

## Research Article

# A Differential Game of Industrial Pollution Management considering Public Participation

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In recent years, with the rapid development of economy, industrial pollution problems have become more and more serious. In this paper, a differential game model is proposed for industrial pollution management, in which public participation is taken into consideration. Then, a feedback Nash equilibrium (FBNE) solution is obtained among the government, enterprises, and the public. Finally, a numerical example is given to illustrate the results. The results show that the public participation will take a positive part in forcing enterprises to reduce emissions. Furthermore, with the increase of the probability of the public reporting the illegal discharge of pollutants by enterprises, the probability of enterprises' active emission reduction will also greatly increase

## 1. Introduction

Recently, owing to the rapid development of economy, environmental pollution has become more and more serious. One of the main sources of pollution is industrial pollution. So, how to effectively control industrial pollution has become an urgent problem to be solved. Therefore, many scholars began to study industrial pollution by using different approaches. In one branch, control theory is employed [1–5]. For example, Plourde and Yeung [4] proceeded to analyze properties of a policy for efficient social management of competitive firms which generate pollution. Then, they established a model of industrial pollution in a stochastic environment. Van and De [5] argued that, if the precommitment was absent, it would lead to lower emission charges, less cleaning-up activities, and more pollution. Lin [2] developed an optimal control model, which considered the firm's spending, employment, and investment. Forster [1] discussed the central problems of industrial pollution and optimal economic activity. Lusky [3] developed a recycling model which both included the effect of the recycled good and the allocation of resources. In another branch, game theory is applied [6–8]. For example, Misiolek [6] provided a model considered the effect of rent seeking costs on the

design of an efficient pollution tax. Yao [7] developed a two-period model which examined the dynamics of standard-setting regulation. Milliman and Prince [8] argued that innovation, diffusion, and optimal agency response were three processes of technological changes in pollution control. However, as Plourde and Yeung [4] illustrated, there were some properties of industrial pollution which made the analysis complicated and difficult. For example, the first property was the intertemporal or dynamic nature of the industrial pollution. A second property of industrial pollution was the uncertain nature of its generation and effects. But, the above researches are mainly in a (static) principal-agent framework; obviously, they ignore the fact that the analysis of industrial pollution management is complicated and difficult.

In recent years, differential game has been flourishing and applied to a large array of topics in industrial organization, labor economics, oligopoly theory, marketing, production and operations management, microeconomics, macroeconomics, innovation and R&D, and so forth. Besides, it has been widely used in pollution control problems. For example, Yeung [9] studied a differential game between the government and a profit maximizing entrepreneur in which production generates pollution for the first time.

Yeung [10] extended industrial pollution into transboundary industrial pollution, so the first study of transboundary industrial pollution management in a stochastic game framework was derived. Li [11] extended the work proposed by Yeung [10]. He added the emission permits trading to the model. Huang et al. [12] provided transboundary pollution problem with a differential game, in which both global impact and regional negative impact of pollution are taken into account. For more literatures on differential game for industrial pollution, see [13–23]. However, all these studies focused on the relationships between the government and enterprises but ignored the importance of public participation. Many studies have shown that the public plays an important role in the process of environmental pollution control [24, 25]. Especially, Peel [24] illustrated that public participation is a necessary prerequisite for effective environmental governance.

Though there are some researches about industrial pollution control under the framework of differential games, most of them mainly focused on the relationships between the government and the enterprises. To the best of our knowledge, there are few research studies on industrial pollution management considering public participation under the differential games model. Therefore, the purpose of this paper is to discuss industrial pollution management problems based on differential games. Our contributions can be summarized as follows: (1) we propose a differential game model for industrial pollution, in which public participation is taken into account. (2) We derive a FBNE solution among the government, enterprises, and the public.

