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*Research Article*

# Analysis of Fractional Kundu-Eckhaus and Massive Thirring Equations Using a Hybridization Scheme

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This paper deals with the study of fractional Kundu-Eckhaus equation (FKEE) and fractional massive Thirring problem (FMTP) that appear in the quantum field theory, weakly nonlinear dispersive water waves, and nonlinear optics. Since the variational iteration method involves integration, the Laplace transform involves convolution theorem in recurrence relation to derive the series solution. To avoid some assumptions and hypothesis, we apply a two-scale approach for such a nonlinear complex model. The fractional differential equation may be transformed into its partner equation using He's fractional complex transform, and then, the nonlinear elements can be readily handled using the homotopy perturbation method (HPM). Numerical results are derived in a rapid converge series form to improve the accuracy of the scheme greatly. Graphical representations and error distribution show that the two-scale approach is a very convenient tool.

## 1. Introduction

In recent years, fractional calculus (FC) has assumed a greater significance in mathematical theory and widely used in many fields including ecology, physics, astronomy, and economics. Researchers are increasingly realizing that the fractional framework may be compatible with a wide range of phenomena in common applied sciences after the concepts of FC were successfully applied to a variety of different features. Mathematical models of many physical processes are developed using fractional differential equations. They are employed not only in mathematics but also in physics, dynamical systems, power systems, and applied science [1–3]. Kundu and Eckhaus [4, 5] introduced the FKKE which is studied in quantum field theory and many dispersion phenomena.

$$iD_{\xi}^{\alpha}\Psi(\varsigma,\xi)+\Psi_{\varsigma\varsigma}+2\Psi\left(|\Psi|^{2}\right)_{\varsigma}+\Psi|\Psi|^{4}=0,\quad 0<\alpha\leq 1. \quad (1)$$

The FKKE is a combination of Lax couples, higher conserved portion, particular soliton solution, and rogue wave solution. It is very essential to develop a scientific design that acts on behalf of ultrashort light pulses in a glass fiber. This model will be used to demonstrate the propagation of light across an optical cable. The fractional massive Thirring problem (FMTP)

$$\begin{aligned} i\left(D_{\xi}^{\alpha}\Psi+\Psi_{\varsigma}\right)+\Phi+\Psi|\Phi|^{2}=0 \\ i\left(D_{\xi}^{\alpha}\Phi+\Phi_{\varsigma}\right)+\Psi+\Phi|\Psi|^{2}=0 \end{aligned} \quad (2)$$

was autonomously introduced in 1958 by Thirring. It is a nonlinear coupled fractional differential equation which appears in the quantum field theory [6, 7]. Feng and Wang [8] discussed the algebraic curve method to obtain the explicit particular solitary solutions for the Kundu equation and the derivative Schrodinger equation. Yi and

Liu [9] employed the bifurcation approach to explore the bifurcations of traveling wave solutions for the Kundu equation. Luo and Nadeem [10] established Mohand transform with HPM to obtain the numerical solution of FKEE and coupled FMTP.

Many authors [11–13] have studied the various features of this equation, its generalities, and relationship with other nonlinear equations. It is classified to a variation of famous integrable equations such as nonlinear Schrodinger equation and also several nonlinear equations through a gauge transformation. Many researchers have studied this equation through various approaches such as gauge transformation [14], Lie symmetry method [15], Bernoulli subequation method [16], Backlund transformation [17], sine-Gordon expansion approach [18], Darboux transformation [19], and rogue wave solutions [20]. A lot of researchers introduced numerous semianalytical and numerical methods to study the fractional derivatives and fractional differential equations. He [21] constructed a technique which is called HPM that does not depend upon a small parameter to estimate the approximate solution of a nonlinear model. Later, Nadeem and Li [22] combined HPM with the Laplace transform to find the approximate solution of nonlinear vibration systems and nonlinear wave equations. It can be seen that HPM is a powerful tool and effective for nonlinear problems [23, 24]. It is however challenging to identify the analytical solutions for the most of the problems, and therefore, these problems can be tended by semianalytical methods. The objective of this paper is to suggest two-scale approach for quantum phenomena in fractal environments. The two-scale approach is the most friendly approach which converts fractional differential equations into its differential partner equations to make it extremely easy for the solution procedure.

