Research Article

Fixed Point Results for Multivalued Contractive Mappings Endowed with Graphic Structure

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The aim of this paper is to introduce the notion of admissible multivalued mappings and to set up fixed point results for such mappings fulfilling generalized locally Ćirić type rational-contractive conditions on a closed ball in complete dislocated b -metric space. Example and application have been given to demonstrate the novelty of our results. Our results combine, extend, and infer several comparable results in the existing literature.

1. Introduction and Preliminaries

FP theory plays a fundamental role in functional analysis. Banach proved significant result for contraction mappings. After that, a huge number of FP theorems have been established by various authors and they made different generalizations of the Banach's result. Shoaib et al. [1], discussed the result related to α_* - ψ -Ćirić type multivalued mappings on an intersection of a closed ball and a sequence with graph. Further FP results on a closed ball can be seen ([2–12]).

Boriceanu [13] proved FP results for multivalued generalized contraction on a set with two b -metrics. After this Aydi et al. [14] established FP theorem for set-valued quasi contraction in b -metric spaces. Nawab et al. [15] established the new idea of dislocated b -metric space as a conception of metric space and proved some common FP results for four mappings fulfilling the generalized weak contractive conditions in partially ordered dislocated b -metric space.

Nadler [16] initiated the study of FP theorems for the multivalued mappings (see also [17]). Several results on multivalued mappings have been observed (see [18–20]). Asl et al. [21] gave the idea of α_* - ψ contractive multifunctions and α_* -admissible mapping and got some FP conclusions for these multifunctions (see also [22, 23]). In 1974, Ćirić [24] introduced quasi contraction. Reference [25] established some new common fixed points of generalized rational-contractive

mappings in dislocated metric spaces with applications. In this paper, the concept of new Ćirić type rational multivalued contraction has been introduced. Now we prove new type of result for a different multivalued rational expression studied by Rasham et al. [6]. Common FP results for such contraction on a closed ball in complete dislocated b -metric space have been established. Example and application have been given. We give the following definitions and results which will be needed in the sequel.

Definition 1 (see [15]). Let M be a nonempty set and let $d_b : M \times M \rightarrow [0, \infty)$ be a function, called a dislocated b -metric (or simply d_b -metric), if for any $g, q, z \in M$ and $t \geq 1$ the following conditions hold:

- (i) if $d_b(g, q) = 0$, then $g = q$;
- (ii) $d_b(g, q) = d_b(q, g)$;
- (iii) $d_b(g, q) \leq t[d_b(g, z) + d_b(z, q)]$.

The pair (M, d_b) is called a dislocated b -metric space. It should be noted that the class of d_b metric spaces is effectively larger than that of d_l metric spaces, since d_b is a d_l metric when $t = 1$.

It is clear that if $d_b(g, q) = 0$, then from (i), $g = q$. But if $g = q$, $d_b(g, q)$ may not be 0. For $g \in M$ and $\varepsilon > 0$, $\overline{B}(g, \varepsilon) =$

$\{q \in M : d_b(g, q) \leq \varepsilon\}$ is a closed ball in (M, d_b) . We use $D.B.M$ space instead of dislocated b -metric space.

Example 2. If $M = \mathbb{R}^+ \cup \{0\}$, then $d_b(g, q) = (g + q)^2$ defines a $D.B.M$ d_b on M .

Definition 3 (see [15]). Let (M, d_b) be a $D.B.M$ space.

(i) A sequence $\{g_n\}$ in (M, d_b) is called Cauchy sequence if, given $\varepsilon > 0$, there corresponds $n_0 \in \mathbb{N}$ such that for all $n, m \geq n_0$ we have $d_b(g_m, g_n) < \varepsilon$ or $\lim_{n, m \rightarrow \infty} d_b(g_n, g_m) = 0$.

(ii) A sequence $\{g_n\}$ dislocated b -converges (for short d_b -converges) to g if $\lim_{n \rightarrow \infty} d_b(g_n, g) = 0$. In this case g is called a d_b -limit of $\{g_n\}$.

(iii) (M, d_b) is called complete if every Cauchy sequence in M converges to a point $g \in M$ such that $d_b(g, g) = 0$.

Definition 4. Let $\widehat{H}$ be a nonempty subset of $D.B.M$ space M and let $g \in M$. An element $q_0 \in \widehat{H}$ is called a best approximation in $\widehat{H}$ if

$$d_b(g, \widehat{H}) = d_b(g, q_0), \quad (1)$$

where $d_b(g, \widehat{H}) = \inf_{q \in \widehat{H}} d_b(g, q)$.

If each $g \in M$ has at least one best approximation in $\widehat{H}$, then $\widehat{H}$ is called a proximal set.

We denote by $P(M)$ the set of all proximal subsets of M . Let Ψ , where $t \geq 1$, denote the family of all nondecreasing functions $\psi : [0, +\infty) \rightarrow [0, +\infty)$ such that $\sum_{k=1}^{+\infty} t^k \psi^k(u) < +\infty$ and $t\psi(u) < u$ for all $u > 0$, where ψ^k is the k^{th} iterate of ψ . Also $t^{n+1}\psi^{n+1}(u) = t^n t\psi(\psi^n(u)) < t^n \psi^n(u)$.

Definition 5 (see [26]). Let $B, A : M \rightarrow P(M)$ be the closed valued multifunctions and $\beta : M \times M \rightarrow [0, +\infty)$ be a function. We say that the pair (B, A) is β_* -admissible if for all $g, q \in M$

$$\beta(g, q) \geq 1 \implies \beta_*(Bg, Aq) \geq 1, \quad (2)$$

and $\beta_*(Ag, Bq) \geq 1$,

where $\beta_*(Ag, Bq) = \inf\{\beta(\bar{a}, b) : \bar{a} \in Ag, b \in Bq\}$. When $B = A$, then we obtain the definition of α_* -admissible mapping given in [21].

