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Research Article

Fixed Point of Almost Contraction in b-Metric Spaces

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In this paper, we introduce a generalized multivalued (α, L)-almost contraction in the b-metric space. Furthermore, we prove the existence and uniqueness of the fixed point for a specific mapping. The result presented in this paper extends some of the earlier results in the existing literature. Moreover, some examples are given to illuminate the usability of the obtained results.

1. Introduction and Preliminaries

The idea of b-metric was initiated from the works of Bourbaki [1] and Bakhtin [2]. A generalized form of the metric space is called a b-metric space. The concept of the b-metric space or metric type space was first introduced by Czerwik [3] as a generalization of the metric space. He provided an axiom which is weaker than the triangular inequality and formally defined a b-metric space with a view of generalizing the Banach contraction mapping principle. Later on, Fagin and Stockmeyer [4] worked on some kind of relaxation in the triangular inequality and called this new distance measure as nonlinear elastic matching (NEM). A similar type of relaxed triangle inequality was also used for trade measure [5] and to measure ice floes [6]. All these applications pushed us to introduce the concept of the b-metric space so that the results obtained for such rich spaces become more viable in different directions of applications. Since then, several authors proved fixed-point results of single-valued and multivalued operators in b-metric spaces [7–10]. Recently, Kamran et al. [11] worked on the b-metric space in which they discussed a generalization of the b-metric space called the extended b-metric space and established some fixed-point theorems for self-mappings defined on such spaces.

Definition 1 (see [3]). Let X be a nonempty set. A function $d\colon X \times X \longrightarrow [0, \infty]$ is said to be a b-metric if it satisfies the following conditions:

(1) $0 \le d(x, y)$ and $d(x, y) = 0$ if and only if $x = y$,
(2) $d(x, y) = d(y, x)$, and
(3) $d(x, z) \le s[d(x, y) + d(y, z)]$ for some $s \ge 1$,
for all $x, y, z \in X$. The pair (X, d) is called a b-metric space with coefficient s.

We observe that every metric space is a b-metric space with $s = 1$. The class of b-metric spaces is larger than the class of metric spaces, and the concept of the b-metric space coincides with the concept of the metric space. This is a weaker concept than that of a metric space. Conditions (1) and (2) are similar to the metric space, but (3) is a key feature of this concept. Therefore, it is important to study how to use (3) effectively. Each b-metric is not a continuous function. To show this statement, the following example was presented in [12].

Example 1. Let (X, d) be a metric space and $\sigma_d\colon X \times X \longrightarrow \mathbb{R}_+$ be defined by

$$\sigma_d(x, y) = [d(x, y)]^p, \tag{1}$$

for all $x, y \in X$, where $p > 1$ is a fixed real number. Then, σ_d is a b-metric with $s = 2^{p-1}$. Indeed, conditions (1) and (2) in Definition 1 are satisfied, and thus, we only have to show that condition (3) holds for σ_d.

It is easy to see that if $1 < p < \infty$, then the convexity of the function $f(x) = x^p$, where $x \geq 0$, implies

$$\left(\frac{a+c}{2}\right)^p \leq \frac{1}{2}\left(a^p + c^p\right), \tag{2}$$

and hence,

$$(a+c)^p \leq 2^{p-1}\left(a^p + c^p\right). \tag{3}$$

Therefore, for each x, y, $z \in X$, we get

$$\begin{aligned}\sigma_d(x,y) &= [d(x,y)]^p \leq [d(x,z) + d(z,y)]^p \\ &\leq 2^{p-1}\left(\left[d(x,z)^p + ((z,y))^p\right]\right) \\ &= 2^{p-1}\left[\sigma_d(x,z) + \sigma_d(z,y)\right].\end{aligned} \tag{4}$$

So, condition (3) in Definition 1 holds, and then σ_d is a b-metric space with coefficient $s = 2^{p-1}$.

Similarly, the concepts of b-convergent sequence, b-Cauchy sequence, and complete b-metric space are defined accordingly.

Definition 2 (see [11]). Let (X, d) be a b-metric space.

