Hindawi Publishing Corporation
Journal of Mathematics
Volume 2013, Article ID 936859, 8 pages
http://dx.doi.org/10.1155/2013/936859

Research Article

Pulsating Different Curves of Zero Velocity around Triangular Equilibrium Points in Elliptical Restricted Three-Body Problem

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Received 30 December 2012; Revised 15 February 2013; Accepted 16 February 2013

Academic Editor: Baoding Liu

The oblateness and the photogravitational effects of both the primaries on the location and the stability of the triangular equilibrium points in the elliptical restricted three-body problem have been discussed. The stability of the triangular points under the photogravitational and oblateness effects of both the primaries around the binary systems Achird, Lyeten, Alpha Cen-AB, Kruger 60, and Xi-Bootis, has been studied using simulation techniques by drawing different curves of zero velocity.

1. Introduction

The present paper is devoted to the analysis of the photogravitational and the oblateness effects of both primaries on the stability of triangular equilibrium points of the planar elliptical restricted three-body problem. The elliptical restricted three-body problem describes the dynamical system more accurately on account that the realistic assumptions of the motion of the primaries are subjected to move along the elliptical orbit. We have attempted to investigate the stability of triangular equilibrium points under the photogravitational and oblateness effects of both the primaries. The bodies of the elliptical restricted three-body problem are generally considered to be spherical in shape, but in actual situations, we have observed that several heavenly bodies are either oblate spheroid or triaxial rigid bodies. The Earth, Jupiter, and Saturn are examples of the oblate spheroid. The lack of sphericity in heavenly bodies causes large perturbation. In addition to the oblateness of heavenly bodies, the triaxiality, the radiation forces of the bodies, the atmospheric drag, and the solar wind are also causes of perturbation.

This motivates studies of stability of triangular equilibrium points under the influence of oblateness and radiation of the primaries in the elliptical restricted three-body problem. The stability of the infinitesimal around the triangular equilibrium points in the elliptical restricted three-body problem described in considerable details is due to [1] and the problem was also studied [2–9]. The stability of motion of infinitesimal around one of the triangular equilibrium points (L_4) also depends on μ and e. Nonlinear stability of the triangular equilibrium points of the elliptical restricted three-body problem with or without radiation pressure was studied [10–12]. Furthermore, the nonlinear stability of the infinitesimal in the orbits or the size of the stable region around L_4 was studied numerically by [11] and the parametric resonance stability around L_4 in the elliptical restricted three-body problem has been studied [10].

The existence of the libration points [13, 14] and their stability in the photogravitational elliptical restricted three-body problem has been studied. The different aspects of the problem in details of elliptical restricted three-body problem have been investigated [15–26].

The influence of the eccentricity of the orbits of the oblate primary bodies with one of the photogravitational effects on the location of collinear and triangular equilibrium points and their stability has been investigated [27–29]. The stability of triangular points in the elliptical restricted three-body problem under the radiating and oblate primaries has been recently discussed [30]. A similar problem has been discussed

in detail by applying different techniques to investigate the stability of the system using simulation technique [31].

The present study aims to examine the motion of the infinitesimal body in the elliptical restricted three-body problem, when the primaries are oblate spheroid and are also a source of radiation. We have obtained the coordinate of the triangular equilibrium points of the problem. For the circular problem, the primaries are fixed with respect to uniformly rotating axes and hence the Hamiltonian does not involve time explicitly. But when the primaries move on elliptical orbits, the introduction of nonuniformly rotating and pulsating coordinate system results again in fixed location of the primaries. The elliptical restricted three-body problem generalizes the original circular restricted three-body problems, while some useful problem of circular model still can be satisfied by the elliptical case. The Hamiltonian, however, does not depend explicitly on independent variable in this case. The dimensionless variables are introduced by using the distance r between primaries given by

$$r = \frac{a\left(1-e^2\right)}{(1+e\cos v)}, \tag{1}$$

where a and e are the semimajor axis and the eccentricity of the elliptical orbit of the primaries, moving along each other and v is the true anomaly of m_1. A co-ordinate system which rotates with the variable angular velocity w is introduced. This angular velocity is given by

$$\frac{d\omega}{dt^*} = \frac{k(m_1+m_2)^{1/2}(1+e\cos v)^2}{a^{3/2}(1-e^2)^{3/2}}, \tag{2}$$

where t^* is dimensionless time.

The equation follows from the principal of the conservation of angular momentum in the problem of two bodies formed by the primaries of masses m_1 and m_2. This principle is expressed by

$$\omega r^2 = \left[a\left(1-e^2\right)k^2\left(m_1+m_2\right)\right]^{1/2}, \tag{3}$$

where $k = k_1 + k_2$, and k_1 and k_2 are the products of the universal gravitational constants with the masses of primaries.

