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Research Article

Approximate Solutions of Multidimensional Wave Problems Using an Effective Approach

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The main goal of this paper is to introduce a new scheme for the approximate solution of 1D, 2D, and 3D wave equations. The recurrence relation is very important to deal with the approximate solution of differential problems. We construct a scheme with the help of the Laplace-Carson integral transform ($\mathbb{L}_c$IT) and the homotopy perturbation method (HPM), called Laplace-Carson homotopy integral transform method ($\mathbb{L}_c$HITM). $\mathbb{L}_c$IT produces the recurrence relation and destructs the restriction of variables whereas HPM gives the successive iteration of the relation using the initial conditions. The convergence analysis is provided to study the wave equation with multiple dimensions. Some numerical examples are considered to show the efficiency of this scheme. Graphical representation and plot distribution between the approximate and the exact solution predict the high rate of convergence of this approach.

1. Introduction

Numerous physical phenomena in real world are modeled using partial differential equations (PDEs) in a variety of applied science fields including fluid dynamics, mathematical biology, quantum physics, chemical kinetics, and linear optics [1–3]. There are various perturbation approaches that can be used to analytically solve the PDEs. Although the calculations for these strategies are pretty straightforward, their limitations are predicated on the assumption of small parameters. As a result, many researchers are searching for novel methods to get around these restrictions. Various researchers and scientists have studied multiple novel methods for getting the analytical solution that are reasonably close to the precise solutions such as homotopy analysis method [4], modified extended tanh method [5], new Kudryashov's method [6], Chun-Hui He's iteration method [7], subequation method [8], exp-function method [9], modified exponential rational method [10], homotopy asymptotic method [11], modified extended tanh expansion [12], fractal variational iteration transform method [13], Laplace homotopy perturbation transform method [14], residual power series (RPS) method [15], and Adomian decomposition method [16]. In the past, many experts and researchers established the application of the homotopy perturbation method (HPM) [17, 18] in various physical problems, because this approach consistently transforms the challenging issue into a straightforward resolution. The method yields a physical problem, because this approach consistently transforms the challenging issue into a straightforward resolution. The method yields a very rapid convergence of the solution perturbation theory and showed the ability to be a very strong mathematical tool.

The wave equation, which describes the wave propagation phenomenon, is a partial differential equation for a scalar function. It is influenced by time and one or more spatial factors. The wave equations perform an important role in different area of engineering, physics, and scientific applications. Wazwaz [19] studied linear and nonlinear problems in bounded and unbounded domains using the variational iteration method. Ghasemi et al. [20] employed the homotopy perturbation method to derive the numerical solution of two-dimensional nonlinear differential equation. Keskin and Oturanc [21] applied reduced differential transform method to various wave equations. Ullah et al. [22] proposed optimal homotopy asymptotic method to obtain the analytic series solution of wave equations. Adwan et al. [23] presented the numerical solution of multidimensional wave equations and showed the accuracy of proposed techniques. Jleli et al. [24] studied the framework of the homotopy perturbation transform method for analytic treatment of wave equations. Mullen and Belytschko [25] provided the finite element scheme for the examination of two-dimensional wave equation and considered some semidiscretizations. These schemes have many limitations and assumptions in finding the approximate solution of the problems. To overcome these limitations and restriction of variable, we introduce a new iterative strategy for the approximate solution of multidimensional wave problem.

The variational iteration method (VIM), Laplace transform, and homotopy analysis method (HAM) have some limitations such as VIM involves the integration and produces the constant of integration, Laplace transform involves the convolution theorem, and HAM also considered some assumptions. The Laplace-Carson integral transform is very easy to implement to the differential problems. The purpose of this paper is to apply $\mathbb{L}_c$HITM with the combination of the Laplace-Carson integral transform and the HPM for wave problems of different dimensions. Less computations, fast convergence, and significant results make this scheme unique and different than other approaches of literature. This strategy derives the series of solution with fast convergence and yields the approximate solution very close to the precise solution. This approach is more useful and reliable for the solution of these problems. This paper is introduced as follows: in Section 2, we give a brief detail of the Laplace-Carson integral transform. In Section 3, we present the formulation of $\mathbb{L}_c$HITM for solving multidimensional problems. We provide the convergence analysis in Section 4. Some numerical applications are demonstrated to show the effectiveness in Section 5, and eventually, we discuss the conclusion in Section 6.

2. Preliminary Definitions of $\mathbb{L}_c$IT

In this section, we describe a few fundamental characteristics and concepts of $\mathbb{L}_c$IT that are very helpful in the formulation of this scheme.

Definition 1. Let $\vartheta(\phi)$ be a function precise for $\sigma \geq 0$; then,

$$\mathscr{L}\{\vartheta(\phi)\} = F(s) = \int_0^\infty \vartheta(\phi) e^{-\sigma\phi} d\phi \tag{1}$$

is called the Laplace transform.

Definition 2. The $\mathbb{L}_c$IT of a function $\vartheta(\phi)$ is defined as [26]

$$\mathbb{L}_c[\vartheta(\phi)] = R(\sigma) = \sigma \int_0^\infty \vartheta(\phi) e^{-\sigma\phi} d\phi, \phi \geq 0, k_1 \leq \sigma \leq k_2, \tag{2}$$

where $\mathbb{L}_c$ represents the symbol of $\mathbb{L}_c$IT, k_1 and k_2 are constants, and σ is the independent variable of the transformed function ϕ. Conversely, since $R(\sigma)$ is the $\mathbb{L}_c$IT of function $\vartheta(\phi)$, then

$$\mathbb{L}_c^{-1}[R(\sigma)] = \vartheta(\phi). \tag{3}$$

$\mathbb{L}_c^{-1}$ is called inverse $\mathbb{L}_c$IT.

