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Research Article

Fuzzy Adaptive Control for a Class of Nonlinear System with Prescribed Performance and Unknown Dead-Zone Inputs

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Based on fuzzy logic systems and prescribed performance technology, this paper investigated the stable tracking problem of a class of nonlinear systems with unknown dead-zone inputs. Firstly, the original system is transformed into an equivalent system. Then, a Nussbaum-type function is added in the predefined performance controller to estimate the unknown control direction, and the boundedness of all signals in the closed-loop system is guaranteed. In addition, tracking errors will always remain within the prescribed performance boundaries. Finally, a comparative simulation example shows the effectiveness of the method proposed in this paper.

1. Introduction

As is known to all, the dead zone may badly limit the system performance, such as electronic circuits and electric servomotors [1–5]. So, many researchers used different control methods (i.e., backstepping technique, fuzzy systems, neural networks, sliding mode control, and so on) to research the stability of nonlinear systems with unknown control directions [6–9]. For example, based on the terminal sliding mode control, Chen and Ren in [6] proposed an adaptive neural control scheme for permanent magnet synchronous motor (PMSM) servo system with dead-zone input. For uncertain discrete-time systems with nonaffine dead-zone input, Liu and Tong [7] used adaptive neural networks tracking control such that the tracking error converges to a small neighborhood of zero. By using high-dimensional integral Lyapunov–Krasovskii functional theory and direct adaptive control scheme, control of uncertain MIMO systems with time-varying delay and dead-zone input was considered in [8]. In [9], the authors investigated an adaptive backstepping control method for an SISO system with unknown dead-zone input. For eliminating the influence of unknown control directions, one of the commonly applied methods is the Nussbaum-type function technique [10–12]. Liu et al. [10] constructed an adaptive neural controller for discrete-time system, which made all signals of the closed-loop system bounded in the condition of unknown dead-zone input. For a class of MIMO stochastic systems in [11], Li and Tong put forward an adaptive fuzzy output-feedback control method by using Nussbaum gain function and fuzzy logic systems. Xu [12] applied the adaptive neural control scheme to guarantee that the flexible hypersonic flight vehicle system is bounded stable with dead-zone input.

However, control methods in aforementioned literature can only guarantee that the tracking error converges to a small residual set, rather than a small residual set with the prescribed performance bounds. For an uncertain nonlinear system [4, 13–15], in order to solve the above problem, the prescribed performance control (PPC) strategy has been proposed. The idea of this method is converting the original system to an equivalent system and ensuring the boundedness of the states of the equivalent system. For example, based on the backstepping technique and the PPC, Wang and Hu [13] proposed an improved PPC for the longitudinal model of an air-breathing hypersonic vehicle (AHV) with unknown dead-zone input nonlinearity. Yang and Chen [14] developed an adaptive neural PPC for near space vehicles (NSVs) with input nonlinearities including saturation and

dead zone. It can be seen from the literature [13, 14] that it has certain practical significance to adopt the PPC method in the practical system. For MIMO nonlinear systems with unknown dead-zone inputs, Shi et al. [4] proposed an adaptive fuzzy PPC scheme such that only three parametric adaptive rules were needed. Cao et al. [16] studied a fractional neural network model with impulse amplitude, time-delay time-varying, and reaction diffusion terms. Many other interesting results about PPC of nonlinear systems can be seen in [17–19].

Inspired by the work of [4, 20], this paper explores the stability problem that the tracking error meets prescribed performance boundary (PPB) with unknown dead-zone input. Compared with related works, the main contributions of this paper are listed as follows: (1) compared with the work of [14, 17, 19], the introduced error transformation variable z_i can be directly differentiated without the participation of the transformation function. (2) The Nussbaum-type function is incorporated into the control method so that the influence of unknown control directions is eliminated. (3) The singular problem in the PPC for nonlinear systems can be avoided by using the proposed method.

The structure of this paper is as follows. In Section 2, the uncertain system model with unknown dead-zone inputs is raised and provides assumption and lemmas. Section 3 includes the designation of fuzzy adaptive prescribed performance control method. The results of simulation are presented in Section 4. In Section 5, a brief conclusion is given.

2. Preliminaries

Consider the following uncertain nonlinear system, which described as

$$\begin{cases} \dot{x}_1 = f_1(t,x) + d_1(t) + b_1\phi_1(u_1), \\ \dot{x}_2 = f_2(t,x) + d_2(t) + b_2\phi_2(u_2), \\ \dots, \\ \dot{x}_n = f_n(t,x) + d_n(t) + b_n\phi_n(u_n), \end{cases} \tag{1}$$

where $x = [x_1, x_2, \dots, x_n]^T$ is the system state vector, $f_i(t,x)$ is an unknown smooth nonlinear function, $d_i(t)$ is an external disturbance, $\phi_i(u_i)$ denotes the control input subject to dead-zone type nonlinear, and b_i is an unknown control coefficient, $i = 1, 2, \dots, n$. Here, $\phi(u_i)$ is described as follows:

$$\phi_i(u_i) = \begin{cases} m_i(u_i - b_{ir}), & \text{if } u_i \geq b_{ir}, \\ 0, & \text{if } b_{il} < u_i < b_{ir}, \\ m_i(u_i + b_{il}), & \text{if } u_i \leq -b_{il}, \end{cases} \tag{2}$$

where m_i, b_{ir}, and b_{il} are unknown positive constants, $i = 1, 2, \dots, n$. Then, we rewrite (2) as

$$\phi_i(u_i) = m_i u_i + \triangle u_i, \tag{3}$$

where

$$\triangle u_i = \begin{cases} -m_i b_{ir}, & \text{if } u_i \geq b_{ir}, \\ -m_i u_i, & \text{if } b_{il} < u_i < b_{ir}, \\ m_i b_{il}, & \text{if } u_i \leq -b_{il}. \end{cases} \tag{4}$$

