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Research Article

On New Type of F-Contractive Mapping for Quasipartial b-Metric Spaces and Some Results of Fixed-Point Theorem and Application

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In this paper, we introduce the concept of new type of F-contractive type for quasipartial b-metric spaces and some definitions and lemmas. Also, we will prove a new fixed-point theorem in quasipartial b-metric spaces for F-contractive type mappings. In addition, we give an application which illustrates a situation when Banach's fixed-point theorem for complete quasipartial b-metric spaces cannot be applied, while the conditions of our theorem are satisfying.

1. Introduction

In 1922, Banach [1] introduced the most significant outcome in spaces identified with metric spaces. This principle has been used in proving fixed-point theorems in different spaces and has been used successfully in proving the existence of the solutions of some nonlinear differential equations, integral equations, nonlinear integral differential equations, etc. A great deal of extensions of Banach's principle have been done, for the most part by generalizing the contraction operator and also by expanding the necessity of completeness or even both (see, for example, [2–4]). The concept of partial metric was introduced by Matthews [5] in 1994.

Furthermore, Matthews introduced the concept of metric spaces via putting self-distances which are not necessarily equal to zero. In the year 1994, Czerwik [6] introduced the notion of a b-metric space as another generalization of metric space. Karapinar et al. [7] defined the concept of quasipartial metric space. This notion is a unification of both quasi metric spaces and partial metric spaces under the same substructure. In the following, we present some results that are related to our paper and show the motivation of our concern.

Wardowski [8] in 2012 Introduced the definition of contraction called F-contraction. After that, in 2017, Górnicki [9] called F-contraction to be Wardowski contraction and also extended fixed-point theorems established by Ran and Reurings [10] and Nieto and Rodríguez-López [11] to CJM contractions on preordered metric spaces, where a preordered binary relation is weaker than a partial order. Recently, in 2019, Goswami et al. [12] defined F-contractive type mappings in b-metric space; they proved fixed-point theorems with suitable examples. After that, they had given F-expanding type mappings and a fixed-point result was obtained. In this paper, we introduce the concept of new type of F-contractive type for quasipartial b-metric space and some definitions and lemmas. Also, we state and prove some fixed-point results in new concept and generalize a notion of F-contractive type mappings in quasipartial b-metric spaces with application.

2. Preliminaries and Definitions

We begin the section with some basic definitions and concepts.

Definition 1 (see [7]). Let ξ be a set. A function $q^p\colon \xi \times \xi \longrightarrow [0,\infty)$ is said to be a quasipartial metric on a nonempty set ξ, such that for all $w,t,r \in \xi$, the following conditions hold:

$$\begin{aligned}&(q^p1)\ \text{If } q^p(w,w) = q^p(w,t) = q^p(t,t), \quad \text{then } w = t,\\&(q^p2) q^p(w,w) \le q^p(w,t),\\&(q^p3) q^p(w,t) = q^p(t,w),\\&(q^p4) q^p(w,t) + q^p(r,r) \le q^p(w,r) + q^p(r,t) \quad \text{for all } w,t,r \in \xi.\end{aligned} \tag{1}$$

A quasipartial metric space is a pair (ξ, q^p) such that ξ is a nonempty set and q^p is a quasipartial metric on ξ. Then, $d_{q^p}(w,t) = q^p(w,t) + q^p(t,w) - q^p(w,w) - q^p(t,t)$ is a metric on ξ.

Lemma 1. *For a quasipartial metric q^p on ξ,*

$$p_{q^p}(w,t) = \frac{1}{2}\left[q^p(w,t) + q^p(t,e)\right], \quad \forall w,t \in \xi, \tag{2}$$

and then p_{q^p} is a partial metric on ξ.

In 2015, Gupta and Gautam [13] introduced the concept of quasipartial b-metric space, lemma, example, and some other definitions.

