

## *Retraction*

# **Retracted: Inverted Length-Biased Exponential Model: Statistical Inference and Modeling**

### **Journal of Mathematics**

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This article has been retracted by Hindawi following an investigation undertaken by the publisher [1]. This investigation has uncovered evidence of one or more of the following indicators of systematic manipulation of the publication process:

- (1) Discrepancies in scope
- (2) Discrepancies in the description of the research reported
- (3) Discrepancies between the availability of data and the research described
- (4) Inappropriate citations
- (5) Incoherent, meaningless and/or irrelevant content included in the article
- (6) Manipulated or compromised peer review

The presence of these indicators undermines our confidence in the integrity of the article's content and we cannot, therefore, vouch for its reliability. Please note that this notice is intended solely to alert readers that the content of this article is unreliable. We have not investigated whether authors were aware of or involved in the systematic manipulation of the publication process.

Wiley and Hindawi regrets that the usual quality checks did not identify these issues before publication and have since put additional measures in place to safeguard research integrity.

We wish to credit our own Research Integrity and Research Publishing teams and anonymous and named external researchers and research integrity experts for contributing to this investigation.

The corresponding author, as the representative of all authors, has been given the opportunity to register their agreement or disagreement to this retraction. We have kept a record of any response received.

### **References**

- [1] W. Almutiry, "Inverted Length-Biased Exponential Model: Statistical Inference and Modeling," *Journal of Mathematics*, vol. 2021, Article ID 1980480, 8 pages, 2021.

## Research Article

# Inverted Length-Biased Exponential Model: Statistical Inference and Modeling

Waleed Almutiry 

Department of Mathematics, College of Science and Arts in Ar Rass, Qassim University, Ar Rass, Saudi Arabia

Correspondence should be addressed to Waleed Almutiry; [wkmtierie@qu.edu.sa](mailto:wkmtierie@qu.edu.sa)

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This research article proposes a new probability distribution, referred to as the inverted length-biased exponential distribution. The hazard rate function (HZRF) and density function (PDF) in the new distribution allow additional flexibility as well as some desired features. It provides a more flexible approach that may be used to represent many forms of real-world data. The quantile function (QuF), moments (MOs), moment generating function (MOGF), mean residual lifespan (MRLS), mean inactivity time (MINT), and probability weighted moments (PRWMOs) are among the mathematical and statistical features of the inverted length-biased exponential distribution. In the case of complete and type II censored samples (TIICS), the maximum likelihood (MLL) strategy can be used to estimate the model parameters. An asymptotic confidence interval (COI) of parameter is constructed at two confidence levels. We perform simulation study to examine the accuracy of estimates depending upon some statistical measures. Simulation results show that there is great agreement between theoretical and empirical studies. We demonstrate the new model's relevance and adaptability by modeling three lifespan datasets. The proposed model is a better fit than the half logistic inverse Rayleigh (HLOIR), type II Topp–Leone inverse Rayleigh (TIITOLIR), and transmuted inverse Rayleigh (TRIR) distributions. We anticipate that the expanded distribution will attract a broader range of applications in a variety of fields of research.

## 1. Introduction

Length-biased exponential (LBE) or moment exponential (ME) distribution is considered as one of the most important univariate and parametric models. It is commonly utilized in the analysis of data collected throughout a lifespan and in problems connected to the modeling of failure processes. There is much to be said for a flexible lifespan distribution model, and this one may be a suitable fit for some sets of failure data. Reference [1] proposed the LBE with the following PDF and distribution function (CDF):

$$g(x; \alpha) = \alpha^2 x e^{-\alpha x}; \quad x \geq 0, \alpha > 0, \quad (1)$$

$$G(x; \alpha) = 1 - (1 + \alpha x)e^{-\alpha x}; \quad x \geq 0, \alpha > 0, \quad (2)$$

where  $\alpha$  is the scale parameter. Different values of the shape parameter lead to different shapes of density function.

Many authors extended new models from the LBE distribution such as exponentiated ME [2], generalized exponentiated ME [3], and Marshall–Olkin (MO) LBE (MOLBE) distributions [4]. MO Kumaraswamy ME model was discussed in [5].

