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Research Article

Energy Consumption Predication in China Based on the Modified Fractional Grey Prediction Model

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China's increasing energy consumption poses challenges to economy and environment. How to predict the energy consumption accurately and regulate the future energy consumption production is a problem worth studying. In this paper, the fractional order cumulative linear time-varying parameter discrete grey prediction model (FTDGM (1, 1) model) is introduced. Firstly, the data are preprocessed by buffer operators, and then, the FTDGM (1, 1) model is established. In this paper, the parameter estimation method and the specific process of model establishment are presented. Finally, the models of energy consumption in China are built. The advantages and prediction accuracy of the model established in this paper are analyzed, and the data in the following years are effectively predicted, so as to provide theoretical support for the government to formulate reasonable energy policies.

1. Introduction

Grey system theory is a small sample modeling method proposed by Professor Deng Julong in 1982, which mainly includes grey prediction theory, grey correlation theory, grey decision theory, grey clustering theory, and grey game theory [1, 2]. Among them, the grey predication theory is an important part of grey system theory, for the reason that the grey prediction theory does not have strict requirements on data. For example, the data do not need to meet certain distribution rule, and the high precision model can be established based on a small amount of data. Therefore, since the model is put forward, it has been widely used in the fields of agriculture, energy, economy, medicine, and so on and has achieved fruitful achievements [3]. Compared with other uncertainty methods, the advantage of grey system theory is that it treats the modeling object as a system and uses the idea of system theory to build the model.

In recent years, many scholars have carried out extensive research studies on the grey prediction model [4–17], whose achievements have played a positive role in improving the grey prediction theory. The GM (1, 1) model requires the transformation of differential equation and difference equation, which will lead to systematic errors. Based on this, Xie and Liu constructed the DGM (1, 1) model and its extended form. The DGM (1, 1) model does not require the conversion of differential equation and difference equation, but directly uses the difference equation for parameter estimation and model solution, so as to effectively reduce the error source within the model [18]. The GM (1, 1) and DGM (1, 1) models construct prediction models by accumulating quasi-exponential rules of sequences. However, accumulative sequences contain linear laws as well as quasi-exponential laws. Based on this, Zhang and Liu proposed a discrete grey prediction model with linear time-varying parameters and discussed the properties of the model and the parameter solving method, and the model has a completely error-free simulation accuracy for sequences with linear laws and exponential laws [19]. Wu et al. constructed time-varying parameters of the GM (1, 1) model by introducing a polynomial function of time [20]. Accumulation is the basis of grey prediction, through which the sequence regularity can be highlighted. Traditional accumulation is a kind of first-order accumulation, so the importance of data has not been differentiated. In view of this, Wu Lifeng puts forward a fractional accumulative grey predication model and discussed the modeling process and parameter-solving method [21, 22].

Time series are generally affected by the shock disturbance of the system. When the shock disturbance exists in the system, there will be relatively large modeling errors in the modeling of time series, and the inconsistency between the qualitative analysis and quantitative calculation will occur. This paper proposes a discrete grey prediction model based on the weakening buffer operator and fractional order accumulation of linear time-varying parameters. Firstly, the buffer operators are used to weaken the impact disturbance of the system, and then, the prediction model is constructed. In this paper, the structure is as follows. Section 2 introduces the differences between existing accumulative methods, the advantages of the fractional order accumulation and its calculation method, and the parameter-solving method. In Section 3, the modeling process of the fractional order accumulative linear time-varying discrete grey prediction model is described. In Section 4, two practical examples are given, and the feasibility and practicability of the proposed method is verified. The conclusion is drawn in Section 5.

2. Construction of Fractional Discrete Grey Models

The grey prediction model is established based on the accumulation sequence, which can effectively reduce the randomness of the system and show the development law of the system. For example, for an original data sequence $X^{(0)} = (1, 4, 2, 5, 2, 3)$, its first-order accumulation sequence is $X^{(1)} = (1, 5, 7, 12, 14, 17)$, as shown in Figure 1.

It can be seen from Figure 1 that the generation of first-order accumulation can transform the oscillation sequence with no obvious regularity into a monotonic increasing sequence, thus highlighting the change law of data and making a good preliminary preparation for the establishment of the grey prediction model.

