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Research Article

On Neutrosophic 2-Metric Spaces with Application

Ali Asghar,[1] Aftab Hussain ,[2] Khaleel Ahmad,[3] Umar Ishtiaq ,[4] Hamed Al Sulami,[2] and Nawab Hussain [2]

[1]*Department of Mathematics and Statistics, The University of Lahore, Lahore, Pakistan*
[2]*Department of Mathematics, King Abdulaziz University, P. O. Box 80203, Jeddah 21589, Saudi Arabia*
[3]*Department of Mathematics, University of Management and technology, Lahore, Pakistan*
[4]*Office of Research, Innovation and Commercialization, University of Management and Technology, Lahore, Pakistan*

Correspondence should be addressed to Umar Ishtiaq; umarishtiaq000@gmail.com

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Classical sets, fuzzy sets, intuitionistic fuzzy sets, and other sets are all generalized into the neutrosophic sets. A neutrosophic set is a mathematical approach that helps with challenges involving data that is inconsistent, indeterminate, or imprecise. The goal of this manuscript is to present the notion of neutrosophic 2-metric spaces. In this situation, we prove various fixed point theorems. The findings support previous methodologies in the literature and are backed up by various examples and an application.

1. Introduction and Preliminaries

There is a lot of imprecision and vagueness or fuzziness in our daily lives when it comes to sharing knowledge and information. Simple examples include comments like "Umar is tall" and "Khaleel is smart." Terms like "tall" and "smart" in the examples above are vague in the sense that they cannot be defined precisely. There was a need to compare terms such as "tall" and "smart." In 1965, Zadeh [1] coined the "fuzzy notion" to depict imprecise terms (such as those in the preceding examples) in stark contrast. In this continuation, Kramosil and Michlek [2] established the approach of fuzzy metric spaces (FMSs). George and Veeramani [3] initiated the approach of FMSs by utilizing continuous t-norms (CTNs). Grabiec [4] gave the fuzzy interpretation of the Banach contraction principle in FMSs. Sharma [5] coined the term fuzzy 2-metric spaces (F2MSs). Cho [6] established a common fixed point theorem for four mappings in FMSs, while Han [7] extended the results to F2MSs. Priyanka and Malviya [8] also derived several common fixed point theorems in F2MSs for occasionally weekly compatible mappings.

The concept of intuitionistic fuzzy 2-metric spaces (IF2MSs) was presented by Mursaleen and Danishlohani [9]. CTNs and continuous triangular conorms (CTCNs) were used to define neutrosophic metric spaces (NMSs). NMSs have been studied for their topological and structural features. Kirişci and Simsek [10] expanded the notion of the intuitionistic fuzzy metric space approach and proposed the notion of NMSs. In the context of NMSs, Simsek and Kirişci [11] and Sowndrarajan et al. [12] demonstrated various fixed point results. Neutrosophic soft linear spaces were first established by Bera and Mahapatra [13]. Bera and Mahapatra [14] established neutrosophic soft normed linear spaces. Ishtiaq et al. [15] introduced the concept of orthogonal neutrosophic metric spaces and proved several interesting fixed point results in the context of orthogonal neutrosophic metric spaces. Jeyaraman and Sowndrarajan [16] used contraction mappings to prove various common fixed point results in the context of neutrosophic metric spaces. Şahin et al. [17] introduced the notion of neutrosophic triple partial metric spaces and proved some fixed point results. Zararsız and Riaz [18] introduced the notion of bipolar fuzzy metric spaces and proved several fixed point results and provided an interesting application towards multiattribute decision-making.

Fathollahi et al. [19] proved several fixed point results for modified weak and rational α-ψ-contractions in ordered 2-metric spaces. Ali et al. [20] solved nonlinear differential equations in the context of neutrosophic metric spaces. Al-Omeri et al. [21] worked on (Φ, Ψ)-weak contractions in the context of neutrosophic cone metric spaces. Naeem et al. [22] worked on strong convergence theorems for a finite family of enriched strictly pseudocontractive mappings and Φ_T-enriched Lipschitzian mappings using a new modified mixed-type Ishikawa iteration scheme with error. Al-Omeri et al. [23, 24] worked on numerous interesting contraction mappings in the context of neutrosophic cone metric spaces. Hussain et al. [25] proved several fixed point results for contraction mappings. Salama and Alblowi [26] worked on neutrosophic topological spaces.

In this manuscript, we replace the triangular inequalities of neutrosophic metric spaces by tetrahedron inequalities and introduce a notion of neutrosophic 2-metric spaces. The main objectives of this manuscript are as follows:

(i) To introduce the notion of neutrosophic 2-metric spaces (N2MSs)

(ii) To prove fixed point results in the context of N2MSs

(iii) To enhance the literature of neutrosophic fixed point theory

(iv) To prove the uniqueness of the solution of integral equations

Now, we provide some definitions that are helpful for readers to understand the main section.

Definition 1 (see [1]). If a binary relation $*$ ($\Diamond$) on the interval [0,1] fulfills the below criteria, then $*$ ($\Diamond$) is known as CTNs (CTCNs):

(a1) $*$ ($\Diamond$) is commutative and associative

(b1) $*$ ($\Diamond$) is continuous

(c1) For all $\gimel \in [0,1]$, $\gimel * 1 = \gimel$ ($\gimel \Diamond 0 = \gimel$)

for all $\gimel, \mathfrak{g}, \mathfrak{u}, \mathfrak{w} \in [0,1]$, $\gimel * \mathfrak{g} = \mathfrak{u} * \mathfrak{w}$ ($\gimel \Diamond \mathfrak{g} \leq \mathfrak{u} \Diamond \mathfrak{w}$) whenever $\gimel \leq \mathfrak{u}$ and $\mathfrak{g} \leq \mathfrak{w}$.

Definition 2 (see [5]). Let $\mathfrak{D} \neq \varnothing$. A function $d : \mathfrak{D} \times \mathfrak{D} \longrightarrow \mathbb{R}$ is said to be 2-metric if d fulfills the below criteria:

(a) To each pair of $\gimel, \mathfrak{g} \in \mathfrak{D} (\gimel \neq \mathfrak{g})$, there is a point $\mathfrak{u} \in \mathfrak{D}$ such that $d(\gimel, \mathfrak{g}, \mathfrak{u}) \neq 0$

(b) $d(\gimel, \mathfrak{g}, \mathfrak{u}) = 0$, when at least two of $\gimel, \mathfrak{g}, \mathfrak{u}$ are equal

(c) $d(\gimel, \mathfrak{g}, \mathfrak{u}) = d(\gimel, \mathfrak{u}, \mathfrak{g}) = d(\mathfrak{g}, \mathfrak{u}, \gimel)$ for all $\gimel, \mathfrak{g}, \mathfrak{u} \in \mathfrak{D}$

(d) $d(\gimel, \mathfrak{g}, \mathfrak{u}) = d(\gimel, \mathfrak{g}, w) + d(\gimel, w, \mathfrak{u}) + d(w, \mathfrak{g}, \mathfrak{u})$ for all $\gimel, \mathfrak{g}, \mathfrak{u}, w \in \mathfrak{D}$

Then, the pair $(\mathfrak{D}, d)$ is called 2-metric space, and d is a 2-metric on $\mathfrak{D}$.

Example 1 (see [5]). Let $\mathfrak{D} = \{0, 1, 1/2, 1/3, \cdots\}$. A mapping $d : \mathfrak{D} \times \mathfrak{D} \times \mathfrak{D} \longrightarrow [0,+\infty)$ defined by

$$d(\gimel, \mathfrak{g}, \mathfrak{u}) = \begin{cases} 1, & \gimel, \mathfrak{g}, \mathfrak{u} \text{ are distinct and } \left\{\frac{1}{n}, \frac{1}{n+1}\right\} \subseteq \{\gimel, \mathfrak{g}, \mathfrak{u}\}, \\ 0, & \text{otherwise,} \end{cases} \tag{1}$$

for all $n \in \mathbb{N}$ and $\gimel, \mathfrak{g}, \mathfrak{u} \in \mathfrak{D}$ is a 2-metric on $\mathfrak{D}$.

Definition 3 (see [9]). Let $\mathfrak{D} \neq \varnothing$ be a set, $*$ be a CTN, $\Diamond$ be a CTCN, and $\mathfrak{w}, \mathfrak{N} : \mathfrak{D} \times \mathfrak{D} \times \mathfrak{D} \times [0,+\infty) \longrightarrow [0,1]$ be mappings. A five tuple ($\mathfrak{D}, \mathfrak{w}, \mathfrak{N}, *$, and $\Diamond$) is called an intuitionistic fuzzy 2-metric space (IF2MS), if the following conditions are satisfied:

(1) $\mathfrak{w}(\gimel, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}) + \mathfrak{N}(\gimel, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}) \leq 1$

(2) $\mathfrak{w}(\gimel, \mathfrak{g}, \mathfrak{u}, 0) = 0$

(3) $\mathfrak{w}(\gimel, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}) = 1$ for all $\mathfrak{z} > 0$ when at least two of $\gimel, \mathfrak{g}, \mathfrak{u}$ are equal

(4) $\mathfrak{w}(\gimel, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}) = \mathfrak{w}(\gimel, \mathfrak{u}, \mathfrak{g}, \mathfrak{z}) = \mathfrak{w}(\mathfrak{g}, \mathfrak{u}, \gimel, \mathfrak{z}) = \mathfrak{w}(\mathfrak{u}, \gimel, \mathfrak{g}, \mathfrak{z})$

(5) $\mathfrak{w}(\gimel, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}_1 + \mathfrak{z}_2 + \mathfrak{z}_3) \geq \mathfrak{w}(\gimel, \mathfrak{g}, \nu, \mathfrak{z}_1) * \mathfrak{w}(\gimel, \nu, \mathfrak{u}, \mathfrak{z}_2) * \mathfrak{w}(\nu, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}_3)$

(6) $\mathfrak{w}(\gimel, \mathfrak{g}, \mathfrak{u}, .)$: $[0,+\infty] \longrightarrow [0,1]$ is left continuous

(7) $\mathfrak{N}(\gimel, \mathfrak{g}, \mathfrak{u}, 0) = 1$

(8) $\mathfrak{N}(\gimel, \mathfrak{g}, \mathfrak{u}, 0) = 0$ for all $\mathfrak{z} > 0$ when at least two of $\gimel, \mathfrak{g}, \mathfrak{u}$ are equal

(9) $\mathfrak{N}(\gimel, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}) = \mathfrak{N}(\gimel, \mathfrak{u}, \mathfrak{g}, \mathfrak{z}) = \mathfrak{N}(\mathfrak{g}, \mathfrak{u}, \gimel, \mathfrak{z}) = \mathfrak{N}(\mathfrak{u}, \gimel, \mathfrak{g}, \mathfrak{z})$

(10) $\mathfrak{N}(\gimel, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}_1 + \mathfrak{z}_2 + \mathfrak{z}_3) \leq \mathfrak{N}(\gimel, \mathfrak{g}, \nu, \mathfrak{z}_1) \Diamond \mathfrak{N}(\gimel, \nu, \mathfrak{u}, \mathfrak{z}_2) \Diamond \mathfrak{N}(\nu, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}_3)$

(11) $\mathfrak{N}(\gimel, \mathfrak{g}, \mathfrak{u}, .)$: $[0,+\infty) \longrightarrow [0,1]$ is left continuous

for all $\gimel, \mathfrak{g}, \mathfrak{u} \in \mathfrak{D}$ and $\mathfrak{z}, \mathfrak{z}_1, \mathfrak{z}_2, \mathfrak{z}_3 > 0$.

