

## Retraction

# Retracted: Correlation Coefficient and Entropy Measures Based on Complex Dual Type-2 Hesitant Fuzzy Sets and Their Applications

### Journal of Mathematics

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This article has been retracted by Hindawi following an investigation undertaken by the publisher [1]. This investigation has uncovered evidence of one or more of the following indicators of systematic manipulation of the publication process:

- (1) Discrepancies in scope
- (2) Discrepancies in the description of the research reported
- (3) Discrepancies between the availability of data and the research described
- (4) Inappropriate citations
- (5) Incoherent, meaningless and/or irrelevant content included in the article
- (6) Manipulated or compromised peer review

The presence of these indicators undermines our confidence in the integrity of the article's content and we cannot, therefore, vouch for its reliability. Please note that this notice is intended solely to alert readers that the content of this article is unreliable. We have not investigated whether authors were aware of or involved in the systematic manipulation of the publication process.

Wiley and Hindawi regrets that the usual quality checks did not identify these issues before publication and have since put additional measures in place to safeguard research integrity.

We wish to credit our own Research Integrity and Research Publishing teams and anonymous and named external researchers and research integrity experts for contributing to this investigation.



The corresponding author, as the representative of all authors, has been given the opportunity to register their agreement or disagreement to this retraction. We have kept a record of any response received.

### References

- [1] T. Mahmood, Z. Ali, H. Garg, L. Zedam, and R. Chinram, "Correlation Coefficient and Entropy Measures Based on Complex Dual Type-2 Hesitant Fuzzy Sets and Their Applications," *Journal of Mathematics*, vol. 2021, Article ID 2568391, 34 pages, 2021.

## Research Article

# Correlation Coefficient and Entropy Measures Based on Complex Dual Type-2 Hesitant Fuzzy Sets and Their Applications

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The theory of complex dual type-2 hesitant fuzzy sets (CDT-2HFSs) is a blend of two different modifications of fuzzy sets (FSs), called complex fuzzy sets (CFSs) and dual type-2 hesitant fuzzy sets (DT-2HFSs). CDT-2HFS is a proficient technique to cope with unpredictable and awkward information in realistic decision problems. CDT-2HFS is composed of the grade of truth and the grade of falsity, and the grade of truth (also for grade of falsity) contains the grade of primary and secondary parts in the form of polar coordinates with the condition that the sum of the maximum of the real part (also for the imaginary part) of the primary grade (also for the secondary grade) cannot exceed the unit interval  $[0, 1]$ . The aims of this manuscript are to discover the novel approach of CDT-2HFS and its operational laws. These operational laws are also justified with the help of an example. Additionally, based on a novel CDT-2HFS, we explored the correlation coefficient (CC) and entropy measures (EMs), and their special cases are also discussed. TOPSIS method based on CDT-2HFS is also explored. Then, we applied our explored measures based on CDT-2HFSs in the environment of the TOPSIS method, medical diagnosis, pattern recognition, and clustering algorithm to cope with the awkward and complicated information in realistic decision issues. Finally, some numerical examples are given to examine the proficiency and validity of the explored measures. Comparative analysis, advantages, and graphical interpretation of the explored measures with some other existing measures are also discussed.

## 1. Introduction

The present decision-making is one of the genuinely basic movements in individuals' everyday life, the reason for existing of which is to rank the limited arrangement of options regarding that they are so solid to the choice maker(s). Multiattribute decision-making (MADM) is a part of decision-making and is viewed as an intellectual-based human movement. People unavoidably are confronted with different decision-making issues, which include numerous

fields [1–3]. The idea of the fuzzy set (FS) proposed by Zadeh [4] modified the method of measuring the vulnerability/fuzziness. Before the development of the FS hypothesis by Zadeh [4], the likelihood hypothesis was the customary instrument to quantify the vulnerability. Be that as it may, to gauge the vulnerability utilizing likelihood, it ought to have been communicated as exact numbers which are its primary constraints. The obscure terms, for instance, “without doubt” and “marginally,” could not be measured utilizing the likelihood hypothesis. To gauge the

vulnerability/fuzziness related to such unclear terms, the FS hypothesis has ended up being a successful apparatus. In the FS hypothesis, every component relating to a specific universe of talk has been appointed an enrollment degree lying somewhere in the range of 0 and 1, which indicates its level of belongingness to the set being referred to called FS. By the goodness of its reasonableness in genuine issues, FSs increased a lot of prevalence with analysts around the world. Endeavors were made to additionally sum up the idea by numerous creators to make it progressively versatile for viable issues.

Notwithstanding, in certain issues including etymological factors, for example, exceptionally low, low, medium, high, and extremely high, the assurance of the participation capacity may not be simple; that is, in an issue, dubious participation capacity might be experienced. To survive such circumstances, the idea of type-2 FSs (T-2FSs) was presented by Zadeh [5], as a distinction from common FSs. Many researchers have utilized T-2FSs in different areas [6–8]. The tale structures, which are speculations and expansions of the FSs, have been proposed by numerous analysts since Zadeh presented the FSs. The fundamental motivation behind these structures is to take out vulnerabilities and to guarantee that specialists settle on choices in a way that is without blunder or with not many mistakes. One of these structures is the idea of hesitant FS (HFS) characterized by Torra [9]. Feng et al. [10] presented the type-2 hesitant fuzzy set (T-2HFS). The idea of dual HFS (DHFS) was first characterized as a speculation of the HFSs characterized by Zhu et al. [11]. A DHFS is distinguished as two distinct capacities called enrollment and nonmembership capacity. This structure permits the leader to make more adaptable, precise, and reasonable remarks about the components under the reluctant zone. In this manner, it limits the blunder edge by giving more solid outcomes than the current structures, as HFSs and interval-valued HFSs. Alcantud et al. [12] characterized the idea of the double broadened HFSs and applied it to a decision-making issue under dual extended hesitant fuzzy data.

