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Research Article

Groups Which Contain the Diffeomorphisms and Superdiffeomorphisms as Proper Subgroups

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We begin by recalling a group which is an enlargement of the diffeomorphisms on an ordinary manifold. We used this larger group to unify the gravitational and electroweak fields, which are mediated by bosons. Unfortunately, we could not include the neutrinos, because geometrical theories based on an ordinary manifold generally cannot include fermions. Therefore, in this paper, we introduce an analogous group that contains the superdiffeomorphisms on a DeWitt supermanifold as a proper subgroup.

1. Introduction

Our group enlargement has been motivated by a prediction of Dirac's [1], a subsequent more specific suggestion of Einstein's [2], a requirement by Pauli [3], and by remarks by DeWitt [4, 5].

In the preface to the first edition of his famous book *Quantum Mechanics*, Dirac made the prophetic statement: "The growth of the use of transformation theory, as applied first to relativity and later to the quantum theory, is the essence of the new method in theoretical physics. Further progress lies in the direction of making our equations invariant under wider and still wider transformations."

In his *Autobiographical Notes*, Einstein made the following statement which is consistent with that of Dirac, but which is more specific: "Our problem is that of finding the field equations for the total field. The desired structure must be a generalization of the symmetric tensor. The group must not be any narrower than that of the continuous transformations of co-ordinates. If one introduces a richer structure, then the group will no longer determine the equations as strongly as in the case of the symmetric tensor as structure. Therefore it would be most beautiful, if one were to succeed in expanding the group once more, analogous to the step which led from special relativity to general relativity."

The group that Einstein called "the continuous transformations of co-ordinates" is presently called the group of diffeomorphisms and is defined by (1). Now, the diffeomorphisms (which led to general relativity) contains the Lorentz group (which led to special relativity) as a proper subgroup. Thus, Einstein's suggestion is that we seek a group which contains the diffeomorphisms as a proper subgroup, but does not suggest how such a group should be found.

Pauli noted that "it is absolutely essential that such a fundamental theorem as the covariance law be derivable from the simplest possible basis assumptions." In Section 2.2, we obtain a covariance group which satisfies Pauli's requirement, by assuming that all observers will agree whether or not any given vector quantity is conserved. The new covariance group is defined by (2) and is called the "conservation group." In previous physics papers [6–13] we used the conservation group in an attempt to unify the gravitational and electroweak interactions.

Our attempt was only partially successful because it does not include fermions. This is a common defect in geometrical theories defined on an ordinary manifold. However, as DeWitt [4] notes, the discovery of Bose-Fermi supersymmetry suggests that we should pass from an ordinary manifold to a supermanifold. DeWitt [5] also states that, in a supermanifold, the proper setting "will be a suitable

elaboration of the geometrical ideas upon which Einstein based general relativity."

For the reasons discussed above, in Section 3 of this paper, we introduce a super conservation group which contains the superdiffeomorphisms as a proper subgroup.

2. Group Which Contains the Diffeomorphisms as a Proper Subgroup

In previous physics papers [6–13], we presented a group which contains the diffeomorphisms as a proper subgroup. We include a discussion of that group in this section, because its inclusion will optimize the clarity of our new group's introduction in Section 3.

2.1. The Diffeomorphism Group. For a diffeomorphism, infinitesimal changes of the coordinates are related by $dx^{\bar{i}} = x^{\bar{i}}_{j} dx^{j}$, where the summation convention has been adopted. The matrix of the transformation coefficients $x^{\bar{i}}_{j}$ must be nonsingular, and the integrability condition

$$x^{\bar{i}}_{k,j} - x^{\bar{i}}_{j,k} = 0 \tag{1}$$

must be satisfied. Partial differentiation is denoted by a comma.

2.2. The Conservation Group. In [6–13], we replaced (1) by the requirement that the transformation coefficients will satisfy the weaker condition

$$x^{k}_{\bar{i}} \left(x^{\bar{i}}_{k,j} - x^{\bar{i}}_{j,k} \right) = 0. \tag{2}$$

Transformations which satisfy (2) are called *conservative*, for a reason that will become clear in Section 2.2.1. Proof that (2) defines a group which contains the diffeomorphisms as a proper subgroup will be recalled in Section 2.2.2.