The force of radiation is given by

$$F = F_g - F_p = \left(1 - \frac{F_p}{F_g}\right)F_g = qF_g, \tag{4}$$

where F_g is the gravitational attraction force; F_p is the radiation pressure; q is the mass radiation factor. For simplicity of calculation, we have considered $q = 1 - \epsilon^i,\ i = 1,2$, where $\epsilon^i \ll 1,\ i = 1,2$.

The present paper deals with the photogravitational and oblateness effects of both the primaries on the stability of infinitesimal by exploiting simulation techniques by drawing the different curves of zero velocity.

The present paper comprises three sections. The first section of the paper describes the variational equation of motion of the system. In the second section, we have described the triangular equilibrium points of the system, and in the third section the paper we have derived an expression which is suitable for tracing of different curves of zero velocity.

The curves of zero velocity around equilibrium points have been presented through simulation techniques, which shows the region of stability. The effect of the oblateness of primaries plays an important role in analyzing the stability of infinitesimal which is obvious from the curves of zero velocity traced.

2. Variational Equation of Motion

The differential equations of motion of infinitesimal in the elliptical restricted three-body problem under the oblate and radiating primaries in barycentric, pulsating, and nondimensional coordinates are represented as follows [31]:

$$\begin{aligned} x'' - 2y' &= \phi\Omega_x, \\ y'' + 2x' &= \phi\Omega_y, \end{aligned} \tag{5}$$

where

$$\Omega = \frac{x^2+y^2}{2} + \frac{1}{1+3\left(\left(A_1+A_2\right)/2\right)} \times \left[\frac{(1-\mu)\,q_1}{r_1} + \frac{\mu q_2}{r_2} + \frac{(1-\mu)\,q_1A_1}{2r_1^3} + \frac{\mu A_2 q_2}{2r_2^3}\right], \tag{6}$$

where

$$\begin{aligned} r_1^2 &= (x+\mu)^2 + y^2, \\ r_2^2 &= (x-1+\mu)^2 + y^2, \end{aligned} \tag{7}$$

$$\phi = \left(\frac{1}{1+e\cos v}\right), \tag{8}$$

where Ω_x denotes the partial differentiation of Ω with respect to x, and Ω_y denotes the differentiation of Ω partially with respect to y, where A_1 and A_2 are the oblateness parameters of the primaries. q_1 and q_2 are the mass radiation factors due to the source of radiation of the bigger primary and the smaller primary.

The co-ordinates of the triangular equilibrium points L_4 and L_5 are determined as follows [31]:

$$\begin{aligned} x_0 &= 1 - \mu - A_1\left(\frac{1}{2} + \frac{\epsilon^{(2)}}{2} - \frac{\epsilon^{(1)}}{2}\right) - A_2\left(\frac{1}{2} + \frac{\epsilon^{(2)}}{2} - \frac{\epsilon^{(1)}}{2}\right) + \frac{\epsilon^{(2)}}{3} - \frac{\epsilon^{(1)}}{3}, \\ y_o &= \pm\frac{\sqrt{3}}{2}\left[1 - \left(\frac{A_1+A_2}{3}\right)\left(1+\epsilon^{(1)}+\epsilon^{(2)}\right) - \frac{2}{9}\left(\epsilon^{(1)}+\epsilon^{(2)}\right)\right]. \end{aligned} \tag{9}$$

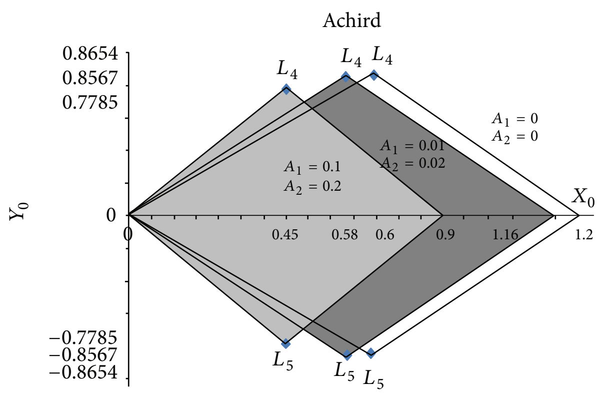

FIGURE 1: Location of triangular points for Achird with $A_1 = A_2 = 0$, $A_1 = 0.01$, $A_2 = 0.02$, and $A_1 = 0.1$, $A_2 = 0.2$.