The rest of this paper is organized as follows. In Section 2, we describe in detail about the differential games model. In Section 3, we obtain a FBNE solution. In Section 4, we discuss the results with a numerical example. In Section 5, we explore the discussion of the paper. Finally, in Section 6, we present the conclusions and future work of this paper

## 2. Game Formulation

In this section, a differential game of industrial pollution control is presented, where there is a region and a local government,  $n$  asymmetric enterprises, and the public. Further, the differential game model is described in detail as follows.

**2.1. Industrial Sector.** Consider a region comprised of  $n$  enterprises. At time instant  $s$ , following Yeung's [10] work, we define the demand function of the output of enterprise  $i \in N \equiv \{1, 2, \dots, n\}$  as follows:

$$P_i(s) = \alpha^i - \beta^i q_i(s), \quad (1)$$

where  $P_i(s)$  is the price of the output of enterprise  $i$  and  $q_i(s)$  is the output of enterprise  $i$ . Moreover,  $\alpha^i$  and  $\beta^i$  are positive constants with  $i \in N \equiv \{1, 2, \dots, n\}$ . To simplify the analysis, we assume that the output of enterprises is homogeneous. In this case, let  $\alpha^i = \alpha^j = \alpha$  and  $\beta^i = \beta^j = \beta$  for all  $i \in N \equiv \{1, 2, \dots, n\}$ .

In order to deal with polluted problems, the government generates some contaminated standards for businesses, especially to limit the discharging of pollutants. To illustrate,  $n_1$  is the emission standard set by the government when the enterprise exceeds the emission limit, and  $m$  is the extra income per unit. To make it clear, we assume that all enterprises will be supervised by the public officials. However, if the amount of the discharge exceeds the limits, the companies will be punished by the government. We assume that the formula demonstrates the probability of report which is  $f$ , where  $0 \leq f \leq 1$ .  $n_2$  is the unit penalty value that the government imposes on enterprises for excessive discharge of pollutants.

The following two cases for industrial profits of enterprises are discussed:

- (1) If the emission exceeds the limit, the industrial profits of enterprise  $i$  at time  $s$  can be expressed as

$$J^E = \int_{t_0}^t \left\{ [\alpha - \beta q_i(s)] q_i(s) + m[q_i(s) - av_i(s) - n_1] - \frac{c_i}{2} [q_i(s)]^2 - \frac{c_i^a}{2} [av_i(s)]^2 - R[q_i(s) - av_i(s)] - fn_2 [q_i(s) - av_i(s) - n_1] \right\} e^{-r(t-t_0)} ds, \quad (2)$$

where  $[\alpha - \beta q_i(s)] q_i(s)$  is the income of the enterprise  $i$ ,  $m[q_i(s) - av_i(s) - n_1]$  is the total benefit when enterprise  $i$  exceeds the emission of pollutants,  $c_i$  is a positive cost parameter,  $c_i^a$  is a positive cost of emission reduction, and  $R$  is the tax imposed on industrial output by the government. In model (2),  $a$  is the utility coefficient of emission reduction and  $v_i(s)$  is the pollutant produced by enterprise  $i$ . Then,  $av_i(s)$  is the reduced pollutant emission through

emission reduction technology by enterprise  $i$ . We suppose that the public will report to the government if the enterprise exceeds the emission standard. So, the enterprise will be punished by the government, the penalty value is  $fn_2 [q_i(s) - av_i(s) - n_1]$ .

- (2) If the emission does not exceed the standard, industrial profits of enterprise  $i$  at time  $s$  can be expressed as

$$J^E = \int_{t_0}^t \left\{ [\alpha - \beta q_i(s)]q_i(s) - \frac{c_i}{2}[q_i(s)]^2 - \frac{c_i^a}{2}[av_i(s)]^2 - R[q_i(s) - av_i(s)] \right\} e^{-r(t-t_0)} ds. \tag{3}$$

To make it more general in this paper, we suppose that the enterprise always exceeds the emission limit.