Definition 4 (see [10]). Suppose $\mathfrak{D} \neq \varnothing$, and assume a six tuple $(\mathfrak{D}, \mathfrak{w}, \mathfrak{N}, \mathfrak{L}, *, \Diamond)$, where $*$ is a CTN, $\Diamond$ is a CTCN, and $\mathfrak{w}$, $\mathfrak{N}$, and $\mathfrak{L}$ are neutrosophic sets (NSs) on $\mathfrak{D} \times \mathfrak{D} \times (0,+\infty)$. If $(\mathfrak{D}, \mathfrak{w}, \mathfrak{N}, \mathfrak{L}, *, \Diamond)$ meets the below conditions for all $\gimel, \mathfrak{g}, \mathfrak{u} \in \mathfrak{D}$ and $\mathfrak{z}, \mathfrak{s} > 0$:

(NS1) $\mathfrak{w}(\gimel, \mathfrak{g}, \mathfrak{z}) + \mathfrak{N}(\gimel, \mathfrak{g}, \mathfrak{z}) + \mathfrak{L}(\gimel, \mathfrak{g}, \mathfrak{z}) \leq 3$

(NS2) $0 \leq \mathfrak{w}(\gimel, \mathfrak{g}, \mathfrak{z}) \leq 1$

(NS3) $\mathfrak{w}(\gimel, \mathfrak{g}, \mathfrak{z}) = 1 \Longleftrightarrow \gimel = \mathfrak{g}$

(NS4) $\mathfrak{w}(\gimel, \mathfrak{g}, \mathfrak{z}) = \mathfrak{w}(\mathfrak{g}, \gimel, \mathfrak{z})$

(NS5) $\mathfrak{w}(\gimel, \mathfrak{u}, (\mathfrak{z} + \mathfrak{s})) \geq \mathfrak{w}(\gimel, \mathfrak{g}, \mathfrak{z}) * \mathfrak{w}(\mathfrak{g}, \mathfrak{u}, \mathfrak{s})$

(NS6) $\mathfrak{w}(\gimel, \mathfrak{g}, \bullet)$: $[0,+\infty) \longrightarrow [0,1]$ is continuous

(NS7) $\lim_{\mathfrak{z} \longrightarrow +\infty} \mathfrak{w}(\gimel, \mathfrak{g}, \mathfrak{z}) = 1$

(NS8) $0 \leq \mathfrak{N}(\gimel, \mathfrak{g}, \mathfrak{z}) \leq 1$

(NS9) $\mathfrak{N}(\gimel, \mathfrak{g}, \mathfrak{z}) = 0 \Longleftrightarrow \gimel = \mathfrak{g}$

(NS10) $\mathfrak{N}(\gimel, \mathfrak{g}, \mathfrak{z}) = \mathfrak{N}(\mathfrak{g}, \gimel, \mathfrak{z})$

(NS11) $\mathfrak{N}(\gimel, \mathfrak{u}, b(\mathfrak{z} + \mathfrak{s})) \leq \mathfrak{N}(\gimel, \mathfrak{g}, \mathfrak{z}) \Diamond \mathfrak{N}(\mathfrak{g}, \mathfrak{u}, \mathfrak{s})$

(NS12) $\mathfrak{N}(\gimel, \mathfrak{g}, \bullet)$: $[0,+\infty) \longrightarrow [0, 1]$ is continuous

(NS13) $\lim_{\mathfrak{z}\longrightarrow+\infty} \mathfrak{N}(\gimel, \mathfrak{g}, \mathfrak{z}) = 0$

(NS14) $0 \le \mathfrak{L}(\gimel, \mathfrak{g}, \mathfrak{z}) \le 1$

(NS15) $\mathfrak{L}(\gimel, \mathfrak{g}, \mathfrak{z}) = 0 \Longleftrightarrow \gimel = \mathfrak{g}$

(NS16) $\mathfrak{L}(\gimel, \mathfrak{g}, \mathfrak{z}) = \mathfrak{L}(\mathfrak{g}, \gimel, \mathfrak{z})$

(NS17) $\mathfrak{L}(\gimel, \mathfrak{u}, (\mathfrak{z} + \mathfrak{s})) \le \mathfrak{L}(\gimel, \mathfrak{g}, \mathfrak{z})\Diamond\mathfrak{L}(\mathfrak{g}, \mathfrak{u}, \mathfrak{s})$

(NS18) $\mathfrak{L}(\gimel, \mathfrak{g}, \bullet)$: $[0,+\infty) \longrightarrow [0, 1]$ is continuous

(NS19) $\lim_{\mathfrak{z}\longrightarrow+\infty} \mathfrak{L}(\gimel, \mathfrak{g}, \mathfrak{z}) = 0$

(NS20) If $\mathfrak{L} \le 0$, then $\mathfrak{w}(\gimel, \mathfrak{g}, \mathfrak{z}) = 0, \mathfrak{N}(\gimel, \mathfrak{g}, \mathfrak{z}) = 1, \mathfrak{L}(\gimel, \mathfrak{g}, \mathfrak{z}) = 1$

Then, $(\mathfrak{D}, \mathfrak{w}, \mathfrak{N}, \mathfrak{L})$ is a neutrosophic metric on $\mathfrak{D}$ and $(\mathfrak{D}, \mathfrak{w}, \mathfrak{N}, \mathfrak{L}, *, \Diamond)$ be a NMS.

2. Neutrosophic 2-Metric Spaces

Definition 5. Let $\mathfrak{D} \ne \emptyset$ be a set, $*$ be a CTN, $\Diamond$ be a CTCN, and $\mathfrak{w}, \mathfrak{N}, \mathfrak{L} : \mathfrak{D} \times \mathfrak{D} \times \mathfrak{D} \times [0,+\infty) \longrightarrow [0, 1]$ be mappings. A six tuple $(\mathfrak{D}, \mathfrak{w}, \mathfrak{N}, \mathfrak{L}, *,$ and $\Diamond)$ is called a N2MS, if the following conditions are satisfied:

(NM1) $\mathfrak{w}(\gimel, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}) + \mathfrak{N}(\gimel, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}) + \mathfrak{L}(\gimel, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}) \le 3$

(NM2) $\mathfrak{w}(\gimel, \mathfrak{g}, \mathfrak{u}, 0) = 0$

(NM3) $\mathfrak{w}(\gimel, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}) = 1$ for all $\mathfrak{z} > 0$ when at least two of $\gimel, \mathfrak{g}, \mathfrak{u}$ are equal

(NM4) $\mathfrak{w}(\gimel, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}) = \mathfrak{w}(\gimel, \mathfrak{u}, \mathfrak{g}, \mathfrak{z}) = \mathfrak{w}(\mathfrak{g}, \mathfrak{u}, \gimel, \mathfrak{z}) = \mathfrak{w}(\mathfrak{u}, \gimel, \mathfrak{g}, \mathfrak{z})$

(NM5) $\mathfrak{w}(\gimel, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}_1 + \mathfrak{z}_2 + \mathfrak{z}_3) \ge \mathfrak{w}(\gimel, \mathfrak{g}, u, \mathfrak{z}_1) * \mathfrak{w}(\gimel, u, \mathfrak{u}, \mathfrak{z}_2) * \mathfrak{w}(u, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}_3)$

(NM6) $\mathfrak{w}(\gimel, \mathfrak{g}, \mathfrak{u}, .)$: $[0,+\infty] \longrightarrow [0, 1]$ is left continuous

(NM7) $\mathfrak{N}(\gimel, \mathfrak{g}, \mathfrak{u}, 0) = 1$

(NM8) $\mathfrak{N}(\gimel, \mathfrak{g}, \mathfrak{u}, 0) = 0$ for all $\mathfrak{z} > 0$ when at least two of $\gimel, \mathfrak{g}, \mathfrak{u}$ are equal

(NM9) $\mathfrak{N}(\gimel, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}) = \mathfrak{N}(\gimel, \mathfrak{u}, \mathfrak{g}, \mathfrak{z}) = \mathfrak{N}(\mathfrak{g}, \mathfrak{u}, \gimel, \mathfrak{z}) = \mathfrak{N}(\mathfrak{u}, \gimel, \mathfrak{g}, \mathfrak{z})$

(NM10) $\mathfrak{N}(\gimel, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}_1 + \mathfrak{z}_2 + \mathfrak{z}_3) \le \mathfrak{N}(\gimel, \mathfrak{g}, u, \mathfrak{z}_1)\Diamond\mathfrak{N}(\gimel, u, \mathfrak{u}, \mathfrak{z}_2)\Diamond\mathfrak{N}(u, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}_3)$

(NM11) $\mathfrak{N}(\gimel, \mathfrak{g}, \mathfrak{u}, .)$: $[0,+\infty) \longrightarrow [0, 1]$ is left continuous

(NM12) $\mathfrak{L}(\gimel, \mathfrak{g}, \mathfrak{u}, 0) = 1$

(NM13) $\mathfrak{L}(\gimel, \mathfrak{g}, \mathfrak{u}, 0) = 0$ for all $\mathfrak{z} > 0$ when at least two of $\gimel, \mathfrak{g}, \mathfrak{u}$ are equal

(NM14) $\mathfrak{L}(\gimel, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}) = \mathfrak{L}(\gimel, \mathfrak{u}, \mathfrak{g}, \mathfrak{z}) = \mathfrak{L}(\mathfrak{g}, \mathfrak{u}, \gimel, \mathfrak{z}) = \mathfrak{L}(\mathfrak{u}, \gimel, \mathfrak{g}, \mathfrak{z})$

(NM15) $\mathfrak{L}(\gimel, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}_1 + \mathfrak{z}_2 + \mathfrak{z}_3) \le \mathfrak{L}(\gimel, \mathfrak{g}, u, \mathfrak{z}_1)\Diamond\mathfrak{L}(\gimel, u, \mathfrak{u}, \mathfrak{z}_2)\Diamond\mathfrak{L}(u, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}_3)$

(NM16) $\mathfrak{L}(\gimel, \mathfrak{g}, \mathfrak{u}, .)$: $[0,+\infty) \longrightarrow [0, 1]$ is left continuous for all $\gimel, \mathfrak{g}, \mathfrak{u} \in \mathfrak{D}$ and $\mathfrak{z}, \mathfrak{z}_1, \mathfrak{z}_2, \mathfrak{z}_3 > 0$.

