y, x)), d(y, 2x) + d(2y, x),] \right)
\]

with \( \varphi : [0, +\infty) \rightarrow [0, 1/2) \) is a nondecreasing function.

In the next, we are available to present the second theorem.

**Theorem 14.** Let \((F, H)\) and \((G, K)\) be occasionally weakly \(H\)-biased and \(K\)-biased of type \(\alpha\), respectively, from a \(d\)-metric-like space \((X, d)\) into itself satisfying

\[
d(Fx, Gy) \leq \frac{1}{k^2} [\varphi(d(Hx, Ky))] \cdot d(Hx, Ky) + \rho(d(Hx, Ky)) \cdot d(Fx, Hx) + \varphi(d(Hx, Ky)) \cdot d(Gy, Ky) + \tau(d(Hx, Ky)) \cdot d(Hy, Ky) + \varphi(d(Hx, Ky)) \cdot d(Fx, Ky).
\]

For all \(x, y \in X\), where \(\kappa \geq 1\) is a real number and \(\alpha, \rho, \omega, \varphi, \tau, \nu : [0, +\infty) \rightarrow [0, 1]\) are nondecreasing functions such that \(\varphi(t) + 2\xi(\rho(t) + \varphi(t)) + \tau(t) + t + \mu(t) < 1\); therefore, the four maps admit only one fixed point.

**Proof.** By the same manner, since \(F\) and \(H\) and \(G\) and \(K\) are occasionally weakly \(H\)-biased and \(K\)-biased of type \(\alpha\), there exist \(m\) and \(n\) such that

\[
Fm = Hm \text{ implies } d(HHm, Fm) \leq d(HHm, Hm),
\]

\[
Gn = Kn \text{ implies } d(KKn, Gn) \leq d(GKn, Kn).
\]

Suppose that \(Fm \neq Gn\), then, using inequality (26), we get

\[
d(Fm, Gn) \leq \frac{1}{k^4} \left(\varphi(d(Hm, Kn)) \cdot d(Hm, Kn) + \rho(d(Hm, Kn)) \cdot d(Fm, Hm) + \varphi(d(Hm, Kn)) \cdot d(Gn, Kn) + \tau(d(Hm, Kn)) \cdot d(Hm, Gn) + \varphi(d(Hm, Kn)) \cdot d(Fm, Kn)\right)
\]

\[
= \frac{1}{k^4} \left[\varphi(d(Fm, Gn)) \cdot d(Fm, Gn) + \rho(d(Fm, Gn)) \cdot d(Fm, Fm) + \varphi(d(Fm, Gn)) \cdot d(Gn, Gn) + \tau(d(Fm, Gn)) \cdot d(Fm, Gn) + \varphi(d(Fm, Gn)) \cdot d(Fm, Gn)\right]
\]

\[
\leq \frac{1}{k^4} \left[\varphi(d(Fm, Gn)) + 2\xi(\rho(d(Fm, Gn)) + \varphi(d(Fm, Gn)) + \tau(d(Fm, Gn)) \cdot d(Fm, Gn) + \varphi(d(Fm, Gn)) \cdot d(Fm, Gn)\right]
\]

\[
= \frac{1}{k^4} \left[\varphi(d(Fm, Gn)) + 2\xi(\rho(d(Fm, Gn)) + \varphi(d(Fm, Gn)) + \tau(d(Fm, Gn)) \cdot d(Fm, Gn) + \varphi(d(Fm, Gn)) \cdot d(Fm, Gn)\right]
\]
By the occasional weakly biased assumption, we obtain

\[
d(Fm, Fm) \leq \frac{1}{k^4} \left[ \omega(d(Fm, Hm))d(Fm, Hm) + \kappa \rho(d(Hm, Hm))d(Fm, Fm) + d(Hm, Hm) + 2\kappa \omega(d(Hm, Hm)) \cdot d(Fm, Fm) + 2\kappa d(Hm, Hm) \cdot \tau(d(Hm, Hm)) \cdot d(Fm, Fm) + \nu(d(Hm, Hm))d(Fm, Fm) \right]d(Fm, Fm) \leq d(Fm, Fm),
\]

which is a contradiction; hence, \( FFm = Fm \), consequently, \( HFm = Fm \).

