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Research Article

Impact of Fractional Derivative and Brownian Motion on the Solutions of the Radhakrishnan-Kundu-Lakshmanan Equation

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The fractional-stochastic Radhakrishnan-Kundu-Lakshmanan equation (FSRKLE) is considered here. To attain new hyperbolic, elliptic, rational, and trigonometric stochastic-fractional solutions, we use two various methods such as the sine-cosine and the Jacobi elliptic function methods. The solutions acquired are important in understanding some interesting physical phenomena due to the significance of the Radhakrishnan-Kundu-Lakshmanan equation in designing the propagation of solitons through an optical fiber. Furthermore, we graph some of the obtained solutions in 3D to display the influence of fractional derivative and multiplicative noise on these solutions. Finally, we show that when the order of fractional derivative decreases, the surface shrinks, while the multiplicative noise stabilizes the solutions of FSRKLE a round zero.

1. Introduction

Partial differential equations (PDEs) are found in several areas of applied science, including quantum mechanics, plasma physics, nonlinear optics, surface of water waves, hydrodynamics, molecular biology, fluid dynamics, elastic media, and biology. Obtaining solutions of PDEs is crucial for understanding physical phenomena. Therefore, many effective methods, including exp-function method [1], auxiliary equation [2], Darboux transformation [3], sine-cosine [4], Jacobi elliptic function [5], exp $(-\phi(\varsigma))$-expansion [6], sine-Gordon expansion [7], (G'/G)-expansion [8–10], generalized Kudryashov [11], perturbation [12–14], extended trial equation [15, 16], Jacobi elliptic function [17, 18], Riccati equation [19], tanh-coth [20], homotopy perturbation [21], modified decomposition [22], and F-expansion [23], have been constructed to attain exact solutions of PDEs.

Researchers and scientists have focused their attention over the last two decades on fractional differential equations (FDEs) that have been found to be more precise than classical differential equations in explaining complex physical phenomena in the real life. The idea of fractional derivative has been used to define various phenomena including fluid dynamics porous medium, signal processing, viscoelastic materials, ocean wave, electromagnetism, photonic, chaotic systems, wave propagation, optical fiber communication, plasma physics, and nuclear physics. Recently, Atangana and Goufo [24] have suggested the new conformable fractional derivative called beta-derivative. From this point, let us define the Atangana conformable derivative (ACD) for the function $\psi : (0,\infty) \longrightarrow \mathbb{R}$ of order $\beta \in (0, 1]$ as follows:

$$\mathbb{D}_x^\beta \psi(x) = \lim_{\varepsilon \longrightarrow 0} \frac{\psi\left(x + \varepsilon\left(x + (1/\Gamma(\beta))\right)^{1-\beta}\right) - \psi(x)}{\varepsilon}. \quad (1)$$

The ACD satisfies the following properties for any constant a and b: (1) $\mathbb{D}_x^\beta[a\varphi(x) + b\psi(x)] = a\mathbb{D}_x^\beta\varphi(x) + b\mathbb{D}_x^\beta\psi(x)$, (2) $\mathbb{D}_x^\beta[a] = 0$, (3) $\mathbb{D}_x^\beta\psi(\theta) = (x + (1/\Gamma(\beta)))^{1-\beta} d\psi/dx$, (4) If $\theta = a/\beta(x + (1/\Gamma(\beta)))^\beta$, then $\mathbb{D}_x^\beta\psi(\theta) = a d\psi/d\theta$.

Stochastic partial differential equations (SPDEs), on the other hand, have been widely addressed as theoretical equations for spatial-temporal physical, chemical, and biological systems related to random perturbations. The significance of involving stochastic impacts in complex system modeling has been emphasized. For example, there is gaining awareness in using SPDEs to mathematically model complex phenomena in information systems, condensed matter physics, biology, climate systems, electrical and mechanical engineering, materials sciences, and finance.

It is worth noting that two forms widely utilized for stochastic integral are Itô and Stratonovich [25]. Modeling problems primarily determine which form is acceptable; however, once that form is selected, an equivalent equation of the other form can be produced using the same solutions. As a result, the following relationship can be utilized to switch between Itô (written as $\int_0^t \phi dW$) and Stratonovich (written as $\int_0^t \phi \circ dW$):

$$\int_0^t \sigma\phi(s)dW(s) = \int_0^t \sigma\phi(s) \circ dW(s) - \frac{\sigma^2}{2}\int_0^t \phi(s)ds, \tag{2}$$

where $W(t)$ is a Brownian motion (BM).

To satisfy a higher degree of quality agreement, the following stochastic Radhakrishnan-Kundu-Lakshmanan equation (FSRKLE) [26–28] is considered:

$$\begin{aligned} &id\varphi + \Big[\gamma_1 \mathbb{D}_{xx}^{\beta}\varphi - i\gamma_2 \mathbb{D}_{x}^{\beta}\varphi + \gamma_3|\varphi|^2\varphi - i\gamma_4\varphi\mathbb{D}_{x}^{\beta}\left(|\varphi|^2\right) \\ &\quad --i\gamma_5\mathbb{D}_{x}^{\beta}\left(|\varphi|^2\varphi\right) + i\gamma_6\mathbb{D}_{xxx}^{\beta}\varphi\Big]dt + i\sigma\varphi \circ dW = 0, \end{aligned} \tag{3}$$

where $\varphi \in \mathbb{C}$, γ_k for $k = 1, 2, 3, 4, 5, 6$ are constants and σ is the noise strength and $\varphi \circ dW$ is multiplicative Brownian motion in the Stratonovich sense. Recently, many investigators have created exact solutions of FSRKLE (3), with $\beta = 0$ and $\sigma = 0$, using different methods such as extended simple equation method [29], first integral method [30], sine-cosine method [31], Lie group analysis [32], and trial equation method [33].