The structure of this paper is formed as follows: in Section 2, we briefly explain the concept of HPM for a nonlinear problem. A two-scale approach with a numerical problem has been presented in Sections 3 and 4. In Section 5, we will explain the obtained results and discussion through our suggested approach. Section 6 will be our conclusions.

## 2. Basic Idea of Homotopy Perturbation Method

We assume the following nonlinear problem to present the concept of HPM [22]:

$$T_1(\Psi) - h(r) = 0, \quad r \in \Omega, \tag{3}$$

with boundary conditions

$$T_2\left(\Psi, \frac{\partial \Psi}{\partial S}\right) = 0, \quad \Psi \in \Gamma, \tag{4}$$

where $T_1$ is particular operators, $T_2$ is a boundary operator, $h(r)$ is a known function, and $\Gamma$ is the boundary of the domain $\Omega$. We can divide operator $T_1$ into two parts, $R$ and $S$ with considering linear and nonlinear operators, respectively. Thus, Equation (2) may also be stated as

$$R(\Psi) + S(\Psi) - h(r) = 0. \tag{5}$$

According to the homotopy strategy, we develop a homotopy $\rho(r, \theta)$: $\Omega \times [0, 1] \longrightarrow \mathbb{R}$ which satisfies

$$H(\Psi, \theta) = (1-\theta)[R(\Psi) - R(\Psi_0)] + \theta[R(\Psi) - S(\Psi) - h(r)], \tag{6}$$

or

$$H(\Psi, \theta) = R(\Psi) - R(\Psi_0) + pL(\Psi_0) + \theta[S(\Psi) - h(r)] = 0, \tag{7}$$

where $\theta \in [0, 1]$ is termed as homotopy parameter and $\Psi_0$ is an initial guess of Equation (2) that complies with the boundary conditions. Since the definition of HPM states that $\theta$ is estimated as a small parameter, so, we may consider the solution of Equation (5) in terms of a power series of $\theta$ such as

$$\Psi = \Psi_0 + \theta\Psi_1 + \theta^2\Psi_2 + \cdots. \tag{8}$$

Choosing $\theta = 1$, the estimated solution of Equation (2) is acquired as

$$\Psi = \lim_{\theta \longrightarrow 1} \Psi = \Psi_0 + \Psi_1 + \Psi_2 + \Psi_3 + \cdots. \tag{9}$$

The nonlinear terms are evaluated as

$$S\Psi(\varsigma, \xi) = \sum_{n=0}^{\infty} \theta^n H_n(\Psi), \tag{10}$$

where polynomials $H_n(\Psi)$ are presented such as

$$H_n(\Psi_0 + \Psi_1 + \cdots + \Psi_n) = \frac{1}{n!}\frac{\partial^n}{\partial \theta^n}\left(S\left(\sum_{i=0}^{\infty}\theta^i\Psi_i\right)\right)_{\theta=0}, \quad n = 0, 1, 2, \cdots. \tag{11}$$

Since the series depends on the nonlinear operator $S$, therefore, the results obtained in Equation (8) are convergent.

## 3. Fractional Complex Transform

In this segment, we illustrate the concept of fractional complex transform in such a way that it concerts a fractional problem into its differential parts such as [25–27]

$$\Delta S = \frac{\Delta \xi^{\alpha}}{\Gamma(1+\alpha)}, \tag{12}$$

where $\Delta S$ is nominated as a slighter scale. On a slighter scale, FKEE reacts discontinuously, in particular at the top of the solitary wave, whereas the heavier scale forecasts a coherent solitary wave. We use this transformation of Equation (12) in a fractional differential problem to change a fractal space in a lighter scale. Thus, a smooth space refers to a smooth space with a heavier scale, also called as the two-scale transform [28, 29].