Definition 6. Let (M, d_b) be a $D.B.M$ space, $B : M \rightarrow P(M)$ be multivalued mapping, and $\alpha : M \times M \rightarrow [0, +\infty)$. Let $\bar{A} \subseteq M$, and we say that the B is semi- α_* -admissible on $\bar{A}$, whenever $\alpha(g, q) \geq 1$ implies that $\alpha_*(Bg, Bq) \geq 1$ for all $g, q \in \bar{A}$, where $\alpha_*(Bg, Bq) = \inf\{\alpha(\bar{a}, b) : \bar{a} \in Bg, b \in Bq\}$. If $\bar{A} = M$, then we say that the B is α_* -admissible on M .

Definition 7. The function $H_{d_b} : P(M) \times P(M) \rightarrow \mathbb{R}^+$, defined by

$$H_{d_b}(\bar{A}, B) = \max \left\{ \sup_{\bar{a} \in \bar{A}} d_b(\bar{a}, B), \sup_{b \in B} d_b(\bar{A}, b) \right\}, \quad (3)$$

is called dislocated Hausdorff b -metric on $P(M)$.

Lemma 8. Let (M, d_b) be a $D.B.M$ space. Let $(P(M), H_{d_b})$ is a dislocated Hausdorff b -metric space on $P(M)$. Then for all $\bar{A}, B \in P(M)$ and for each $\bar{a} \in \bar{A}$ there exists $b_{\bar{a}} \in B$ satisfying $d_b(\bar{a}, B) = d_b(\bar{a}, b_{\bar{a}})$; then $H_{d_b}(\bar{A}, B) \geq d_b(\bar{a}, b_{\bar{a}})$.

2. Main Result

Let (M, d_b) be a $D.B.M$ space, and let $g_0 \in M$ and $B : M \rightarrow P(M)$ be the multifunctions on M . Then there exist $g_1 \in Bg_0$ such that $d_b(g_0, Bg_0) = d_b(g_0, g_1)$. Let $g_2 \in Bg_1$ be such that $d_b(g_1, Bg_1) = d_b(g_1, g_2)$. Continuing this process, we construct a sequence g_n of points in M such that $g_{n+1} \in Bg_n$, $d_b(g_n, Bg_n) = d_b(g_n, g_{n+1})$. We denote this iterative sequence by $\{MB(g_n)\}$. We say that $\{MB(g_n)\}$ is a sequence in M generated by g_0 .

Theorem 9. Let (M, d_b) be a complete $D.B.M$ space, $\acute{r} > 0$, $g_0 \in \overline{B_{d_b}(g_0, \acute{r})}$, and $B : M \rightarrow P(M)$ be a semi- α_* -admissible multifunction on $\overline{B_{d_b}(g_0, \acute{r})}$; $\{MB(g_n)\}$ is a sequence in M generated by g_0 , $\alpha(g_0, g_1) \geq 1$. Assume that, for some $\psi \in \Psi$ and

$$D_b(g, q) = \max \left\{ d_b(g, q), \frac{d_b(g, Bg) \cdot d_b(q, Bq)}{\bar{a} + d_b(g, q)}, d_b(g, Bg), d_b(q, Bq) \right\}, \quad (4)$$

where $\bar{a} > 0$, the following hold:

$$\alpha_*(Bg, Bq) H_{d_b}(Bg, Bq) \leq \psi(D_b(g, q)) \quad (5)$$

$\forall g, q \in \overline{B_{d_b}(g_0, \acute{r})} \cap \{MB(g_n)\}$

$$\sum_{i=0}^n B^{i+1} \{\psi^i(d_b(g_0, g_1))\} \leq \acute{r} \quad \forall n \in \mathbb{N} \cup \{0\}. \quad (6)$$

Then, $\{MB(g_n)\}$ is a sequence in $\overline{B_{d_b}(g_0, \acute{r})}$, $\alpha(g_n, g_{n+1}) \geq 1$, and $\{MB(g_n)\} \rightarrow g^* \in \overline{B_{d_b}(g_0, \acute{r})}$. Also if $\alpha(g_n, g^*) \geq 1$ or $\alpha(g^*, g_n) \geq 1$, for all $n \in \mathbb{N} \cup \{0\}$ and the inequality (5) holds for all $g, q \in \overline{B_{d_b}(g_0, \acute{r})} \cap \{MB(g_n)\} \cup \{g^*\}$, then B has a common fixed point g^* in $\overline{B_{d_b}(g_0, \acute{r})}$.

Proof. Consider a sequence $\{MB(g_n)\}$ generated by g_0 . Then, we have $g_n \in Bg_{n-1}$, and $d_b(g_{n-1}, Bg_{n-1}) = d_b(g_{n-1}, g_n)$, for all $n \in \mathbb{N}$. By Lemma 8, we have $d_b(g_n, g_{n+1}) \leq H_{d_b}(Bg_{n-1}, Bg_n)$ for all $n \in \mathbb{N}$. If $g_0 = g_1$, then g_0 is a fixed point in $\overline{B_{d_b}(g_0, \acute{r})}$ of B . Let $g_0 \neq g_1$. From (6), we have

$$d_b(g_0, g_1) \leq \sum_{i=0}^n \psi^i(d_b(g_0, g_1)) \leq \acute{r}. \quad (7)$$

It follows that

$$g_1 \in \overline{B_{d_b}(g_0, \acute{r})}. \quad (8)$$

If $g_1 = g_2$, then g_1 is a fixed point in $\overline{B_{d_b}(g_0, \acute{r})}$ of B . Let $g_1 \neq g_2$. Since $\alpha(g_0, g_1) \geq 1$ and B is semi- α_* -admissible

multifunction on $\overline{B_{d_b}(g_0, \hat{r})}$, then $\alpha_*(Bg_0, Bg_1) \geq 1$. As $\alpha_*(Bg_0, Bg_1) \geq 1$, $g_1 \in Bg_0$ and $g_2 \in Bg_1$, so $\alpha(g_1, g_2) \geq 1$. Let $g_2, \dots, g_j \in \overline{B_{d_b}(g_0, \hat{r})}$ for some $j \in \mathbb{N}$. As