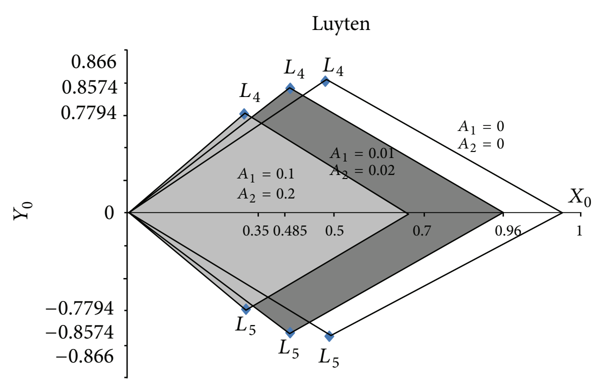

FIGURE 2: Location of triangular points for Luyten with $A_1 = A_2 = 0$, $A_1 = 0.01$, $A_2 = 0.02$, and $A_1 = 0.1$, $A_2 = 0.2$.

Thus by the coordinates of the triangular equilibrium points we obtain up to the first-order terms in the parameter A_1, A_2, q_1, and q_2, which is represented by (9), the location of equilibrium points is shown Figures 1, 2, 3, 4, and 5 for the infinitesimal moving around various binary systems.

3. Different Curves of Zero Velocity

In order to discuss the different curves of zero velocity of the infinitesimal in the elliptical restricted three-body problem, when both the primaries are oblate spheroid and radiating, multiplying the first equation of (5) by x' and the second equation by y' and adding, we get;

we obtain

$$x'x'' + y'y'' = \frac{\partial\Omega}{\partial x}x' + \frac{\partial\Omega}{\partial y}y',$$
$$\frac{1}{2}\frac{\partial}{\partial v}\left[x'^2 + y'^2\right] = \left(\frac{\partial\Omega}{\partial v}\right). \tag{10}$$

Since Ω does not contain the time (true anomaly) explicitly, (10) can be integrated to give

$$\frac{1}{2}\left[x'^2 + y'^2\right] = \int \frac{\partial\Omega}{1 + e\cos v} + c. \tag{11}$$

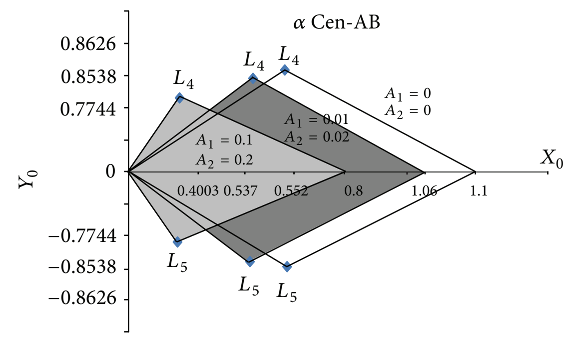

FIGURE 3: Location of triangular points for Achird with $A_1 = A_2 = 0$, $A_1 = 0.01$, $A_2 = 0.02$, and $A_1 = 0.1$, $A_2 = 0.2$.

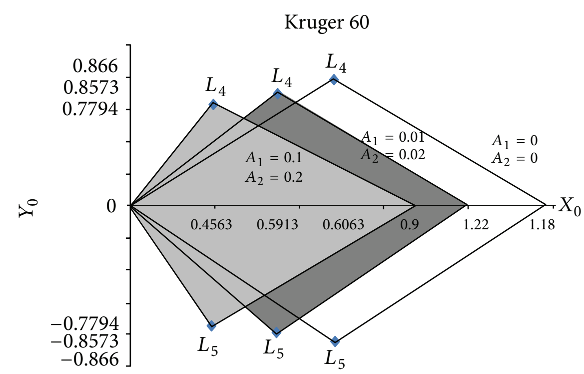

FIGURE 4: Location of triangular points for Kruger 60 with $A_1 = A_2 = 0$, $A_1 = 0.01$, $A_2 = 0.02$, and $A_1 = 0.1$, $A_2 = 0.2$.

Due to the presence of $(1 + e\cos v)$ in the denominator of (11), the equation is not possible to integrate to any defined form. Hence, in elliptical restricted three-body problem, it does not adjust the Jacobi integral of the classical circular problem at least in its usual sense.

The elliptical restricted three-body problem is different from the classical restricted problem in the sense that the Jacobi integral does not exist [16], and energy along any orbit is a time-dependent quantity. As we know no exact, complete, and general solution to the elliptical restricted three-body problem, Ω can be obtained unlike in classical restricted three-body problem, but this mathematical inconvenience is overcome along investigation of certain special cases of the problem based on simplifying the mathematical model under consideration [2]. Now, consider the potential function which is represented as follows:

$$\overline{\Omega}(x, y) = \frac{\Omega(x, y)}{1 + e\cos v} + c. \tag{12}$$

Hence, $\Omega(x, y)$ depends not only on the position coordinate of the infinitesimal but also on an independent variable. We select the initial point $v = 0$ and we consider only a part of the trajectory $v = 0$ and $v = \delta$, where δ is arbitrary sufficiently small time interval, during which the primaries