Definition 2. Let ξ be a set. A function $q_b^p\colon \xi \times \xi \longrightarrow [0,\infty)$ is said to be a quasipartial b-metric on a nonempty set ξ, such that for all $w,t,r \in \xi$, the following conditions hold:

$$\begin{aligned}&\left(q_b^p1\right)\ \text{If } q_b^p(w,w) = q_b^p(w,t) = q_b^p(t,t) \Longrightarrow w = t,\\&\left(q_b^p2\right) q_b^p(w,w) \le q_b^p(w,t),\\&\left(q_b^p3\right) q_b^p(w,t) = q_b^p(t,w),\\&\left(q_b^p4\right) q_b^p(w,t) + q_b^p(r,r) \le \eta\left[q_b^p(w,r) + q_b^p(r,t)\right], \text{ for all } w,t,r \in \xi.\end{aligned} \tag{3}$$

A quasipartial b-metric space is a pair (ξ, q_b^p) such that ξ is a nonempty set and (ξ, q_b^p) is a quasipartial b-metric space on ξ. The number η is called the coefficient of (ξ, q_b^p). Further, q_b^p is a quasipartial b-metric space on the set ξ. Then,

$$d_{q_b^p}(w,t) = q^p(w,t) + q^p(t,w) - q^p(w,w) - q^p(t,t) \tag{4}$$

is a metric on ξ.

Lemma 2. *Every (ξ, q^p) is a (ξ, q_b^p). But the opposite does not have to be true.*

Example 1. Let $\xi = [0,1]$; then, $q_b^p(w,t) = |w - t| + w$. Hence, (ξ, q_b^p) is quasipartial b-metric space with $\eta > 0$.

Definition 3. Let (ξ, q_b^p) be a quasipartial b-metric space which satisfies the following:

(i) A sequence $\{w_n\} \subset \xi$ converges to $G^{(2)}$ $w \in \xi$ iff

$$q_b^p(w,w) = \lim_{n\longrightarrow\infty} q_b^p(w,w_n) = \lim_{n\longrightarrow\infty} q_b^p(w_n,w). \tag{5}$$

(ii) A sequence $\{w_n\} \subset \xi$ is said to be a Cauchy sequence iff

$$\begin{aligned}&\lim_{n,m\longrightarrow\infty} q_b^p(w_n,w_m),\\&\lim_{n,m\longrightarrow\infty} q_b^p(w_m,w_n).\end{aligned} \tag{6}$$

exist and are finite.

(iii) (ξ, q_b^p) is called complete if every Cauchy sequence $\{w_n\} \subset \xi$ converges with respect to $\tau_{q_b^p}$ to a point $w \in \xi$ so that

$$q_b^p(w,w) = \lim_{n,m\longrightarrow\infty} q_b^p(w_n,w_m) = \lim_{n,m\longrightarrow\infty} q_b^p(w_m,w_n). \tag{7}$$

(iv) A mapping $\Lambda\colon \xi \longrightarrow \xi$ is called continuous at $w_0 \subset \xi$ if, for every $\varepsilon > 0$, there is $\delta > 0$, so that $\Lambda(B(w-0,\delta)) \subset B(\Lambda(w-0),\varepsilon)$.

Remark 1. Let (ξ, q_b^p) be a quasipartial b-metric space. Then, the following holds:

(i) If $q_b^p(w,t) = 0 \Longrightarrow w = t$.

(ii) If $w \ne t \Longrightarrow q_b^p(w,t) > 0$ and $q_b^p(t,w) > 0$.

Definition 4 (see [8]). Let $F\colon (0,\infty) \longrightarrow R$ be a map satisfying the following conditions:

(F_1) F is strictly increasing.

(F_2) $\lim_{n\longrightarrow\infty}\sigma_n = 0 \Longleftrightarrow \lim_{n\longrightarrow\infty} F(\sigma_n) = -\infty$, for each sequence $\{\sigma_n\}_{n\in N}$ of positive numbers.

(F_3) If there exists $l \in (0,1)$, then $\lim_{\sigma\longrightarrow 0^+} \sigma l F(\sigma) = 0$.

Let $(\xi,\, d)$ be a metric space. A mapping $\Lambda\colon \xi \longrightarrow \xi$ is called a Wardowski F-contraction if there exists $\tau > 0$ such that

$$d(\Lambda w, \Lambda t) > 0 \Longrightarrow \tau + F\, d(\Lambda w, \Lambda t) \le F\, d(w,t), \tag{8}$$

for all $w,t \in \xi$.

After that, in 2015, Cosentino et al. [14] extended the notion of quasipartial b-metric space with the following condition added to these concepts.