As for the above existing examinations, it has been dissected that they have researched the decision-making issues under the FS, IFS, or its speculations, which are just ready to manage the vulnerability and dubiousness existing in the information. These models cannot speak to the fractional obliviousness of the information and its changes at a given period of time. Be that as it may, in complex informational collections, vulnerability and ambiguity in the information happen simultaneously with changes to the stage (periodicity) of the information. Instances of complex informational indexes incorporate a lot of information that is created from clinical research, just as government databases for biometric and facial acknowledgments, sound, and pictures, all of which may contain a lot of deficient, dubious, and ambiguous data. To deal with these kinds of issues, the theory of complex FS (CFS) was discovered by Ramot et al. [13]. CFS contains the grade of membership in the form of a complex number belonging to a unit disc in a complex plane. Various scholars utilized CFS in different fields [14–16].

Correlation examination shows a direct connection between two sets and it has a very significant spot for dynamics. In this way, numerous researchers in various fields have considered the relationship coefficients. Additionally, the FS and its speculations have a significant job in dynamics, so CCs have drawn in the consideration of scientists examining the FS and its speculations. For instance, Chiang and Lin [17] and Chaudhuri and Bhattacharya [18] examined the correlation between two FSs. Gerstenkorn and Mańko [19] worked the relationship and CC of the intuitionistic FSs (IFSs). The entropy of FSs is a proportion of fuzziness between FSs. De Luca and Termini [20] first presented the aphorism development for the entropy of FSs concerning Shannon's likelihood entropy. Yager [21] characterized fuzziness proportions of FSs as far as a need of differentiation between the FS and its nullification based on  $L_p$  standard. Kosko [22] gave a proportion of fuzziness between FSs utilizing a proportion of separation between the FS and its closest set to the separation between the FS and its farthest set. Xuecheng [23] gave some aphorism definitions of entropy and furthermore characterized a  $\sigma$ -entropy. Pal and Pal [24] proposed exponential entropy. Meanwhile Fan and Ma [25] gave some new fuzzy entropy equations. The technique for establishing order preference by similarity to the ideal solution (TOPSIS) technique as a strategy for building up request inclination by likeness to the perfect arrangement, started by Hwang and Yoon [26], is one of the best and beneficial methods for decision-making. The basic idea of TOPSIS strategy is to pick the elective that has the briefest good way from the positive perfect arrangement (PIS) and the greatest good way from the negative perfect arrangement (NIS). There exists a tremendous writing including study and utilization of TOPSIS hypothesis in a wide scope of MCDM just as multicriteria group decision-making (MCGDM) issues [27–29].

Dual type-2 hesitant fuzzy set contains the grade of truth and the grade of falsity in the form of the subset of the unit interval with the condition that the sum of the maximum of the truth grade and the maximum of the falsity grade cannot exceed the unit interval. The complex dual type-2 hesitant fuzzy set is a generalization of the dual type-2 hesitant fuzzy set, in which the amplitude term provides the extent of belonging of an object, while the phase term describes the periodicity. These phase terms distinguish the complex dual type-2 hesitant fuzzy set from the traditional dual type-2 hesitant fuzzy set theories. In dual type-2 hesitant fuzzy set theory, the data are managed with the compensation of only the degree of belonging, while the part of periodicity is completely ignored. Hence, this may result in the loss of information during the decision-making processes in some certain cases. To further illustrate the concept of phase terms, we take an example. Suppose that a person wants to purchase a car under crucial factors such as its model and its production date. Since the model of each car moves with the evolution of the production dates, to make a selection or decision regarding choosing the optimal car is a decision-making process taking these two factors into account simultaneously. Moreover, it is quite obvious that such types of problems cannot be modeled accurately with traditional theories. However, complex dual type-2 hesitant fuzzy set theory is

well suited for such classes of problems, where the amplitude terms may be used to provide a decision about the model of a car, while the phase term concerns its production dates. Henceforth, a complex dual type-2 hesitant fuzzy set is a more generalized continuation of the existing theories, such as type-2 hesitant fuzzy sets and dual type-2 hesitant fuzzy sets.

When a decision-maker gives  $(0.4e^{i2\pi(0.3)}, 0.3e^{i2\pi(0.2)})$  and  $(0.41e^{i2\pi(0.31)}, 0.31e^{i2\pi(0.21)})$  for the grade of complex-valued supporting and the grade of complex-valued supporting against in the form of primary and secondary information with the condition that the sum of the maximum of the real part (also for the imaginary part) of the complex-valued supporting (also for supporting against) grade for primary (also for secondary) information cannot exceed the unit interval, There exist notions like FSs, T-2FSs, HFSs, DHFSs, CFSs, and DT-2HFSs. Handling such kind of issues is very difficult, but when a decision-maker provides such kind of information in the form of the finite subset of unit interval, then it is very complicated for a decision-maker to handle it. For coping with such kind of issues, in this manuscript, the novel approach of CDT-2HFS, which is a mixture of two different modifications of FS, that is, CFS and DT-2HFS, is explored. CDT-2HFS is a proficient technique to cope with unpredictable and awkward information in realistic decision problems. CDT-2HFS composes the grade of truth and the grade of falsity, and the grade truth (also for falsity grade) contains the grade of primary and secondary parts in the form of polar coordinates with the condition that the sum of the maximum of the real part (also for the imaginary part) of the primary grade (also for the secondary grade) cannot exceed the unit interval. The aims of this manuscript are to discover the novel approach of CDT-2HFS and its operational laws. These operational laws are also justified with the help of an example. Additionally, based on a novel CDT-2HFS, we explored the correlation coefficient (CC) and entropy measures (EMs), and their special cases are discussed. TOPSIS method based on CDT-2HFS is also explored. Then, we applied our explored measures based on CDT-2HFSs in the environment of the TOPSIS method, medical diagnosis, pattern recognition, and clustering algorithm to cope with awkward and complicated information in realistic decision issues. Finally, four numerical examples are resolved to examine the proficiency and validity of the explored measures. Comparative analysis, advantages, and graphical interpretation of the explored measures with some other existing measures are also discussed.