$\alpha_*(Bg_1, Bg_2) \geq 1$, we have $\alpha(g_2, g_3) \geq 1$, which further implies $\alpha_*(Bg_2, Bg_3) \geq 1$. Continuing this process, we have $\alpha_*(Bg_{j-1}, Bg_j) \geq 1$. Now, by using Lemma 8

$$\begin{aligned} d_b(g_j, g_{j+1}) &\leq H_{d_b}(Bg_{j-1}, Bg_j) \leq \alpha_*(Bg_{j-1}, Bg_j) H_{d_b}(Bg_{j-1}, Bg_j) \leq \psi(D_b(g_{j-1}, g_j)) \\ &= \psi\left(\max\left\{d_b(g_{j-1}, g_j), \frac{d_b(g_{j-1}, Bg_{j-1}) \cdot d_b(g_j, Bg_j)}{\bar{a} + d_b(g_{j-1}, g_j)}, d_b(g_{j-1}, Bg_{j-1}), d_b(g_j, Bg_j)\right\}\right) \\ &= \psi\left(\max\left\{d_b(g_{j-1}, g_j), \frac{d_b(g_{j-1}, g_j) \cdot d_b(g_j, g_{j+1})}{\bar{a} + d_b(g_{j-1}, g_j)}, d_b(g_{j-1}, g_j), d_b(g_j, g_{j+1})\right\}\right) \\ &= \psi(\max\{d_b(g_{j-1}, g_j), d_b(g_j, g_{j+1})\}). \end{aligned} \quad (9)$$

If $\max\{d_b(g_{j-1}, g_j), d_b(g_j, g_{j+1})\} = d_b(g_j, g_{j+1})$, then $d_b(g_j, g_{j+1}) \leq \psi(d_b(g_j, g_{j+1}))$. This is contradiction to the fact that $\psi(u) < u$ for all $u > 0$. Hence, we obtain $\max\{d_b(g_{j-1}, g_j), d_b(g_j, g_{j+1})\} = d_b(g_{j-1}, g_j)$. Therefore, we have

$$\begin{aligned} d_b(g_j, g_{j+1}) &\leq \psi(d_b(g_{j-1}, g_j)) \leq \dots \\ &\leq \psi^j(d_b(g_0, g_1)). \end{aligned} \quad (10)$$

Now, by using triangular inequality and by (10), we have

$$\begin{aligned} d_b(g_0, g_{j+1}) &\leq td_b(g_0, g_1) + t^2 d_b(g_1, g_2) + \dots \\ &\quad + t^{j+1} d_b(g_j, g_{j+1}) \\ &\leq td_b(g_0, g_1) + t^2 \psi(d_b(g_0, g_1)) + \dots \\ &\quad + t^{j+1} \psi^j(d_b(g_0, g_1)) \\ &\leq \sum_{i=0}^j t^{i+1} \{\psi^i(d_b(g_0, g_1))\} \leq \hat{r}. \end{aligned} \quad (11)$$

Thus $g_{j+1} \in \overline{B_{d_b}(g_0, \hat{r})}$. Hence, by induction, $g_n \in \overline{B_{d_b}(g_0, \hat{r})}$. As $\alpha_*(Bg_{j-1}, Bg_j) \geq 1$, $g_j \in Bg_j$, $g_{j+1} \in Bg_j$, then we have $\alpha(g_j, g_{j+1}) \geq 1$. Also B is semi- α_* -admissible multifunction on $\overline{B_{d_b}(g_0, \hat{r})}$, and therefore $\alpha_*(Bg_j, Bg_{j+1}) \geq 1$. This further implies that $\alpha(g_{j+1}, g_{j+2}) \geq 1$. Continuing this process, we have $\alpha(g_n, g_{n+1}) \geq 1$ for all $n \in \mathbb{N}$. Now, inequality (10) can be written as

$$d_b(g_n, g_{n+1}) \leq \psi^n(d_b(g_0, g_1)) \quad \forall n \in \mathbb{N}. \quad (12)$$

Fix $\epsilon > 0$ and let $k_1(\epsilon) \in \mathbb{N}$, such that

$$\sum_{k \geq k_1(\epsilon)} t^k \psi^k(d_b(g_0, g_1)) < \epsilon. \quad (13)$$

Let $n, m \in \mathbb{N}$ with $m > n > k_1(\epsilon)$. Now, we have

$$\begin{aligned} d_b(g_n, g_m) &\leq \sum_{k=n}^{m-1} d_b(g_k, g_{k+1}) \\ &\leq \sum_{k=n}^{m-1} t^k \psi^k(d_b(g_0, g_1)), \quad \text{by (12)} \end{aligned} \quad (14)$$

$$d_b(g_n, g_m) \leq \sum_{k \geq k_1(\epsilon)} t^k \psi^k(d_b(g_0, g_1)) < \epsilon.$$

