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*Research Article*

# A Special Mutation Operator in the Genetic Algorithm for Fixed Point Problems

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Over the past century, the fixed point theory has emerged as a very useful and efficient tool in the study of nonlinear problems. This study introduced a progressed genetic algorithm (GA) based on a particular mutation operator applying on a subdivided search space where integer label and relative coordinates are used. This algorithm eventually categorizes each fixed point as its solution in appropriate set. Extensive computational experiments are conducted to assess the performance of the proposed technique with a standard GA for solving some nonlinear numerical functions from the literature.

## 1. Introduction

The fixed point theory was introduced scientifically in the 20th century. The basis of this theory is the principle of the Picard-Banach-Caccioppoli, which led to important lines of research and applications of this theory [1]. Fixed point theory is used and is important in various theoretical and practical fields. Theoretical fields such as variable and linear inequalities, theory of approximation, nonlinear analysis, equations, integrals, and differential components, theory of dynamic systems, fractals theory, financial mathematic, and game theory and applied fields such as biology, chemistry, management and economics, engineering in various disciplines, computer science, physics, geometry, astronomy, fluid mechanics, and image processing.

Riehl et al. [2] considered fixed points of discrete systems in large networks and optimized them. In this study, the equilibrium fixed points of discrete systems in distributed networks were considered; and by using appropriate partitions, they recursively decompose the main problem into a set of smaller and simpler problems and combine their solutions to gain a set of fixed points. The results showed the proposed algorithm with examples in two areas of calculating the number of fixed points in brain networks and finding the minimum energy combinations of network-based protein folding models.

Lael et al. [3] introduced a method for the Caristi-Kirk fixed point result for single mappings in conical metric spaces with a simple yet complete argument. The results of this research showed that the Caristi-Kirk fixed point in conical metric spaces turns into a result similar to traditional methods in reduced metric spaces. Bakery and Mohamed [4] proposed a new definition of the variable exponent of the Cesàro complex function space using the official power series. In this space, by utilizing s-numbers produced prequasi-ideal and then presented the topological and geometric structures of this class of ideal.

Metric space developed with the introduction of the Banach contraction principle and found more applications. One of the concepts presented in this space was the concept of F-metric [5]. Asif et al. [6] considered f-metric and create common fixed point results of Reich-type contraction. The results of this definition and its development showed that a unique common fixed point can be obtained if the contraction conditions are limited to only one closed ball subset of the total F-metric space. In addition, some significant implications are exploited from the significant results that characterize the fixed point outcomes for a single mapping. Among