The aims of this manuscript are summarized as follows: in Section 2, we review some basic notions like FSs, T-2FSs, HFSs, DHFSs, CFSs, and their basic laws. In Section 3, the theory of CDT-2HFS, which is a mixture of two different modifications of FS, that is, CFS and DT-2HFS, is presented. CDT-2HFS is a proficient technique to cope with unpredictable and awkward information in realistic decision problems. CDT-2HFS is composed of the grade of truth and the grade of falsity, and the grade truth (also for falsity grade) contains the grade of primary and secondary parts in the form of polar coordinates with the condition that the sum of the maximum of the real part (also for the imaginary part) of

the primary grade (also for secondary grade) cannot exceed the unit interval. The aims of this manuscript were to discover the novel approach of CDT-2HFS and its operational laws. These operational laws are also justified with the help of examples. In Sections 4 and 5, based on a novel CDT-2HFS, we explored the correlation coefficient (CC) and entropy measures (EMs), and their special cases are discussed. In Section 6, TOPSIS method based on CDT-2HFS is also explored. Then, we applied our explored measures based on CDT-2HFSs in the environment of TOPSIS method, medical diagnosis, pattern recognition, and clustering algorithm to cope with awkward and complicated information in realistic decision issues. Finally, four numerical examples are resolved to examine the proficiency and validity of the explored measures. Comparative analysis, advantages, and graphical interpretation of the explored measures with some other existing measures are also discussed. The conclusion of this paper is discussed in Section 7.

## 2. Preliminaries

Basic notions of FSs, T-2FSs, HFSs, DHFSs, CFSs, and their operational laws are briefly reviewed in this study. Throughout this manuscript, the symbol  $\mathcal{X}_{\text{UNI}}$  denotes the fixed set.

*Definition 1* (see [4]). A FS is an object of the form

$$\mathcal{Q}_{\text{FS}} = \{(\tilde{x}, M_{\mathcal{Q}_{\text{FS}}}(\tilde{x})) : \tilde{x} \in \mathcal{X}_{\text{UNI}}\}, \quad (1)$$

where  $M_{\mathcal{Q}_{\text{FS}}}$  represents the grade of supporting with the condition that  $0 \leq M_{\mathcal{Q}_{\text{FS}}} \leq 1$ .

*Definition 2* (see [5]). A T-2FS is an object of the form

$$\mathcal{Q}_{\text{T-2FS}} = \{((\tilde{x}, \tilde{x}'), M_{\mathcal{Q}_{\text{T-2FS}}}(\tilde{x}, \tilde{x}')) : \forall \tilde{x} \in \mathcal{X}_{\text{UNI}}, \tilde{x}' \in J_{\tilde{x}} \subseteq [0, 1]\}, \quad (2)$$

where  $M_{\mathcal{Q}_{\text{T-2FS}}}(\tilde{x}, \tilde{x}')$  represents the grade of type-2 supporting with the condition that  $0 \leq M_{\mathcal{Q}_{\text{T-2FS}}}(\tilde{x}, \tilde{x}') \leq 1$ .

*Definition 3* (see [9]). A HFS is an object of the form

$$\mathcal{Q}_{\text{HFS}} = \{(\tilde{x}, M_{\mathcal{Q}_{\text{HFS}}}(\tilde{x})) : \tilde{x} \in \mathcal{X}_{\text{UNI}}\}, \quad (3)$$

where  $M_{\mathcal{Q}_{\text{HFS}}}$  represents the grade of supporting in the form of the subset of the unit interval, with the condition that  $0 \leq \text{Max}(M_{\mathcal{Q}_{\text{HFS}}}) \leq 1$ , whenever  $M_{\mathcal{Q}_{\text{HFS}}} \subseteq [0, 1]$ .

*Definition 4* (see [11]). A DHFS is an object of the form

$$\mathcal{Q}_{\text{DHFS}} = \{(\tilde{x}, M_{\mathcal{Q}_{\text{DHFS}}}(\tilde{x}), N_{\mathcal{Q}_{\text{DHFS}}}(\tilde{x})) : \tilde{x} \in \mathcal{X}_{\text{UNI}}\}, \quad (4)$$

where  $M_{\mathcal{Q}_{\text{DHFS}}}$  and  $N_{\mathcal{Q}_{\text{DHFS}}}$  represent the grade of supporting and the grade of supporting against with the condition that  $0 \leq \max(M_{\mathcal{Q}_{\text{DHFS}}}) + \max(N_{\mathcal{Q}_{\text{DHFS}}}) \leq 1$ , whenever  $M_{\mathcal{Q}_{\text{DHFS}}}, N_{\mathcal{Q}_{\text{DHFS}}} \subseteq [0, 1]$ .