The motivation of this article is to attain the exact solutions for FSRKLE (3). We use two separate approaches, the sine-cosine and the Jacobi elliptic function methods, to provide a wide range of solutions, including hyperbolic, trigonometric, rational, and elliptic functions. The acquired solutions are helpful for understanding several fascinating scientific events because of the significance of the RKL in describing the propagation of solitons through an optical fiber. Also, by creating 3D representations of the obtained FSRKLE (3) solutions, we examine the effect of BM on these solutions.

The article is in the following format: in Section 2, we determine the wave equation of the FSRKLE (3) by applying a suitable wave transformation. To develop the analytical solutions for the FSRKLE in Section 3, we use two different approaches (3). In Section 4, the impact of the BM on the solutions obtained is examined. The final section of the document is the conclusion.

2. Wave Equation for FSRKLE

To obtain the wave equation of the FSRKLE (3), the following transformation is utilized:

$$\begin{aligned} \varphi(x,t) &= \psi(\zeta)e^{\left(iq(x,t)-\sigma W(t)-\sigma^2 t\right)}, \\ \zeta &= \frac{1}{\beta}\left(x + \frac{1}{\Gamma(\beta)}\right)^{\beta} - \nu t, \\ q(x,t) &= -\frac{k}{\beta}\left(x + \frac{1}{\Gamma(\beta)}\right)^{\beta} + \omega t, \end{aligned} \tag{4}$$

where the function ψ is deterministic and ν,k, and ω are unknown constants. Putting Equation (4) into Equation (3) and utilizing

$$\begin{aligned} d\varphi &= \left[\left(-\nu\psi' + i\omega\psi + \frac{1}{2}\sigma^2\psi - \sigma^2\psi\right)dt - \sigma\psi dW\right]e^{\left[iq(x,t)-\sigma W(t)-\sigma^2 t\right]}, \\ &= \left[\left(-\nu\psi' + i\omega\psi\right)dt - \sigma\psi \circ dW\right]e^{\left(iq(x,t)-\sigma W(t)-\sigma^2 t\right)}, \end{aligned} \tag{5}$$

where $(1/2)\sigma^2\psi$ is the Itô correction term, and

$$\begin{aligned} \mathbb{D}_{x}^{\beta}\varphi &= \left(\psi' - ik\psi\right)e^{\left(iq(x,t)-\sigma W(t)-\sigma^2 t\right)}, \\ \mathbb{D}_{xx}^{\beta}\varphi &= \left[\psi'' - 2ik\psi' - k^2\psi\right]e^{\left(iq(x,t)-\sigma W(t)-\sigma^2 t\right)}, \\ \mathbb{D}_{xxx}^{\beta}\varphi &= \left[\psi''' - 3ik\psi'' - 3k^2\psi' + ik^3\psi\right]e^{\left(iq(x,t)-\sigma W(t)-\sigma^2 t\right)} \\ \varphi\mathbb{D}_{x}^{\beta}\left(|\varphi|^2\right) &= 2\psi^2\psi' e^{\left(iq(x,t)-3\sigma W(t)-3\sigma^2 t\right)}, \\ \mathbb{D}_{x}^{\beta}\left(|\varphi|^2\varphi\right) &= \left(3\psi^2\psi' - ik\psi^3\right)e^{\left(iq(x,t)-3\sigma W(t)-3\sigma^2 t\right)}, \end{aligned} \tag{6}$$

we get for imaginary part

$$\begin{aligned} &\gamma_6 k^3\psi''' - \left(3\gamma_6 k^2 + \gamma_2 + 2k\gamma_1 + \nu\right)\psi' \\ &\quad - (3\gamma_5 + 2\gamma_4)\psi^2\psi' e^{\left(-2\sigma W(t)-2\sigma^2 t\right)} = 0, \end{aligned} \tag{7}$$

and for real part

$$\begin{aligned} &(\gamma_1 + 3k\gamma_6)\psi'' - \left(k^2\gamma_1 + k\gamma_2 - k^3\gamma_6\right)\psi \\ &\quad + (\gamma_3 - k\gamma_5)\psi^3 e^{\left(-2\sigma W(t)-2\sigma^2 t\right)} = 0. \end{aligned} \tag{8}$$

Taking expectation $\mathbb{E}(\cdot)$ on both sides for Equations (7) and (8) and using

$$\mathbb{E}\left(e^{\sigma W(t)}\right) = e^{\left(\sigma^2/2\right)t}, \tag{9}$$

we have

$$\gamma_6 k^3 \psi''' - \left(3\gamma_6 k^2 + \gamma_2 + 2k\gamma_1 + \nu\right)\psi' - (3\gamma_5 + 2\gamma_4)\psi^2\psi' = 0, \tag{10}$$

$$(\gamma_1 + 3k\gamma_6)\psi'' - \left(\omega + k^2\gamma_1 + k\gamma_2 + k^3\gamma_6\right)\psi - (k\gamma_5 - \gamma_3)\psi^3 = 0. \tag{11}$$

Integrating Equation (10), we get

$$\gamma_6 k^3 \psi'' - \left(3\gamma_6 k^2 + \gamma_2 + 2k\gamma_1 + \nu\right)\psi - \left(\gamma_5 + \frac{2}{3}\gamma_4\right)\psi^3 = 0. \tag{12}$$

