Research Article

An Improved Data Fusion Method Based on Weighted Belief Entropy considering the Negation of Basic Probability Assignment

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Uncertainty in data fusion applications has received great attention. Due to the effectiveness and flexibility in handling uncertainty, Dempster–Shafer evidence theory is widely used in numerous fields of data fusion. However, Dempster–Shafer evidence theory cannot be used directly for conflicting sensor data fusion since counterintuitive results may be attained. In order to handle this issue, a new method for data fusion based on weighted belief entropy and the negation of basic probability assignment (BPA) is proposed. First, the negation of BPA is applied to represent the information in a novel view. Then, by measuring the uncertainty of the evidence, the weighted belief entropy is adopted to indicate the relative importance of evidence. Finally, the ultimate weight of each body of evidence is applied to adjust the mass function before fusing by the Dempster combination rule. The validity of the proposed method is demonstrated in accordance with an experiment on artificial data and an application on fault diagnosis.

1. Introduction

In recent years, considerable attention has been paid to multisensor data fusion technology or information acquisition and environment sensing, such as the wireless network [1], fault detection [2], condition monitoring [3], and image processing [4–6]. Due to the complexity of the targets, the data collected from a single sensor perform poorly when making decisions. Besides, multisensors may be affected by the complicated application environment so that they may make wrong decisions. Hence, multisource information modeling and fusion are important issues in many real applications [7]. Nevertheless, the uncertainty and imprecision are ineluctable for the real applications [8]. How to model and handle these kinds of imprecise and uncertain information remains an ongoing challenge [9, 10]. To address this issue, numerous methods have been proposed for multisensor modeling and data fusion, including rough sets theory [11], belief function theory [12], Dempster–Shafer evidence theory [13], fuzzy set theory [14, 15], Z-numbers [16], and D-numbers [17–19].

Dempster–Shafer evidence theory is an effective method in uncertain information modeling and processing, which

was firstly proposed by Dempster [20] and had been developed by Shafer [21]. Dempster–Shafer evidence theory has been widely applied in considerable fields, such as decision-making [22–24], risk analysis [25, 26], evidence analysis [27], pattern recognition [28–30], fault diagnosis [31, 32], human reliability analysis [33], supplier selection [34], and failure mode and effects analysis [35–38]. Despite a number of advantages of Dempster–Shafer evidence theory, the classical Dempster combination rule cannot be directly used for conflicting sensor data fusion, especially when fusing highly conflicting data [39], which may lead to counterintuitive results [40]. Therefore, many researchers tend to preprocess the bodies of evidence to resolve the problem of fusing the highly conflicting evidence, e.g., to quantify the uncertainty before data fusion [41–43]. However, how to measure the uncertainty for practical applications in the framework of Dempster–Shafer evidence theory has to be identified [44–46].

As a well-known theory for uncertainty measure in the probabilistic framework, Shannon entropy attracts much attention in real applications [47–49]. Shannon entropy has also been introduced to many other fields, such as the network entropy [50, 51]. Nevertheless, Shannon entropy is

unable to be used directly among mass functions in the framework of Dempster–Shafer evidence theory because a mass function is a generalized probability assigned on the power set of the frame of discernment (FOD) [52]. To handle this issue, many methods for uncertainty measure in the Dempster–Shafer framework are claimed, including Yager’s dissonance measure [53], Klir and Ramer’s discord measure [54], Hohle’s confusion measure [55], the weighted Hartley entropy [56], Klir and Parviz’s strife measure [57], and George and Pal’s conflict measure [58–60]. However, the existed methods may have limited efficiency in some situations [61]. In recent years, an uncertainty measure named Deng entropy was proposed [61]. Although Deng entropy has successfully overcome the weaknesses of the above methods [62] to some extent, Deng entropy fails to take the scale of FOD into consideration, which contributes to the loss of substantial and valuable information in information processing. By taking this into account, Tang et al. [52] introduced the weighted belief entropy to be related to the scale of FOD. But there is still some room for improvement to achieve more accurate fusing results.

Recently, Yin et al. [63] proposed a novel method to obtain the negation of the basic probability assignment (BPA), which can be used to measure the uncertainty of BPA. Based on that, this paper proposes an improved data fusion method by integrating the negation of BPA with the weighted belief entropy. The proposed method considers both the uncertainty measure of the negation of BPA and the uncertainty measure on the weight so that it can acquire more suitably weighted average evidence before applying the Dempster combination rule. Accordingly, the proposed method consists of the following steps. Firstly, Yin et al.’s model is used to obtain the negation of the BPA. After that, the relative importance of the evidence is attained by making use of the weighted belief entropy to obtain the uncertainty measure of the negation of the BPA. Moreover, the final weight for each body of evidence (BOE) is presented by the modified credibility degree of each body of evidence. Based on that, the weighted average evidence can be calculated. Then, the Dempster combination rule will be used to fuse the weighted average evidence. Furthermore, this new data fusion method is applied on fault diagnosis of a motor rotor to validate its capacity in real applications.