**2.2. Pollution Dynamics.** The enterprise emits pollutants into the environment, and the amount of pollution created by different enterprises' output may vary from each other. Thus, the government adopts its pollution abatement policy to reduce pollutants existed in the environment, while each enterprise adopts its own pollution abatement technology to reduce pollutants emission. Therefore, let  $x(s) \in R^+$  denote the level of pollution at time  $s$ . For simplicity, we assume that one unit of product produces one unit of pollutant. In this case,  $q_i(s)$  also denotes the amount of pollutants produced by enterprise  $i$ . So, the dynamics of pollution stock is governed by the differential equation

$$\dot{x}(s) = \sum_{i=1}^n [q_i(s) - av_i(s)] - b\mu(s) - \delta x(s), \quad x(t_0) = x_0, \tag{4}$$

where  $q_i(s) - av_i(s)$  is the pollutants discharged by enterprises,  $b\mu(s)$  is the amount of pollution removed by the government,  $\delta$  is the natural rate of decay of the pollutants, and the initial level of pollution at time  $t_0$  is given as  $x_0$ .

**2.3. The Government Objectives.** The government needs to adopt its pollution abatement policy to reduce pollutants. The income includes gains of the entrepreneurs' production, the tax revenue, and the penalty value for the enterprises. Moreover, while the government is actively trying to reduce emissions, it needs to pay a certain cost and will suffer the environment damage. Besides, we suppose that the government will reward the public if they timely report the enterprise's excessive emission behavior.

We assume that the discount is  $r$ . So, we can express the problem as a differential game in which the government attempts to obtain

$$J^G = \int_{t_0}^t \left\{ \sum_{i=1}^n \left\{ [\alpha - \beta q_i(s)]q_i(s) - \frac{c_i}{2}[q_i(s)]^2 \right\} + \sum_{i=1}^n \{R[q_i(s) - av_i(s)]\} + \sum_{i=1}^n \{fn_2[q_i(s) - av_i(s) - n_1]\} - d_1x(s) - fR_2 - \frac{c_i^u}{2}[b\mu(s)]^2 \right\} e^{-r(s-t_0)} ds, \tag{5}$$

where  $\sum_{i=1}^n \{[\alpha - \beta q_i(s)]q_i(s) - (c_i/2)[q_i(s)]^2\}$  denotes the gains of the entrepreneur's production,  $\sum_{i=1}^n \{R[q_i(s) - av_i(s)]\}$  is the tax revenue of the government,  $\sum_{i=1}^n \{fn_2[q_i(s) - av_i(s) - n_1]\}$  is the sum of the penalty value to the enterprises,  $fR_2$  is the government's reward to the public, and  $(c_i^u/2)[b\mu(s)]^2$  is the cost to the government of reducing emissions.

**2.4. The Public Objectives.** The public which obtain rewards from the government and at the same time suffer from industrial pollution attempt to maximize the payoff function

$$J^P = \int_{t_0}^t \left\{ \lambda \sum_{i=1}^n \left\{ [\alpha - \beta q_i(s)]q_i(s) - \frac{c_i}{2}[q_i(s)]^2 \right\} + fR_2 - d_2x(s) \right\} e^{-r(s-t_0)} ds, \tag{6}$$

where  $\lambda$  denotes the conversion rate of social welfare and  $d_2x(s)$  is the damage caused by industrial pollution to the public.

### 3. Feedback Nash Equilibrium Solution

In this section, a set of feedback strategies  $[v_i^*(s), \mu^*(s), q_i^*(s)]$  provides a Nash equilibrium solution to the game (2) and (4)–(6) if there exist a set of bounded continuous differentiable value functions  $V_i(x): [t_0, t] \times R \rightarrow R, i = 1, 2, 3$ , satisfying the Hamilton–Jacobi–Bellman (HJB) equations:

