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Research Article

Nonfragile Synchronization of Semi-Markovian Jumping Neural Networks with Time Delays via Sampled-Data Control and Application to Chaotic Systems

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This study discusses the synchronization problem for delayed neural networks with semi-Markovian jumping parameters. With the support of Jenson's inequality and Writinger-based integral inequality, a suitable Lyapunov–Krasovskii functional was constructed, and a synchronization criterion for the considered system was derived in the form of LMIs. In order to cope with system uncertainties, the nonfragile controller was taken into account. Also, sampled-data controller is used to improve the effectiveness of the bandwidth usage. In order to achieve the benefits of both control techniques, the nonfragile sampled-data controller was considered for synchronization of semi-Markovian jumping neural networks and to assure that the error system is asymptotically stable. At last, numerical simulations are exhibited to validate the proposed technique.

1. Introduction

Due to their effective implementation in cryptography, image analysis, associative memory, model identification, and so on, more attention has been given to different neural network (NN) models over the past centuries [1–4]. In latest years, the stability assessment of the constructed networks has appeared as a significant study subject due to the dynamic nature of these applications. In many of the engineering and neural systems, the communication transmission in a network is frequently interrupted by some exterior factors, which may direct to adverse dynamic behaviors such as oscillation and instability, and there arises time-delay. In addition to the fact that time delays in NNs are unavoidable, the literature has explored the stability assessment of delayed NNs well [5–8]. For example, synchronization problem for coupled inertial neural networks with reaction diffusion terms and time-varying delays via pinning sampled-data control has been considered in [5]. Finite time synchronization of drive response networks with discontinuous nodes and noise distribution has been analyzed in [9]. H_∞ filtering problem for fuzzy stochastic NNs with mixed time-delays has been investigated in [10].

In the actual application, since the system's state equation tends to have some randomness, a linear time-invariant system cannot generally describe such systems. The Markov jump system, however, can describe such dynamic systems accurately, which has led to extensive research by scholars [11, 12]. Since the jump time of a Markov chain is exponentially distributed, Markovian jumping parameters have severe limitations in applications. A semi-Markovian process is a continuous stochastic process whose sojourn

time fits a variety of probability distributions, such as the Weibull and Gaussian distributions. As a result of the relaxed conditions on the probability distributions, semi-Markovian jumping parameters are more general than Markovian jumping parameters in modelling realistic systems. Semi-MJSs have a fixed matrix of transition probabilities and a matrix of sojourn time probability density functions, while Markovian jumping systems (MJSs) have constant transition rates. This means that the transition rates in S-MJSs are time-varying. Due to these advantages, researchers paid their attention towards semi-Markovian jumping systems [13–16]. Recently, in [17], stochastic synchronization problem for semi-Markovian jumping Lure's system with packet dropouts subject to multiple sampling periods has been discussed. In [18], network-based nonlinear semi-Markovian jump systems with randomly occurring parameter uncertainties and transmission delay have been taken into account. Event-triggered synchronization problem for semi-Markovian jumping complex dynamical networks with a reliable control technique has been investigated in [19].

To stabilize the NNs, there are many control techniques such as impulsive control [20, 21], feedback control [22], sampled-data control [23–25], intermittent control [26, 27], and so on. It should be noted that the sampled-data control will improve the efficacy of the use of bandwidth by radically deducing the amount of information transmitted. Analog signal processing techniques are frequently replaced by digital signal processing techniques to achieve improved efficiency, reliability, and precision, in addition to the hasty developments in discrete measurement and intelligent instruments. Sampled-data control systems are continuous-time systems operated by a digital controller, and they are usually made up of continuous-time plants to be controlled, a discrete-time controller to control them, and an ideal sampler and ZOH to transform continuous-time signals into discrete-time signals and vice versa. The sampled-data systems are hybrid in nature because they operate in the continuous-time domain with both continuous-time and discrete-time signals. One of the discrete controllers, the sampled-data control system, allows control signals to adjust only at discrete sampling, resulting in a significant reduction in communication traffic and energy savings. As a result, sampled-data controllers have received a lot of attention in recent decades, with related findings published in the literature [28, 29].

Practically, the designed controller should be able to tolerate some uncertainties in its coefficients because uncertainty cannot be avoided for many reasons, such as the inherent imprecision in analog systems and additional parameter tuning in the final implementation of the controller. Due to this fact, the nonfragile controller has been studied by many researchers [30–32]. Nonfragile synchronization for chaotic time-delay neural networks with semi-Markovian jump parameters has been discussed in [32]. Synchronization stability criteria for delay-coupled fractional-order complex Cohen–Grossberg neural networks under parameter uncertainties are discussed in [33].

By the impact of the preceding facts, this manuscript discusses the synchronization problem for SMJNN with hybrid control strategy. At first, synchronization analysis has been performed to consider SMJNNs with recently introduced integral inequality techniques and proposed control strategy. Later, the synchronization criteria for SMJNNs have been explored with the nonfragile control technique. Finally, in numerical simulations, chaotic NNs are considered to verify the designed control technique.

The main contributions and features of this study are presented as follows:

(i) Distinguished from the previous works, this article aims to study the synchronization issue for SMJNNs with time delays by using nonfragile sampled-data control

(ii) Moreover, in an aim to enjoy the benefits of the nonfragile control technique and sampled-data control technique, the nonfragile sampled-data control which has the features of both control techniques has been adopted for achieving synchronization

(iii) By utilizing novel integral inequalities and designed controller, synchronization criteria have been given in the form of LMIs. The resulting LMIs are solved with the help of Matlab LMI toolbox.

(iv) Finally, numerical simulations are presented to validate the correctness of the proposed control technique

2. Problem Formulation

Let $\{\beta(t), t \geq 0\}$ be a discrete-state continuous-time semi-Markov process and assume the values in the finite set $= \{1, 2, \ldots, N\}$ are given by

$$\Pr\{\beta(t+l) = j \mid \beta(t) = i\} = \begin{cases} \alpha_{ij}(l)l + o(l) & i \neq j \\ 1 + \alpha_{ii}(l)l + o(l) & i = j \end{cases}, \tag{1}$$

where $\Delta = \alpha_{ij}(l)$ denotes the transition probability matrix, $\lim_{l \longrightarrow 0}(o(l)/l) = 0$, and $\alpha_{ij}(l) \geq 0$, for $i \neq j$, is the transition rate from mode i at time t to mode j at time $t + l$, and $\alpha_{ii}(l) = \sum_{j \in S, j \neq i} \alpha_{ij}(l)$.

