

## Research Article

# Dynamical Behaviour of the Light Pulses through the Optical Fiber: Two Nonlinear Atangana Conformable Fractional Evolution Equations

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This study, using the extended simplest method of equation, examines the explicit movement solutions of both the Schwarzian Korteweg-de Vries (SKdV) and (2 + 1)-Ablowitz-Kaup-Newell-Segur (AKNS.) equation. These models show the movement of the waves in optical fiber mathematically. The SKdV equation explains the movement of the isolated waves in diverse fields and on the site in a small space microsection. Some solutions obtained have been developed to show the physical and dynamic behaviors of these solutions in the obtained wave.

## 1. Introduction

Partial differential equations (PDEs) have been playing an essential role in describing and studying some complex phenomena in distinct branches of science [1–5]. These phenomena have been formulated in nonlinear PDEs with an integer order or fractional order [6–8]. Studying these mathematical models have been forcing many research groups in physics, chemistry, mathematics, and so on to derive practical and powerful computational schemes (analytical, semianalytical, and numerical techniques) for constructing exact and numerical solutions [9–15]. These schemes include the modified and generalized Kudryashov methods, the extended tanh-function method, the improved tan( $\phi/2$ ) expansion method, the novel, improved, extended, and generalized ( $G'/G$ ) expansion method, the extended and generalized  $e^{-\phi(\xi)}$  expansion method, the Khater method, the modified Khater method, the Adomian decomposing method, the B-spline schemes, and so on [16–24].

In this research, we investigate two primary mathematical models in the optical fiber via the extended simplest equation method. The first model is Atangana conformable fractional SKdV equation that was derived by Krichever and Novikov in the following form [25]:

$$\frac{\mathcal{D}_t^q \mathcal{U}}{\mathcal{U}_x} + \left( \frac{\mathcal{U}_{xx}}{\mathcal{U}_x} \right)_x - \frac{1}{2} \left( \frac{\mathcal{U}_{xx}}{\mathcal{U}_x} \right)^2 = 0, \quad (0 < q < 1), \quad (1)$$

where  $\mathcal{U} = \mathcal{U}(x, t)$  satisfies Newton's equation of motion in a cubic potential. Equation (1) is also given by [26]

$$\begin{aligned} \mathcal{D}_t^q \mathcal{G} + \frac{1}{4} \mathcal{G}_{xxx} - \frac{\mathcal{G}_x \mathcal{G}_{xz}}{2\mathcal{G}} - \frac{\mathcal{G}_{xx} \mathcal{G}_z}{4\mathcal{G}} + \frac{\mathcal{G}_x^2 \mathcal{G}_z}{2\mathcal{G}^2} \\ - \frac{\mathcal{G}_x}{8} \int \left( \frac{\mathcal{G}_x^2}{\mathcal{G}^2} \right) dx = 0. \end{aligned} \quad (2)$$

Equation (2) has an essential role in a right-moving soliton and the nonlocal form. However, we study a new form of equation (2) that is given in the following system [27]:

$$\begin{cases} \mathcal{L}_1 \mathcal{B}^2 \mathcal{C}_x - \mathcal{L}_1 \mathcal{B} \mathcal{B}_x \mathcal{C} + \mathcal{B}^2 \mathcal{C}_{xxz} - \mathcal{B} \mathcal{B}_{xx} \mathcal{B}_z - \mathcal{L}_2 \mathcal{B} \mathcal{B}_x \mathcal{B}_{xz} + \mathcal{L}_2 \mathcal{B}_x^2 \mathcal{B}_z - \mathcal{B}^4 \mathcal{B}_z = 0, \\ \mathcal{D}_t^q \mathcal{B} - \mathcal{D}_t^q \mathcal{C} = 0, \end{cases} \tag{3}$$

where  $\mathcal{B} = \mathcal{B}(x, t)$ ,  $\mathcal{C} = \mathcal{C}(x, z, t)$ . Additionally,  $\mathcal{L}_1, \mathcal{L}_2$  are arbitrary constants. Using the following wave transformation  $\mathcal{B}(x, t) = \mathcal{B}(b)$ ,  $\mathcal{C}(x, z, t) = v(f)$ ,  $b = x + z - (c/q)(t + 1/\Gamma(q))^q$  and then integrating the second equation of the transformed system once with zero constant of integration and substituting the result into the first equation of the same system lead to

$$\mathcal{B}^2 \mathcal{B}''' - (1 + \mathcal{L}_2) \mathcal{B} \mathcal{B} \mathcal{B}' \mathcal{B}'' + \mathcal{L}_2 \mathcal{B}^3 - \mathcal{B}^4 \mathcal{B}' = 0. \tag{4}$$