Thus, $\{MB(g_n)\}$ is a Cauchy sequence in $(\overline{B_{d_b}(g_0, \hat{r})}, d_b)$. As every closed ball in a complete $D.B.M$ space is complete, there exist $g^* \in \overline{B_{d_b}(g_0, \hat{r})}$ such that $\{MB(g_n)\} \rightarrow g^*$, and

$$\lim_{n \rightarrow \infty} d_b(g_n, g^*) = 0. \quad (15)$$

By assumption, we have $\alpha(g_n, g^*) \geq 1$ for all $n \in \mathbb{N} \cup \{0\}$. Thus, $\alpha_*(Bg_n, Bg^*) \geq 1$. Now, we have

$$\begin{aligned} d_b(g^*, Bg^*) &\leq td_b(g^*, g_{n+1}) + td_b(g_{n+1}, Bg^*) \\ &\leq td_b(g^*, g_{n+1}) + tH_{d_b}(Bg_n, Bg^*) \quad \text{by Lemma 8} \\ &\leq td_b(g^*, g_{n+1}) + t\{\alpha_*(Bg_n, Bg^*)H_{d_b}(Bg_n, Bg^*)\} \\ &\leq td_b(g^*, g_{n+1}) + t\psi\left(\max\left\{d_b(g_n, g^*), \frac{d_b(g_n, Bg_n) \cdot d_b(g^*, Bg^*)}{\bar{a} + d_b(g_n, g^*)}, d_b(g_n, Bg_n), d_b(g^*, Bg^*)\right\}\right) \\ &\leq td_b(g^*, g_{n+1}) + t\psi\left(\max\left\{d_b(g_n, g^*), \right. \right. \end{aligned}$$

$$\frac{d_b(g_n, g_{n+1}) \cdot d_b(g^*, Bg^*)}{\bar{a} + d_b(g_n, g^*)}, d_b(g_n, g_{n+1}), \\ d_b(g^*, Bg^*) \Big\} \Bigg). \quad (16)$$

Letting $n \rightarrow \infty$ and by using inequality (15), we obtain $(1-t)d_b(g^*, Bg^*) \leq 0$. So $(1-t) \neq 0$, and then $d_b(g^*, Bg^*) = 0$. Hence $g^* \in Bg^*$. So B has a fixed point in $\overline{B_{d_b}(g_0, \hat{r})}$.

Let M be a nonempty set. Then $(M, \leq, d_b)$ is called a preordered $D.B.M$ space if d_b is called $D.B.M$ on M . Let $(M, \leq, d_b)$ be a preordered $D.B.M$ space and $H, K \subseteq M$. We say that $H \leq_{\hat{r}} K$ whenever for each $\bar{a} \in H$ there exist $b \in K$ such that $\bar{a} \leq b$. Also, we say that $H \leq_{\hat{r}} K$ whenever, for each $\bar{a} \in H$ and $b \in K$, we have $\bar{a} \leq b$. $\square$

Corollary 10. Let $(M, \leq, d_b)$ be a preordered complete $D.B.M$ space, $\hat{r} > 0$, $g_0 \in \overline{B_{d_b}(g_0, \hat{r})}$, and $B : M \rightarrow P(M)$ be a multifunction on $\overline{B_{d_b}(g_0, \hat{r})}$; $\{MB(g_n)\}$ is a sequence generated by g_0 , with $g_0 \leq g_1$. Assume that, for some $\psi \in \Psi$ and

$$D_b(g, q) = \max \left\{ d_b(g, q), \frac{d_b(g, Bg) \cdot d_b(q, Bq)}{\bar{a} + d_b(g, q)}, \right. \\ \left. d_b(g, Bg), d_b(q, Bq) \right\} \quad (17)$$

where $\bar{a} > 0$, the following hold:

$$H_{d_b}(Bg, Bq) \leq \psi(D_b(g, q)) \\ \forall g, q \in \overline{B_{d_b}(g_0, \hat{r})} \cap \{MB(g_n)\} \text{ with } g \leq q \quad (18)$$

$$\text{and } \sum_{i=0}^n t^{i+1} \{\psi^i(d_b(g_0, g_1))\} \leq \hat{r} \quad \forall n \in \mathbb{N} \cup \{0\}. \quad (19)$$

If $g, q \in \overline{B_{d_b}(g_0, \hat{r})}$, such that $g \leq q$ implies $Bg \leq_{\hat{r}} Bq$. Then $\{MB(g_n)\}$ is a sequence in $\overline{B_{d_b}(g_0, \hat{r})}$, $g_n \leq g_{n+1}$, and $\{MB(g_n)\} \rightarrow g^* \in \overline{B_{d_b}(g_0, \hat{r})}$. Also if $g^* \leq g_n$ or $g_n \leq g^*$, for all $n \in \mathbb{N} \cup \{0\}$, and inequality (18) holds for all $g, q \in \overline{B_{d_b}(g_0, \hat{r})} \cap \{MB(g_n)\} \cup \{g^*\}$. Then g^* is a fixed point of B in $\overline{B_{d_b}(g_0, \hat{r})}$.

Corollary 11. Let $(M, \leq, d_b)$ be a preordered complete $D.B.M$ space, $\hat{r} > 0$, $g_0 \in \overline{B_{d_b}(g_0, \hat{r})}$, and $B : M \rightarrow P(M)$ be a multifunction on $\overline{B_{d_b}(g_0, \hat{r})}$; $\{MB(g_n)\}$ is a sequence generated by g_0 , with $g_0 \leq g_1$. Assume that, for some $k \in [0, 1)$ and

$$D_b(g, q) = \max \left\{ d_b(g, q), \frac{d_b(g, Bg) \cdot d_b(q, Bq)}{\bar{a} + d_b(g, q)}, \right. \\ \left. d_b(g, Bg), d_b(q, Bq) \right\} \quad (20)$$

where $\bar{a} > 0$, the following hold:

$$H_{d_b}(Bg, Bq) \leq k(D_b(g, q)) \\ \forall g, q \in \overline{B_{d_b}(g_0, \hat{r})} \cap \{MB(g_n)\} \text{ with } g \leq q \quad (21)$$

$$\text{and } \sum_{i=0}^n t^{i+1} \{k^i(d_b(g_0, g_1))\} \leq \hat{r} \quad \forall n \in \mathbb{N} \cup \{0\}. \quad (22)$$

If $g, q \in \overline{B_{d_b}(g_0, \hat{r})}$, such that $g \leq q$ implies $Bg \leq_{\hat{r}} Bq$. Then $\{MB(g_n)\}$ is a sequence in $\overline{B_{d_b}(g_0, \hat{r})}$, $g_n \leq g_{n+1}$, and $\{MB(g_n)\} \rightarrow g^* \in \overline{B_{d_b}(g_0, \hat{r})}$. Also if $g^* \leq g_n$ or $g_n \leq g^*$, for all $n \in \mathbb{N} \cup \{0\}$, and inequality (21) holds for all $g, q \in \overline{B_{d_b}(g_0, \hat{r})} \cap \{MB(g_n)\} \cup \{g^*\}$. Then g^* is a fixed point of B in $\overline{B_{d_b}(g_0, \hat{r})}$.