From (3), system (1) is described as

$$\begin{cases} \dot{x}_1 = f_1(t,x) + d_1(t) + b_1 m_1 u_1 + b_1 \triangle u_1, \\ \dot{x}_2 = f_2(t,x) + d_2(t) + b_2 m_2 u_2 + b_2 \triangle u_2, \\ \dots, \\ \dot{x}_n = f_n(t,x) + d_n(t) + b_n m_n u_n + b_n \triangle u_n. \end{cases} \tag{5}$$

Designing two adaptive prescribed performance control methods is the control objective of this paper. The goal of this paper is that the tracking error e_i satisfies PPB, and all closed-loop system signals are bounded.

In order to meet the objective, we proposed the following assumptions.

Assumption 1. The system state x, the reference signal x_d, and its derivative $\dot{x}_d$ are available and bounded.

Assumption 2. The nonlinear function $f_i(t,x)$, the control coefficient b_i, and the external disturbance $d_i(t)$ are unknown but bounded, $i = 1, 2, \dots, n$.

Remark 1. Assumptions 1 and 2 are commonly used to address the problem of unknown control direction [21–23]. And owing to the boundedness of $b_i, \Delta u_i$, and $d_i(t)$, we know that there exists an unknown positive constant D_i^*, such that $|b_i \triangle u_i + d_i(t)| \leq D_i^*, i = 1, 2, \dots, n$.

Let $\overline{b}_i = b_i m_i$ and $D_i(t) = b_i \triangle u_i + d_i(t)$, then $|D_i(t)| \leq D_i^*$. The error state is defined as $e = [e_1, e_2, \dots, e_n]^T = [x_1 - x_{1d}, x_2 - x_{2d}, \dots, x_n - x_{nd}]^T$, and the following error system is acquired:

$$\begin{cases} \dot{e}_1 = f_1(t,x) + D_1(t) + \overline{b}_1 u_1(t) - \dot{x}_{1d}, \\ \dot{e}_2 = f_2(t,x) + D_2(t) + \overline{b}_2 u_2(t) - \dot{x}_{2d}, \\ \dots, \\ \dot{e}_n = f_n(t,x) + D_n(t) + \overline{b}_n u_n(t) - \dot{x}_{nd}. \end{cases} \tag{6}$$

The following definition and lemmas used in this paper are to overcome the problem of unknown control directions.

Definition 1. A continuous function $N(\cdot)$ is called a Nussbaum function, if $N(\cdot)$ satisfies

$$\begin{aligned} \lim_{t \longrightarrow +\infty} \sup \frac{1}{t} \int_0^t N(s) \mathrm{d}s &= +\infty, \\ \lim_{t \longrightarrow +\infty} \inf \frac{1}{t} \int_0^t N(s) \mathrm{d}s &= -\infty. \end{aligned} \tag{7}$$

Remark 2. Generally, the selection of the Nussbaum functions for the controller design with unknown control direction is $\xi^2 \cos(\xi)$ or $e^{\xi^2} \cos((\Pi/2)\xi)$ or $\ln(\xi + 1)$

$\cos(\sqrt[2]{\ln(\xi+1)})$ and so on. In this paper, we choose the even Nussbaum function $\xi^2\cos(\xi)$ to design the PPC controller.

Lemma 1 (see [15]). *Let $V(t)$ and $\xi(t)$ be smooth functions, $V(t)\geq 0$ for $\forall t\in[0,t_f)$, and $N(\cdot)$ be an even smooth Nussbaum function. If the following inequality holds:*

$$V(t)\leq r_0+e^{-r_1t}\int_0^t(\alpha(x(s))N(\xi(s))+1)\cdot\dot{\xi}(s)e^{-r_1s}\mathrm{d}s,\quad \forall t\in\left[0,t_f\right), \tag{8}$$

where r_0 and r_1 are constants with $r_1>0$, $\alpha(x(t))$ is a time-varying parameter which takes values in the unknown closed intervals $I=:[l^-,l^+]$ with $0\overline{\in}I$, then $V(t)$, $\xi(t)$, and $\int_0^t(\alpha(x(s))N(\xi(s))+1)\dot{\xi}(s)e^{-r_1s}ds$ must be bounded on $[0,t_f)$.