2.2.1. Transformation of a Conservation Law. A conservation law is an expression of the form

$$V^{i}_{,i} = 0, \tag{3}$$

where V^{i} is a vector density of weight +1. It is surprising and very important that (3) is invariant, not only under the diffeomorphisms, but also under the more general transformations defined by (2). This may be seen in the following way.

The transformation law for a vector density of weight +1 is

$$V^{\bar{i}} = \frac{\partial x}{\partial \bar{x}} x^{\bar{i}}_{j} V^{j}, \tag{4}$$

where $\partial x / \partial \bar{x}$ is the Jacobian determinant of $x^{j}_{\bar{i}}$. If we differentiate (4) with respect to $x^{\bar{i}}$, a short calculation gives

$$V^{\bar{i}}_{,\bar{i}} = \frac{\partial x}{\partial \bar{x}} V^{i}_{,i} - V^{j} \frac{\partial x}{\partial \bar{x}} x^{k}_{\bar{i}} \left(x^{\bar{i}}_{k,j} - x^{\bar{i}}_{j,k} \right). \tag{5}$$

Equation (5) shows that (3) is invariant, not only under the diffeomophisms, but also under the larger group defined by (2). This is clear because V^{j} is arbitrary and $\partial x / \partial \bar{x} \neq 0$. Thus it follows from (5) that we have both $V^{i}_{,i} = 0$ and $V^{\bar{i}}_{,\bar{i}} = 0$ (i.e., a conservation law is an invariant statement) if and only if the transformation coefficients satisfy (2). We therefore call transformations that satisfy (2) "conservative."

2.2.2. Proof That the Conservative Transformations Form a Group Which Contains the Diffeomorphisms as a Proper Subgroup. First, we note that the identity transformation $x^{\bar{i}} = x^{i}$ is a conservative transformation. Next, we consider the result of following a transformation from x^{i} to $x^{\bar{i}}$ by a transformation from $x^{\bar{i}}$ to $x^{\hat{i}}$. Upon differentiating $x^{\hat{i}}_{k} = x^{\hat{i}}_{\bar{r}} x^{\bar{r}}_{k}$ with respect to x^{j}, subtracting the corresponding expression with j and k interchanged, and multiplying by $x^{k}_{\hat{i}}$, we obtain

$$\begin{aligned} & x^{k}_{\hat{i}} \left(x^{\hat{i}}_{k,j} - x^{\hat{i}}_{j,k} \right) \\ & \quad = x^{\bar{r}}_{\hat{i}} \left(x^{\hat{i}}_{\bar{r},\bar{s}} - x^{\hat{i}}_{\bar{s},\bar{r}} \right) x^{\bar{s}}_{j} + x^{k}_{\bar{r}} \left(x^{\bar{r}}_{k,j} - x^{\bar{r}}_{j,k} \right). \end{aligned} \tag{6}$$

We see from (6) that if the quantities $x^{\bar{r}}_{\hat{i}} (x^{\hat{i}}_{\bar{r},\bar{s}} - x^{\hat{i}}_{\bar{s},\bar{r}}) x^{\bar{s}}_{j}$ and $x^{k}_{\bar{r}} (x^{\bar{r}}_{k,j} - x^{\bar{r}}_{j,k})$ both vanish, then the quantity $x^{k}_{\hat{i}} (x^{\hat{i}}_{k,j} - x^{\hat{i}}_{j,k})$ vanishes. Thus, if the transformations from x^{i} to $x^{\bar{i}}$ and from $x^{\bar{i}}$ to $x^{\hat{i}}$ are conservative, then the product transformation from x^{i} to $x^{\hat{i}}$ is conservative. If we let $x^{\hat{i}} = x^{i}$, we see from (6) that the inverse of a conservative transformation is conservative. From $x^{k}_{\hat{i}} = x^{\bar{i}}_{\bar{r}} x^{\bar{r}}_{k}$, we see that the product of matrices $x^{\hat{i}}_{\bar{r}}$ and $x^{\bar{r}}_{k}$ (which represent the transformations from $x^{\bar{i}}$ to $x^{\hat{i}}$ and from $x^{\bar{i}}$ to x^{i}, resp.) equals the matrix $x^{\hat{i}}_{k}$ (which represents the product transformation from $x^{\hat{i}}$ to x^{i}). It is obvious, and well known, that if products admit a matrix representation in this sense, then the associative law is satisfied. This completes the proof that the conservative transformations form a group.

