Hindawi Publishing Corporation
Journal of Mathematics
Volume 2013, Article ID 812153, 6 pages
http://dx.doi.org/10.1155/2013/812153

Research Article

A Triple Fixed Point Theorem for Multimap in a Hausdorff Fuzzy Metric Space

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Received 25 September 2012; Revised 30 November 2012; Accepted 5 December 2012

Academic Editor: Pierpaolo D'Urso

We obtain two triple fixed point theorems for a multimap in a Hausdorff fuzzy metric space.

1. Introduction and Preliminaries

The concept of fuzzy sets was introduced by Zadeh [1] in 1965 as a mathematical tool to represent vagueness in every day life. Since then, it was developed extensively by many authors, which also include interesting applications of this theory in diverse areas. To use this concept in topology and analysis, several researchers have defined fuzzy metric spaces in several ways (e.g., [2–4]). George and Veeramani [2] have modified the concept of fuzzy metric space introduced by Kramosil and Michálek [3] and also have succeeded in inducing a Hausdroff topology on such fuzzy metric space which is often used in current research these days. Later Grabiec [5] proved the contraction principle in the setting of fuzzy metric spaces introduced in [2]. Fuzzy metric spaces have many applications, for example, the various concepts of fuzzy topology have already been found in vital applications in quantum particle physics particularly in connection with both string and $\epsilon^{(\infty)}$ theory which were studied and formulated by El Naschie [6] and also most recently Gregori et al. [7] have furnished several interesting examples of fuzzy metrics in the sense of George and Veeramani [2] and have also utilized such fuzzy metrics to color image processing. For fixed point theorems in fuzzy metric spaces some of the interesting references are in [2, 5, 8–16].

The theory of set valued maps has applications in control theory, convex optimization, differential inclusions, and economics. The study of fixed points for multivalued contraction mappings using the Hausdorff metric was initiated by Nadler [17]. In 2004, Rodríguez-López and Romaguera [18] introduced Hausdorff's fuzzy metric on the set of the nonempty compact subsets of a given fuzzy metric space. Later several authors proved some fixed point theorems for multivalued maps in fuzzy metric spaces (e.g., [19–22]). The existence of fixed points for various multivalued contractive mappings has been studied by many authors under different conditions. For details, we refer the reader to [3, 17, 23–29] and the references therein. In 2006, Gnana Bhaskar and Lakshmikantham [30] introduced the notion of a coupled fixed point in partially ordered metric spaces, also discussed some problems of the uniqueness of a coupled fixed point, and applied their results to the problems of the existence and uniqueness of a solution for the periodic boundary value problems. In 2011, Samet and Vetro [31] extended the coupled fixed point theorems for a multivalued mapping and later several authors, namely, Hussain and Alotaibi [32], Aydi et al. [33], and Abbas et al. [34] proved coupled coincidence point theorems in partially ordered metric spaces. Borcut [35] observed that the coupled fixed points technique cannot solve a system with the following form: $x^2+2yz-6x+3=0$, $y^2+2zx-6y+3=0$, $z^2+2xy-6z+3=0$ and hence Berinde and Borcut [36] introduced the concept of triple fixed points and obtained a tripled fixed point theorem for a single valued map in partially ordered metric spaces. Moreover, these results could be used to study the existence of solutions of periodic boundary value problem involving $y^{11} = f(t, y, y^1)$.

In this paper, we obtain a triple fixed point theorem for a multimap in a Hausdorff fuzzy metric space and using it, we obtain a common triple fixed point for a multi- and single valued maps.

In the sequel, we need the following.

Definition 1 (see [37]). A binary operation $* : [0,1]\times[0,1] \to [0,1]$ is a continuous t-norm if it satisfies the following conditions:

(1) $*$ is associative and commutative,

(2) $*$ is continuous,

(3) $a * 1 = a$ for all $a \in [0,1]$,

(4) $a * b \le c * d$ whenever $a \le c$ and $b \le d$, for each $a, b, c, d \in [0,1]$.

Two typical examples of continuous t-norm are $a * b = ab$ and $a * b = \min\{a, b\}$.