Consider the SMJ-delayed neural networks:

$$\dot{\zeta}(t) = -D(\beta(t))\zeta(t) + A(\beta(t))f(\zeta(t)) + B(\beta(t))f(\zeta(t - \varrho(t))) + J(t), \tag{2}$$

where $\zeta(t) = (\zeta_1(t), \zeta_2(t), \ldots, \zeta_n(t))$ represents the state vector. $f(x(t)) = (f_1(x(t)), \ldots, f_n(x(t)))$ stands for neuron activation function and $\tau(t)$ is the time-varying delay with $0 \le \varrho(t) \le \varrho$. $D = \mathrm{diag}\{d_1, d_2, \ldots, d_n\}$ with $d_i > 0$. A and B are connection and delayed connection weight matrices. The external input is denoted by $J(t)$. The slave system is described as

$$\dot{\xi}(t) = -D(\beta(t))\xi(t) + A(\beta(t))f(\xi(t)) + B(\beta(t))f(\xi(t-\varrho(t))) + J(t) + u(t), \tag{3}$$

where $u(t) \in R^n$ is the control input. By letting the error as $\psi(t) = \zeta(t) - \xi(t)$, the error system is

$$\dot{\psi}(t) = -D(\beta(t))\psi(t) + A(\beta(t))f(\psi(t)) + B(\beta(t))f(\psi(t-\varrho(t))) + u(t). \tag{4}$$

Consider the nonfragile sampled-data control as

$$u_i(t) = (K + \Delta K(t))\psi(t_k) = \overline{K}\psi(t_k), \tag{5}$$

where K is the gain matrix to be designed, and t_k denotes the sampling instant and satisfies $0 = t_0 < t_1 < \cdots < t_k < \cdots < \lim_{k\longrightarrow\infty} t_k$. $\Delta K(t)$ stands for the controller gain fluctuations. It takes the form

$$\Delta K(t) = MF(t)N, \tag{6}$$

where $F^T(t)F(t) \le I$. Using the input-delay approach $t - t_k = \sigma(t)$, $t_k \le t < t_{k+1}$, and $0 < \sigma(t) < \sigma$, then the controller becomes

$$u_i(t) = \overline{K}\psi(t - \sigma(t)). \tag{7}$$

Then, (4) can be written as

$$\dot{\psi}(t) = -D(\beta(t))\psi(t) + A(\beta(t))f(\psi(t)) + B(\beta(t))f(\psi(t-\varrho(t))) + \overline{K}\psi(t-\sigma(t)). \tag{8}$$

Lemma 1 (see [34]). *For any two scalars $v_2 \ge v_1 > 0$, constant matrix $H \in \mathbb{R}^{n\times n}$, $H = H^T > 0$, such that the integrations concerned are well defined:*

$$-(v_2 - v_1)\int_{t-v_2}^{t-v_1} \xi^T(s)H\xi(s)\mathrm{d}s \le -\left(\int_{t-v_2}^{t} \xi(s)\mathrm{d}s\right)^T H\left(\int_{t}^{t-v_1} \xi(s)\mathrm{d}s\right). \tag{9}$$

Lemma 2 (see [35]). *For any matrix $E \in \mathbb{R}^{n\times m}$, $E = E^T > 0$, differentiable function θ from $[a,b] \longrightarrow \mathbb{R}^n$, the succeeding inequality holds:*

$$\int_a^b \dot{z}^T(s)E\dot{z}(s)\mathrm{d}s \ge \frac{\vartheta^T\left[\Upsilon_1^T E\Upsilon_1 + \pi^2\Upsilon_2^T E\Upsilon_2\right]\vartheta}{b-a}, \tag{10}$$

where $\vartheta = [z^T(b)z^T(a)\int_a^b((z^T(s))/(b-a))ds]^T$, $\Upsilon_1 = [I - I0]$, *and* $\Upsilon_2 = [(I/2)(I/2) - I]$.

Lemma 3 (see [36]). *Let $S = S^T, U$ and V be the real constant matrices of appropriate dimensions $S + UF(t)V + V^T F^T(t)U^T < 0$ for F, satisfying $F^T(t)F(t) = I$ if and only if there exists a scalar $\varepsilon > 0$, such that $S + \varepsilon^{-1}UU^T + \varepsilon V^T V < 0$.*

Assumption 1. Each activation function $f_i(\cdot)$ is continuous and bounded and there exist constants F_i^- and F_i^+ such that

$$F_i^- \le \frac{f_i(k_1) - f_i(k_2)}{k_1 - k_2} \le F_i^+, \quad i = 1, 2, \ldots, n, \tag{11}$$

where $k_1, k_2 \in R$ and $k_1 \ne k_2$.

3. Nonfragile Sampled-Data Synchronization

This section derives some sufficient conditions for the synchronization of considered system (8), which can be seen through the subsequent theorem.

Theorem 1. *The system (8) is asymptotically synchronized, if there exist matrices $\mathfrak{P} > 0, \mathfrak{Q}_1 > 0, \mathfrak{Q}_2 > 0, \mathfrak{R}_1 > 0, \mathfrak{R}_2 > 0, \mathfrak{Z}_1 > 0, \mathfrak{Z}_2 > 0$ and matrices S, L, G, for a given scalar γ, such that the following LMI holds:*

$$\Upsilon(\delta) < 0, \tag{12}$$

where

$$
\begin{aligned}
\Upsilon_{1,1} &= 2\mathfrak{P} + \mathfrak{Q}_1 + \mathfrak{Q}_3 + \tau_1^2\mathfrak{R}_1 + \varrho\mathfrak{R}_2 + \frac{\varrho^2}{4}\mathfrak{T}_1 + \mathfrak{Z}_1 \\
&\quad - 2GD_i - \left(\mathfrak{Z}_2 + \frac{\pi^2}{4}\mathfrak{Z}_2\right) \\
&\quad - \left(\mathfrak{R}_1 + \frac{\pi^2}{4}\mathfrak{R}_1\right) - F_1\Lambda_{1i} + \sum_{j=1}^{N}\alpha_{ij}(\delta)\mathfrak{P}_j, \\
\Upsilon_{1,2} &= GA_i + F_2\Lambda_{1i}, \\
\Upsilon_{1,3} &= GB_i, \\
\Upsilon_{1,4} &= G\overline{K} + \mathfrak{Z}_2 - \frac{\pi^2}{4}\mathfrak{Z}_2, \\
\Upsilon_{1,5} &= \mathfrak{R}_1 - \frac{\pi^2}{4}\mathfrak{R}_1, \\
\Upsilon_{1,7} &= -G + \mathfrak{P}_i, \\
\Upsilon_{1,9} &= \frac{\pi^2}{4}\frac{1}{\varrho}\mathfrak{R}_1, \\
\Upsilon_{1,12} &= -\frac{\pi^2}{2}\mathfrak{Z}_2, \\
\Upsilon_{2,2} &= -\Lambda_{1i} + \mathfrak{Q}_2, \\
\Upsilon_{2,5} &= F_2\widetilde{\Lambda}_{2i}, \\
\Upsilon_{2,7} &= GA_i, \\
\Upsilon_{3,3} &= (1-\mu)\mathfrak{Q}_2, \\
\Upsilon_{3,7} &= \gamma GB_i, \\
\Upsilon_{4,4} &= 2\left(-\mathfrak{Z}_2 - \frac{\pi^2}{4}\mathfrak{Z}_2\right), \\
\Upsilon_{4,7} &= G\overline{K}, \\
\Upsilon_{5,5} &= -\mathfrak{Q}_1 - 2\left(\mathfrak{R}_1 + \frac{\pi^2}{4}\mathfrak{R}_1\right) \\
\Upsilon_{5,6} &= \mathfrak{R}_1 - \frac{\pi^2}{4}\mathfrak{R}_1, \\
\Upsilon_{5,7} &= \gamma GB_i, \\
\Upsilon_{5,8} &= \frac{\pi^2}{4}\mathfrak{R}_1, \\
\Upsilon_{5,9} &= \frac{\pi^2}{2}\mathfrak{R}_1, \\
\Upsilon_{6,6} &= -\mathfrak{Q}_3 - \left(\mathfrak{Z}^2 + \frac{\pi^2}{4}\mathfrak{Z}_2\right), \\
\Upsilon_{7,7} &= \tau^2\mathfrak{R}_1 + \eta^2\mathfrak{Z}_2 - 2\gamma G, \\
\Upsilon_{8,8} &= -\pi^2\mathfrak{R}_1, \\
\Upsilon_{9,9} &= -\pi^2\mathfrak{R}_1, \\
\Upsilon_{10,10} &= -\mathfrak{Z}_1 - \mathfrak{Z}_2 - \frac{\pi^2}{4}\mathfrak{Z}_2, \\
\Upsilon_{10,11} &= \frac{\pi^2}{2}\mathfrak{Z}_2, \\
\Upsilon_{11,11} &= -\pi^2\mathfrak{Z}_2, \\
\Upsilon_{12,12} &= -\pi^2\mathfrak{Z}_2, \\
\Upsilon_{13,13} &= -\mathfrak{T}_1.
\end{aligned}
\tag{13}
$$