The disadvantages of the integer order cumulative grey prediction model are analyzed by taking DGM (1, 1) as an example. For a sequence $X^{(0)} = (1, 2, 6, 10, 14, 19)$, the first-order cumulative DGM (1, 1) model is established. And, we can calculate the parameters as follows: $\beta_1 = 1.49$ and $\beta_2 = 3.85$. The simulation sequence is $\widehat{X}^{(0)} = (1.00, 4.34, 6.46, 9.61, 14.31, 21.30)$, and the average relative error is 23.78%. In comparison, if the 0.2 order cumulative DGM (1, 1) model is established, then we can get $\beta_1 = 1.27$ and $\beta_2 = 2.51$, the simulation sequence $\widehat{X}^{(0)} = (3.00, 4.85, 7.38, 10.71, 15.11, 20.93, 28.64)$, and the average relative error is 3.07%.

Assume that the original value of the sequence is changed, and there is a sequence $X^{(0)} = (3, 2, 6, 10, 14, 19)$; then, there are $\beta_1 = 1.49$ and $\beta_2 = 2.87$, the simulation sequence = (3.00, 4.34, 6.46, 9.61, 14.31, 21.30), and the average relative error remains 23.78%. In comparison, when the 0.2-order cumulative DGM (1, 1) model is established, we can get $\beta_1 = 1.27$ and $\beta_2 = 2.51$, the simulation sequence $\widehat{X}^{(0)} = (1.00, 3.58, 6.46, 9.94, 14.24, 19.61, 26.34)$, and the average relative error changes to 10.38%.

It can be seen from the above results that, for an integer order accumulation at the DGM (1, 1) model, the change of the original value will not influence the modeling results of the sequence, which means that the first value has not been effective utilized, which is a serious waste for small sample modeling. As to the fractional order accumulation at the DGM (1, 1) model, the change of the original value can lead to sequence change and the corresponding change of simulation precision, which means that fractional-order accumulation can effectively use the original value. Moreover, with the adjustment of the order, the fractional-order cumulative DGM (1, 1) model can obtain better modeling accuracy than the integer order cumulative DGM (1, 1) model. The same conclusion can be reached for other grey prediction models.

3. Construction of Novel Fractional Discrete Grey Model

For the series with disturbance information if the grey prediction model is established based on the original data, the simulation and prediction results obtained will be quite different from the actual trend of the system. This paper uses a classical weakening buffer operator to preprocess the data.

Definition 1. Assume a nonnegative sequence $X^{(1)} = \{x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(n)\}$ is the first-order accumulative sequence of $X^{(0)}$, where

$$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i), \quad k = 1, 2, \ldots, n. \tag{1}$$

Then,

$$x^{(1)}(k+1) = \beta_1 x^{(1)}(k) + \beta_2, \quad k = 1, 2, \ldots, n-1, \tag{2}$$

is called the discrete grey prediction model (DGM (1, 1) model).

Theorem 1. *The parameters of the DGM (1, 1) model can be solved by using the following least squares estimation:*

$$\begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} = \left(B^T B\right)^{-1} B^T Y. \tag{3}$$

Among them

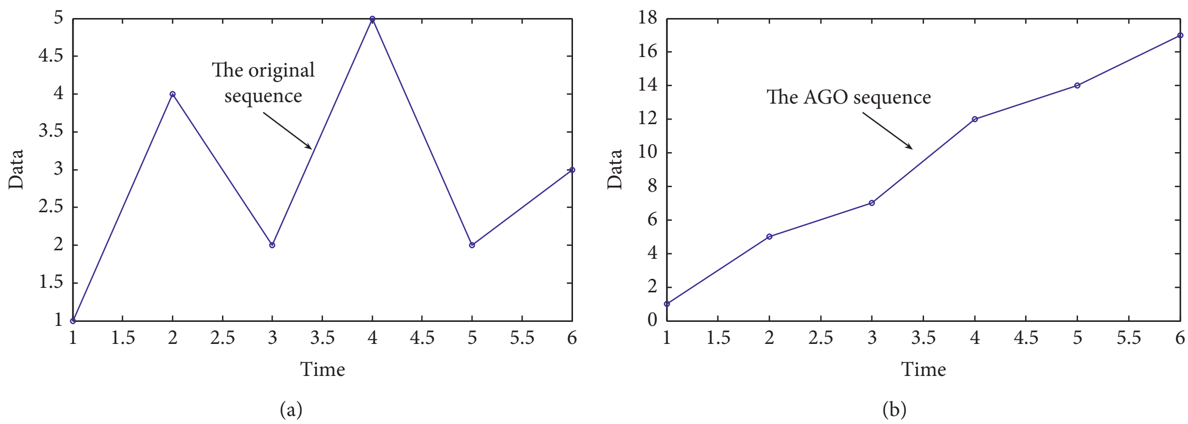

Figure 1: Comparison between the original data sequence and its first-order accumulation.