Definition 2 (see [2]). A 3-tuple $(X, M, *)$ is called a fuzzy metric space if X is an arbitrary (nonempty) set, $*$ is a continuous t-norm, and M is a fuzzy set on $X^2 \times (0,\infty)$, satisfying the following conditions for each $x, y, z \in X$ and each t and $s > 0$,

(1) $M(x, y, t) > 0$,

(2) $M(x, y, t) = 1$ if and only if $x = y$,

(3) $M(x, y, t) = M(y, x, t)$,

(4) $M(x, y, t) * M(y, z, s) \le M(x, z, t + s)$,

(5) $M(x, y, \cdot) : (0,\infty) \to [0,1]$ is continuous.

Let $(X, M, *)$ be a fuzzy metric space. For $t > 0$, the open ball $B(x, r, t)$ with centre $x \in X$ and radius $0 < r < 1$ is defined by $B(x, r, t) = \{y \in X : M(x, y, t) > 1 - r\}$.

A subset $A \subset X$ is called open if for each $x \in A$, there exist $t > 0$ and $0 < r < 1$ such that $B(x, r, t) \subset A$. Let τ denote the family of all open subsets of X. Then τ is called the topology on X induced by the fuzzy metric M. This topology is Hausdorff and first countable. A subset A of X is said to be F-bounded if there exist $t > 0$ and $0 < r < 1$ such that $M(x, y, t) > 1 - r$ for all $x, y \in A$.

Grabiec [5] obtained the following important lemma.

Lemma 3 (see [5]). *Let $(X, M, *)$ be a fuzzy metric space. Then $M(x, y, t)$ is nondecreasing with respect to t, for all x, y in X.*

Rodríguez-López and Romaguera [18] defined the continuity of fuzzy metric M and obtained the following lemma relating to the continuity of M.

Definition 4. Let $(X, M, *)$ be a fuzzy metric space. M is said to be continuous on $X^2 \times (0,\infty)$ if

$$\lim_{n\to\infty} M(x_n, y_n, t_n) = M(x, y, t), \quad (1)$$

whenever a sequence $\{(x_n, y_n, t_n)\}$ in $X^2 \times (0,\infty)$ converges to a point $(x, y, t) \in X^2 \times (0,\infty)$, that is, whenever

$$\begin{gathered} \lim_{n\to\infty} M(x_n, x, t) = \lim_{n\to\infty} M(y_n, y, t) = 1, \\ \lim_{n\to\infty} M(x, y, t_n) = M(x, y, t). \end{gathered} \quad (2)$$

Lemma 5 (see [18]). *Let $(X, M, *)$ be a fuzzy metric space. Then M is a continuous function on $X^2 \times (0,\infty)$.*

From now onwards, (A) will denote the following condition:

$$\lim_{t\to\infty} M(x, y, t) = 1, \quad \forall x, y \in X. \quad (A)$$

In 1994, Mishra et al. [13] proved the following lemma relating to Cauchy sequences in fuzzy metric spaces.

Lemma 6 (see [13]). *Let $\{y_n\}$ be a sequence in fuzzy metric space $(X, M, *)$ satisfying (A). If there exists a positive number $k < 1$ such that*

$$M(y_n, y_{n+1}, kt) \ge M(y_{n-1}, y_n, t), \quad t > 0, \ n = 1, 2, \ldots, \quad (3)$$

then $\{y_n\}$ is a Cauchy sequence in X.

Definition 7 ([18], Definition 2.2). Let B be a nonempty subset of a fuzzy metric space $(X, M, *)$. For $a \in X$ and $t > 0$, define $M(a, B, t) = \sup\{M(a, b, t)/b \in B\}$.

Throughout the paper, let $K(X)$ denote the class of all nonempty compact subsets of X.

Lemma 8 ([18], Lemma 1). *Let $(X, M, *)$ be a fuzzy metric space. Then for each $a \in X, B \in K(X)$ and $t > 0$, there exists $b \in B$ such that $M(a, B, t) = M(a, b, t)$.*

Definition 9 (see [18]). Let $(X, M, *)$ be a fuzzy metric space. For each $A, B \in K(X)$ and $t > 0$, set

$$H_M(A, B, t) = \min\left\{\inf_{x\in A} M(x, B, t), \inf_{y\in B} M(A, y, t)\right\}. \quad (4)$$

The 3-tuple $(K(X), H_M, *)$ is called a Hausdorff fuzzy metric space.