Additionally, we defined some operational laws based on DHFSs. For any two DHFSs  $\mathcal{Q}_{\text{DHFS-1}} = (M_{\mathcal{Q}_{\text{DHFS-1}}}^{6(1)}(\tilde{x}), N_{\mathcal{Q}_{\text{DHFS-1}}}^{6(1)}(\tilde{x}))$  and  $\mathcal{Q}_{\text{DHFS-2}} = (M_{\mathcal{Q}_{\text{DHFS-2}}}^{6(1)}(\tilde{x}), N_{\mathcal{Q}_{\text{DHFS-2}}}^{6(1)}(\tilde{x}))$ , we have

$$\begin{aligned} \mathcal{Q}_{\text{DHFS-1}} \cup \mathcal{Q}_{\text{DHFS-2}} &= \left( \max \left( M_{\mathcal{Q}_{\text{DHFS-1}}}^{\delta(1)}(\tilde{x}), M_{\mathcal{Q}_{\text{DHFS-2}}}^{\delta(1)}(\tilde{x}) \right), \min \left( N_{\mathcal{Q}_{\text{DHFS-1}}}^{\delta(1)}(\tilde{x}), N_{\mathcal{Q}_{\text{DHFS-2}}}^{\delta(1)}(\tilde{x}) \right) \right), \\ \mathcal{Q}_{\text{DHFS-1}} \cap \mathcal{Q}_{\text{DHFS-2}} &= \left( \min \left( M_{\mathcal{Q}_{\text{DHFS-1}}}^{\delta(1)}(\tilde{x}), M_{\mathcal{Q}_{\text{DHFS-2}}}^{\delta(1)}(\tilde{x}) \right), \max \left( N_{\mathcal{Q}_{\text{DHFS-1}}}^{\delta(1)}(\tilde{x}), N_{\mathcal{Q}_{\text{DHFS-2}}}^{\delta(1)}(\tilde{x}) \right) \right). \end{aligned} \tag{5}$$

*Definition 5* (see [13]). A CFS is an object of the form

$$\mathcal{Q}_{\text{CFS}} = \left\{ \left( \tilde{x}, M_{\mathcal{Q}_{\text{CFS}}}(\tilde{x}) \right) : \tilde{x} \in \mathcal{X}_{\text{UNI}} \right\}, \tag{6}$$

where  $M_{\mathcal{Q}_{\text{CFS}}} = M_{\mathcal{Q}_{\text{CFRP}}}(\tilde{x})e^{i2\pi(M_{\mathcal{Q}_{\text{CEIP}}}(\tilde{x}))}$  represents the grade of complex-valued supporting with the condition that  $0 \leq M_{\mathcal{Q}_{\text{CFRP}}}, M_{\mathcal{Q}_{\text{CFIP}}} \leq 1$ .

### 3. Complex Dual Type-2 Hesitant Fuzzy Sets

Based on the existing drawbacks [30], in this study, we discovered the new theory of CDT-2HFSs and their operational laws. The presented operational laws are also justified with the help of some examples.

*Definition 6.* A CDT-2HFS is an object of the form

$$\mathcal{Q}_{\text{CDTH}} = \left\{ \left( \tilde{x}, \left( M_{\mathcal{Q}_{\text{CDTH}}}(\tilde{x}), N_{\mathcal{Q}_{\text{CDTH}}}(\tilde{x}) \right) \right) : \tilde{x} \in \mathcal{X}_{\text{UNI}} \right\}. \tag{7}$$

where  $M_{\mathcal{Q}_{\text{CDTH}}}(\tilde{x}) = \left\{ \left( M_{\mathcal{Q}_{\text{CDTHRP-P}}}^{\delta(j)}(\tilde{x}) \cdot e^{i2\pi(M_{\mathcal{Q}_{\text{CDTHRP-P}}}^{\delta(j)}(\tilde{x}))}, M_{\mathcal{Q}_{\text{CDTHRP-S}}}^{\delta(j)}(\tilde{x}) \cdot e^{i2\pi(M_{\mathcal{Q}_{\text{CDTHRP-S}}}^{\delta(j)}(\tilde{x}))} \right) : j = 1, 2, 3, \dots, n \right\}$  and  $N_{\mathcal{Q}_{\text{CDTH}}}(\tilde{x}) = \left\{ \left( N_{\mathcal{Q}_{\text{CDTHRP-P}}}^{\delta(k)}(\tilde{x}) \cdot e^{i2\pi(N_{\mathcal{Q}_{\text{CDTHRP-P}}}^{\delta(k)}(\tilde{x}))}, N_{\mathcal{Q}_{\text{CDTHRP-S}}}^{\delta(k)}(\tilde{x}) \cdot e^{i2\pi(N_{\mathcal{Q}_{\text{CDTHRP-S}}}^{\delta(k)}(\tilde{x}))} \right) : k = 1, 2, 3, \dots, m \right\}$  represent the grade of complex-valued supporting and the grade of complex-valued supporting against in the form of complex type-2 hesitant fuzzy elements (CT-2HFEs) with the following conditions:  $0 \leq \max(M_{\mathcal{Q}_{\text{CDTHRP-P}}}^{\delta(j)}) + \max(N_{\mathcal{Q}_{\text{CDTHRP-P}}}^{\delta(k)}) \leq 1, 0 \leq \max(M_{\mathcal{Q}_{\text{CDTHRP-S}}}^{\delta(j)}) + \max(N_{\mathcal{Q}_{\text{CDTHRP-S}}}^{\delta(k)}) \leq 1$  and  $0 \leq \max(M_{\mathcal{Q}_{\text{CDTHRP-P}}}^{\delta(j)}) + \max(N_{\mathcal{Q}_{\text{CDTHRP-S}}}^{\delta(k)}) \leq 1$ . The complex dual type-2 hesitant fuzzy set is expressed by