The rest of this paper is organized as follows: in Section 2, the preliminaries on Dempster–Shafer evidence theory, Shannon entropy, weighted belief entropy, negation of BPA, and some uncertainty measures in the Dempster–Shafer framework are briefly introduced; then, an improved data fusion method which is based on the weighted belief entropy and the negation of BPA is proposed in Section 3; Section 4 illustrates a numerical example to show the effectiveness of the proposed method; in Section 5, the proposed sensor data fusion method is adopted to an application in fault diagnosis; finally, Section 6 gives a conclusion.

2. Preliminaries

In this section, some preliminaries are briefly introduced, including Dempster–Shafer evidence theory [20, 21],

Shannon entropy [64], weighted belief entropy [52], the negation of BPA [63], and some other typical uncertainty measures in Dempster–Shafer framework [53–58].

2.1. Dempster–Shafer Evidence Theory. Let $\Omega = \{\Theta_1, \Theta_2, \dots, \Theta_i, \dots, \Theta_N\}$ be a finite nonempty set with N mutually exclusive and exhaustive events, and Ω is called the frame of discernment (FOD). The power set of Ω , denoted as 2^Ω , is composed of 2^N elements denoted as follows:

$$2^\Omega = \{\emptyset, \{\theta_1\}, \{\theta_2\}, \dots, \{\theta_N\}, \{\theta_1, \theta_2\}, \dots, \{\theta_1, \theta_2, \dots, \theta_i\}, \dots, \Omega\}. \quad (1)$$

A mass function m is defined as a mapping from the power set 2^Ω to the interval $[0, 1]$, which satisfies the following conditions [20, 21]:

$$\begin{aligned} m(\emptyset) &= 0, \\ \sum_{A \in 2^\Omega} m(A) &= 1. \end{aligned} \quad (2)$$

If $m(A) > 0$, then A is called a focal element, and the mass function $m(A)$ represents how strongly the evidence supports the proposition A .

A body of evidence, also known as a basic probability assignment (BPA) or basic belief assignment (BBA), is represented by the focal sets and their associated mass value:

$$(\mathfrak{R}, m) = \{A, m(A): A \in 2^\Omega, m(A) > 0\}, \quad (3)$$

where $\mathfrak{R}$ is a subset of the power set 2^Ω and each $A \in \mathfrak{R}$ has an associated nonzero mass value $m(A)$.

A BPA m can also be represented by its associate belief function Bel and plausibility function Pl , respectively, defined as follows:

$$\text{Bel}(A) = \sum_{\emptyset \neq B \subseteq A} m(B), \quad \text{Pl}(A) = \sum_{B \cap A \neq \emptyset} m(B). \quad (4)$$

In Dempster–Shafer evidence theory, two independent mass functions, denoted as m_1 and m_2 , can be combined with Dempster combination rule defined as follows [20, 21]:

$$m(A) = (m_1 \oplus m_2)(A) = \frac{1}{1-k} \sum_{B \cap C = A} m_1(B)m_2(C), \quad (5)$$

where k is a normalization constant representing the degree of conflict between m_1 and m_2 and k is defined as follows [20, 21]:

$$k = \sum_{B \cap C = \emptyset} m_1(B)m_2(C). \quad (6)$$

2.2. Shannon Entropy. As an uncertainty measure of information volume in a system or process, Shannon entropy plays a central role in information theory. Shannon entropy indicates that the information volume of each piece of information is directly connected to its uncertain degree.

Shannon entropy, as the information entropy, is defined as follows [64]:

$$H = \sum_{i=1}^N p_i \log_b p_i, \quad (7)$$

where N is the number of basic states, p_i is the probability of state i , and p_i satisfies $\sum_{i=1}^N p_i = 1$. If the unit of information is bit, then $b = 2$.

2.3. Weighted Belief Entropy. Weighted belief entropy is an improved measure of Deng entropy in the Dempster–Shafer framework. Compared with the original Deng entropy, weighted belief entropy focuses on the uncertain information represented by not only the mass function but also the scale of the FOD, which means less information loss in information processing. Weighted belief entropy, denoted as E_{wd} , is defined as follows [52]:

$$E_{wd}(m) = - \sum_{A \subseteq X} \frac{|A|m(A)}{|X|} \log_2 \frac{m(A)}{2^{|A|} - 1}, \quad (8)$$

where X is the FOD, A is the focal element of the mass function, $|A|$ denotes the cardinality of the proposition A , and $|X|$ is the number of elements in FOD. If and only if the mass value is assigned to single elements, weighted belief entropy can be degenerated to Shannon entropy; in this case, the form of weighted belief entropy is as follows:

$$\begin{aligned} E_{wd}(m) &= - \sum_{A \subseteq X} \frac{|A|m(A)}{|X|} \log_2 \frac{m(A)}{2^{|A|} - 1} \\ &= - \sum_{A \subseteq X} m(A) \log_2 m(A). \end{aligned} \quad (9)$$

For more details about weighted belief entropy, refer to [52].