We use the following lemma proved by Haghi et al. [38] in Theorem 15 in Section 2.

Lemma 10 (see [38]). *Let X be a nonempty set and $g : X \to X$ be a mapping. Then there exists a subset $E \subseteq X$ such that $g(E) = g(X)$ and $g : E \to X$ is one one.*

Now, we give the following definitions for a hybrid pair of mappings (see also [39]).

Definition 11. Let X be a nonempty set, $T : X \times X \times X \to 2^X$ (collection of all nonempty subsets of X) and $f : X \to X$.

(i) The point $(x, y, z) \in X \times X \times X$ is called a tripled fixed of T if

$$\begin{aligned} x &\in T(x, y, z), \\ y &\in T(y, x, y), \\ z &\in T(z, y, x). \end{aligned} \tag{5}$$

(ii) The point $(x, y, z) \in X \times X \times X$ is called a tripled coincident point of T and f if

$$\begin{aligned} fx &\in T(x, y, z), \\ fy &\in T(y, x, y), \\ fz &\in T(z, y, x). \end{aligned} \tag{6}$$

(iii) The point $(x, y, z) \in X \times X \times X$ is called a tripled common fixed point of T and f if

$$\begin{aligned} x &= fx \in T(x, y, z), \\ y &= fy \in T(y, x, y), \\ z &= fz \in T(z, y, x). \end{aligned} \tag{7}$$

Definition 12. Let $T : X \times X \times X \to 2^X$ be a multivalued map and f be a self-map on X. The Hybrid pair $\{T, f\}$ is called w-compatible if $f(T(x, y, z)) \subseteq T(fx, fy, fz)$ whenever (x, y, z) is a tripled coincidence point of T and f.

2. Main Result

First we prove a slightly different result from Lemma 6 which is essential in proving our main result.

Lemma 13. *Let $\{x_n\}$, $\{y_n\}$, and $\{z_n\}$ be sequences in fuzzy metric space $(X, M, *)$ satisfying (A). If there exists a positive number $k < 1$ such that*

$$\begin{aligned} &\min\{M(x_n, x_{n+1}, kt), M(y_n, y_{n+1}, kt), M(z_n, z_{n+1}, kt)\} \\ &\quad \geq \min\{M(x_{n-1}, x_n, t), M(y_{n-1}, y_n, t), M(z_{n-1}, z_n, t)\}, \end{aligned} \tag{8}$$

for all $t > 0$, $n = 1, 2, \ldots$, then $\{x_n\}$, $\{y_n\}$ and $\{z_n\}$ are Cauchy sequences in X.

Proof. We have

$$\begin{aligned} &\min\{M(x_n, x_{n+1}, kt), \\ &\qquad M(y_n, y_{n+1}, kt), M(z_n, z_{n+1}, kt)\} \\ &\quad \geq \min\{M(x_{n-1}, x_n, t), \\ &\qquad\qquad M(y_{n-1}, y_n, t), M(z_{n-1}, z_n, t)\} \\ &\quad \geq \min\left\{M\left(x_{n-2}, x_{n-1}, \frac{t}{k^2}\right),\right. \\ &\qquad\qquad \left. M\left(y_{n-2}, y_{n-1}, \frac{t}{k^2}\right), M\left(z_{n-2}, z_{n-1}, \frac{t}{k^2}\right)\right\} \\ &\quad \vdots \\ &\quad \geq \min\left\{M\left(x_0, x_1, \frac{t}{k^n}\right), M\left(y_0, y_1, \frac{t}{k^n}\right),\right. \\ &\qquad\qquad \left. M\left(z_0, z_1, \frac{t}{k^n}\right)\right\}. \end{aligned} \tag{9}$$

Hence,

$$\begin{aligned} &M(x_n, x_{n+1}, t) \\ &\quad \geq \min\left\{M\left(x_0, x_1, \frac{t}{k^n}\right), M\left(y_0, y_1, \frac{t}{k^n}\right),\right. \\ &\qquad\qquad \left. M\left(z_0, z_1, \frac{t}{k^n}\right)\right\}. \end{aligned} \tag{10}$$