We note that if (1) is satisfied, then (2) is satisfied; that is, the conservation group contains the diffeomorphisms as a subgroup. Thus, to show that it contains the diffeomorphisms as a *proper* subgroup, we need only to exhibit transformation coefficients which satisfy (2), but do not satisfy (1). Such transformation coefficients are

$$x^{\bar{i}}_{k} = \delta^{i}_{k} + \delta^{i}_{3} \delta^{2}_{k} x^{1}, \qquad x^{k}_{\bar{i}} = \delta^{k}_{i} - \delta^{k}_{3} \delta^{2}_{i} x^{1}, \tag{7}$$

where δ^{i}_{k} is the usual Kronecker delta. If we take the partial derivative of $x^{\bar{i}}_{k} = \delta^{i}_{k} + \delta^{i}_{3} \delta^{2}_{k} x^{1}$ with respect to x^{j} and subtract from the result the corresponding expression with j and k interchanged, we get

$$x^{\bar{i}}_{k,j} - x^{\bar{i}}_{j,k} = \delta^{i}_{3} \left(\delta^{2}_{k} \delta^{1}_{j} - \delta^{2}_{j} \delta^{1}_{k} \right). \tag{8}$$

If, in (8), we choose $\bar{i} = 3$, $k = 2$, and $j = 1$, we get $x^{\bar{3}}_{2,1} - x^{\bar{3}}_{1,2} = \delta^{3}_{3} (\delta^{2}_{2} \delta^{1}_{1} - \delta^{2}_{1} \delta^{1}_{2}) = 1$, so the transformation coefficients in (7)

do not satisfy (1). However, upon multiplying (8) by $x^k_{\bar{i}} = \delta^k_i - \delta^k_3\delta^2_i x^1$, we find that the transformation coefficients in (7) *do* satisfy (2). q.e.d.

3. Group Which Contains the Superdiffeomorphisms as a Proper Subgroup

In [6–13] we used the conservation group as the covariance group for a geometrical theory which unifies the gravitational and electroweak *fields,* which are mediated by *bosons*. This theory, however, fails to completely unify the gravitational and electroweak *interactions,* because it does not include the neutrinos, which are *fermions*. Geometrical theories based on an ordinary manifold generally cannot include fermions. However both bosons and fermions are included in theories based on a supermanifold. We therefore investigate a generalization of the conservation group to a setting which is a supermanifold.

3.1. Supermanifolds. As Rogers [14] has stated, "a supermanifold is a set, more specifically it is a manifold modelled on some flat 'superspace' so that it has local coordinates some of which take values in the even and some in the odd part of a Grassmann algebra." "Even" quantities commute with all quantities, while "odd" quantities anticommute among themselves.

We adopt the standard notation of DeWitt [15, pages 2 and 3 and Section 2], in which the rules for commutation relations and for shifting tensor indices are accommodated by using coefficients that are powers of (-1). When the first edition of DeWitt's book was published in 1984, the journal *Nature* stated "*Supermanifolds* is destined to become the standard work for all serious study of super-symmetric theories of physics." We assume that a reader interested in this section is somewhat familiar with DeWitt's notation. However, since we wish to optimize readability, we shall give more details than what is customary in the remainder of this paper.

3.2. The Superdiffeomorphism Group. For a superdiffeomorphism, the transformation coefficients $x^{\bar{i}}_{j}$ in the relation $dx^{\bar{i}} = x^{\bar{i}}_{j}dx^j$ must be nonsingular and must satisfy the condition

$$x^{\bar{i}}_{k,j} - (-1)^{jk} x^{\bar{i}}_{j,k} = 0. \tag{9}$$

If (9) is replaced by the weaker condition

$$(-1)^k x^k_{\bar{i}} \left[x^{\bar{i}}_{k,j} - (-1)^{jk} x^{\bar{i}}_{j,k} \right] = 0, \tag{10}$$

the transformation will be called *superconservative*, for a reason which will become clear in Section 3.2.1. Proof that (10) defines a group which contains the super-diffeomorphisms as a proper subgroup will be given in Section 3.2.2.