2.4. The Negation of the BPA. The negation method provides a novel view to investigate the property of the mass functions. And the negation method can achieve the maximum entropy allocation in a manner [65]. Additionally, the negation method can highlight the rare focal elements in the BPA, which is significant to study, because the rare event may influence the system dramatically in some circumstances [66].

Recently, Yin et al. [63] proposed a novel method to obtain the negation of the basic probability assignment (BPA) for measuring the uncertainty of BPA. Unlike some other existing negation methods, Yin et al.'s method can be applied to not only the probability distributions but also the BPA. The main concepts are defined as follows.

Let us consider an event E and e_i denotes the i th focal element. For each focal element e_i , $m(e_i)$ is the belief of the i th focal element of the initial mass function and the negation of $m(e_i)$ is denoted by $\bar{m}(e_i)$. The general formula of the negation of the mass function can be derived as

$$\bar{m}(e_i) = \frac{1 - m(e_i)}{n - 1}, \quad (10)$$

where n is the number of focal element.

Actually, the negation of BPA makes use of the complement of each focal element to generate the negative one. Then, the belief of each focal element is normalized by making the sum of them equal to 1. The negation method helps to assign the belief equally to each focal element, and if the mass function consists of only two focal elements, the negation method will exchange the belief of the two focal elements after the negation process. An example is given to show the procedure of the negation process.

Example 1. Assume the FOD = $\{a, b, c\}$, for a mass function $m(a) = 0.2$, $m(b) = 0.7$, and $m(c) = 0.1$, then

$$\begin{aligned} \bar{m}(a) &= \frac{1 - m(a)}{n - 1} = 0.4\bar{m}(b) = \frac{1 - m(b)}{n - 1} = 0.15\bar{m}(c) \\ &= \frac{1 - m(c)}{n - 1} = 0.45. \end{aligned} \quad (11)$$

For more details about weighted belief entropy, refer to [63].

2.5. Uncertainty Measures in Dempster–Shafer Framework. In this section, some other typical uncertainty measures in the framework of Dempster–Shafer evidence theory are briefly introduced. Assume that X is the FOD, A and B are focal elements of the mass function, and $|A|$ denotes the cardinality of A . Then, the definitions of different uncertainty measures are shown as follows.

2.5.1. Hohle's Confusion Measure. Hohle's confusion measure, denoted as C_H , is defined as follows [55]:

$$C_H(m) = - \sum_{A \subseteq X} m(A) \log_2 \text{Bel}(A). \quad (12)$$

2.5.2. Yager's Dissonance Measure. Yager's dissonance measure, denoted as E_Y , is defined as follows [53]:

$$E_Y(m) = - \sum_{A \subseteq X} m(A) \log_2 \text{Pl}(A). \quad (13)$$

2.5.3. Dubois and Prade's Weighted Hartley Entropy. Dubois and Prade's weighted Hartley entropy, denoted as EDP, is defined as follows [56]:

$$E_{DP}(m) = \sum_{A \subseteq X} m(A) \log_2 |A|. \quad (14)$$

2.5.4. Klir and Ramer's Discord Measure. Klir and Ramer's discord measure, denoted as DKR, is defined as follows [54]:

$$D_{KR}(m) = - \sum_{A \subseteq X} m(A) \log_2 \sum_{B \subseteq X} m(B) \frac{|A \cap B|}{|B|}. \quad (15)$$

2.5.5. Klir and Parviz's Strife Measure. Klir and Parviz's strife measure, denoted as SKP, is defined as follows [57]:

$$S_{KP}(m) = - \sum_{A \subseteq X} m(A) \log_2 \sum_{B \subseteq X} m(B) \frac{|A \cap B|}{|A|}. \quad (16)$$

2.5.6. George and Pal's Conflict Measure. The total conflict measure proposed by George and Pal, denoted as TC_{GP} , is defined as follows [58]:

$$TC_{GP}(m) = \sum_{A \subseteq X} m(A) \sum_{B \subseteq X} m(B) 1 - \left(\frac{|A \cap B|}{|A \cup B|} \right). \quad (17)$$

3. The Proposed Method

In this section, an improved data fusion approach is presented. The proposed method is based on the weighted belief entropy and the negation of BPA.

After uncertainty measure with the negation of BPA and the weighted belief entropy, the modified BOEs are fused with Dempster combination rule. Finally, decision-making is based on the fused results. The procedures for data fusion based on the weighted belief entropy and the negation of BPA are designed in Figure 1.