Now, for any positive integer p,

$$\begin{aligned} &M(x_n, x_{n+p}, t) \\ &\quad \geq M\left(x_n, x_{n+1}, \frac{t}{p}\right) * M\left(x_{n+1}, x_{n+2}, \frac{t}{p}\right) \\ &\qquad * \cdots * M\left(x_{n+p-1}, x_{n+p}, \frac{t}{p}\right) \\ &\quad \geq \min\left\{M\left(x_0, x_1, \frac{t}{pk^n}\right),\right. \\ &\qquad\qquad \left. M\left(y_0, y_1, \frac{t}{pk^n}\right), M\left(z_0, z_1, \frac{t}{pk^n}\right)\right\} \\ &\qquad * \min\left\{M\left(x_0, x_1, \frac{t}{pk^{n+1}}\right),\right. \\ &\qquad\qquad \left. M\left(y_0, y_1, \frac{t}{pk^{n+1}}\right), M\left(z_0, z_1, \frac{t}{pk^{n+1}}\right)\right\} \end{aligned}$$

$$* \cdots * \min \left\{ M\left(x_0, x_1, \frac{t}{pk^{n+p-1}}\right), M\left(y_0, y_1, \frac{t}{pk^{n+p-1}}\right), M\left(z_0, z_1, \frac{t}{pk^{n+p-1}}\right) \right\}. \tag{11}$$

Letting $n \to \infty$ and using (A), we have

$$\lim_{n\to\infty} M\left(x_n, x_{n+p}, t\right) \geq 1 * 1 * \cdots * 1 = 1. \qquad \text{Hence, } \lim_{n\to\infty} M\left(x_n, x_{n+p}, t\right) = 1. \tag{12}$$

Thus $\{x_n\}$ is a Cauchy sequence in X. Similarly, we can show that $\{y_n\}$ and $\{z_n\}$ are also Cauchy sequences in X. □

Now, we are ready to prove our first main result.

Theorem 14. *Let $(X, M, *)$ be a complete fuzzy metric space satisfying condition (A) and $F : X \times X \times X \to \mathcal{K}(X)$ be a set valued mapping satisfying*

$$H_M\left(F\left(x, y, z\right), F\left(u, v, w\right), kt\right) \geq \min\left\{M\left(x, u, t\right), M\left(y, v, t\right), M\left(z, w, t\right)\right\} \tag{13}$$

for each $x, y, z, u, v, w \in X$, $t > 0$, where $0 < k < 1$.

Then F has a tripled fixed point.

Proof. Let $x_0, y_0, z_0 \in X$.

Choose $x_1 \in F(x_0, y_0, z_0)$, $y_1 \in F(y_0, x_0, y_0)$, $z_1 \in F(z_0, y_0, x_0)$.

Since F is compact valued, by Lemma 8, there exists $x_2 \in F(x_1, y_1, z_1)$ such that

$$\begin{aligned} M\left(x_1, x_2, kt\right) &= \sup_{x\in F(x_1,y_1,z_1)} M\left(x_1, x, kt\right) \\ &\geq H_M\left(F\left(x_0, y_0, z_0\right), F\left(x_1, y_1, z_1\right), kt\right) \\ &\geq \min\left\{M\left(x_0, x_1, t\right), M\left(y_0, y_1, t\right), M\left(z_0, z_1, t\right)\right\}. \end{aligned} \tag{14}$$

Since F is compact valued, by Lemma 8, there exists $y_2 \in F(y_1, x_1, y_1)$ such that

$$\begin{aligned} M\left(y_1, y_2, kt\right) &= \sup_{y\in F(y_1,x_1,y_1)} M\left(y_1, y, kt\right) \\ &\geq H_M\left(F\left(y_0, x_0, y_0\right), F\left(y_1, x_1, y_1\right), kt\right) \\ &\geq \min\left\{M\left(y_0, y_1, t\right), M\left(x_0, x_1, t\right), M\left(y_0, y_1, t\right)\right\} \\ &\geq \min\left\{M\left(x_0, x_1, t\right), M\left(y_0, y_1, t\right), M\left(z_0, z_1, t\right)\right\}. \end{aligned} \tag{15}$$