$$B = \begin{bmatrix} x^{(1)}(1) & 1 \\ x^{(1)}(2) & 1 \\ \vdots & \vdots \\ x^{(1)}(n-2) & 1 \\ x^{(1)}(n-1) & 1 \end{bmatrix}, \quad Y = \begin{bmatrix} x^{(1)}(2) \\ x^{(1)}(3) \\ \vdots \\ x^{(1)}(n-1) \\ x^{(1)}(n) \end{bmatrix}. \tag{4}$$

The time response formula is as follows:

$$x^{(1)}(k+1) = \beta_1^k x^{(0)}(1) + \frac{1-\beta_1^k}{1-\beta_1}\beta_2, \quad k = 1, 2, \ldots, n-1. \tag{5}$$

From the above modeling, it can be seen that the DGM (1, 1) model avoids the transformation from differential equation to check score equation, so it can reduce the system error. The DGM (1, 1) model is an exponential model, which reflects the exponential law generated by accumulation. After accumulating, the sequence has both a trend of index and a linear trend. The linear time-varying parameters' discrete grey prediction model is inclusive for its complete linear specification. One of the problems for a linear time-varying discrete grey prediction model lies in ineffective use of the original value. Based on this, this paper carried out modeling by the fractional-order cumulative linear time-varying parameter discrete grey prediction model.

Definition 2. Assume the nonnegative sequence is $X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)\}$. $X^{(1)} = \{x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(n)\}$ is the first-order accumulative sequence of $X^{(0)}$.

Among them

$$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i), \quad k = 1, 2, \ldots, n. \tag{6}$$

The equation,

$$x^{(1)}(k+1) = (\beta_1 k + \beta_2)x^{(1)}(k) + \beta_3 k + \beta_4, \quad k = 1, 2, \ldots, n-1, \tag{7}$$

is called linear time-varying parameters' discrete grey model TDGM (1,1) model.

Theorem 2. *The parameters of the TDGM (1, 1) model can be solved by using the following least squares estimation:*

$$\begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{bmatrix} = \left(C^T C\right)^{-1} C^T Y. \tag{8}$$

Among them

$$C = \begin{bmatrix} x^{(1)}(1) & x^{(1)}(1) & 1 & 1 \\ 2x^{(1)}(2) & x^{(1)}(2) & 2 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ (n-2)x^{(1)}(n-2) & x^{(1)}(n-2) & n-2 & 1 \\ (n-1)x^{(1)}(n-1) & x^{(1)}(n-1) & n-1 & 1 \end{bmatrix}, \quad Y = \begin{bmatrix} x^{(1)}(2) \\ x^{(1)}(3) \\ \vdots \\ x^{(1)}(n-1) \\ x^{(1)}(n) \end{bmatrix}. \tag{9}$$

The time response formula is

$$\hat{x}^{(1)}(k+1) = (\beta_1 k + \beta_2)\hat{x}^{(1)}(k) + \beta_3 k + \beta_4, \quad k = 1, 2, \ldots, n-1. \tag{10}$$

Theorem 3. *For sequence $y = ab^k$, parameters $\beta_1 = \beta_3 = 0$, and the TDGM (1, 1) model is degraded to the DGM (1, 1) model. For the sequence $y = ax + b$, parameter $\beta_1 = 0$, and the TDGM (1, 1) model was degraded to the NDGM (1, 1) model. For sequence $y = ab^k + c$, parameter $\beta_1 = 0$, and the TDGM (1, 1) model was degraded to the NDGM (1, 1) model.*

It can be seen from Theorem 2 that the DGM (1, 1) model and NDGM (1, 1) model are uniform special cases of the TDGM (1, 1) model.