## 4. Numerical Application

*Example 1.* We may rewrite Equation (1) such as

$$\frac{\partial^\alpha \Psi}{\partial \xi^\alpha} = i\Psi_{\varsigma\varsigma} + i2\Psi\left(|\Psi|^2\right)_\varsigma + i\Psi|\Psi|^4, \tag{13}$$

with the following initial conditions

$$\Psi(\varsigma, 0) = \mu e^{i\varsigma}. \tag{14}$$

Now, we use Equation (12) to convert it in differential parts. So, Equation (13) can be written as

$$\frac{\partial \Psi}{\partial S} = i\Psi_{\varsigma\varsigma} + 2i\Psi\left(|\Psi|^2\right)_\varsigma + i\Psi|\Psi|^4. \tag{15}$$

We may write it as follows:

$$\frac{\partial \Psi}{\partial S} = i\Psi_{\varsigma\varsigma} + 2i\left(\Psi\Psi_\varsigma\bar{\Psi} + \Psi^2\bar{\Psi}_\varsigma\right) + i\Psi^3\bar{\Psi}^2, \tag{16}$$

where $|\Psi|^2 = \Psi\bar{\Psi}$ and $\bar{\Psi}$ is the conjugate of $\Psi$.

We can select $\Psi(\varsigma, 0) = \mu e^{i\varsigma}$ by using the given initial values. Thus, HPM can be employed to Equation (16) to get the following series:

$$\begin{aligned}
\frac{\partial \Psi_1}{\partial S} &= i\Psi_{0\varsigma\varsigma} + 2i\left(\Psi_0\Psi_{0\varsigma}\bar{\Psi}_0 + \Psi^2{}_0\bar{\Psi}_{0\varsigma}\right) + i\Psi^3{}_0\bar{\Psi}^2{}_0, \\
\frac{\partial \Psi_2}{\partial S} &= i\Psi_{1\varsigma\varsigma} + 2i\Big(\Psi_0\Psi_{0\varsigma}\bar{\Psi}_1 + \Psi_0\Psi_{1\varsigma}\bar{\Psi}_0 + \Psi_1\Psi_{0\varsigma}\bar{\Psi}_0 \\
&\quad + \Psi^2{}_0\bar{\Psi}_{1\varsigma} + 2\Psi_0\Psi_{1\varsigma}\bar{\Psi}_{0\varsigma} \\
&\quad + i\left(2\bar{\Psi}_0\bar{\Psi}_1\Psi^3{}_0 + 3\Psi^2{}_0\Psi_1\bar{\Psi}^2{}_0\right)\Big), \\
\frac{\partial \Psi_3}{\partial S} &= i\Psi_{2\varsigma\varsigma} + 2i\Big(\Psi_0\Psi_{0\varsigma}\bar{\Psi}_2 + \Psi_0\Psi_{1\varsigma}\bar{\Psi}_1 + \Psi_0\Psi_{2\varsigma}\bar{\Psi}_0 \\
&\quad + \Psi_1\Psi_{0\varsigma}\bar{\Psi}_1 + \Psi_1\Psi_{1\varsigma}\bar{\Psi}_0 + \Psi_2\Psi_{0\varsigma}\bar{\Psi}_0 + \Psi^2{}_0\bar{\Psi}_{2\varsigma} \\
&\quad + 2\Psi_0\Psi_1\bar{\Psi}_{1\varsigma} + \Psi^2{}_1\bar{\Psi}_{0\varsigma}\Big) + i\Big(\bar{\Psi}^2{}_1\Psi^3{}_0 + 2\bar{\Psi}_0\bar{\Psi}_2\Psi^3{}_0 \\
&\quad + 6\Psi^2{}_0\Psi_1\bar{\Psi}_0\bar{\Psi}_1 + 3\Psi_0\Psi^2{}_1\bar{\Psi}^2{}_0 + 3\Psi^2{}_0\Psi_2\bar{\Psi}^2{}_0\Big).
\end{aligned} \tag{17}$$

Hence, the derived results are obtained as follows:

$$\begin{aligned}
\Psi_0 &= \mu e^{i\varsigma}, \\
\Psi_1 &= i\mu e^{i\varsigma}\left(\mu^4 - 1\right)S, \\
\Psi_2 &= -\mu e^{i\varsigma}\left(\mu^4 - 1\right)^2\frac{S^2}{2}, \\
\Psi_3 &= \mu e^{i\varsigma}\left(i + 4\mu^2 - i\mu^4\right)\left(\mu^4 - 1\right)^2\frac{S^3}{6}.
\end{aligned} \tag{18}$$