$$\mathcal{Q}_{\text{CDTH}} = \left\{ \left( \left( \left( M_{\mathcal{Q}_{\text{CDTHRP-P}}}^{\delta(j)}(\tilde{x}) \cdot e^{i2\pi(M_{\mathcal{Q}_{\text{CDTHRP-P}}}^{\delta(j)}(\tilde{x}))}, M_{\mathcal{Q}_{\text{CDTHRP-S}}}^{\delta(j)}(\tilde{x}) \cdot e^{i2\pi(M_{\mathcal{Q}_{\text{CDTHRP-S}}}^{\delta(j)}(\tilde{x}))} \right), \left( N_{\mathcal{Q}_{\text{CDTHRP-P}}}^{\delta(k)}(\tilde{x}) \cdot e^{i2\pi(N_{\mathcal{Q}_{\text{CDTHRP-P}}}^{\delta(k)}(\tilde{x}))}, N_{\mathcal{Q}_{\text{CDTHRP-S}}}^{\delta(k)}(\tilde{x}) \cdot e^{i2\pi(N_{\mathcal{Q}_{\text{CDTHRP-S}}}^{\delta(k)}(\tilde{x}))} \right) \right) \right), j, k = 1, 2, 3, \dots, n, m \right\}. \tag{8}$$

Additionally, we defined some operational laws based on CDT-2HFEs. For any two CDT-2HFEs  $\mathcal{Q}_{\text{CDTH-1}} = \left( \left( M_{\mathcal{Q}_{\text{CDTHRP-P-1}}}^{\delta(1)}(\tilde{x}) \cdot e^{i2\pi(M_{\mathcal{Q}_{\text{CDTHRP-P-1}}}^{\delta(1)}(\tilde{x}))}, M_{\mathcal{Q}_{\text{CDTHRP-S-1}}}^{\delta(1)}(\tilde{x}) \cdot e^{i2\pi(M_{\mathcal{Q}_{\text{CDTHRP-S-1}}}^{\delta(1)}(\tilde{x}))} \right), \left( N_{\mathcal{Q}_{\text{CDTHRP-P-1}}}^{\delta(1)}(\tilde{x}) \cdot e^{i2\pi(N_{\mathcal{Q}_{\text{CDTHRP-P-1}}}^{\delta(1)}(\tilde{x}))}, N_{\mathcal{Q}_{\text{CDTHRP-S-1}}}^{\delta(1)}(\tilde{x}) \cdot e^{i2\pi(N_{\mathcal{Q}_{\text{CDTHRP-S-1}}}^{\delta(1)}(\tilde{x}))} \right) \right)$  and  $\mathcal{Q}_{\text{CDTH-2}} =$

$\left( \left( M_{\mathcal{Q}_{\text{CDTHRP-P-2}}}^{\delta(1)}(\tilde{x}) \cdot e^{i2\pi(M_{\mathcal{Q}_{\text{CDTHRP-P-2}}}^{\delta(1)}(\tilde{x}))}, M_{\mathcal{Q}_{\text{CDTHRP-S-1}}}^{\delta(1)}(\tilde{x}) \cdot e^{i2\pi(M_{\mathcal{Q}_{\text{CDTHRP-S-1}}}^{\delta(1)}(\tilde{x}))} \right), \left( N_{\mathcal{Q}_{\text{CDTHRP-S-1}}}^{\delta(1)}(\tilde{x}) \cdot e^{i2\pi(N_{\mathcal{Q}_{\text{CDTHRP-S-1}}}^{\delta(1)}(\tilde{x}))}, N_{\mathcal{Q}_{\text{CDTHRP-P-2}}}^{\delta(1)}(\tilde{x}) \cdot e^{i2\pi(N_{\mathcal{Q}_{\text{CDTHRP-P-2}}}^{\delta(1)}(\tilde{x}))} \right) \right)$ , we have

$$\mathcal{Q}_{\text{CDTH-1}} \cup \mathcal{Q}_{\text{CDTH-2}} = \left\{ \left( \left( \left( \max \left( M_{\mathcal{Q}_{\text{CDTHRP-P-1}}}^{\delta(1)}(\tilde{x}), M_{\mathcal{Q}_{\text{CDTHRP-P-2}}}^{\delta(1)}(\tilde{x}) \right) e^{i2\pi \left( \max \left( M_{\mathcal{Q}_{\text{CDTHRP-P-1}}}^{\delta(1)}(\tilde{x}), M_{\mathcal{Q}_{\text{CDTHRP-P-2}}}^{\delta(1)}(\tilde{x}) \right) \right)}, \max \left( M_{\mathcal{Q}_{\text{CDTHRP-S-1}}}^{\delta(1)}(\tilde{x}), M_{\mathcal{Q}_{\text{CDTHRP-S-2}}}^{\delta(1)}(\tilde{x}) \right) e^{i2\pi \left( \max \left( M_{\mathcal{Q}_{\text{CDTHRP-S-2}}}^{\delta(1)}(\tilde{x}), M_{\mathcal{Q}_{\text{CDTHRP-S-1}}}^{\delta(1)}(\tilde{x}) \right) \right)} \right), \left( \min \left( N_{\mathcal{Q}_{\text{CDTHRP-P-1}}}^{\delta(1)}(\tilde{x}), N_{\mathcal{Q}_{\text{CDTHRP-S-2}}}^{\delta(1)}(\tilde{x}) \right) e^{i2\pi \left( \min \left( N_{\mathcal{Q}_{\text{CDTHRP-P-1}}}^{\delta(1)}(\tilde{x}), N_{\mathcal{Q}_{\text{CDTHRP-S-2}}}^{\delta(1)}(\tilde{x}) \right) \right)}, \min \left( N_{\mathcal{Q}_{\text{CDTHRP-S-1}}}^{\delta(1)}(\tilde{x}), N_{\mathcal{Q}_{\text{CDTHRP-P-2}}}^{\delta(1)}(\tilde{x}) \right) e^{i2\pi \left( \min \left( N_{\mathcal{Q}_{\text{CDTHRP-S-2}}}^{\delta(1)}(\tilde{x}), N_{\mathcal{Q}_{\text{CDTHRP-S-1}}}^{\delta(1)}(\tilde{x}) \right) \right)} \right) \right) \right\}, \tag{9}$$