Remark 6. The inequalities (NM5), (NM10), and (NM15) correspond to tetrahedron inequality in 2-metric space. The function values of $\mathfrak{w}(\gimel, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}), \mathfrak{N}(\gimel, \mathfrak{g}, \mathfrak{u}, \mathfrak{z})$, and $\mathfrak{L}(\gimel, \mathfrak{g}, \mathfrak{u}, \mathfrak{z})$ be interpreted as the probability that the area of triangle is less than $\mathfrak{z}$.

Remark 7.

(i) Every N2MS is nonnegative

(ii) We may assume that every N2MS contains at least three distinct points

Example 2. Let $(\mathfrak{D}, d)$ be a 2-metric space, $a * b = ab$ and $a \Diamond b = \min\{1, a + b\}$. Let $\mathfrak{w}, \mathfrak{N}, \mathfrak{L} : \mathfrak{D}^3 \times [0,+\infty) \longrightarrow [0, 1]$ be three mappings defined by

$$\begin{aligned} \mathfrak{w}(\gimel, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}) &= \begin{cases} \dfrac{\mathfrak{z}}{\mathfrak{z} + d(\gimel, \mathfrak{g}, \mathfrak{u})}, & \mathfrak{z} > 0, \\ 0, & \mathfrak{z} = 0, \end{cases} \\ \mathfrak{N}(\gimel, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}) &= \begin{cases} \dfrac{d(\gimel, \mathfrak{g}, \mathfrak{u})}{\mathfrak{z} + d(\gimel, \mathfrak{g}, \mathfrak{u})}, & \mathfrak{z} > 0, \\ 0, & \mathfrak{z} = 0, \end{cases} \\ \mathfrak{L}(\gimel, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}) &= \begin{cases} \dfrac{d(\gimel, \mathfrak{g}, \mathfrak{u})}{\mathfrak{z}}, & \mathfrak{z} > 0, \\ 0, & \mathfrak{z} = 0, \end{cases} \end{aligned} \tag{2}$$

for all $\gimel, \mathfrak{g}, \mathfrak{u} \in \mathfrak{D}$. Then, $(\mathfrak{D}, \mathfrak{w}, \mathfrak{N}, \mathfrak{L}, *, \Diamond)$ is known as N2MS.

Definition 8. Assume $\{\gimel_n\}$ be a sequence in N2MS. Then,

(1) $\gimel_n \subseteq \mathfrak{D}$ is known as convergent to $\gimel \in \mathfrak{D}$ if $\lim_{n\longrightarrow+\infty} \mathfrak{w}(\gimel_n, \gimel, \mathfrak{u}, \mathfrak{z}) = 1$, $\lim_{n\longrightarrow+\infty} \mathfrak{N}(\gimel_n, \gimel, \mathfrak{u}, \mathfrak{z}) = 0$, and $\lim_{n\longrightarrow+\infty} \mathfrak{L}(\gimel_n, \gimel, \mathfrak{u}, \mathfrak{z}) = 0$ for all $\mathfrak{u} \in \mathfrak{D}$ and $\mathfrak{z} > 0$. It is denoted by $\lim_{n\longrightarrow+\infty} \gimel_n = \gimel$ or $\gimel_n \longrightarrow \gimel$

(2) $\gimel_n \subseteq \mathfrak{D}$ is known as the Cauchy sequence if $\lim_{n,m\longrightarrow+\infty} \mathfrak{w}(\gimel_n, \gimel_m, \mathfrak{u}, \mathfrak{z}) = 1$, $\lim_{n,m\longrightarrow+\infty} \mathfrak{N}(\gimel_n, \gimel_m, \mathfrak{u}, \mathfrak{z}) = 0$ and $\lim_{n,m\longrightarrow+\infty} \mathfrak{L}(\gimel_n, \gimel_m, \mathfrak{u}, \mathfrak{z}) = 0$ for all $\mathfrak{u} \in \mathfrak{D}$ and $\mathfrak{z} > 0$

(3) If each Cauchy sequence in N2MS $(\mathfrak{D}, \mathfrak{w}, \mathfrak{N}, \mathfrak{L}, *, \Diamond)$ is convergent, then N2MS is said to be complete

Note 9. From now, we will assume that $(\mathfrak{D}, \mathfrak{w}, \mathfrak{N}, \mathfrak{L}, *, \Diamond)$ is a N2MS with the condition

$$\begin{aligned} \lim_{\mathfrak{z}\longrightarrow+\infty} \mathfrak{w}(\gimel_n, \gimel, \mathfrak{u}, \mathfrak{z}) &= 1, \\ \lim_{\mathfrak{z}\longrightarrow+\infty} \mathfrak{N}(\gimel_n, \gimel, \mathfrak{u}, \mathfrak{z}) &= 0, \\ \lim_{\mathfrak{z}\longrightarrow+\infty} \mathfrak{N}(\gimel_n, \gimel, \mathfrak{u}, \mathfrak{z}) &= 0, \end{aligned} \tag{3}$$

for all $\gimel, \mathfrak{u}, \mathfrak{z} \in \mathfrak{D}$.

3. Main Results

We establish the basic properties of N2MSs and demonstrate some fixed point findings in this section.

Lemma 10. *If $\gimel_n$ is a sequence for all $n \in \mathbb{N}$ in a given N2MS $(\mathfrak{D}, \mathfrak{w}, \mathfrak{N}, \mathfrak{L}, *, \Diamond)$, then the below inequalities hold for all $\mathfrak{u} \in \mathfrak{D}$, $\mathfrak{z} > 0$ and $\partial > 0$:*

$$\begin{aligned}
\mathfrak{w}(\daleth_n, \daleth_{n+\partial}, \mathfrak{u}, \mathfrak{z}) \geq{} & \mathfrak{w}\left(\daleth_n, \daleth_{n+1}, \daleth_{n+\partial}, \frac{\mathfrak{z}}{2(\partial-1)+1}\right) \\
& * \mathfrak{w}\left(\daleth_n, \daleth_{n+2}, \daleth_{n+\partial}, \frac{\mathfrak{z}}{2(\partial-1)+1}\right) \\
& * \cdots * \mathfrak{w}\left(\daleth_{n+\partial-2}, \daleth_{n+\partial-1}, \daleth_{n+\partial}, \frac{\mathfrak{z}}{2(\partial-1)+1}\right) \\
& * \mathfrak{w}\left(\daleth_n, \daleth_{n+1}, \mathfrak{u}, \frac{\mathfrak{z}}{2(\partial-1)+1}\right) \\
& * \cdots * \mathfrak{w}\left(\daleth_{n+\partial-1}, \daleth_{n+\partial}, \mathfrak{u}, \frac{\mathfrak{z}}{2(\partial-1)+1}\right) \\
& * \mathfrak{w}\left(\daleth_{n+\partial-1}, \daleth_{n+\partial}, \mathfrak{u}, \frac{\mathfrak{z}}{2(\partial-1)+1}\right), \\
\mathfrak{N}(\daleth_n, \daleth_{n+\partial}, \mathfrak{u}, \mathfrak{z}) \leq{} & \mathfrak{N}\left(\daleth_n, \daleth_{n+1}, \daleth_{n+\partial}, \frac{\mathfrak{z}}{2(\partial-1)+1}\right) \\
& \Diamond \mathfrak{N}\left(\daleth_n, \daleth_{n+2}, \daleth_{n+\partial}, \frac{\mathfrak{z}}{2(\partial-1)+1}\right) \\
& \Diamond \cdots \Diamond \mathfrak{N}\left(\daleth_{n+\partial-2}, \daleth_{n+\partial-1}, \daleth_{n+\partial}, \frac{\mathfrak{z}}{2(\partial-1)+1}\right) \\
& \Diamond \mathfrak{N}\left(\daleth_n, \daleth_{n+1}, \mathfrak{u}, \frac{\mathfrak{z}}{2(\partial-1)+1}\right) \\
& \Diamond \cdots \Diamond \mathfrak{N}\left(\daleth_{n+\partial-1}, \daleth_{n+\partial}, \mathfrak{u}, \frac{\mathfrak{z}}{2(\partial-1)+1}\right) \\
& \Diamond \mathfrak{N}\left(\daleth_{n+\partial-1}, \daleth_{n+\partial}, \mathfrak{u}, \frac{\mathfrak{z}}{2(\partial-1)+1}\right), \\
\mathfrak{L}(\daleth_n, \daleth_{n+\partial}, \mathfrak{u}, \mathfrak{z}) \leq{} & \mathfrak{L}\left(\daleth_n, \daleth_{n+1}, \daleth_{n+\partial}, \frac{\mathfrak{z}}{2(\partial-1)+1}\right) \\
& \Diamond \mathfrak{L}\left(\daleth_n, \daleth_{n+2}, \daleth_{n+\partial}, \frac{\mathfrak{z}}{2(\partial-1)+1}\right) \\
& \Diamond \cdots \Diamond \mathfrak{L}\left(\daleth_{n+\partial-2}, \daleth_{n+\partial-1}, \daleth_{n+\partial}, \frac{\mathfrak{z}}{2(\partial-1)+1}\right) \\
& \Diamond \mathfrak{L}\left(\daleth_n, \daleth_{n+1}, \mathfrak{u}, \frac{\mathfrak{z}}{2(\partial-1)+1}\right) \\
& \Diamond \cdots \Diamond \mathfrak{L}\left(\daleth_{n+\partial-1}, \daleth_{n+\partial}, \mathfrak{u}, \frac{\mathfrak{z}}{2(\partial-1)+1}\right) \\
& \Diamond \mathfrak{L}\left(\daleth_{n+\partial-1}, \daleth_{n+\partial}, \mathfrak{u}, \frac{\mathfrak{z}}{2(\partial-1)+1}\right).
\end{aligned} \tag{4}$$

Lemma 11. *Let $\{\daleth_n\}$be a Cauchy sequence in N2MS $(\mathfrak{O}, \mathfrak{w}, \mathfrak{N}, \mathfrak{L}, *, \Diamond)$ such that $\daleth_n \neq \daleth_m$ whenever $m, n \in \mathbb{N}$ with $n \neq m$. Then, the sequence $\{\daleth_n\}$ converges to at most one limit point.*

Lemma 12. *Let $(\mathfrak{O}, \mathfrak{w}, \mathfrak{N}, \mathfrak{L}, *, \Diamond)$ be a N2MS. If for some $0 < \text{\H{u}} < 1$ and for any $\daleth, \mathfrak{g} \in \mathfrak{O}, \mathfrak{z} > 0$,*

$$\begin{aligned}
\mathfrak{w}(\daleth, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}) &\geq \mathfrak{w}\left(\daleth, \mathfrak{g}, \mathfrak{u}, \frac{\mathfrak{z}}{\text{\H{u}}}\right), \\
\mathfrak{N}(\daleth, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}) &\leq \mathfrak{N}\left(\daleth, \mathfrak{g}, \mathfrak{u}, \frac{\mathfrak{z}}{\text{\H{u}}}\right), \\
\mathfrak{L}(\daleth, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}) &\leq \mathfrak{L}\left(\daleth, \mathfrak{g}, \mathfrak{u}, \frac{\mathfrak{z}}{\text{\H{u}}}\right).
\end{aligned} \tag{5}$$

Then, $\daleth = \mathfrak{g}$.