(F_4) Let $\eta \geq 1$ be a real number. For each sequence $\{\sigma_n\}_{n\in N}$ of positive numbers,

$$\begin{aligned}&\tau + F(\eta\sigma_n) \geq F(\sigma_{n-1})\\ &\Longrightarrow \tau + F(\eta^n\sigma_n) \geq F(\eta^{n-1}\sigma_{n-1}),\end{aligned} \tag{9}$$

for all $n \in N$ and some $\tau > 0$. Alsulami et al. in 2015, [15] defined a generalized F-Suzuki type in a b-metric space (ξ, d, η) as a mapping $\Lambda\colon \xi \longrightarrow \xi$; then, there exists $\tau > 0$ with $w = t$, such that

$$\begin{aligned}&2\eta\, d(w, \Lambda w) < d(w,t)\\ &\Longrightarrow \tau + F\, d(\Lambda w, \Lambda t) \leq \sigma F\, d(w,t) + \delta F\, d(w, \Lambda w) + \rho F\, d(t, \Lambda t),\end{aligned} \tag{10}$$

for all $w, t \in \xi$, since $\sigma, \delta \in [0,1]$ and $\rho \in [0,1)$ under condition $\sigma + \delta + \rho = 1$ with σ and F satisfying conditions F_1 and F_2.

Lemma 3 (see [16]). *Let (ξ, d, η) be a b-metric space and $\{w_n\}$ be a convergent sequence in ξ with $\lim_{n\longrightarrow\infty} w_n = wl$. Therefore, $t \in \xi$, and we have*

$$\eta^{-1} d(w,t) \leq \lim_{n\longrightarrow\infty} \inf d(w_n, t) \leq \lim_{n\longrightarrow\infty} \sup d(w_n, t) \leq \eta\, d(w,t). \tag{11}$$

The present paper aims to establish a similar type of result for F-contractive type mappings in a quasipartial b-metric space. In our results, we consider that a quasipartial b-metric is not a continuous functional.

3. Main Convergent Results

Lemma 4. *Let (ξ, q_b^p, η) be a quasipartial b-metric space and $\{w_n\}$ be a convergent sequence in ξ with $\lim_{n\longrightarrow\infty} w_n = wl$, for all $t \in \xi$; we have*

$$\left(\frac{1}{\eta}\right) q_b^p(w,t) \leq \lim_{n\longrightarrow\infty} \inf q_b^p(w_n, t) \leq \lim_{n\longrightarrow\infty} \sup q_b^p(w_n, t) \leq \eta q_b^p(w,t), \tag{12}$$

$$\left(\frac{1}{\eta}\right) q_b^p(t,w) \leq \lim_{n\longrightarrow\infty} \inf q_b^p(t, w_n) \leq \lim_{n\longrightarrow\infty} \sup q_b^p(t, w_n) \leq \eta q_b^p(t,w). \tag{13}$$

Proof. We will apply twice the relaxed triangle inequality $(q_b^p 4)$; then, for every $n \in N$, we obtain

$$\begin{aligned}\left(\frac{1}{\eta}\right) q_b^p(w,t) &\leq q_b^p(w, w_n) + q_b^p(w_n, t) - q_b^p(w_n, w_n),\\ &\leq q_b^p(w, w_n) + q_b^p(w_n, t),\end{aligned} \tag{14}$$

and then we have

$$\begin{aligned}\left(\frac{1}{\eta}\right) q_b^p(w,t) - q_b^p(w, w_n) &\leq q_b^p(w_n, w) + q_b^p(w,t) - q_b^p(w,w),\\ &\leq q_b^p(w_n, w) + q_b^p(w,t).\end{aligned} \tag{15}$$

Therefore, we take on the left-hand side inequality and on the right-hand side inequality, and we obtain equation (12); in the same way, we get equation (13). □