Since F is compact valued, by Lemma 8, there exists $z_2 \in F(z_1, y_1, x_1)$ such that

$$\begin{aligned} M\left(z_1, z_2, kt\right) &= \sup_{z\in F(z_1,y_1,x_1)} M\left(z_1, z, kt\right) \\ &\geq H_M\left(F\left(z_0, y_0, x_0\right), F\left(z_1, y_1, x_1\right), kt\right) \\ &\geq \min\left\{M\left(z_0, z_1, t\right), M\left(y_0, y_1, t\right), M\left(x_0, x_1, t\right)\right\} \\ &= \min\left\{M\left(x_0, x_1, t\right), M\left(z_0, z_1, t\right), M\left(y_0, y_1, t\right)\right\}. \end{aligned} \tag{16}$$

Thus,

$$\begin{aligned} &\min\left\{M\left(x_1, x_2, kt\right), M\left(y_1, y_2, kt\right), M\left(z_1, z_2, kt\right)\right\} \\ &\quad\geq \min\left\{M\left(x_0, x_1, t\right), M\left(y_0, y_1, t\right), M\left(z_0, z_1, t\right)\right\}. \end{aligned} \tag{17}$$

Continuing in this way, we can obtain sequences $\{x_n\}$, $\{y_n\}$ and $\{z_n\}$ in X such that $x_{n+1} \in F(x_n, y_n, z_n)$, $y_{n+1} \in F(y_n, x_n, y_n)$ and $z_{n+1} \in F(z_n, y_n, x_n)$ such that

$$\begin{aligned} &\min\left\{M\left(x_n, x_{n+1}, kt\right) M\left(y_n, y_{n+1}, kt\right) M\left(z_n, z_{n+1}, kt\right)\right\} \\ &\quad\geq \min\left\{M\left(x_{n-1}, x_n, t\right), M\left(y_{n-1}, y_n, t\right), M\left(z_{n-1}, z_n, t\right)\right\}. \end{aligned} \tag{18}$$

Hence, by Lemma 13, $\{x_n\}$ $\{y_n\}$ and $\{z_n\}$ are Cauchy sequences in X.

Since X is complete, there exist $x, y, z \in X$ such that $\lim_{n\to\infty}\{x_n\} = x$, $\lim_{n\to\infty}\{y_n\} = y$ and $\lim_{n\to\infty}\{z_n\} = z$. Consider

$$\begin{aligned} &H_M\left(F\left(x_n, y_n, z_n\right), F\left(x, y, z\right), kt\right) \\ &\quad\geq \min\left\{M\left(x_n, x, t\right), M\left(y_n, y, t\right), M\left(z_n, z, t\right)\right\}. \end{aligned} \tag{19}$$

Letting $n \to \infty$, we get

$$\begin{aligned} &\lim_{n\to\infty} H_M\left(F\left(x_n, y_n, z_n\right), F\left(x, y, z\right), kt\right) = 1 \text{ so that} \\ &\lim_{n\to\infty} H_M\left(F\left(x_n, y_n, z_n\right), F\left(x, y, z\right), t\right) = 1. \end{aligned} \tag{20}$$

Similarly, we can show that

$$\begin{aligned} &\lim_{n\to\infty} H_M\left(F\left(y_n, x_n, y_n\right), F\left(y, x, y\right), t\right) = 1, \\ &\lim_{n\to\infty} H_M\left(F\left(z_n, y_n, x_n\right), F\left(z, y, x\right), t\right) = 1. \end{aligned} \tag{21}$$

Since $x_{n+1} \in F(x_n, y_n, z_n)$, $y_{n+1} \in F(y_n, x_n, y_n)$, and $z_{n+1} \in F(z_n, y_n, x_n)$, from (20) and (21), we have

$$\begin{aligned} &\lim_{n\to\infty} \sup_{a\in F(x,y,z)} M\left(x_{n+1}, a, t\right) = 1, \\ &\lim_{n\to\infty} \sup_{b\in F(y,x,y)} M\left(y_{n+1}, b, t\right) = 1, \\ &\lim_{n\to\infty} \sup_{c\in F(z,y,x)} M\left(z_{n+1}, c, t\right) = 1. \end{aligned} \tag{22}$$

Hence there exist sequences $p_n \in F(x, y, z)$, $q_n \in F(y, x, y)$ and $r_n \in F(z, y, x)$ such that

$$\lim_{n\to\infty} M(x_{n+1}, p_n, t) = 1,$$
$$\lim_{n\to\infty} M(y_{n+1}, q_n, t) = 1,$$
$$\lim_{n\to\infty} M(z_{n+1}, r_n, t) = 1, \tag{23}$$
$$\text{for each } t > 0.$$