(1) A sequence $\{x_n\}$ in X is called b-convergent if there exists $x \in X$ such that $d(x_n, x) \longrightarrow 0$ as $n \longrightarrow \infty$. In this case, we write $\lim_{n\longrightarrow\infty} x_n = x$.

(2) A sequence $\{x_n\}$ in X is called b-Cauchy if $d(x_n, x_m) \longrightarrow 0$ as $n, m \longrightarrow \infty$.

(3) A b-metric space (X, d) is said to be a complete b-metric space if every b-Cauchy sequence in X is b-convergent.

1.1. Hausdorff Metric. The concept of Hausdorff metric or Hausdorff distance was first introduced by Hausdorff in his book *Grundzuge der Mengenlehre* [13]. The second name of Hausdorff distance is Pompeiu–Hausdorff distance. The Hausdorff distance has many applications in the computer field. The use of Hausdorff distance is to find a given template in an arbitrary target image in computer vision. The most important application of the Hausdorff metric in computer graphics is to measure the difference between two different representations of the same 3D object specifically when generating the level of detail for efficient display of complex 3D models.

Let (X, d) be a b-metric space and $\mathrm{CB}(X)$ denote the family of all nonempty, closed, and bounded subsets of X.

For $M, N \subset X$ and $a \in X$, the distance between a and N is defined as

$$d(a, N) = \inf\{d(a,b)\colon b \in N\}. \tag{5}$$

The diameter of M and N is defined as

$$\begin{aligned}\delta(M,N) &= \sup\{d(a,b)\colon a \in M, b \in N\}, \\ H(M,N) &= \max\left\{\sup_{a\in M} d(a,N), \sup_{b\in N} d(b,M)\right\},\end{aligned} \tag{6}$$

which is a Hausdorff metric on $\mathrm{CB}(X)$ induced by d. Then, $\mathrm{CB}(X)$ is a b-metric space under the Hausdorff distance H.

If (X, d) is a complete b-metric space, then $(\mathrm{CB}(X), H)$ is a complete b-metric space, too. If M and N have the same closures, then $H = 0$.

1.2. (α, L)-Almost Contraction Mappings. Berinde [14] extended the notion of almost contraction from single-valued mappings to multivalued mappings. There are many examples of contractive conditions which imply the almost contractiveness condition, for instance, Mizoguchi and Takahashi [15] used the concept of almost contraction in their work.

Theorem 1 (see [16]). *Let (X, d) be a complete metric space and $T\colon X \longrightarrow CB(X)$ be a generalized multivalued (α, L)-almost contraction, i.e., a mapping for which there exists a function $\alpha\colon [0,\infty] \longrightarrow [0,1]$ satisfying*

$$\lim_{r\longrightarrow t^+} \sup \alpha(r) < 1, \tag{7}$$

for every $t \in (0,\infty)$, such that

$$H\left(T_x, T_y\right) \leq \alpha(d(x,y))d(x,y) + LN, \tag{8}$$

where

$$N = \min\left\{d\left(x, T_x\right), d\left(y, T_y\right), d\left(x, T_y\right), d\left(y, T_x\right)\right\}, \tag{9}$$

for all $x, y \in X$ and $T_x, T_y \in CB(X)$. Then, T has a fixed point.

Lemma 1 (see [4]). *Let (X, d) be a complete b-metric space with coefficient s and $T\colon X \longrightarrow K(X)$ be a mapping, where $K(X)$ is a nonempty compact subset of X. Assume that there exists a function $\alpha\colon (0,\infty) \longrightarrow [0,1]$ such that, for each $t \in (0,\infty)$,*

$$\begin{aligned}&\lim_{r\longrightarrow t^+} \sup \alpha(r) < 1, \\ &H\left(T_x, T_y\right) \leq \alpha(d(x,y))d(x,y), \quad \forall x, y \in X, x \neq y.\end{aligned} \tag{10}$$

Then, T has a fixed point.

Lemma 2 (see [15]). *Let (X, d) be a complete b-metric space and $T\colon X \longrightarrow CB(X)$ be a mapping. Assume that there exists a function $\alpha\colon (0,\infty) \longrightarrow [0,1]$ such that, for each $t \in (0,\infty)$,*

$$\begin{aligned}&\lim_{r\longrightarrow t^+} \sup \alpha(r) < 1, \\ &H\left(T_x, T_y\right) \leq \alpha(d(x,y))d(x,y), \forall x, y \in X, x \neq y.\end{aligned} \tag{11}$$

Then, T has a fixed point.