We obtain the next constraint conditions where the same function ψ achieves both Equations (11) and (12):

$$\frac{\gamma_1 + 3k\gamma_6}{\gamma_6} = \frac{\omega + k^2\gamma_1 + k\gamma_2 + k^3\gamma_6}{3\gamma_6 k^2 + \gamma_2 + 2k\gamma_1 + \nu} = \frac{3(k\gamma_5 - \gamma_3)}{3\gamma_5 + 2\gamma_4}, \tag{13}$$

whenever

$$\gamma_3 = -\frac{3\gamma_5\gamma_1 + \gamma_1\gamma_4 + 6k\gamma_6\gamma_5 + 3k\gamma_6\gamma_4}{3\gamma_6}, \tag{14}$$

$$\omega = \frac{8k^3\gamma_6^2 + 8k^2\gamma_1\gamma_6 + 2k\gamma_1^2 + 2k\gamma_2\gamma_6 + \gamma_1\gamma_2 + \nu(3k\gamma_6 + \gamma_1)}{\gamma_6}. \tag{15}$$

Plugging Equation (14) into Equation (11), we have the wave equation as follows:

$$\psi'' - \hbar_1\psi^3 - \hbar_2\psi = 0, \tag{16}$$

where

$$\hbar_1 = \frac{3\gamma_5\gamma_1 + \gamma_1\gamma_4 + 9k\gamma_6\gamma_5 + 3k\gamma_6\gamma_4}{3\gamma_6(\gamma_1 + 3k\gamma_6)}, \tag{17}$$

$$\hbar_2 = \frac{9k^3\gamma_6^2 + 9k^2\gamma_1\gamma_6 + 2k\gamma_1^2 + 3k\gamma_2\gamma_6 + \gamma_1\gamma_2 + \nu(3k\gamma_6 + \gamma_1)}{\gamma_6(\gamma_1 + 3k\gamma_6)}. \tag{18}$$

3. The Exact Solutions of the FSRKLE

We employ two various methods such as the Jacobi elliptic function [18] and sine-cosine [4], to determine the exact solutions to Equation (16). As a consequence, we can obtain the solutions of the FSRKLE (3).

3.1. Jacobi Elliptic Function Method. We suppose the solutions of Equation (16) has the type

$$\psi(\zeta) = a + b\,sn(\theta\zeta), \tag{19}$$

where $sn(\theta\zeta) = sn(\theta\zeta, m)$, for $0 < m < 1$, is Jacobi elliptic sine function and a, b, and θ are undefined constants. Differentiate Equation (19) twice, we get

$$\psi''(\zeta) = -\left(m^2 + 1\right)b\theta^2 sn(\theta\zeta) + 2m^2 b\theta^2 sn^3(\theta\zeta). \tag{20}$$

Putting Equations (19) and (20) into Equation (16), we obtain

$$\begin{aligned} &\left(2m^2 b\theta^2 - \hbar_1 b^3\right)sn^3(\theta\zeta) - 3\hbar_1 a b^2 sn^2(\theta\zeta) \\ &\quad - \left[\left(m^2 + 1\right)b\theta^2 + 3\hbar_1 a^2 b + \hbar_2 b\right]sn(\theta\zeta) \\ &\quad - \left(\hbar_1 a^3 + a\hbar_2\right) = 0. \end{aligned} \tag{21}$$

Equating each coefficient of $[sn(\theta\zeta)]^n$ to zero, we have for $n = 0, 1, 2, 3,$

$$\begin{aligned} \hbar_1 a^3 + a\hbar_2 &= 0, \\ \left(m^2 + 1\right)b\theta^2 + 3\hbar_1 a^2 b + \hbar_2 b &= 0, \\ 3\hbar_1 a b^2 sn^2 &= 0, \\ 2m^2 b\theta^2 - \hbar_1 b^3 &= 0. \end{aligned} \tag{22}$$

The outcomes of solving the previous equations are

$$\begin{aligned} a &= 0, \\ b &= \pm\sqrt{\frac{-2m^2\hbar_2}{(m^2 + 1)\hbar_1}}, \\ \theta^2 &= \frac{-\hbar_2}{(m^2 + 1)}. \end{aligned} \tag{23}$$

As a result, using (19), the solution of Equation (16) is

$$\psi(\zeta) = \pm\sqrt{\frac{-2m^2\hbar_2}{(m^2 + 1)\hbar_1}}\,sn\left(\sqrt{\frac{-\hbar_2}{(m^2 + 1)}}\zeta\right). \tag{24}$$

Hence, the exact solution of the FSRKLE (3) is

$$\varphi(x, t) = \pm\sqrt{\frac{-2m^2\hbar_2}{(m^2 + 1)\hbar_1}}\,sn\left(\sqrt{\frac{-\hbar_2}{(m^2 + 1)}}\left(\frac{1}{\beta}\left(x + \frac{1}{\Gamma(\beta)}\right)^\beta - \nu t\right)\right)e^{\left[iq(x,t) - \sigma W(t) - \sigma^2 t\right]}, \tag{25}$$

for $\hbar_2 < 0$ and $\hbar_1 > 0$. If $m \longrightarrow 1$, then solution (25) tends to

$$\varphi(x, t) = \pm\sqrt{\frac{-\hbar_2}{\hbar_1}}\tanh\left(\sqrt{\frac{-\hbar_2}{2}}\left(\frac{1}{\beta}\left(x + \frac{1}{\Gamma(\beta)}\right)^\beta - \nu t\right)\right)e^{\left[iq(x,t) - \sigma W(t) - \sigma^2 t\right]}. \tag{26}$$