$$rV_1(x) = \max_{v_i(s)} \left\{ [\alpha - \beta q_i(s)]q_i(s) + m[q_i(s) - av_i(s) - n_1] - \frac{c_i}{2}[q_i(s)]^2 - \frac{c_i^a}{2}[av_i(s)]^2 \right. \\ \left. - R[q_i(s) - av_i(s)] - fn_2[q_i(s) - av_i(s) - n_1] + V_1'(x) \left\{ \sum_{i=1}^n [q_i(s) - av_i(s)] - b\mu(s) - \delta x(s) \right\} \right\}, \quad (7)$$

$$rV_2(x) = \max_{\mu(s)} \left\{ \sum_{i=1}^n \left\{ [\alpha - \beta q_i(s)]q_i(s) - \frac{c_i}{2}[q_i(s)]^2 \right\} + \sum_{i=1}^n \{R[q_i(s) - av_i(s)]\} + \sum_{i=1}^n \{fn_2[q_i(s) - av_i(s) - n_1]\} - d_1x(s) \right. \\ \left. - fR_2 - \frac{c_i^u}{2}[b\mu(s)]^2 + V_2'(x) \left\{ \sum_{i=1}^n [q_i(s) - av_i(s)] - b\mu(s) - \delta x(s) \right\} \right\}, \quad (8)$$

$$rV_3(x) = \max_{q_i(s)} \left\{ \lambda \sum_{i=1}^n \left\{ [\alpha - \beta q_i(s)]q_i(s) - \frac{c_i}{2}[q_i(s)]^2 \right\} + fR_2 \right. \\ \left. - d_2x(s) + V_3'(x) \left\{ \sum_{i=1}^n [q_i(s) - av_i(s)] - b\mu(s) - \delta x(s) \right\} \right\}. \quad (9)$$

Performing the indicated maximization in (7)–(9) yields

$$\begin{cases} -ma + Ra - a^2c_i^a v_i(s) + afn_2 - naV_1'(x) = 0, \\ -c_i^u b^2 \mu(s) - bV_2'(x) = 0, \\ \lambda[n\alpha - 2n\beta q_i(s)] - c_i q_i(s) + nV_3'(x) = 0, \end{cases} \quad (10)$$

for  $i \in N$ .

To get linear value functions in (11), we set  $V_i(s) = A_i + B_i x(s)$ ,  $i = 1, 2, 3$ , where  $A_i$  and  $B_i$  are constants. Then, we can obtain

$$\begin{cases} V_1'(x) = B_1, \\ V_2'(x) = B_2, \\ V_3'(x) = B_3. \end{cases} \quad (12)$$

Solving (10) yields

$$\begin{cases} v_i^*(s) = -\frac{nV_1'(x) - fn_2 + m - R}{ac_i^a}, \\ \mu^*(s) = -\frac{V_2'(x)}{c_i^u b}, \\ q_i^*(s) = \frac{\lambda n\alpha + nV_3'(x)}{2n\beta\lambda + c_i}. \end{cases} \quad (11)$$

Substituting the results in (12) into (7)–(9), we obtain

$$r[A_1 + B_1 x(s)] = \max_{v_i(s)} \left\{ [\alpha - \beta q_i(s)]q_i(s) + m[q_i(s) - av_i(s) - n_1] - \frac{c_i}{2}[q_i(s)]^2 \right. \\ \left. - \frac{c_i^a}{2}[av_i(s)]^2 - R[q_i(s) - av_i(s)] - fn_2[q_i(s) - av_i(s) - n_1] + B_1 \left\{ \sum_{i=1}^n [q_i(s) - av_i(s)] - b\mu(s) - \delta x(s) \right\} \right\}, \quad (13)$$