Proposition 3. *Let* $\mathbb{L}_c\{\vartheta_1(\phi)\} = R_1(\sigma)$ *and* $\mathbb{L}_c\{\vartheta_2(\phi)\} = R_2(\sigma)$*; then [27]*

$$\begin{aligned} &\mathbb{L}_c\{au_1(\phi) + bu_2(\phi)\} = aS\{\vartheta_1(\phi)\} + bS\{\vartheta_2(\phi)\}, \\ &\Rightarrow \mathbb{L}_c\{au_1(\phi) + bu_2(\phi)\} = aR_1(\sigma) + bR_2(\sigma). \end{aligned} \tag{4}$$

Proposition 4. *If* $\mathbb{A}\{\vartheta(\phi)\} = R(\sigma)$*, the differential properties are defined as [26, 27]*

$$\begin{aligned} &\mathbb{L}_c\left\{\vartheta'(\phi)\right\} = \sigma R(\sigma) - \sigma\vartheta(0), \\ &\mathbb{L}_c\left\{\vartheta''(\phi)\right\} = \sigma^2 R(\sigma) - \sigma^2\vartheta(0) - \sigma\vartheta'(0), \\ &\mathbb{L}_c\{\vartheta^m(\phi)\} = \sigma^m R(\sigma) - \sigma^m\vartheta(0) - \sigma^{m-1}\vartheta'(0) - \cdots - \sigma\vartheta^{m-1}(0). \end{aligned} \tag{5}$$

3. Formulation of $\mathbb{L}_c$HITM

In this segment, we formulate the strategy of $\mathbb{L}_c$HITM for finding the approximate solutions of 1D, 2D, and 3D wave equation flows. We observe that this strategy is independent of integration and any hypothesis during the formulation of this scheme. We consider a differential problem such that

$$\vartheta''(\varsigma, \phi) = \vartheta(\varsigma, \phi) + g(\vartheta) + g(\varsigma, \phi), \tag{6}$$

with initial condition

$$\begin{aligned} \vartheta(\varsigma, 0) &= a_1, \\ \vartheta_\phi(\varsigma, 0) &= a_2, \end{aligned} \tag{7}$$

where ϑ denotes the function in region of time ϕ, $g(\vartheta)$ is considered as a nonlinear term, and $g(\varsigma, \phi)$ is source term arbitrary constant a. Employing $\mathbb{L}_c$IT on Equation (6), it yields

$$\mathbb{L}_c\left[\vartheta''(\varsigma, \phi)\right] = \mathbb{L}_c[\vartheta(\varsigma, \phi) + g(\vartheta) + g(\varsigma, \phi)]. \tag{8}$$

Using proposition (5) of $\mathbb{L}_c$IT, we obtain

$$\sigma^2 R(\sigma) - \sigma^2 \vartheta(\varsigma, 0) - \sigma \vartheta'(\varsigma, 0) = \mathbb{L}_c[\vartheta(\varsigma, \phi) + g(\vartheta) + g(\varsigma, \phi)]. \tag{9}$$

Hence, $R(\sigma)$ is evaluated such as

$$R[\sigma] = \vartheta(\varsigma, 0) + \frac{\vartheta'(\varsigma, 0)}{\sigma} + \frac{1}{\sigma^2}\mathbb{L}_c[\vartheta(\varsigma, \phi) + g(\vartheta) + g(\varsigma, \phi)]. \tag{10}$$

Operating inverse $\mathbb{L}_c$IT on Equation (10), we get

$$\vartheta(\varsigma, \phi) = \vartheta(\varsigma, 0) + \phi\vartheta'(\varsigma, 0) + \mathbb{L}_c^{-1}\left[\frac{1}{\sigma^2}\mathbb{L}_c\{\vartheta(\varsigma, \phi) + g(\vartheta) + g(\varsigma, \phi)\}\right]. \tag{11}$$

Using initial conditions, we get

$$\vartheta(\varsigma, \phi) = a_1 + \phi a_2 + \mathbb{L}_c^{-1}\left[\frac{1}{\sigma^2}\mathbb{L}_c\{\vartheta(\varsigma, \phi) + g(\vartheta) + g(\varsigma, \phi)\}\right]. \tag{12}$$

Using proposition (4), we obtain

$$\vartheta(\varsigma, \phi) = a_1 + \phi a_2 + \mathbb{L}_c^{-1}\left[\frac{1}{\sigma^2}\mathbb{L}_c\{g(\varsigma, \phi)\}\right] + \mathbb{L}_c^{-1}\left[\frac{1}{\sigma^2}\mathbb{L}_c[\vartheta(\varsigma, \phi) + g(\vartheta)]\right]. \tag{13}$$

This implies that

$$\vartheta(\varsigma, \phi) = G(\varsigma, \phi) + \mathbb{L}_c^{-1}\left[\frac{1}{\sigma^2}\mathbb{L}_c[\vartheta(\varsigma, \phi) + g(\vartheta)]\right]. \tag{14}$$

Equation (14) is called the formulation of $\mathbb{L}_c$HITM of Equation (6) and

$$G(\varsigma, \phi) = a_1 + \phi a_2 + \mathbb{L}_c^{-1}\left[\frac{1}{\sigma^2}\mathbb{L}_c\{g(\varsigma, \phi)\}\right]. \tag{15}$$

We introduce HPM in such a way that

$$\vartheta(\phi) = \sum_{i=0}^{\infty} p^i \vartheta_i(n) = \vartheta_0 + p^1\vartheta_1 + p^2\vartheta_2 + \cdots, \tag{16}$$

and nonlinear terms $g(\vartheta)$ are evaluated by considering an algorithm:

$$g(\vartheta) = \sum_{i=0}^{\infty} p^i H_i(\vartheta) = H_0 + p^1 H_1 + p^2 H_2 + \cdots, \tag{17}$$

where H_n polynomials are derived as

$$H_n(\vartheta_0 + \vartheta_1 + \cdots + \vartheta_n) = \frac{1}{n!}\frac{\partial^n}{\partial p^n}\left(g\left(\sum_{i=0}^{\infty} p^i \vartheta_i\right)\right)_{p=0}, \quad n = 0, 1, 2, \cdots. \tag{18}$$

Use Equations (16)–(18) in Equation (14) to compare the identical power of p such as

$$\begin{aligned}
p^0 &: \vartheta_0(\varsigma, \phi) = G(\varsigma, \phi),\\
p^1 &: \vartheta_1(\varsigma, \phi) = \mathbb{L}_c^{-1}\left[\frac{1}{\sigma^2}\mathbb{L}_c\{\vartheta_0(\varsigma, \phi) + H_0(\vartheta)\}\right],\\
p^2 &: \vartheta_2(\varsigma, \phi) = \mathbb{L}_c^{-1}\left[\frac{1}{\sigma^2}\mathbb{L}_c\{\vartheta_1(\varsigma, \phi) + H_1(\vartheta)\}\right],\\
p^3 &: \vartheta_3(\varsigma, \phi) = \mathbb{L}_c^{-1}\left[\frac{1}{\sigma^2}\mathbb{L}_c\{\vartheta_2(\varsigma, \phi) + H_2(\vartheta)\}\right],\\
&\vdots .
\end{aligned} \tag{19}$$

On proceeding this process, this yields

$$\vartheta(\varsigma, \phi) = \vartheta_0 + \vartheta_1 + \vartheta_2 + \cdots = \sum_{i=0}^{\infty} \vartheta_i. \tag{20}$$

Thus, Equation (20) is the approximate result of the differential problem (6).