Lemma 3 (see [17]). *Let (X,d) be a b-metric space. Let $A,B\subset X$ and $q>1$. Then, for every $x\in A$, there exists $y\in B$ such that*

$$d(x,y)\le qH(A,B). \tag{12}$$

2. Generalized Multivalued Almost Contraction

In this section, we present and prove our main results on the existence of fixed points for (α,L)-almost contractions in the b-metric space. The list of following results is reestablished as the extension of the multivalued almost contraction in the b-metric space.

Lemma 4. *Let (X,d) be a b-metric space and $T\colon X\longrightarrow C(X)$ be a mapping. Then, for every $x\in X$ with $d(x,Tx)>0$ and any $b\in(0,1)$, there exists $y\in Tx$ $(y\ne x)$ such that*

$$b^s d(x,y)\le b\, d(x,T_x). \tag{13}$$

Lemma 5. *Let (X,d) be a b-metric space with $s\ge 1$ and $T\colon X\longrightarrow C(X)$ be a mapping such that the following conditions hold:*

(1) The function $f\colon X\longrightarrow\mathbb{R}$, $f(x)=d(x,T_x)$, is lower semicontinuous.

(2) There exist $b\in(0,1)$ and $\alpha\colon(0,\infty)\longrightarrow[0,1]$ such that, for all $t\in(0,\infty)$,

$$\lim_{r\longrightarrow t^+}\sup\alpha(r)<1. \tag{14}$$

If for all $x\in X$, there exists $y\in I_b^x=\{y\in T_x\colon b\, d(x,y)\le d(x,T_x)\}$ satisfying

$$b^s d(y,T_y)\le b\alpha(d(x,y))d(x,y), \tag{15}$$

then T has a fixed point.

Lemma 6. *Let (X,d) be a complete b-metric space with coefficient $s\ge 1$, and assume that there exist a constant $b\in(0,1)$ and a set-valued α-contraction $T\colon X\longrightarrow CB(X)$, i.e., a mapping for which there exists a constant $\alpha\in(0,\infty)$ such that*

$$b^s H(T_x,T_y)\le\alpha(d(x,y)),\quad \forall x,y\in X. \tag{16}$$

Then, T has a fixed point.

2.1. (α,L)-Almost Contraction Mappings in the b-Metric Space. Let (X,d) be a complete b-metric space with $s\ge 1$ and $T\colon X\longrightarrow \mathrm{CB}(X)$ be a generalized multivalued (α, L)-almost contraction, i.e., a mapping for which there exist a function $\alpha\colon[0,\infty]\longrightarrow[0,1]$ and a constant $L>0$ and $b\in(0,1)$ such that

$$b^s H(T_x,T_y)\le\alpha(d(x,y))d(x,y)+LN, \tag{17}$$

where

$$N=\min\{d(x,T_x),d(y,T_y),d(x,T_y),d(y,T_x)\}, \tag{18}$$

for all $x,y\in X$ and $T_x,T_y\in\mathrm{CB}(X)$.

Theorem 2. *Let (X,d) be a b-metric space with $s\ge 1$ and $T\colon X\longrightarrow C(X)$ satisfy the following conditions:*

(1) The function $f\colon X\longrightarrow\mathbb{R}$, $f(x)=d(x,T_x)$, $x\in X$, is lower semicontinuous.

(2) There exist $L\ge 0, b\in(0,1)$ and $\alpha\colon(0,\infty)\longrightarrow[0,b]$ such that, for all $t\in(0,\infty)$,

$$\lim_{r\longrightarrow t^+}\sup\alpha(r)<b. \tag{19}$$

If for all $x\in X$, there exists $y\in I_b^x=\{y\in T_x\colon b\, d(x,y)\le d(x,T_x)\}$ satisfying

$$b^s d(y,T_y)\le\alpha(d(x,y))d(x,y)+LN, \tag{20}$$

where

$$N=\min\{d(x,T_x),d(y,T_y),d(x,T_y),d(y,T_x)\}, \tag{21}$$

for all $x,y\in X$, then T has a fixed point.