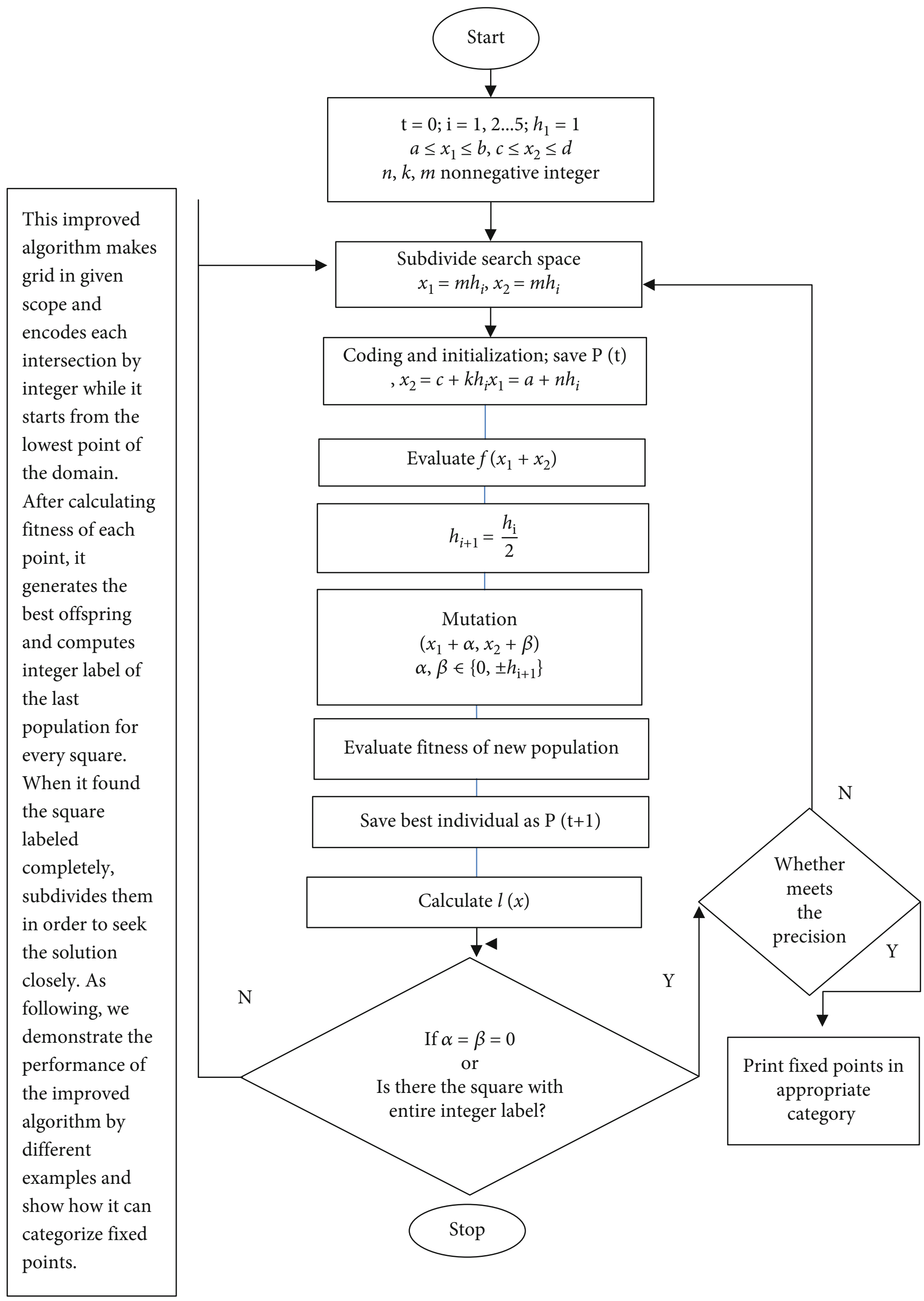


FIGURE 1: Improvement genetic algorithm flow chart.

nonlinear maps, nonexpansion maps are of particular importance. Expansion maps are maps that have a Lipschitz constant equal to one. Shukla and Panickar [7] assumed a nonexpansion map and they gained a number of fixed point theorems for these maps in geodetic spaces.

When we consider different optimization methods and compare them with the genetic algorithm, we find that the genetic algorithm (GA) by simulating the evolutionary process in organisms can provide an effective solution to find the optimal point in most cases [8, 9]. Mutation is used for avoiding of premature convergence and consequently escaping from local optimal. The GAs have been very successful in handling combinatorial optimization problems which are difficult [10].

Tang et al. [11], in order to prevent premature convergence in the GA, utilized the idea of flight behavior in the bird swarm algorithm to maintain population diversity and reduce the probability of falling to the local optimal. Mutation and the mutation probability ($p_m$) are important parameters in GAs. The mutation operator generates a new string by altering one or more bits of a string. By applying the mutation operator to a string, muting each bit of the string independently from the other bits is considered. So,

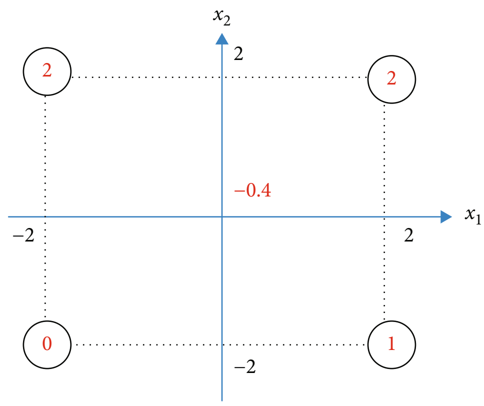


Figure 2: Initial population of $f_1$.