Definition 5. Let (ξ, q_b^p, η) be a quasipartial b-metric space, and a mapping $\Lambda\colon \xi \longrightarrow \xi$ is said to be an F-contractive type mapping if there exists $\tau > 0$ such that $q_b^p(w, \Lambda w), q_b^p(t, \Lambda t) \neq 0$ and $q_b^p(\Lambda w, w), q_b^p(\Lambda t, t) \neq 0$ imply

$$\tau + F\left(\eta q_b^p(\Lambda w, \Lambda t)\right) \leq \left(\frac{1}{3}\right)\left\{F\left(q_b^p(w,t)\right) + F\left(q_b^p(w, \Lambda W)\right) + F\left(q_b^p(t, \Lambda t)\right)\right\}, \tag{16}$$

$$\tau + F\left(\eta q_b^p(\Lambda t, \Lambda w)\right) \leq \left(\frac{1}{3}\right)\left\{F\left(q_b^p(t,w)\right) + F\left(q_b^p(t, \Lambda t)\right) + F\left(q_b^p(w, \Lambda w)\right)\right\}. \tag{17}$$

Also, if $q_b^p(w,\Lambda w), q_b^p(t,\Lambda t)=0$ and $q_b^p(\Lambda w,w), q_b^p(\Lambda t,t)=0$, we have the same equations ((16) and (17)) for all $w,t\in\xi$.

Lemma 5. *Let (ξ,q_b^p,η) be a quasipartial b-metric space and let $(\xi,d_{q_b^p},\eta)$ be the corresponding b-metric space. Then, the following statements are equivalent:*

(i) (ξ,q_b^p,η) is complete.

(ii) $(\xi,d_{q_b^p},\eta)$ is complete.

Moreover,

$$\begin{aligned}&\lim_{n\longrightarrow\infty} d_{q_b^p}(z,x_n)=0\Longleftrightarrow q_b^p(z,z)=\lim_{n\longrightarrow\infty}q_b^p(z,x_n)=\lim_{n\longrightarrow\infty}q_b^p(x_n,x_m),\\&\lim_{n\longrightarrow\infty} d_{q_b^p}(x_n,z)=0\Longleftrightarrow q_b^p(z,z)=\lim_{n\longrightarrow\infty}q_b^p(x_n,z)=\lim_{n\longrightarrow\infty}q_b^p(x_m,x_n),\end{aligned}\tag{18}$$

since $z,x_n,x_n\in\xi$, for all $m,n\in N$ and $m>n$.

Definition 6. Let (ξ,q_b^p,η) be a quasipartial b-metric space and Λ: $\xi\longrightarrow\xi$ be a mapping; then, there exists $\tau>0$ with $w=t$, for all $w,t\in\xi$, such that

$$\begin{aligned}&2\eta q_b^p(w,\Lambda w)<d(w,t)\\&\Longrightarrow\tau+Fq_b^p(\Lambda w,\Lambda t)\le\sigma Fq_b^p(w,t)+\delta Fq_b^p(w,\Lambda w)+\rho Fq_b^p(t,\Lambda t),\\&2\eta q_b^p(\Lambda w,w)<q_b^p(t,w)\\&\Longrightarrow\tau+Fq_b^p(\Lambda t,\Lambda w)\le\sigma Fq_b^p(t,w)+\delta Fq_b^p(\Lambda w,w)+\rho Fq_b^p(\Lambda t,t),\\&q_b^p(w,\Lambda w)q_b^p(t,\Lambda y)=q_b^p(\Lambda w,w)q_b^p(\Lambda t,t)=0.\end{aligned}\tag{19}$$

Hence, we have

$$\begin{aligned}&\tau+Fq_b^p(\Lambda w,\Lambda t)\le\sigma Fq_b^p(w,t)+\delta Fq_b^p(w,\Lambda w)+\rho Fq_b^p(t,\Lambda t),\\&\Longrightarrow\tau+Fq_b^p(\Lambda t,\Lambda w)\le\sigma Fq_b^p(t,w)+\delta Fq_b^p(\Lambda w,w)+\rho Fq_b^p(\Lambda t,t).\end{aligned}\tag{20}$$

Since $\sigma,\delta\in[0,1]$ and $\rho\in[0,1)$ under conditionsince $\sigma+\delta+\rho=1$ with σ, F satisfies each conditions between $(F-F_4)$.

Now, we establish a new type of result for F-contractive type mappings in a quasipartial b-metric space by appealing to Lemma 4.