While the second model is the Atangana conformable fractional (2 + 1) AKNS equation which is so close to the first model. This model is given by [28–30]

$$4\mathcal{D}_t^q \mathcal{Q}_x + \mathcal{Q}_{xxxz} + 8\mathcal{Q}_{xz} \mathcal{Q}_x + 4\mathcal{Q}_z \mathcal{Q}_{xx} = 0, \tag{5}$$

where  $\mathcal{Q} = \mathcal{Q}(x, z, t)$ . Applying the next wave transformation  $\mathcal{Q} = \mathcal{Q}(b)$ ,  $b = x + z - (c/q)(t + (1/\Gamma(q)))^q$  to equation (5) gives

$$-4c\mathcal{Q}' + \mathcal{Q}''' + 6\mathcal{Q}^2 = 0. \tag{6}$$

Balancing the terms in equations (4) and (6) based on the next principle of homogenous rule,

$$D \left[ \frac{d^\varepsilon \mathcal{P}(b)}{db^\varepsilon} \right] = N + \varepsilon, \tag{7}$$

$$D \left[ \mathcal{P}^\varepsilon \left( \frac{d^\varepsilon \mathcal{P}(b)}{d\mathcal{P}^\varepsilon} \right)^s \right] = \varepsilon N + s(N + \varepsilon),$$

leads to  $N = 1$ .

The rest sections are order in the following order. Section 2 gives the implementation of the extended simplest equation method for the Atangana conformable fractional SKdV equation (8) and the Atangana conformable fractional (2 + 1)-AKNS equation (9). Also, some solutions are sketched to illustrate the physical behaviour of the wave solutions. Section 3 produces a conclusion of our paper.

## 2. Application

In this section, we apply the extended simplest equation method to the SKdV equation and the (2 + 1)-AKNS equation for constructing the exact traveling and solitary wave solutions.

**2.1. The SKdV Equation.** According to the extended simplest equation method and value of homogenous balance value, we get

$$\mathcal{B}(b) = \sum_{i=-N}^N a_i f(b)^i = \frac{a_{-1}}{f(b)} + a_0 + f(b)a_1, \tag{8}$$

where  $a_{-1}, a_0$ , and  $a_1$  are arbitrary constants. Also,  $f(b)$  follows the next ODE:

$$f'(b) = \alpha + \lambda f(b) + \mu f(b)^2, \tag{9}$$

where  $\alpha, \lambda$ , and  $\mu$  are the arbitrary constants. Substituting equation (8) and its derivative along equation (9) into equation (4) and collecting all terms with the same power of  $f^i(b), i = (0, 1, 2, \dots)$  lead to a system of equations. Solving this system with any computer software yields family one:

$$\begin{aligned} a_0 &\longrightarrow \frac{1}{2} \left( -\lambda - \sqrt{\lambda^2 - 4\alpha\mu} \right), & a_1 &\longrightarrow 0, & a_{-1} &\longrightarrow \\ & & & & & -\alpha, & \mathcal{L}_2 &\longrightarrow 3. \end{aligned} \tag{10}$$

Subsequently, the explicit solutions of the fractional SKdV equation are given as follows.

When  $\lambda = 0, \alpha\mu < 0$ ,

$$\begin{aligned} \mathcal{B}_1(x, t) &= -\sqrt{-\alpha\mu} + \sqrt{-\alpha\mu} \operatorname{Coth} \left[ \mathcal{H} \sqrt{-\alpha\mu} \mp \frac{\operatorname{Log}[\vartheta]}{2} \right], \\ \mathcal{B}_2(x, t) &= -\sqrt{-\alpha\mu} + \sqrt{-\alpha\mu} \operatorname{Tanh} \left[ \mathcal{H} \sqrt{-\alpha\mu} \mp \frac{\operatorname{Log}[\vartheta]}{2} \right]. \end{aligned} \tag{11}$$