On continuing this process, we can achieve the following series:

$$\begin{aligned}
\Psi(\varsigma, S) &= \mu e^{i\varsigma} + i\mu e^{i\varsigma}\left(\mu^4 - 1\right)S - \mu e^{i\varsigma}\left(\mu^4 - 1\right)^2\frac{S^2}{2} \\
&\quad + \mu e^{i\varsigma}\left(i + 4\mu^2 - i\mu^4\right)\left(\mu^4 - 1\right)^2\frac{S^3}{6} + \cdots.
\end{aligned} \tag{19}$$

Using Equation (12), we can get

$$\begin{aligned}
\Psi(\varsigma, \xi) &= \mu e^{i\varsigma} + i\mu e^{i\varsigma}\left(\mu^4 - 1\right)\frac{\xi^\alpha}{\Gamma(1+\alpha)} \\
&\quad - \frac{\mu e^{i\varsigma}}{2}\left(\mu^4 - 1\right)^2\left(\frac{\xi^\alpha}{\Gamma(1+\alpha)}\right)^2 \\
&\quad + \frac{\mu e^{i\varsigma}}{6}\left(i + 4\mu^2 - i\mu^4\right)\left(\mu^4 - 1\right)^2\left(\frac{\xi^\alpha}{\Gamma(1+\alpha)}\right)^3 + \cdots,
\end{aligned} \tag{20}$$

which can be in closed form of [30, 31] at $\alpha = 1$

$$\Psi(\varsigma, \xi) = \frac{e^{i\varsigma}}{\left[1 + \left(1/\mu^4 - 1\right)e^{4i\xi}\right]^{1/4}}. \tag{21}$$

*Example 2.* We may rewrite Equation (2) such as

$$\begin{aligned}
\left(\frac{\partial^\alpha \Psi}{\partial \xi^\alpha} + \frac{\partial \Psi}{\partial \varsigma}\right) - i\Phi - i\Psi|\Phi|^2 &= 0, \\
\left(\frac{\partial^\alpha \Phi}{\partial \xi^\alpha} + \frac{\partial \Phi}{\partial \varsigma}\right) - i\Psi - i\Phi|\Psi|^2 &= 0,
\end{aligned} \tag{22}$$

with the following initial conditions:

$$\begin{aligned}
\Psi(\varsigma, 0) &= \mu e^{i\varsigma}, \\
\Phi(\varsigma, 0) &= \eta e^{i\varsigma}.
\end{aligned} \tag{23}$$

Now, we use Equation (12) to convert it in differential parts. So, the above system of Equation (22) becomes as

$$\begin{aligned} &\frac{\partial \Psi}{\partial S}+\frac{\partial \Psi}{\partial \varsigma}-i\Phi-i\Psi|\Phi|^2=0,\\ &\frac{\partial \Phi}{\partial S}+\frac{\partial \Phi}{\partial \varsigma}-i\Psi-i\Phi|\Psi|^2=0. \end{aligned} \tag{24}$$

We may write it as follows:

$$\begin{aligned} &\frac{\partial \Psi}{\partial S}+\frac{\partial \Psi}{\partial \varsigma}-i\Phi-i\Psi\Phi\bar{\Phi}=0,\\ &\frac{\partial \Phi}{\partial S}+\frac{\partial \Phi}{\partial \varsigma}-i\Psi-i\Phi\Psi\bar{\Psi}=0, \end{aligned} \tag{25}$$

where $|\Psi|^2=\Psi\bar{\Psi}$ and $|\Phi|^2=\Phi\bar{\Phi}$ with $\bar{\Psi}$ and $\bar{\Phi}$ are the conjugate of $\Psi$ and $\Phi$, respectively.