Definition 3 (see [20–22]). Assume the nonnegative sequence is $X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)\}$. $X^{(r)} = \{x^{(r)}(1), x^{(r)}(2), \ldots, x^{(r)}(n)\}$ is called the fractional-order accumulative sequence of $X^{(0)}$

Among them

$$x^{(r)}(k) = \sum_{i=1}^{k} C_{k-i+r-1}^{k-i} x^{(0)}(i), C_{r-1}^{0} = 1, C_{k-1}^{k} = 0, \quad k = 1, 2, \ldots, n. \tag{11}$$

Definition 4 (see [23]). Assume the nonnegative sequence $X^{(0)}$, $X^{(r)}$ is defined as Definition 3. The equation

$$x^{(r)}(k+1) = (\beta_1 k + \beta_2) x^{(r)}(k) + \beta_3 k + \beta_4, \quad k = 1, 2, \ldots, n-1, \tag{12}$$

is called the fractional-order accumulative linear time-varying parameters' discrete grey model FTDGM (1,1) model.

Theorem 4. *The parameters of the FTDGM (1, 1) model can be solved by using the following least squares estimation:*

$$\begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{bmatrix} = \left(D^T D\right)^{-1} D^T W. \tag{13}$$

Among them

$$D = \begin{bmatrix} x^{(r)}(1) & x^{(r)}(1) & 1 & 1 \\ 2x^{(r)}(2) & x^{(r)}(2) & 2 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ (n-2)x^{(r)}(n-2) & x^{(r)}(n-2) & n-2 & 1 \\ (n-1)x^{(r)}(n-1) & x^{(r)}(n-1) & n-1 & 1 \end{bmatrix},$$

$$W = \begin{bmatrix} x^{(r)}(2) \\ x^{(r)}(3) \\ \vdots \\ x^{(r)}(n-1) \\ x^{(r)}(n) \end{bmatrix}. \tag{14}$$

The predicted value of the FTDGM (1, 1) model is as follows:

$$\hat{x}^{(r)}(k+1) = (\beta_1 k + \beta_2)\hat{x}^{(r)}(k) + \beta_3 k + \beta_4. \tag{15}$$

According to the calculation formula of fractional order accumulation, it is not difficult to calculate the reduced value of the predicted sequence as follows:

$$\hat{x}^{(0)}(k) = \hat{x}^{(r)}(k) - \sum_{i=1}^{k} C_{k-i+r-1}^{k-i} \hat{x}^{(r)}(i), \quad k = 1, 2, \ldots, n. \tag{16}$$

It can be seen from the modeling process that the FTDGM (1, 1) model can better reflect the change rule of the system through the adjustment and optimization of the cumulative order. The model not only has a good modeling effect for practice series with exponential trend but also has a good trend tracking ability for time series with linear trend and nonlinear trend superposition. The optimal parameters can be determined by intelligent optimization algorithm, and genetic optimization algorithm is adopted in this paper.

For the time series with system impact disturbance, the modeling results of the model become very poor, and even the qualitative analysis and the theorem analysis are inconsistent. Professor Liu Sifeng deeply analyzed this phenomenon and believed that it was not due to the choice of model, but due to the impact disturbance of the system, which made the data series unable to show the real change rule of the system. Therefore, it was necessary to restore the real face of the system. Based on this, Professor Liu Sifeng put forward the concept of buffer operator, analyzed its properties, and carried out a lot of applications.

In reality, the system is relatively complex, and many systems can be regarded as complex systems with shock disturbance. For such a system, the first step of modeling is to present the real rules of the system and then build the model. In this paper, the weakening buffer operator proposed by Professor Liu Sifeng is selected for data preprocessing. The definition of the weakening buffer operator is as follows.

Table 1: China's total energy consumption between 2009 and 2019 (10000 tce).

Year	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019
Consumption	336126	360648	387043	402138	416913	428334	434113	441492	455827	471925	487000

Table 2: Results from different grey predication models.

Original value	DGM (1, 1) predication	Predication error (%)	FTDGM (1, 1) predicated value ($r = 0.88$)	Predication value (%)	B-FTDGM (1, 1) predicated value ($r = 0.11$)	Predication error (%)
441492	457233.2	3.57	444420.4	0.66	444771.5	0.74
455827	473641.2	3.91	451907.1	0.86	461055.2	1.15
471925	490637.9	3.97	458769.2	2.79	455037.9	3.58
487000	508244.7	4.36	465125.7	4.49	486801.3	0.04
MAPE		3.95		2.20		1.38

MAPE (mean absolute percentage error) = $100\% (1/n) \sum_{k=1}^{n} |(x(k) - \hat{x}(k)/x(k))|$.

Table 3: Per capita consumption of domestic electricity in Beijing from 2011 to 2019.