Lemma 2 (see [4]). *Let $A\in R^{n\times n}$ be a symmetric matrix and $x\in R^n$ be a nonzero vector. Let $\alpha=(x^TAx/x^Tx)$. Then, there exist eigenvalues of A such that $\lambda_1\in(-\infty,\alpha]$ and $\lambda_2\in[\alpha,+\infty)$.*

Remark 3. In this paper, we will construct the time-varying parameter $\alpha(t)$ by using Lemma 2. For example, let $A=\mathrm{diag}(a_1,a_2,\ldots,a_n)$ and z is a nonzero vector; we can design $\alpha(t)$ as $\alpha(t)=(z^TAz/z^Tz)$, and then there exist two constants l^- and l^+ such that

$$l^-\leq\min_i\{a_i\}\leq\alpha(t)\leq\max_i\{a_i\}\leq l^+. \tag{9}$$

In this paper, fuzzy logic systems will be used to approximate unknown nonlinear functions. Generally, the output of fuzzy logic system can be expressed as [24–32]

$$\widehat{f}=\frac{\sum_{i\in I}w_i(t)\mu_i(x(t))}{\sum_{i\in I}\mu_i(x(t))}, \tag{10}$$

where $x=[x_1(t),\ldots,x_n(t)]^T\in C^1[\mathscr{I},\Omega]\,(\forall t\in\mathscr{I})$ is the input vector. $\widehat{f}\colon\Omega\mapsto\mathscr{R}$ is a Lipschitz continuous mapping, $\Omega\subseteq\mathscr{R}^n$ is a compact subset, and $\mathscr{R}$ is a real line. $I=\prod_{j=1}^n\mathscr{F}_j$, where $\mathscr{F}_j$ consists of N_j fuzzy sets $(1\leq j\leq n)$. The membership function of the i-th rule is $\mu_i\,(i\in I)$, where $\mu_i\colon\mathscr{R}^n\mapsto[0,1]$. $w_i\colon\mathscr{I}\mapsto\mathscr{R}$ $(i\in 1,2,\ldots,N)$, which represents the centroid of the i-th consequent set. Let $\theta(t)=[w_1(t),\ldots,w_N(t)]^T$ and $\varphi(t)=[\varphi_1(x(t)),\varphi_2(x(t)),\ldots,\varphi_N(x(t))]^T$, where φ_i $(i\in I)$ is the i-th fuzzy basis function and φ_i is a continuous mapping, then φ_i can be defined as

$$\varphi_i(x(t))=\frac{\mu_i(t)}{\sum_{s\in I}\mu_s(x(t))}. \tag{11}$$

Thus, (10) can be rearranged as

$$\widehat{f}(x(t))=\theta^T(t)\varphi(t). \tag{12}$$

3. Synchronization Controller Design

3.1. Prescribed Performance. The steady-state performance of e_i can be preserved by setting the performance constraint condition as

$$-\lambda_2\mu(t)<e_i<\lambda_1\mu(t),\quad i=1,2,\ldots,n, \tag{13}$$

where λ_1 and λ_2 are positive constants, which can regulate the error boundaries. $\mu(t)$ is denoted by

$$\mu(t)=(\mu_0-\mu_\infty)e^{-\overline{\lambda}t}+\mu_\infty, \tag{14}$$

where $\overline{\lambda}>0$ and $\mu_0,\mu_\infty>0$. Obviously, $\mu(t)$ is a decreasing function and $\mu(0)=\mu_0$, $\lim_{t\longrightarrow+\infty}\mu(t)=\mu_\infty$.

We abbreviate $\mu(t)$ to μ and give an error transformation as

$$z_i=\frac{\lambda_1e_i}{\lambda_1\mu-e_i}l(e_i)+\frac{\lambda_2e_i}{\lambda_2\mu+e_i}(1-l(e_i)), \tag{15}$$

where $l(e_i)$ satisfies

$$l(e_i)=\begin{cases}1, & e_i\geq 0,\\ 0, & e_i<0.\end{cases} \tag{16}$$

Obviously, if we choose the initial condition $e_i(0)$ as $-\lambda_2\mu(0)<e_i(0)<\lambda_1\mu(0)$ and z_i is bounded, then the error state e_i will satisfy the inequality that $-\lambda_2\mu<e_i<\lambda_1\mu$. Because when e_i approaches $-\lambda_2\mu(0)$ or $\lambda_1\mu(0)$, z_i will approach infinity, which contradicts the boundedness of z_i. So, here, the work for us is to prove that the error transformation variable z_i is bounded, $z_i i=1,2,\ldots,n$.

Remark 4. The proposed transformation variable z_i in this paper can be directly expressed by e_i and parameters λ_1 and λ_2, see (15), while the traditional transformation variable z_i in [4, 14] needs to rely on the transformation function $\tanh(z_i)=(e_i/\mu)$ to transform. Therefore, the proposed error transformation variable z_i is simpler and is more convenient for the design of the controller u_i.

3.2. Control Design and Stability Analysis. The time derivative of z_i is

$$\dot{z}_i=\mu\Pi_i\dot{e}_i-\dot{\mu}\Pi_ie_i, \tag{17}$$

where $\Pi_i=(\lambda_1^2/(\lambda_1\mu-e_i)^2)l(e_i)+(\lambda_2^2/(\lambda_2\mu+e_i)^2)(1-l(e_i))$. So, we obtain the following error transformation dynamic system.