$$\mathcal{Q}_{CDTH-1} \cap \mathcal{Q}_{CDTH-2} = \left\{ \left( \begin{array}{l} \left( \min(M_{\mathcal{Q}_{CDTHRP-P-1}}^{\delta(1)}(\tilde{x}), M_{\mathcal{Q}_{CDTHRP-P-2}}^{\delta(1)}(\tilde{x}))e^{i2\pi(\min(M_{\mathcal{Q}_{CDTHIP-P-1}}^{\delta(1)}(\tilde{x}), M_{\mathcal{Q}_{CDTHIP-P-2}}^{\delta(1)}(\tilde{x}))), \right) \\ \left( \min(M_{\mathcal{Q}_{CDTHRP-S-1}}^{\delta(1)}(\tilde{x}), M_{\mathcal{Q}_{CDTHRP-S-2}}^{\delta(1)}(\tilde{x}))e^{i2\pi(\min(M_{\mathcal{Q}_{CDTHIP-S-1}}^{\delta(1)}(\tilde{x}), M_{\mathcal{Q}_{CDTHIP-S-2}}^{\delta(1)}(\tilde{x}))) \right) \\ \left( \max(N_{\mathcal{Q}_{CDTHRP-P-1}}^{\delta(1)}(\tilde{x}), N_{\mathcal{Q}_{CDTHRP-P-2}}^{\delta(1)}(\tilde{x}))e^{i2\pi(\max(N_{\mathcal{Q}_{CDTHIP-P-1}}^{\delta(1)}(\tilde{x}), N_{\mathcal{Q}_{CDTHIP-P-2}}^{\delta(1)}(\tilde{x}))) \right) \\ \left( \max(N_{\mathcal{Q}_{CDTHRP-S-1}}^{\delta(1)}(\tilde{x}), N_{\mathcal{Q}_{CDTHRP-S-2}}^{\delta(1)}(\tilde{x}))e^{i2\pi(\max(N_{\mathcal{Q}_{CDTHIP-S-1}}^{\delta(1)}(\tilde{x}), N_{\mathcal{Q}_{CDTHIP-S-2}}^{\delta(1)}(\tilde{x}))) \right) \end{array} \right) \right\}. \quad (10)$$

*Example 1.* For any two CDT-2HFSs, all their entries in the form of complex numbers are stated as follows:

$$\mathcal{Q}_{CDTH-1} = \left( \left\{ (0.1e^{i2\pi(0.2)}, 0.2e^{i2\pi(0.3)}), (0.2e^{i2\pi(0.3)}, 0.3e^{i2\pi(0.4)}), (0.3e^{i2\pi(0.4)}, 0.4e^{i2\pi(0.5)}) \right\}, \left\{ (0.01e^{i2\pi(0.02)}, 0.02e^{i2\pi(0.03)}), (0.03e^{i2\pi(0.04)}, 0.05e^{i2\pi(0.06)}) \right\} \right), \quad (11)$$

$$\mathcal{Q}_{CDTH-2} = \left( \left\{ (0.7e^{i2\pi(0.6)}, 0.6e^{i2\pi(0.5)}), (0.6e^{i2\pi(0.5)}, 0.5e^{i2\pi(0.4)}), (0.5e^{i2\pi(0.4)}, 0.4e^{i2\pi(0.3)}) \right\}, \left\{ (0.07e^{i2\pi(0.08)}, 0.09e^{i2\pi(0.01)}), (0.01e^{i2\pi(0.03)}, 0.22e^{i2\pi(0.03)}) \right\} \right).$$

Then, by using equations (9) and (10), we get

$$\mathcal{Q}_{CDTH-1} \cup \mathcal{Q}_{CDTH-2} = \left( \left\{ (0.7e^{i2\pi(0.6)}, 0.6e^{i2\pi(0.5)}), (0.6e^{i2\pi(0.5)}, 0.5e^{i2\pi(0.4)}), (0.5e^{i2\pi(0.4)}, 0.4e^{i2\pi(0.5)}) \right\}, \left\{ (0.01e^{i2\pi(0.02)}, 0.02e^{i2\pi(0.01)}), (0.01e^{i2\pi(0.03)}, 0.05e^{i2\pi(0.03)}) \right\} \right), \quad (12)$$

$$\mathcal{Q}_{CDTH-1} \cap \mathcal{Q}_{CDTH-2} = \left( \left\{ (0.1e^{i2\pi(0.2)}, 0.2e^{i2\pi(0.3)}), (0.2e^{i2\pi(0.3)}, 0.3e^{i2\pi(0.4)}), (0.3e^{i2\pi(0.4)}, 0.4e^{i2\pi(0.3)}) \right\}, \left\{ (0.07e^{i2\pi(0.08)}, 0.09e^{i2\pi(0.03)}), (0.03e^{i2\pi(0.04)}, 0.22e^{i2\pi(0.06)}) \right\} \right).$$

The explored notions, which are stated in the form of equations (7), (9), and (10), are more proficient and more modified than the existing drawbacks; for instance, if we choose the imaginary part of equations (7), (9), and (10) to be zero, then equations (7), (9), and (10) convert it for DT-2HFS [30].

#### 4. Correlation Coefficient for Complex Dual Type-2 Hesitant Fuzzy Sets

The aim of this study is to present the novel correlation, correlation coefficient (CC), maximum-based CC (MCC), weighted CC (WCC), and maximum-based WCC (MWCC). The special cases of the explored measures are also explored.