Several univariate continuous distributions have been extensively used in environmental, engineering, financial, and biomedical sciences, among other areas for modeling lifetime data. However, there is still a strong need for a significant improvement of the classical distributions through different techniques for modeling several data lifetime. In this regard, the inverted (or inverse) (I) distribution is one procedure that explores extra properties of the phenomenon which cannot be produced from noninverted distributions. Applications of inverted distributions include econometrics as well as the engineering sciences as well as biology and survey sampling as well as medical research among others. In the literature, several studies related to

inverted distributions have been handled by several researchers; for instance, Reference [6] introduced the I Weibull distribution. Reference [7] studied the I Pareto type 1 distribution. Reference [8] investigated the I Pareto type 2 distribution. Reference [9] handled exponentiated I Weibull distribution. Reference [10] provided the I Lindley distribution. Reference [11] suggested the I Kumaraswamy model. Reference [12] presented the I Nadarajah-Haghighi model. Reference [13] studied the I power Lomax model. Reference [14] suggested I exponentiated Lomax model. Reference [15] discussed the Weibull I Lomax model. Reference [16] suggested the power transmuted I Rayleigh model. Reference [17] investigated the I Topp-Leone distribution, and half logistic I Topp-Leone distribution was studied in [18].

Our motivation here is (i) introducing a new distribution, referred to as the inverted length-biased exponential (ILBE), (ii) studying some of the main properties, (iii) providing point and interval estimators for the model parameter from complete and censored samples, and (iv) examining its applicability using three real datasets.

The inverted LBE (ILBE) distribution is constructed by using the random variable  $T=1/X$  where  $X$  follows (2). The ILBE distribution's CDF is described as

$$F(t; \alpha) = \left(1 + \frac{\alpha}{t}\right) e^{-\alpha/t}; \quad t \geq 0, \alpha > 0. \quad (3)$$

The ILBE distribution's PDF is specified as

$$f(t; \alpha) = \frac{\alpha^2}{t^3} e^{-\alpha/t}; \quad t \geq 0, \alpha > 0. \quad (4)$$

The survival function (SRF) and HZRF of the ILBE distribution are provided by

$$\begin{aligned} \bar{F}(t; \alpha) &= 1 - \left(1 + \frac{\alpha}{t}\right) e^{-\alpha/t}, \\ h(t; \alpha) &= \frac{\alpha^2 e^{-\alpha/t}}{t^3 [1 - (1 + \alpha/t) e^{-\alpha/t}]}. \end{aligned} \quad (5)$$

Figure 1 depicts PDF and HZRF plots for the ILBE distribution. According to Figure 1, the density of the suggested distribution is highly flexible in nature and can take on a number of forms, including positively skewed and unimodal. Through the parameter space, the HZRF can take on many forms, such as decreasing, rising, or upside down.

This paper is organized as follows. In Section 2, the basic characteristics of the ILBE distribution are obtained. The MLL estimators for the ILBE model are described in Section 3 and are established on complete and censored samples, accompanied by a simulation analysis. The application to actual data collection is covered in Section 4. Section 5 concludes the paper with some remarks.

## 2. Fundamental Mathematical Properties of ILBE Distribution

Here, we give some essential properties of the ILBE distribution, like QuF, MOs, PRWMOs, incomplete MOs, and inverse MOs.

**2.1. Quantile Function.** A generated random number from the ILBE distribution is obtained by solving the following equation numerically:

$$Q(u) = \frac{\alpha}{-1 - W_{-1}(-e^{-1}u)}, \quad 0 < u < 1, \quad (6)$$

where  $W_{-1}$  denotes the negative branch of the Lambert  $W$  function (i.e., the solution of the equation  $W(Z)e^{W(Z)} = z$ ). The median, say  $Q_2$ , is achieved by adjusting  $u = 0.5$  in (6), and the first quartile and third quartile, denoted by  $Q_1$  and  $Q_3$ , are obtained by setting  $u = 0.25$  and  $0.75$ , respectively, in (6). Note that equation (6) is solved numerically by using Mathematica 9.

$$\begin{aligned} Q_1 &= \frac{\alpha}{-1 - W_{-1}(-0.25e^{-1})}, \\ Q_2 &= \frac{\alpha}{-1 - W_{-1}(-0.5e^{-1})}, \\ Q_3 &= \frac{\alpha}{-1 - W_{-1}(-0.75e^{-1})}. \end{aligned} \quad (7)$$

**2.2. Moments.** Due to its relevance in any statistical study, we shall give the  $n$ -th MO of the ILBE distribution here. For the ILBE model, the  $n$ -th MO of  $T$  about the origin is computed as follows:

$$\mu'_n = E(T^n) = \int_0^\infty t^n \frac{\alpha^2}{t^3} e^{-\alpha/t} dt = \alpha^n \Gamma(2-n), \quad n < 2. \quad (8)$$

The following formula may be used to determine the MOGF of the ILBE distribution:

$$M_x(t) = \sum_{n=0}^\infty \frac{t^n}{n!} \mu'_n = \sum_{n=0}^\infty \frac{t^n}{n!} \alpha^n \Gamma(2-n), \quad n < 2. \quad (9)$$

The incomplete (IN) MO, say  $c_n(x)$ , is

$$c_n(t) = \alpha^2 t \int_0^t t^{n-3} e^{-\alpha/t} dt = \alpha^n \gamma\left(2-n, \frac{\alpha}{t}\right), \quad n < 2, \quad (10)$$

where  $\gamma(\cdot, t)$  is the upper IN gamma function.