$$r[A_2 + B_2 x(s)] = \max_{\mu(s)} \left\{ \sum_{i=1}^n \left\{ [\alpha - \beta q_i(s)]q_i(s) - \frac{c_i}{2}[q_i(s)]^2 \right\} + \sum_{i=1}^n \{R[q_i(s) - av_i(s)]\} \right\} \\ + \sum_{i=1}^n \{fn_2[q_i(s) - av_i(s) - n_1]\} - d_1x(s) - fR_2 - \frac{c_i^u}{2}[b\mu(s)]^2 + B_2 \left\{ \sum_{i=1}^n [q_i(s) - av_i(s)] - b\mu(s) - \delta x(s) \right\}, \quad (14)$$

$$r[A_3 + B_3x(s)] = \max_{q_i(s)} \left\{ \lambda \sum_{i=1}^n \left\{ [\alpha - \beta q_i(s)]q_i(s) - \frac{c_i}{2}[q_i(s)]^2 \right\} + fR_2 - d_2x(s) + B_3 \left\{ \sum_{i=1}^n [q_i(s) - av_i(s)] - b\mu(s) - \delta x(s) \right\} \right\}. \tag{15}$$

Solving (13)—(15) yields

$$\begin{cases} B_1 = 0, \\ B_2 = -\frac{d_1}{r + \delta}, \\ B_3 = -\frac{d_2}{r + \delta}. \end{cases} \tag{16}$$

Substituting the results in (12) and (16) into (11), we obtain a set of feedback strategies  $[v_i^*(s), \mu^*(s), q_i^*(s)]$  as

$$\begin{cases} v_i^*(s) = \frac{fn_2 + R - m}{ac_i^a}, \\ \mu^*(s) = \frac{d_1}{c_i^\mu b(r + \delta)}, \\ q_i^*(s) = \frac{\lambda n\alpha - (d_2n/r + \delta)}{2n\beta\lambda + c_i}. \end{cases} \tag{17}$$

According to the results in (17), we can get several propositions as follows.

**Proposition 1.**  $v_i^*(s)$  is positively related to  $f, n_2$ , and  $R$ .

*Proof.* According to the positive and negative of the above symbols, we can obtain that

$$\begin{aligned} \frac{\partial v_i^*(s)}{\partial f} &= \frac{n_2}{ac_i^a} > 0, \\ \frac{\partial v_i^*(s)}{\partial n_2} &= \frac{f}{ac_i^a} > 0, \\ \frac{\partial v_i^*(s)}{\partial R} &= \frac{1}{ac_i^a} > 0. \end{aligned} \tag{18}$$

The intuition behind Proposition 1 is straightforward. First, if the unit penalty value and the tax rise, the companies would try as much as possible to reduce the polluted discharges. Second, the public will take a positive role in forcing enterprises to reduce emissions. Thus, with the increase of the probability that the public reports the illegal enterprises, the probability of enterprises' emission reduction will be gradually raised.

**Proposition 2.**  $v_i^*(s)$  is positively related to  $a$  and  $c_i^a$  if  $fn_2 + R - m < 0$ ; it is negatively related to  $a$  and  $c_i^a$  if  $fn_2 + R - m > 0$ .

*Proof.* According to the positive and negative of the above symbols, we can obtain that

$$\begin{aligned} \frac{\partial v_i^*(s)}{\partial a} &= \frac{fn_2 + R - m}{a^2c_i^a} > 0, \quad \text{if } fn_2 + R - m < 0, \\ \frac{\partial v_i^*(s)}{\partial a} &= \frac{fn_2 + R - m}{a^2c_i^a} < 0, \quad \text{if } fn_2 + R - m > 0, \\ \frac{\partial v_i^*(s)}{\partial c_i^a} &= \frac{fn_2 + R - m}{a(c_i^a)^2} > 0, \quad \text{if } fn_2 + R - m < 0, \\ \frac{\partial v_i^*(s)}{\partial c_i^a} &= \frac{fn_2 + R - m}{a(c_i^a)^2} < 0, \quad \text{if } fn_2 + R - m > 0. \end{aligned} \tag{19}$$

The symbol of the two derivatives can be positive or negative in general due to the strategic interactions between  $f, n_2$ , and  $R$ . If  $fn_2 + R - m < 0$ , with the increase of  $a$  and  $c_i^a$ , the probability of enterprises' emission reduction will be certainly increased.