*Definition 7.* For any two CDT-2HFSs,  $\mathcal{Q}_{CDTH-1} = \left\{ \left( (M_{\mathcal{Q}_{CDTHRP-P-1}}^{\delta(j)}(\tilde{x}_i) \cdot e^{i2\pi(M_{\mathcal{Q}_{CDTHIP-P-1}}^{\delta(j)}(\tilde{x}_i))}, M_{\mathcal{Q}_{CDTHRP-S-1}}^{\delta(j)}(\tilde{x}_i) \cdot e^{i2\pi(M_{\mathcal{Q}_{CDTHIP-S-1}}^{\delta(j)}(\tilde{x}_i))}, (N_{\mathcal{Q}_{CDTHRP-P-1}}^{\delta(k)}(\tilde{x}_i) \cdot e^{i2\pi(N_{\mathcal{Q}_{CDTHIP-P-1}}^{\delta(k)}(\tilde{x}_i))}, (N_{\mathcal{Q}_{CDTHRP-S-1}}^{\delta(k)}(\tilde{x}_i) \cdot e^{i2\pi(N_{\mathcal{Q}_{CDTHIP-S-1}}^{\delta(k)}(\tilde{x}_i))}, j, k = 1, 2, 3, \dots, n, m \right) \right\}$  and  $\mathcal{Q}_{CDTH-2} = \left\{ \left( (M_{\mathcal{Q}_{CDTHRP-P-2}}^{\delta(j)}(\tilde{x}_i) \cdot e^{i2\pi(M_{\mathcal{Q}_{CDTHIP-P-2}}^{\delta(j)}(\tilde{x}_i))}, M_{\mathcal{Q}_{CDTHRP-S-2}}^{\delta(j)}(\tilde{x}_i) \cdot e^{i2\pi(M_{\mathcal{Q}_{CDTHIP-S-2}}^{\delta(j)}(\tilde{x}_i))}, (N_{\mathcal{Q}_{CDTHRP-P-2}}^{\delta(k)}(\tilde{x}_i) \cdot e^{i2\pi(N_{\mathcal{Q}_{CDTHIP-P-2}}^{\delta(k)}(\tilde{x}_i))}, (N_{\mathcal{Q}_{CDTHRP-S-2}}^{\delta(k)}(\tilde{x}_i) \cdot e^{i2\pi(N_{\mathcal{Q}_{CDTHIP-S-2}}^{\delta(k)}(\tilde{x}_i))}, j, k = 1, 2, 3, \dots, \tilde{l}_{\tilde{x}_i}, \sim \tilde{l}_{\tilde{x}_i} \right\}$ , the correlation is of the form



**Proposition 1.** For any two CDT-2HFSs, QCDTH-1 and QCDTH-2, the CC among CDT-2HFSs satisfies the following axioms:

- (1)  $0 \leq \zeta_{\text{CDTHF-cc}}(\mathcal{Q}_{\text{CDTH-1}}, \mathcal{Q}_{\text{CDTH-2}}) \leq 1$
- (2)  $\zeta_{\text{CDTHF-cc}}(\mathcal{Q}_{\text{CDTH-1}}, \mathcal{Q}_{\text{CDTH-2}}) = 1 \Leftrightarrow \mathcal{Q}_{\text{CDTH-1}} = \mathcal{Q}_{\text{CDTH-2}}$

$$(3) \zeta_{\text{CDTHF-cc}}(\mathcal{Q}_{\text{CDTH-1}}, \mathcal{Q}_{\text{CDTH-2}}) = \zeta_{\text{CDTHF-cc}}(\mathcal{Q}_{\text{CDTH-2}}, \mathcal{Q}_{\text{CDTH-1}})$$

*Proof.* We prove the three above conditions by using equation (14). By using the inequality, it is clear that  $0 \leq \zeta_{\text{CDTHF-cc}}(\mathcal{Q}_{\text{CDTH-1}}, \mathcal{Q}_{\text{CDTH-2}})$ ; then we only prove that  $\zeta_{\text{CDTHF-cc}}(\mathcal{Q}_{\text{CDTH-1}}, \mathcal{Q}_{\text{CDTH-2}}) \leq 1$ . For this, we choose that