Due to the fact that $f_i(t,x)$ is unknown, we need to employ fuzzy logic system to approximate $f_i(t,x)$. So, the approximate of function $f_i(t,x)$ can be expressed as $\widehat{f}_i(x,\widehat{\theta}_{f_i})=\widehat{\theta}_{f_i}^T\varphi_{f_i}(x), i=1,2,\ldots,n$, where $\widehat{\theta}_{f_i}$ is the parameter vector and the fuzzy function vector is $\varphi_{f_i}(x)$. For $f_i(t,x)$, there is an optimal parameter vector $\theta_{f_i}^*$ such that

$$f_i(t,x)=\theta_{f_i}^{*T}\varphi_{f_i}(x)+\varepsilon_{f_i}(x), \tag{18}$$

where $\theta^*_{f_i} = \arg\min_{\widehat{\theta}_{f_i}} \left\{ \sup |f_i(t,x) - \widehat{\theta}^T_{f_i} \varphi_{f_i}(x)| \right\}$ and $\varepsilon_{f_i}(x)$ is the approximation error, which is bounded as $|\varepsilon_{f_i}(x)| \le \varepsilon^*_{f_i}$, where $\varepsilon^*_{f_i}$ is a positive constant, $i = 1, 2, \ldots, n$.

Let $\overline{D}_i(t) = \varepsilon_{f_i}(x) + D_i(t)$, then $|\overline{D}(t)| \le \varepsilon^*_{f_i} + D^*_i \triangleq \overline{D}^*_i$. The error transformation dynamic system is rewritten as follows:

$$\begin{cases} \dot{z}_1 = \mu\Pi_1\left(f_1(t,x) + D_1(t) + \overline{b}_1 u_1(t) - \dot{x}_{1d}\right) - \dot{\mu}\Pi_1 e_1, \\ \dot{z}_2 = \mu\Pi_2\left(f_2(t,x) + D_2(t) + \overline{b}_2 u_2(t) - \dot{x}_{2d}\right) - \dot{\mu}\Pi_2 e_2, \\ \ldots, \\ \dot{z}_n = \mu\Pi_n f_n(t,x) + D_n(t) + \overline{b}_n u_n(t) - \dot{x}_{nd}) - \dot{\mu}\Pi_n e_n. \end{cases} \tag{19}$$

Next, we design the controller u_i to guarantee the boundedness of z_i. Let $z = [z_1, z_2, \ldots, z_n]^T$, $\Pi = [\Pi_1, \Pi_2, \ldots, \Pi_n]^T$, and another controller as follows:

$$\begin{aligned} u_i &= \frac{z_i N(\xi)\overline{u}}{\mu\Pi_i}, \\ \overline{u} &= \frac{\Delta^2}{\Delta\|z\| + \sigma}, \\ \dot{\xi} &= \|z\|^2 \overline{u}, \end{aligned} \tag{20}$$

where $\Delta = \Delta_1 + \Delta_2$, $\Delta_1 = \mu\widehat{\theta}\|\Pi\|\sqrt{\sum_{i=1}^n \|\varphi_{f_i}(x)\|^2}$, $\Delta_2 = \mu\|\Pi\| \widehat{D} + \sum_{i=1}^n |\dot{\mu}|\Pi_i|e_i| + (1/2)\|z\|$, $\theta^* \triangleq \sqrt{\sum_{i=1}^n \|\theta^*_{f_i}\|^2}$, $D^* = \sum_{i=1}^n \overline{D}^*_i$, $\widehat{\theta}$ and $\widehat{D}$ are the estimates of θ^* and D^*, respectively, and the parameters adaptive laws are given as

$$\dot{\widehat{\theta}} = \mu\|\Pi\|\sqrt{\sum_{i=1}^n \left\|\varphi_{f_i}(x)\right\|^2} \cdot \|z\| - \gamma_1\widehat{\theta}, \dot{\widehat{D}} = \mu\|\Pi\|\|z\| - \gamma_2\widehat{D}, \tag{21}$$

where γ_1 and γ_2 are positive constants.

Now, consider the Lyapunov function:

$$V = \frac{1}{2}z^T z + \frac{1}{2}\widetilde{\theta}^2 + \frac{1}{2}\widetilde{D}^2, \tag{22}$$

where $\widetilde{\theta} = \theta^* - \widehat{\theta}$ and $\widetilde{D} = D^* - \widehat{D}$. By differentiating V, one achieves

$$\begin{aligned} \dot{V} &= -\frac{1}{2}z^T z + z^T\left(\dot{z} + \frac{z}{2}\right) + \widetilde{\theta}\dot{\widetilde{\theta}} + \widetilde{D}\dot{\widetilde{D}} \\ &= -\frac{1}{2}z^T z + \sum_{i=1}^n \left(z_i\mu\Pi_i\theta^{*T}_{f_i}\varphi_{f_i}(x) + z_i\mu\Pi_i\overline{D}_i(t) - \dot{\mu}z_i\Pi_i e_i - z_i\dot{x}_{id}\mu\Pi_i + \frac{z_i^2}{2}\right) + \sum_{i=1}^n z_i\mu\Pi_i\overline{b}_i u_i + \widetilde{\theta}\dot{\widetilde{\theta}} + \widetilde{D}\dot{\widetilde{D}} \\ &\le -\frac{1}{2}z^T z + \sum_{i=1}^n \left(|z_i|\mu\Pi_i\left\|\theta^*_{f_i}\right\|\left\|\varphi_{f_i}(x)\right\| + |z_i|\mu\Pi_i|\overline{D}_i(t)| + |\dot{\mu}||z_i|\Pi_i|e_i| + |z_i|\left|\dot{x}_{id}\right|\mu\Pi_i + \frac{z_i^2}{2}\right) + \sum_{i=1}^n z_i\mu\Pi_i\overline{b}_i u_i + \widetilde{\theta}\dot{\widetilde{\theta}} + \widetilde{D}\dot{\widetilde{D}} \\ &\le -\frac{1}{2}z^T z + \mu\|z\|\|\Pi\| \cdot \sum_{i=1}^n \left\|\theta^*_{f_i}\right\|\left\|\varphi_{f_i}(x)\right\| + \|z\|\mu\|\Pi\| \underbrace{\sum_{i=1}^n D^*_i}_{D^* = \widetilde{D} + \widehat{D}} + \|z\| \cdot \sum_{i=1}^n |\dot{\mu}|\Pi_i|e_i| + \|z\|\sum_{i=1}^n \left|\dot{x}_{id}\right|\mu\Pi_i + \frac{1}{2}\|z\|^2 \\ &\quad + \sum_{i=1}^n z_i\mu\Pi_i\overline{b}_i u_i + \widetilde{\theta}\dot{\widetilde{\theta}} + \widetilde{D}\dot{\widetilde{D}}. \end{aligned} \tag{23}$$