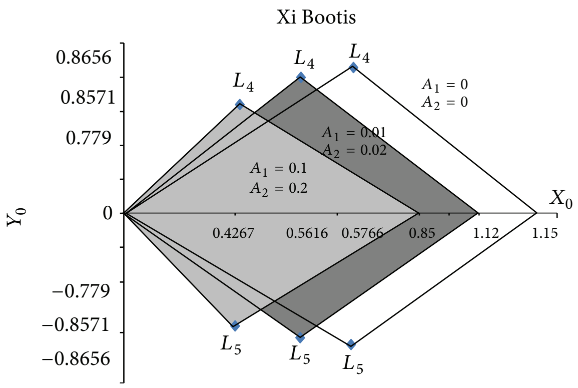

FIGURE 5: Location of triangular points for Xi Bootis with $A_1 = A_2 = 0$, $A_1 = 0.01$, $A_2 = 0.02$, and $A_1 = 0.1$, $A_2 = 0.2$.

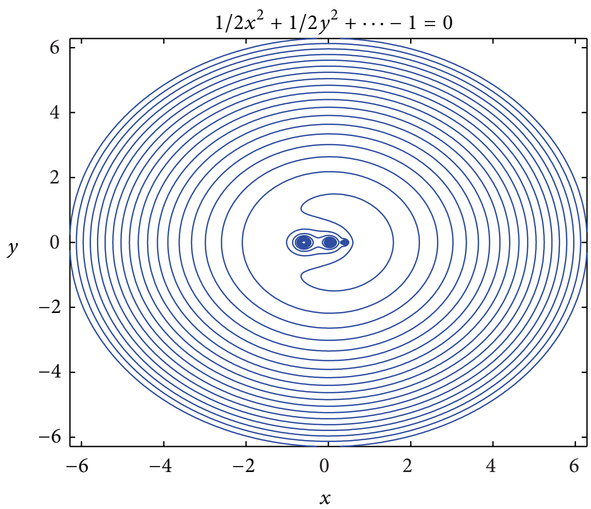

FIGURE 6: Curve of zero velocity (Archid-1) $q_1 = 0.9971$, $q_2 = 0.9997$, and $A_1 = 0$, $A_2 = 0$.

describe the small one. We may define a Jacobi constant in elliptical case as follows:

$$x'^2 + y'^2 - \frac{x^2 + y^2}{(1 + e\cos v)} - \frac{2}{(1 + e\cos v)} \times \left[\frac{1}{1 + 3\left((A_1 + A_2)/2\right)} \times \left\{ \frac{(1-\mu) q_1}{r_1} + \frac{\mu q_2}{r_2} + \frac{(1-\mu) q_1 A_1}{2r_1^3} + \frac{\mu A_2 q_2}{2r_2^3} \right\} \right] = c. \tag{13}$$

Equation (13) describes different curves of zero velocity, at each given instant of time of elliptical restricted three-body problem. The zero velocity curves are now pulsating with frequency of the nominal elliptical motion. Therefore, in

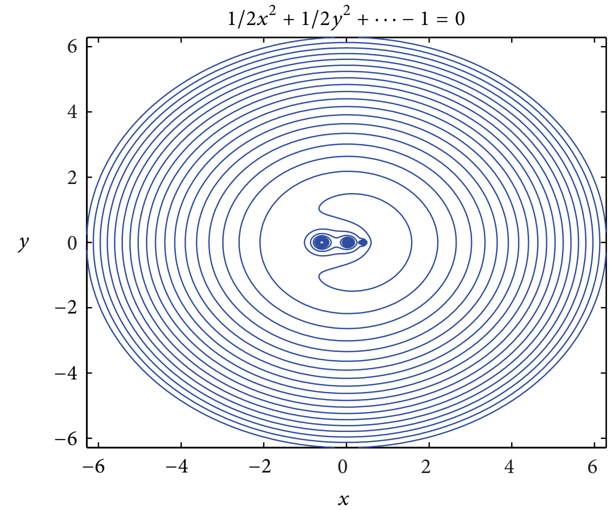

FIGURE 7: Curve of zero velocity (Archid-2) $q_1 = 0.9971$, $q_2 = 0.9997$, and $A_1 = 0.01$, $A_2 = 0.02$.