Proof. It is easy to show on the lines of Lemma 1 in [15]. □

Definition 13. Let $(\mathfrak{O}, \mathfrak{w}, \mathfrak{N}, \mathfrak{L}, *, \Diamond)$ be a N2MS and ζ be a self-mapping on $(\mathfrak{O}, \mathfrak{w}, \mathfrak{N}, \mathfrak{L}, *, \Diamond)$. If there is $k \in (0, 1)$ such that

$$\begin{aligned}
\mathfrak{w}(\zeta\daleth, \zeta\mathfrak{g}, \mathfrak{u}, k\mathfrak{z}) &\geq \mathfrak{w}(\daleth, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}), \\
\mathfrak{N}(\zeta\daleth, \zeta\mathfrak{g}, \mathfrak{u}, k\mathfrak{z}) &\leq \mathfrak{N}(\daleth, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}), \\
\mathfrak{L}(\zeta\daleth, \zeta\mathfrak{g}, \mathfrak{u}, k\mathfrak{z}) &\leq \mathfrak{L}(\daleth, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}).
\end{aligned} \tag{6}$$

For all $\daleth, \mathfrak{g}, \mathfrak{u} \in \mathfrak{O}$, $\mathfrak{z} > 0$, then ζ is known as contractive mapping.

Theorem 14. *Let $(\mathfrak{O}, \mathfrak{w}, \mathfrak{N}, \mathfrak{L}, *, \Diamond)$ be a complete N2MS with $0 < \text{\H{u}} < 1$ and suppose that*

$$\begin{aligned}
\lim_{\mathfrak{z} \longrightarrow +\infty} \mathfrak{w}(\daleth, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}) &= 1, \\
\lim_{\mathfrak{z} \longrightarrow +\infty} \mathfrak{N}(\daleth, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}) &= 0, \\
\lim_{\mathfrak{z} \longrightarrow +\infty} \mathfrak{L}(\daleth, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}) &= 0,
\end{aligned} \tag{7}$$

for all $\daleth, \mathfrak{g} \in \mathfrak{O}$ and $\mathfrak{z} > 0$. Let $\zeta : \mathfrak{O} \longrightarrow \mathfrak{O}$ be a contractive mapping in the above definition. Then, ζ has a unique fixed point.

Proof. It is easy to show on the lines of Theorem 1 in [15]. □

Example 3. Let $\mathfrak{O} = [0, 1]$, $a * b = ab$, and $a \Diamond b = \max\{a, b\}$. Let $\mathfrak{w}, \mathfrak{N}, \mathfrak{L} : \mathfrak{O}^3 \times [0, +\infty) \longrightarrow [0, 1]$ be three mappings defined by

$$\begin{aligned}
\mathfrak{w}(\daleth, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}) &= \begin{cases} \dfrac{\mathfrak{z}}{\mathfrak{z} + \max\{|\daleth - \mathfrak{g}|, |\mathfrak{g} - \mathfrak{u}|, |\daleth - \mathfrak{u}|\}}, & \mathfrak{z} > 0, \\ 1, & \mathfrak{z} = 0, \end{cases} \\
\mathfrak{N}(\daleth, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}) &= \begin{cases} \dfrac{\max\{|\daleth - \mathfrak{g}|, |\mathfrak{g} - \mathfrak{u}|, |\daleth - \mathfrak{u}|\}}{\mathfrak{z} + \max\{|\daleth - \mathfrak{g}|, |\mathfrak{g} - \mathfrak{u}|, |\daleth - \mathfrak{u}|\}}, & \mathfrak{z} > 0, \\ 0, & \mathfrak{z} = 0, \end{cases} \\
\mathfrak{L}(\daleth, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}) &= \begin{cases} \dfrac{\max\{|\daleth - \mathfrak{g}|, |\mathfrak{g} - \mathfrak{u}|, |\daleth - \mathfrak{u}|\}}{\mathfrak{z}}, & \mathfrak{z} > 0, \\ 0, & \mathfrak{z} = 0, \end{cases}
\end{aligned} \tag{8}$$

for all $\daleth, \mathfrak{g}, \mathfrak{u} \in \mathfrak{O}$. Then, $(\mathfrak{O}, \mathfrak{w}, \mathfrak{N}, \mathfrak{L}, *, \Diamond)$ is a complete N2MS.

Now, define a self-mapping $\zeta : \mathfrak{O} \longrightarrow \mathfrak{O}$ by

$$\zeta(\daleth) = \frac{\daleth}{6}. \tag{9}$$

Then, all the conditions of Theorem 14 are fulfilled, and 0 is a unique fixed point.

Example 4. Let $\mathfrak{D} = [0, 1]$, $a * b = ab$, and $a \Diamond b = \max\{a, b\}$. Let $\mathfrak{w}, \mathfrak{N}, \mathfrak{L} : \mathfrak{D}^3 \times [0, +\infty) \longrightarrow [0, 1]$ be three mappings defined by

$$\mathfrak{w}(\beth, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}) = \begin{cases} \dfrac{\mathfrak{z}}{\mathfrak{z} + |\beth - \mathfrak{g} - \mathfrak{u}|}, & \mathfrak{z} > 0, \\ 1, & \mathfrak{z} = 0, \end{cases}$$
$$\mathfrak{N}(\beth, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}) = \begin{cases} 1 - \dfrac{\mathfrak{z}}{\mathfrak{z} + |\beth - \mathfrak{g} - \mathfrak{u}|}, & \mathfrak{z} > 0, \\ 0, & \mathfrak{z} = 0, \end{cases} \quad (10)$$
$$\mathfrak{L}(\beth, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}) = \begin{cases} \dfrac{|\beth - \mathfrak{g} - \mathfrak{u}|}{\mathfrak{z}}, & \mathfrak{z} > 0, \\ 0, & \mathfrak{z} = 0, \end{cases}$$

for all $\beth, \mathfrak{g}, \mathfrak{u} \in \mathfrak{D}$. Then, $(\mathfrak{D}, \mathfrak{w}, \mathfrak{N}, \mathfrak{L}, *, \Diamond)$ is a complete N2MS.

Now, define a self-mapping $\zeta : \mathfrak{D} \longrightarrow \mathfrak{D}$ by

$$\zeta(\beth) = \frac{\beth}{2}. \quad (11)$$

Then, all the conditions of Theorem 14 are fulfilled for $k = [1/2, 1)$, and 0 is a unique fixed point.

Definition 15. Let $(\mathfrak{D}, \mathfrak{w}, \mathfrak{N}, \mathfrak{L}, *, \Diamond)$ be a N2MS and ζ be a self-mapping on $(\mathfrak{D}, \mathfrak{w}, \mathfrak{N}, \mathfrak{L}, *, \Diamond)$. Then,

(I) ζ is known as continuous at $\beth_0 \in \mathfrak{D}$, if for all $\beth_n \longrightarrow \beth_0$ implies $\zeta\beth_n \longrightarrow \zeta\beth_0$

(II) let $\varphi > 0$ and $0 < \lambda < 1$. ζ is known as (φ, λ) uniform locally contractive if

$$\begin{aligned} \mathfrak{w}(\beth, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}) > 1 - \varphi &\Rightarrow \mathfrak{w}(\zeta\beth, \zeta\mathfrak{g}, \mathfrak{u}, \mathfrak{z}) \geq \mathfrak{w}\left(\beth, \mathfrak{g}, \mathfrak{u}, \frac{\mathfrak{z}}{\lambda}\right), \\ \mathfrak{N}(\beth, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}) < \varphi &\Rightarrow \mathfrak{N}(\zeta\beth, \zeta\mathfrak{g}, \mathfrak{u}, \mathfrak{z}) < \mathfrak{N}\left(\beth, \mathfrak{g}, \mathfrak{u}, \frac{\mathfrak{z}}{\lambda}\right), \\ \mathfrak{L}(\beth, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}) < \varphi &\Rightarrow \mathfrak{L}(\zeta\beth, \zeta\mathfrak{g}, \mathfrak{u}, \mathfrak{z}) < \mathfrak{L}\left(\beth, \mathfrak{g}, \mathfrak{u}, \frac{\mathfrak{z}}{\lambda}\right), \end{aligned} \quad (12)$$

for all $\beth, \mathfrak{g}, \mathfrak{u} \in \mathfrak{D}$, $\mathfrak{z} > 0$. Clearly, each (φ, λ) uniform locally contractive mapping ζ is continuous.

Example 5. Suppose $(\mathfrak{D}, \mathfrak{w}, \mathfrak{N}, \mathfrak{L}, *, \Diamond)$ be a N2MS and ζ be a self-mapping on $(\mathfrak{D}, \mathfrak{w}, \mathfrak{N}, \mathfrak{L}, *, \Diamond)$ given by $\zeta\beth = c$ for all $\beth \in \mathfrak{D}$ (c is constant). Let $\varphi > 0$ and $0 < \lambda < 1$. Assume that $\mathfrak{w}(\beth, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}) > 1 - \varphi$, $\mathfrak{N}(\beth, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}) < \varphi$ and $\mathfrak{L}(\beth, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}) < \varphi$ for all $\beth, \mathfrak{g}, \mathfrak{u} \in \mathfrak{D}$, $\mathfrak{z} > 0$. Therefore, we have that $1 = \mathfrak{w}(\zeta\beth, \zeta\mathfrak{g}, \mathfrak{u}, \mathfrak{z}) \geq \mathfrak{w}(\beth, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}/\lambda)$, $0 = \mathfrak{N}(\zeta\beth, \zeta\mathfrak{g}, \mathfrak{u}, \mathfrak{z}) \leq \mathfrak{N}(\beth, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}/\lambda)$, and $0 = \mathfrak{L}(\zeta\beth, \zeta\mathfrak{g}, \mathfrak{u}, \mathfrak{z}) \leq \mathfrak{L}(\beth, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}/\lambda)$ for all $\beth, \mathfrak{g}, \mathfrak{u} \in \mathfrak{D}$, $\mathfrak{z} > 0$. Hence, ζ is a (φ, λ) uniform locally contractive mapping.