According to Cauchy–Schwarz inequality,

$$\begin{aligned}\sum_{i=1}^{n}\left\|\theta_{f_i}^{*}\right\|\left\|\varphi_{f_i}(x)\right\| &\le \sqrt{\sum_{i=1}^{n}\left\|\theta_{f_i}^{*}\right\|^2}\cdot\sqrt{\sum_{i=1}^{n}\left\|\varphi_{f_i}(x)\right\|^2}\\ &= \theta^{*}\sqrt{\sum_{i=1}^{n}\left\|\varphi_{f_i}(x)\right\|^2}\\ &= (\widetilde{\theta}+\widehat{\theta})\sqrt{\sum_{i=1}^{n}\left\|\varphi_{f_i}(x)\right\|^2}.\end{aligned} \tag{24}$$

Considering its adaptive law (21), it is concluded that

$$\begin{aligned}\widetilde{\theta}\dot{\widetilde{\theta}} &= -\widetilde{\theta}\dot{\widehat{\theta}}\\ &= -\widetilde{\theta}\left(\mu\|\Pi\|\sqrt{\sum_{i=1}^{n}\left\|\varphi_{f_i}(x)\right\|^2}\cdot\|z\|-\gamma_1\widehat{\theta}\right)\\ &= -\widetilde{\theta}\mu\|\Pi\|\sqrt{\sum_{i=1}^{n}\left\|\varphi_{f_i}(x)\right\|^2}\cdot\|z\|+\gamma_1\widetilde{\theta}\widehat{\theta}\\ &= -\widetilde{\theta}\mu\|\Pi\|\sqrt{\sum_{i=1}^{n}\left\|\varphi_{f_i}(x)\right\|^2}\cdot\|z\|+\gamma_1\widetilde{\theta}\left(\theta^{*}-\widetilde{\theta}\right)\\ &\le -\frac{\gamma_1}{2}\widetilde{\theta}^2-\widetilde{\theta}\mu\|\Pi\|\sqrt{\sum_{i=1}^{n}\left\|\varphi_{f_i}(x)\right\|^2}\cdot\|z\|+\frac{\gamma_1}{2}\theta^{*2}.\end{aligned} \tag{25}$$

Similarly,

$$\widetilde{D}\dot{\widetilde{D}}\le -\frac{\gamma_2}{2}\widetilde{D}^2-\widetilde{D}\mu\|\Pi\|\|z\|+\frac{\gamma_2}{2}D^{*2}. \tag{26}$$

So, one gets

$$\begin{aligned}\dot{V}\le &-\overline{\overline{\gamma}}V+\|z\|\left(\mu\|\Pi\|\widehat{\theta}\sum_{i=1}^{n}\sqrt{\left\|\varphi_{f_i}(x)\right\|^2}\right)\\ &+\mu\|\Pi\|\widehat{D}+\sum_{i=1}^{n}|\dot{\mu}|\Pi_i|e_i|+\sum_{i=1}^{n}\left|\dot{x}_{id}\right|\mu\Pi_i\\ &+\frac{\|z\|}{2}\Bigg)+\sum_{i=1}^{n}z_i\overline{b}z_iN(\xi)\overline{u}+\overline{\overline{R}}_0,\end{aligned} \tag{27}$$

where $\overline{\overline{\gamma}}=\min\{1,\gamma_1,\gamma_2\}$ and $\overline{\overline{R}}_0=(\gamma_1/2)\theta^{*2}+(\gamma_2/2)D^{*2}$. For $z_i\neq 0$, we define $\alpha(t)=((\sum_{i=1}^{n}z_i\overline{b}_iz_i)/(\sum_{i=1}^{n}z_i^2))=(z^TUz/z^Tz)$, where $U=\operatorname{diag}(\overline{b}_1,\overline{b}_2,\ldots,\overline{b}_n)$. By using Lemma 2, there are two constants l^+ and l^- such that $l^-\le\min_i\overline{b}_i\le\alpha(t)\le\max_i b_i\le l^+$. So,

$$\sum_{i=1}^{n}z_i\overline{b}z_i=\|z\|^2\alpha(t). \tag{28}$$

Obviously, if $z_i=0$, (28) also holds. By substituting $u_i(t)$ in (20) into (27), it yields

$$\dot{V}\le -\overline{\overline{\gamma}}V+(\alpha(t)N(\xi)+1)\dot{\xi}+\overline{\overline{\overline{R}}}_0, \tag{29}$$

where $\overline{\overline{\overline{R}}}_0=\overline{\overline{R}}_0+\sigma$. Integrating (29) over $[0,t]$, one obtains

$$V\le V(0)+\frac{\overline{\overline{\overline{R}}}_0}{\overline{\overline{\gamma}}}+\int_0^t(1+\alpha(s)N(\xi))\dot{\xi}e^{\overline{\overline{\gamma}}(s-t)}ds. \tag{30}$$

According to Lemma 2, $V(t)$ is bounded into $[0,t_f)$. So, z_i is bounded for any given positive constant t_f, and then the boundedness of z_i is true for $t_f=+\infty$ (see [20]).