Proof. Consider the following Lyapunov–Krasovskii functional:

$$
\mathfrak{V}(t) = \sum_{i=1}^{5}\mathfrak{V}_i(t), \tag{14}
$$

where

$$\begin{aligned}
\mathfrak{V}_1(t) &= \psi^T(t)\mathfrak{P}_i\psi(t),\\
\mathfrak{V}_2(t) &= \int_{t-\varrho(t)}^{t}\psi^T(s)\mathfrak{Q}_1\psi(s)\mathrm{d}s + \int_{t-\varrho(t)}^{t} g^T(\psi(s))\mathfrak{Q}_2 g(\psi(s))\mathrm{d}s\\
&\quad + \int_{t-\varrho}^{t}\psi^T(s)\mathfrak{Q}_3\psi(s)\mathrm{d}s,\\
\mathfrak{V}_3(t) &= \varrho\int_{-\varrho}^{0}\int_{t+\theta}^{t}\dot{\psi}^T(s)\mathfrak{R}_1\dot{\psi}(s)\mathrm{d}s\mathrm{d}\theta + \int_{-\varrho}^{0}\int_{t+\theta}^{t}\psi^T(s)\mathfrak{R}_2\psi(s)\mathrm{d}s\mathrm{d}\theta,\\
\mathfrak{V}_4(t) &= \frac{\varrho}{2}\int_{-\varrho}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t}\psi^T(s)\mathfrak{T}_1\psi(s)\mathrm{d}s\mathrm{d}\lambda\mathrm{d}\theta,\\
\mathfrak{V}_5(t) &= \int_{t-\sigma_m}^{t}\psi^T(s)\mathfrak{Z}_1\psi(s)\mathrm{d}s + \sigma_m\int_{-\sigma_m}^{0}\int_{t+\theta}^{t}\dot{\psi}^T(s)\mathfrak{Z}_2\dot{\psi}(s)\mathrm{d}s\mathrm{d}\theta.
\end{aligned} \tag{15}$$

Calculating the time-derivative of (14) along (8) gives

$$\begin{aligned}
\mathfrak{L}\mathfrak{V}_1(t) &= -2\psi^T(t)\mathfrak{P}_i\psi(t) + \xi^T(t)\sum_{j=1}^{N}\pi_{ij}(\delta)\mathfrak{P}_j\psi(t),\\
\mathfrak{L}\mathfrak{V}_2(t) &= \psi^T(t)Q_1\psi(t) - (1-\mu)\psi^T(t-\varrho(t))\mathfrak{Q}_1\psi(t-\varrho(t)) + g^T(\psi(t))\mathfrak{Q}_2 g(\psi(t))\\
&\quad - (1-\mu)g^T(t-\tau(t))\mathfrak{Q}_2 g(t-\varrho(t)) + \psi^T(t)\mathfrak{Q}_3\psi(t) - \psi^T(t-\varrho)\mathfrak{Q}_3\psi(t-\varrho),
\end{aligned} \tag{16}$$

$$\begin{aligned}
\mathfrak{L}\mathfrak{V}_3(t) &= \varrho^2\dot{\psi}^T(t)\mathfrak{R}_1\dot{\psi}(t) - \varrho\int_{t-\varrho}^{t}\dot{\psi}^T(s)\mathfrak{R}_1\dot{\psi}(s)\mathrm{d}s + \varrho\psi^T(t)\mathfrak{R}_2\psi(t)\\
&\quad - \int_{t-\varrho}^{t}\psi^T(s)\mathfrak{R}_2\psi(s)\mathrm{d}s,
\end{aligned} \tag{17}$$

$$\mathfrak{L}\mathfrak{V}_4(t) = \frac{\varrho^4}{4}\psi^T(t)\mathfrak{T}_1\psi(t) - \int_{-\varrho}^{0}\int_{t+\theta}^{t}\psi^T(s)\mathrm{d}s\mathrm{d}\theta\mathfrak{T}_1\int_{-\varrho}^{0}\int_{t+\theta}^{t}\psi(s)\mathrm{d}s\mathrm{d}\theta, \tag{18}$$

$$\begin{aligned}
\mathfrak{L}\mathfrak{V}_5(t) &= \psi^T(s)\mathfrak{Z}_1\psi(s) - \psi^T(t-\sigma_m)\mathfrak{Z}_1\psi(t-\sigma_m) + \sigma_m^2\dot{\psi}^T(t)\mathfrak{Z}_2\dot{\psi}(t)\\
&\quad - \sigma_m\int_{t-\sigma_M}^{t}\dot{\psi}^T(s)\mathfrak{Z}_2\dot{\psi}(s)\mathrm{d}s.
\end{aligned} \tag{19}$$

From Lemma 1, it follows from equation (19) that

$$-\int_{t-\varrho}^{t}\psi^T(s)\mathfrak{R}_2\psi(s)\mathrm{d}s \le -\frac{1}{\varrho}\begin{bmatrix}\int_{t-\varrho}^{t-\varrho(t)}\psi(s)\mathrm{d}s\\ \int_{t-\varrho(t)}^{t}\psi(s)\mathrm{d}s\end{bmatrix}^T\begin{bmatrix}\mathfrak{R}_2 & 0\\ 0 & \mathfrak{R}_2\end{bmatrix}\begin{bmatrix}\int_{t-\varrho}^{t-\varrho(t)}\psi(s)\mathrm{d}s\\ \int_{t-\varrho(t)}^{t}\psi(s)\mathrm{d}s\end{bmatrix}. \tag{20}$$