Year	2011	2012	2013	2014	2015	2016	2017	2018	2019
Consumption	727.2	791.8	750.6	793.5	808.7	899.9	1004	1185.5	1168.1

Definition 5. Assume $X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n))$ is the original sequence and $X^{(0)}d_1$ is the buffered sequence and where

$$x^{(0)}(k)d_1 = \frac{x^{(0)}(k) + x^{(0)}(k+1) + \cdots + x^{(0)}(n)}{n-k+1}. \tag{17}$$

Then, $X^{(0)}d$ is called the average weakening buffer operator.

Definition 6. Assume $X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n))$ is the original sequence, $X^{(0)}d_2$ is the buffered sequence and $X^{(0)}d_2 = (x^{(0)}(1)d_2, x^{(0)}(2)d_2, \ldots, x^{(0)}(n)d_2)$, where

$$x^{(0)}(k)d_2 = \frac{(n-k+1)x^{(0)}(k)}{x^{(0)}k + x^{(0)}(k+1) + \cdots + x^{(0)}n} x^{(0)}k. \tag{18}$$

Then, $X^{(0)}d_2$ is called the average strengthening buffer operator.

Weakening buffer operator can be used for real time series with fast change in the front and slow change in the back, while strengthening buffer operator can be used for real time series with slow change in the front and fast change in the back. $X^{(0)}d_1$ and $X^{(0)}d_2$ are the most commonly used buffer operators.

Therefore, the modeling ideas proposed in this paper are as follows. Firstly, the system data are buffered to restore the real law of the system. Then, the FTDGM (1, 1) model was established to calculate and optimize the parameters. In order to illustrate the practicability and effectiveness of the new model, two case studies are is carried out as follows.

4. Case Study

Case 1: In recent years, with the development of society and economy, China's total energy consumption grows rapidly, which poses higher requirements to energy production and natural environment. Accurate predictions for China's energy consumption can help the government and enterprises take reasonable measures to cope with the situation. Therefore, energy consumption prediction in China is of great value. China's total energy consumption between 2009 and 2019 is shown in Table 1.

We take the data from 2009 to 2015 as the modeling data and the data from 2016 to 2019 as the prediction data to test the prediction effect of different grey prediction models. Among them, genetic algorithm is used to determine the optimal order, and MATLAB software is used for programming calculation.

It can be seen from Table 2 that the prediction error of the B-FTDGM (1, 1) model has been effectively reduced. There is no obvious phenomenon that the error increases with the increase of step size. Thus, it can overcome the problem that the traditional grey prediction model can only be used for short-term prediction.

Case 2: In recent years, with the progress of China's industrialization, electricity, as a very important energy source, become the foundation of economic development. Therefore, it become of great significance to forecast power consumption timely and accurately for economic and social development. The per capita consumption of domestic electricity in Beijing from 2011 to 2019 is shown in Table 3.

Data from 2011 to 2017 are used as modeling data, and data from 2018 to 2019 are used as prediction data to test the prediction effect of different gray prediction models. Calculation results of different grey prediction models are shown in Table 4.

As can be seen from Table 4, due to the impact disturbance of the system, the prediction accuracy of DGM

Table 4: Predication results of different grey predication models.

Original value	DGM (1, 1) predication	Predication error (%)	FTDGM (1, 1) predication ($r = 1.21$)	Predication error (%)	B-FTDGM (1, 1) predication ($r = 0.43$)	Predication error (%)
1185.5	1010.14	14.79	1095.63	7.58	1087.62	8.26
1168.1	1065.65	8.77	1226.11	4.97	1168.16	0.00
MAPE		11.78		6.27		4.13

(1, 1) is not high, while the accuracy of the FTDGM (1, 1) model is improved to some extent. The 2-step average prediction error can be further reduced by using the sequence strengthened by the strengthening operator for modeling.

5. Conclusion

In this paper, a discrete grey prediction model of linear time-varying combination of buffering operator and fractional-order accumulation is constructed, and its properties, parameter-solving method, and modeling process are discussed. The model is applied to the prediction of energy consumption and per capita electricity consumption in China, and high prediction accuracies are obtained, showing the obvious superiority of the model. The discussion of other time-varying parameter grey prediction models and multivariable time-varying parameter grey prediction models is a direction that needs to be further studied in the future.

Data Availability

The data used to support the findings of this study have been deposited in http://www.stats.gov.cn/tjsj/ndsj and http://tjj.beijing.gov.cn.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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