We can select $\Psi(\varsigma,0)=\mu e^{i\varsigma}$ and $\Phi(\varsigma,0)=\eta e^{i\varsigma}$ by using the given initial values. Thus, HPM can be employed to Equation (25) to get the following series:

$$\begin{aligned} &\frac{\partial \Psi_1}{\partial S}+\frac{\partial \Psi_0}{\partial \varsigma}-i\Phi_0-i\Psi_0\Phi_0\bar{\Phi}_0=0, \quad \Psi_1(\varsigma,0)=0,\\ &\frac{\partial \Phi_1}{\partial S}+\frac{\partial \Phi_0}{\partial \varsigma}-i\Psi_0-i\Phi_0\Psi_0\bar{\Psi}_0=0, \quad \Psi_1(\varsigma,0)=0,\\ &\frac{\partial \Psi_2}{\partial S}+\frac{\partial \Psi_1}{\partial \varsigma}-i\Phi_1-i\left(\Psi_0\Phi_0\bar{\Phi}_1+\Psi_0\Phi_1\bar{\Phi}_0+\Psi_1\Phi_0\bar{\Phi}_0\right)=0,\\ &\qquad \Psi_2(\varsigma,0)=0,\\ &\frac{\partial \Phi_2}{\partial S}+\frac{\partial \Phi_1}{\partial \varsigma}-i\Psi_1-i\left(\Phi_0\Psi_0\bar{\Psi}_1+\Phi_0\Psi_1\bar{\Psi}_0+\Phi_1\Psi_0\bar{\Psi}_0\right)=0,\\ &\qquad \Phi_2(\varsigma,0)=0,\\ &\frac{\partial \Psi_3}{\partial S}+\frac{\partial \Psi_2}{\partial \varsigma}-i\Phi_2-i\left(\Psi_0\Phi_0\bar{\Phi}_2+\Psi_0\Phi_1\bar{\Phi}_2\right.\\ &\qquad \left.+\Psi_0\Phi_2\bar{\Phi}_0+\Psi_1\Phi_0\bar{\Phi}_1+\Psi_1\Phi_1\bar{\Phi}_0+\Psi_2\Phi_0\bar{\Phi}_0\right)=0,\\ &\qquad \Psi_3(\varsigma,0)=0,\\ &\frac{\partial \Phi_3}{\partial S}+\frac{\partial \Phi_2}{\partial \varsigma}-i\Psi_2-i\left(\Phi_0\Psi_0\bar{\Psi}_2+\Phi_0\Psi_1\bar{\Psi}_1+\Phi_0\Psi_2\bar{\Psi}_0\right.\\ &\qquad \left.+\Phi_1\Psi_0\bar{\Psi}_1+\Phi_1\Psi_1\bar{\Psi}_0+\Phi_2\Psi_0\bar{\Psi}_0\right)=0, \quad \Phi_3(\varsigma,0)=0. \end{aligned} \tag{26}$$

Hence, the derived results are obtained as follows:

$$\begin{aligned} &\Psi(\varsigma,0)=\mu e^{i\varsigma},\\ &\Phi(\varsigma,0)=\eta e^{i\varsigma},\\ &\Psi_1(\varsigma,S)=ie^{i\varsigma}\left[\eta-\mu+\eta^2\mu\right]S,\\ &\Phi_1(\varsigma,S)=ie^{i\varsigma}\left[\mu-\eta+\mu^2\eta\right]S,\\ &\Psi_2(\varsigma,S)=i^2e^{i\varsigma}\left[\eta^3+2\mu+\eta^4\mu+2\eta^2\mu\left(-2+\mu^2\right)\right.\\ &\qquad \left.+\eta\left(-2+3\mu^2\right)\right]\frac{S^2}{2},\\ &\Phi_2(\varsigma,S)=i^2e^{i\varsigma}\left[\mu^3+2\eta+\mu^4\eta+2\mu^2\eta\left(-2+\eta^2\right)\right.\\ &\qquad \left.+\mu\left(-2+3\eta^2\right)\right]\frac{S^2}{2}. \end{aligned} \tag{27}$$

On continuing this process, we can achieve the following series:

$$\begin{aligned} \Psi(\varsigma,S)&=\mu e^{i\varsigma}+ie^{i\varsigma}\left[\eta-\mu+\eta^2\mu\right]S+i^2e^{i\varsigma}\left[\eta^3+2\mu+\eta^4\mu\right.\\ &\quad \left.+2\eta^2\mu\left(-2+\mu^2\right)+\eta\left(-2+3\mu^2\right)\right]\frac{S^2}{2}+\cdots,\\ \Phi(\varsigma,S)&=\eta e^{i\varsigma}+ie^{i\varsigma}\left[\mu-\eta+\mu^2\eta\right]S+i^2e^{i\varsigma}\left[\mu^3+2\eta+\mu^4\eta\right.\\ &\quad \left.+2\mu^2\eta\left(-2+\eta^2\right)+\mu\left(-2+3\eta^2\right)\right]\frac{S^2}{2}+\cdots. \end{aligned} \tag{28}$$