From Lemma 2, (19) becomes

$$-\sigma_m \int_{t-\sigma_m}^{t-\sigma(t)} \dot{\psi}^T(s)\mathfrak{Z}_2\dot{\psi}(s)\mathrm{d}s \leq -\begin{bmatrix} \psi(t-\sigma(t)) \\ \psi(t-\sigma) \\ \frac{1}{\sigma}\int_{t-\sigma}^{t-\sigma(t)} \psi(s)\mathrm{d}s \end{bmatrix}^T \times \begin{bmatrix} \mathfrak{Z}_2 + \frac{\pi^2}{4}\mathfrak{Z}_2 & -Z_2 + \frac{\pi^2}{4}\mathfrak{Z}_2 & -\frac{\pi^2}{4}\mathfrak{Z}_2 \\ -\mathfrak{Z}_2 + \frac{\pi^2}{4}\mathfrak{Z}_2 & Z_2 + \frac{\pi^2}{4}\mathfrak{Z}_2 & \frac{\pi^2}{2}\mathfrak{Z}_2 \\ -\frac{\pi^2}{4}\mathfrak{Z}_2 & -\frac{\pi^2}{4}\mathfrak{Z}_2 & \pi^2\mathfrak{Z}_2 \end{bmatrix} \begin{bmatrix} \psi(t-\sigma(t)) \\ \psi(t-\sigma) \\ \frac{1}{\sigma}\int_{t-\sigma}^{t-\sigma(t)} \psi(s)\mathrm{d}s \end{bmatrix} \tag{21}$$

and

$$-\sigma_m \int_{t-\sigma(t)}^{t} \dot{\psi}^T(s)\mathfrak{Z}_2\dot{\psi}(s)\mathrm{d}s \leq \begin{bmatrix} \psi(t) \\ \psi(t-\sigma(t)) \\ \frac{1}{\sigma}\int_{t-\sigma(t)}^{t} \psi(s)\mathrm{d}s \end{bmatrix}^T \times \begin{bmatrix} \mathfrak{Z}_2 + \frac{\pi^2}{4}\mathfrak{Z}_2 & -\mathfrak{Z}_2 + \frac{\pi^2}{4}\mathfrak{Z}_2 & -\frac{\pi^2}{4}\mathfrak{Z}_2 \\ -\mathfrak{Z}_2 + \frac{\pi^2}{4}\mathfrak{Z}_2 & \mathfrak{Z}_2 + \frac{\pi^2}{4}\mathfrak{Z}_2 & \frac{\pi^2}{2}\mathfrak{Z}_2 \\ -\frac{\pi^2}{4}\mathfrak{Z}_2 & -\frac{\pi^2}{4}\mathfrak{Z}_2 & \pi^2\mathfrak{Z}_2 \end{bmatrix} \begin{bmatrix} \psi(t) \\ \psi(t-\sigma(t)) \\ \frac{1}{\sigma}\int_{t-\sigma(t)}^{t} \psi(s)\mathrm{d}s \end{bmatrix}. \tag{22}$$

From (17), we have

$$\int_{t-\varrho}^{t-\varrho(t)} \dot{\psi}^T(s)\mathfrak{R}_1\dot{\psi}(s)\mathrm{d}s \geq \frac{1}{\varrho}\begin{bmatrix} \psi(t-\varrho(t)) \\ \psi(t-\varrho) \\ \frac{1}{\varrho}\int_{t-\varrho(t)}^{t-\varrho(t)} \psi(s)\mathrm{d}s \end{bmatrix}^T \times \begin{bmatrix} \mathfrak{R}_1 + \frac{\pi^2}{4}\mathfrak{R}_1 & -\mathfrak{R}_1 + \frac{\pi^2}{4}\mathfrak{R}_1 & -\frac{\pi^2}{4}\mathfrak{R}_1 \\ -\mathfrak{R}_1 + \frac{\pi^2}{4}\mathfrak{R}_1 & \mathfrak{R}_1 + \frac{\pi^2}{4}\mathfrak{R}_1 & \frac{\pi^2}{2}\mathfrak{R}_1 \\ -\frac{\pi^2}{4}\mathfrak{R}_1 & -\frac{\pi^2}{4}\mathfrak{R}_1 & \pi^2\mathfrak{R}_1 \end{bmatrix} \begin{bmatrix} \psi(t-\varrho(t)) \\ \psi(t-\varrho) \\ \frac{1}{\varrho}\int_{t-\varrho(t)}^{t-\varrho(t)} \psi(s)\mathrm{d}s \end{bmatrix}, \tag{23}$$

and

$$\int_{t-\varrho(t)}^{t} \dot{\psi}^T(s)\mathfrak{R}_1\dot{\psi}(s)\mathrm{d}s \geq \frac{1}{\varrho}\begin{bmatrix} \psi(t) \\ \psi(t-\varrho(t)) \\ \frac{1}{\varrho}\int_{t-\varrho(t)}^{t-\varrho(t)} \psi(s)ds \end{bmatrix}^T \times \begin{bmatrix} \mathfrak{R}_1 + \frac{\pi^2}{4}\mathfrak{R}_1 & -\mathfrak{R}_1 + \frac{\pi^2}{4}\mathfrak{R}_1 & -\frac{\pi^2}{4}\mathfrak{R}_1 \\ -\mathfrak{R}_1 + \frac{\pi^2}{4}\mathfrak{R}_1 & \mathfrak{R}_1 + \frac{\pi^2}{4}\mathfrak{R}_1 & \frac{\pi^2}{2}\mathfrak{R}_1 \\ -\frac{\pi^2}{4}\mathfrak{R}_1 & -\frac{\pi^2}{4}\mathfrak{R}_1 & \pi^2\mathfrak{R}_1 \end{bmatrix} \begin{bmatrix} \psi(t) \\ \psi(t-\varrho(t)) \\ \frac{1}{\varrho}\int_{t-\varrho(t)}^{t-\varrho(t)} \psi(s)\mathrm{d}s \end{bmatrix}. \tag{24}$$

By Assumption 1, for diagonal matrices Λ_{1i} and $\Lambda_2 i$,

$$\begin{aligned} 0 \leq & -\psi^T(t)\mathfrak{F}_1\Lambda_{1i}\psi(t) + \psi^T(t)\mathfrak{F}_2\Lambda_{1i}g(\psi(t)) + g^T(\psi(t))\mathfrak{F}_2\widetilde{\Lambda}_{1i}\psi(t) \\ & - g^T(\psi(t))\widetilde{\Lambda_{1i}}g(\psi(t)), \end{aligned} \tag{25}$$

$$\begin{aligned} 0 \leq & -\psi^T(t-\tau(t))\mathfrak{F}_1\Lambda_{2i}\psi(t-\tau(t)) + \psi^T(t-\tau(t))\mathfrak{F}_2\widetilde{\Lambda}_{2i}g(\psi(t-\varrho(t))) \\ & + g(\psi(t-\varrho(t)))\mathfrak{F}_2\widetilde{\Lambda}_{2i}\psi(t-\varrho(t)) - g^T(\psi(t-\varrho(t)))\widetilde{\Lambda}_{2i}g(\psi(t-\varrho(t))). \end{aligned} \tag{26}$$

For any matrix G, we have

$$0 = 2\left[\psi^T(t)G + \dot{\psi}^T(t)G\right][-\dot{\psi}(t) - D(\beta(t))\psi(t) + A(\beta(t))f(\psi(t)) + B(\beta(t))f(\psi(t-\varrho(t))) + \overline{K}\psi(t-\sigma(t))]. \tag{27}$$

From equations (15)–(27), we have

$$E\{LV(t)\} \le \chi^T(t)\Upsilon\chi(t) < 0, \tag{28}$$

where

$$\chi(t) = [\psi(t)\, g(\psi(t))\, g(\psi(t-\varrho(t)))\, \psi(t-\sigma(t))\, \psi(t-\varrho(t))\, \psi(t-\varrho)\, \dot{\psi}(t) \int_{t-\varrho}^{t-\varrho(t)} \psi(s)\mathrm{d}s \int_{t-\varrho(t)}^{t} \psi(s)\mathrm{d}s\, \psi(t-\sigma)\, \frac{1}{\sigma}\int_{t-\sigma}^{t-\sigma(t)} \psi(s)\mathrm{d}s\, \frac{1}{\sigma}\int_{t-\sigma(t)}^{t} \psi(s)\mathrm{d}s \int_{-\varrho}^{0}\int_{t+\theta}^{t} \psi(s)\mathrm{d}s\mathrm{d}\theta\Big], \tag{29}$$

and the elements of the matrix Υ are given in the statement of theorem.