Remark 16. In a N2MS $(\mathfrak{D}, \mathfrak{w}, \mathfrak{N}, \mathfrak{L}, *, \Diamond)$, a contractive mapping can be considered as a (φ, λ) uniform locally contractive mapping.

Definition 17. A N2MS $(\mathfrak{D}, \mathfrak{w}, \mathfrak{N}, \mathfrak{L}, *, \Diamond)$ is said to be metrically convex if for each $\beth, \mathfrak{g}, \mathfrak{u} \in \mathfrak{D}$, there is a $\beth \neq \mathfrak{g}, \mathfrak{u}, \mathfrak{z}$ for which $\mathfrak{w}(\beth, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}) = \mathfrak{w}(\beth, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}_0) * \mathfrak{w}(\beth, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}_1)$, $\mathfrak{N}(\beth, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}) = \mathfrak{N}(\beth, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}_0) \Diamond \mathfrak{N}(\beth, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}_1)$, and $\mathfrak{L}(\beth, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}) = \mathfrak{L}(\beth, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}_0) \Diamond \mathfrak{L}(\beth, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}_1)$, where $\mathfrak{z} = \mathfrak{z}_0 + \mathfrak{z}_1$ for all $\mathfrak{z}_0, \mathfrak{z}_1 > 0$.

Theorem 18. *Let $(\mathfrak{D}, \mathfrak{w}, \mathfrak{N}, \mathfrak{L}, *, \Diamond)$ be a metrically convex N2MS. If a self-mapping ζ on $(\mathfrak{D}, \mathfrak{w}, \mathfrak{N}, \mathfrak{L}, *, \Diamond)$ is (φ, λ) uniform locally contractive, then ζ is a contractive mapping with the fuzzy contractive constant λ.*

Proof. Let $\beth, \mathfrak{g}, \mathfrak{u} \in \mathfrak{D}$. Since $(\mathfrak{D}, \mathfrak{w}, \mathfrak{N}, *, \Diamond)$ is metrically convex, there are points $\beth = \beth_0, \beth_1, \beth_3, \cdots, \beth_{n-1}, \beth_n = \mathfrak{g}$ and $\mathfrak{z}_0, \mathfrak{z}_1, \cdots, \mathfrak{z}_n > 0$ such that

$$\begin{aligned} \mathfrak{w}(\beth, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}) &= \mathfrak{w}(\beth_0, \beth_1, \mathfrak{u}, \mathfrak{z}_0) * \mathfrak{w}(\beth_1, \beth_2, \mathfrak{u}, \mathfrak{z}_1) \\ &\quad * \cdots * \mathfrak{w}(\beth_{n-1}, \beth_n, \mathfrak{u}, \mathfrak{z}_n), \\ \mathfrak{N}(\beth, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}) &= \mathfrak{N}(\beth_0, \beth_1, \mathfrak{u}, \mathfrak{z}_0) \Diamond \mathfrak{N}(\beth_1, \beth_2, \mathfrak{u}, \mathfrak{z}_1) \\ &\quad \Diamond \cdots \Diamond \mathfrak{N}(\beth_{n-1}, \beth_n, \mathfrak{u}, \mathfrak{z}_n), \\ \mathfrak{L}(\beth, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}) &= \mathfrak{L}(\beth_0, \beth_1, \mathfrak{u}, \mathfrak{z}_0) \Diamond \mathfrak{L}(\beth_1, \beth_2, \mathfrak{u}, \mathfrak{z}_1) \\ &\quad \Diamond \cdots \Diamond \mathfrak{L}(\beth_{n-1}, \beth_n, \mathfrak{u}, \mathfrak{z}_n), \end{aligned} \quad (13)$$

where $\mathfrak{z} = \mathfrak{z}_0 + \mathfrak{z}_1 + \cdots + \mathfrak{z}_n$, $\mathfrak{w}(\beth_{i-1}, \beth_i, \mathfrak{u}, \mathfrak{z}) > 1 - \varphi$, $\mathfrak{N}(\beth_{i-1}, \beth_i, \mathfrak{u}, \mathfrak{z}) < \varphi$ and $\mathfrak{L}(\beth_{i-1}, \beth_i, \mathfrak{u}, \mathfrak{z}) < \varphi$ for $i = 1, 2, \cdots, n$. Also,

$$\begin{aligned} \mathfrak{w}(\zeta\beth, \zeta\mathfrak{g}, \mathfrak{u}, \mathfrak{z}) &= \mathfrak{w}(\zeta\beth_0, \zeta\beth_1, \mathfrak{u}, \mathfrak{z}_0) * \mathfrak{w}(\zeta\beth_1, \zeta\beth_2, \mathfrak{u}, \mathfrak{z}_1) \\ &\quad * \cdots * \mathfrak{w}(\zeta\beth_{n-1}, \zeta\beth_n, \mathfrak{u}, \mathfrak{z}_n), \\ \mathfrak{N}(\zeta\beth, \zeta\mathfrak{g}, \mathfrak{u}, \mathfrak{z}) &= \mathfrak{N}(\zeta\beth_0, \zeta\beth_1, \mathfrak{u}, \mathfrak{z}_0) \Diamond \mathfrak{N}(\zeta\beth_1, \zeta\beth_2, \mathfrak{u}, \mathfrak{z}_1) \\ &\quad \Diamond \cdots \Diamond \mathfrak{N}(\zeta\beth_{n-1}, \zeta\beth_n, \mathfrak{u}, \mathfrak{z}_n), \\ \mathfrak{L}(\zeta\beth, \zeta\mathfrak{g}, \mathfrak{u}, \mathfrak{z}) &= \mathfrak{L}(\zeta\beth_0, \zeta\beth_1, \mathfrak{u}, \mathfrak{z}_0) \Diamond \mathfrak{L}(\zeta\beth_1, \zeta\beth_2, \mathfrak{u}, \mathfrak{z}_1) \\ &\quad \Diamond \cdots \Diamond \mathfrak{L}(\zeta\beth_{n-1}, \zeta\beth_n, \mathfrak{u}, \mathfrak{z}_n). \end{aligned} \quad (14)$$

As ζ is (φ, λ) uniform locally contractive, we have

$$\begin{aligned} \mathfrak{w}(\zeta\beth_{i-1}, \zeta\beth_i, \mathfrak{u}, \mathfrak{z}_{i-1}) &\geq \mathfrak{w}\left(\beth_{i-1}, \beth_i, \mathfrak{u}, \frac{\mathfrak{z}_{i-1}}{\lambda}\right), \\ \mathfrak{N}(\zeta\beth_{i-1}, \zeta\beth_i, \mathfrak{u}, \mathfrak{z}_{i-1}) &\leq \mathfrak{N}\left(\beth_{i-1}\beth, \beth_i, \mathfrak{u}, \frac{\mathfrak{z}_{i-1}}{\lambda}\right), \\ \mathfrak{L}(\zeta\beth_{i-1}, \zeta\beth_i, \mathfrak{u}, \mathfrak{z}_{i-1}) &\leq \mathfrak{L}\left(\beth_{i-1}, \beth_i, \mathfrak{u}, \frac{\mathfrak{z}_{i-1}}{\lambda}\right), \end{aligned} \quad (15)$$

for $i = 1, 2, \cdots, n$. Hence, we have

$$\begin{aligned}
\mathfrak{w}(\zeta\gimel, \zeta\mathfrak{g}, \mathfrak{u}, \mathfrak{z}) &= \mathfrak{w}(\zeta\gimel_0, \zeta\gimel_1, \mathfrak{u}, \mathfrak{z}_0) * \mathfrak{w}(\zeta\gimel_1, \zeta\gimel_2, \mathfrak{u}, \mathfrak{z}_1) \\
&\quad * \cdots * \mathfrak{w}(\zeta\gimel_{n-1}, \zeta\gimel_n, \mathfrak{u}, \mathfrak{z}_n) \geq \mathfrak{w}\left(\gimel_0, \gimel_1, \mathfrak{u}, \frac{\mathfrak{z}_0}{\lambda}\right) \\
&\quad * \mathfrak{w}\left(\gimel_1, \gimel_2, \mathfrak{u}, \frac{\mathfrak{z}_1}{\lambda}\right) * \cdots * \mathfrak{w}\left(\gimel_{n-1}, \gimel_n, \mathfrak{u}, \frac{\mathfrak{z}_n}{\lambda}\right) \\
&= \mathfrak{w}\left(\gimel, \mathfrak{g}, \mathfrak{u}, \frac{\mathfrak{z}_n}{\lambda}\right), \\
\mathfrak{N}(\zeta\gimel, \zeta\mathfrak{g}, \mathfrak{u}, \mathfrak{z}) &= \mathfrak{N}(\zeta\gimel_0, \zeta\gimel_1, \mathfrak{u}, \mathfrak{z}_0) \Diamond \mathfrak{N}(\zeta\gimel_1, \zeta\gimel_2, \mathfrak{u}, \mathfrak{z}_1) \\
&\quad \Diamond \cdots \Diamond \mathfrak{N}(\zeta\gimel_{n-1}, \zeta\gimel_n, \mathfrak{u}, \mathfrak{z}_n) \\
&\leq \mathfrak{N}\left(\gimel_0, \gimel_1, \mathfrak{u}, \frac{\mathfrak{z}_0}{\lambda}\right) \Diamond \mathfrak{N}\left(\gimel_1, \gimel_2, \mathfrak{u}, \frac{\mathfrak{z}_1}{\lambda}\right) \\
&\quad \Diamond \cdots \Diamond \mathfrak{N}\left(\gimel_{n-1}, \gimel_n, \mathfrak{u}, \frac{\mathfrak{z}_n}{\lambda}\right) = \mathfrak{N}\left(\gimel, \mathfrak{g}, \mathfrak{u}, \frac{\mathfrak{z}_n}{\lambda}\right), \\
\mathfrak{L}(\zeta\gimel, \zeta\mathfrak{g}, \mathfrak{u}, \mathfrak{z}) &= \mathfrak{L}(\zeta\gimel_0, \zeta\gimel_1, \mathfrak{u}, \mathfrak{z}_0) \Diamond \mathfrak{L}(\zeta\gimel_1, \zeta\gimel_2, \mathfrak{u}, \mathfrak{z}_1) \\
&\quad \Diamond \cdots \Diamond \mathfrak{L}(\zeta\gimel_{n-1}, \zeta\gimel_n, \mathfrak{u}, \mathfrak{z}_n) \\
&\leq \mathfrak{L}\left(\gimel_0, \gimel_1, \mathfrak{u}, \frac{\mathfrak{z}_0}{\lambda}\right) \Diamond \mathfrak{L}\left(\gimel_1, \gimel_2, \mathfrak{u}, \frac{\mathfrak{z}_1}{\lambda}\right) \\
&\quad \Diamond \cdots \Diamond \mathfrak{L}\left(\gimel_{n-1}, \gimel_n, \mathfrak{u}, \frac{\mathfrak{z}_n}{\lambda}\right) = \mathfrak{L}\left(\gimel, \mathfrak{g}, \mathfrak{u}, \frac{\mathfrak{z}_n}{\lambda}\right).
\end{aligned} \tag{16}$$