Through the above analysis and discussion, the following theorem is obtained.

Theorem 1. *Consider the uncertain nonlinear system (1) with dead-zone inputs and unknown control directions. The proposed control method (20) can guarantee the boundedness of all the signals in the closed-loop system. Furthermore, for any $e_i(0)$ satisfies $-\lambda_2\mu(0)<e_i(0)<\lambda_1\mu(0)$, the error e_i will satisfy the prescribed performance (13).*

4. Simulation Studies

Consider the Chen chaotic system as a simulation example:

$$\begin{cases}\dot{x}_1=\underbrace{35x_2-35x_1}_{f_1(x)}+d_1(t)+b_1\phi_1(u_1),\\ \dot{x}_2=\underbrace{-x_1x_3-7x_1+28x_2}_{f_2(x)}+d_2(t)+b_2\phi_2(u_2),\\ \dot{x}_3=\underbrace{-3x_3+x_1x_2}_{f_3(x)}+d_3(t)+b_3\phi_3(u_3),\end{cases} \tag{31}$$

where $d_1(t)=d_2(t)=d_3(t)=2\sin(2t)$, $b_1=2, b_2=-2, b_3=2$, and $\phi_i(u_i)$ is described as

$$\phi_i(u_i)=\begin{cases}3(u_i-2), & u_i\ge 2,\\ 0, & -2<u_i<2,\\ 3(u_i+2), & u_i\le -2,\end{cases} \tag{32}$$

$i=1,2,3$. One reference signal $x_d=[\sin(t)\sin(t)\sin(t)]^T$. Choose the fuzzy membership function as.

Firstly, we use the method in [15] to control Chen chaotic system (31), and controller is designed as follows:

$$\begin{cases}u_i=\xi_i^2\cos(\xi_i)\overline{u}_i,\\ \overline{u}_i=\widehat{\eta}_{f_i}^T\varphi_{f_i}(x)+\widehat{\rho}\tanh\left(\dfrac{e_i}{\varepsilon_i}\right)+c_ie_i,\\ \dot{\xi}_i=e_i\overline{u}_i,\\ \dot{\widehat{\eta}}_{f_i}=e_i\varphi_{f_i}(x)-\gamma_{1i}\widehat{\eta}_{f_i},\\ \dot{\widehat{\rho}}_i=e_i\tanh\left(\dfrac{e_i}{\varepsilon}\right)-\gamma_{2i}\widehat{\rho}_i,\quad i=1,2,3,\end{cases} \tag{33}$$

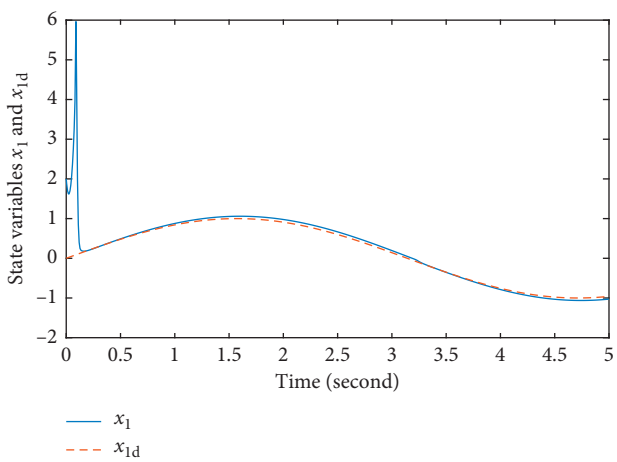

FIGURE 1: x_1 and x_{1d} with control scheme (33).

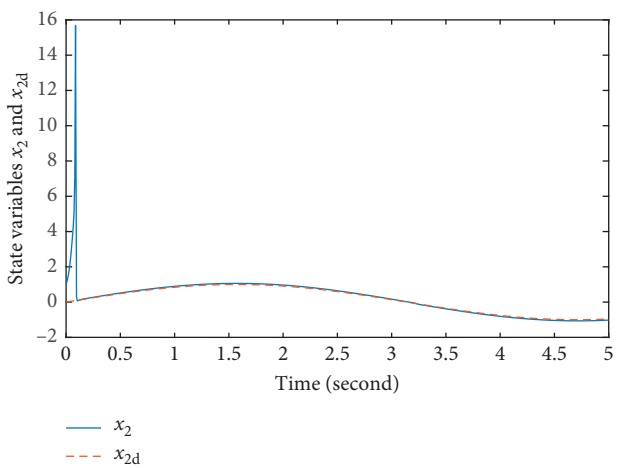

FIGURE 2: x_2 and x_{2d} with control scheme (33).