Further, the conditional MO, say  $\omega_s(x)$ , is

$$\omega_s(t) = \alpha^2 \int_t^\infty t^{\alpha-3} e^{-\alpha/t} dt = \alpha^n \Gamma\left(2-n, \frac{\alpha}{t}\right), \quad n < 2, \quad (11)$$

where  $\Gamma(\cdot, t)$  is the lower IN gamma function.

For the ILBE distribution, the  $n$ -th inverse MO is calculated on the basis:

$$\tau_k(x) = \alpha^2 \int_0^\infty t^{-n-3} e^{-\alpha/t} dt = \alpha^{-k} \Gamma(n+2). \quad (12)$$

For  $n=1$ , we get the harmonic mean of the ILBE distribution.

The Lorenz and Bonferroni curves are obtained as follows.

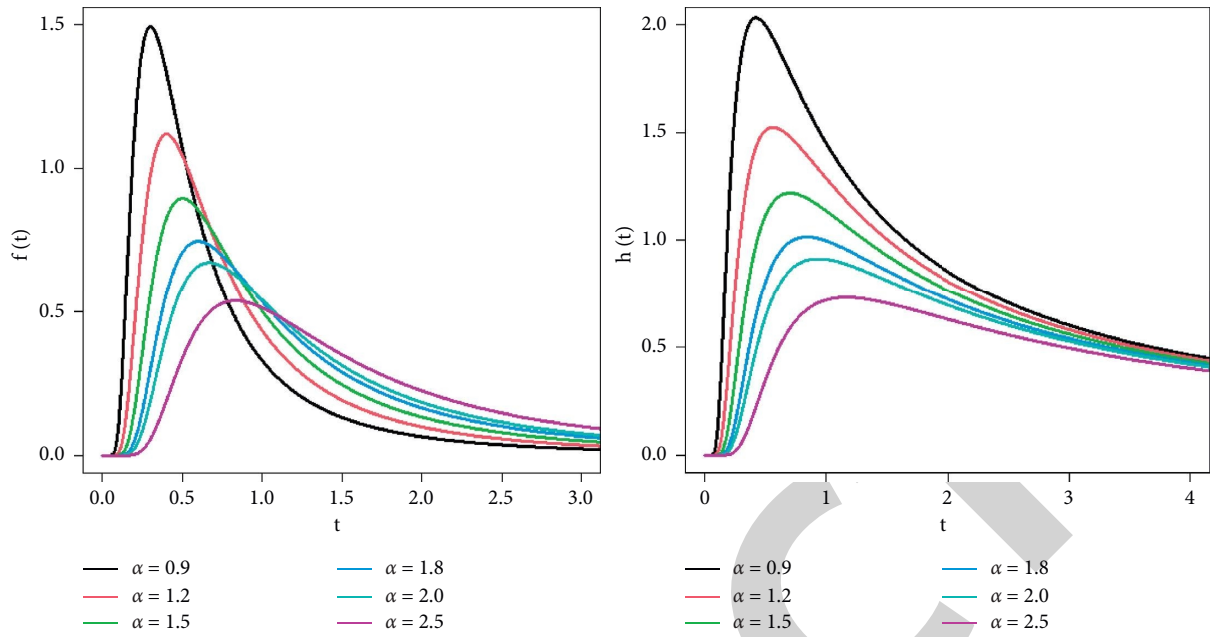


FIGURE 1: Plots of PDF and HZRF for the ILBE distribution.

$$L_F(t) = \frac{\zeta_1(t)}{E(T)} = \gamma\left(1, \frac{\alpha}{t}\right), \tag{13}$$

$$B_F(t) = \frac{L_F(t)}{F(t)} = \frac{\gamma\left(1, (\alpha/t)\right)}{\left(1 + (\alpha/t)\right)e^{-\alpha/t}}. \tag{14}$$

2.3. *Order Statistics.* Let  $T_1, T_2, \dots, T_n$  be  $r$  samples from the ILBE model with order statistics  $T_{(1)}, T_{(2)}, \dots, T_{(n)}$ . The PDF of  $T_{(k)}$  of order statistics is given by

$$f_{T_{(k)}}(t) = \frac{n!}{(k-1)!(n-k)!} F^{k-1}(t) f(t) (1-F(t))^{n-k}. \tag{15}$$