So ζ is a contractive mapping. □

Definition 19. Let $(\mathfrak{D}, \mathfrak{w}, \mathfrak{N}, \mathfrak{L}, *, \Diamond)$ be a N2MS and $\varphi > 0$. A finite sequence

$$\gimel = \gimel_0, \gimel_1, \gimel_3, \cdots, \gimel_{n-1}, \gimel_n = \mathfrak{g} \tag{17}$$

is known as φ-chain from $\gimel$ to $\mathfrak{g}$ if

$$\begin{aligned}
&\mathfrak{w}(\gimel_{i-1}, \gimel_i, \mathfrak{u}, \mathfrak{z}) > 1 - \varphi, \\
&\mathfrak{N}(\gimel_{i-1}, \gimel_i, \mathfrak{u}, \mathfrak{z}) < \varphi, \\
&\mathfrak{L}(\gimel_{i-1}, \gimel_i, \mathfrak{u}, \mathfrak{z}) < \varphi \text{ for all } \mathfrak{u} \in \mathfrak{D}, \mathfrak{z} > 0,
\end{aligned} \tag{18}$$

and $i = 1, 2, \cdots, n$. A N2MS $(\mathfrak{D}, \mathfrak{w}, \mathfrak{N}, \mathfrak{L}, *, \Diamond)$ is known as φ-chainable if for each $\gimel, \mathfrak{g} \in \mathfrak{D}$, there is a φ-chain from $\gimel$ to $\mathfrak{g}$.

Theorem 20. *Let $(\mathfrak{D}, \mathfrak{M}, \mathfrak{N}, \mathfrak{L}, *, \Diamond)$ be a complete and φ -chainable N2MS. If a self-mapping ζ on $(\mathfrak{D}, \mathfrak{w}, \mathfrak{N}, \mathfrak{L}, *, \Diamond)$ is a (φ, λ)-uniform locally contractive, then ζ has a unique fixed point in $\mathfrak{D}$.*

Proof. Without loss of generality, let $\gimel \in \mathfrak{D}$ and $\zeta\gimel \neq \gimel$. Since $(\mathfrak{D}, \mathfrak{w}, \mathfrak{N}, *, \Diamond)$ is φ-chainable, there is a φ-chain $\gimel = \gimel_0, \gimel_1, \gimel_3, \cdots, \gimel_{n-1}, \gimel_n = \zeta\gimel$ from $\gimel$ to $\zeta\gimel$. From here, we have

$$\begin{aligned}
&\mathfrak{w}(\gimel_{i-1}, \gimel_i, \mathfrak{u}, \mathfrak{z}) > 1 - \varphi, \\
&\mathfrak{N}(\gimel_{i-1}, \gimel_i, \mathfrak{u}, \mathfrak{z}) < \varphi, \\
&\mathfrak{L}(\gimel_{i-1} < \varphi, \gimel_i, \mathfrak{u}, \mathfrak{z}) \text{ for all } \mathfrak{u} \in \mathfrak{D}, \mathfrak{z} > 0,
\end{aligned} \tag{19}$$

and $i = 1, 2, \cdots, n$. Utilizing induction, we get

$$\begin{aligned}
&\mathfrak{w}(\zeta^m\gimel_{i-1}, \zeta^m\gimel_i, \mathfrak{u}, \mathfrak{z}_{i-1}) \geq \mathfrak{w}\left(\gimel_{i-1}, \gimel_i, \mathfrak{u}, \frac{\mathfrak{z}_{i-1}}{\lambda^m}\right), \\
&\mathfrak{N}(\zeta^m\gimel_{i-1}, \zeta^m\gimel_i, \mathfrak{u}, \mathfrak{z}_{i-1}) \leq \mathfrak{N}\left(\gimel_{i-1}, \gimel_i, \mathfrak{u}, \frac{\mathfrak{z}_{i-1}}{\lambda^m}\right), \\
&\mathfrak{L}(\zeta^m\gimel_{i-1}, \zeta^m\gimel_i, \mathfrak{u}, \mathfrak{z}_{i-1}) \leq \mathfrak{L}\left(\gimel_{i-1}, \gimel_i, \mathfrak{u}, \frac{\mathfrak{z}_{i-1}}{\lambda^m}\right),
\end{aligned} \tag{20}$$

for all $\mathfrak{z} > 0$, $\mathfrak{u} \in \mathfrak{D}$, $m \in \mathbb{N}$, $i = 1, 2, \cdots, n$. Now, we deduce

$$\begin{aligned}
&1 \geq \mathfrak{w}(\zeta^m\gimel, \zeta^m\gimel, \mathfrak{u}, \mathfrak{z}_{i-1}) = \mathfrak{w}(\zeta^m\gimel_0, \zeta^m\gimel_n, \mathfrak{u}, \mathfrak{z}_{i-1}), \\
&0 \leq \mathfrak{N}(\zeta^m\gimel, \zeta^m\gimel, \mathfrak{u}, \mathfrak{z}_{i-1}) = \mathfrak{N}(\zeta^m\gimel_0, \zeta^m\gimel_n, \mathfrak{u}, \mathfrak{z}_{i-1}), \\
&0 \leq \mathfrak{L}(\zeta^m\gimel, \zeta^m\gimel, \mathfrak{u}, \mathfrak{z}_{i-1}) = \mathfrak{L}(\zeta^m\gimel_0, \zeta^m\gimel_n, \mathfrak{u}, \mathfrak{z}_{i-1}),
\end{aligned} \tag{21}$$

for all $\mathfrak{z} > 0$, $\mathfrak{u} \in \mathfrak{D}$, $m \in \mathbb{N}$, $i = 1, 2, \cdots, n$. Utilizing the Lemma 10, we examine that $\{\zeta^m\gimel\}$ is a Cauchy sequence in $\mathfrak{D}$. As $(\mathfrak{D}, \mathfrak{w}, \mathfrak{N}, \mathfrak{L}, *, \Diamond)$ is complete, there is a point $\mathfrak{g} \in \mathfrak{D}$ such that $\lim_{n \longrightarrow +\infty} \zeta^m\gimel = \mathfrak{g}$. Since ζ is continuous, we have $\lim_{n \longrightarrow +\infty} \zeta^{m+1}\gimel = \zeta\mathfrak{g}$. Hence, $\zeta\mathfrak{g} = \mathfrak{g}$ and $\mathfrak{g}$ is a fixed point of ζ. To show uniqueness, assume $\zeta\mathfrak{L} = \mathfrak{L}$ for some $\mathfrak{L} \in \mathfrak{D}(\mathfrak{L} \neq \mathfrak{g})$. Since $(\mathfrak{D}, \mathfrak{w}, \mathfrak{N}, \mathfrak{L}, *, \Diamond)$ is φ-chainable, there is a φ-chain $\mathfrak{g} = \mathfrak{L}_0, \mathfrak{L}_1, \mathfrak{L}_3, \cdots, \mathfrak{L}_{n-1}, \mathfrak{L}_n = \mathfrak{L}$ from $\mathfrak{g}$ to $\mathfrak{L}$. Now, for any $\mathfrak{m} \in \mathbb{N}$, we have

$$\begin{aligned}
&1 \geq \mathfrak{w}(\mathfrak{g}, \mathfrak{L}, \mathfrak{u}, \mathfrak{z}) = \mathfrak{w}(\zeta^{\mathfrak{m}}\gimel, \zeta^{\mathfrak{m}}\gimel, \mathfrak{u}, \mathfrak{z}) = \mathfrak{w}(\zeta^{\mathfrak{m}}\gimel_0, \zeta^{\mathfrak{m}}\gimel_n, \mathfrak{u}, \mathfrak{z}), \\
&1 \leq \mathfrak{N}(\mathfrak{g}, \mathfrak{L}, \mathfrak{u}, \mathfrak{z}) = \mathfrak{N}(\zeta^{\mathfrak{m}}\gimel, \zeta^{\mathfrak{m}}\gimel, \mathfrak{u}, \mathfrak{z}) = \mathfrak{N}(\zeta^{\mathfrak{m}}\gimel_0, \zeta^{\mathfrak{m}}\gimel_n, \mathfrak{u}, \mathfrak{z}), \\
&1 \leq \mathfrak{L}(\mathfrak{g}, \mathfrak{L}, \mathfrak{u}, \mathfrak{z}) = \mathfrak{L}(\zeta^{\mathfrak{m}}\gimel, \zeta^{\mathfrak{m}}\gimel, \mathfrak{u}, \mathfrak{z}) = \mathfrak{L}(\zeta^{\mathfrak{m}}\gimel_0, \zeta^{\mathfrak{m}}\gimel_n, \mathfrak{u}, \mathfrak{z}),
\end{aligned} \tag{22}$$

for all $\mathfrak{z} > 0$, $\mathfrak{u} \in \mathfrak{D}$, $m \in \mathbb{N}$, $i = 1, 2, \cdots, n$. We have

$$\begin{aligned}
&\mathfrak{w}(\mathfrak{g}, \mathfrak{L}, \mathfrak{u}, \mathfrak{z}) = 1, \mathfrak{N}(\mathfrak{g}, \mathfrak{L}, \mathfrak{u}, \mathfrak{z}) = 0, \\
&\mathfrak{L}(\mathfrak{g}, \mathfrak{L}, \mathfrak{u}, \mathfrak{z}) = 0,
\end{aligned} \tag{23}$$

for all $\mathfrak{z} > 0$, $\mathfrak{u} \in \mathfrak{D}$. So, we obtain that $\mathfrak{g} = \mathfrak{L}.\gimel$ □

Definition 21. Let $(\mathfrak{D}, \mathfrak{w}, \mathfrak{N}, \mathfrak{L}, *, \Diamond)$ be a N2MS and A, B be two self-mappings on $(\mathfrak{D}, \mathfrak{w}, \mathfrak{N}, \mathfrak{L}, *, \Diamond)$. A pair (A, B) is said to be weak compatible if $A\gimel = B\gimel$ for some $\gimel \in \mathfrak{D}$ implies $A B\gimel = BA\gimel$.