**Proposition 3.**  $\mu^*(s)$  is positively related to  $d_1$  and negatively related to  $c_i^\mu$ ; on the contrary,  $q_i^*(s)$  is negatively related to  $d_2$ .

*Proof.* According to the positive and negative of the above symbols, we can obtain that

$$\begin{aligned} \frac{\partial \mu^*(s)}{\partial d_1} &= \frac{1}{c_i^\mu b(r + \delta)} > 0, \\ \frac{\partial \mu^*(s)}{\partial c_i^\mu} &= -\frac{d_1}{(c_i^\mu)^2 b(r + \delta)} < 0, \\ \frac{\partial q_i^*(s)}{\partial d_2} &= -\frac{n}{(r + \delta)(2n\beta\lambda + c_i)} < 0. \end{aligned} \tag{20}$$

It is obvious that the more serious the damage caused by industrial pollution to the government, the more active the government will be in dealing with industrial pollution. Also, as the cost of pollution control increases, the government will be less motivated to control it. Moreover, the more serious the damage caused by industrial pollution to the public, the less pollutants will be in the environment.

### 4. Numerical Example

Consider an economy which consists of one region, there are three industrial firms in this region, so  $n = 3$ ; also, let us use the following initial parameters:  $\alpha = 100, \beta = 0.5, a = 0.4, b = 0.2, \delta = 0.01, m = 2, c_i = 0.1, c_i^a = 0.2, f = 0.5, n_2 = 5, R = 0.5, r = 0.005, c_i^\mu = 0.3, \lambda = 0.2$ , and  $d_1 = d_2 = 0.02$ .

Substituting the parameters into (17), we obtain a set of feedback strategies  $[v_i^*(s), \mu^*(s), q_i^*(s)]$  as  $v_i^*(s) = 12.5$ ,

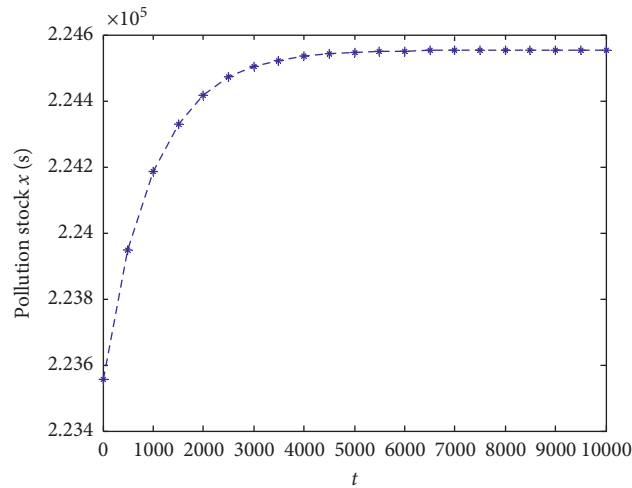


FIGURE 1: The pollution stock that changes over time.