The details of the six steps in Figure 1 are presented as follows:

Step 1. Evidence from sensor report is modeled as the BOE.

In real applications, due to the diversity styles of information, the first step of information processing in the frame of Dempster-Shafer evidence theory mainly focuses on modeling uncertain information with BPAs.

Step 2. Calculate the negation of BPA.

In order to obtain more accurate fusing results, the negation of BPA is adopted to represent the information in this paper. In the proposed method, Yin et al.'s method is used to obtain the negation of BPA. For the i th focal element e_i in each BOE, the general formula of the negation of the mass function $\bar{m}(e_i)$ is calculated as follows:

$$\bar{m}(e_i) = \frac{1 - m(e_i)}{n - 1}, \quad (18)$$

where n is the number of focal elements.

Step 3. Uncertainty measure of the negation of BPA with weighted belief entropy.

The uncertainty of the negation of BPA modeled in the 2nd step needs to be measured with an appropriate method before further processing. In this paper, the weighted belief entropy is used to measure the uncertain degree of each BOE. For the i th BOE ($i = 1, 2,$

$\dots, m$), its corresponding uncertain degree with the weighted belief entropy E_{Wd} is calculated as follows:

$$E_{Wd}(m_i) = - \sum_{A \subseteq X} \frac{|A| \bar{m}_i(A)}{|X|} \log_2 \frac{\bar{m}_i(A)}{2^{|A|} - 1}. \quad (19)$$

Step 4. Calculate the weight of each BOE.

The weight of each BOE can be calculated based on the value of weighted belief entropy. It is commonly accepted that the bigger the entropy is, the higher the uncertain degree will be [67]. For the i th BOE ($i = 1, 2, \dots, m$), its relative weight among all the available m BOEs, denoted as w_i , is calculated as follows:

$$w_i = \frac{E_{Wd}(m_i)}{\sum_{i=1}^m E_{Wd}(m_i)}. \quad (20)$$

Step 5. Calculate the modified mass functions.

The modified mass function of each proposition is calculated for the final data fusion. For each proposition A in the BOE, the modified mass function can be calculated as follows:

$$m_w(A) = \sum_{i=1}^m w_i m_i(A). \quad (21)$$

Step 6. Combination of the modified BPAs.

In the proposed method, the conflict among different evidences is transformed and measured by the negation of BPA and the weighted belief entropy, and now, data fusion can be accomplished via the Dempster combination rule. For each proposition A in the BOE, the combination result can be got by calculating Dempster combination rule with $(m - 1)$ times:

$$m(A) = (((m_w \oplus m_w)_1 \oplus m_w)_2 \cdots \oplus m_w)_{(m-2)} \oplus m_w)_{(m-1)}(A), \quad m \geq 2. \quad (22)$$

4. Experiment with Artificial Data

With the purpose of demonstrating the effectiveness and rationality of the proposed data fusion method, a numerical example is illustrated in this section. The experiment in [68] is recalled for the convenience of comparing with some other methods.

In target recognition, decision-making is based on reports from sensors occasionally. Considering a multisensor-based target recognition problem, three potential targets are, respectively, denoted as A , B , and C in the FOD, denoted as $X = \{a, b, c\}$. As is shown in Table 1, the evidence reported by five sensors is modeled as BPAs, which are denoted as m_1 , m_2 , m_3 , m_4 , and m_5 . Intuitively, the report from the 2nd sensor is contrary to the other four sensors, which may come

FIGURE 1: Flow chart of data fusion based on the weighted belief entropy and the negation of BPA.

TABLE 1: BPAs of artificial data.

BPA	$m(A)$	$m(B)$	$m(C)$	$m(A, C)$
1 st sensor report: $m_1(\cdot)$	0.41	0.29	0.3	0
2 nd sensor report: $m_2(\cdot)$	0	0.9	0.1	0
3 rd sensor report: $m_3(\cdot)$	0.58	0.07	0	0.35
4 th sensor report: $m_4(\cdot)$	0.55	0.1	0	0.35
5 th sensor report: $m_5(\cdot)$	0.6	0.1	0	0.3

Execute the proposed method in Figure 1.

from a bad sensor, and A will be the right target with the highest belief.

For the 1st step, the result is adopted from [68], and the BPAs are provided in Table 1.

For the 2nd step, with equation (18), the negation of BPA coming from the 1st sensor is calculated as follows:

$$\begin{aligned}\bar{m}(A) &= \frac{1 - m(A)}{n - 1} = 0.295, \\ \bar{m}(B) &= \frac{1 - m(B)}{n - 1} = 0.355, \\ \bar{m}(C) &= \frac{1 - m(C)}{n - 1} = 0.350.\end{aligned}\quad (23)$$

Similarly, the negation of BPAs of the 2nd sensor to the 5th sensor can be calculated, and the results are presented in Table 2.