3.2.1. Transformation of a Superconservation Law. A discussion by DeWitt [15, page 113] shows that a super-conservation law is an expression of the form

$$(-1)^i V^i_{,i} = 0, \tag{11}$$

where V^i is a supervector density of weight +1, and that the transformation law for a super vector density is

$$V^{\bar{i}} = \frac{\partial x}{\partial \bar{x}} V^j {}_j x^{\bar{i}}, \tag{12}$$

where $\partial x / \partial \bar{x}$ is the Jacobian superdeterminant of $x^j_{\bar{i}}$. (note: DeWitt uses the symbol μX^i instead of our V^i, and the symbol J^{-1} instead of our $\partial x/\partial \bar{x}$.).

If we differentiate (12) with respect to $x^{\bar{i}}$ and multiply by $(-1)^i$, we get

$$(-1)^i V^{\bar{i}}_{,\bar{i}} = (-1)^{i+j+ij} \left(\frac{\partial x}{\partial \bar{x}} V^j x^{\bar{i}}_{j} \right)_{,\bar{i}}, \tag{13}$$

where we have used ${}_j x^{\bar{i}} = (-1)^{j(j+i)} x^{\bar{i}}_{j}$, which corresponds to DeWitt's equation (1.7.25). Continuing, we have

$$\begin{aligned}
(-1)^i V^{\bar{i}}_{,\bar{i}} &= (-1)^{i+j+ij+i(i+j)} \left(\frac{\partial x}{\partial \bar{x}} V^j \right)_{,\bar{i}} x^{\bar{i}}_{j} \\
&\quad + (-1)^{i+j+ij} \frac{\partial x}{\partial \bar{x}} V^j x^{\bar{i}}_{j,\bar{i}} \\
&= (-1)^i \left(\frac{\partial x}{\partial \bar{x}} V^i \right)_{,i} \\
&\quad + (-1)^{i+j+ij} \frac{\partial x}{\partial \bar{x}} V^j x^{\bar{i}}_{j,k} x^k_{\bar{i}} \\
&= \frac{\partial x}{\partial \bar{x}} (-1)^i V^i_{,i} + \left(\frac{\partial x}{\partial \bar{x}} \right)_{,j} V^j \\
&\quad + (-1)^{j+k+jk} \frac{\partial x}{\partial \bar{x}} V^j x^k_{\bar{i}} x^{\bar{i}}_{j,k},
\end{aligned} \tag{14}$$

and since $(\partial x/\partial \bar{x})_{,j} = -(-)^k (\partial x/\partial \bar{x}) x^k_{\bar{i}} x^{\bar{i}}_{j,k}$, we get

$$\begin{aligned}
(-1)^i V^{\bar{i}}_{,\bar{i}} &= \frac{\partial x}{\partial \bar{x}} (-1)^i V^i_{,i} - (-)^k \frac{\partial x}{\partial \bar{x}} x^k_{\bar{i}} x^{\bar{i}}_{k,j} V^j \\
&\quad + (-1)^{j+k+jk} \frac{\partial x}{\partial \bar{x}} V^j x^k_{\bar{i}} x^{\bar{i}}_{j,k} \\
&= \frac{\partial x}{\partial \bar{x}} (-1)^i V^i_{,i} - (-)^{j+k} \frac{\partial x}{\partial \bar{x}} V^j x^k_{\bar{i}} x^{\bar{i}}_{k,j} \\
&\quad + (-1)^{j+k+jk} \frac{\partial x}{\partial \bar{x}} V^j x^k_{\bar{i}} x^{\bar{i}}_{j,k}.
\end{aligned} \tag{15}$$

Thus, we have

$$\begin{aligned}
(-1)^i V^{\bar{i}}_{,\bar{i}} &= \frac{\partial x}{\partial \bar{x}} (-1)^i \, V^i_{,i} \\
&\quad - (-1)^{j+k} \frac{\partial x}{\partial \bar{x}} V^j x^k_{\bar{i}} \left[x^{\bar{i}}_{k,j} - (-1)^{jk} x^{\bar{i}}_{j,k} \right].
\end{aligned} \tag{16}$$