Now, for each $n \in \mathcal{N}$, we have

$$M(p_n, x, t) \geq M\left(p_n, x_{n+1}, \frac{t}{2}\right) * M\left(x_{n+1}, x, \frac{t}{2}\right). \tag{24}$$

Letting $n \to \infty$, we obtain

$$\lim_{n\to\infty} M(p_n, x, t) = 1 \quad \text{so that } \lim_{n\to\infty} p_n = x. \tag{25}$$

Similarly, we can show that

$$\lim_{n\to\infty} q_n = y, \qquad \lim_{n\to\infty} r_n = z. \tag{26}$$

Since $F(x, y, z)$, $F(y, x, y)$, and $F(z, y, x)$ are compact, we have $x \in F(x, y, z)$, $y \in F(y, x, y)$, and $z \in F(z, y, x)$.

Thus, (x, y, z) is a tripled fixed point of F. □

Using the above theorem, we now prove a tripled coincidence and common fixed point theorem for a hybrid pair of multivalued and single-valued mapping.

Theorem 15. *Let $(X, M, *)$ be a complete fuzzy metric space satisfying condition (A) and $F : X \times X \times X \to \mathcal{K}(X)$ and $g : X \to X$ be a mappings satisfying*

$$H_M(F(x, y, z), F(u, v, w), kt) \geq \min\{M(gx, gu, t), M(gy, gv, t), M(gz, gw, t)\}, \tag{27}$$

for all $x, y, z, u, v, w \in X$, $t > 0$ and $0 < k < 1$. Further assume that $F(X\times X\times X) \subseteq g(X)$, then F and g have a tripled coincidence point. Moreover, F and g have a tripled common fixed point if one of the following conditions holds.

(a) *The pair (F, g) is w-compatible and there exist $\alpha, \beta, \gamma \in X$ such that $\lim_{n\to\infty} g^n x = \alpha$, $\lim_{n\to\infty} g^n y = \beta$, and $\lim_{n\to\infty} g^n z = \gamma$, whenever (x, y, z) is a tripled coincidence point of F and g and g is continuous at α, β, γ.*

(b) *There exist $\alpha, \beta, \gamma \in X$ such that $\lim_{n\to\infty} g^n\alpha = x$, $\lim_{n\to\infty} g^n\beta = y$ and $\lim_{n\to\infty} g^n\gamma = z$, whenever (x, y, z) is a tripled coincidence point of F and g and g is continuous at x, y, z.*

Proof. By Lemma 10, There exists $E \subseteq X$ such that $g : E \to X$ is one to one and $g(E) = g(X)$.

Now, define $G : g(E) \times g(E) \times g(E) \to \mathcal{K}(X)$ by $G(gx, gy, gz) = F(x, y, z)$ for all $gx, gy, gz \in g(E)$. Since g is one-one on E, G is well defined.

Now,

$$\begin{aligned} H_M&(G(gx, gy, gz), G(gu, gv, gw), kt) \\ &= H_M(F(x, y, z), F(u, v, w), kt) \\ &\geq \min\{M(gx, gu, t), M(gy, gv, t), M(gz, gw, t)\}. \end{aligned} \tag{28}$$

Hence G satisfies (13) and all the conditions of Theorem 14.

By Theorem 14, G has a tripled fixed point $(u, v, w) \in g(E) \times g(E) \times g(E)$. Thus,

$$\begin{aligned} u &\in G(u, v, w), \\ v &\in G(v, u, v), \\ w &\in G(w, v, u). \end{aligned} \tag{29}$$

Since $F(X \times X \times X) \subseteq g(X)$, there exist $u, v, w \in X \times X \times X$ such that $gu_1 = u$, $gv_1 = v$, and $gw_1 = w$. So from (29), we have

$$\begin{aligned} gu_1 &\in G(gu_1, gv_1, gw_1) \\ &= F(u_1, v_1, w_1), \\ gv_1 &\in G(gv_1, gu_1, gv_1) \\ &= F(v_1, u_1, v_1), \\ gw_1 &\in G(gw_1, gv_1, gu_1) \\ &= F(w_1, v_1, u_1). \end{aligned} \tag{30}$$

This implies that $(u_1, v_1, w_1) \in X \times X \times X$ is a tripled coincidence point of F and g.