The PDF of  $T_{(k)}$  can be expressed as

$$f_{T_{(k)}}(t) = \frac{n! \alpha^2}{(k-1)!(n-k)!} t^{-3} \left(1 + \frac{\alpha}{t}\right)^{k-1} e^{-k\alpha/t} \left(1 - \left(1 + \frac{\alpha}{t}\right)e^{-\alpha/t}\right)^{n-k}. \tag{16}$$

Particularly, PDF of the first and largest order statistics can be calculated as

$$f_{T_{(1)}}(t) = n\alpha^2 t^{-3} e^{-\alpha/t} \left(1 - \left(1 + \frac{\alpha}{t}\right)e^{-\alpha/t}\right)^{n-1}, \tag{17}$$

$$f_{T_{(n)}}(t) = n\alpha^2 t^{-3} \left(1 + \frac{\alpha}{t}\right)^{n-1} e^{-n\alpha/t}, \tag{18}$$

respectively.

2.4. *Mean Residual Life Function.* It has an important application of the MOs of residual lifetime function. The MRLS of ILBE distribution is

$$\begin{aligned} v(t) &= E(T-t|T>t) = \frac{1}{\bar{F}(t)} \int_t^\infty xf(x)dt - t \\ &= \alpha\Gamma\left(1, \frac{\alpha}{t}\right) \left[1 - \left(1 + \frac{\alpha}{t}\right)e^{-\alpha/t}\right]^{-1} - t. \end{aligned} \tag{19}$$

The MINT represents the amount of time that has passed after an item has failed, assuming that this failure has occurred. The MINT of ILBE distribution is

$$\begin{aligned} \omega(t) &= E(T-t|T\leq t) = t - \frac{1}{F(t)} \int_0^t xf(x)dx \\ &= t - \alpha\gamma\left(1, \frac{\alpha}{t}\right) \left(1 + \frac{\alpha}{t}\right)^{-1} e^{\alpha/t}. \end{aligned} \tag{20}$$

2.5. *Probability Weighted Moments.* The PRWMOs are often used to investigate additional aspects of the probability distribution. The PRWMOs of the random variable  $T$ , denoted by  $S_{r,p}$ , are defined as

$$S_{r,p} = \int_{-\infty}^\infty t^r f(t) [F(t)]^p dt, \tag{21}$$

where  $r$  and  $p$  are positive integers. Substituting (3) and (4) into (21) yields the PRWMOs of the ILBE distribution as follows:

TABLE 1: MLE,  $\wp$ ,  $\mathfrak{F}$ ,  $\bar{\sigma}$ ,  $\mathfrak{H}$ , and  $\sqsupset$  of the ILBE distribution for  $\alpha = 1.2$  under TIIC.

n	$t_r$ (%)	MLE	$\wp$	$\mathfrak{F}$	$\bar{\sigma}$	90%		95%		
						$\mathfrak{H}$	$\sqsupset$	$\bar{\sigma}$	$\mathfrak{H}$	$\sqsupset$
100	60	1.6091	0.4091	0.2111	1.3068	1.9113	0.6045	1.2489	1.9692	0.7203
	80	1.3548	0.1548	0.0546	1.0803	1.6294	0.5490	1.0278	1.6819	0.6542
	100	1.2316	0.0316	0.0263	0.9700	1.4931	0.5231	0.9199	1.5432	0.6233
200	60	1.5648	0.3648	0.1575	1.3372	1.7924	0.4552	1.2937	1.8360	0.5424
	80	1.3195	0.1195	0.0319	1.1125	1.5266	0.4142	1.0728	1.5663	0.4935
	100	1.1993	0.0007	0.0146	1.0020	1.3965	0.3946	0.9642	1.4343	0.4701
300	60	1.5777	0.3777	0.1551	1.4154	1.7399	0.3245	1.3844	1.7710	0.3866
	80	1.3307	0.1307	0.0256	1.1831	1.4784	0.2953	1.1548	1.5067	0.3519
	100	1.2097	0.0097	0.0071	1.0690	1.3504	0.2814	1.0420	1.3773	0.3353

TABLE 2: MLE,  $\wp$ ,  $\mathfrak{F}$ ,  $\bar{\sigma}$ ,  $\mathfrak{H}$ , and  $\sqsupset$  of the ILBE distribution for  $\alpha = 1.5$  under TIIC.