Theorem 1. *Let (ξ,q_b^p,η) be a complete quasipartial b-metric space and Λ: $\xi\longrightarrow\xi$ be an F-contractive type mapping. Then, Λ satisfies a Picard operator.*

Proof. We start our proof by supposing that $w_0\in\xi$ is arbitrary (but fixed) and considering the sequence $\{w_n\}$; since $w_n=\Lambda w_n$, $n\in N$. Suppose that $\lambda_n>0$ for all $n\in N$ and denote $q_b^p(w_n,w_{n+1})$, $q_b^p(w_{n+1},w_n)$ by λ_n; then, we have

$$\begin{aligned}F\left(\eta q_b^p(w_n,w_{n+1})\right)&=F(\eta\lambda_n)\\&\le\frac{1}{3}\left\{F\left(q_b^p(w_{n-1},w_n)\right)+F\left(q_b^p(w_{n-1},w_n)\right)+F\left(q_b^p(w_n,w_{n+1})\right)\right\}-\tau,\end{aligned}\tag{21}$$

$$\begin{aligned}F\left(\eta q_b^p(w_{n+1},w_n)\right)&=F(\eta\lambda_n)\\&\le\frac{1}{3}\left\{F\left(q_b^p(w_n,w_{n-1})\right)+F\left(q_b^p(w_n,w_{n+1})\right)+F\left(q_b^p(w_{n-1},w_n)\right)\right\}-\tau.\end{aligned}\tag{22}$$

From (21), we can write

$$F(\eta\lambda_n)\le F(\eta\lambda_{n-1})-\left(\frac{3}{2}\right)\tau.\tag{23}$$

By condition (F_4), we have

$$F(\eta^n\lambda_n)\le F\left(\eta^{n-1}\lambda_{n-1}\right)-\left(\frac{3}{2}\right)\tau.\tag{24}$$

Then, by induction, we get

$$F(\eta^n\lambda_n)\le F\left(\eta^{n-1}\lambda_{n-1}\right)-\left(\frac{3}{2}\right)\tau\le\cdots\le F(\lambda_0)-\left(\frac{3}{2}\right)n.\tag{25}$$

By taking a limit as $n\longrightarrow\infty$, we have

$$\lim_{n\longrightarrow\infty}F(\eta^n\lambda_n)=-\infty\Longrightarrow\lim_{n\longrightarrow\infty}\eta^n\lambda_n=0.\tag{26}$$

Therefore, from condition (F_3), there exists $l\in(0,1)$ such that

$$\lim_{n\longrightarrow\infty}(\eta^n\lambda_n)^l F(\eta^n\lambda_n)=0.\tag{27}$$

Multiplication of (25) with $(\eta^n\lambda_n)^l$ yields

$$0 \le (\eta^n\lambda_n)^l F(\eta^n\lambda_n) + \left(\frac{3}{2}\right) n (\eta^n\lambda_n)^l \tau \le (\eta^n\lambda_n)^l F(\lambda_0). \tag{28}$$

By taking a limit as $n \longrightarrow \infty$, we have

$$\lim_{n\longrightarrow\infty} n(\eta^n\lambda_n)^l = 0. \tag{29}$$

Now, we can show that $\{w_n\}$ is a Cauchy sequence. Since (ξ, q_b^p, η) is a complete quasipartial b-metric space, there exists $r \in \xi$ such that

$$\lim_{n\longrightarrow\infty} w_n = r. \tag{30}$$

By applying Lemma 4, we have

$$\lim_{n\longrightarrow\infty} q_b^p(r, w_n) = \lim_{n\longrightarrow\infty} \sup q_b^p(r, w_n) \le \eta q_b^p(r, r) \ge 0. \tag{31}$$

On the other hand, let m and n be natural numbers with $m > n$; then, using (q^p4), we get

$$\begin{aligned}
q_b^p(w_n, w_{n+2}) &\le \eta\left\{q_b^p(w_n, w_{n+1}) + q_b^p(w_{n+1}, w_{n+2})\right\} - q_b^p(w_{n+1}, w_{n+1}),\\
&\le \eta\left\{q_b^p(w_n, w_{n+1}) + q_b^p(w_{n+1}, w_{n+2})\right\},\\
q_b^p(w_n, w_{n+3}) &\le \eta\left\{q_b^p(w_n, w_{n+2}) + q_b^p(w_{n+2}, w_{n+3})\right\} - q_b^p(w_{n+2}, w_{n+2}),\\
&\le \eta\left(\eta\left(q_b^p(w_n, w_{n+1}) + q_b^p(w_{n+1}, w_{n+2})\right)\right) + \eta q_b^p(w_{n+2}, w_{n+3}),\\
&\le \eta^2 q_b^p(w_n, w_{n+1}) + \eta^2 q_b^p(w_{n+1}, w_{n+2}) + \eta q_b^p(w_{n+2}, w_{n+3}).
\end{aligned} \tag{32}$$