4. Convergence Analysis

Let P and Q be Banach spaces where $X : P \longrightarrow Q$ is a nonlinear mapping. If the series produced by HPM is

$$\vartheta_n(P, \varsigma) = X(\vartheta_{n-1}(P, \varsigma)) = \sum_{i=0}^{n-1} \vartheta_i(P, \varsigma), \quad n = 1, 2, 3 \cdots, \tag{21}$$

the following conditions must be true:

(1) $\|\vartheta_n(P, \varsigma) - \vartheta(P, \varsigma)\| \le \varphi^n \|\vartheta(P, \varsigma) - \vartheta(P, \varsigma)\|$

(2) $\vartheta_n(P, \varsigma)$ is forever in the neighbourhood of $\vartheta(P, x)$ meaning $\vartheta_n(P, \varsigma) \in B(\vartheta(P, \varsigma), r) = \{\vartheta^*(P, \varsigma) / \|\vartheta^*(P, \varsigma) - \vartheta(P, \varsigma)\|\}$

(3) $\lim_{n \longrightarrow \infty} \vartheta_n(P, x) = \vartheta(P, \varsigma)$

Proof.

(1) We demonstrate condition (1) by recognition on n, such as $\|\vartheta_1 - \vartheta\| = \|G(\vartheta_0) - \vartheta\|$, and the Banach fixed point theorem states that X has a fixed point ϑ, i.e.,

$X(\vartheta) = \vartheta$; therefore,

$$\begin{aligned} \|\vartheta_1 - \vartheta\| &= \|G(\vartheta_0) - \vartheta\| = \|G(\vartheta_0) - G(\vartheta)\| \\ &\le \varphi\|\vartheta_0 - \vartheta\| = \varphi\|\vartheta(P, \varsigma) - \vartheta\|, \end{aligned} \tag{22}$$

where X is a nonlinear mapping. Consider that $\|\vartheta_{n-1} - \vartheta\| \le \varphi^{n-1}\|\vartheta(P, 0) - \vartheta(P, x)\|$ is an induction hypothesis; then,

$$\|\vartheta_n - \vartheta\| = \|G(\vartheta_{n-1}) - G(\vartheta)\| \le \varphi\|\vartheta_{n-1} - \vartheta\| \le \varphi\varphi^{n-1}\|\vartheta(P, \varsigma) - \vartheta\| \tag{23}$$

(2) Our initial challenge is to demonstrate the $\vartheta(P, \varsigma) \in B(\vartheta(P, \varsigma), r)$, which is attained by replacing m. Thus, $m = 1$, $\|\vartheta(P, \varsigma) - \vartheta(P, \varsigma)\| = \|\vartheta(P, 0) - \vartheta(P, \varsigma)\| \le r$ with $\vartheta(P, 0)$ as an initial condition. Consider that $\|\vartheta(P, x) - \vartheta(P, \varsigma)\| \le r$ for $m - 2$ is an induction theory, so

$$\begin{aligned} \|\vartheta(P, \varsigma) - \vartheta(P, \varsigma)\| &= \vartheta_{m-2}(P, \varsigma) - \left.\frac{f_m(P)}{\Gamma(\delta - m + 1)} x^{\delta-m}\right\| \\ &\le \|\vartheta_{m-1}(P, \varsigma) - \vartheta(P, \varsigma)\| \\ &\quad + \left\|\frac{f_m(P)}{\Gamma(\delta - m + 1)} x^{\delta-m}\right\| = r \end{aligned} \tag{24}$$

Now, $\forall n \ge 1$, using (1) we get

$$\|\vartheta_n - \vartheta\| \le \varphi^n\|\vartheta(P, \varsigma) - \vartheta\| \le \varphi^n r \le r. \tag{25}$$

(3) Using condition (2) and $\lim_{n\longrightarrow\infty}\varphi^n = 0$, it provides that $\lim_{n\longrightarrow\infty}\|\vartheta_n - \vartheta\| = 0$; hence,

$$\lim_{n\longrightarrow\infty} \vartheta_n = \vartheta \tag{26}$$

Thus, ϑ converges. □

5. Numerical Applications

We illustrate some numerical applications to check the validity and authenticity of $\mathbb{L}_c$HITM. We observe that this strategy is extremely convenient to utilize and generate the series of convergence much easier than other schemes. We also study the physical behaviors of these surface solutions. The error distribution is obtained graphically to show that the results obtained by $\mathbb{L}_c$HITM are very close to the precise results.

5.1. Example 1. Suppose a one-dimensional wave equation

$$\frac{\partial^2\vartheta}{\partial\phi^2} = \frac{\partial^2\vartheta}{\partial\varsigma^2} - 3\vartheta, \tag{27}$$

with the initial condition

$$\begin{aligned} \vartheta(\varsigma, 0) &= 0, \\ \vartheta_\phi(\varsigma, 0) &= 2\cos(\varsigma), \end{aligned} \tag{28}$$

and boundary condition

$$\begin{aligned} \vartheta(0, \phi) &= \sin(2\phi), \\ \vartheta_\varsigma(\pi, \phi) &= -\sin(2\phi). \end{aligned} \tag{29}$$

Using $\mathbb{L}_c$IT on Equation (27), we obtain $R(\sigma)$ such as

$$R[\sigma] = \vartheta(\varsigma, 0) + \frac{\vartheta'(\varsigma, 0)}{\sigma} + \frac{1}{\sigma^2}\mathbb{A}\left[\frac{\partial^2\vartheta}{\partial\varsigma^2} - 3\vartheta\right]. \tag{30}$$

Using inverse $\mathbb{L}_c$IT, it yields

$$\vartheta(\varsigma, \phi) = \vartheta(\varsigma, 0) + \phi\vartheta_\phi(\varsigma, 0) + \mathbb{L}_c^{-1}\left[\frac{1}{\sigma^2}\mathbb{L}_c\left\{\frac{\partial^2\vartheta}{\partial\varsigma^2} - 3\vartheta\right\}\right]. \tag{31}$$