n	$t_r$ (%)	MLE	$\wp$	$\mathfrak{F}$	$\bar{\sigma}$	90%		95%		
						$\mathfrak{H}$	$\sqsupset$	$\bar{\sigma}$	$\mathfrak{H}$	$\sqsupset$
100	60	1.9815	0.4815	0.2846	1.6091	2.3538	0.7447	1.5378	2.4251	0.8873
	80	1.6720	0.1720	0.0666	1.3332	2.0107	0.6775	1.2683	2.0756	0.8073
	100	1.5196	0.0196	0.0313	1.1969	1.8423	0.6454	1.1351	1.9041	0.7690
200	60	1.9785	0.4785	0.2638	1.6907	2.2664	0.5757	1.6356	2.3215	0.6859
	80	1.6695	0.1695	0.0544	1.4075	1.9315	0.5240	1.3573	1.9817	0.6243
	100	1.5170	0.0170	0.0215	1.2674	1.7665	0.4991	1.2196	1.8143	0.5946
300	60	1.9562	0.4562	0.2273	1.7550	2.1573	0.4023	1.7165	2.1958	0.4793
	80	1.6486	0.1486	0.0359	1.4656	1.8315	0.3659	1.4306	1.8666	0.4359
	100	1.4988	0.0012	0.0114	1.3245	1.6731	0.3487	1.2911	1.7065	0.4154

$$\begin{aligned} \mathbb{S}_{r,p} &= \alpha^2 \int_0^\infty t^{r-3} \left(1 + \frac{\alpha}{t}\right)^p e^{-(p+1)\alpha/t} \\ &= \sum_{j=0}^p \binom{p}{j} \alpha^{j+2} \int_0^\infty t^{r-3-j} e^{-(p+1)\alpha/t} dt. \end{aligned} \tag{22}$$

As a result of the simplification, the PRWMOs of the ILBE distribution assume the following structure:

$$\mathbb{S}_{r,p} = \sum_{j=0}^p \binom{p}{j} \alpha^r \frac{\Gamma(j-r+2)}{(p+1)^{j-r+2}}. \tag{23}$$

### 3. Statistical Inference

3.1. *MLL Estimator Based on TIIC.* Assume  $T_{(1)}, T_{(2)}, \dots, T_{(n)}$  are the recorded TIICS of size  $r$ , whose lifetimes have the ILBE distribution with PDF (4), and the experiment is completed once the  $r$ -th object fails for just some fixed values of  $r$ . The log-likelihood function (LLF), according to TIIC, is provided by

$$\begin{aligned} \ln l_2 &= \ln C + 2r \ln \alpha - 3 \sum_{i=1}^r \ln t_i - \sum_{i=1}^r \frac{\alpha}{t_i} \\ &+ (n-r) \ln \left[ 1 - \left(1 + \frac{\alpha}{t_r}\right) e^{-\alpha/t_r} \right], \end{aligned} \tag{24}$$

and for the sake of simplification, we abbreviate  $t_i$  rather than  $t_{(i)}$ . As a result, the partial derivatives of the LLF with regard to the component of the score  $U(\alpha) = \partial \ln l_2 / \partial \alpha$  may be computed as follows:

$$U(\alpha) = \frac{2r}{\alpha} - \sum_{i=1}^r \frac{1}{t_i} + \frac{(n-r)\alpha e^{-\alpha/t_r}}{t_r^2 [1 - ((1 + (\alpha/t_r))e^{-\alpha/t_r})]}. \tag{25}$$

The model parameters' MLL estimator is produced by numerically solving equation (18) after assigning it to zero. In the case of a complete sample, we acquire the MLL estimators of the model parameters for  $r = n$ .

3.2. *Simulation Results.* A simulation is used to evaluate the estimators' behavior considering a set of parameter choices. Mean square error ( $\wp$ ), bias ( $\mathfrak{F}$ ), lower limit ( $\bar{\sigma}$ ) of the COIs, upper bound ( $\mathfrak{H}$ ) of the COIs, and average length ( $\sqsupset$ ) of 90% and 95% are among the metrics computed. All numerical calculations are made using the R programming (R 4.1.1). The following algorithm are used:

- (i) On aggregate, the ILBE distribution produces 1000 random samples with sizes of  $n = 100, 200,$  and  $300$ .
- (ii) Values for a few parameters are  $\alpha = 1.2$  and  $\alpha = 1.5$ .
- (iii) There are three degrees of censorship:  $r = 60\%, 80\%$  (TIIC), and  $100\%$  (complete sample).
- (iv)  $\wp, \mathfrak{F}, \bar{\sigma}, \mathfrak{H},$  and  $\sqsupset$  of estimates are computed.

Tables 1 and 2 include the numerical findings for the complete and TIIC measurements, respectively.