Corollary 12. Let $(M, \leq, d_l)$ be a preordered $D.M$ space, $\hat{r} > 0$, $g_0 \in \overline{B_{d_l}(g_0, \hat{r})}$, and $B : M \rightarrow P(M)$ be a multifunction on $\overline{B_{d_l}(g_0, \hat{r})}$; $\{MB(g_n)\}$ is a sequence generated by g_0 , with $g_0 \leq g_1$. Assume that, for some $\psi \in \Psi$ and

$$D_l(g, q) = \max \left\{ d_l(g, q), \frac{d_l(g, Bg) \cdot d_l(q, Bq)}{\bar{a} + d_l(g, q)}, \right. \\ \left. d_l(g, Bg), d_l(q, Bq) \right\} \quad (23)$$

where $\bar{a} > 0$, the following hold:

$$H_{d_l}(Bg, Bq) \leq \psi(D_l(g, q)) \\ \forall g, q \in \overline{B_{d_l}(g_0, \hat{r})} \cap \{MB(g_n)\} \text{ with } g \leq q \quad (24)$$

$$\text{and } \sum_{i=0}^n \psi^i(d_l(g_0, g_1)) \leq \hat{r} \quad \forall n \in \mathbb{N} \cup \{0\}. \quad (25)$$

If $g, q \in \overline{B_{d_l}(g_0, \hat{r})}$, such that $g \leq q$ implies $Bg \leq_{\hat{r}} Bq$. Then $\{MB(g_n)\}$ is a sequence in $\overline{B_{d_l}(g_0, \hat{r})}$, $g_n \leq g_{n+1}$, and $\{MB(g_n)\} \rightarrow g^* \in \overline{B_{d_l}(g_0, \hat{r})}$. Also if $g^* \leq g_n$ or $g_n \leq g^*$, for all $n \in \mathbb{N} \cup \{0\}$, and inequality (24) holds for all $g, q \in \overline{B_{d_l}(g_0, \hat{r})} \cap \{MB(g_n)\} \cup \{g^*\}$. Then g^* is a fixed point of B in $\overline{B_{d_l}(g_0, \hat{r})}$.

Corollary 13. Let $(M, \leq, d_l)$ be a preordered complete $D.M$ space, $\hat{r} > 0$, $g_0 \in \overline{B_{d_l}(g_0, \hat{r})}$, and $B : M \rightarrow P(M)$ be a multifunction on $\overline{B_{d_l}(g_0, \hat{r})}$; $\{MB(g_n)\}$ is a sequence generated by g_0 , with $g_0 \leq g_1$. Assume that, for some $k \in [0, 1)$ and

$$D_l(g, q) = \max \left\{ d_l(g, q), \frac{d_l(g, Bg) \cdot d_l(q, Bq)}{\bar{a} + d_l(g, q)}, \right. \\ \left. d_l(g, Bg), d_l(q, Bq) \right\} \quad (26)$$

where $\bar{a} > 0$, the following hold:

$$H_{d_1}(Bg, Bq) \leq k(D_1(g, q)) \quad (27)$$

$$\forall g, q \in \overline{B_{d_1}(g_0, \hat{r})} \cap \{MB(g_n)\} \text{ with } g \leq q$$

$$\text{and } \sum_{i=0}^n k^i(d_1(g_0, g_1)) \leq \hat{r} \quad \forall n \in \mathbb{N} \cup \{0\}. \quad (28)$$

If $g, q \in \overline{B_{d_1}(g_0, \hat{r})}$, such that $g \leq q$ implies $Bg \leq_r Bq$. Then $\{MB(g_n)\}$ is a sequence in $\overline{B_{d_1}(g_0, \hat{r})}$, $g_n \leq g_{n+1}$, and $\{MB(g_n)\} \rightarrow g^* \in \overline{B_{d_1}(g_0, \hat{r})}$. Also if $g^* \leq g_n$ or $g_n \leq g^*$, for all $n \in \mathbb{N} \cup \{0\}$, and inequality (27) holds for all $g, q \in \overline{B_{d_1}(g_0, \hat{r})} \cap \{MB(g_n)\} \cup \{g^*\}$. Then g^* is a fixed point of B in $\overline{B_{d_1}(g_0, \hat{r})}$.

Example 14. Let $M = Q^+ \cup \{0\}$ and let $d_b : M \times M \rightarrow M$ be the D.B.M space on M defined by

$$d_b(g, q) = (g + q)^2 \quad \forall g, q \in M \quad (29)$$

with parameter $t > 1$. Define the multivalued mappings, $B : M \times M \rightarrow P(M)$ by

$$Bg = \begin{cases} \left[\frac{g}{3}, \frac{2g}{3}\right] & \text{if } g \in [0, 9] \cap M \\ [g, g + 1] & \text{if } g \in (9, \infty) \cap M. \end{cases} \quad (30)$$

Considering $g_0 = 1$, $\hat{r} = 100$, and $\bar{a} = 1$, $b = 2$, then $\overline{B_{d_b}(g_0, \hat{r})} = [0, 9] \cap M$. Now $d_b(g_0, Bg_0) = d_b(1, B1) = d_b(1, 1/3) = 16/9$. So we obtain a sequence $\{MB(g_n)\} = \{1, 1/3, 1/9, 1/27, \dots\}$ in M generated by g_0 . Let $t = 1.2$, $\psi(t) = 4t/5$, and then $t\psi(t) < t$. Define

$$\alpha(g, q) = \begin{cases} 1 & \text{if } g, q \in [0, 9] \cap M \\ \frac{3}{2} & \text{otherwise.} \end{cases} \quad (31)$$