Now, apply HPM to obtain He's polynomials

$$\sum_{i=0}^{\infty} p^i\vartheta_i(\varsigma, \phi) = 2\phi\cos(\varsigma) + \mathbb{L}_c^{-1}\left[\frac{1}{\sigma^2}\mathbb{L}_c\left\{\sum_{i=0}^{\infty} p^i\frac{\partial^2\vartheta_i}{\partial\varsigma^2} - 3\sum_{i=0}^{\infty} p^i\vartheta\right\}\right]. \tag{32}$$

Evaluating similar components of p, we obtain

$$\begin{aligned} p^0 &: \vartheta_0(\varsigma, \phi) = \vartheta(\varsigma, 0) = 2\phi\cos(\varsigma), \\ p^1 &: \vartheta_1(\varsigma, \phi) = \mathbb{L}_c^{-1}\left[\frac{1}{\sigma^2}\mathbb{L}_c\left\{\frac{\partial^2\vartheta_0}{\partial\varsigma^2} - 3\vartheta_0\right\}\right] = -\frac{(2\phi)^3}{3!}\cos(\varsigma), \\ p^2 &: \vartheta_2(\varsigma, \phi) = \mathbb{L}_c^{-1}\left[\frac{1}{\sigma^2}\mathbb{L}_c\left\{\frac{\partial^2\vartheta_1}{\partial\varsigma^2} - 3\vartheta_1\right\}\right] = \frac{(2\phi)^5}{5!}\cos(\varsigma), \\ p^3 &: \vartheta_3(\varsigma, \phi) = \mathbb{L}_c^{-1}\left[\frac{1}{\sigma^2}\mathbb{L}_c\left\{\frac{\partial^2\vartheta_2}{\partial\varsigma^2} - 3\vartheta_2\right\}\right] = -\frac{(2\phi)^7}{7!}\cos(\varsigma), \\ p^4 &: \vartheta_4(\varsigma, \phi) = \mathbb{L}_c^{-1}\left[\frac{1}{\sigma^2}\mathbb{L}_c\left\{\frac{\partial^2\vartheta_3}{\partial\varsigma^2} - 3\vartheta_3\right\}\right] = \frac{(2\phi)^9}{9!}\cos(\varsigma), \\ &\vdots. \end{aligned} \tag{33}$$

In the similar way, we can consider the approximate series such as

$$\vartheta(\varsigma,\phi)=\vartheta_0(\varsigma,\phi)+\vartheta_1(\varsigma,\phi)+\vartheta_2(\varsigma,\phi)+\vartheta_3(\varsigma,\phi)+\vartheta_4(\varsigma,\phi)+\cdots,$$
$$(\varsigma,\phi)=\cos(\varsigma)\left(2\phi-\frac{(2\phi)^3}{3!}+\frac{(2\phi)^5}{5!}-\frac{(2\phi)^7}{7!}+\frac{(2\phi)^9}{9!}\right)+\cdots, \tag{34}$$

which can approach to

$$\vartheta(\varsigma,\phi)=\cos(\varsigma)\sin(2\phi). \tag{35}$$

Figure 1 contains two diagrams: (a) the $\mathbb{L}_c$HITM results of $\vartheta(\varsigma,\phi)$ and (b) the exact results of $\vartheta(\varsigma,\phi)$ at $-2\le\varsigma\le 2$ and $0\le\phi\le 0.5$ for 1D wave problem. Figure 2 represents the graphical error of 1D wave equation between the approximate and the precise solutions at $0\le\varsigma\le 20$ with $\phi=0.5$. We observe that the current approach demonstrates the strong agreement with the precise answer to the problem (5.1) only after a few iterations. The rate of convergence shows that $\mathbb{L}_c$HITM is a relatable approach for $\vartheta(\varsigma,\phi)$. It states that we can effectively model any surface in accordance with the desired physical processes appearing in science and engineering.

5.2. Example 2. Suppose a two-dimensional wave equation

$$\frac{\partial^2\vartheta}{\partial\phi^2}=2\left(\frac{\partial^2\vartheta}{\partial\varsigma^2}+\frac{\partial^2\vartheta}{\partial\xi^2}\right)+6\phi+2\varsigma+4\xi, \tag{36}$$

with the initial condition

$$\vartheta(\varsigma,\xi,0)=0,\quad \vartheta_\phi(\varsigma,\xi,0)=2\sin(\varsigma)\sin(\xi), \tag{37}$$

and boundary condition

$$\begin{aligned}\vartheta(0,\xi,\phi)&=\phi^3+2\phi^2\xi,\\ \vartheta_\varsigma(\pi,\xi,\phi)&=\phi^3+\pi\phi^2+2\phi^2\xi,\\ \vartheta(\varsigma,0,\phi)&=\phi^3+\phi^2\varsigma,\\ \vartheta_\varsigma(\varsigma,\pi,\phi)&=\phi^3+2\pi\phi^2+\phi^2\varsigma.\end{aligned} \tag{38}$$

Apply $\mathbb{L}_c$IT on

$$\mathbb{L}_c\left[\frac{\partial^2\vartheta}{\partial\phi^2}\right]=\mathbb{L}_c\left[2\left(\frac{\partial^2\vartheta}{\partial\varsigma^2}+\frac{\partial^2\vartheta}{\partial\xi^2}\right)+6\phi+2\varsigma+4\xi\right]. \tag{39}$$

Using the property functions of $\mathbb{L}_c$IT, we obtain

$$\begin{aligned}\sigma^2R(\sigma)-\vartheta(\varsigma,0)-\frac{\vartheta'(\varsigma,0)}{\sigma}&=\mathbb{L}_c\left[2\left(\frac{\partial^2\vartheta}{\partial\varsigma^2}+\frac{\partial^2\vartheta}{\partial\xi^2}\right)+6\phi+2\varsigma+4\xi\right],\\ \sigma^2R(\sigma)-\vartheta(\varsigma,0)-\frac{\vartheta'(\varsigma,0)}{\sigma}&=\mathbb{L}_c\left[2\left(\frac{\partial^2\vartheta}{\partial\varsigma^2}+\frac{\partial^2\vartheta}{\partial\xi^2}\right)\right]+6\mathbb{L}_c[\phi]+2\varsigma\mathbb{L}_c[1]+4\xi\mathbb{L}_c[1].\end{aligned} \tag{40}$$