Analogously, we can replace sn in (19) with $cn(\xi) = cn(\xi, m)$ and $dn(\xi, m) = dn(\xi, m)$ to obtain the following solutions of Equation (16):

$$\begin{aligned}\psi(\zeta) &= \pm\sqrt{\frac{-2m^2\hbar_2}{(2m^2-1)\hbar_1}}cn\left(\sqrt{\frac{\hbar_2}{(2m^2-1)}}\zeta\right),\\ \psi(\zeta) &= \pm\sqrt{\frac{-2m^2\hbar_2}{(2-m^2)\hbar_1}}dn\left(\sqrt{\frac{\hbar_2}{(2-m^2)}}\zeta\right).\end{aligned} \tag{27}$$

Thus, the exact solutions of the FSRKLE (3) are as follows:

$$\varphi(x,t) = \pm\sqrt{\frac{-2m^2\hbar_2}{(2m^2-1)\hbar_1}}cn\left(\sqrt{\frac{-\hbar_2}{(2m^2-1)}}\left(\frac{1}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta}-\nu t\right)\right)e^{\left[iq(x,t)-\sigma W(t)-\sigma^2 t\right]}, \tag{28}$$

for $\hbar_2/(2m^2-1)>0$, $\hbar_1<0$, and

$$\varphi(x,t) = \pm\sqrt{\frac{-2m^2\hbar_2}{(2-m^2)\hbar_1}}dn\left(\sqrt{\frac{\hbar_2}{(2-m^2)}}\left(\frac{1}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta}-\nu t\right)\right)e^{\left[iq(x,t)-\sigma W(t)-\sigma^2 t\right]}, \tag{29}$$

for $\hbar_2>0$, $\hbar_1<0$, respectively. If $m \longrightarrow 1$, then the Equations (28) and (29) tends to

$$\varphi(x,t) = \pm\sqrt{\frac{-2\hbar_2}{\hbar_1}}\,\mathrm{sech}\left(\sqrt{\hbar_2}\left(\frac{1}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta}-\nu t\right)\right)e^{\left(iq(x,t)-\sigma W(t)-\sigma^2 t\right)}, \tag{30}$$

for $\hbar_2>0$, $\hbar_1<0$.

3.2. Sine-Cosine Method. Suppose the solution ψ of Equation (16) takes the form

$$\psi(\zeta) = A\mathbb{Y}^n, \tag{31}$$

where

$$\mathbb{Y} = \cos(B\zeta) \text{ or } \mathbb{Y} = \sin(B\zeta). \tag{32}$$

Setting Equation (31) into Equation (16), we get

$$-AB^2\left[-n^2\mathbb{Y}^n + n(n-1)\mathbb{Y}^{n-2}\right] - \hbar_1 A^3\mathbb{Y}^{3n} - \hbar_2 A\mathbb{Y}^n = 0, \tag{33}$$

rewriting the above equation

$$\left(\hbar_2 A - AB^2n^2\right)\mathbb{Y}^n + n(n-1)AB^2\mathbb{Y}^{n-2} + \hbar_1 A^3\mathbb{Y}^{3n} = 0. \tag{34}$$

Balancing the term of $\mathbb{Y}$ in Equation (34), we obtain

$$n-2 = 3n \Rightarrow n = -1. \tag{35}$$

Plugging Equation (35) into Equation (34),

$$\left(\hbar_2 A - AB^2\right)\mathbb{Y}^{-1} + \left(\hbar_1 A^3 + 2AB^2\right)\mathbb{Y}^{-3} = 0. \tag{36}$$

We get by setting each coefficient of $\mathbb{Y}^{-3}$ and $\mathbb{Y}^{-1}$ equal to zero

$$\hbar_2 A - AB^2 = 0, \tag{37}$$

$$\hbar_1 A^3 + 2AB^2 = 0. \tag{38}$$

By solving Equations (37) and (38), we get

$$B = \sqrt{\hbar_2} \text{ and } A = \sqrt{\frac{-2\hbar_2}{\hbar_1}}. \tag{39}$$

Hence, the solution of Equation (16) is

$$\psi(\zeta) = A\sec(B\zeta) \text{ or } \psi(\zeta) = A\csc(B\zeta). \tag{40}$$

Depending on the sign of $\hbar_1$ and $\hbar_2$, there are numerous cases:

Case 1. If $\hbar_2>0$ and $\hbar_1<0$, then FSRKLE (3) has the solutions

$$\varphi(x,t) = \sqrt{\frac{-2\hbar_2}{\hbar_1}}\sec\left[\sqrt{\hbar_2}\left(\frac{1}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta}-\nu t\right)\right]e^{\left(i\left(-(k/\beta)(x+(1/\Gamma(\beta)))^{\beta}+\omega t\right)-\sigma W(t)-\sigma^2 t\right)}, \tag{41}$$

or

$$\varphi(x,t) = \sqrt{\frac{-2\hbar_2}{\hbar_1}}\csc\left[\sqrt{\hbar_2}\left(\frac{1}{\beta}x^{\beta}-\nu t\right)\right]e^{\left(i\left(-(k/\beta)(x+(1/\Gamma(\beta)))^{\beta}+\omega t\right)-\sigma W(t)-\sigma^2 t\right)}. \tag{42}$$