Next, assume that \( GGn \neq Gn \), then, we have

\[
d(Fm, GGn) \leq \frac{1}{k^4} \left[ \omega(d(Hm, KGn))d(Hm, KGn) + \rho(d(Hm, KGn))d(Fm, Hm) + \tau(d(Hm, KGn))d(Hm, GGn) \right]d(Fm, GGn) \leq \frac{1}{k^4} \left[ \omega(d(Hm, KGn))d(Hm, KGn) + \rho(d(Hm, KGn))d(Hm, KGn) + \tau(d(Hm, KGn))d(Hm, KGn) \right]d(Fm, GGn),
\]

i.e.,

\[
d(Gn, GGn) \leq \frac{1}{k^4} \left[ \omega(d(Gn, KGn))d(Gn, KGn) + \kappa \rho(d(Gn, KGn))d(Gn, Gn) + d(Gn, GGn) + \tau(d(Gn, KGn))d(Gn, GGn) \cdot d(Gn, GGn) + \nu(d(Gn, KGn))d(Gn, GGn) \right]d(Gn, KGn).
\]

A contradiction, so \( \zeta = \xi \)

The next example supports and illustrates the above result.

**Example 2.** Endow \( X = (-\pi/2, \pi/2) \) with the \( b \)-metric-like \( d(x, y) = (|x| + |y|)^2 \), where \( \kappa = 2 \). Define

\[
F_{x} = \begin{cases} 
0 & \text{if } x \in \left( -\frac{\pi}{2}, 0 \right], \\
\frac{x - \pi}{100} & \text{if } x \in \left( 0, \frac{\pi}{2} \right).
\end{cases}
\]

\[
G_{x} = \begin{cases} 
0 & \text{if } x \in \left( -\frac{\pi}{2}, 0 \right], \\
\frac{x - \pi}{200} & \text{if } x \in \left( 0, \frac{\pi}{2} \right).
\end{cases}
\]

\[
H_{x} = \begin{cases} 
-\pi & \text{if } x \in \left( -\frac{\pi}{2}, 0 \right], \\
\frac{-\pi}{3} & \text{if } x \in \left( 0, \frac{\pi}{2} \right),
\end{cases}
\]

\[
K_{x} = \begin{cases} 
-\pi & \text{if } x \in \left( -\frac{\pi}{2}, 0 \right], \\
\frac{-\pi}{6} & \text{if } x \in \left( 0, \frac{\pi}{2} \right),
\end{cases}
\]

taking \( \omega = 3/4 \), \( \rho = \kappa = \tau = \nu = 1/41\sin(t) \), we get

\[
\frac{1}{k^4} \left[ \omega(d(Gn, GGn)) + 2\kappa \rho(d(Gn, GGn)) + \tau(d(Gn, GGn)) + \nu(d(Gn, GGn)) \right]d(Gn, GGn) \leq d(Gn, GGn).
\]
(1) For \( x, y \in (-\pi/2, 0] \), \( Fx = 0 \), \( Gy = 0 \), \( Hx = -x \), \( Ky = -y \), and

\[
d(Fx, Gy) = 0 \leq \frac{1}{16} \left[ \frac{\pi}{4} (-x - y)^2 \right] + \frac{2}{41} \left| \sin \left( -x - y \right) \right| (x^2 + y^2)
\]

\[
= \frac{1}{\kappa^2} \left[ \omega(d(Hx, Ky))d(Hx, Ky) \right.
+ \rho(d(Hx, Ky))d(Fx, Hx)
+ \psi(d(Hx, Ky))d(Gy, Ky)
+ \tau(d(Hx, Ky))d(Hx, Gy)
+ \nu(d(Hx, Ky))d(Fx, Ky)]
\]

\[
(37)
\]