On generalization, we have

$$\begin{aligned}
q_b^p(w_n, w_m) &\le \eta^{m-n-1}\left\{q_b^p(w_n, w_{n+1}) + q_b^p(w_{n+1}, w_{n+2})\right\} + \eta^{m-n-2}\left\{q_b^p(w_{n+2}, w_{n+3}) + \cdots + \eta q_b^p(w_{m-1}, w_m)\right\},\\
&= \sum_{i=n+1}^{m-1} \eta^{m-i}\left\{q_b^p(w_i, w_{i+1})\right\} + \eta^{m-n-1}\left\{q_b^p(w_n, w_{n+1})\right\},\\
&= \sum_{i=n}^{m-1} \eta^{m-i}\left\{q_b^p(w_i, w_{i+1})\right\} - \eta^{m-n}\left\{q_b^p(w_n, w_{n+1})\right\}\left(1 - \left(\frac{1}{\eta}\right)\right),\\
&\le \sum_{i=n}^{m-1} \eta^{m-i}\left\{q_b^p(w_i, w_{i+1})\right\},\\
&\le \sum_{i=n}^{m-1} \eta^{m-i} \cdot l^i\left\{q_b^p(w_0, w_1)\right\},\\
&\le \sum_{i=n}^{m-1} \left(\frac{l}{\eta}\right)^i \eta^m\left\{q_b^p(w_0, w_1)\right\}.
\end{aligned} \tag{33}$$

By taking limit as $n \longrightarrow \infty$ in (33), since m holding fixed, we have

$$\lim_{n\longrightarrow\infty} q_b^p(w_n, w_m) \le 0. \tag{34}$$

But, we know

$$\lim_{n\longrightarrow\infty} q_b^p(w_n, w_m) \ge 0. \tag{35}$$

Thus,

$$\lim_{n\longrightarrow\infty} q_b^p(w_n, w_m) = 0. \tag{36}$$

Similarly, we can show that

$$\lim_{n\longrightarrow\infty} q_b^p(w_m, w_n) = 0. \tag{37}$$

Due to (18) in Lemma 5 and (36), we get

$$\begin{aligned}
\lim_{n\longrightarrow\infty} d_{q_b^p}(r, w_n) = 0 \Longrightarrow q_b^p(r, r) &= \lim_{n\longrightarrow\infty} \sup q_b^p(r, w_n)\\
&= \lim_{n\longrightarrow\infty} \sup q_b^p(w_n, w_m) = 0.
\end{aligned} \tag{38}$$

Also, using (16), we have for all $n \longrightarrow N$,

$$\tau + F\left(\eta q_b^p(\Lambda r, \Lambda w_n)\right) \leq \left(\frac{1}{3}\right)\left\{F\left(q_b^p(r, w_n)\right) + F\left(q_b^p(r, \Lambda r)\right) + F\left(q_b^p(w_n, \Lambda w_n)\right)\right\}, \tag{39}$$

since. Then, by taking a limit as $n \longrightarrow \infty$ in last equation, we get

$$\tau + \lim_{n\longrightarrow\infty} F\left(\eta q_b^p(\Lambda r, \Lambda w_n)\right) \leq -\infty. \tag{40}$$

This implies

$$\lim_{n\longrightarrow\infty} q_b^p(\Lambda r, w_{n+1}) = \lim_{n\longrightarrow\infty} q_b^p(\Lambda r, \Lambda w_n) = 0. \tag{41}$$

Since the sequence $\{w_n\}$ converges to both r and Λr, we have $\Lambda r = r$. Finally to show the uniqueness of the fixed point, let x be another fixed point of Λ with $r \neq x$; then, from (16),

$$\tau + F\left(\eta q_b^p(\Lambda r, \Lambda x)\right) \leq \left(\frac{1}{3}\right)\left\{F\left(q_b^p(r, x)\right) + F\left(q_b^p(r, \Lambda r)\right) + F\left(q_b^p(x, \Lambda x)\right)\right\}, \tag{42}$$

or

$$F\left(\eta q_b^p(\Lambda r, \Lambda x)\right) < F\left(q_b^p(\Lambda r, \Lambda x)\right). \tag{43}$$

This is a contradiction. Hence, we proved the theorem. □

4. Application in Fixed-Point Theorem

The example illustrates a situation when Banach's fixed-point theorem for complete quasipartial b-metric spaces cannot be applied, while the conditions of Theorem 1 are satisfied.