Case 2. If $\hbar_2<0$ and $\hbar_1<0$, then the analytical solutions of FSRKLE (3) have the form

$$\varphi(x,t) = i\sqrt{\frac{2\hbar_2}{\hbar_1}}\sec h\left[\sqrt{-\hbar_2}\left(\frac{1}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta}-\nu t\right)\right]e^{\left(i\left(-(k/\beta)(x+(1/\Gamma(\beta)))^{\beta}+\omega t\right)-\sigma W(t)-\sigma^2 t\right)}, \tag{43}$$

or

$$\varphi(x,t) = \sqrt{\frac{2\hbar_2}{\hbar_1}}\csc h\left[\sqrt{-\hbar_2}\left(\frac{1}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta}-\nu t\right)\right]e^{\left(i\left(-(k/\beta)(x+(1/\Gamma(\beta)))^{\beta}+\omega t\right)-\sigma W(t)-\sigma^2 t\right)}. \tag{44}$$

Case 3. If $\hbar_2<0$ and $\hbar_1>0$, then the solutions of FSRKLE (3) are

$$\varphi(x,t) = \sqrt{\frac{-2\hbar_2}{\hbar_1}}\sec h\left[\sqrt{-\hbar_2}\left(\frac{1}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta}-\nu t\right)\right]e^{\left(i\left(-(k/\beta)(x+(1/\Gamma(\beta)))^{\beta}+\omega t\right)-\sigma W(t)-\sigma^2 t\right)}, \tag{45}$$

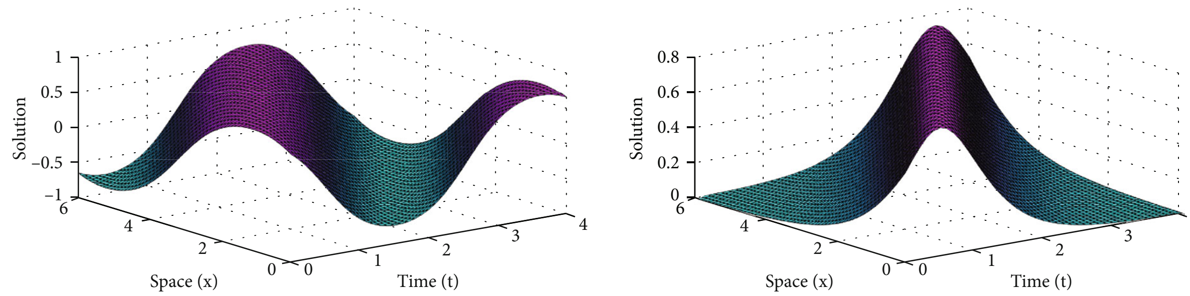

FIGURE 1: 3D diagram of Equations (25) and (45) with $\sigma = 0$.

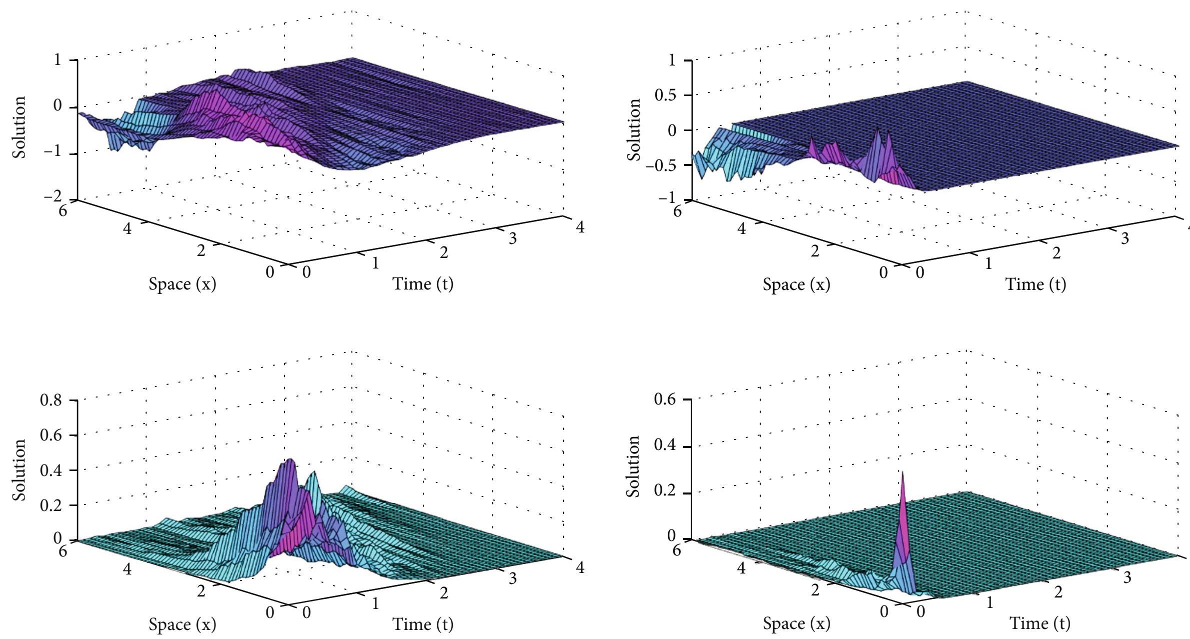

FIGURE 2: 3D diagram of Equations (25) and (45) with $\sigma = 1, 2$.

or

$$\varphi(x,t) = -i\sqrt{\frac{-2\hbar_2}{\hbar_1}}\operatorname{csc} h\left[\sqrt{-\hbar_2}\left(\frac{1}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta} - \nu t\right)\right] e^{\left(i\left(-(k/\beta)(x+(1/\Gamma(\beta)))^{\beta}+\omega t\right)-\sigma W(t)-\sigma^2 t\right)}. \tag{46}$$