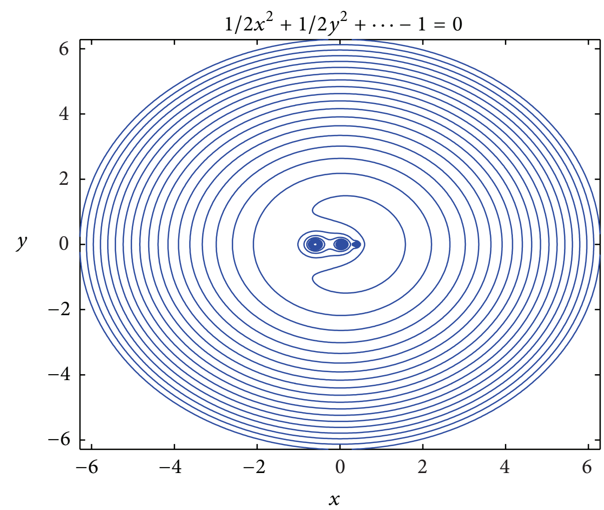

FIGURE 8: Curve of zero velocity (Archid-3) $q_1 = 0.9971$, $q_2 = 0.9997$, and $A_1 = 0.1$, $A_2 = 0.2$.

the planar elliptical restricted three-body problem, the zero velocity curves are obtained from the following equation:

$$x^2 + y^2 - 2\left[\frac{1}{1 + 3\left((A_1 + A_2)/2\right)} \times \left\{ \frac{(1-\mu) q_1}{r_1} + \frac{\mu q_2}{r_2} + \frac{(1-\mu) q_1 A_1}{2r_1^3} + \frac{\mu A_2 q_2}{2r_2^3} \right\} \right] + c = 0 \tag{14}$$

with the help of (14), the different curves of zero velocity have been traced using the software MATLAB 7.1 of the infinitesimal around the binary system Achird, Luyten, Alpha Cen AB, Kruger 60 and Xi Bootis, taking into account various values of

Table 1: Location of triangular equilibrium points mentioned in the research paper of Jagdish and Aishetu [30].

Binary system	Mass ratio (μ)	Critical mass value		Oblateness		Location of triangular points	
		q_1	q_2	A_1	A_2	X_0	$\pm Y_0$
				0	0	0.604	0.8654
Achird	0.3949	0.9971	0.9997	0.01	0.02	0.5891	0.8567
				0.1	0.2	0.4544	0.7785
				0	0	0.5	0.866
Luyten	0.5	0.999998	0.99999	0.01	0.02	0.485	0.8574
				0.1	0.2	0.35	0.7794
				0	0	0.5421	0.8626
α Cen-AB	0.4519	0.9971	0.985	0.01	0.02	0.527	0.8538
				0.1	0.2	0.3903	0.7744
				0	0	0.6063	0.866
Kruger 60	0.3937	0.99992	0.99996	0.01	0.02	0.5913	0.8573
				0.1	0.2	0.4563	0.7794
				0	0	0.5764	0.8658
Xi Bootis	0.4231	0.9988	0.9998	0.01	0.02	0.5614	0.8571
				0.1	0.2	0.4267	0.779

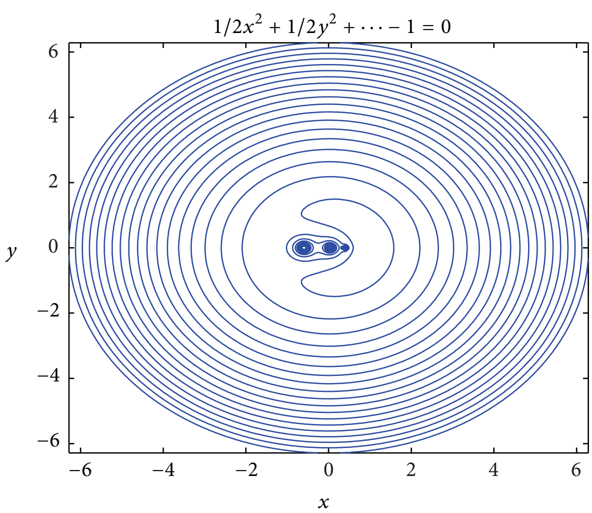

Figure 9: Curve of zero velocity (Luyten-1) $q_1 = 0.9971$, $q_2 = 0.9997$, and $A_1 = 0.1$, $A_2 = 0.2$.

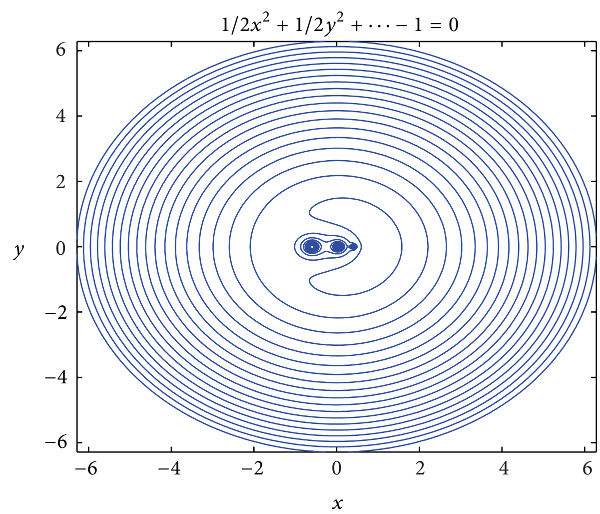

Figure 10: Curve of zero velocity (Luyten-2) $q_1 = 0.999998$, $q_2 = 0.99999$, and $A_1 = 0.01$, $A_2 = 0.02$.