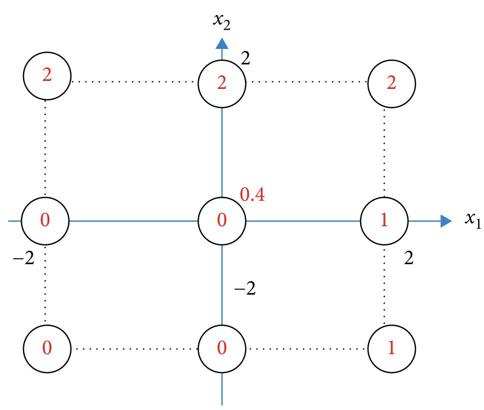


Figure 3: First generation of $f_1$.

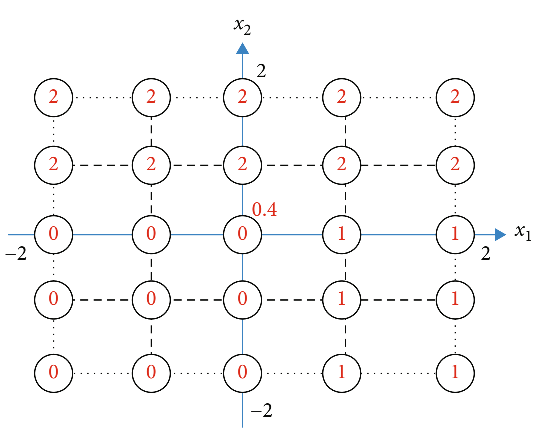


Figure 4: Second generation of $f_1$.

the mutation operator is more likely to significantly disrupt the allocation of trials to high order schemata than to low order ones. The efficiency of the mutation operator as a means of exploring the search space is questionable. A GA using mutation as the only genetic operator would be a random search that is biased toward sampling good hyper planes rather than poor ones [12].

The relationship between the genetic algorithm and the fixed points is a two-way relationship. In this sense, in some studies, fixed point properties have been used to improve the performance of genetic algorithms [13–18], and in some studies, updated models of genetic algorithms have been used to solve fixed point problems [19–22].

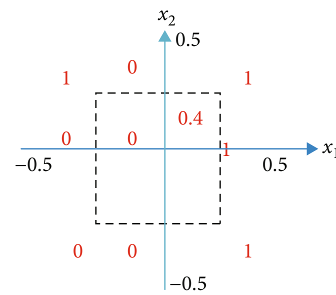


Figure 5: Third generation of $f_1$.

The concepts of fixed point and subdivision theory are used in some researches for improving GA. Gao et al. [13] introduced a GA based on fixed point algorithm and subdivision theory of continuous self-mapping in Euclidean space. They used subdivision of searching space and generate the integer labels and then these labels utilized for operators in GA. Pop [14] introduced a new developed GA based on the fixed point theorem and triangulation technique. Researcher utilized the crossover and mutation and increased the dimension genetic operators to avoid of premature convergence. Also, they utilized a custom increase dimension operator that expressively increases the total fitness.

Wolfram [23] used GA for controlling fixed point optimization. The researcher considers the floating point and fixed point display error in the optimization. Since both methods allow weighing between the theoretical and actual simulation, error occurred. Due to the script features of the simulation system, this can be easily automated. Zhang et al. [15] introduced triangulation theory into GA by the virtue of the concept of relative coordinate genetic coding and designed corresponding crossover and mutation operator. Hayes and Gedeon [17] considered the infinite population model for GA where the generation of the algorithm corresponds to a generation of a map. They showed that for a typical mixing operator, all the fixed points are hyperbolic.