For the 3rd step, the weighted belief entropy for each sensor report can be calculated with equation (8), and the calculation results are as follows:

TABLE 2: The negation of BPA.

BPA	$\bar{m}(A)$	$\bar{m}(B)$	$\bar{m}(C)$	$\bar{m}(A, C)$
1 st sensor report: $m_1(\cdot)$	0.295	0.2355	0.35	0
2 nd sensor report: $m_2(\cdot)$	0	0.1	0.9	0
3 rd sensor report: $m_3(\cdot)$	0.21	0.465	0	0.325
4 th sensor report: $m_4(\cdot)$	0.225	0.45	0	0.325
5 th sensor report: $m_5(\cdot)$	0.2	0.45	0	0.35

$$E_{Wd}(m_1) = - \sum_{A \subseteq X} \frac{|A| \bar{m}_1(A)}{|X|} \log_2 \frac{\bar{m}_1(A)}{2^{|A|} - 1} = 0.4811,$$

$$E_{Wd}(m_2) = - \sum_{A \subseteq X} \frac{|A| \bar{m}_2(A)}{|X|} \log_2 \frac{\bar{m}_2(A)}{2^{|A|} - 1} = 0.2282,$$

$$E_{Wd}(m_3) = - \sum_{A \subseteq X} \frac{|A| \bar{m}_3(A)}{|X|} \log_2 \frac{\bar{m}_3(A)}{2^{|A|} - 1} = 0.8546, \quad (24)$$

$$E_{Wd}(m_4) = - \sum_{A \subseteq X} \frac{|A| \bar{m}_4(A)}{|X|} \log_2 \frac{\bar{m}_4(A)}{2^{|A|} - 1} = 0.8567,$$

$$E_{Wd}(m_5) = - \sum_{A \subseteq X} \frac{|A| \bar{m}_5(A)}{|X|} \log_2 \frac{\bar{m}_5(A)}{2^{|A|} - 1} = 0.8760.$$

For the 4th step, with equation (19), the weight of each BOE is calculated as follows:

$$\begin{aligned}
w_1 &= \frac{E_{Wd}(m_1)}{\sum_{i=1}^5 E_{Wd}(m_i)} = 0.1459, \\
w_2 &= \frac{E_{Wd}(m_2)}{\sum_{i=1}^5 E_{Wd}(m_i)} = 0.0439, \\
w_3 &= \frac{E_{Wd}(m_3)}{\sum_{i=1}^5 E_{Wd}(m_i)} = 0.2712, \\
w_4 &= \frac{E_{Wd}(m_4)}{\sum_{i=1}^5 E_{Wd}(m_i)} = 0.2789, \\
w_5 &= \frac{E_{Wd}(m_5)}{\sum_{i=1}^5 E_{Wd}(m_i)} = 0.2593.
\end{aligned} \tag{25}$$

For the 5th step, the weighted mass function of each proposition in Table 2 can be calculated with equation (21), and the calculation results are as follows:

$$\begin{aligned}
m(A) &= (((m_w \oplus m_w)_1 \oplus m_w)_2 \oplus m_w)_3 \oplus m_w)_4(A) = 0.9890, \\
m(B) &= (((m_w \oplus m_w)_1 \oplus m_w)_2 \oplus m_w)_3 \oplus m_w)_4(B) = 0.0006, \\
m(C) &= (((m_w \oplus m_w)_1 \oplus m_w)_2 \oplus m_w)_3 \oplus m_w)_4(C) = 0.0061, \\
m(A, C) &= (((m_w \oplus m_w)_1 \oplus m_w)_2 \oplus m_w)_3 \oplus m_w)_4(A, C) = 0.0043.
\end{aligned} \tag{27}$$

The fusion results with different methods are presented in Table 3 and Figure 2.

With the proposed method, it can be inferred that target A is the right target even though the evidence m_2 has a high conflict with other evidence. With the highest belief (98.90%), as shown in Figure 2, the proposed method works more efficiently in dealing with the conflicting evidence than the other methods in [68–70]. The reason is that the proposed method not only focuses on the scale of the FOD to reduce information loss in information processing but also considers the negation of BPA, which addresses more available uncertain information in BOE. After taking above aspects into consideration, the reliable evidence's weight is increased, while unreliable evidence's weight is decreased; therefore, its negative effect is relieved on the final fusing results than other methods.

5. Application

In this section, the proposed method is applied to an application on fault diagnosis for a motor rotor, where the practical data in [62] are used for the comparison with the related method.

5.1. Problem Description. According to [62], suppose the frame of discernment Θ consists of three types of fault, denoted as $F1 = \{\text{rotor unbalance}\}$, $F2 = \{\text{rotor misalignment}\}$, and $F3 =$

TABLE 3: Experimental results with different methods.