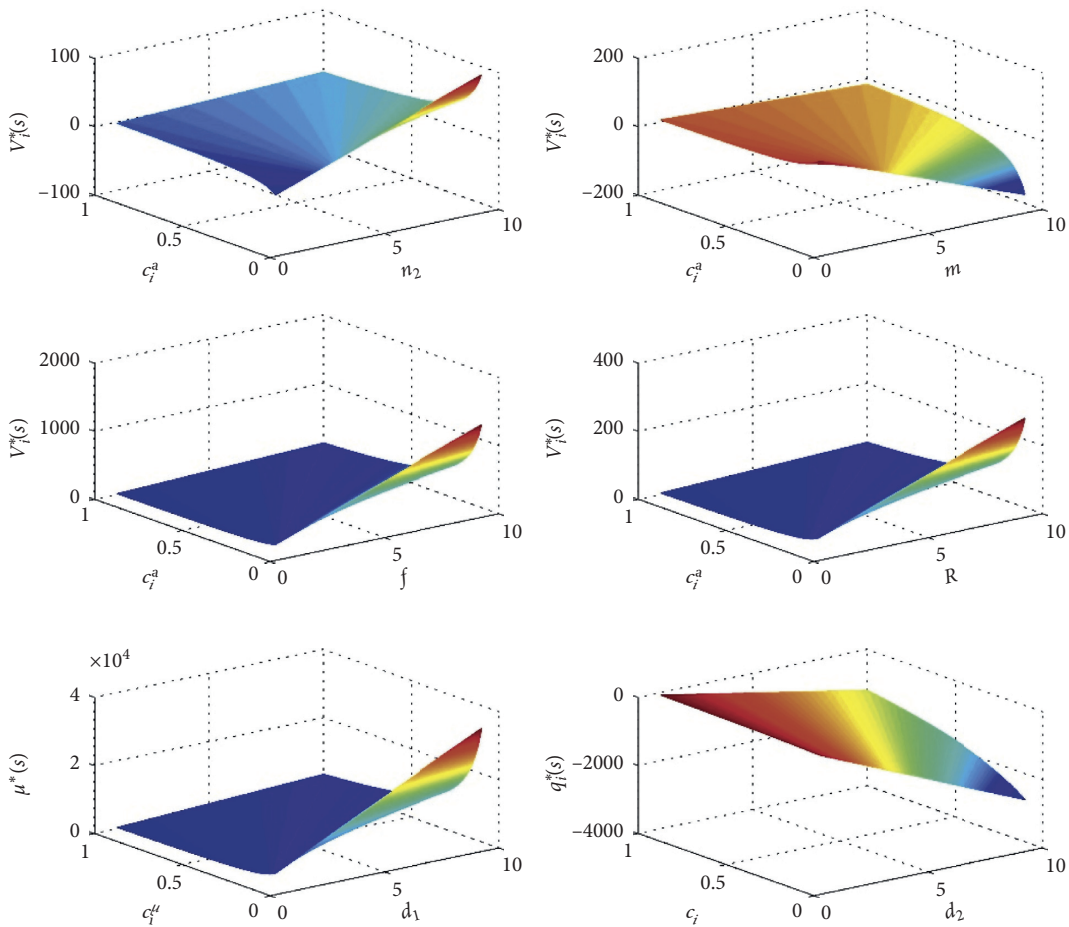


FIGURE 2: The relationship between the set of feedback strategies and the multiple factor variable.

$\mu^*(s) = 2.22, q_i^*(s) = 80$ . Further, substituting the results of  $[v_i^*(s), \mu^*(s), q_i^*(s)]$  into (4), we obtain that the dynamics of pollution stock is governed by the following equation:

$$x = 224556 - 1000 \exp(-0.01s). \quad (21)$$

According to the formula in (21), we can get Figure 1.

From Figure 1, we can get the following conclusions: with the increase of time, the growth rate of the stock of industrial pollutants will gradually decrease, and the stock of industrial pollutants will reach the historical maximum. At this time, the pollutant stock will also reach a stable state. So, the government will actively supervise and control the

emission of enterprises at a certain level. It can be seen that the model we established is reasonable and effective.

In Propositions 1–3, we describe the relationship between a feedback of Nash strategy and a single variable. To obtain more efficient results, we extract some key variables from  $[v_i^*(s), \mu^*(s), q_i^*(s)]$  and substitute the parameters into (17). Then, we obtain

$$\left\{ \begin{array}{l} v_i^*(s) = \frac{0.5n_2 - 1.5}{0.4c_i^a}, \\ v_i^*(s) = \frac{3 - m}{0.4c_i^a}, \\ v_i^*(s) = \frac{5f - 1.5}{0.4c_i^a}, \\ v_i^*(s) = \frac{0.5 + R}{0.4c_i^a}, \\ \mu^*(s) = \frac{d_1}{0.003c_i^\mu}, \\ q_i^*(s) = \frac{60 - 200d_2}{0.6 + c_i} \end{array} \right. \quad (22)$$

According to the results in (22), we can get Figure 2.