Now,

$$\begin{aligned} \alpha_*(B10, B11) H_{d_b}(B10, B11) &= \left(\frac{3}{2}\right) (484) \\ &> \psi(D_b(g, q)) = \frac{4}{5} (484). \end{aligned} \quad (32)$$

So the contractive condition does not hold on M . Now, for all $g, q \in \overline{B_{d_b}(g_0, \hat{r})} \cap \{MB(g_n)\}$, we have

$$\begin{aligned} \alpha_*(Bg, Bq) H_{d_b}(Bg, Bq) &= 1 \left[\max \left\{ \sup_{\bar{a} \in Bg} d_b(\bar{a}, Bq), \right. \right. \\ &\quad \left. \left. \sup_{b \in Bq} d_b(Bg, b) \right\} \right] = \max \left\{ \sup_{\bar{a} \in Bg} d_b\left(\bar{a}, \left[\frac{q}{3}, \frac{2q}{3}\right]\right), \right. \\ &\quad \left. \sup_{b \in Bq} d_b\left(\left[\frac{g}{3}, \frac{2g}{3}\right], b\right) \right\} = \max \left\{ \sup_{\bar{a} \in Bg} d_b\left(\frac{2g}{3}, \right. \right. \\ &\quad \left. \left. \left[\frac{q}{3}, \frac{2q}{3}\right]\right), \sup_{b \in Bq} d_b\left(\left[\frac{g}{3}, \frac{2g}{3}\right], \frac{2q}{3}\right) \right\} \end{aligned}$$

$$\begin{aligned} &= \max \left\{ d_b\left(\frac{2g}{3}, \frac{q}{3}\right), d_b\left(\frac{g}{3}, \frac{2q}{3}\right) \right\} \\ &= \max \left\{ \frac{(2g+q)^2}{9}, \frac{(g+2q)^2}{9} \right\} \\ &\leq \psi \left(\max \left\{ (g+q)^2, \frac{256g^2q^2}{81(1+(g+q)^2)}, \frac{16g^2}{9}, \right. \right. \\ &\quad \left. \left. \frac{16q^2}{9} \right\} \right) = \psi(d_b(g, q)). \end{aligned} \quad (33)$$

So the contractive condition holds on $\overline{B_{d_b}(g_0, \hat{r})} \cap \{MB(g_n)\}$. As $t = 1.2 > 1$, then

$$\begin{aligned} \sum_{i=0}^n t^{i+1} \{\psi^i(d_b(g_0, g_1))\} &= \frac{16}{9} \times \frac{6}{5} \sum_{i=0}^n \left(\frac{24}{25}\right)^i < 100 \\ &= \hat{r}. \end{aligned} \quad (34)$$

Hence, all the conditions of Theorem 9 are satisfied. Now, we have that $\{MB(g_n)\}$ is a sequence in $\overline{B_{d_b}(g_0, \hat{r})}$, $\alpha(g_n, g_{n+1}) \geq 1$ and $\{MB(g_n)\} \rightarrow 0 \in \overline{B_{d_b}(g_0, \hat{r})}$. Moreover, 0 is a fixed point of B .

3. Fixed Point Results For Graphic Contractions

In this section, we present an application of Theorem 9 in graph theory. Jachymski [27] proved the result concerning contraction mappings on metric space with a graph. Hussain et al. [28] discussed the fixed points theorem for graphic contraction and gave an application. A graph $\dot{G}$ is affix if there is a way among any two vertices (see for details [29, 30]).

Definition 15. Let M be a nonempty set and $\dot{G} = (V(\dot{G}), Q(\dot{G}))$ be a graph such that $V(\dot{G}) = M$, and $B : M \rightarrow P(M)$ is said to be multigraph preserving if $(g, q) \in Q(\dot{G})$, and then $(w, p) \in Q(\dot{G})$ for all $w \in Bg$ and $p \in Bq$.

Theorem 16. Let (M, d_b) be a complete D.B.M space endowed with a graph $\dot{G}$. Suppose there exists a function $\alpha : M \times M \rightarrow [0, \infty)$. Let, $\hat{r} > 0$, $g_0 \in \overline{B_{d_b}(g_0, \hat{r})}$, $B : M \rightarrow P(M)$, and let for a sequence $\{MB(g_n)\}$ in M generated by g_0 , with $(g_0, g_1) \in Q(\dot{G})$. Suppose that the following are satisfy:

- (i) B is a graph preserving for all $g, q \in \overline{B_{d_b}(g_0, \hat{r})} \cap \{MB(g_n)\}$;
- (ii) there exists $\psi \in \Psi$ and

$$\begin{aligned} D_b(g, q) &= \max \left\{ d_b(g, q), \frac{d_l(g, Bg) \cdot d_l(q, Bq)}{\bar{a} + d_l(g, q)}, \right. \\ &\quad \left. d_b(g, Bg), d_b(q, Bq) \right\} \end{aligned} \quad (35)$$

where $\bar{a} > 0$ such that

$$H_{d_b}(Bg, Bq) \leq \psi(D_b(g, q)), \quad (36)$$

for all $g, q \in \overline{B_{d_b}(g_0, \hat{r})} \cap \{MB(g_n)\}$ and $(g, q) \in Q(G)$;

(iii) $\sum_{i=0}^n t^{i+1} \{\psi^i(d_b(g_0, Bg_0))\} \leq \hat{r}$ for all $n \in \mathbb{N} \cup \{0\}$ and $t > 1$.

Then, $\{MB(g_n)\}$ is a sequence in $\overline{B_{d_b}(g_0, \hat{r})}$, $(g_n, g_{n+1}) \in Q(G)$ and $\{MB(g_n)\} \rightarrow g^*$. Also, if and inequality (36) holds for g^* and $(g_n, g^*) \in Q(G)$ or $(g^*, g_n) \in Q(G)$ for all $n \in \mathbb{N} \cup \{0\}$, then B has a fixed point g^* in $\overline{B_{d_b}(g_0, \hat{r})}$.