(2) For \( x, y \in (0, \pi/2) \), \( Fx = (x - \pi)/100 \), \( Gy = (y - \pi)/200 \), \( Hx = -\pi/3 \), \( Ky = -\pi/6 \), and

\[
d(Fx, Gy) = \left( \frac{x - \pi}{100} + \frac{y - \pi}{200} \right)^2
\]

\[
\leq \frac{1}{16} \left[ \frac{\pi}{4} \left( \frac{x^2}{2} + \frac{y^2}{2} \right) \right] + \frac{1}{41} \left[ \sin \left( \frac{\pi}{2} \right) \right]^2
\]

\[
= \frac{1}{\kappa^2} \left[ \omega(d(Hx, Ky))d(Hx, Ky) \right.
+ \rho(d(Hx, Ky))d(Fx, Hx)
+ \psi(d(Hx, Ky))d(Gy, Ky)
+ \tau(d(Hx, Ky))d(Hx, Gy)
+ \nu(d(Hx, Ky))d(Fx, Ky)]
\]

\[
(38)
\]

(3) For \( x \in (-\pi/2, 0] \), \( y \in (0, \pi/2) \), \( Fx = 0 \), \( Gy = y - \pi/100 \), \( Hx = -x \), \( Ky = -\pi/6 \) and

\[
d(Fx, Gy) = \left( \frac{y - \pi}{200} \right)^2 \leq \frac{1}{16} \left[ \frac{3 \pi}{4} \left( \frac{y - x}{2} \right)^2 \right] + \frac{1}{41} \left[ \sin \left( \frac{\pi}{6} - x \right) \right]^2
\]

\[
= \frac{1}{\kappa^2} \left[ \omega(d(Hx, Ky))d(Hx, Ky) \right.
+ \rho(d(Hx, Ky))d(Fx, Hx)
+ \psi(d(Hx, Ky))d(Gy, Ky)
+ \tau(d(Hx, Ky))d(Hx, Gy)
+ \nu(d(Hx, Ky))d(Fx, Ky)]
\]

\[
(39)
\]

(4) For \( x \in (0, \pi/2) \), \( y \in (-\pi/2, 0] \), \( Fx = x - \pi/100 \), \( Gy = 0 \), \( Hx = -\pi/3 \), \( Ky = -y \), and

\[
d(Fx, Gy) = \left( \frac{x - \pi}{100} \right)^2 \leq \frac{1}{16} \left[ \frac{3 \pi}{4} \left( \frac{\pi}{3} - y \right)^2 \right] + \frac{1}{41} \left[ \sin \left( \frac{\pi}{3} - y \right) \right]^2
\]

\[
= \frac{1}{\kappa^2} \left[ \omega(d(Hx, Ky))d(Hx, Ky) \right.
+ \rho(d(Hx, Ky))d(Fx, Hx)
+ \psi(d(Hx, Ky))d(Gy, Ky)
+ \tau(d(Hx, Ky))d(Hx, Gy)
+ \nu(d(Hx, Ky))d(Fx, Ky)]
\]

\[
(40)
\]

In addition, mention that the occasional weakly biased hypothesis is satisfied, consequently, all conditions of Theorem 14 are achieved and 0 is the unique common fixed point of the four maps.

Note that

\[
FX = \left( -\frac{\pi}{100}, \frac{\pi}{200} \right) \cup \{0\} \notin \left[ 0, \frac{\pi}{2} \right] \cup \left( -\frac{\pi}{3} \right) = KX,
\]

\[
GX = \left( -\frac{\pi}{200}, -\frac{\pi}{400} \right) \cup \{0\} \notin \left[ 0, \frac{\pi}{2} \right] \cup \left( -\frac{\pi}{6} \right) = HX.
\]

\[
(41)
\]

4. Conclusion

As we know, the concept of occasionally weakly biased maps has an advantage over the concepts of weak compatibility and occasional weakly compatible. One conclude this work by mentioning that our presented outcomes expand and ameliorate several existing results in the fixed point domain, among them, for a few, the main results of [9, 27]. Again, we know that metric-like spaces are b-metric-like spaces, and b-metric spaces yield partial b-metric spaces yield b-metric-like spaces. Consequently, our theorems improve and extend many similar results on the four mentioned spaces.

Data Availability

No real data were used to support this study. The data used in this study are hypothetical.

Disclosure

This work was conducted during our work at Hodeidah University.
Conflicts of Interest
No conflicts of interest are related to this work.

References