Based on the above theorem, now we are in a position to design the gain matrix of the derived controller. □

Theorem 2. *The system (8) is asymptotically synchronized, if there exist matrices $\mathfrak{P} > 0, \mathfrak{Q}_1 > 0, \mathfrak{Q}_2 > 0, \mathfrak{R}_1 > 0, \mathfrak{R}_2 > 0, \mathfrak{Z}_1 > 0, \mathfrak{Z}_2 > 0$ and matrices S, L, G, for a given scalar γ, such that the following LMI holds:*

$$\begin{bmatrix} \Upsilon(\delta) & \mathfrak{U}_1 & \epsilon\mathfrak{B}_1 & \mathfrak{U}_2 & \varepsilon\mathfrak{B}_2 \\ * & -\epsilon_1 I & 0 & 0 & 0 \\ * & * & -\epsilon_1 I & 0 & 0 \\ * & * & * & -\epsilon_2 I & 0 \\ * & * & * & * & -\epsilon_2 I \end{bmatrix} < 0, \tag{30}$$

where

$$\Upsilon_{1,1} = 2\mathfrak{P} + \mathfrak{Q}_1 + \mathfrak{Q}_3 + \tau_1^2\mathfrak{R}_1 + \varrho\mathfrak{R}_2 + \frac{\varrho^2}{4}\mathfrak{T}_1 + \mathfrak{Z}_1 - 2GD_i - \left(\mathfrak{Z}_2 + \frac{\pi^2}{4}\mathfrak{Z}_2\right) - \left(\mathfrak{R}_1 + \frac{\pi^2}{4}\mathfrak{R}_1\right) - F_1\Lambda_{1i} + \sum_{j=1}^{N}\pi_{ij}(\delta)\mathfrak{P}_j,$$

$$\Upsilon_{1,2} = GA_i + F_2\Lambda_{1i},$$

$$\Upsilon_{1,3} = GB_i,$$

$$\Upsilon_{1,4} = GK + \mathfrak{Z}_2 - \frac{\pi^2}{4}\mathfrak{Z}_2,$$

$$\Upsilon_{1,5} = \mathfrak{R}_1 - \frac{\pi^2}{4}\mathfrak{R}_1,$$

$$\Upsilon_{1,7} = -G + \mathfrak{P}_i,$$

$$\Upsilon_{1,9} = \frac{\pi^2}{4}\frac{1}{\varrho}\mathfrak{R}_1,$$

$$\Upsilon_{1,12} = -\frac{\pi^2}{2}\mathfrak{Z}_2,$$

$$\Upsilon_{2,2} = -\Lambda_{1i} + \mathfrak{Q}_2,$$

$$\Upsilon_{2,5} = F_2\widetilde{\Lambda}_{2i},$$

$$\Upsilon_{2,7} = GA_i,$$

$$\Upsilon_{3,3} = (1-\mu)\mathfrak{Q}_2,$$

$$\Upsilon_{3,7} = \gamma GB_i,$$

$$\Upsilon_{4,4} = 2\left(-\mathfrak{Z}_2 - \frac{\pi^2}{4}\mathfrak{Z}_2\right),$$

$$\Upsilon_{4,7} = GK,$$

$$\Upsilon_{5,5} = -\mathfrak{Q}_1 - 2\left(\mathfrak{R}_1 + \frac{\pi^2}{4}\mathfrak{R}_1\right),$$

$$\Upsilon_{5,6} = \mathfrak{R}_1 - \frac{\pi^2}{4}\mathfrak{R}_1,$$

$$\Upsilon_{5,7} = \gamma GB_i,$$

$$\Upsilon_{5,8} = \frac{\pi^2}{4}\mathfrak{R}_1,$$

$$\Upsilon_{5,9} = \frac{\pi^2}{2}\mathfrak{R}_1,$$

$$\Upsilon_{6,6} = -\mathfrak{Q}_3 - \left(\mathfrak{Z}^2 + \frac{\pi^2}{4}\mathfrak{Z}_2\right),$$

$$\Upsilon_{7,7} = \tau^2\mathfrak{R}_1 + \eta^2\mathfrak{Z}_2 - 2\gamma G,$$

$$\Upsilon\Upsilon_{8,8} = -\pi^2\mathfrak{R}_1,$$

$$\begin{aligned}
\Upsilon_{9,9} &= -\pi^2\mathfrak{R}_1,\\
\Upsilon_{10,10} &= -\mathfrak{Z}_1 - \mathfrak{Z}_2 - \frac{\pi^2}{4}\mathfrak{Z}_2,\\
\Upsilon_{10,11} &= \frac{\pi^2}{2}\mathfrak{Z}_2,\\
\Upsilon_{11,11} &= -\pi^2\mathfrak{Z}_2,\\
\Upsilon_{12,12} &= -\pi^2\mathfrak{Z}_2,\\
\Upsilon_{13,13} &= -\mathfrak{T}_1.
\end{aligned} \tag{31}$$

Moreover, the desired gain matrices are computed by $K = LG^{-1}$.

Proof. By making use of $\Delta K(t) = \mathfrak{U}F(t)\mathfrak{V}$, LMI in (12) can be written as

$$\Upsilon_{(i,j),\overline{w}} + \mathfrak{U}_1F(t)\mathfrak{V}_1 + \mathfrak{V}_1^TF(t)\mathfrak{U}_1^T + \mathfrak{U}_2F(t)\mathfrak{V}_2 + \mathfrak{V}_2^TF(t)\mathfrak{U}_2, \tag{32}$$

where

$$\begin{aligned}
\mathfrak{U}_1 &= \left[\underbrace{0\cdots0}_{3}\ \ G\mathfrak{U}\ \ \underbrace{0\cdots0}_{9}\right],\\
\mathfrak{U}_2 &= \left[\underbrace{0\cdots0}_{3}\ \ G\mathfrak{U}\ \ \underbrace{0\cdots0}_{9}\right],\\
\mathfrak{V}_1 &= \left[\mathfrak{V}\ \ \underbrace{0\cdots0}_{12}\right],\\
\mathfrak{V}_2 &= \left[\underbrace{0\cdots0}_{6}\ \ \mathfrak{V}\ \ \underbrace{0\cdots0}_{6}\right].
\end{aligned} \tag{33}$$

From Lemma 3, we have

$$\Upsilon_{(i,j),\overline{w}} + \varsigma_1^{-1}\mathfrak{U}_1\mathfrak{U}_1^T + \varsigma_1\mathfrak{V}_1^T\mathfrak{V}_1 + \varsigma_2^{-1}\mathfrak{U}_2\mathfrak{U}_2^T + \varsigma_2\mathfrak{V}_2^T\mathfrak{V}_2. \tag{34}$$

Thus, one can get

$$\begin{bmatrix} \Upsilon_{(i,j),\overline{w}} & \mathfrak{U}_1 & \varsigma_1\mathfrak{V}_1 & \mathfrak{U}_2 & \varsigma_2\mathfrak{V}_2\\ * & -\varsigma_1 I & 0 & 0 & 0\\ * & * & -\varsigma_1 I & 0 & 0\\ * & * & * & -\varsigma_2 I & 0\\ * & * & * & * & -\varsigma_2 I \end{bmatrix}, \quad i = 1, 2, \ldots, s. \tag{35}$$