From Figure 2, we can see that the higher the unit penalty value as well as the lower the cost of reducing emissions, the more pollutants the enterprise will reduce. An important reason for enterprises to discharge industrial pollution is that the cost of crime is too low and the cost of emission reduction is high, so they will not actively reduce emissions. Therefore, when the government increases the punishment for illegal emission, the amount of pollutants discharged will be greatly reduced. Also, after enterprises actively reduce the cost of emission reduction, the emission of pollutants will be greatly reduced. Moreover, the higher the probability of the public reporting illegal emission of enterprises, the higher the pollution tax levied by the government, and the greater the intensity of enterprises' emission reduction. Obviously, the public's supervision of illegal pollution by enterprises will play a positive role in reducing pollutants. Also, we can see that the higher the  $d_1$ , the lower the  $\mu^*(s)$ , but the higher the  $d_2$ , the higher the  $q_i^*(s)$ .

### 5. Discussion

In this paper, we present a differential game among the government, enterprises, and the public in the process of industrial pollution management. The public participation is considered in this model. Compared with other related literatures, this study has some important contributions in terms of research perspective, methods, and results. These contributions are shown as follows.

Firstly, in terms of research perspective, a great amount of attention has been paid to the development of the

industrial pollution management between the enterprises and governments [26–29]. These researches mainly discussed the development of the industrial pollution problems or some policies between enterprises and governments. However, they ignore the importance of public participation in the process of industrial pollution control. Therefore, our objective is to conduct a multiagent differential game among the government, enterprises, and the public to explore some useful policies to control the industrial pollution. Then, based on the simulation analysis, we can give more convincing results and some more conducive policies.

Secondly, in terms of methods, differential game extends the game theory to continuous time. Game players can change their strategies in an infinitesimally small period of time. Based on these advantages, the differential game has seized a mounting attention in recent years. It has been applied to many areas; also, it is a better way to handle environment pollution problems [9–14]. Therefore, according to our research problem, it is very appropriate to build a multiagent model with the method of differential game.

Thirdly, in terms of results, other scholars mainly discussed qualitative theoretical analysis, lacking quantitative analysis. This study is based on the differential game. Then, we present a numerical simulation to make the result more convincing. Combined with the results of other related studies, the findings in this paper have more theoretical significance and practical value in reality.

### 6. Conclusions and Future Work

This paper explores a differential game of industrial pollution which takes public participation into consideration. A feedback Nash equilibrium (FBNE) solution is derived among the government, enterprises, and the public. In addition, a numerical example is given to illustrate the results; we discuss the relationships between the equilibrium strategies and model parameters. The public participation will take a positive part in forcing enterprises to reduce emissions. Moreover, with the increase of the probability of the public reporting the illegal discharge of pollutants by enterprises, the probability of enterprises' active emission reduction will also greatly increase.

There are also some limitations about our studies. Firstly, we just use the simulation data to examine the result rather than the real data in reality. Secondly, to simplify the differential game model, cooperation in environmental control is not taken into account. So, a further research direction would be needed to examine the situations where the cooperation among the government, enterprises, and the public are taken into account. Moreover, we need the real cases in reality to analysis whether our study is really convincing.

### Data Availability

The data the authors used in the manuscript are simulated data.



## Conflicts of Interest

The authors declare that they have no conflicts of interest.

## Authors' Contributions

Chuansheng Wang developed the idea of the study and Fulei Shi built the model and wrote the original draft of the manuscript. Cuiyou Yao revised the manuscript. All authors read and approved the final manuscript.

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