where $\hat{\eta}_{f_i}^T \varphi_{f_i}(x)$ is the estimate of $f_i(x)$ and $f_i(x) = \eta_{f_i}^{*T} \varphi_{f_i}(x) + \varepsilon_{f_i}(x)$, where $\varepsilon_{f_i}(x)$ is the bounded approximation error. Let $|d_i(t) + \varepsilon_{f_i}(x) + \dot{x}_{id}| \le \rho_i$, where $\hat{\rho}_i$ is the estimate of ρ_i. The parameters are $\varepsilon = 0.5$, $\gamma_{i1} = 3, \gamma_{i2} = 2$, $i = 1, 2, 3$. The initial values are $x_1(0) = 2, x_2(0) = 1, x_3(0) = -2, \xi_1(0) = 0.01, \xi_2(0) = 0.1, \xi_3(0) = 0.1, \hat{\rho}_1(0) = 0.05, \hat{\rho}_2(0) = -0.3, \hat{\rho}_3(0) = 0.4, \hat{\eta}_{f_1}(0) = \hat{\eta}_{f_2}(0) = \hat{\eta}_{f_3}(0) = 0, \mu(t) = 3e^{-2.5t} + 0.1$. By using control scheme (33), the trajectories of x_1, x_2, and x_3 are shown in Figures 1–3. The corresponding error trajectories are shown in Figure 4. It can be seen that, in the initial control stage, e_1 and e_2 will have a jump beyond the prescribed performance boundary (PPB) $[-\mu(t), \mu(t)]$ and then remain within the BBP $[-\mu(t), \mu(t)]$, while e_3, on the contrary, is within the PPB $[-\mu(t), \mu(t)]$ at the initial stage and stays outside the PPB $[-\mu(t), \mu(t)]$ after 3 seconds.

In order to compare with the control scheme (33), we employ our proposed control scheme (20). The parameters are $\lambda_1 = \lambda_2 = 1$ and $\sigma = 0.5$, and the initial values are $\hat{\bar{D}}_1 = 0.2, \hat{\bar{D}}_2 = 0.3$, and $\hat{\bar{D}}_3 = 0.4$, Other parameters and initial values in this example are same as those of the control scheme (33). The simulation results are shown in Figures 5–8. The states x_1, x_2, and x_3 can track the reference

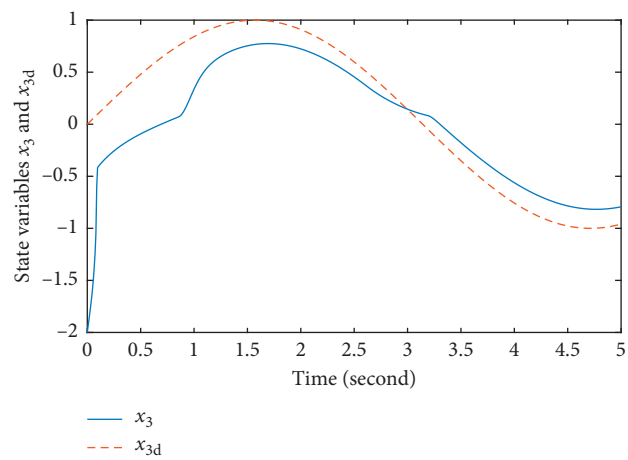

FIGURE 3: x_3 and x_{3d} with control scheme (33).

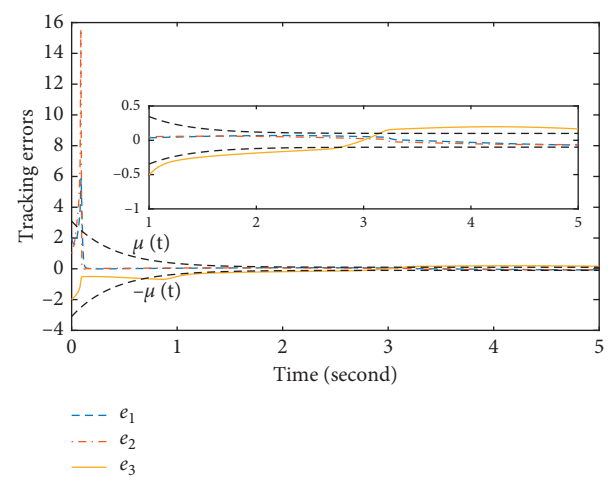

FIGURE 4: e_1, e_2, and x_3 with control scheme (33).

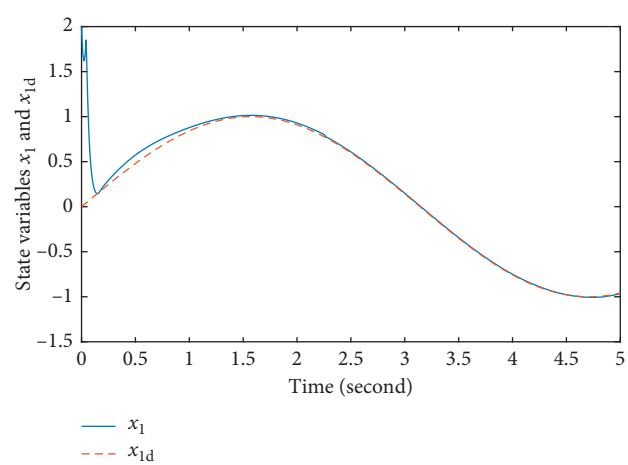

FIGURE 5: x_1 and x_{1d} with proposed control scheme (20).