Case 4. If $\hbar_2 > 0$ and $\hbar_1 > 0$, then FSRKLE (3) has the solutions

$$\varphi(x,t) = i\sqrt{\frac{2\hbar_2}{\hbar_1}}\sec\left[\sqrt{\hbar_2}\left(\frac{1}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta} - \nu t\right)\right] e^{\left(i\left(-(k/\beta)(x+(1/\Gamma(\beta)))^{\beta}+\omega t\right)-\sigma W(t)-\sigma^2 t\right)}, \tag{47}$$

or

$$\varphi(x,t) = i\sqrt{\frac{2\hbar_2}{\hbar_1}}\csc\left[\sqrt{\hbar_2}\left(\frac{1}{\beta}\left(x+\frac{1}{\Gamma(\beta)}\right)^{\beta} - \nu t\right)\right] e^{\left[i\left(-(k/\beta)(x+(1/\Gamma(\beta)))^{\beta}+\omega t\right)-\sigma W(t)-\sigma^2 t\right]}, \tag{48}$$

where $\hbar_1$,$\hbar_2$ are defined in (17).

Remark 1. Setting $\beta = 1$ and $\sigma = 0$ in Equations (41), (42), and (45), we get the identical solutions as asserted in [31].

4. Effect of BM and Fractional Derivative on the Solutions

Here, the impact of BM and the fractional derivative on the exact solutions of the FSRKLE (3) is described. Fix the constants $\gamma_1 = \gamma_2 = \gamma_4 = \gamma_5 = \gamma_6 = 1, k = -1, \nu = 3$, and $m = 0.5$. Hence, $\gamma_3 = 5/3, \hbar_1 = 4/3$, and $\hbar_2 = -1$. Now, we present some diagrams for various value of σ (intensity of noise) and for $t \in [0, 5]$, $x \in [0, 6]$. We apply the MATLAB to simulate the solutions of Equation (3).

Firstly, the impact of noise: in Figure 1, when $\sigma = 0$, we note that the surface fluctuates.

In Figure 2, if the noise appeared, then after small transit behaviors, the surface gets more planer when the intensity of noise increases as follows:

Secondly, the impact of fractional derivative: in Figures 3 and 4, if $\sigma = 0$, we can observe that as β increases, the surface extends:

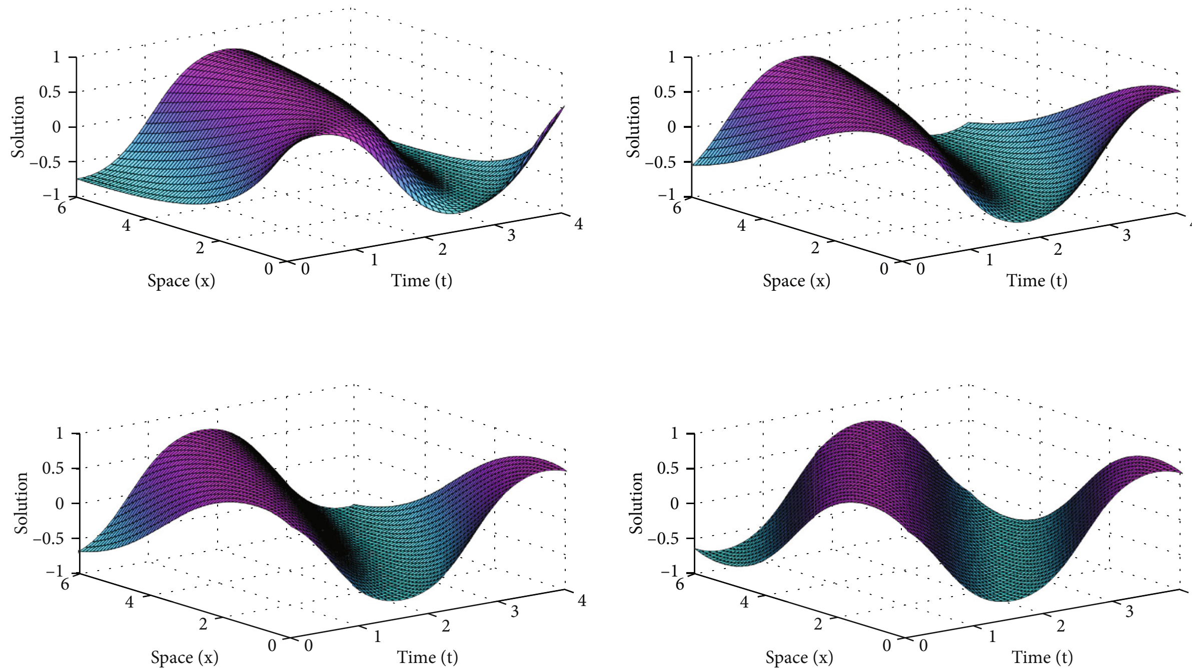

Figure 3: 3D diagram of Equation (25) with $\sigma = 0$ and various β.