oblateness parameters A_1 and A_2 and critical mass parameters q_1 and q_2 from Table 1. We have traced different curves of zero velocity of the infinitesimal about triangular equilibrium points. Figures 6, 7, and 8 represent the same around binary system Achird, and likewise Figures 9, 10, and 11 for binary system Luyten, Figures 12, 13, and 14 for binary system Alpha Cen-AB, Figures 15, 16, and 17 for binary system Kruger 60, and Figures 18, 19, and 20 for binary system Xi Bootis.

Hence, we observe typical behavior of the infinitesimal around the binary system, Achird, Luyten, Alpha Cen-AB, Kruger 60, and Xi Bootis.

4. Discussion and Conclusion

The oblateness and photogravitational effects of the primaries on the location and the stability of the triangular Lagrangian points in elliptical restricted three-body problem have been studied. The problem was studied under the assumption that the eccentricity of the orbit of the gravitating bodies is small. The oblateness of the more massive primary does not affect the motion of the smaller primary due to its large mass, whereas it affects the motion of infinitesimal body.

The differential equation governing the motion and stability of triangular equilibrium points of the elliptical restricted three-body problem under the oblate and radiating primaries has been analyzed, and configurations of the triangular equilibrium points are described. The stability of the triangular points under the photogravitational and oblateness effects of both the primaries around the binary systems Achird, Lyeten, Aipha Cen-AB, Kruger 60, and Xi Bootis have been studied using simulation technique by drawing different curves

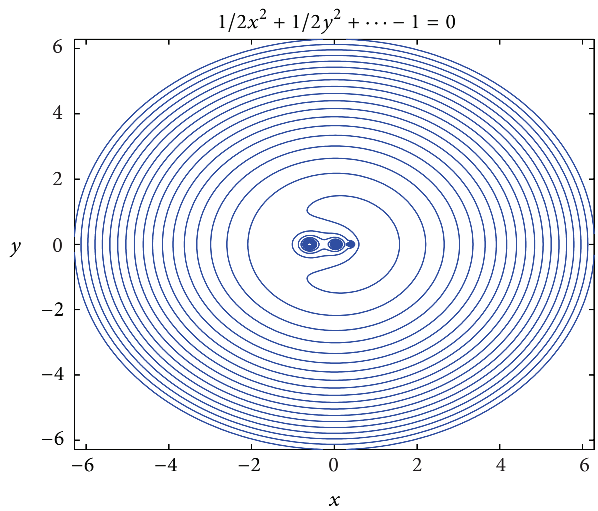

FIGURE 11: Curve of zero velocity (Luyten-3) $q_1 = 0.999998$, $q_2 = 0.99999$, and $A_1 = 0.01$, $A_2 = 0.02$.

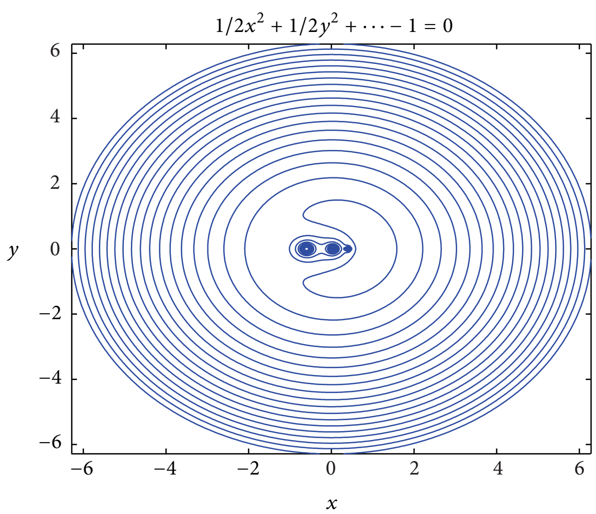

FIGURE 12: Curve of zero velocity (α CenAB-1) $q_1 = 0.9971$, $q_2 = 0.985$, and $A_1 = 0$, $A_2 = 0$.