Proof. The main idea of the proof begins with the concept of the Banach contraction principle in which we take a Cauchy sequence in a complete b-metric space. Every Cauchy sequence is convergent in a complete b-metric space, and the converging point of that sequence is proved to be a fixed point of contraction.

If there exists $x\in X$ such that $d(x,T_x)=0$, then $x\in T_x$, i.e., x is a fixed point of T. Since the range of T is closed, for each $b\in(0,1)$ and any $x\in X$ with $d(x,T_x)>0$, there exists $y\in T_x$ such that $y\in I_b^x$, that is,

$$b^s d(x,y)\le b\, d(x,T_x). \tag{22}$$

So, we can assume that we have $y\in I_b^x, y\ne x$; otherwise, $y=x\in T_x$ will be a fixed point of T, and the proof is done.

Let $x_1\in X$ be arbitrary but fixed with $d(x_1,T_{x_1})>0$. By (16), there exists $x_2\in T_{x_1}, x_2\ne x_1$, satisfying the inequality

$$b^s d(x_1,x_2)\le b\, d(x_1,T_{x_1}). \tag{23}$$

By (22),

$$b^s d(x_2,T_{x_2})\le\alpha(d(x_1,x_2))d(x_1,x_2), \tag{24}$$

$$<b\, d(x_1,x_2). \tag{25}$$

Since $d(x_2,T_{x_1})=0$, by (23) and (24), we have

$$\begin{aligned} b\, d(x_1,T_{x_1})-b^s d(x_2,T_{x_2})&\ge b^s d(x_1,x_2)\\ &\quad-\alpha(d(x_1,x_2))d(x_1,x_2)\\ &=[b^s-\alpha(d(x_1,x_2))]d((x_1,x_2)>0. \end{aligned} \tag{26}$$

Now, for x_2, there exists $x_3\in T_{x_2}, x_3\ne x_2$, satisfying

$$b^s d(x_2, x_3) \le b\ d(x_2, T_{x_2}), \tag{27}$$

$$b^s d(x_3, T_{x_3}) \le \alpha(d(x_2, x_3)) d(x_2, x_3) < b\ d(x_2, x_3). \tag{28}$$

By (27) and (28), we get

$$\begin{aligned} b\ d(x_2, T_{x_2}) - b^s d(x_3, T_{x_3}) &\ge b^s d(x_2, x_3) - \alpha(d(x_2, x_3)) d(x_2, x_3) \\ &= [b^s - \alpha\ d(x_2, x_3)] d(x_2, x_3) > 0. \end{aligned} \tag{29}$$

Using (27) and (28), we obtain

$$\begin{aligned} d(x_2, x_3) &\le \frac{b}{b^s} d(x_2, T_{x_2}) \\ &\le \frac{b}{b^s} \alpha(d(x_1, x_2)) d(x_1, x_2) < d(x_1, x_2). \end{aligned} \tag{30}$$

By induction, for x_n, $n > 1$, obtained in the previous way, there exists $x_{n+1} \in T_{x_n}$, $x_n \ne x_{n+1}$, such that

$$b^s d(x_n, x_{n+1}) \le b\ d(x_n, x_{n+1}), \tag{31}$$

$$b^s d(x_{n+1}, T_{x_{n+1}}) \le \alpha(d(x_n, x_{n+1})) d(x_n, x_{n+1}) < b\ d(x_n, x_{n+1}). \tag{32}$$

By the above pattern, we get

$$\begin{aligned} b\ d(x_n, T_{x_n}) - b^s d(x_{n+1}, T_{x_{n+1}}) &\ge b^s d(x_n, x_{n+1}) - \alpha(d(x_n, x_{n+1})) d(x_n, x_{n+1}) \\ &= [b^s - \alpha(d(x_n, x_{n+1}))] d(x_n, x_{n+1}) > 0, \end{aligned} \tag{33}$$

and we know that, in a complete b-metric space, every Cauchy sequence is convergent, and so,

$$d(x_n, x_{n+1}) < d(x_n, x_{n-1}). \tag{34}$$

From (33) and (34), we can see that $d(x_n, T_{x_n})$ and $d(x_n, x_{n+1})$ are convergent and decreasing sequences, which are positive numbers. From assumption (2) in the theorem, it follows that there exists $\theta \in [0, b]$ such that

$$\lim_{n\longrightarrow\infty} \sup \alpha(d(x_n, x_{n+1})) = \theta. \tag{35}$$