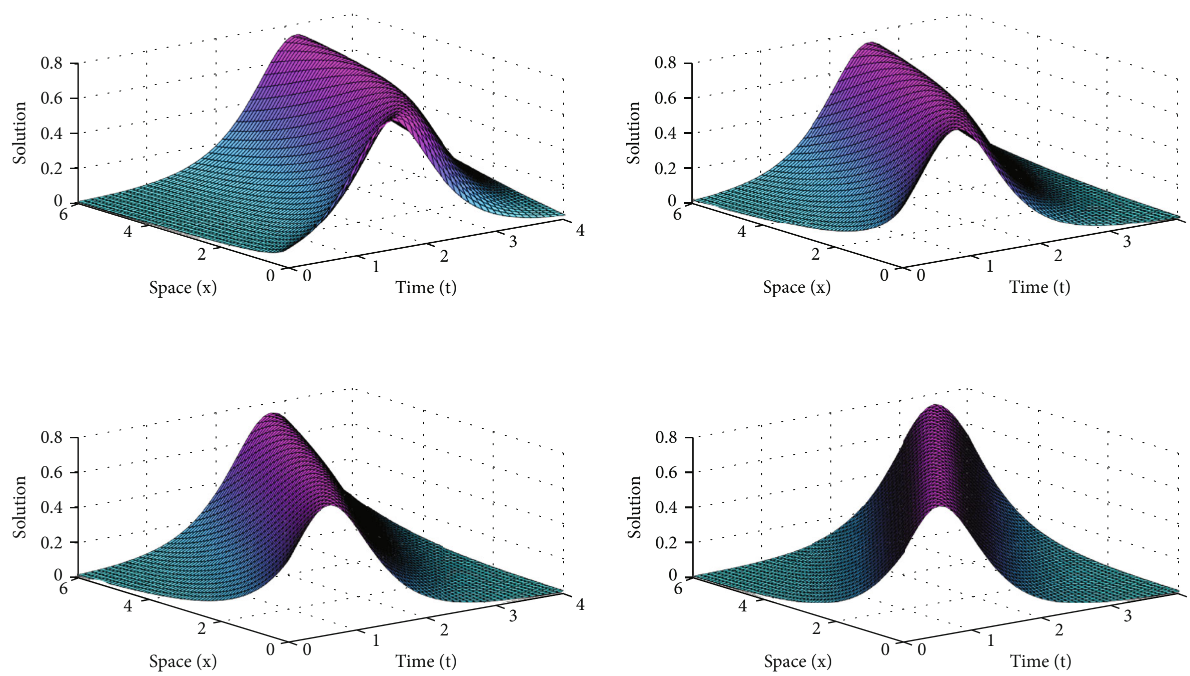

Figure 4: 3D diagram of Equation (45) with $\sigma = 0$ and various β.

5. Conclusions

In this paper, we obtained the exact solutions of the fractional-stochastic Radhakrishnan-Kundu-Lakshmanan Equation (3). To obtain rational, elliptic, trigonometric, and hyperbolic stochastic solutions, we used two different methods: the Jacobi elliptic function and the sine-cosine. Because of the priority of the FSRKLE in fluid dynamics and plasma physics, the results produced are useful for understanding some exciting physical phenomena. Finally, we plotted the obtained solutions using MATLAB tools to provide a number 3D diagram to demonstrate the impact of fractional derivative and multiplicative noise on these solutions. In future work, we can consider the FSRKLE (3) with additive noise.

Data Availability

All data are available in this paper.

Conflicts of Interest

The authors declare that they have no competing interests.

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References

[1] Y. Gurefe and E. Misirli, "Exp-function method for solving nonlinear evolution equations with higher order nonlinearity," *Computers & Mathematics with Applications*, vol. 61, no. 8, pp. 2025–2030, 2011.

[2] A. Akbulut and M. Kaplan, "Auxiliary equation method for time-fractional differential equations with conformable derivative," *Computers & Mathematcs with Applications*, vol. 75, no. 3, pp. 876–882, 2018.

[3] W. X. Ma and S. Batwa, "A binary darboux transformation for multicomponent NLS equations and their reductions," *Analysis and Mathematical Physics*, vol. 11, no. 2, p. 44, 2021.

[4] A. M. Wazwaz, "A sine-cosine method for handlingnonlinear wave equations," *Mathematical and Computer Modelling*, vol. 40, no. 5-6, pp. 499–508, 2004.

[5] S. Alshammari, W. W. Mohammed, S. K. Samura, and S. Faleh, "The analytical solutions for the stochastic-fractional Broer–Kaup equations," *Mathematical Problems in Engineering*, vol. 2022, Article ID 6895875, 9 pages, 2022.

[6] K. Khan and M. A. Akbar, "The exp(−$\Phi(\xi)$)-expansion method for finding travelling wave solutions of Vakhnenko-Parkes equation," *International Journal of Dynamical Systems and Differential Equations*, vol. 5, no. 1, pp. 72–83, 2014.

[7] H. K. Barman, A. R. Seadawy, M. A. Akbar, and D. Baleanu, "Competent closed form soliton solutions to the Riemann wave equation and the Novikov-Veselov equation," *Results in Physics*, vol. 17, article 103131, 2020.

[8] M. L. Wang, X. Z. Li, and J. L. Zhang, "The (G'G)-expansion method and travelling wave solutions of nonlinear evolution equations in mathematical physics," *Physics Letters A*, vol. 372, no. 4, pp. 417–423, 2008.

[9] H. Zhang, "New application of the (G'/G)-expansion method," *Communications in Nonlinear Science and Numerical Simulation*, vol. 14, no. 8, pp. 3220–3225, 2009.

[10] W. W. Mohammed, M. Alesemi, S. Albosaily, N. Iqbal, and M. El-Morshedy, "The exact solutions of stochastic fractional-space Kuramoto-Sivashinsky equation by using (G'/G)-expansion Method," *Mathematics*, vol. 9, no. 21, p. 2712, 2021.