We see from (16) that we have both $(-1)^i V^i_{,i} = 0$ and $(-1)^{\bar{i}} V^{\bar{i}}_{,\bar{i}} = 0$ (i.e., a super-conservation law is an invariant statement) if and only if $(-1)^{j+k}(\partial x/\partial\bar{x})V^j x^k_{\bar{i}}[x^{\bar{i}}_{k,j} - (-1)^{jk}x^{\bar{i}}_{j,k}] = 0$. Since V^j is arbitrary and $\partial x/\partial\bar{x} \neq 0$, we must have $(-1)^{j+k}x^k_{\bar{i}}\,[x^{\bar{i}}_{k,j} - (-1)^{jk}x^{\bar{i}}_{j,k}] = 0$. The index j is not summed, so we divide by $(-1)^j$ to obtain (10). We therefore call transformations that satisfy (10) "super-conservative."

3.2.2. Proof That the Super-Conservative Transformations Form a Group Which Contains the Superdiffeomorphisms as a Proper Subgroup. We begin by noting that the identity transformation $x^{\bar{i}} = x^i$ is a super-conservative transformation that is, it satisfies (10). Next, we consider the result of following a transformation from x^i to $x^{\bar{i}}$ by a transformation from $x^{\bar{i}}$ to $x^{\hat{i}}$. Upon differentiating $x^{\hat{i}}_k = x^{\hat{i}}_{\bar{r}}x^{\bar{r}}_k$ with respect to x^j, we get

$$\begin{aligned} x^{\hat{i}}_{k,j} &= (-1)^{j(k+r)}x^{\hat{i}}_{\bar{r},j}x^{\bar{r}}_k + x^{\hat{i}}_{\bar{r}}x^{\bar{r}}_{k,j} \\ &= (-1)^{jk+jr}x^{\hat{i}}_{\bar{r},\bar{s}}x^{\bar{s}}_j x^{\bar{r}}_k + x^{\hat{i}}_{\bar{r}}x^{\bar{r}}_{k,j} \\ &= (-1)^{ki+kr+ri+r}x^{\bar{r}}_k x^{\hat{i}}_{\bar{r},\bar{s}}x^{\bar{s}}_j + x^{\hat{i}}_{\bar{r}}x^{\bar{r}}_{k,j}, \end{aligned} \tag{17}$$

and we multiply by $(-1)^k x^k_{\hat{i}}$ to get

$$\begin{aligned} (-1)^k x^k_{\hat{i}}x^{\hat{i}}_{k,j} &= (-1)^{ki+kr+ri+r+k}x^k_{\hat{i}}x^{\bar{r}}_k x^{\hat{i}}_{\bar{r},\bar{s}}x^{\bar{s}}_j \\ &\quad + (-1)^k x^k_{\hat{i}}x^{\hat{i}}_{\bar{r}}x^{\bar{r}}_{k,j} \\ &= (-1)^r x^{\bar{r}}_k x^k_{\hat{i}}x^{\hat{i}}_{\bar{r},\bar{s}}x^{\bar{s}}_j \\ &\quad + (-1)^k x^k_{\bar{r}}x^{\bar{r}}_{k,j}. \end{aligned} \tag{18}$$

Thus,

$$\begin{aligned} &(-1)^k x^k_{\hat{i}}x^{\hat{i}}_{k,j} \\ &\quad = (-1)^r x^{\bar{r}}_{\hat{i}}x^{\hat{i}}_{\bar{r},\bar{s}}x^{\bar{s}}_j + (-1)^k x^k_{\bar{r}}x^{\bar{r}}_{k,j}. \end{aligned} \tag{19}$$

We interchange j and k in the middle line of (17) to obtain

$$\begin{aligned} x^{\hat{i}}_{j,k} &= (-1)^{jk+kr}x^{\hat{i}}_{\bar{r},\bar{s}}x^{\bar{s}}_k x^{\bar{r}}_j + x^{\hat{i}}_{\bar{r},}x^{\bar{r}}_{j,k} \\ &= (-1)^{jk+r+rs+ri+kr+ki}x^{\bar{r}}_k x^{\hat{i}}_{\bar{s},\bar{r}}x^{\bar{s}}_j + x^{\hat{i}}_{\bar{r}}x^{\bar{r}}_{j,k}, \end{aligned} \tag{20}$$