When  $4\alpha\mu > \lambda^2$ ,

$$\begin{aligned} \mathcal{B}_3(x, t) &= -\frac{\lambda}{2} - \frac{1}{2} \sqrt{\lambda^2 - 4\alpha\mu} + \frac{2\alpha\mu}{\lambda - \sqrt{-\lambda^2 + 4\alpha\mu} \operatorname{Tan} \left[ (1/2) (\mathcal{H} + \vartheta) \sqrt{-\lambda^2 + 4\alpha\mu} \right]}, \\ \mathcal{B}_4(x, t) &= -\frac{\lambda}{2} - \frac{1}{2} \sqrt{\lambda^2 - 4\alpha\mu} + \frac{2\alpha\mu}{\lambda - \sqrt{-\lambda^2 + 4\alpha\mu} \operatorname{Cot} \left[ (1/2) (\mathcal{H} + \vartheta) \sqrt{-\lambda^2 + 4\alpha\mu} \right]}, \end{aligned} \tag{12}$$

where  $\mathcal{H} = x + z - (c/q)(t + (1/\Gamma(q)))^q$ .

Family two:

$$a_0 \longrightarrow \frac{1}{2} \left( \lambda - \sqrt{\lambda^2 - 4\alpha\mu} \right), \quad a_1 \longrightarrow \mu, \quad a_{-1} \longrightarrow 0, \quad \mathcal{L}_2 \longrightarrow 3. \tag{13}$$

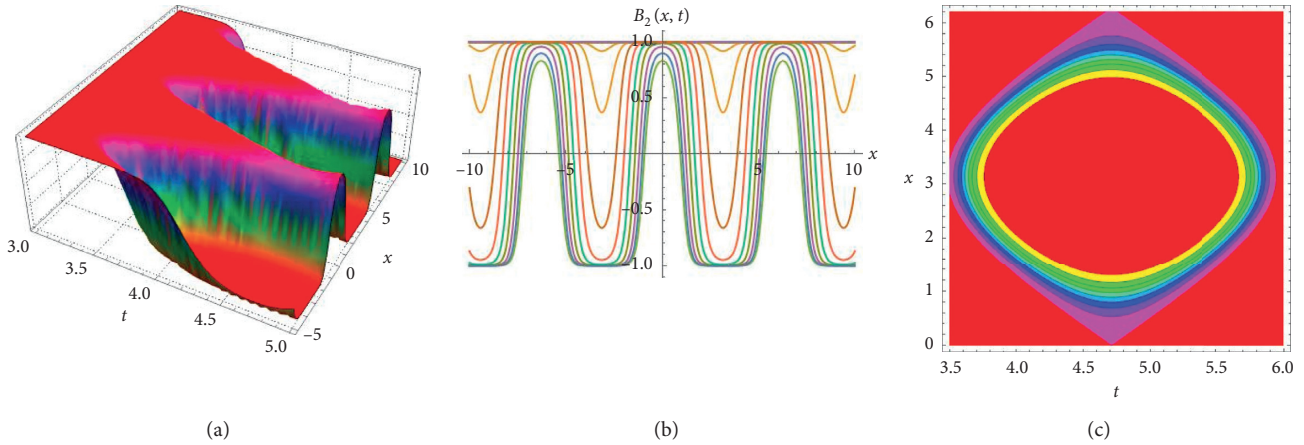


FIGURE 1: Periodic kink solitary wave solutions of equation (11) in three-dimensional, two-dimensional, and contour plot 3D for  $\lambda = 6, a_1 = 3, \mu = -9, c = -3,$  and  $\vartheta = 2.$