Hence, $R(\sigma)$ is evaluated. Using $\mathbb{L}_c$IT on Equation (36), we obtain $R(\sigma)$ such as

$$R[\sigma]=\frac{6}{\sigma^3}+\frac{2\varsigma}{\sigma^2}+\frac{4\xi}{\sigma^2}+\vartheta(\varsigma,0)+\frac{\vartheta'(\varsigma,0)}{\sigma}+\frac{1}{\sigma^2}\mathbb{L}_c\left[2\left(\frac{\partial^2\vartheta}{\partial\varsigma^2}+\frac{\partial^2\vartheta}{\partial\xi^2}\right]\right.. \tag{41}$$

Using inverse $\mathbb{L}_c$IT, it yields

$$\vartheta(\varsigma,\xi,\phi)=\phi^3+\varsigma\phi^2+2\xi\phi^2+\vartheta(\varsigma,0)+\phi\vartheta_\phi(\varsigma,0)+\mathbb{L}_c^{-1}\left[\frac{1}{\sigma^2}\mathbb{L}_c\left\{2\left(\frac{\partial^2\vartheta}{\partial\varsigma^2}+\frac{\partial^2\vartheta}{\partial\xi^2}\right\}\right]\right.. \tag{42}$$

Now, apply HPM to obtain He's polynomials

$$\sum_{i=0}^{\infty}p^i\vartheta_i(\varsigma,\xi,\phi)=\phi^3+\varsigma\phi^2+2\xi\phi^2+2\phi\sin(\varsigma)\sin(\xi)+\mathbb{L}_c^{-1}\left[\frac{1}{\sigma^2}\mathbb{L}_c\left\{2\left(\sum_{i=0}^{\infty}p^i\frac{\partial^2\vartheta_i}{\partial\varsigma^2}+\sum_{i=0}^{\infty}p^i\frac{\partial^2\vartheta_i}{\partial\xi^2}\right)\right\}\right]. \tag{43}$$

Evaluating similar components of p, we obtain

$$\begin{aligned}p^0&:\vartheta_0(\varsigma,\xi,\phi)=\vartheta(\varsigma,0)=\phi^3+\varsigma\phi^2+2\xi\phi^2+2\phi\sin(\varsigma)\sin(\xi),\\ p^1&:\vartheta_1(\varsigma,\xi,\phi)=\mathbb{L}_c^{-1}\left[\frac{1}{\sigma^2}\mathbb{L}_c\left\{\frac{\partial^2\vartheta_0}{\partial\varsigma^2}+\frac{\partial^2\vartheta_0}{\partial\xi^2}\right\}\right]=-\frac{(2\phi)^3}{3!}\sin(\varsigma)\sin(\xi),\\ p^2&:\vartheta_2(\varsigma,\xi,\phi)=\mathbb{L}_c^{-1}\left[\frac{1}{\sigma^2}\mathbb{L}_c\left\{\frac{\partial^2\vartheta_1}{\partial\varsigma^2}+\frac{\partial^2\vartheta_1}{\partial\xi^2}\right\}\right]=\frac{(2\phi)^5}{5!}\sin(\varsigma)\sin(\xi),\\ p^3&:\vartheta_3(\varsigma,\xi,\phi)=\mathbb{L}_c^{-1}\left[\frac{1}{\sigma^2}\mathbb{L}_c\left\{\frac{\partial^2\vartheta_2}{\partial\varsigma^2}+\frac{\partial^2\vartheta_2}{\partial\xi^2}\right\}\right]=-\frac{(2\phi)^7}{7!}\sin(\varsigma)\sin(\xi),\\ p^4&:\vartheta_4(\varsigma,\xi,\phi)=\mathbb{L}_c^{-1}\left[\frac{1}{\sigma^2}\mathbb{L}_c\left\{\frac{\partial^2\vartheta_3}{\partial\varsigma^2}+\frac{\partial^2\vartheta_3}{\partial\xi^2}\right\}\right]=\frac{(2\phi)^9}{9!}\sin(\varsigma)\sin(\xi),\\ &\vdots.\end{aligned} \tag{44}$$

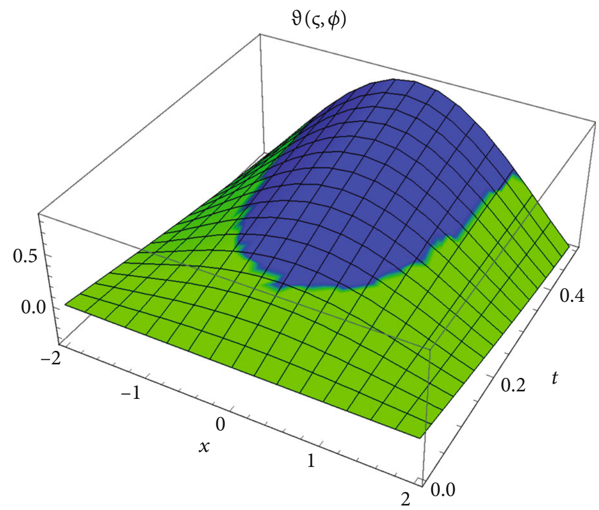
In the similar way, we can consider the approximate series such as

$$\begin{aligned}\vartheta(\varsigma,\xi,\phi)&=\vartheta_0(\varsigma,\xi,\phi)+\vartheta_1(\varsigma,\xi,\phi)+\vartheta_2(\varsigma,\xi,\phi)+\vartheta_3(\varsigma,\xi,\phi)+\vartheta_4(\varsigma,\xi,\phi)+\cdots,\\ \vartheta(\varsigma,\xi,\phi)&=\phi^3+\varsigma\phi^2+2\xi\phi^2+\sin(\varsigma)\sin(\xi)\cdot\left(2\phi-\frac{(2\phi)^3}{3!}+\frac{(2\phi)^5}{5!}-\frac{(2\phi)^7}{7!}+\frac{(2\phi)^9}{9!}\right)+\cdots,\end{aligned} \tag{45}$$