Ren et al. [24] introduced the fixed point theory in PSO optimization and proposed an improved FP-PSO (fixed point PSO) algorithm. In the FP-PSO algorithm, the objective function is converted to a set of fixed point equations and the set of solutions obtained by the simple algorithm (SA) is used as the initial population of the PSO algorithm. Therefore, the remaining parameters can be obtained based on this choice of the classical PSO algorithm. Zhang et al. [16] introduced a GA that the population of individual is regarded as the triangulation of the point. They used the vertex label information of the individual simplex of individual to design selection operator, crossover, and mutation operators.

Zhang and Shang [25] proposed an improved multiobjective genetic algorithm based on Pareto front and fixed

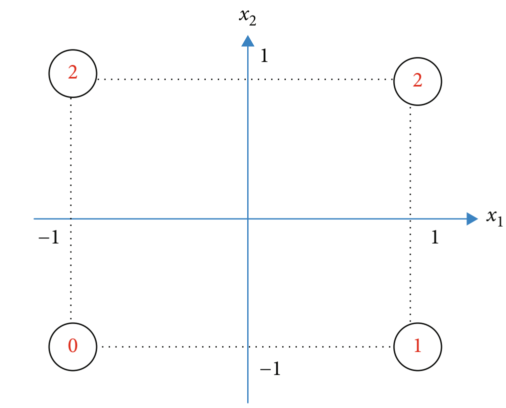


Figure 6: Initial population of $f_2$.

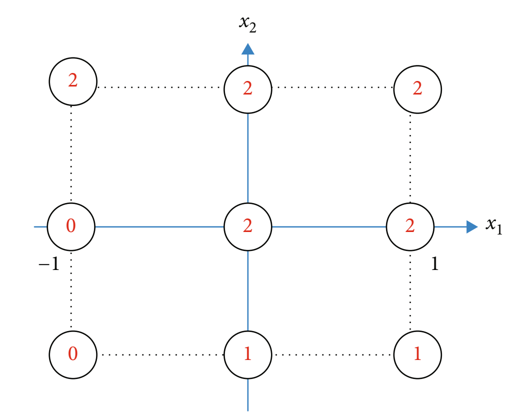


Figure 7: First generation of $f_2$.

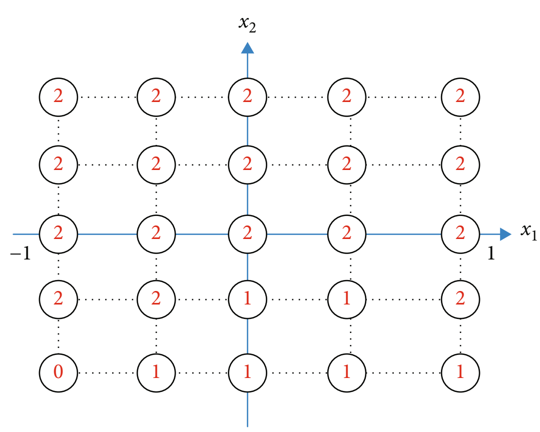


Figure 8: Second generation of $f_2$.

point theory. In this algorithm, the fixed point theory is introduced to a multiobjective optimization questions, and K1 triangulation is carried on to solutions for the weighting function constructed by all subfunctions, so the optimal problems are transferred to fixed point problems. Yang et al. [11] introduced the van der Laan-Talman algorithm to the GA to design convergence criteria objectively and to solve the convergence problem in the later period. The par-