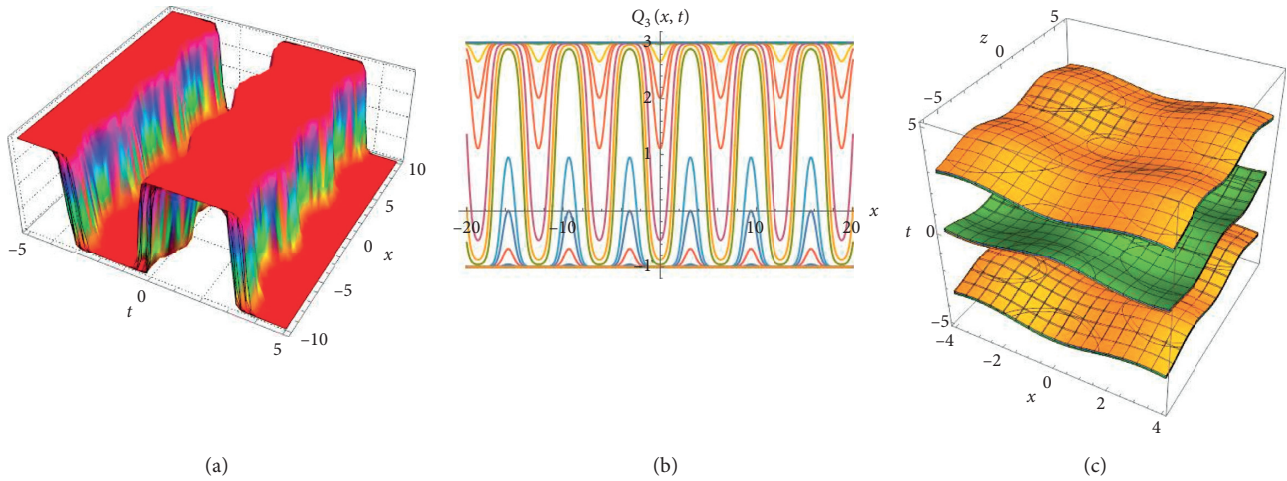


FIGURE 2: Kink solitary wave solutions of equation (21) in three-dimensional, two-dimensional and contour plot 3D for  $a_0 = 1, \alpha = -1, \mu = 4,$  and  $\vartheta = 3.$

Subsequently, the explicit solutions of the fractional SKdV equation are given as follows.

When  $\lambda = 0, \alpha\mu < 0,$

$$\mathcal{B}_5(x, t) = \sqrt{-\alpha\mu} + \sqrt{-\alpha\mu} \operatorname{Tanh} \left[ \mathcal{H} \sqrt{-\alpha\mu} \mp \frac{\operatorname{Log}[\vartheta]}{2} \right],$$

$$\mathcal{B}_6(x, t) = -\sqrt{-\alpha\mu} + \sqrt{-\alpha\mu} \operatorname{Coth} \left[ \mathcal{H} \sqrt{-\alpha\mu} \mp \frac{\operatorname{Log}[\vartheta]}{2} \right].$$

(14)

When  $\alpha = 0,$

For  $\lambda > 0,$

$$\mathcal{B}_7(x, t) = \frac{\lambda}{2} - \frac{\sqrt{\lambda^2}}{2} + \frac{\lambda}{1 - e^{(\mathcal{H} + \vartheta)\lambda} \mu}. \tag{15}$$

For  $\lambda < 0,$

$$\mathcal{B}_8(x, t) = \frac{\lambda}{2} - \frac{\sqrt{\lambda^2}}{2} - \mu + \frac{\mu}{1 + e^{(\mathcal{H} + \vartheta)\lambda} \mu}. \tag{16}$$

When  $4\alpha\mu > \lambda^2,$

$$\begin{aligned} \mathcal{B}_9(x, t) &= -\frac{1}{2} \sqrt{\lambda^2 - 4\alpha\mu} + \frac{1}{2} \sqrt{-\lambda^2 + 4\alpha\mu} \operatorname{Tan} \left[ \frac{1}{2} (\mathcal{H} + \vartheta) \sqrt{-\lambda^2 + 4\alpha\mu} \right], \\ \mathcal{B}_{10}(x, t) &= -\frac{1}{2} \sqrt{\lambda^2 - 4\alpha\mu} + \frac{1}{2} \sqrt{-\lambda^2 + 4\alpha\mu} \operatorname{Cot} \left[ \frac{1}{2} (\mathcal{H} + \vartheta) \sqrt{-\lambda^2 + 4\alpha\mu} \right], \end{aligned} \tag{17}$$

where  $\mathcal{H} = x + z - (c/q)(t + (q/\Gamma(q)))^q$

2.2. *The (2 + 1)-AKNS Equation.* According to the extended simplest equation method and value of homogenous balance value, we get

$$\mathcal{Q}(b) = \sum_{i=-N}^N a_i f(b)^i = \frac{a_{-1}}{f(b)} + a_0 + f(b)a_1, \quad (18)$$

where  $a_{-1}, a_0,$  and  $a_1$  are the arbitrary constants. Also,  $f(b)$  satisfies equation (9). Substituting equation (18) and its derivative along (9) into equation (6) and collecting all terms with the same power of  $f^i(b), i = (0, 1, 2, \dots)$  lead to a system of algebraic equations. Solving this system with any computer software yields

$$a_{-1} \longrightarrow 0, a_1 \longrightarrow -\mu, c \longrightarrow \frac{1}{4}(\lambda^2 - 4\alpha\mu). \quad (19)$$

Subsequently, the explicit solutions of the fractional (2 + 1)-AKNS equation are given as follows.