Using Equation (12), we can get

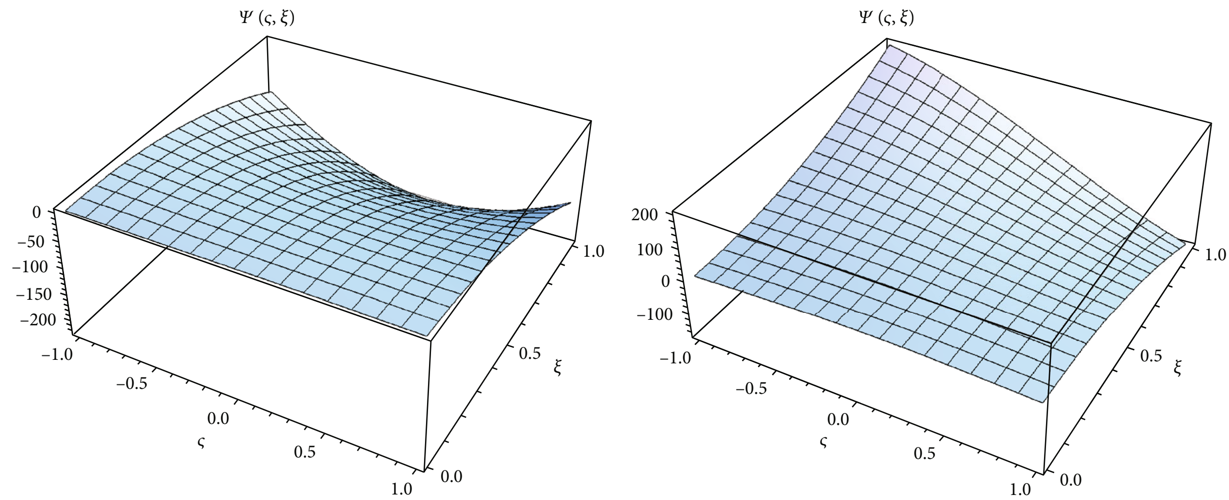
$$\begin{aligned} \Psi(\varsigma,\xi)&=\mu e^{i\varsigma}+ie^{i\varsigma}\left[\eta-\mu+\eta^2\mu\right]\eta^\alpha+\frac{i^2}{2}e^{i\varsigma}\left[\eta^3+2\mu+\eta^4\mu\right.\\ &\quad \left.+2\eta^2\mu\left(-2+\mu^2\right)+\eta\left(-2+3\mu^2\right)\right]\left(\frac{\eta^\alpha}{\Gamma(1+\alpha)}\right)^2+\cdots,\\ \Phi(\varsigma,\xi)&=\eta e^{i\varsigma}+ie^{i\varsigma}\left[\mu-\eta+\mu^2\eta\right]\eta^\alpha+\frac{i^2}{2}e^{i\varsigma}\left[\mu^3+2\eta+\mu^4\eta\right.\\ &\quad \left.+2\mu^2\eta\left(-2+\eta^2\right)+\mu\left(-2+3\eta^2\right)\right]\left(\frac{\eta^\alpha}{\Gamma(1+\alpha)}\right)^2+\cdots. \end{aligned} \tag{29}$$

By solving the above equations and using the approximate solution,

$$\begin{aligned} \Psi(\varsigma,\xi)&=\sum_{i=0}^{N}\Psi_i(\varsigma,\eta)\left(\frac{1}{n}\right)^i,\\ \Phi(\varsigma,\xi)&=\sum_{i=0}^{N}\Phi_i(\varsigma,\eta)\left(\frac{1}{n}\right)^i. \end{aligned} \tag{30}$$