Example 2. Let (ξ, q_b^p, η) be a quasipartial b-metric space, where

$$q_b^p(w, t) = \min\{|x - y| + x, 2\} \tag{44}$$

is complete with $\eta = 1$. Suppose the mapping $\Lambda\colon \xi \longrightarrow \xi$ is defined by

$$\Lambda w = \begin{cases} \dfrac{1}{2} & : 0 \leq w < 1, \\ 0 & : w = 1, \\ \left(\dfrac{1}{2}\right) - \left(\dfrac{1}{w}\right) & : w \geq 2. \end{cases} \tag{45}$$

It is clear that Λ is not continuous at $w = 1$. Now, for $w, t \geq 2$ with $w \neq t$, we have

$$\begin{aligned} q_b^p(\Lambda w, \Lambda t)^3 &= \min\left\{\left|\left(\frac{1}{2}\right) - \left(\frac{1}{w}\right) - \left(\frac{1}{2}\right) + \left(\frac{1}{t}\right)\right| + \left(\frac{1}{2}\right) - \left(\frac{1}{w}\right), 2\right\}^3, \\ &= \min\left\{\left|-\left(\frac{1}{w}\right) + \left(\frac{1}{t}\right)\right| + \left(\frac{1}{2}\right) - \left(\frac{1}{w}\right), 2\right\}^3 < 1, \\ q_b^p(w,t)q_b^p(w,\Lambda w)q_b^p(t,\Lambda t) &= \min\{|w - t| + w, 2\}\min\left\{\left|w - \left(\frac{1}{2}\right) + \left(\frac{1}{w}\right)\right| + w, 2\right\}\min\left\{\left|t - \left(\frac{1}{2}\right) + \left(\frac{1}{t}\right)\right| + t, 2\right\} = 8. \end{aligned} \tag{46}$$

Again, if $w \in [0, 1)$ and $t \geq 2$ or conversely, we get

$$\begin{aligned} q_b^p(\Lambda w, \Lambda t)^3 = q_b^p\left(\left(\frac{1}{2}\right), \left(\frac{1}{2}\right) - \left(\frac{1}{y}\right)\right)^3 &= \min\left\{\left|\left(\frac{1}{2}\right) - \left(\frac{1}{2}\right) + \left(\frac{1}{t}\right)\right| + \left(\frac{1}{2}\right), 2\right\}^3, \\ &= \min\left\{\left|\frac{1}{t}\right| + \left(\frac{1}{2}\right)\right\}^3 < 1, \\ q_b^p(w,t)q_b^p(w,\Lambda w)q_b^p(t,\Lambda t) &= \min\{|w - t| + w, 2\}\min\left\{\left|w - \left(\frac{1}{2}\right)\right| + w, 2\right\}\min\left\{\left|t - \left(\frac{1}{2}\right) + \left(\frac{1}{t}\right)\right| + t, 2\right\}, \\ &= 2\left(\left|w - \left(\frac{1}{2}\right)\right| + w\right)2 = 4\left(\left|w - \left(\frac{1}{2}\right)\right| + w\right) \geq 2. \end{aligned} \tag{47}$$

Also, for $w \in [0,1)$,

$$\begin{aligned} q_b^p(\Lambda w, \Lambda 1)^3 &= q_b^p\left(\left(\frac{1}{2}\right), 0\right)^3 = \min\left\{\left|\left(\frac{1}{2}\right) - 0\right| + \left(\frac{1}{2}\right), 2\right\}^3 = \left(\frac{1}{64}\right), \\ q_b^p(w,1) q_b^p(w, \Lambda w) q_b^p(1, \Lambda 1) &= \min\{|w-1| + w, 2\} \min\left\{\left|w - \left(\frac{1}{2}\right)\right| + w, 2\right\} \min\{|1-0| + 1, 2\} \geq 2. \end{aligned} \tag{48}$$