When  $\lambda = 0,$   
For  $\alpha\mu > 0,$

$$\begin{aligned} \mathcal{Q}_1(x, t) &= a_0 - \sqrt{\alpha\mu} \operatorname{Tan} \left[ \sqrt{\alpha\mu} \left( x + z + \vartheta + \frac{\alpha\mu(t + (1/\Gamma[q]))^q}{q} \right) \right], \\ \mathcal{Q}_2(x, t) &= -\sqrt{\alpha\mu} \operatorname{Cot} \left[ \sqrt{\alpha\mu} \left( x + z + \vartheta + \frac{\alpha\mu(t + (1/\Gamma[q]))^q}{q} \right) \right] + a_0. \end{aligned} \quad (20)$$

For  $\alpha\mu < 0,$

$$\mathcal{Q}_3(x, t) = a_0 - \sqrt{-\alpha\mu} \operatorname{Tanh} \left[ \sqrt{-\alpha\mu} \left( x + z + \frac{\alpha\mu(t + (1/\Gamma[q]))^q}{q} \right) \mp \frac{\operatorname{Log}[\vartheta]}{2} \right], \quad (21)$$

$$\mathcal{Q}_4(x, t) = -\sqrt{-\alpha\mu} \operatorname{Coth} \left[ \sqrt{-\alpha\mu} \left( x + z + \frac{\alpha\mu(t + (1/\Gamma[q]))^q}{q} \right) \mp \frac{\operatorname{Log}[\vartheta]}{2} \right] + a_0. \quad (22)$$

When  $\alpha = 0,$   
For  $\lambda > 0,$

$$\mathcal{Q}_5(x, t) = \lambda + \frac{\lambda}{-1 + e^{\lambda(x+z+\vartheta - (\lambda^2(t+(1/\Gamma[q]))^q/4q))\mu}} + a_0. \quad (23)$$

For  $\lambda < 0,$

$$\mathcal{Q}_6(x, t) = \mu - \frac{\mu}{1 + e^{\lambda(x+z+\vartheta - (\lambda^2(t+(1/\Gamma[q]))^q/4q))\mu}} + a_0. \quad (24)$$

When  $4\alpha\mu > \lambda^2,$

$$\begin{aligned} \mathcal{Q}_7(x, t) &= \frac{\lambda}{2} + a_0 - \frac{1}{2} \sqrt{-\lambda^2 + 4\alpha\mu} \operatorname{Tan} \left[ \frac{1}{2} \sqrt{-\lambda^2 + 4\alpha\mu} \left( x + z + \vartheta - \frac{(\lambda^2 - 4\alpha\mu)(t + (1/\Gamma[q]))^q}{4q} \right) \right], \\ \mathcal{Q}_8(x, t) &= \frac{\lambda}{2} - \frac{1}{2} \sqrt{-\lambda^2 + 4\alpha\mu} \operatorname{Cot} \left[ \frac{1}{2} \sqrt{-\lambda^2 + 4\alpha\mu} \left( x + z + \vartheta - \frac{(\lambda^2 - 4\alpha\mu)(t + (1/\Gamma[q]))^q}{4q} \right) \right] + a_0. \end{aligned} \quad (25)$$

### 3. Conclusion

This paper has investigated the exact traveling and solitary wave solutions of the fractional SKdV equation and the fractional (2 + 1)-AKNS equation. The extended simplest equation method has successfully been implemented and some new distinct optical solitary wave solutions are obtained for both models. Some solutions have been sketched in three types (three-dimensional, two-dimensional, and contour plots) (Figures 1 and 2). The powerful effect of the used method is illustrated. Moreover, the ability of applying to different types of nonlinear evolution equations has been verified.

### Data Availability

The data that support the findings of this study are available from the corresponding author (Mostafa M. A. Khater) upon reasonable request.

### Conflicts of Interest

The authors declare that there are no conflicts of interest.

### Authors' Contributions

All the authors conceived the study, participated in its design and coordination, drafted the manuscript, participated in the sequence alignment, and read and approved the final manuscript.

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