Proof. Define, $\alpha : M \times M \rightarrow [0, \infty)$ by

$$\alpha(g, q) = \begin{cases} 1, & \text{if } g \in \overline{B_{d_b}(g_0, \hat{r})}, (g, q) \in Q(G) \\ 0, & \text{otherwise.} \end{cases} \quad (37)$$

As $\{MB(g_n)\}$ is a sequence in M generated by g_0 with $(g_0, g_1) \in Q(G)$, we have $\alpha(g_0, g_1) \geq 1$. Let $\alpha(g, q) \geq 1$, and then $(g, q) \in Q(G)$. From (i), we have $(w, p) \in Q(G)$ for all $w \in Bg$ and $p \in Bq$. This implies that $\alpha(w, p) = 1$ for all $w \in Bg$ and $p \in Bq$. This implies that $\inf\{\alpha(w, p) : w \in Bg, p \in Bq\} = 1$. So, $B : M \rightarrow P(M)$ is a semi- α_* -admissible multifunction on $\overline{B_{d_b}(g_0, \hat{r})}$. Moreover, inequality (36) can be written as

$$\alpha_*(Bg, Bq) H_{d_b}(Bg, Bq) \leq \psi(D_b(g, q)), \quad (38)$$

for all elements g, q in $\overline{B_{d_b}(g_0, \hat{r})} \cap \{MB(g_n)\}$ with either $\alpha(g, q) \geq 1$ or $\alpha(q, g) \geq 1$. Also, (iii) holds. Then, by Theorem 9, we have that $\{MB(g_n)\}$ is a sequence in $\overline{B_{d_b}(g_0, \hat{r})}$ and $\{MB(g_n)\} \rightarrow g^* \in \overline{B_{d_b}(g_0, \hat{r})}$. Now, $g_n, g^* \in \overline{B_{d_b}(g_0, \hat{r})}$ and either $(g_n, g^*) \in Q(G)$ or $(g^*, g_n) \in Q(G)$ for all $n \in \mathbb{N} \cup \{0\}$, and inequality (36) holds for all $g, q \in \overline{B_{d_b}(g_0, \hat{r})} \cap \{MB(g_n)\} \cup \{g^*\}$. Then we have $\alpha(g_n, g^*) \geq 1$ or $\alpha(g^*, g_n) \geq 1$ for all $n \in \mathbb{N} \cup \{0\}$ and inequality (5) holds for all $g, q \in \overline{B_{d_b}(g_0, \hat{r})} \cap \{MB(g_n)\} \cup \{g^*\}$. So, all the conditions of Theorem 9 are satisfied. Hence, by Theorem 9, B has a common fixed point g^* in $\overline{B_{d_b}(g_0, \hat{r})}$ and $d_b(g^*, g^*) = 0$. $\square$

4. Fixed Point Results for Single-Valued Mapping

In this section we discussed some fixed point results for self-mapping in complete $D.B.M$ space. Let (M, d_b) be a $D.B.M$ space, $g_0 \in M$, and $B : M \rightarrow M$ be a mapping. Let $g_1 = Bg_0$, $g_2 = Bg_1$. Continuing this process, we construct a sequence g_n of points in M such that $g_{n+1} = Bg_n$. We denote this iterative sequence by $\{g_n\}$. We say that $\{g_n\}$ is a sequence in M generated by g_0 .

Theorem 17. Let (M, d_b) be a complete $D.B.M$ space, $\hat{r} > 0$, $g_0 \in \overline{B_{d_b}(g_0, \hat{r})}$, and $B : M \rightarrow M$ be a semi- α -admissible

function on $\overline{B_{d_b}(g_0, \hat{r})}$; $\{g_n\}$ is a sequence in M generated by g_0 , $\alpha(g_0, g_1) \geq 1$. Assume that, for some $\psi \in \Psi$ and

$$D_b(g, q) = \max \left\{ d_b(g, q), \frac{d_b(g, Bg) \cdot d_b(q, Bq)}{\bar{a} + d_b(g, q)}, d_b(g, Bg), d_b(q, Bq) \right\}, \quad (39)$$

where $\bar{a} > 0$, the following hold:

$$\alpha(Bg, Bq) H_{d_b}(Bg, Bq) \leq \psi(D_b(g, q)) \quad \forall g, q \in \overline{B_{d_b}(g_0, \hat{r})} \cap \{g_n\} \quad (40)$$

$$\sum_{i=0}^n B^{i+1} \{\psi^i(d_b(g_0, g_1))\} \leq \hat{r} \quad \forall n \in \mathbb{N} \cup \{0\}. \quad (41)$$

Then, $\{g_n\}$ is a sequence in $\overline{B_{d_b}(g_0, \hat{r})}$, $\alpha(g_n, g_{n+1}) \geq 1$, and $\{g_n\} \rightarrow g^* \in \overline{B_{d_b}(g_0, \hat{r})}$. Also if $\alpha(g_n, g^*) \geq 1$ or $\alpha(g^*, g_n) \geq 1$, for all $n \in \mathbb{N} \cup \{0\}$, and inequality (40) holds for all $g, q \in \overline{B_{d_b}(g_0, \hat{r})} \cap \{g_n\} \cup \{g^*\}$. Then B has a common fixed point g^* in $\overline{B_{d_b}(g_0, \hat{r})}$.