## 5. Results and Discussion

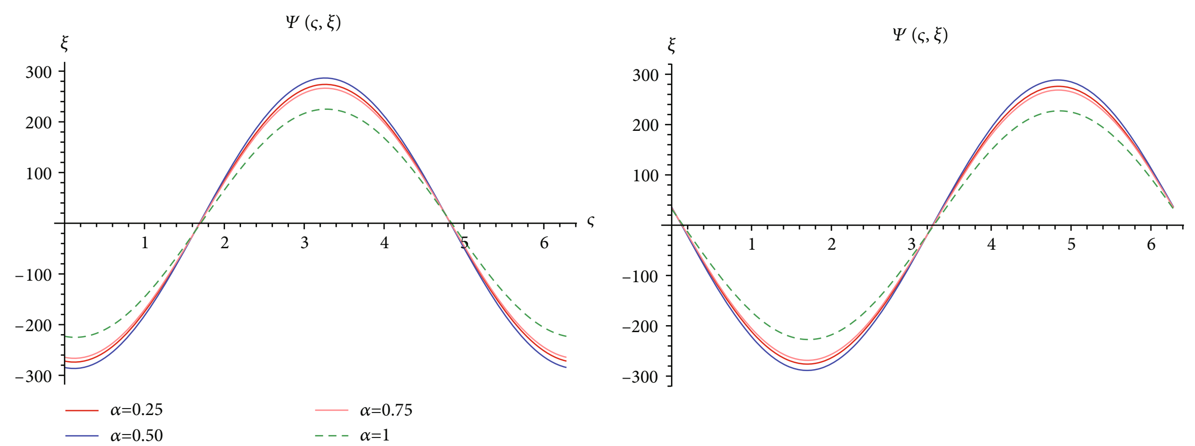
This segment presents the results and discussion for the analytical solution of the FKKE and FMTP. It is believed that after a small number of repetitions, the predicted results quickly approach the exact solution. Figure 1 have been demonstrated into two parts: (a) the real part of the surface solution and (b) the imaginary part of the surface solution at $-1\le\varsigma\le 1$ and $0\le\xi\le 1$ with $\alpha=1$. Figure 2 provides (a) real part of plot distribution and (b) imaginary part of plot distribution for $\alpha=0.25, 0.50, 0.75, 1$ at $\xi=1$. Similarly, Figure 3




(a) Real part of surface solution

(b) Imaginary part of surface solution

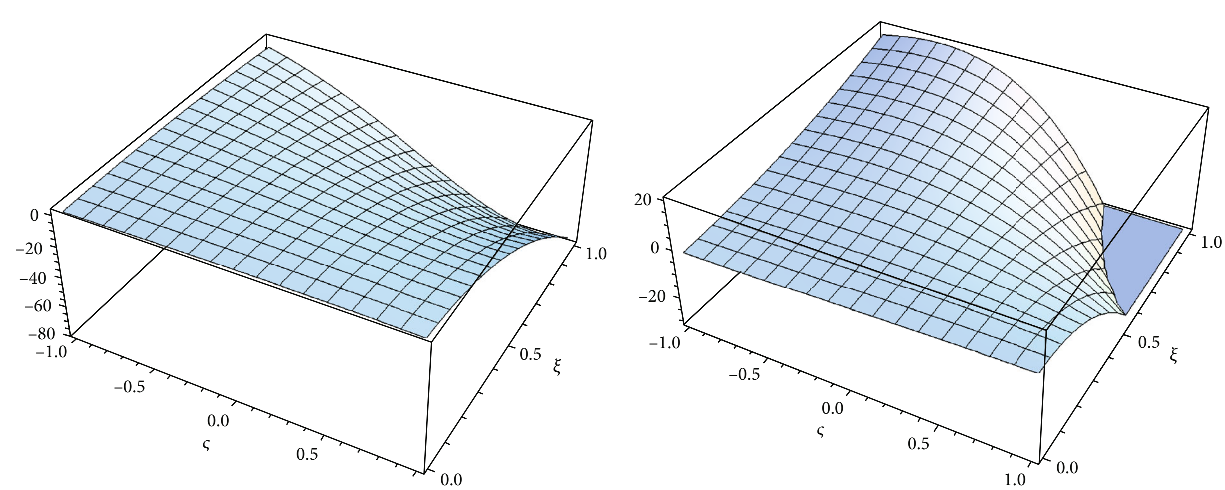
Figure 1: Surface solution of Equation (13) when $\alpha = 1$.