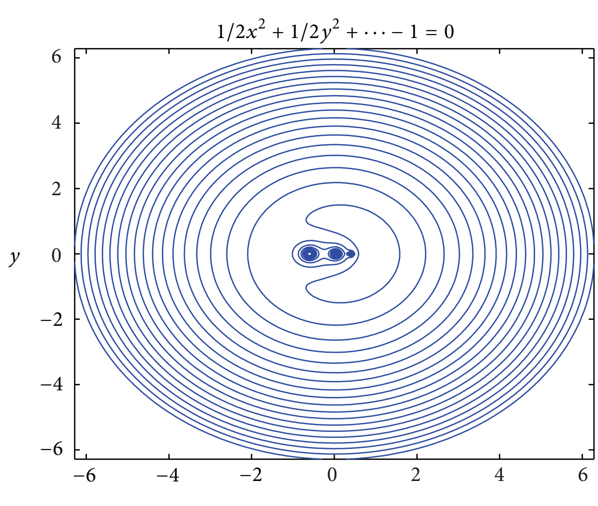

FIGURE 13: Curve of zero velocity (α CenAB-2) $q_1 = 0.9971$, $q_2 = 0.985$, and $A_1 = 0.01$, $A_2 = 0.02$.

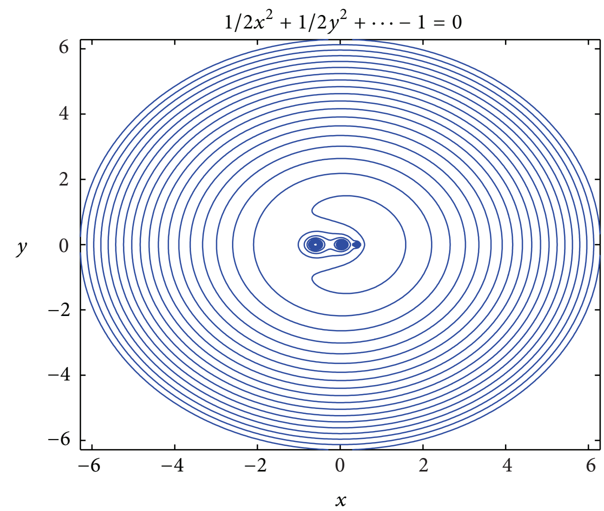

FIGURE 14: Curve of zero velocity (α CenAB-3) $q_1 = 0.9971$, $q_2 = 0.985$, and $A_1 = 0.1$, $A_2 = 0.2$.

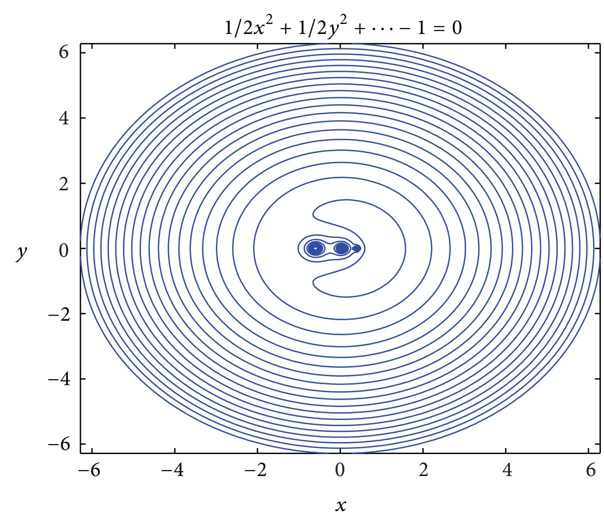

FIGURE 15: Curve of zero velocity (Kruger60-1) $q_1 = 0.99998$, $q_2 = 0.99996$, and $A_1 = 0$, $A_2 = 0$.

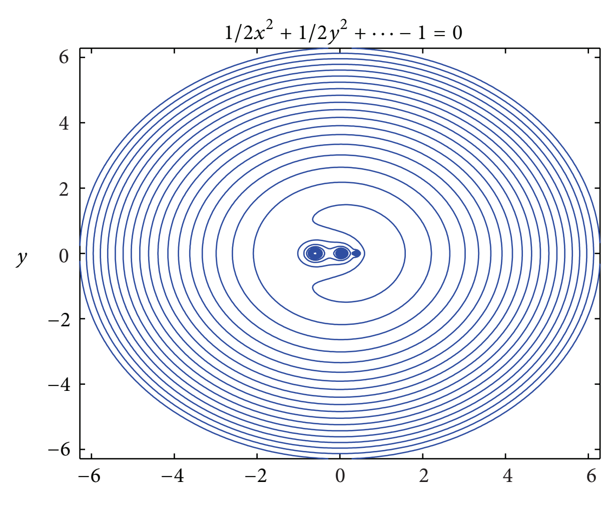

FIGURE 16: Curve of zero velocity (Kruger60-2) $q_1 = 0.99998$, $q_2 = 0.99996$, and $A_1 = 0.01$, $A_2 = 0.02$.