Theorem 22. *Let $(\mathfrak{D}, \mathfrak{w}, \mathfrak{N}, \mathfrak{L}, *, \Diamond)$ be a complete φ-chainable N2MS and the self-mapping A, B, S, ζ on $(\mathfrak{D}, \mathfrak{w}, \mathfrak{N}, \mathfrak{L}, *, \Diamond)$ fulfilling the following criteria:*

(1) $A\mathfrak{D} \subseteq \zeta\mathfrak{D}$ and $B\mathfrak{D} \subseteq S\mathfrak{D}$

(2) There exist $k \in (0, 1)$ such that $\mathfrak{w}(A\gimel, B\mathfrak{g}, \mathfrak{u}, k\mathfrak{z}) \geq \mathfrak{w}(S\gimel, \zeta\mathfrak{g}, \mathfrak{u}, \mathfrak{z}) * \mathfrak{w}(A\gimel, S\mathfrak{g}, \mathfrak{u}, \mathfrak{z}) * \mathfrak{w}(B\gimel, \zeta\mathfrak{g}, \mathfrak{u}, \mathfrak{z}) * \mathfrak{w}(A\gimel, \zeta\mathfrak{g}, \mathfrak{u}, \mathfrak{z})$, $\mathfrak{N}(A\gimel, B\mathfrak{g}, \mathfrak{u}, k\mathfrak{z}) \leq \mathfrak{N}(S\gimel, \zeta\mathfrak{g}, \mathfrak{u}, \mathfrak{z}) \Diamond \mathfrak{N}(A\gimel, S\mathfrak{g}, \mathfrak{u}, \mathfrak{z}) \Diamond \mathfrak{N}(B\gimel, \zeta\mathfrak{g}, \mathfrak{u}, \mathfrak{z}) \Diamond \mathfrak{N}(A\gimel, \zeta\mathfrak{g}, \mathfrak{u}, \mathfrak{z})$, and $\mathfrak{L}(A\gimel, B\mathfrak{g}, \mathfrak{u}, k\mathfrak{z}) \leq \mathfrak{L}(S\gimel, \zeta\mathfrak{g}, \mathfrak{u}, \mathfrak{z}) \Diamond \mathfrak{L}(A\gimel, S\mathfrak{g}, \mathfrak{u}, \mathfrak{z}) \Diamond \mathfrak{L}(B\gimel, \zeta\mathfrak{g}, \mathfrak{u}, \mathfrak{z}) \Diamond \mathfrak{N}(A\gimel, \zeta\mathfrak{g}, \mathfrak{u}, \mathfrak{z})$ for all $\gimel, \mathfrak{g}, \mathfrak{u} \in \mathfrak{D}$ and $\mathfrak{z} > 0$

(3) The pairs (A, S) and (B, ζ) are weakly compatible

(4) A and S are continuous

Then, A, B, S, and ζ have a unique common fixed point in $\mathfrak{D}$.

Corollary 23. *Let $(\mathfrak{D}, \mathfrak{w}, \mathfrak{N}, \mathfrak{L}, *, \Diamond)$ be a complete φ-chainable N2MS and the self-mapping A, B, S, ζ on $(\mathfrak{D}, \mathfrak{w}, \mathfrak{N}, \mathfrak{L}, *, \Diamond)$ fulfilling the following criteria:*

(C1) $A\mathfrak{D} \subseteq \zeta\mathfrak{D}$ and $B\mathfrak{D} \subseteq S\mathfrak{D}$

(C2) There exist $k \in (0, 1)$ such that $\mathfrak{w}(A\daleth, B\mathfrak{g}, \mathfrak{u}, k\mathfrak{z}) \geq \mathfrak{w}(S\daleth, \zeta\mathfrak{g}, \mathfrak{u}, \mathfrak{z})$, $\mathfrak{N}(A\daleth, B\mathfrak{g}, \mathfrak{u}, k\mathfrak{z}) \leq \mathfrak{N}(S\daleth, \zeta\mathfrak{g}, \mathfrak{u}, \mathfrak{z})$, and $\mathfrak{L}(A\daleth, B\mathfrak{g}, \mathfrak{u}, k\mathfrak{z}) \leq \mathfrak{L}(S\daleth, \zeta\mathfrak{g}, \mathfrak{u}, \mathfrak{z})$ for all $\daleth, \mathfrak{g}, \mathfrak{u} \in \mathfrak{D}$ and $\mathfrak{z} > 0$

(C3) The pairs (A, S) and (B, ζ) are weakly compatible

(C4) A and S are continuous

Then, A, B, S, and ζ have a unique common fixed point in $\mathfrak{D}$.

If assume $S, \zeta = I$ in the preceding corollary, we deduce the below result.

Corollary 24. *Let $(\mathfrak{D}, \mathfrak{w}, \mathfrak{N}, \mathfrak{L}, *, \Diamond)$ be a complete φ-chainable N2MS and the self-mapping A, B on $(\mathfrak{D}, \mathfrak{w}, \mathfrak{N}, \mathfrak{L}, *, \Diamond)$ fulfilling the following criteria.*

There exist $k \in (0, 1)$ such that

$$\begin{aligned}\mathfrak{w}(A\daleth, B\mathfrak{g}, \mathfrak{u}, k\mathfrak{z}) &\geq \mathfrak{w}(\daleth, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}),\\ \mathfrak{N}(A\daleth, B\mathfrak{g}, \mathfrak{u}, k\mathfrak{z}) &\leq \mathfrak{N}(\daleth, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}),\\ \mathfrak{L}(A\daleth, B\mathfrak{g}, \mathfrak{u}, k\mathfrak{z}) &\leq \mathfrak{L}(\daleth, \mathfrak{g}, \mathfrak{u}, \mathfrak{z}),\end{aligned} \tag{24}$$

for all $\daleth, \mathfrak{g}, \mathfrak{u} \in \mathfrak{D}$ and $\mathfrak{z} > 0$ and A and B are continuous. Then, A and B have a unique common fixed point in $\mathfrak{D}$.

If we suppose $A = B$ in the preceding corollary, then we get the below result.

4. Application

Let $\mathfrak{D} = C([\sigma, \tau], \mathbb{R})$ be the set of all continuous functions with domain of real values and defined on $[\sigma, \tau]$.

Now, we let the neutrosophic integral equation:

$$\daleth(\mathfrak{m}) = \mathfrak{A}(\mathfrak{n}) + \int_\sigma^\tau \mathfrak{B}(\mathfrak{m}, \mathfrak{n})\daleth(\mathfrak{m})\mathfrak{d}\mathfrak{n} \text{ for } \mathfrak{m}, \mathfrak{n} \in [\sigma, \tau], \tag{25}$$

where $\mathfrak{A}(\mathfrak{n})$ be a neutrosophic function of $\mathfrak{n} : \mathfrak{n} \in [\sigma, \tau]$ and $\mathfrak{B} \in \mathfrak{D}$. Define $\mathfrak{w}, \mathfrak{N}$, and $\mathfrak{L}$ by

$$\begin{aligned}&\mathfrak{w}(\daleth(\mathfrak{m}), \mathfrak{g}(\mathfrak{m}), \mathfrak{u}(\mathfrak{m}), \mathfrak{z})\\ &\quad = \sup_{\mathfrak{m}\in[\sigma,\tau]} \frac{\mathfrak{z}}{\mathfrak{z} + |\daleth(\mathfrak{m}) - \mathfrak{g}(\mathfrak{m}) - \mathfrak{u}(\mathfrak{m})|}\\ &\quad \text{for all } \daleth, \mathfrak{g} \in \mathfrak{D} \text{ and } \mathfrak{z} > 0,\end{aligned} \tag{26}$$

$$\begin{aligned}&\mathfrak{N}(\daleth(\mathfrak{m}), \mathfrak{g}(\mathfrak{m}), \mathfrak{u}(\mathfrak{m}), \mathfrak{z})\\ &\quad = 1 - \sup_{\mathfrak{m}\in[\sigma,\tau]} \frac{\mathfrak{z}}{\mathfrak{z} + |\daleth(\mathfrak{m}) - \mathfrak{g}(\mathfrak{m}) - \mathfrak{u}(\mathfrak{m})|}\\ &\quad \text{for all } \daleth, \mathfrak{g} \in \mathfrak{D} \text{ and } \mathfrak{z} > 0,\end{aligned} \tag{27}$$

$$\begin{aligned}&\mathfrak{L}(\daleth(\mathfrak{m}), \mathfrak{g}(\mathfrak{m}), \mathfrak{u}(\mathfrak{m}), \mathfrak{z})\\ &\quad = \sup_{\mathfrak{m}\in[\sigma,\tau]} \frac{|\daleth(\mathfrak{m}) - \mathfrak{g}(\mathfrak{m}) - \mathfrak{u}(\mathfrak{m})|}{\mathfrak{z}}\\ &\quad \text{for all } \daleth, \mathfrak{g} \in \mathfrak{D} \text{ and } \mathfrak{z} > 0,\end{aligned} \tag{28}$$

with CTN and CTCN define by $a * b = a.b$ and $a\Diamond b = \max\{a, b\}$. Then, $(\mathfrak{D}, \mathfrak{w}, \mathfrak{N}, \mathfrak{L}, *, \Diamond)$ be a complete N2MS.

Assume that $|\mathfrak{B}(\mathfrak{m}, \mathfrak{n})\daleth(\mathfrak{m}) - \mathfrak{B}(\mathfrak{m}, \mathfrak{n})\mathfrak{g}(\mathfrak{m}) - (\mathfrak{u}(\mathfrak{m})/\int_\sigma^\tau \mathfrak{d}\mathfrak{n})| \leq |\daleth(\mathfrak{m}) - \mathfrak{g}(\mathfrak{m}) - \mathfrak{u}(\mathfrak{m})|$ for $\daleth, \mathfrak{g} \in \mathfrak{D}$, $\acute{\ddot{u}} \in (0, 1)$ and for all $\mathfrak{m}, \mathfrak{n} \in [\sigma, \tau]$. Also, consider $\int_\sigma^\tau \mathfrak{d}\mathfrak{n} \leq \acute{\ddot{u}} < 1$. Then, neutrosophic integral Equation (25) has a unique solution.

Proof. Define $\zeta : (\mathfrak{D}, \mathfrak{w}, \mathfrak{N}, \mathfrak{L}, *, \Diamond) \longrightarrow (\mathfrak{D}, \mathfrak{w}, \mathfrak{N}, \mathfrak{L}, *, \Diamond)$ by

$$\zeta\daleth(\mathfrak{m}) = \mathfrak{A}(\mathfrak{n}) + \int_\sigma^\tau \mathfrak{B}(\mathfrak{m}, \mathfrak{n})\sigma(\mathfrak{m})\mathfrak{d}\mathfrak{n} \text{ for all } \mathfrak{m}, \mathfrak{n} \in [\sigma, \tau]. \tag{29}$$

Scrutinize that survival of a fixed of the operator ζ has come to the survival of solution of a neutrosophic integral equation.