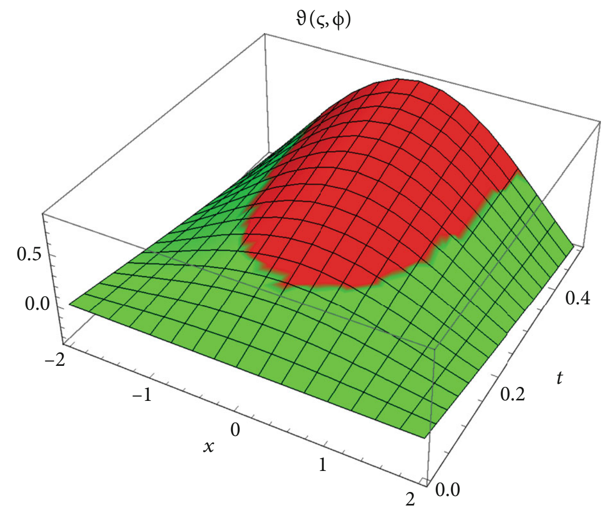
which can approach to

$$\vartheta(\varsigma,\xi,\phi)=\phi^3+\varsigma\phi^2+2\xi\phi^2+\sin(\varsigma)\sin(\xi)\sin(2\phi). \tag{46}$$

Figure 3 contains two diagrams: (a) the $\mathbb{L}_c$HITM results of $\vartheta(\varsigma,\xi,\phi)$ and (b) the exact results of $\vartheta(\varsigma,\xi,\phi)$ at $-1\le\varsigma\le 1$



(a) Surface plot for approximate results



(b) Surface plot for precise results

FIGURE 1: Surface solutions of 1D wave equation.

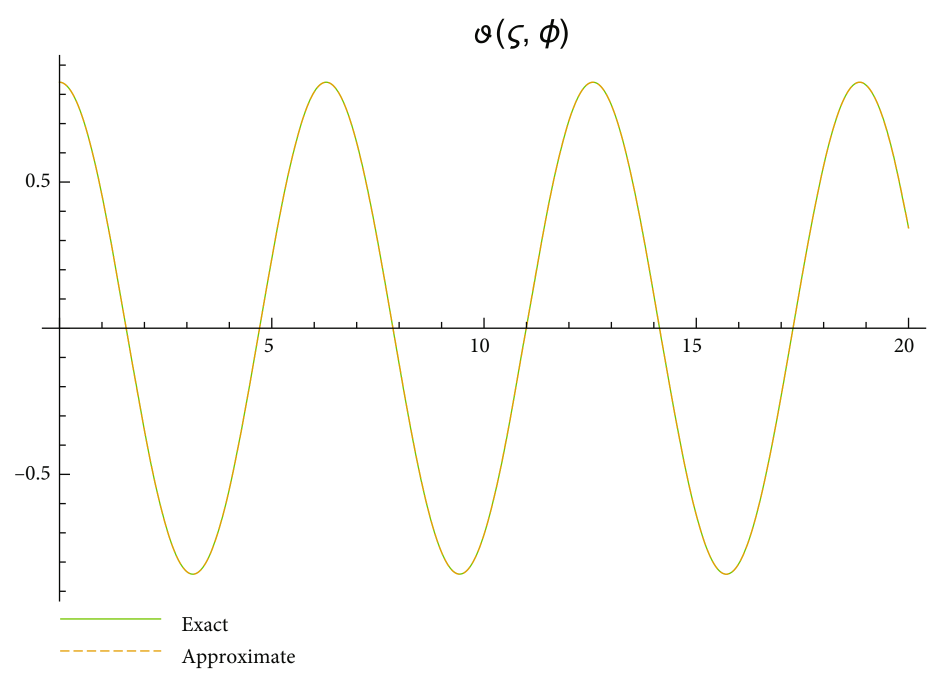


FIGURE 2: Graphical error between the approximate and the precise results of $\vartheta(\varsigma, \phi)$.

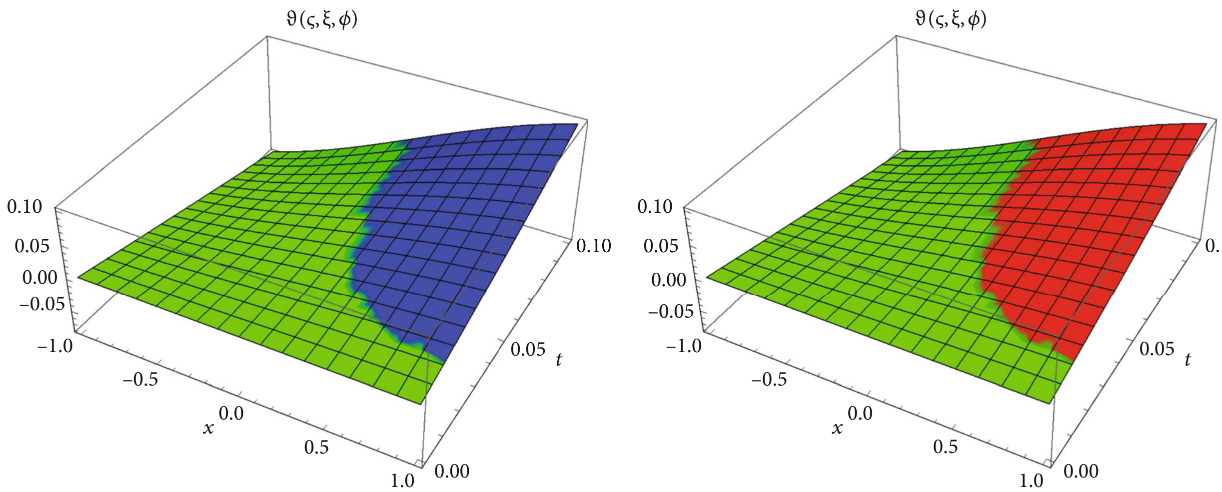
and $0 \leq \phi \leq 0.1$ with $\xi = 0.5$ for 2D wave problem. Figure 4 represents the graphical error of 2D wave equation between the approximate and the precise solutions at $0 \leq \varsigma \leq 20$ with $\xi = 0.01$ and $\phi = 0.01$. We observe that current approach demonstrates the strong agreement with the precise answer to the problem (5.2) only after a few iterations. The rate of convergence shows that $\mathbb{L}_c$HITM is a reliable approach for $\vartheta(\varsigma, \xi, \phi)$. It states that we can effectively model any surface in accordance with the desired physical processes appearing in nature.