and we multiply by $(-1)^{k+jk}\ x^k{}_{\hat{i}}$ to get

$$\begin{aligned} (-1)^{k+jk}x^k_{\hat{i}}x^{\hat{i}}_{j,k} &= (-1)^{k+r+rs+ri+kr+ki}x^k_{\hat{i}}x^{\bar{r}}_k\ x^{\hat{i}}_{\bar{s},\bar{r}}x^{\bar{s}}_j \\ &\quad + (-1)^{k+jk}x^k_{\hat{i}}x^{\hat{i}}_{\bar{r}}x^{\bar{r}}_{j,k} \\ &= (-1)^{r+rs}x^{\bar{r}}_k x^k_{\hat{i}}x^{\hat{i}}_{\bar{s},\bar{r}}x^{\bar{s}}_j \\ &\quad + (-1)^{k+jk}x^k_{\hat{i}}x^{\hat{i}}_{\bar{r}}x^{\bar{r}}_{j,k}, \end{aligned} \tag{21}$$

so

$$\begin{aligned} &(-1)^{k+jk}x^k_{\hat{i}}x^{\hat{i}}_{j,k} \\ &\quad = (-1)^{r+rs}x^{\bar{r}}_{\hat{i}}x^{\hat{i}}_{\bar{s},\bar{r}}x^{\bar{s}}_j + (-1)^{k+jk}x^k_{\bar{r}}x^{\bar{r}}_{j,k}. \end{aligned} \tag{22}$$

Upon subtracting (22) from (19), we obtain

$$\begin{aligned} &(-1)^k x^k_{\hat{i}}\left[x^{\hat{i}}_{k,j} - (-1)^{jk}x^{\hat{i}}_{j,k}\right] \\ &\quad = (-1)^r x^{\bar{r}}_{\hat{i}}\left[x^{\hat{i}}_{\bar{r},\bar{s}} - (-1)^{rs}x^{\hat{i}}_{\bar{s},\bar{r}}\right]x^{\bar{s}}_j \\ &\qquad + (-1)^k x^k_{\bar{r}}\left[x^{\bar{r}}_{k,j} - (-1)^{jk}\ x^{\bar{r}}_{j,k}\right]. \end{aligned} \tag{23}$$

Equation (23) shows that if the quantities $(-1)^r x^{\bar{r}}_{\hat{i}}[x^{\hat{i}}_{\bar{r},\bar{s}} - (-1)^{rs}x^{\hat{i}}_{\bar{s},\bar{r}}]$ and $(-1)^k x^k_{\bar{r}}[x^{\bar{r}}_{k,j} - (-1)^{jk}x^{\bar{r}}_{j,k}]$ both vanish, then it follows that the quantity $(-1)^k x^k_{\hat{i}}[x^{\hat{i}}_{k,j} - (-1)^{jk}x^{\hat{i}}_{j,k}]$ vanishes. Thus, if the transformations from x^i to $x^{\bar{i}}$ and from $x^{\bar{i}}$ to $x^{\hat{i}}$ are super-conservative, then the product transformation from x^i to $x^{\hat{i}}$ is super-conservative. If we let $x^{\hat{i}} = x^i$, we see from (23) that the inverse of a super-conservative transformation is super-conservative. As in Section 2.2.2, the relation $x^{\hat{i}}_k = x^{\hat{i}}_{\bar{r}}x^{\bar{r}}_k$ guarantees that the associative law is satisfied. This completes the proof that the super-conservative transformations form a group.

We note that if (9) is satisfied, then (10) is satisfied; that is, the super-conservation group contains the super-diffeomorphisms as a subgroup. Thus, to show that it contains the super-diffeomorphisms as a *proper* subgroup, we need only to exhibit transformation coefficients which satisfy (10), but do not satisfy (9). Such transformation coefficients are the same as those in (7). This is clear, because the quantities in (7) are all ordinary numbers. Therefore, as DeWitt notes, they are in the *even* part of our supermanifold. Hence, in this case, the exponents j and k in (9) and (10) are even numbers. Thus, for the transformation coefficients in (7), (9), and (10) reduce to (1) and (2). We have shown previously in Section 2.2.2 that the transformation coefficients in (7) do not satisfy (1), but satisfy (2). q.e.d.

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