Proof. The proof of the above theorem is similar to Theorem 17. $\square$

Corollary 18. Let $(M, \leq, d_b)$ be a preordered complete $D.B.M$ space, $\hat{r} > 0$, $g_0 \in \overline{B_{d_b}(g_0, \hat{r})}$, and $B : M \rightarrow M$ be a self-mapping on $\overline{B_{d_b}(g_0, \hat{r})}$; $\{g_n\}$ is a sequence generated by g_0 , with $g_0 \leq g_1$. Assume that, for some $k \in [0, 1)$ and

$$D_b(g, q) = \max \left\{ d_b(g, q), \frac{d_b(g, Bg) \cdot d_b(q, Bq)}{\bar{a} + d_b(g, q)}, d_b(g, Bg), d_b(q, Bq) \right\}, \quad (42)$$

where $\bar{a} > 0$, the following hold:

$$H_{d_b}(Bg, Bq) \leq k(D_b(g, q)) \quad \forall g, q \in \overline{B_{d_b}(g_0, \hat{r})} \cap \{g_n\} \text{ with } g \leq q \quad (43)$$

$$\text{and } \sum_{i=0}^n t^{i+1} \{k^i(d_b(g_0, g_1))\} \leq \hat{r} \quad \forall n \in \mathbb{N} \cup \{0\}. \quad (44)$$

If $g, q \in \overline{B_{d_b}(g_0, \hat{r})}$, such that $g \leq q$ implies $Bg \leq Bq$. Then $\{g_n\}$ is a sequence in $\overline{B_{d_b}(g_0, \hat{r})}$, $g_n \leq g_{n+1}$, and $\{g_n\} \rightarrow g^* \in \overline{B_{d_b}(g_0, \hat{r})}$. Also if $g^* \leq g_n$ or $g_n \leq g^*$, for all $n \in \mathbb{N} \cup \{0\}$, and inequality (43) holds for all $g, q \in \overline{B_{d_b}(g_0, \hat{r})} \cap \{g_n\} \cup \{g^*\}$. Then g^* is a fixed point of B in $\overline{B_{d_b}(g_0, \hat{r})}$.

Corollary 19. Let $(M, \leq, d_l)$ be a preordered complete $D.M$ space, $\hat{r} > 0$, $g_0 \in \overline{B_{d_l}(g_0, \hat{r})}$, and $B : M \rightarrow M$ be a

self-mapping on $\overline{B_{d_1}(g_0, \hat{r})}$; $\{g_n\}$ is a sequence generated by g_0 , with $g_0 \leq g_1$. Assume that, for some $\psi \in \Psi$ and

$$D_l(g, q) = \max \left\{ d_l(g, q), \frac{d_l(g, Bg) \cdot d_l(q, Bq)}{\bar{a} + d_l(g, q)}, d_l(g, Bg), d_l(q, Bq) \right\} \quad (45)$$

where $\bar{a} > 0$, the following hold:

$$H_{d_1}(Bg, Bq) \leq \psi(D_l(g, q)) \quad (46)$$

$$\forall g, q \in \overline{B_{d_1}(g_0, \hat{r})} \cap \{g_n\} \text{ with } g \leq q$$

$$\text{and } \sum_{i=0}^n \psi^i(d_l(g_0, g_1)) \leq \hat{r} \quad \forall n \in \mathbb{N} \cup \{0\}. \quad (47)$$

If $g, q \in \overline{B_{d_1}(g_0, \hat{r})}$, such that $g \leq q$ implies $Bg \leq_{\hat{r}} Bq$. Then $\{g_n\}$ is a sequence in $\overline{B_{d_1}(g_0, \hat{r})}$, $g_n \leq g_{n+1}$, and $\{g_n\} \rightarrow g^* \in \overline{B_{d_1}(g_0, \hat{r})}$. Also if $g^* \leq g_n$ or $g_n \leq g^*$, for all $n \in \mathbb{N} \cup \{0\}$, and inequality (46) holds for all $g, q \in \overline{B_{d_1}(g_0, \hat{r})} \cap \{g_n\} \cup \{g^*\}$. Then g^* is a fixed point of B in $\overline{B_{d_1}(g_0, \hat{r})}$.

Recall that if $(M, \leq)$ is a preordered set and $A : M \rightarrow M$ is such that for $g, q \in M$, with $g \leq q$ implying $Ag \leq Aq$, then the mapping A is said to be nondecreasing.

Corollary 20. Let $(M, \leq, d_l)$ be a preordered complete D.M space, $\hat{r} > 0$, g_0 be an arbitrary point in $\overline{B_{d_1}(g_0, \hat{r})}$, $B : M \rightarrow M$ be a self-mapping on $\overline{B_{d_1}(g_0, \hat{r})}$, and $\{g_n\}$ be a Picard sequence in M with initial guess g_0 , with $g_0 \leq g_1$. For some $k \in [0, 1)$ and

$$D_l(g, q) = \max \left\{ d_l(g, q), \frac{d_l(g, Bg) \cdot d_l(q, Bq)}{\bar{a} + d_l(g, q)}, d_l(g, Bg), d_l(q, Bq) \right\} \quad (48)$$

where $\bar{a} > 0$, the following hold:

$$d_l(Bg, Bq) \leq k(D_l(g, q)) \quad (49)$$

$$\forall g, q \in \overline{B_{d_1}(g_0, \hat{r})} \cap \{g_n\} \text{ with } g \leq q$$

$$\text{and } \sum_{i=0}^j k^i(d_l(g_0, g_1)) \leq \hat{r} \quad \forall j \in \mathbb{N} \cup \{0\}. \quad (50)$$

Then, $\{g_n\}$ is a sequence in $\overline{B_{d_1}(g_0, \hat{r})}$, such that $g_n \leq g_{n+1}$ and $\{g_n\} \rightarrow g^* \in \overline{B_{d_1}(g_0, \hat{r})}$. Also if $g^* \leq g_n$ or $g_n \leq g^*$, for all $n \in \mathbb{N} \cup \{0\}$, and inequality (49) holds for all $g, q \in \overline{B_{d_1}(g_0, \hat{r})} \cap \{g_n\} \cup \{g^*\}$. Then g^* is a fixed point of B in $\overline{B_{d_1}(g_0, \hat{r})}$.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

Each author equally contributed to this paper and read and approved the final manuscript.

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