Then, for any $b_0 \in (0, b)$, there exists $n_0 \in \mathbb{N}$ such that

$$\alpha(d(x_n, x_{n+1})) < b_0, \quad \forall n > n_0. \tag{36}$$

By using (33) and by taking $a = b - b_0$, we get

$$d(x_n, T_{x_n}) - d(x_{n+1}, T_{x_{n+1}}) \ge a\ d(x_n, x_{n+1}), \quad \forall n > n_0. \tag{37}$$

By (31) and (32) for any $n > n_0$, we get

$$\begin{aligned} d(x_{n+1}, T_{x_{n+1}}) &\le \alpha(d(x_n, x_{n+1})) \\ &\le \frac{\alpha(d(x_n, x_{n+1}))}{b} d(x_n, T_{x_n}) \\ &\le \frac{\alpha(d(x_n, x_{n+1})), \ldots, \alpha(d(x_1, x_2))}{b^n} d(x_1, T_{x_1}) \\ &= \frac{\alpha(d(x_n, x_{n+1})), \ldots, \alpha(d(x_{n_{0+1}}, x_{n_{0+2}}))}{b^{n-n_0}} \frac{\alpha(d(x_{n_0}, x_{n_{0+1}})), \ldots, \alpha(d(x_1, x_2))}{b^{n_0}} d(x_1, T_{x_1}) \\ &< \left(\frac{b_0}{b}\right)^{n-n_0} \frac{\alpha(d(x_{n_0}, x_{n_{0+1}})), \ldots, \alpha(d(x_1, x_2))}{b^{n_0}} d(x_1, T_{x_1}). \end{aligned} \tag{38}$$

Since $b_0 < b$, we have

$$\lim_{n\longrightarrow\infty} \left(\frac{b_0}{b}\right)^{n-n_0} = 0. \tag{39}$$

Thus,

$$\lim_{n\longrightarrow\infty} d(x_n, T_{x_n}) = 0. \tag{40}$$

Now, we have to show that $\{x_n\}$ is a Cauchy sequence. For this, using condition (3) in Definition 1 and (37), for $n, p \in \mathbb{N}$, $n, p > n_0$, we have

$$\begin{aligned} d(x_n, x_{n+p}) &\le s\left[d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+p})\right] \\ &\le s\, d(x_n, x_{n+1}) + s^2 d(x_{n+1}, x_{n+2}) + s^2 d(x_{n+2}, x_{n+p}) \\ &\vdots \\ &\le s\, d(x_n, x_{n+1}) + s^2 d(x_{n+1}, x_{n+2}) +, \cdots, + s^p d(x_{n+p-1}, x_{n+p}) \\ &\le s\frac{1}{a}\left[d(x_n, x_{n+1}) - d(x_{n+1}, x_{n+2})\right] + s^2\frac{1}{a}\left[d(x_{n+1}, x_{n+2}) - d(x_{n+2}, x_{n+3})\right] +, \ldots, \\ &\quad + s^p\frac{1}{a}\left[d(x_{n+p-2}, x_{n+p-1}) - d(x_{n+p-1}, x_{n+p})\right] \\ &= \frac{s}{a} d(x_n, x_{n+1}) + \frac{s(s-1)}{a} d(x_{n+1}, x_{n+2}) +, \cdots, + \frac{s^{n+p-1}(s-1)}{a} d(x_{n+p-1}, x_{n+p}) \xrightarrow{n}, p \longrightarrow \infty\ 0. \end{aligned} \tag{41}$$

Hence, $\{x_n\}$ is a Cauchy sequence. Since (X, d) is complete, $\{x_n\}$ is a convergent sequence in the b-metric space (X, d). By using the definition of the convergent sequence, there exists $x^* \in X$ such that

$$\lim_{n\longrightarrow\infty} x_n = x^*. \tag{42}$$

As a result, we get the following:

$$\begin{aligned} 0 \le d(x^*, T_{x^*}) &\le \lim_{n\longrightarrow\infty}\inf\, d(x_n, T_{x_n}) \\ &= \lim_{n\longrightarrow\infty} d(x_n, T_{x_n}) = 0. \end{aligned} \tag{43}$$

By the closeness property of T_{x^*}, $x^* \in T_{x^*}$, which is the definition of the fixed point, and so, x^* is a fixed point.