5.3. Example 3. Consider the three-dimensional wave problem

$$\frac{\partial^2 \vartheta}{\partial \phi^2} = \frac{\varsigma^2}{18}\frac{\partial^2 \vartheta}{\partial \varsigma^2} + \frac{\xi^2}{18}\frac{\partial^2 \vartheta}{\partial \xi^2} + \frac{\eta^2}{18}\frac{\partial^2 \vartheta}{\partial \eta^2} - \vartheta, \tag{47}$$

with the initial condition

$$\begin{gathered} \vartheta(\varsigma, \xi, \eta, 0) = 0, \\ \vartheta_\phi(\varsigma, \xi, \eta, 0) = \varsigma^4 \xi^4 \eta^4, \end{gathered} \tag{48}$$



(a) Surface plot for approximate results

(b) Surface plot for precise results

FIGURE 3: Surface solutions of 2D wave equation.

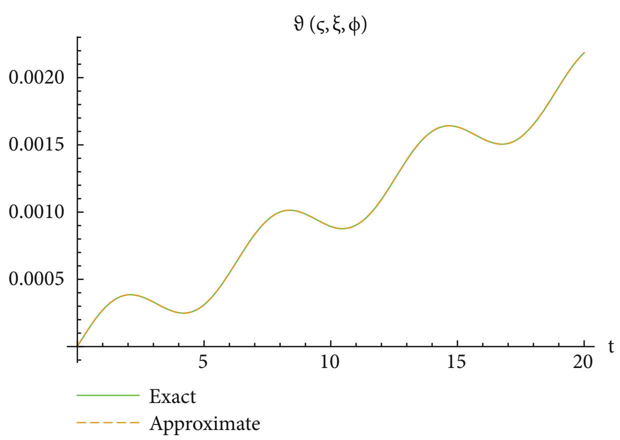


FIGURE 4: Graphical error between the approximate and the precise results of $\vartheta(\varsigma, \xi, \phi)$.

and boundary condition

$$\begin{aligned}
&\vartheta(0, \xi, \eta, \phi) = 0, \\
&\vartheta(1, \xi, \eta, \phi) = \xi^4 \eta^4 \sinh(\phi), \\
&\vartheta(\varsigma, 0, \eta, \phi) = 0, \\
&\vartheta(\varsigma, 1, \eta, \phi) = \varsigma^4 \eta^4 \sinh(\phi), \\
&\vartheta(\varsigma, \xi, 0, \phi) = 0, \\
&\vartheta(\varsigma, \xi, 1, \phi) = \varsigma^4 \xi^4 \sinh(\phi).
\end{aligned} \tag{49}$$

Using $\mathbb{L}_c$IT on Equation (47), we obtain $R(\sigma)$ such as

$$R[\sigma] = \vartheta(\varsigma, 0) + \frac{\vartheta'(\varsigma, 0)}{\sigma} + \frac{1}{\sigma^2} \mathbb{L}_c \left[\frac{\varsigma^2}{18} \frac{\partial^2 \vartheta}{\partial \varsigma^2} + \frac{\xi^2}{18} \frac{\partial^2 \vartheta}{\partial \xi^2} + \frac{\eta^2}{18} \frac{\partial^2 \vartheta}{\partial \eta^2} - \vartheta \right]. \tag{50}$$

Using inverse $\mathbb{L}_c$IT, it yields

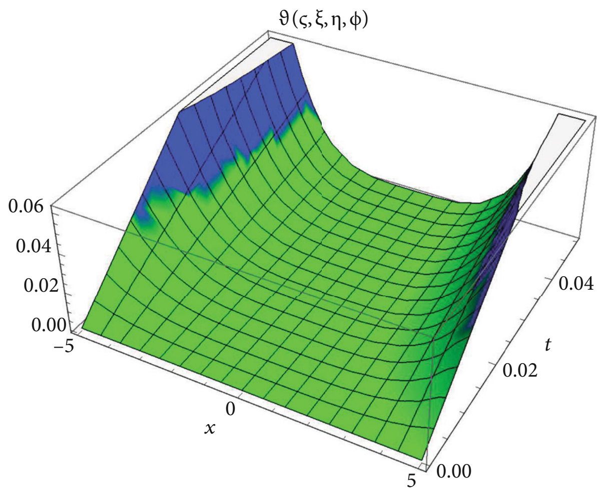
$$\vartheta(\varsigma, \xi, \eta, \phi) = \vartheta(\varsigma, 0) + \phi \vartheta_\phi(\varsigma, 0) + \mathbb{L}_c^{-1} \cdot \left[\frac{1}{\sigma^2} \mathbb{L}_c \left\{ \frac{\varsigma^2}{18} \frac{\partial^2 \vartheta}{\partial \varsigma^2} + \frac{\xi^2}{18} \frac{\partial^2 \vartheta}{\partial \xi^2} + \frac{\eta^2}{18} \frac{\partial^2 \vartheta}{\partial \eta^2} - \vartheta \right\} \right]. \tag{51}$$

Now, apply HPM to obtain He's polynomials

$$\sum_{i=0}^{\infty} p^i \vartheta(\varsigma, \xi, \eta, \phi) = \phi \varsigma^4 \xi^4 \eta^4 + \mathbb{L}_c^{-1} \left[\frac{1}{\sigma^2} \mathbb{L}_c \left\{ \sum_{i=0}^{\infty} p^i \frac{\varsigma^2}{18} \frac{\partial^2 \vartheta_i}{\partial \varsigma^2} + \sum_{i=0}^{\infty} p^i \frac{\xi^2}{18} \frac{\partial^2 \vartheta_i}{\partial \xi^2} + \sum_{i=0}^{\infty} p^i \frac{\eta^2}{18} \frac{\partial^2 \vartheta_i}{\partial \eta^2} - \sum_{i=0}^{\infty} p^i \vartheta \right\} \right]. \tag{52}$$