Now for all $\daleth, \mathfrak{g} \in \mathfrak{D}$, we get

$$\begin{aligned}&\mathfrak{M}\left(\zeta\daleth(\mathfrak{m}), \zeta\mathfrak{g}(\mathfrak{m}), \mathfrak{u}(\mathfrak{m}), \acute{\ddot{u}}\mathfrak{z}\right)\\ &\quad = \sup_{\mathfrak{m}\in[\sigma,\tau]} \frac{\acute{\ddot{u}}\mathfrak{z}}{\acute{\ddot{u}}\mathfrak{z} + |\zeta\daleth(\mathfrak{m}) - \zeta\mathfrak{g}(\mathfrak{m}) - \mathfrak{u}(\mathfrak{m})|} = \sup_{\mathfrak{m}\in[\sigma,\tau]} \frac{\acute{\ddot{u}}\mathfrak{z}}{\acute{\ddot{u}}\mathfrak{z} + \left|\mathfrak{U}(\mathfrak{n}) + \int_\sigma^\tau \mathfrak{B}(\mathfrak{m}, \mathfrak{n})\daleth(\mathfrak{m})\mathfrak{d}\mathfrak{n} - \mathfrak{U}(\mathfrak{n}) - \int_\sigma^\tau \mathfrak{B}(\mathfrak{m}, \mathfrak{n})\mathfrak{g}(\mathfrak{m})\mathfrak{d}\mathfrak{n} - \mathfrak{u}(\mathfrak{m})\right|}\\ &\quad = \sup_{\mathfrak{m}\in[\sigma,\tau]} \frac{\acute{\ddot{u}}\mathfrak{z}}{\acute{\ddot{u}}\mathfrak{z} + \left|\int_\sigma^\tau \mathfrak{B}(\mathfrak{m}, \mathfrak{n})\daleth(\mathfrak{m})\mathfrak{d}\mathfrak{n} - \int_\sigma^\tau \mathfrak{B}(\mathfrak{m}, \mathfrak{n})\mathfrak{g}(\mathfrak{m})\mathfrak{d}\mathfrak{n} - \mathfrak{u}(\mathfrak{m})\right|}\\ &\quad = \sup_{\mathfrak{m}\in[\sigma,\tau]} \frac{\acute{\ddot{u}}\mathfrak{z}}{\acute{\ddot{u}}\mathfrak{z} + \left|\mathfrak{B}(\mathfrak{m}, \mathfrak{n})\daleth(\mathfrak{m}) - \mathfrak{B}(\mathfrak{m}, \mathfrak{n})\mathfrak{g}(\mathfrak{m}) - \left(\mathfrak{u}(\mathfrak{m})/\int_\sigma^\tau \mathfrak{d}\mathfrak{n}\right)\right|\left(\int_\sigma^\tau \mathfrak{d}\mathfrak{n}\right)} \geq \sup_{\mathfrak{m}\in[\sigma,\tau]} \frac{\mathfrak{z}}{\mathfrak{z} + |\daleth(\mathfrak{m}) - \mathfrak{g}(\mathfrak{m}) - \mathfrak{u}(\mathfrak{m})|}\\ &\quad \geq \mathfrak{w}(\daleth(\mathfrak{m}), \mathfrak{g}(\mathfrak{m}), \mathfrak{z}),\end{aligned}$$

$$
\begin{aligned}
&\mathfrak{N}\left(\zeta\beth(\mathfrak{m}), \zeta\mathfrak{g}(\mathfrak{m}), \mathfrak{u}(\mathfrak{m}), \acute{\mathfrak{u}}\mathfrak{z}\right)\\
&\quad = 1 - \sup_{\mathfrak{m}\in[\sigma,\tau]} \frac{\acute{\mathfrak{u}}\mathfrak{z}}{\acute{\mathfrak{u}}\mathfrak{z} + \left|\zeta\beth(\mathfrak{m}) - \zeta\mathfrak{g}(\mathfrak{m}) - \mathfrak{u}(\mathfrak{m})\right|}\\
&\quad = 1 - \sup_{\mathfrak{m}\in[\sigma,\tau]} \frac{\acute{\mathfrak{u}}\mathfrak{z}}{\acute{\mathfrak{u}}\mathfrak{z} + \left|\mathfrak{U}(\mathfrak{n}) + \int_\sigma^\tau \mathfrak{B}(\mathfrak{m},\mathfrak{n})\beth(\mathfrak{m})\mathfrak{d}\mathfrak{n} - \mathfrak{U}(\mathfrak{n}) - \int_\sigma^\tau \mathfrak{B}(\mathfrak{m},\mathfrak{n})\mathfrak{g}(\mathfrak{m})\mathfrak{d}\mathfrak{n} - \mathfrak{u}(\mathfrak{m})\right|}\\
&\quad = 1 - \sup_{\mathfrak{m}\in[\sigma,\tau]} \frac{\acute{\mathfrak{u}}\mathfrak{z}}{\acute{\mathfrak{u}}\mathfrak{z} + \left|\int_\sigma^\tau \mathfrak{B}(\mathfrak{m},\mathfrak{n})\beth(\mathfrak{m})\mathfrak{d}\mathfrak{n} - \int_\sigma^\tau \mathfrak{B}(\mathfrak{m},\mathfrak{n})\mathfrak{g}(\mathfrak{m})\mathfrak{d}\mathfrak{n} - \mathfrak{u}(\mathfrak{m})\right|}\\
&\quad = 1 - \sup_{\mathfrak{m}\in[\sigma,\tau]} \frac{\acute{\mathfrak{u}}\mathfrak{z}}{\acute{\mathfrak{u}}\mathfrak{z} + \left|\mathfrak{B}(\mathfrak{m},\mathfrak{n})\beth(\mathfrak{m}) - \mathfrak{B}(\mathfrak{m},\mathfrak{n})\mathfrak{g}(\mathfrak{m}) - \left(\mathfrak{u}(\mathfrak{m})/\int_\sigma^\tau \mathfrak{d}\mathfrak{n}\right)\right|\left(\int_\sigma^\tau \mathfrak{d}\mathfrak{n}\right)} \geq 1 - \sup_{\mathfrak{m}\in[\sigma,\tau]} \frac{\mathfrak{z}}{\mathfrak{z} + \left|\beth(\mathfrak{m}) - \mathfrak{g}(\mathfrak{m}) - \mathfrak{u}(\mathfrak{m})\right|}\\
&\quad \geq \mathfrak{N}(\beth(\mathfrak{m}), \mathfrak{g}(\mathfrak{m}), \mathfrak{u}(\mathfrak{m}), \mathfrak{z}),\\
&\mathfrak{L}\left(\zeta\beth(\mathfrak{m}), \zeta\mathfrak{g}(\mathfrak{m}), \acute{\mathfrak{u}}\,\mathfrak{z}\right)\\
&\quad = \sup_{\mathfrak{m}\in[\sigma,\tau]} \frac{\left|\zeta\beth(\mathfrak{m}) - \zeta\mathfrak{g}(\mathfrak{m}) - \mathfrak{u}(\mathfrak{m})\right|}{\acute{\mathfrak{u}}\mathfrak{z}} = \sup_{\mathfrak{m}\in[\sigma,\tau]} \frac{\left|\mathfrak{U}(\mathfrak{n}) + \int_\sigma^\tau \mathfrak{B}(\mathfrak{m},\mathfrak{n})\beth(\mathfrak{m})\mathfrak{d}\mathfrak{n} - \mathfrak{U}(\mathfrak{n}) - \int_\sigma^\tau \mathfrak{B}(\mathfrak{m},\mathfrak{n})\mathfrak{g}(\mathfrak{m})\mathfrak{d}\mathfrak{n} - \mathfrak{u}(\mathfrak{m})\right|}{\acute{\mathfrak{u}}\mathfrak{z}}\\
&\quad = \sup_{\mathfrak{m}\in[\sigma,\tau]} \frac{\left|\int_\sigma^\tau \mathfrak{B}(\mathfrak{m},\mathfrak{n})\beth(\mathfrak{m})\mathfrak{d}\mathfrak{n} - \int_\sigma^\tau \mathfrak{B}(\mathfrak{m},\mathfrak{n})\mathfrak{g}(\mathfrak{m})\mathfrak{d}\mathfrak{n} - \mathfrak{u}(\mathfrak{m})\right|}{\acute{\mathfrak{u}}\mathfrak{z}}\\
&\quad = \sup_{\mathfrak{m}\in[\sigma,\tau]} \frac{\left|\mathfrak{B}(\mathfrak{m},\mathfrak{n})\beth(\mathfrak{m}) - \mathfrak{B}(\mathfrak{m},\mathfrak{n})\mathfrak{g}(\mathfrak{m}) - \left(\mathfrak{u}(\mathfrak{m})/\int_\sigma^\tau \mathfrak{d}\mathfrak{n}\right)\right|\left(\int_\sigma^\tau \mathfrak{d}\mathfrak{n}\right)}{\acute{\mathfrak{u}}\mathfrak{z}} \leq \sup_{\mathfrak{m}\in[\sigma,\tau]} \frac{\left|\beth(\mathfrak{m}) - \mathfrak{g}(\mathfrak{m}) - \mathfrak{u}(\mathfrak{m})\right|}{\mathfrak{z}}\\
&\quad \leq \mathfrak{L}((\mathfrak{m}), \mathfrak{g}(\mathfrak{m}), \mathfrak{u}(\mathfrak{m}), \mathfrak{z}).
\end{aligned} \tag{30}
$$

That is, the neutrosophic integral equation satisfied the criteria of Theorem 14. Hence, the neutrosophic integral equation has a unique solution. □

5. Conclusions

In this manuscript, we established the notion of neutrosophic 2-metric space by replacing the triangular inequalities of neutrosophic metric spaces by tetrahedron inequalities and introduce a notion of neutrosophic 2-metric spaces and proved some interesting results in the context of neutrosophic 2-metric spaces. These results boost the approaches of existing ones in the literature. Several examples and an application to examine the uniqueness of the solution of the integral equation are also imparted. This work can easily be extended in various structures like neutrosophic-controlled 2-metric spaces, neutrosophic triple partial 2-metric spaces, and neutrosophic 3-metric spaces. In the future, we will work on fixed point results for more than two self-mappings and solve differential and integral equations by utilizing neutrosophic 2-metric spaces.

Data Availability

On request, the data used to support the findings of this study can be obtained from the corresponding author.

Conflicts of Interest

There are no competing interests declared by the authors.

Authors' Contributions

This article was written in collaboration by all of the contributors. The final manuscript was read and approved by all writers.

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