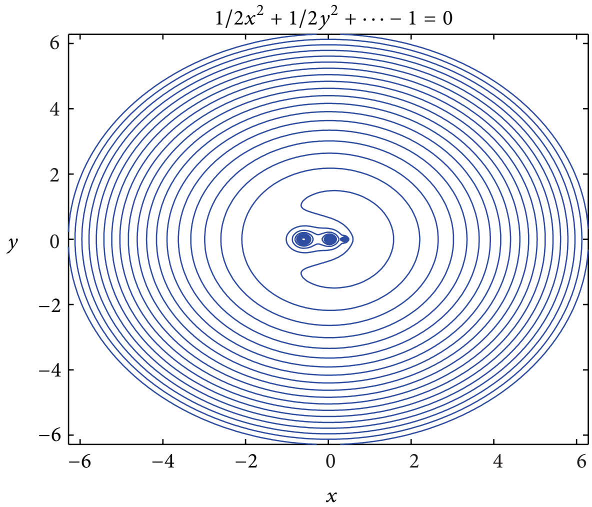

FIGURE 17: Curve of zero velocity (Kruger60-3) $q_1 = 0.99998$, $q_2 = 0.99996$, and $A_1 = 0.1$, $A_2 = 0.2$.

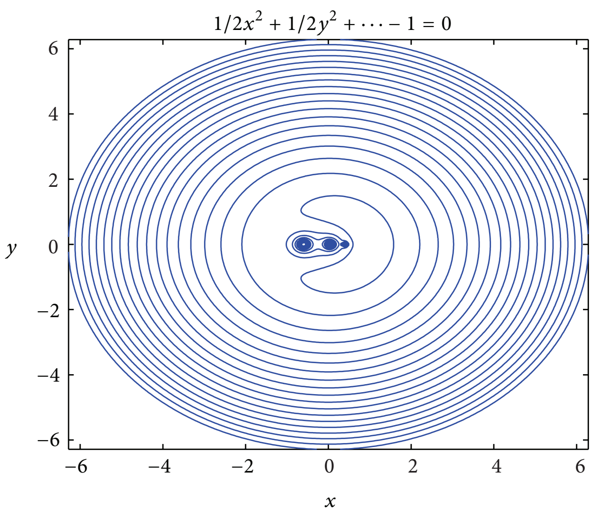

FIGURE 18: Curve of zero velocity (Xi Bootis-1) $q_1 = 0.9988$, $q_2 = 0.9988$, and $A_1 = 0$, $A_2 = 0$.

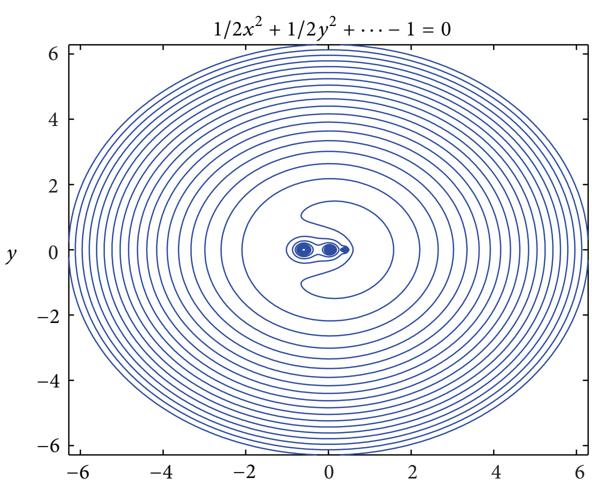

FIGURE 19: Curve of zero velocity (Xi Bootis-2) $q_1 = 0.9988$, $q_2 = 0.9988$, and $A_1 = 0.01$, $A_2 = 0.02$.

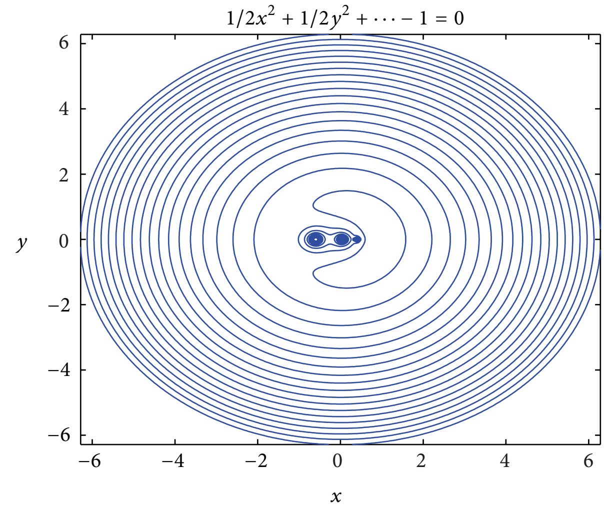

FIGURE 20: Curve of zero velocity (Xi Bootis-3) $q_1 = 0.9988$, $q_2 = 0.9988$, and $A_1 = 0.1$, $A_2 = 0.2$.

of zero velocity around triangular equilibrium point. It is observed that the region within the curves, the infinitesimal will remain stable.

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