(a) Real part of plot distribution

(b) Imaginary part of plot distribution

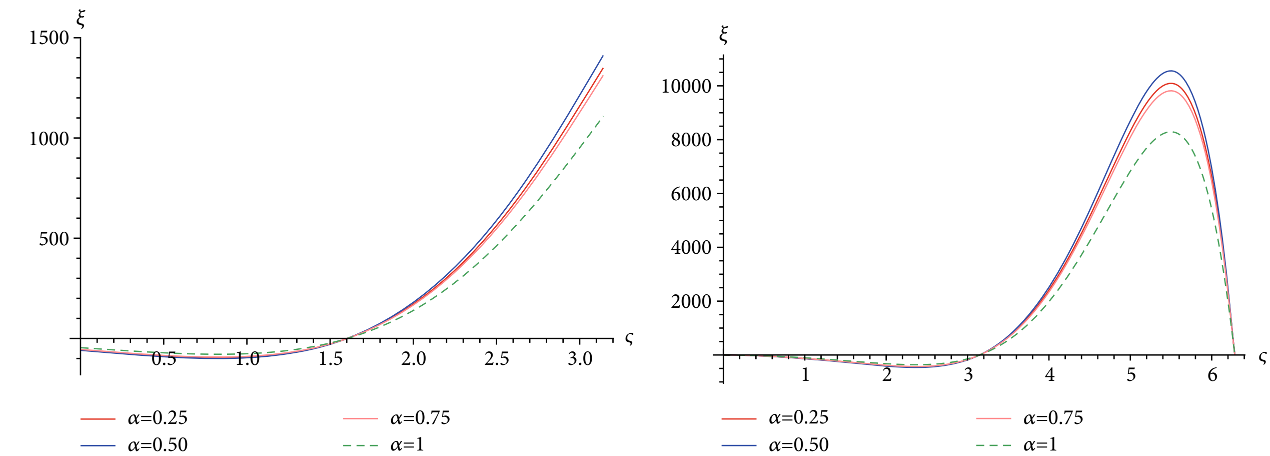
Figure 2: Plot distribution for different values of $\alpha$ at $\xi = 1$.




(a) Real part of $\Psi$ and $\Phi$

(b) Imaginary part of $\Psi$ and $\Phi$

Figure 3: Surface solution of Equation (22) when $\alpha = 1$.




(a) Real part of plot distribution for $\Psi$ and $\Phi$

(b) Imaginary part of plot distribution for $\Psi$ and $\Phi$

FIGURE 4: Plot distribution for different values of $\alpha$ at $\xi = 1$.

has been divided into two parts: (a) real part of $\Psi$ and $\Phi$ and (b) imaginary part of $\Psi$ and $\Phi$ at $-1 \leq \varsigma \leq 1$ and $0 \leq \xi \leq 5$ with $\alpha = 1$. Figure 4 provides (a) real part of plot distribution for $\Psi$ and $\Phi$ and (b) imaginary part of plot distribution for $\Psi$ and $\Phi$ for $\alpha = 0.25, 0.50, 0.75, 1$ at $\xi = 1$. We consider $\mu = \eta = 2$ for the graphical representation in both examples.

## 6. Conclusion

In the present work, we have successfully applied a two-scale approach for the analytical solution of the FKKE and FMTP that arises in quantum field theory. This two-scale approach is capable to handle the PDES of fractional order without any small perturbation theory. We converted the fractional derivative into classical form and implemented the scheme of HPM. The obtained results declare that the two-scale approach possesses a high level of accuracy. The leading novelty of the suggested approach consists of the following beauty that it can deal promptly without any discretization. We used Mathematica 11 to represent the graphical structures and the iterative results. The graphical representations and plot distributions reveal that this approach has an excellent performance in finding the analytical solution of the FKKE and FMTP. In the future, we believe that the two-scale approach is suitable and feasible for other fractional differential problems arising in science and engineering.

## Data Availability

All the data are available within the article.

## Conflicts of Interest

The authors claim to have no conflicts of interest.

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