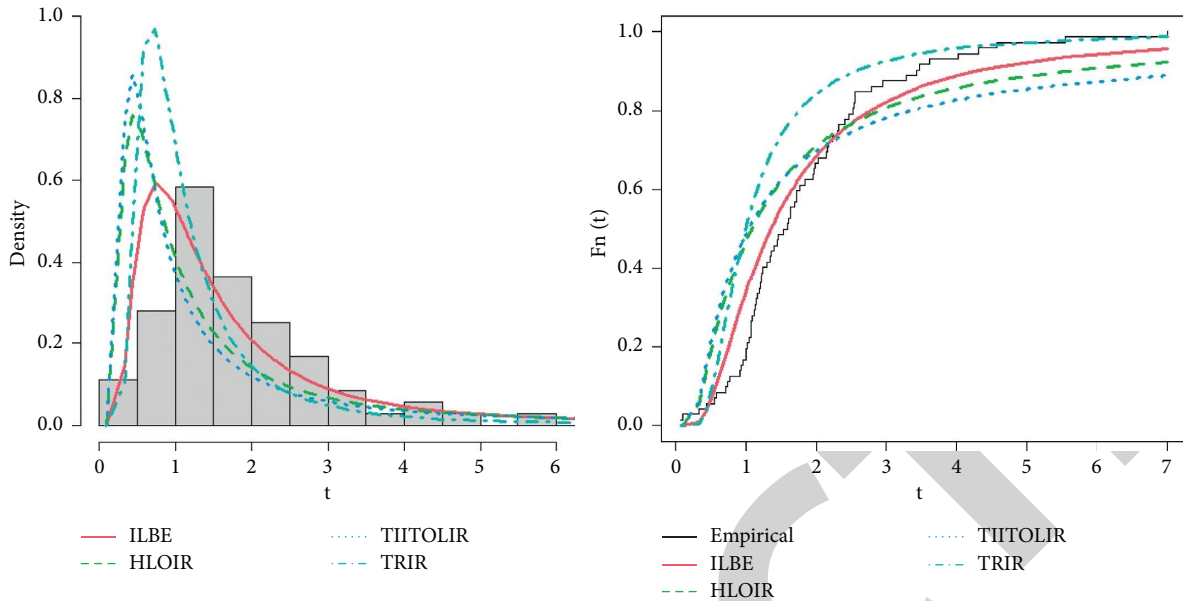


FIGURE 2: Fitted PDFs and CDFs of comparison models for the first dataset.

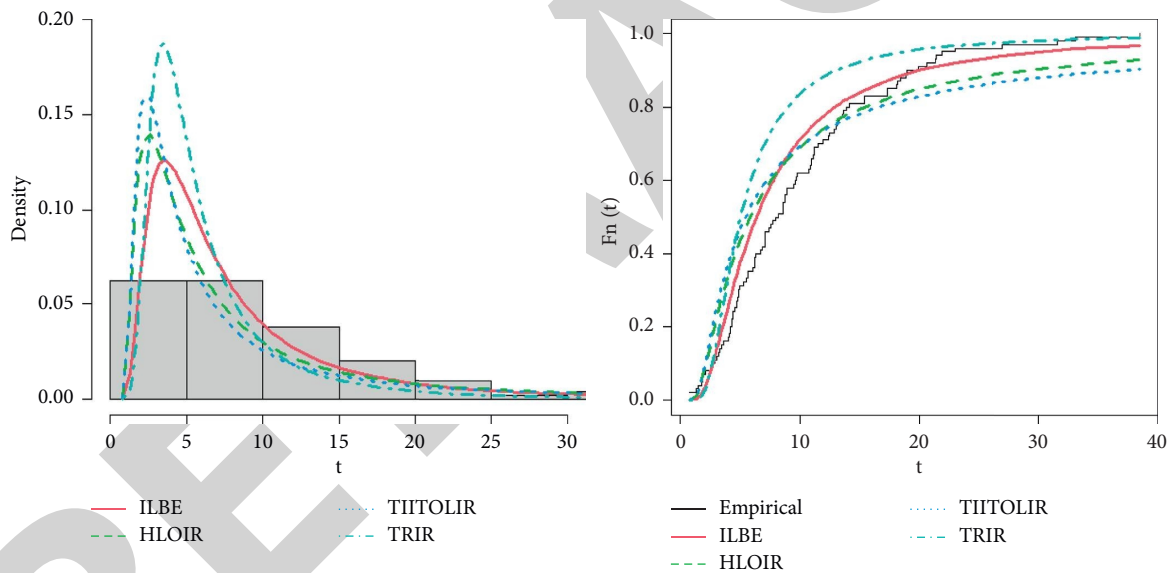


FIGURE 3: Fitted PDFs and CDFs of comparison models for the second dataset.