Theorem 3. *The system (8) is asymptotically synchronized, if there exist matrices* $\mathfrak{P} > 0, \mathfrak{Q}_1 > 0, \mathfrak{Q}_2 > 0, \mathfrak{R}_1 > 0, \mathfrak{R}_2 > 0, \mathfrak{Z}_1 > 0, \mathfrak{Z}_2 > 0$ *and matrices* S, L, G, *for a given scalar* γ, *such that the following LMI holds:*

$$\begin{bmatrix} \Upsilon(\delta) & M_1 & \varsigma_1N_1 & M_2 & \varsigma_2N_2\\ * & -\varsigma_1 I & 0 & 0 & 0\\ * & * & -\varsigma_1 I & 0 & 0\\ * & * & * & -\varsigma_2 I & 0\\ * & * & * & * & -\varsigma_2 I \end{bmatrix} < 0, \tag{36}$$

where

$$\begin{aligned}
\Upsilon_{1,1} &= 2\mathfrak{P} + \mathfrak{Q}_1 + \mathfrak{Q}_3 + \tau_1^2\mathfrak{R}_1 + \varrho\mathfrak{R}_2 + \frac{\varrho^2}{4}\mathfrak{T}_1 + \mathfrak{Z}_1\\
&\quad - 2GD_i - \left(\mathfrak{Z}_2 + \frac{\pi^2}{4}\mathfrak{Z}_2\right)\\
&\quad - \left(\mathfrak{R}_1 + \frac{\pi^2}{4}\mathfrak{R}_1\right) - F_1\Lambda_{1i} + \sum_{j=1}^{N}\pi_{ij}(\delta)\mathfrak{P}_j,\\
\Upsilon_{1,2} &= GA_i + F_2\Lambda_{1i},\\
\Upsilon_{1,3} &= GB_i,\\
\Upsilon_{1,4} &= GK + \mathfrak{Z}_2 - \frac{\pi^2}{4}\mathfrak{Z}_2,\\
\Upsilon_{1,5} &= \mathfrak{R}_1 - \frac{\pi^2}{4}\mathfrak{R}_1,\\
\Upsilon_{1,7} &= -G + \mathfrak{P}_i,\\
\Upsilon_{1,9} &= \frac{\pi^2}{4}\frac{1}{\varrho}\mathfrak{R}_1,\\
\Upsilon_{1,12} &= -\frac{\pi^2}{2}\mathfrak{Z}_2,\\
\Upsilon_{2,2} &= -\Lambda_{1i} + \mathfrak{Q}_2,\\
\Upsilon_{2,5} &= F_2\widetilde{\Lambda}_{2i},\\
\Upsilon_{2,7} &= GA_i,\\
\Upsilon_{3,3} &= (1-\mu)\mathfrak{Q}_2,\\
\Upsilon_{3,7} &= \gamma GB_i,\\
\Upsilon_{4,4} &= 2\left(-\mathfrak{Z}_2 - \frac{\pi^2}{4}\mathfrak{Z}_2\right),\\
\Upsilon_{4,7} &= GK,\\
\Upsilon_{5,5} &= -\mathfrak{Q}_1 - 2\left(\mathfrak{R}_1 + \frac{\pi^2}{4}\mathfrak{R}_1\right),\\
\Upsilon_{5,6} &= \mathfrak{R}_1 - \frac{\pi^2}{4}\mathfrak{R}_1,\\
\Upsilon_{5,7} &= \gamma GB_i,\\
\Upsilon_{5,8} &= \frac{\pi^2}{4}\mathfrak{R}_1,\\
\Upsilon_{5,9} &= \frac{\pi^2}{2}\mathfrak{R}_1,
\end{aligned}$$

$$\begin{aligned}
\Upsilon_{6,6} &= -\mathfrak{Q}_3 - \left(\mathfrak{Z}^2 + \frac{\pi^2}{4}\mathfrak{Z}_2\right),\\
\Upsilon_{7,7} &= \tau^2\mathfrak{R}_1 + \eta^2\mathfrak{Z}_2 - 2\gamma G,\\
\Upsilon_{8,8} &= -\pi^2\mathfrak{R}_1,\\
\Upsilon_{9,9} &= -\pi^2\mathfrak{R}_1,\\
\Upsilon_{10,10} &= -\mathfrak{Z}_1 - \mathfrak{Z}_2 - \frac{\pi^2}{4}\mathfrak{Z}_2,\\
\Upsilon_{10,11} &= \frac{\pi^2}{2}\mathfrak{Z}_2,\\
\Upsilon_{11,11} &= -\pi^2\mathfrak{Z}_2,\\
\Upsilon_{12,12} &= -\pi^2\mathfrak{Z}_2,\\
\Upsilon_{13,13} &= -\mathfrak{T}_1.
\end{aligned} \tag{37}$$

Moreover, the desired gain matrices are given by $K = LG^{-1}$.

Let us assume $\Delta K(t) = 0$; then, the controller in (7) will reduce to the form of sampled-data control. Then, the above theorem can be rewritten.

Theorem 4. *The system (8) is asymptotically synchronized, if there exist matrices* $\mathfrak{P} > 0, \mathfrak{Q}_1 > 0, \mathfrak{Q}_2 > 0, \mathfrak{R}_1 > 0, \mathfrak{R}_2 > 0, \mathfrak{Z}_1 > 0, \mathfrak{Z}_2 > 0$ *and matrices* S, L, G, *for a given scalar* γ, *such that the following LMI holds:*