Methods	$m(A)$	$m(B)$	$m(C)$	$m(A, C)$
Deng et al.'s method [68]	0.9820	0.0039	0.0107	0.0034
Zhang et al.'s method [69]	0.9820	0.0033	0.0115	0.0032
Yuan et al.'s method [70]	0.9886	0.0002	0.0072	0.0039
The proposed method	0.9890	0.0006	0.0061	0.0043

$$\begin{aligned}
m_w(A) &= \sum_{i=1}^5 w_i m_i(A) = 0.5152, \\
m_w(B) &= \sum_{i=1}^5 w_i m_i(B) = 0.1753, \\
m_w(C) &= \sum_{i=1}^5 w_i m_i(C) = 0.0507, \\
m_w(A, C) &= \sum_{i=1}^5 w_i m_i(A, C) = 0.8261.
\end{aligned} \tag{26}$$

Finally, for the 6th step, with Dempster combination rule and equation (22), each of the new weighted mass function is combined with four times. The fusion results are shown as follows:

{pedestal looseness}, respectively. Three vibration acceleration sensors, which are placed in different installation positions, are used to collect the vibration signal. The acceleration vibration frequency amplitudes at the frequencies of Freq1, Freq2, and Freq3 are taken as the fault feature variables. The collected sensor reports are provided in Table 4, where $m_{s1}(\cdot)$, $m_{s2}(\cdot)$, and $m_{s3}(\cdot)$, respectively, denote the BOEs reported from these three vibration acceleration sensors.

5.2. Data Fusion Based on the New Method. Execute the proposed method to solve the fault diagnosis problem mentioned above:

Step 1. Evidence from sensor report is modeled as the BOE.

In this paper, BPAs of sensor reports are directly adopted from [62], as is represented in Table 4. In practical applications, uncertain information modeling with BPAs has yet to be identified [71], which is not the domain of this paper. For more details about generating BPAs of Table 4, refer to [62].

Step 2. Calculate the negation of BPA.

In the proposed method, Yin et al.'s method is used to obtain the negation of BPA. With equation (18), the negation of BPA of each BOE under the vibration acceleration frequency of Freq1 is calculated as follows:

FIGURE 2: Comparison of fusion results generated by different methods in experiment: (a) fusion result for different objectives; (b) fusion result for target A.

$$\begin{aligned}
 \bar{m}_{s1}(\{F2\}) &= \frac{1 - m_{s1}(\{F2\})}{n - 1} = 0.0608, \\
 \bar{m}_{s1}(\{F3\}) &= \frac{1 - m_{s1}(\{F3\})}{n - 1} = 0.3332, \\
 \bar{m}_{s1}(\{F1, F2\}) &= \frac{1 - m_{s1}(\{F1, F2\})}{n - 1} = 0.2816, \\
 \bar{m}_{s1}(\{F1, F2, F3\}) &= \frac{1 - m_{s1}(\{F1, F2, F3\})}{n - 1} = 0.3244, \\
 \bar{m}_{s2}(\{F2\}) &= \frac{1 - m_{s2}(\{F2\})}{n - 1} = 0.1447, \\
 \bar{m}_{s2}(\{F3\}) &= \frac{1 - m_{s2}(\{F3\})}{n - 1} = 0.3330, \\
 \bar{m}_{s2}(\{F1, F2\}) &= \frac{1 - m_{s2}(\{F1, F2\})}{n - 1} = 0.3118, \\
 \bar{m}_{s2}(\{F1, F2, F3\}) &= \frac{1 - m_{s2}(\{F1, F2, F3\})}{n - 1} = 0.2104, \\
 \bar{m}_{s3}(\{F2\}) &= \frac{1 - m_{s3}(\{F2\})}{n - 1} = 0.2532, \\
 \bar{m}_{s3}(\{F3\}) &= \frac{1 - m_{s3}(\{F3\})}{n - 1} = 0.3332, \\
 \bar{m}_{s3}(\{F1, F2\}) &= \frac{1 - m_{s3}(\{F1, F2\})}{n - 1} = 0.3286, \\
 \bar{m}_{s3}(\{F1, F2, F3\}) &= \frac{1 - m_{s3}(\{F1, F2, F3\})}{n - 1} = 0.0849.
 \end{aligned} \tag{28}$$

Similarly, the negation of BPA of sensor reports under Freq2 and Freq3 can be calculated, and the results are shown in Table 5.

Step 3. Uncertainty measure of the negation of BPA with weighted belief entropy.