After that, for $w \geq 2$, we get

$$\begin{aligned} q_b^p(\Lambda w, \Lambda 1)^3 &= q_b^p\left(\left(\frac{1}{2}\right) - \left(\frac{1}{w}\right), 0\right)^3 = \min\left\{\left|\left(\frac{1}{2}\right) - \left(\frac{1}{w}\right) - 0\right| + \left(\frac{1}{2}\right) - \left(\frac{1}{w}\right), 2\right\}^3 = \left(\frac{1}{64}\right), \\ q_b^p(w,1) q_b^p(w, \Lambda w) q_b^p(1, \Lambda 1) &= \min\{|w-1| + w, 2\} \min\left\{\left|w - \left(\frac{1}{2}\right)\right| + w, 2\right\} \min\{|1-0| + 1, 2\} \geq 2. \end{aligned} \tag{49}$$

Thus, from the example in [12], if $q_b^p(w, \Lambda w) q_b^p(t, \Lambda t) \neq 0$ is satisfied for $e^{-3\tau} = 1/16$. In addition,

$$\begin{aligned} q_b^p\left(\Lambda 1, \Lambda\left(\frac{1}{2}\right)\right)^3 &= q_b^p\left(0, \left(\frac{1}{2}\right)\right)^3 = \min\left\{\left|0 - \left(\frac{1}{2}\right)\right| + 0, 2\right\}^3 = \left(\frac{1}{8}\right), \\ q_b^p\left(1, \left(\frac{1}{2}\right)\right) q_b^p\left(1, \Lambda\left(\frac{1}{2}\right)\right) q_b^p\left(\Lambda 1, \left(\frac{1}{2}\right)\right) &= \min\left\{\left|1 - \left(\frac{1}{2}\right)\right| + 1, 2\right\} \min\{|1-0| + 1, 2\} \min\left\{\left|0 - \left(\frac{1}{2}\right)\right| + 0, 2\right\}, \\ &= \left(\frac{3}{2}\right)(2)\left(\frac{1}{2}\right) = \left(\frac{6}{4}\right) = \left(\frac{12}{8}\right). \end{aligned} \tag{50}$$

Finally, for $w \geq 2$,

$$\begin{aligned} q_b^p\left(\Lambda w, \Lambda\left(\frac{1}{2}\right)\right)^3 &= \min\left\{\left|\left(\frac{1}{2}\right) - \left(\frac{1}{x}\right) - \left(\frac{1}{2}\right)\right| + \left(\frac{1}{2}\right) - \left(\frac{1}{x}\right), 2\right\}^3 \leq 1, \\ q_b^p\left(1, \left(\frac{1}{2}\right)\right) q_b^p\left(1, \Lambda\left(\frac{1}{2}\right)\right) q_b^p\left(\Lambda 1, \left(\frac{1}{2}\right)\right) &= \min\left\{\left|1 - \left(\frac{1}{2}\right)\right| + 1, 2\right\} \min\{|1-0| + 1, 2\} \min\left\{\left|0 - \left(\frac{1}{2}\right)\right| + 0, 2\right\}, \\ &= \left(\frac{3}{2}\right)(2)\left(\frac{1}{2}\right) = \left(\frac{6}{4}\right) = \left(\frac{12}{8}\right). \end{aligned} \tag{51}$$

Hence, Λ is an F-contractive type mapping and, by inspection, we see that Λ has a unique fixed point w (1/2).

5. Concluding Remarks and Future Works

We introduced the notion of type of F-contractive type mapping for quasipartial b-metric space and some definitions and lemmas. Also, we proved the fixed-point theorem for F-contractive type mappings in a quasipartial b-metric space with application. In fact, applications of F-contractive type mapping are not limited just to the topic mentioned above.

As future work, we are, joining with others, planning to

(1) Extend the results given in the above sections to quasipartial b-symmetric space.

(2) Study the applications of our results to the mathematical modelling of quality.

Remark 2. It will be interesting to find more applications to our current paper in other fields (see [17–31]).

Data Availability

The reference data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Authors' Contributions

AMZ was responsible for conceptualization, writing, and original draft preparation. AAR contributed to data curation and methodology. All authors have read and agreed to the published version of the manuscript.

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