$$\Upsilon(\delta) < 0, \tag{38}$$

where

$$\begin{aligned}
\Upsilon_{1,1} &= 2\mathfrak{P} + \mathfrak{Q}_1 + \mathfrak{Q}_3 + \tau_1^2\mathfrak{R}_1 + \varrho\mathfrak{R}_2 + \frac{\varrho^2}{4}\mathfrak{T}_1 + \mathfrak{Z}_1\\
&\quad - 2G\,D - \left(\mathfrak{Z}_2 + \frac{\pi^2}{4}\mathfrak{Z}_2\right)\\
&\quad - \left(\mathfrak{R}_1 + \frac{\pi^2}{4}\mathfrak{R}_1\right) - F_1\Lambda_{1i} + \sum_{j=1}^{N}\pi_{ij}(\delta)\mathfrak{P}_j,\\
\Upsilon_{1,2} &= GA + F_2\Lambda_{1i},\\
\Upsilon_{1,3} &= GB,\\
\Upsilon_{1,4} &= GK + \mathfrak{Z}_2 - \frac{\pi^2}{4}\mathfrak{Z}_2,\\
\Upsilon_{1,5} &= \mathfrak{R}_1 - \frac{\pi^2}{4}\mathfrak{R}_1,\\
\Upsilon_{1,7} &= -G + \mathfrak{P},\\
\Upsilon_{1,9} &= \frac{\pi^2}{4}\frac{1}{\varrho}\mathfrak{R}_1,\\
\Upsilon_{1,12} &= -\frac{\pi^2}{2}\mathfrak{Z}_2,\\
\Upsilon_{2,2} &= -\Lambda_{1i} + \mathfrak{Q}_2,\\
\Upsilon_{2,5} &= F_2\tilde{\Lambda}_{2i},\\
\Upsilon_{2,7} &= GA,\\
\Upsilon_{3,3} &= (1-\mu)\mathfrak{Q}_2,\\
\Upsilon_{3,7} &= \gamma GB,\\
\Upsilon_{4,4} &= 2\left(-\mathfrak{Z}_2 - \frac{\pi^2}{4}\mathfrak{Z}_2\right),\\
\Upsilon_{4,7} &= GK,\\
\Upsilon_{5,5} &= -\mathfrak{Q}_1 - 2\left(\mathfrak{R}_1 + \frac{\pi^2}{4}\mathfrak{R}_1\right),\\
\Upsilon_{5,6} &= \mathfrak{R}_1 - \frac{\pi^2}{4}\mathfrak{R}_1,\\
\Upsilon_{5,7} &= \gamma GB,\\
\Upsilon_{5,8} &= \frac{\pi^2}{4}\mathfrak{R}_1,\\
\Upsilon_{5,9} &= \frac{\pi^2}{2}\mathfrak{R}_1,\\
\Upsilon_{6,6} &= -\mathfrak{Q}_3 - \left(\mathfrak{Z}^2 + \frac{\pi^2}{4}\mathfrak{Z}_2\right),\\
\Upsilon_{7,7} &= \tau^2\mathfrak{R}_1 + \eta^2\mathfrak{Z}_2 - 2\gamma G,\\
\Upsilon_{8,8} &= -\pi^2\mathfrak{R}_1,\\
\Upsilon_{9,9} &= -\pi^2\mathfrak{R}_1,\\
\Upsilon_{10,10} &= -\mathfrak{Z}_1 - \mathfrak{Z}_2 - \frac{\pi^2}{4}\mathfrak{Z}_2,\\
\Upsilon_{10,11} &= \frac{\pi^2}{2}\mathfrak{Z}_2,\\
\Upsilon_{11,11} &= -\pi^2\mathfrak{Z}_2,\\
\Upsilon_{12,12} &= -\pi^2\mathfrak{Z}_2,\\
\Upsilon_{13,13} &= -\mathfrak{T}_1.
\end{aligned} \tag{39}$$

Along with, the controller gain matrices are defined as $G = KL^{-1}$.

Remark 1. In the literature, one can find many control methods for achieving synchronization of neural networks such as pinning control, feedback control, adaptive control, impulsive control, and sampled-data control. Different from

the previous literature, in this work, we employed the novel control technique, namely, nonfragile sampled-data control which includes the benefits of both control techniques for synchronization of SMJNNs. Moreover, switching topology or jump connection often arises in a network due to link failures or new development. In Markovian jumping systems, the jump time is exponentially distributed and also irrelevant to sojourn time which leads some restrictions in the utilization of the network. Thus, semi-Markovian jumping parameters which generalizes Markovian jumping parameters are also taken into account. This shows the novelty of the work.

Remark 2. It is worth noting that fractional calculus has a nearly identical background to traditional calculus. Its applications in physics and engineering, on the other hand, are a relatively new source of interest. Fractional-order models would be more suitable for describing memory and inherited properties of different materials than conventional integer-order models. Many known structures that exhibit fractional dynamics have been found to be useful in interdisciplinary fields such as viscoelasticity, dielectric polarization, electromagnetic waves, and complex system quantum evolution. Due to this reason, fractional-order system has been one of the most promising research topics in recent times. In our future work, the qualitative behaviors of fractional-order systems will be considered.

4. Numerical Simulation

This section displays two numerical examples to highlight the advantages of the theoretical results.

Example 1. Consider the NNs with

$$\begin{aligned} D_1 &= \begin{bmatrix} 1.2 & 0 \\ 0 & 0.7 \end{bmatrix}, \\ D_2 &= \begin{bmatrix} 1.1 & 0 \\ 0 & 0.6 \end{bmatrix}, \\ A_1 &= \begin{bmatrix} -1.2 & -2.3 \\ -2.1 & -1.4 \end{bmatrix}, \\ A_2 &= \begin{bmatrix} 1.5 & -1.14 \\ 5.0 & -2.6 \end{bmatrix}, \\ B_1 &= \begin{bmatrix} 3.2 & -5.3 \\ 3.1 & 4.1 \end{bmatrix}, \\ B_2 &= \begin{bmatrix} 2.1 & -1.2 \\ 5.15 & -1.05 \end{bmatrix}. \end{aligned} \tag{40}$$

The nonlinear activation functions are taken as

$$f_1(\omega) = f_2(\omega) = \tanh(\omega(t)), \tag{41}$$

with $\mathfrak{F}_1^+ = \mathfrak{F}_2^+ = 1$ and $\mathfrak{F}_1^- = \mathfrak{F}_2^- = 0$. Then,

$$\begin{aligned} F_1 &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \\ F_2 &= \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}. \end{aligned} \tag{42}$$

The transition rates are assumed to be $\alpha_{11}(\delta) \in [2.2, 1.8]$, $\alpha_{22}(\delta) \in [-1.9, -1.5]$. From that, we have $\alpha_{11,1} = -2.2$, $\alpha_{11,2} = -1.8$, $\alpha_{22,1} = -1.9$, and $\alpha_{22,2} = -1.5$.

By resolving the LMIs obtained in Theorem 1 and by using the help of Matlab LMI toolbox with the parameters $\varrho = 0.2$, the gain matrix is attained as

$$K = 10^{-3} \times \begin{bmatrix} -0.2654 & 0.3954 \\ 0.3608 & 0.2513 \end{bmatrix}. \tag{43}$$

Figure 1 displays the chaotic nature of the master system and Figure 2 shows the chaotic behavior of the slave system. By applying the designed nonfragile sampled-data control technique, the phase portraits of the error system are shown in Figure 3.

Example 2. Consider the NNs with

$$\begin{aligned} D_1 &= \begin{bmatrix} 1.2 & 0 \\ 0 & 1.8 \end{bmatrix}, \\ D_2 &= \begin{bmatrix} 1 & 0 \\ 0 & 1.5 \end{bmatrix}, \\ A_1 &= \begin{bmatrix} -3.7 + \frac{\pi}{4} & 4.1 \\ 3.0 & -4.8 + \frac{\pi}{4} \end{bmatrix}, \\ A_2 &= \begin{bmatrix} -3.7 + \frac{\pi}{4} & 4.1 \\ 3.0 & -4.8 + \frac{\pi}{4} \end{bmatrix} \\ B_1 &= \begin{bmatrix} -0.1\sqrt{(2)}\ \frac{\pi}{4} & -3.6 \\ 1.8 & -3.1\sqrt{(2)}\ \frac{\pi}{4} \end{bmatrix}, \\ B_2 &= \begin{bmatrix} -0.1\sqrt{(2)}\ \frac{\pi}{4} & -3.6 \\ 1.8 & -3.1\sqrt{(2)}\ \frac{\pi}{4} \end{bmatrix}. \end{aligned} \tag{44}$$

The nonlinear activation function and transition rates are taken as in the previous example. From the LMIs in Theorem 4 with the parameters $\varsigma_1 = \varsigma_2 = 0.2, \varrho = 0.5$, one can get the gain matrix

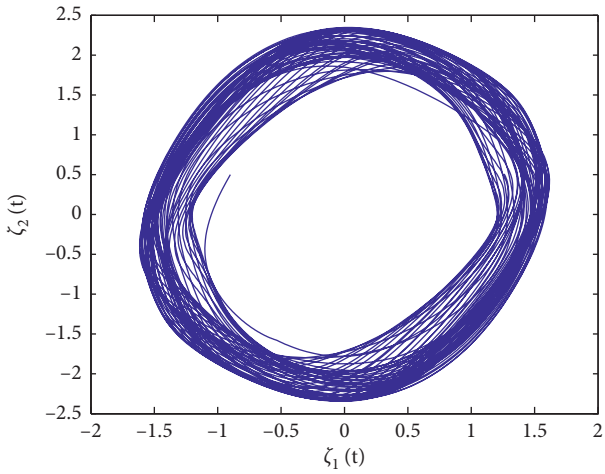

Figure 1: Chaotic behavior of the master system.