The uncertainty of sensor reports is measured based on the weighted belief entropy in the proposed method. With equation (19), the weighted belief entropy of each BOE under the vibration acceleration frequency of Freq1 is calculated as follows:

$$\begin{aligned}
 E_{Wd}(m_{s1}) &= - \sum_{A \subseteq X} \frac{|A| \bar{m}_{s1}(A)}{|X|} \log_2 \frac{\bar{m}_{s1}(A)}{2^{|A|} - 1} = 2.3362, \\
 E_{Wd}(m_{s2}) &= - \sum_{A \subseteq X} \frac{|A| \bar{m}_{s2}(A)}{|X|} \log_2 \frac{\bar{m}_{s2}(A)}{2^{|A|} - 1} = 2.0535, \\
 E_{Wd}(m_{s3}) &= - \sum_{A \subseteq X} \frac{|A| \bar{m}_{s3}(A)}{|X|} \log_2 \frac{\bar{m}_{s3}(A)}{2^{|A|} - 1} = 1.5829.
 \end{aligned} \tag{29}$$

The weighted belief entropy of sensor reports under Freq2 and Freq3 can be calculated, and the results are shown in Table 6.

Step 4. Calculate the weight of each BOE.

With equation (19), for the vibration acceleration frequency of Freq1, the weight of each BOE for evidence modification is calculated as follows:

TABLE 4: Data for fault diagnosis modeled as BPAs.

	Freq1				Freq2			Freq3		
	{F2}	{F3}	{F1, F2}	{F1, F2, F3}	{F2}	{F1, F2, F3}	{F1}	{F2}	{F1, F2}	{F1, F2, F3}
$m_{s1}(\cdot)$	0.8176	0.0003	0.1553	0.0268	0.6229	0.3771	0.3666	0.4563	0.1185	0.0586
$m_{s2}(\cdot)$	0.5658	0.0009	0.0646	0.3687	0.7660	0.2341	0.2793	0.4151	0.2652	0.0404
$m_{s3}(\cdot)$	0.2403	0.0004	0.0141	0.7452	0.8598	0.1402	0.2897	0.4331	0.2470	0.0302

TABLE 5: The negation of BPAs.

	Freq1				Freq2			Freq3		
	{F2}	{F3}	{F1, F2}	{F1, F2, F3}	{F2}	{F1, F2, F3}	{F1}	{F2}	{F1, F2}	{F1, F2, F3}
$m_{s1}(\cdot)$	0.0608	0.3332	0.2816	0.3244	0.3771	0.6229	0.2111	0.1812	0.2938	0.3138
$m_{s2}(\cdot)$	0.1447	0.3330	0.3118	0.2104	0.2341	0.7659	0.2402	0.1950	0.2449	0.3199
$m_{s3}(\cdot)$	0.2532	0.3332	0.3286	0.0849	0.1402	0.8598	0.2367	0.1890	0.2511	0.3232

TABLE 6: Weighted belief entropy of sensor reports under different frequencies.

$E_{Wd}(\cdot)$	Freq1	Freq2	Freq3
$E_{Wd}(m_{s1})$	2.3363	2.3509	2.3690
$E_{Wd}(m_{s2})$	2.0535	2.6083	2.3322
$E_{Wd}(m_{s3})$	1.5829	2.7336	2.3486

$$\begin{aligned}
w_{S_1} &= \frac{E_{Wd}(m_{s_1})}{\sum_{i=1}^3 E_{Wd}(m_{s_i})} = \frac{2.3363}{2.3363 + 2.0535 + 1.5829} = 0.3912, \\
w_{S_2} &= \frac{E_{Wd}(m_{s_2})}{\sum_{i=1}^3 E_{Wd}(m_{s_i})} = \frac{2.0535}{2.3363 + 2.0535 + 1.5829} = 0.3438, \\
w_{S_3} &= \frac{E_{Wd}(m_{s_3})}{\sum_{i=1}^3 E_{Wd}(m_{s_i})} = \frac{1.5829}{2.3363 + 2.0535 + 1.5829} = 0.2650.
\end{aligned} \tag{30}$$

The weight of different sensor reports under Freq2 and Freq3 is shown in Table 7.

Step 5. Calculate the modified mass functions.

With equation (21), the modified mass function for each judgement on fault types with respect to Freq1 can be calculated as follows:

$$\begin{aligned}
m_w(\{F2\}) &= \sum_{i=1}^3 w_{s_i} m_i(\{F2\}) = 0.3911 * 0.8176 + 0.3438 * 0.5658 + 0.2650 * 0.2403 = 0.5780, \\
m_w(\{F3\}) &= \sum_{i=1}^3 w_{s_i} m_i(\{F3\}) = 0.3911 * 0.0003 + 0.3438 * 0.0009 + 0.2650 * 0.0004 = 0.0005, \\
m_w(\{F1, F2\}) &= \sum_{i=1}^3 w_{s_i} m_i(\{F1, F2\}) = 0.3911 * 0.1553 + 0.3438 * 0.0646 + 0.2650 * 0.0141 = 0.0867, \\
m_w(\{F1, F2, F3\}) &= \sum_{i=1}^3 w_{s_i} m_i(\{F1, F2, F3\}) = 0.3911 * 0.0268 + 0.3438 * 0.3687 + 0.2650 * 0.7452 = 0.3347.
\end{aligned} \tag{31}$$

The modified mass function for Freq2 and Freq3 can also be calculated with equation (21), and the result is shown in Table 8.