signal x_{1d}, x_{2d}, and x_{3d} quickly after a transient response process in Figures 5–7. From Figure 8, errors e_1, e_2, and e_3 are not beyond the PPB $[-\mu(t), \mu(t)]$. In Figure 9, two

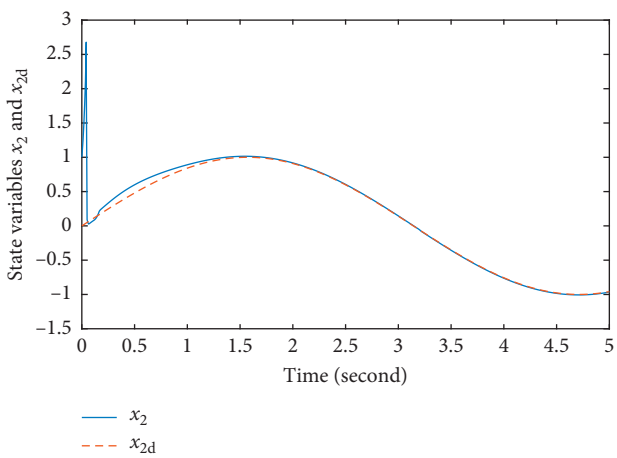

Figure 6: x_2 and x_{2d} with proposed control scheme (20).

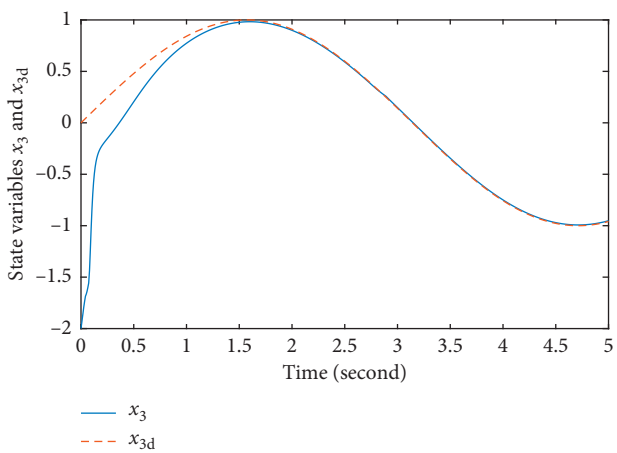

Figure 7: x_3 and x_{3d} with proposed control scheme (20).

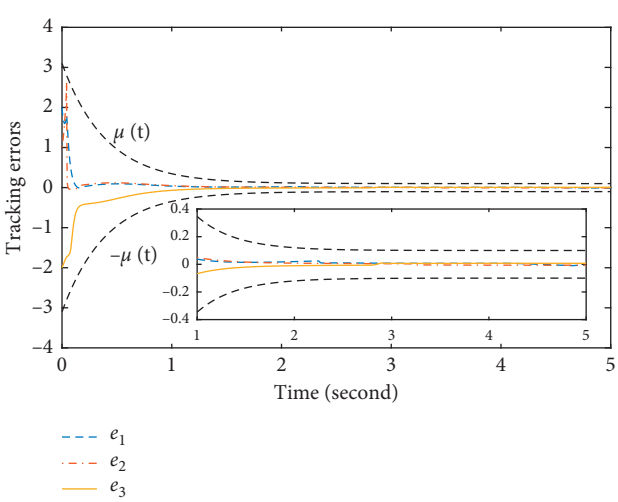

Figure 8: e_1, e_2, and x_3 with proposed control scheme (20).

control effects of error state $\|e\|$ are compared directly. So, these results verify that the proposed control scheme (20) has a better control effect than control scheme (33).

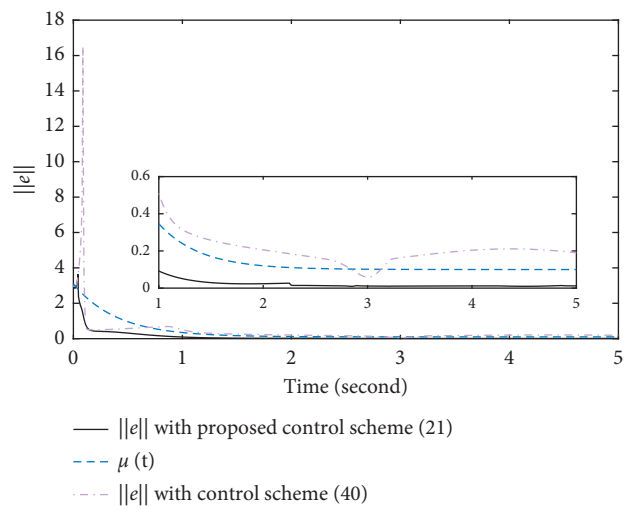

Figure 9: $\|e\|$ with control schemes (20) and (33).

5. Conclusions

The problem of robust fuzzy adaptive prescribed performance control for uncertain nonlinear system is investigated. The proposed control approach is to make sure that all signals in closed-loop system are bounded and tracking errors converge to a small neighborhood of zero with the prescribed performance bounds. The Nussbaum-type function is incorporated into two control methods such that the influence of unknown control directions is eliminated. It is unnecessary to use the inverse operation in the proposed methods. Simulation results verify the effectiveness and robustness of the proposed method. How to effectively estimate the unknown function of the nonlinear system is the future research direction.

Data Availability

All datasets generated for this study are included within the manuscript.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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