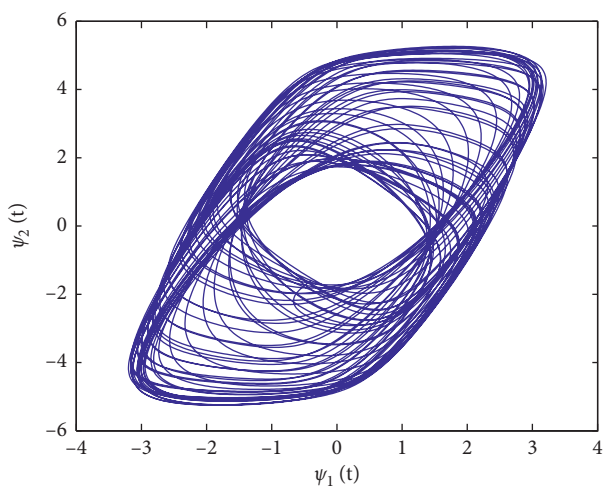

Figure 2: Chaotic behavior of slave system.

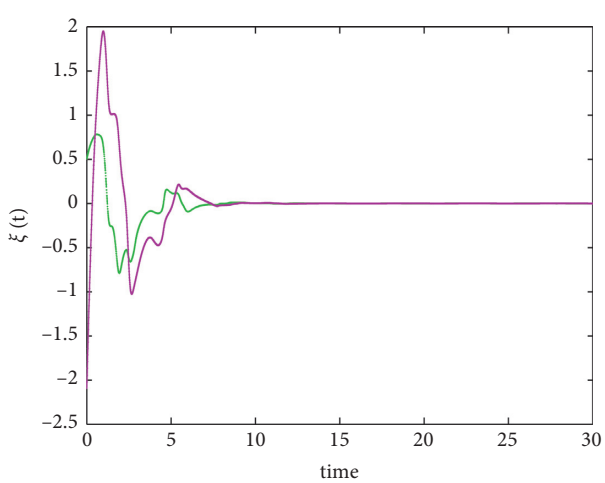

Figure 3: State response of the error system.

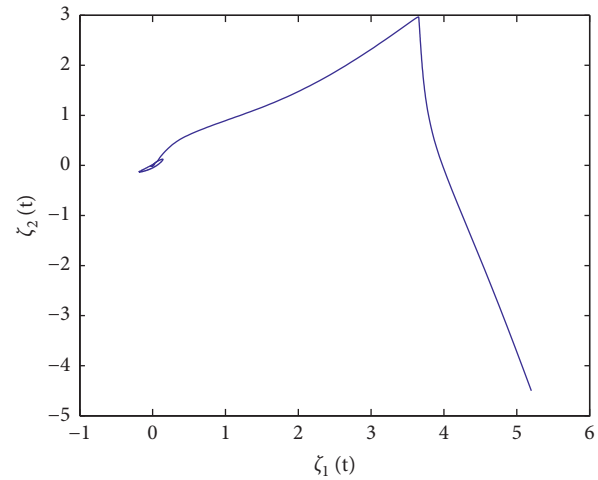

Figure 4: Chaotic nature of the master system.

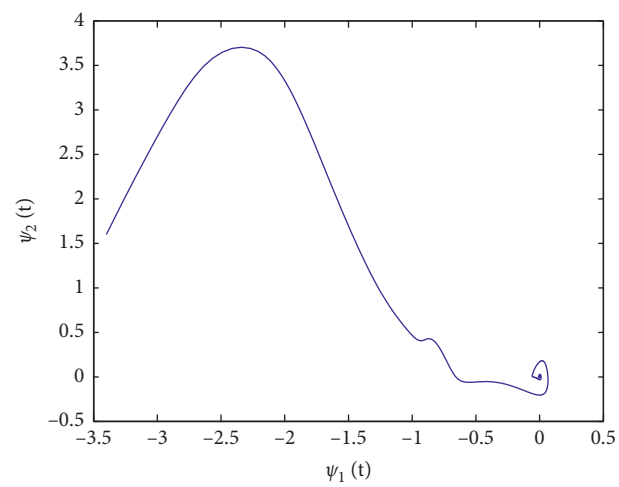

Figure 5: Chaotic nature of the slave system.

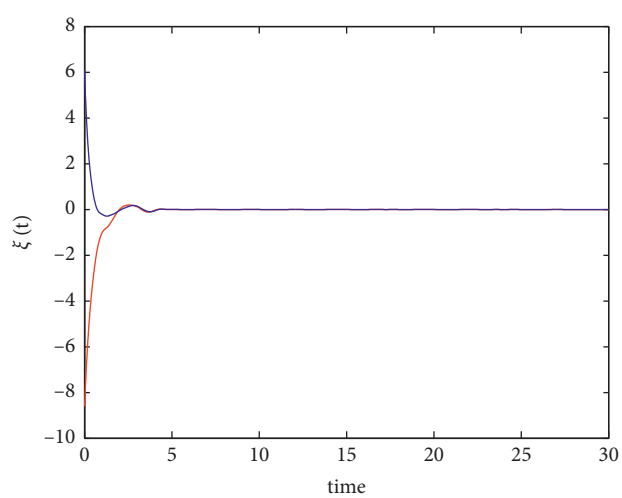

Figure 6: State trajectories of the error system.

$$K = \begin{bmatrix} 0.0351 & 0 \\ 0 & 0.0552 \end{bmatrix}. \tag{45}$$

In Figure 4, the chaotic nature of the master system is presented, and Figure 5 shows the chaotic behavior of the slave system. By making use of the nonfragile sampled-data control technique, the state trajectories of the error system are shown in Figure 6.

5. Conclusion

In this work, the synchronization of SMJNNs has been analyzed through the hybrid control technique, namely, nonfragile sampled-data control. Some criteria that ensure the synchronization of investigated SMJNNs with and without uncertainties in the control technique have been derived in the form of LMIs using Lyapunov stability theory and Writinger-based integral inequality approaches. The acquired LMIs are solved with Matlab LMI toolbox. Last, numerical simulations are granted to validate the designed controllers. Quaternion-valued neural network is the generalization of complex-valued neural networks, in which state, connection weight, and activation function are all quaternion numbers. Compared to real-valued NNs and complex-valued NNs, quaternion-valued NNs show significant advantages in multidimensional data processing. Recently, fractional-order quaternion-valued neural networks have gained great attention among the researchers due to its applications in many fields such as attitude control, image processing, computer graphics, prediction of three-dimensional wind processing, and so on. Due to its growing applications, it is important to analyze the qualitative behaviors of the fractional-order quaternion-valued NN. This will be our future work.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

All authors contributed equally to this work.

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