Now, following as in [39] except from the inequalities satisfied by M we can show that F and g have a tripled common fixed point. □

Acknowledgment

The authors are thankful to the referee for his valuable suggestions.

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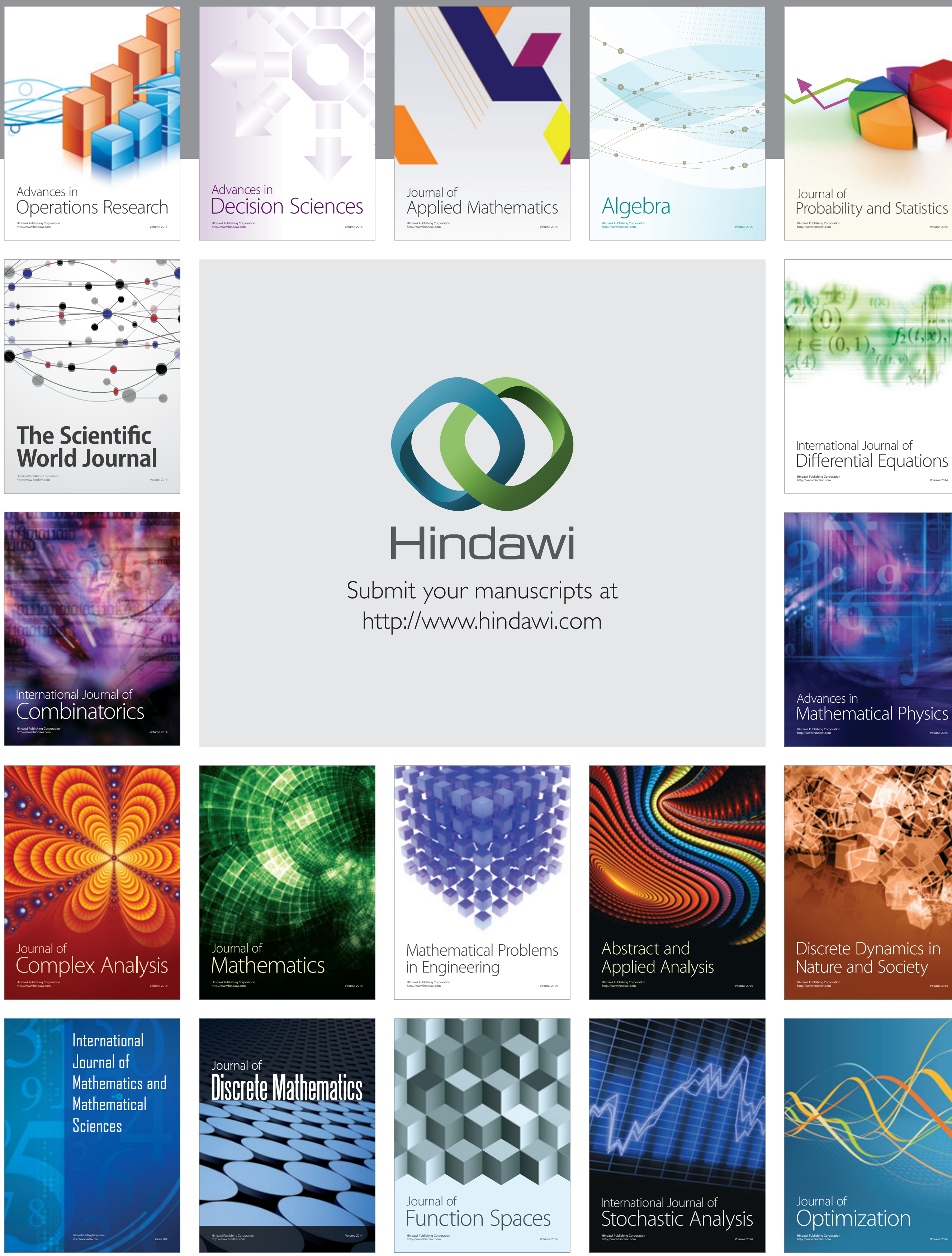
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