To give the relation between our main result and works of Berinde, Nadler, and Mizoguchi [4, 15, 18–20], the following examples are provided.

Example 2. Let $X = \{0, 1, 2, 4, 5, 7, 9\}$ be a finite set, a b-metric d be defined on set X by $d(x, y) = (x - y)^2$ with coefficient $s = 2$, and $C(X) = \{0, 2, 4, 5, 7, 9\}$ be the closed set of X. The mapping $T\colon X \longrightarrow C(X)$ is piecewisely defined by

$$T(x) = \begin{cases} 9 - x, & \text{if } x \in \{2, 4, 7, 9\} \\ x + 4, & \text{if } x \in \{0, 1, 5\}. \end{cases} \tag{44}$$

The values of x and y are 0 and 2, whereas $b = 0.2$. The function α is defined as $\alpha(t) = t/1 + t$.

By taking the notion of Theorem 2 under consideration, Example 2 is elaborated. To choose the value of y,

$$y \in I_b^x = \{y \in T_x\colon b\ d(x, y) \le d(x, T_x)\}. \tag{45}$$

For $x = 0$, $T(x) = 4$, and so,

$$\begin{aligned} (0.2)d(0, 2) &\le d(0, 4), \\ (0.2)(4) &\le 16, \\ 0.8 &\le 16. \end{aligned} \tag{46}$$

Thus, $y = 2$. Now, by substituting values, we get

$$b^S d(y, Ty) \le \alpha(d(x, y))d(x, y) + LN, \tag{47}$$

where

$$N = \min\{d(x, Tx), d(y, Ty), d(x, Ty), d(y, Tx)\}. \tag{48}$$

After substitution, we get

$$1 \le 3.6. \tag{49}$$

Thus, we get the desired result.

The concept that "every metric space is a b-metric space with $s = 1$" can be explained by the following example. The values of x, y, α, and b are also changed so that the concept of the theorem can be cleared in a more effective way. In the following example, the b-metric space is defined on an infinite set which is a generalization of the previous example.

Example 3. Let $X = \mathbb{R}$ and a b-metric d on X be defined by $d(x, y) = \sqrt{|x - y|}$. Note that $d(x, y)$ is a b-metric with coefficient $s = 1$. The closed set of X is taken to be $\mathbb{R}$ itself. The mapping $T\colon X \longrightarrow C(X)$ is defined as $T(x) = x/2$ and $\alpha(t) = 1 + t/2$. The values of x, y, and b are 4, 3, and 0.1, respectively. Since

$$\begin{aligned} x = 4, T(x) &= \frac{4}{2} = 2, \\ y \in I_b^x &= \{y \in T_x\colon b\ d(x, y) \le d(x, T_x)\}, \end{aligned} \tag{50}$$

we have

$$\begin{aligned}(0.1)\, d\,(4,3) &\leq d\,(4,2),\\ 0.1 &\leq 1.4142.\end{aligned} \tag{51}$$

Thus, $y = 3$. By substituting values, we get

$$b^s d(y, Ty) \leq \alpha(d(x, y))d(x, y) + LN, \tag{52}$$

where

$$N = \min\{d(x, Tx), d(y, Ty), d(x, Ty), d(y, Tx)\}. \tag{53}$$

After substitution, we obtain

$$0.12 \leq 1. \tag{54}$$

So, this is the desired result.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest concerning the publication of this article.

Authors' Contributions

The authors equally conceived the study, participated in its design and coordination, drafted the manuscript, participated in the sequence alignment, and read and approved the final manuscript.

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