From these tables, we conclude the following:

- (i) As the sample size grows,  $\varphi$ ,  $\mathfrak{S}$ , and  $\sqsupset$  of all estimates decrease.
- (ii)  $\varphi$ ,  $\mathfrak{S}$ , and  $\sqsupset$  of all estimates decrease as  $r$  decreases.
- (iii)  $\sqsupset$  of the COIs increases as the confidence levels increase from 90% to 95%.

#### 4. Applications to Real Data

In this part, we demonstrate the ILBE model's adaptability by examining three real-world datasets. Comparing the fit of

the ILBE model with known distributions such as the HLOIR [19], TIITOLIR [20], and TRIR [21] distributions, the ILBE model performs better. The PDFs of competitive models are

$$f_{HLOIR}(t) = \frac{4\lambda\alpha^2 t^{-3} \exp(-(\alpha/t)^2) (1 - \exp(-(\alpha/t)^2))^{\lambda-1}}{(1 + (1 - \exp(-(\alpha/t)^2))^\lambda)^2},$$

$$f_{TIITOLIR}(t) = 4\theta\alpha^2 t^{-3} \exp(-2(\alpha/t)^2) (1 - \exp(-2(\alpha/t)^2))^{\theta-1}, \tag{26}$$

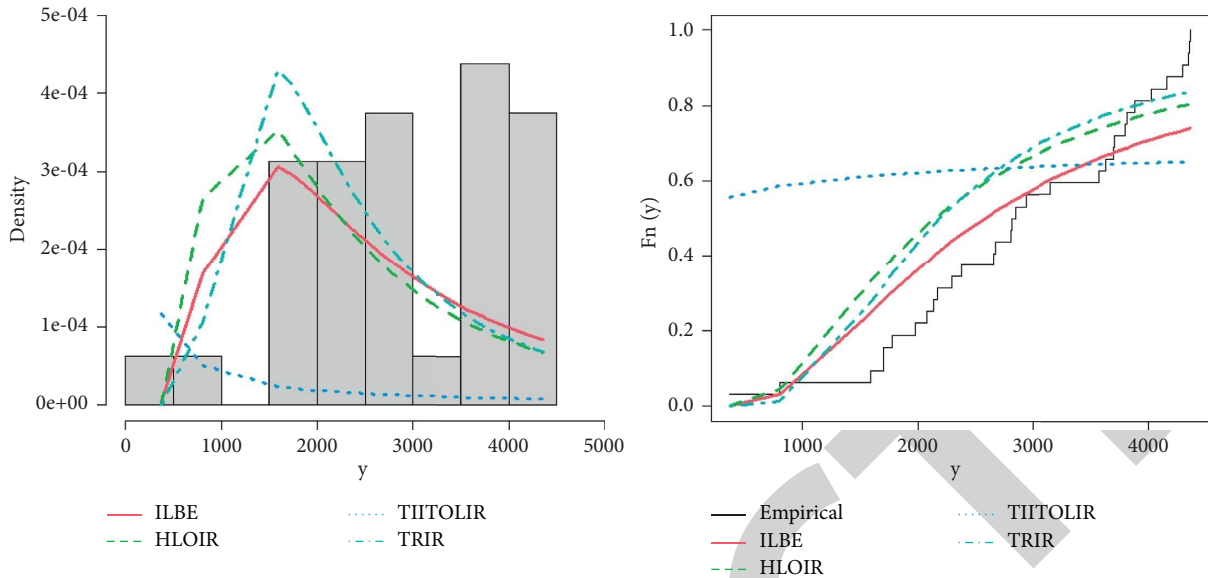


FIGURE 4: Fitted PDFs and CDFs of comparison models for the third dataset.

TABLE 3: Numerical results of MLE, SE,  $\wedge 1$ ,  $\wedge 2$ ,  $\wedge 3$ ,  $\wedge 4$ , and  $\wedge 5$  for the first dataset.

Model	MLEs and SE		$\wedge 1$	$\wedge 2$	$\wedge 4$	$\wedge 5$	$\wedge 3$
ILBE ( $\alpha$ )	2.272 (0.189)		224.111	226.111	225.969	227.018	226.168
HLIR ( $\alpha, \lambda$ )	0.436 (0.05)	0.579 (0.07)	260.586	264.586	264.301	266.399	264.76
TIITLIR ( $\alpha, \lambda$ )	0.325 (0.036)	0.404 (0.058)	280.492	284.492	284.207	286.305	284.666
TIR ( $\alpha, \lambda$ )	0.352 (0.426)	-0.942 (0.351)	280.538	284.538	284.253	286.351	284.712

TABLE 4: Numerical results of MLE, SE,  $\wedge 1$ ,  $\wedge 2$ ,  $\wedge 3$ ,  $\wedge 4$ , and  $\wedge 5$  for the second dataset.