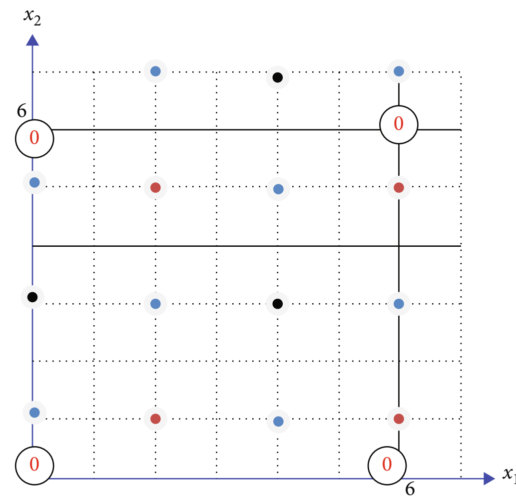


Figure 9: First generation of $f_3$.

allel GA of multibody model vehicle suspension optimization implemented through establishing the interface between ADAMS software and the GA. Wright et al. [26] developed a dynamical system model of a GA that uses gene pool crossover, proportional selection, and mutation. They introduced the concept of bistability for GA and they showed that it is possible for a GA to have two stable fixed points on a single-peak fitness landscape. These can correspond to a metastable finite populations.

Gedeon et al. [27] showed that for an arbitrary selection mechanism and a typical mixing operator, their composition has finitely many fixed points. Qian et al. [28] proposed a GA to treat with such constrained integer programming problem for the sake of efficiency. Then the fixed point evolved (E)-UTRA PRACH detector presented, which further underlines the feasibility and convenience of applying this methodology to practice. Wright et al. [29] considered the dynamic system model of Wright and Vose [18] and showed that with the increase of mutation percentage, the hyperbolic asymptotic fixed points are directed towards the simplex, and the hyperbolic unstable asymptotic fixed points are directed out of the simplex.

Thianwan [30] introduced a new iteration scheme of mixed type for two asymptotically nonexpansive self-mappings and two asymptotically nonexpansive non-self-mappings. After introducing this method, some convergence theorems based on the proposed iterative scheme in uniformly convex Banach spaces have been presented, proved, and compared with previous results on some problems. A new mixed type iteration process for approximating a common fixed point from two asymptotic self-expansion mappings and two nonasymptotic self-expansion mappings was introduced by Thianwan [31]. In the continuation of this research, a convergence theorem was proposed in a uniform convex hyperbolic space and using the introduced method, the presented results showed that the presented model has better results than the previous models.

This paper investigates the concepts of fixed point and square labels with a special mutation operator for improving

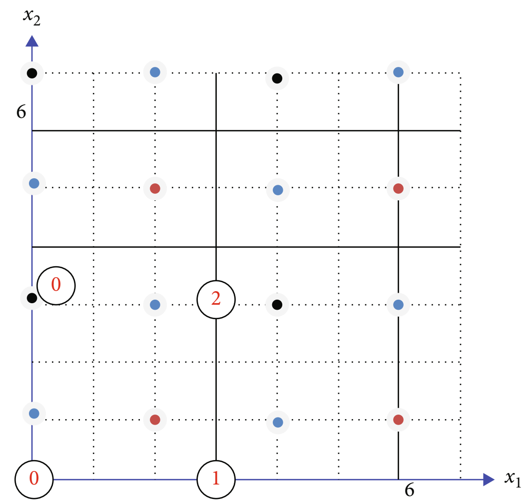


FIGURE 10: Second generation of $f_3$.

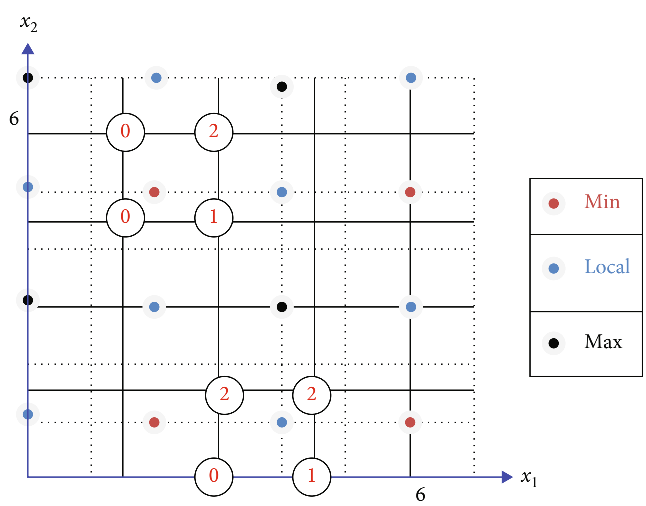


FIGURE 11: Third generation of $f_3$.