[11] M. Kaplan, B. Bekir, and A. Akbulut, "A generalized Kudryashov method to some nonlinear evolution equations in mathematical physics," *Nonlinear Dynamics*, vol. 85, no. 4, pp. 2843–2850, 2016.

[12] W. W. Mohammed, "Fast-diffusion limit for reaction–diffusion equations with degenerate multiplicative and additive noise," *Journal of Dynamics and Differential Equations*, vol. 33, no. 1, pp. 577–592, 2021.

[13] W. W. Mohammed and D. Blömker, "Fast-diffusion limit for reaction-diffusion equations with multiplicative noise," *Journal of Mathematical Analysis and Applications*, vol. 496, no. 2, article 124808, 2021.

[14] K. Klepel, D. Blömker, and W. W. Mohammed, "Amplitude equation for the generalized swift Hohenberg equation with noise," *Zeitschrift für Angewandte Mathematik und Physik ZAMP*, vol. 65, no. 6, pp. 1107–1126, 2014.

[15] Y. Gurefe, E. Misirli, A. Sonmezoglu, and M. Ekici, "Extended trial equation method to generalized nonlinear partial differential equations," *Applied Mathematics and Computation*, vol. 219, no. 10, pp. 5253–5260, 2013.

[16] Y. Pandir, Y. Gurefe, and E. Misirli, "The extended trial equation method for some time fractional differential equations," *Discrete Dynamics in Nature and Society*, vol. 2013, Article ID 491359, 13 pages, 2013.

[17] Z. L. Yan, "Abundant families of Jacobi elliptic function solutions of the (2+1)-dimensional integrable Davey-Stewartson-type equation via a new method," *Chaos, Solitons and Fractals*, vol. 18, no. 2, pp. 299–309, 2003.

[18] E. Fan and J. Zhang, "Applications of the Jacobi elliptic function method to special-type nonlinear equations," *Physics Letters A*, vol. 305, no. 6, pp. 383–392, 2002.

[19] F. M. Al-Askar, W. W. Mohammed, and M. Alshammari, "Impact of Brownian motion on the analytical solutions of the space-fractional stochastic approximate long water wave equation," *Symmetry*, vol. 14, no. 4, p. 740, 2022.

[20] W. Malfliet and W. Hereman, "The tanh method. I. Exact solutions of nonlinear evolution and wave equations," *Physica Scripta*, vol. 54, no. 6, pp. 563–568, 1996.

[21] N. Iqbal, A. M. Albalahi, M. S. Abdo, and W. W. Mohammed, "Analytical analysis of fractional-order Newell-Whitehead-Segel equation: a modified homotopy perturbation transform method," *Journal of Function Spaces*, vol. 2022, Article ID 3298472, 10 pages, 2022.

[22] M. Alshammari, N. Iqbal, W. W. Mohammed, and T. Botmart, "The solution of fractional-order system of KdV equations with exponential- decay kernel," *Results in Physics*, vol. 38, article 105615, 2022.

[23] A. Filiz, M. Ekici, and A. Sonmezoglu, "Expansion method and new exact solutions of the Schrödinger-KdV equation," *Scientific World Journal*, vol. 2014, article 534063, 14 pages, 2014.

[24] A. Atangana and E. F. D. Goufo, "Extension of matched asymptotic method to fractional boundary layers problems," *Mathematical Problems in Engineering*, vol. 2014, Article ID 107535, 7 pages, 2014.

[25] P. E. Kloeden and E. Platen, *Numerical Solution of Stochastic Differential Equations*, SpringerVerlag, New York, 1995.

[26] D. D. Ganji, A. Asgari, and Z. Z. Ganji, "Exp-function based solution of nonlinear Radhakrishnan, Kundu and Laskshmanan (RKL) equation," *Acta applicandae mathematicae*, vol. 104, no. 2, article 201209, 2008.

[27] A. Biswas, "1-soliton solution of the generalized Radhakrishnan, Kundu, Lakshmanan equation," *Physics Letters A*, vol. 373, no. 30, pp. 2546–2548, 2009.

[28] B. Sturdevant, D. A. Lott, and A. Biswas, "Topological 1-soliton solution of the generalized Radhakrishnan, Kundu, Lakshmanan equation with nonlinear dispersion," *Modern Physics Letters B*, vol. 24, no. 16, pp. 1825–1831, 2010.

[29] D. Lu, A. R. Seadawy, and M. M. Khater, "Dispersive optical soliton solutions of the generalized Radhakrishnan-Kundu-Lakshmanan dynamical equation with power law nonlinearity and its applications," *Optik*, vol. 164, pp. 54–64, 2018.

[30] S. S. Singh, "Solutions of Kudryashov-Sinelshchikov equation and generalized Radhakrishnan-Kundu-Lakshmanan equation by the first integral method," *International Journal of Physical Research*, vol. 4, no. 2, pp. 37–42, 2016.

[31] J.-L. Zhang and M.-L. Wang, "Various exact solutions for two special type RKL models, chaos," *Solitons Fractals*, vol. 37, no. 1, article 215226, 2008.

[32] A. Bansal, A. Biswas, M. F. Mahmood et al., "Optical soliton perturbation with Radhakrishnan-Kundu-Lakshmanan equation by Lie group analysis," *Optik*, vol. 163, pp. 137–141, 2018.

[33] A. Biswas, Y. Yildirim, E. Yasar et al., "Optical soliton perturbation for Radhakrishnan-Kundu-Lakshmanan equation with a couple of integration schemes," *Optik*, vol. 163, pp. 126–136, 2018.