Step 6. Combination of the modified BPAs.

With equation (22), for the vibration acceleration frequency of Freq1, the modified mass function will be fused with Dempster combination rule by 2 times, shown as follows:

$$\begin{aligned}
m(\{F2\}) &= ((m_w \oplus m_w)_1 \oplus m_w)_2(\{F2\}) = 0.9248, \\
m(\{F3\}) &= ((m_w \oplus m_w)_1 \oplus m_w)_2(\{F3\}) = 0.0002, \\
m(\{F1, F2\}) &= ((m_w \oplus m_w)_1 \oplus m_w)_2(\{F1, F2\}) = 0.0374, \\
m(\{F1, F2, F3\}) &= ((m_w \oplus m_w)_1 \oplus m_w)_2(\{F1, F2, F3\}) = 0.0376.
\end{aligned} \tag{32}$$

The fusion results for Freq2 and Freq3 are shown in Table 9.

TABLE 7: Weighted belief entropy of sensor reports under different frequencies.

ω_{Si}	Freq1	Freq2	Freq3
ω_{Si}	0.3911	0.3056	0.3360
ω_{Si}	0.3438	0.3391	0.3308
ω_{Si}	0.2650	0.3553	0.3331

TABLE 8: Modified mass function.

	Freq1				Freq2			Freq3		
	{F2}	{F3}	{F1, F2}	{F1, F2, F3}	{F2}	{F1, F2, F3}	{F1}	{F2}	{F1, F2}	{F1, F2, F3}
$m_w(-)$	0.5780	0.0005	0.0870	0.3347	0.7556	0.2444	0.3121	0.4349	0.2098	0.0431

TABLE 9: Sensor data fusion results for fault diagnosis.

	Freq1				Freq2			Freq3		
	{F2}	{F3}	{F1, F2}	{F1, F2, F3}	{F2}	{F1, F2, F3}	{F1}	{F2}	{F1, F2}	{F1, F2, F3}
Fusion result	0.9248	0.0002	0.0374	0.0376	0.9855	0.0146	0.3353	0.6316	0.0329	0.0002

TABLE 10: Sensor data fusion results of different methods.

	Freq1				Freq2			Freq3		
Method	{F2}	{F3}	{F1, F2}	{F1, F2, F3}	{F2}	{F1, F2, F3}	{F1}	{F2}	{F1, F2}	{F1, F2, F3}
Method in [62]	0.8861	0.0002	0.0582	0.0555	0.9621	0.0371	0.3384	0.5904	0.0651	0.0061
Proposed method	0.9248	0.0002	0.0374	0.0376	0.9855	0.0146	0.3353	0.6316	0.0329	0.0002

5.3. Discussion. According to Table 9, the result of fault diagnosis is that $F2$ is the recognized target. The conflict of sensor reports in the problem is overcome with the proposed method. For example, under Freq2, the belief on $F2$ is 0.8176, 0.5658, and 0.2403, respectively, while the fusion result is 0.9248. Moreover, the fusion result is in accordance with the method in [62] based on Table 10. Additionally, the proposed method has a higher belief degree on fault type $F2$ (92.48%) than Jiang et al.'s method, as shown in Table 10.

A few reasons that contribute to the superiority and effectiveness of the new data fusion approach can be summarized as follows. First of all, the sensor data are preprocessed properly before applying the combination rules. This is very critical in combining conflicting evidence. Secondly, the new method based on the negation of BPA and the weighted belief entropy is capable to address more uncertain information in the Dempster–Shafer evidence theory framework, which contributes to a more accurate experiment result in comparison with [62]. Finally, the advantages of Dempster combination rule, such as satisfying the commutativity and associativity, guarantee the rationality of the fusion result.

6. Conclusions

In this paper, by considering both the negation of BPA and the effect of the uncertainty of evidence as the weight, a novel method for data fusion is proposed. The proposed method consists of three main procedures. Firstly, the negation of the BPA is obtained, which aims to measure the uncertainty of BPA in a novel view. Secondly, the weighted evidence based

on weighted belief entropy is calculated for indicating the relative importance of evidence. Thirdly, based on the above two steps, the weighted evidence is obtained by computing the final weight of evidence. More importantly, the weighted evidence can be combined with the Dempster combination rule. Finally, a numerical example and an application on fault diagnosis are presented to demonstrate the superiority and effectiveness of the proposed method. Both numerical example and the application indicate that the method involving negation of BPA produces a more accurate sensor data fusion result than some existed methods by taking into consideration more uncertain information in the BOE. The proposed approach can be applied to solve data fusion problems in industrial applications.

Data Availability

All relevant data are within the paper.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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