performance of the GA. The performance of proposed algorithm on some nonlinear numerical optimization problems shows this algorithm converge to a reasonable results in a few numbers of generations.

## 2. Construing of Optimal Problems as Fixed Point Problem

In genetic algorithm like other evolutionary algorithm, its optimal solutions are points that the algorithm improves, keeps, or returns to them after a certain number of iterations because these points meet the required criteria of the algorithm. When infinite population is used in GA, the algorithm must converge, and the average population fitness increase from one generation to the next. The consequence for a finite population simple genetic algorithm (SGA) is that the expected population fitness increases from one generation to the next. Moreover, the only stable fixed point of the expected next population operator corresponds to the population consisting entirely of the optimal string. This result is then extended by way of a perturbation argument to allow nonzero mutation. Supposing that algorithm is searching a point $x$, which can make continuous function of $f$ to achieve its minimum. The necessary and sufficient condition of extreme point is that this point gradient is 0, that is, $\nabla f(x) = 0$.

For self-mapping $g : \mathbb{R}^n \longrightarrow \mathbb{R}^n$, we say, $x \in \mathbb{R}^n$ is a fixed point of $g$ if $g(x) = x$, then we can convert the solution of zero point problems to fixed point ones of function $g(x) = x + \nabla f(x)$.

## 3. Subdivision and Relative Coordinates

Supposing that definition domain of $f(x_1, x_2)$ is that $a \leq x_1 \leq b, c \leq x_2 \leq d$ and dividing the domain into many squares with two groups of straight lines of $\{x_1 = mh_i\}, \{x_2 = mh_i\}$ in which $m$ is a not negative integer and $h_i$ is a positive quantity relating to precision of the problem; as a result, we can code each point of intersection as $x_1 = a + nh_i$, $x_2 = c + kh_i$ where $n, k$ are not negative integers, so $(n, k)$ is called the relative coordinates of points. Consequently, by changing $n, k$ relative coordinates of each point in search space is determined.

Supposing that $x$ is a vertex of a square that will be labeled as the following [23]:

$$l(x) = \begin{cases} 0, & g(x_1) - x_1 \geq 0, g(x_2) - x_2 \geq 0, \\ 1, & g(x_1) - x_1 < 0, g(x_2) - x_2 \geq 0, \\ 2, & g(x_2) - x_2 < 0. \end{cases} \quad (1)$$

The square with all different kinds of integer label is called a completely labeled unite, when $h_i \longrightarrow 0$ within iteration stages, vertices of that square approximately converge to one point which is a fixed point.

## 4. Mutation Operator

For each point coded $(n, k)$, the GA is trying to improve it to reach optimal solution by mutation operator searching all points surrounding it in certain step determined by $h_{i+1}$. Thus, mutation probability $p_m = 1$.

For instance, $(n, k)$ in $P(0)$, initial population, addressing $(x_1 + nh_i, x_2 + kh_i)$ will be changed as $(x_1 + \alpha, x_2 + \beta)$, $\alpha, \beta \in \{0, \pm h_{i+1}\}$. Subsequently, the algorithm saves the best-mutated individual among all possible offspring. Therefore, this operator produces new population located on intersection of the next grid. Because of this, coming squares are specified to evaluate and label. Furthermore, the next generation is producing from the previous one. For instance, in example 1, we show that the operator mutates (-2, 2) to (-2, 0), (2, 0), and (0, 0) in the given scope, then (0, 0) is selected as the best offspring.