Evaluating similar components of p, we obtain

$$\begin{aligned}
p^0 &: \vartheta_0(\varsigma, \xi, \eta, \phi) = \vartheta(\varsigma, \xi, \eta, 0) = \phi \varsigma^4 \xi^4 \eta^4, \\
p^1 &: \vartheta_1(\varsigma, \xi, \phi) = \mathbb{L}_c^{-1} \left[\frac{1}{\sigma^2} \mathbb{L}_c \left\{ \frac{\varsigma^2}{18} \frac{\partial^2 \vartheta_0}{\partial \varsigma^2} + \frac{\xi^2}{18} \frac{\partial^2 \vartheta_0}{\partial \xi^2} + \frac{\eta^2}{18} \frac{\partial^2 \vartheta_0}{\partial \eta^2} - \vartheta_0 \right\} \right] = \frac{\phi^3}{3!} \varsigma^4 \xi^4 \eta^4, \\
p^2 &: \vartheta_2(\varsigma, \xi, \phi) = \mathbb{L}_c^{-1} \left[\frac{1}{\sigma^2} \mathbb{L}_c \left\{ \frac{\varsigma^2}{18} \frac{\partial^2 \vartheta_1}{\partial \varsigma^2} + \frac{\xi^2}{18} \frac{\partial^2 \vartheta_1}{\partial \xi^2} + \frac{\eta^2}{18} \frac{\partial^2 \vartheta_1}{\partial \eta^2} - \vartheta_1 \right\} \right] = \frac{\phi^5}{5!} \varsigma^4 \xi^4 \eta^4, \\
p^3 &: \vartheta_3(\varsigma, \xi, \phi) = \mathbb{L}_c^{-1} \left[\frac{1}{\sigma^2} \mathbb{L}_c \left\{ \frac{\varsigma^2}{18} \frac{\partial^2 \vartheta_2}{\partial \varsigma^2} + \frac{\xi^2}{18} \frac{\partial^2 \vartheta_2}{\partial \xi^2} + \frac{\eta^2}{18} \frac{\partial^2 \vartheta_2}{\partial \eta^2} - \vartheta_2 \right\} \right] = \frac{\phi^7}{7!} \varsigma^4 \xi^4 \eta^4, \\
p^4 &: \vartheta_4(\varsigma, \xi, \phi) = \mathbb{L}_c^{-1} \left[\frac{1}{\sigma^2} \mathbb{L}_c \left\{ \frac{\varsigma^2}{18} \frac{\partial^2 \vartheta_3}{\partial \varsigma^2} + \frac{\xi^2}{18} \frac{\partial^2 \vartheta_3}{\partial \xi^2} + \frac{\eta^2}{18} \frac{\partial^2 \vartheta_3}{\partial \eta^2} - \vartheta_3 \right\} \right] = \frac{\phi^9}{9!} \varsigma^4 \xi^4 \eta^4, \\
&\vdots .
\end{aligned} \tag{53}$$



(a) Surface plot for approximate results

(b) Surface plot for precise results

FIGURE 5: Surface solutions of 3D wave equation.

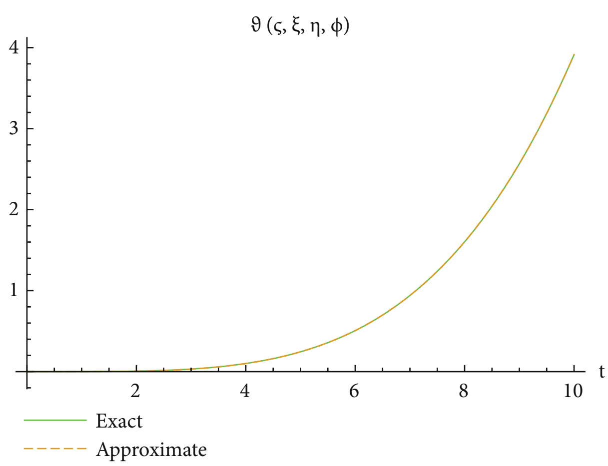


FIGURE 6: Graphical error between the approximate and the precise results of $\vartheta(\varsigma, \xi, \eta, \phi)$.

In the similar way, we can consider the approximate series such as

$$\vartheta(\varsigma, \xi, \eta, \phi) = \vartheta_0(\varsigma, \xi, \eta, \phi) + \vartheta_1(\varsigma, \xi, \eta, \phi) + \vartheta_2(\varsigma, \xi, \eta, \phi) + \vartheta_3(\varsigma, \xi, \eta, \phi) + \vartheta_4(\varsigma, \xi, \eta, \phi) + \cdots,$$

$$\vartheta(\varsigma, \xi, \eta, \phi) = \varsigma^4 \xi^4 \eta^4 \left(\phi + \frac{\phi^3}{3!} + \frac{\phi^5}{5!} + \frac{\phi^7}{7!} + \frac{\phi^9}{9!}\right) + \cdots, \quad (54)$$

which can approach to

$$\vartheta(\varsigma, \xi, \eta, \phi) = \varsigma^4 \xi^4 \eta^4 \sinh(\phi). \quad (55)$$

Figure 5 contains two diagrams: (a) the $\mathbb{L}_c$HITM results of $\vartheta(\varsigma, \xi, \eta, \phi)$ and (b) the exact results of $\vartheta(\varsigma, \xi, \eta, \phi)$ at $-5 \leq \varsigma \leq 5$ and $0 \leq \phi \leq 0.05$ with $\xi = 0.5$ and $\eta = 0.5$ for 3D wave problem. Figure 6 represents the graphical error of 3D wave equation between the approximate and the precise solutions at $0 \leq \varsigma \leq 10$ with $\xi = 0.5$, $\varsigma = 0.5$, and $\phi = 0.1$. We observe that the current approach demonstrates the strong agreement with the precise answer to the problem (5.3) only after a few iterations. The rate of convergence shows that $\mathbb{L}_c$HITM is a reliable approach for $\vartheta(\varsigma, \xi, \eta, \phi)$. It states that we can effectively model any surface in accordance with the desired physical processes appearing in nature.

6. Conclusion

In this paper, we construct a new scheme known as the Laplace-Carson homotopy integral transform method ($\mathbb{L}_c$HITM) for obtaining the approximate solution of 1D, 2D, and 3D wave equations. The main advantage of $\mathbb{L}_c$IT is that the recurrence relation produces the iteration without any assumption of a small parameter. HPM helps to produce successive iterations in the recurrence relation. The obtained results show that this approach is very simple to utilize and derive the series solution in the convergence form. Some graphical results are demonstrated to show the physical nature of these wave problems. The graphical error of plot distortion shows that $\mathbb{L}_c$HITM has the best agreement with the exact solution. We encourage the readers can extend this scheme for the numerical solution of a nonlinear coupled system of fractional order in science and engineering for their future work.

Data Availability

All the data are available within the article.

Conflicts of Interest

The authors declare that they have no competing of interest.

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