Model	MLEs and SE		$\wedge 1$	$\wedge 2$	$\wedge 4$	$\wedge 5$	$\wedge 3$
ILBE ( $\alpha$ )	10.696 (0.7563)		664.794	666.794	666.794	667.848	666.834
HLIR ( $\alpha, \lambda$ )	2.404 (0.226)	0.589 (0.06)	680.806	684.806	684.806	686.915	684.93
TIITLIR ( $\alpha, \lambda$ )	1.824 (0.162)	0.43 (0.051)	700.214	704.214	704.214	706.323	704.338
TIR ( $\alpha, \lambda$ )	9.978 (1.136)	-0.812 (0.085)	720.665	724.665	724.665	726.774	724.706

TABLE 5: Numerical results of MLE, SE,  $\wedge 1$ ,  $\wedge 2$ ,  $\wedge 3$ ,  $\wedge 4$ , and  $\wedge 5$  for the third dataset.

Model	MLEs and S.E		$\wedge 1$	$\wedge 2$	$\wedge 4$	$\wedge 5$	$\wedge 3$
ILBE ( $\alpha$ )	4326 (540.7191)		567.141	569.141	568.647	569.627	569.275
HLIR ( $\alpha, \lambda$ )	1237 (184.326)	0.866 (0.172)	575.242	579.242	578.252	580.214	579.656
TIITLIR ( $\alpha, \lambda$ )	0.069 (0.041)	0.049 (0.0091)	716.204	720.204	719.214	721.176	720.618
TIR ( $\alpha, \lambda$ )	1821000 (346600)	-0.859 (0.126)	575.303	579.303	578.313	580.275	579.717

$$f_{TRIR}(t) = 2\theta t^{-3} \exp\left(-\frac{\theta}{t^2}\right) \left(1 + \lambda - 2\lambda \exp\left(-\frac{\theta}{t^2}\right)\right). \tag{27}$$

In order to make a comparison between various models, some information criteria (INC) like maximized likelihood (?1), Akaike INC (?2), consistent Akaike INC (?3), Bayesian INC (?4), and Hannan–Quinn INC (?5) are used. According to the given data, the optimal model is one with the lowest value of ?1, ?2, ?3, ?4, and ?5.

**The first dataset** [22]: it describes 72 guinea pigs infected with highly pathogenic tubercle bacilli and their survival periods (in days).

**The second dataset**: acquired and documented in [23], the dataset comprises the waiting times (in minutes) of 100 bank clients.

**The third dataset** [24]: it offers 32 observations on the failure time for vertical boring machines.

Figures 2–4 indicate the fitted PDFs, fitted CDFs of the ILBE distribution, and those of the comparison models (HLOIR, TIITOLIR, and TRIR) for the three datasets.

It can be observed from Figures 2–4 that the ILBE distribution exhibits good matches, attesting its applicability for the three datasets.

Tables 3–5 show the ML estimates (MLEs) and standard errors (SEs) for the ILBE model when compared to various known distributions such like HLOIR, TIITOLIR, and TRIR. They also include the relevant measures of fit statistic.

Furthermore, Tables 3–5 show that the ILBE distribution is the best match among the other models for the three datasets, since the ILBE distribution has the lowest values of the suggested metrics.

## 5. Conclusions

This paper developed a new one-parameter lifetime distribution, named as inverse length-biased exponential distribution. The new model is quite flexible in nature and can acquire a variety of shapes of density and hazard rate functions. MOs, PRWMOs, inverse MOs, incomplete MOs, MRLS, and MINT are all explored as key characteristics of the new distribution. In both complete and censored samples, the maximum likelihood methodology is developed to calculate the parameters of the new distribution. To investigate the conduct of estimations, a simulation analysis is discussed. Three real-world examples show that the inverse length exponential distribution gives a pretty good fit and may be used as a competitive model to fit real-world data. It is hoped that this distribution would be helpful to scholars in a variety of disciplines. In the future, we plan to use the new proposed model to study the statistical inference of it under different censored schemes, using various methods of estimation to assess the performance of its parameters. Also, researchers can extend and generalized it because this model is very simple and has more flexibility to fitting more datasets.

## Data Availability

Interested parties can reach out to the author in order to receive the numerical dataset used to perform the research described in the paper.

## Conflicts of Interest

The author declares that there are no conflicts of interest.

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