## 5. The Improved Genetic Algorithm

This improved algorithm makes grid in given scope and encodes each intersection by integer while it starts from the lowest point of the domain. After calculating fitness of each point, it generates the best offspring and computes integer label of the last population for every square. When it found the square labeled completely, it subdivides them in order to seek the solution closely (the process of this method is shown in Figure 1). As following, we demonstrate the performance of the improved algorithm by different examples and show how it can categorize fixed points.

## 6. Computational Experiments

In this section, we present the computational results of the proposed algorithm for solving some nonlinear numerical functions.

*6.1. Test Problem 1.* This function is a continuous and unimodal function taken from [32]. The optimization problem is

$$\min \mathrm{f}(x_1, x_2) = {x_1}^2 + (x_2 - 0.4)^2 - 2 < x_\mathrm{i} < 2, i = 1, 2. \quad (2)$$

The function achieves the minimum when $x_1 = 0$ and $x_2 = 0.4$. In this example, $h_i \in \{4, 2, 1, 0.5, 0.25\}$, mutation probability $p_m = 1$. The completely label square obtains through the iteration, the search scope for both $x_1$ and $x_2$ are (-2, 2), (0, 2), (0, 1), and finally (0, 0.5), respectively (as show from Figures 2–5). During iterations, squares are contracting to (0, 0.5) gradually, if we started from $h_1 = 1$, we got closer answer, i.e., (0, 0.4).

*6.2. Test Problem 2.* The optimization problem considered here is also a nonlinear function problem taken from [32]. The problem is

$$\min\ \mathrm{f}(x_1, x_2) = {x_1}^3 + {x_2}^3 - 1 < x_\mathrm{i} < 1, i = 1, 2. \quad (3)$$

The best obtained solution is $x_1 = -1$ and $x_2 = -1$ with $f(x_1, x_2) = -2$. In this example, $h_i \in \{2, 1, 0.5\}$ mutation probability $p_m = 1$. The completely label square obtains through the iteration, the search scope for both $x_1$ and $x_2$ are (-1, 1), (-1, 0), and (-1, -0.5).

During iterations, squares are contracting to (-1, -1) gradually, which is a boundary point for this increasing function (as show from Figures 6–8).

*6.3. Test Problem 3.* In this problem, we choose a nonlinear optimization problem with two continuous variables. It was also taken from [32].

$$\min \mathrm{f}(x_1, x_2) = \cos \frac{\pi}{2} x_1 - \sin \frac{\pi}{2} x_2 - 7 < x_\mathrm{i} < 7, i = 1, 2. \quad (4)$$

This multimodal function has many local optimal in its domain. The GA keeps each local and global optimal one found in squares labeled completely. In this example, for $h_i \in \{6, 3, 1.5, 0.75\}$ while mutation probability $p_m = 1$, as shown in figure 7, these points can be gotten. Three following generation have been shown in the first quarter of the coordinates system (see Figures 9–11).

## 7. Conclusion

In this paper, we show that labeling technique and the mutation operator producing later generation on the next gridding points have some advantages. First of all, making network on search space provides integer-coding system that simplifies locating of all individuals in the future and present generation, so we can easily label each vertex of square and investigate the possibility of finding every optimal solution. Moreover, the algorithm is capable of starting from a fixed point located in domain boundary; hence, it overcomes weakness of man-made initial point. Second, finding square completely labeled avoids missing local answers because the algorithm focuses on such squares when it is trying to seek global minimum inside of not entirely labeled squares or in other completely ones. Third, this mutation operator works systematically in order to estimate better solution. In other words, it does not work so randomly that loses possible fixed points in an area as it is clear in Figure 3. In addition, the algorithm moves toward obtaining the best solution among likely offspring. Consequently, it performs more quickly and effectively because it eliminates unneeded iterations and calculations. Finally, it categorizes different fixed points at the end of its run.

## Data Availability

Specific data has not been used for this research and only a few numerical functions whose references are given in the text have been used.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

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