$$\zeta_{\text{CDTHF-c}}(\mathcal{Q}_{\text{CDTH-1}}, \mathcal{Q}_{\text{CDTH-2}}) = \left( \begin{aligned} & \frac{1}{\tilde{L}_{x_i}} \sum_{j=1}^{x_i} \left( M_{\mathcal{Q}_{\text{CDTHP-P-1}}}^{\delta(j)}(\tilde{x}_i) M_{\mathcal{Q}_{\text{CDTHP-P-2}}}^{\delta(j)}(\tilde{x}_i) + M_{\mathcal{Q}_{\text{CDTHP-S-1}}}^{\delta(j)}(\tilde{x}_i) M_{\mathcal{Q}_{\text{CDTHP-S-2}}}^{\delta(j)}(\tilde{x}_i) \right) + \\ & \frac{1}{\tilde{L}_{x_i}} \sum_{j=1}^{x_i} \left( M_{\mathcal{Q}_{\text{CDTHP-P-1}}}^{\delta(j)}(\tilde{x}_i) M_{\mathcal{Q}_{\text{CDTHP-P-2}}}^{\delta(j)}(\tilde{x}_i) + M_{\mathcal{Q}_{\text{CDTHP-S-1}}}^{\delta(j)}(\tilde{x}_i) M_{\mathcal{Q}_{\text{CDTHP-S-2}}}^{\delta(j)}(\tilde{x}_i) \right) + \\ & \frac{1}{\tilde{L}_{x_i}} \sum_{k=1}^{x_i} \left( N_{\mathcal{Q}_{\text{CDTHP-P-1}}}^{\delta(k)}(\tilde{x}_i) N_{\mathcal{Q}_{\text{CDTHP-P-2}}}^{\delta(k)}(\tilde{x}_i) + N_{\mathcal{Q}_{\text{CDTHP-S-1}}}^{\delta(k)}(\tilde{x}_i) N_{\mathcal{Q}_{\text{CDTHP-S-2}}}^{\delta(k)}(\tilde{x}_i) \right) + \\ & \frac{1}{\tilde{L}_{x_i}} \sum_{k=1}^{x_i} \left( N_{\mathcal{Q}_{\text{CDTHP-P-1}}}^{\delta(k)}(\tilde{x}_i) N_{\mathcal{Q}_{\text{CDTHP-P-2}}}^{\delta(k)}(\tilde{x}_i) + N_{\mathcal{Q}_{\text{CDTHP-S-1}}}^{\delta(k)}(\tilde{x}_i) N_{\mathcal{Q}_{\text{CDTHP-S-2}}}^{\delta(k)}(\tilde{x}_i) \right) \end{aligned} \right) \\ = \left( \begin{aligned} & \frac{1}{\tilde{L}_{x_1}} \sum_{j=1}^{x_1} \left( M_{\mathcal{Q}_{\text{CDTHP-P-1}}}^{\delta(j)}(\tilde{x}_1) M_{\mathcal{Q}_{\text{CDTHP-P-2}}}^{\delta(j)}(\tilde{x}_1) + M_{\mathcal{Q}_{\text{CDTHP-S-1}}}^{\delta(j)}(\tilde{x}_1) M_{\mathcal{Q}_{\text{CDTHP-S-2}}}^{\delta(j)}(\tilde{x}_1) \right) + \\ & \dots + \frac{1}{\tilde{L}_{x_n}} \sum_{j=1}^{x_n} \left( M_{\mathcal{Q}_{\text{CDTHP-P-1}}}^{\delta(j)}(\tilde{x}_n) M_{\mathcal{Q}_{\text{CDTHP-P-2}}}^{\delta(j)}(\tilde{x}_n) + M_{\mathcal{Q}_{\text{CDTHP-S-1}}}^{\delta(j)}(\tilde{x}_n) M_{\mathcal{Q}_{\text{CDTHP-S-2}}}^{\delta(j)}(\tilde{x}_n) \right) \\ & \frac{1}{\tilde{L}_{x_1}} \sum_{j=1}^{x_1} \left( M_{\mathcal{Q}_{\text{CDTHP-P-1}}}^{\delta(j)}(\tilde{x}_1) M_{\mathcal{Q}_{\text{CDTHP-P-2}}}^{\delta(j)}(\tilde{x}_1) + M_{\mathcal{Q}_{\text{CDTHP-S-1}}}^{\delta(j)}(\tilde{x}_1) M_{\mathcal{Q}_{\text{CDTHP-S-2}}}^{\delta(j)}(\tilde{x}_1) \right) + \\ & \dots + \frac{1}{\tilde{L}_{x_n}} \sum_{j=1}^{x_n} \left( M_{\mathcal{Q}_{\text{CDTHP-P-1}}}^{\delta(j)}(\tilde{x}_n) M_{\mathcal{Q}_{\text{CDTHP-P-2}}}^{\delta(j)}(\tilde{x}_n) + M_{\mathcal{Q}_{\text{CDTHP-S-1}}}^{\delta(j)}(\tilde{x}_n) M_{\mathcal{Q}_{\text{CDTHP-S-2}}}^{\delta(j)}(\tilde{x}_n) \right) \\ & \frac{1}{\tilde{L}_{x_1}} \sum_{k=1}^{x_1} \left( N_{\mathcal{Q}_{\text{CDTHP-P-1}}}^{\delta(k)}(\tilde{x}_1) N_{\mathcal{Q}_{\text{CDTHP-P-2}}}^{\delta(k)}(\tilde{x}_1) + N_{\mathcal{Q}_{\text{CDTHP-S-1}}}^{\delta(k)}(\tilde{x}_1) N_{\mathcal{Q}_{\text{CDTHP-S-2}}}^{\delta(k)}(\tilde{x}_1) \right) + \\ & \dots + \frac{1}{\tilde{L}_{x_n}} \sum_{k=1}^{x_n} \left( N_{\mathcal{Q}_{\text{CDTHP-P-1}}}^{\delta(k)}(\tilde{x}_n) N_{\mathcal{Q}_{\text{CDTHP-P-2}}}^{\delta(k)}(\tilde{x}_n) + N_{\mathcal{Q}_{\text{CDTHP-S-1}}}^{\delta(k)}(\tilde{x}_n) N_{\mathcal{Q}_{\text{CDTHP-S-2}}}^{\delta(k)}(\tilde{x}_n) \right) \\ & \frac{1}{\tilde{L}_{x_1}} \sum_{k=1}^{x_1} \left( N_{\mathcal{Q}_{\text{CDTHP-P-1}}}^{\delta(k)}(\tilde{x}_1) N_{\mathcal{Q}_{\text{CDTHP-P-2}}}^{\delta(k)}(\tilde{x}_1) + N_{\mathcal{Q}_{\text{CDTHP-S-1}}}^{\delta(k)}(\tilde{x}_1) N_{\mathcal{Q}_{\text{CDTHP-S-2}}}^{\delta(k)}(\tilde{x}_1) \right) + \\ & \dots + \frac{1}{\tilde{L}_{x_n}} \sum_{k=1}^{x_n} \left( N_{\mathcal{Q}_{\text{CDTHP-P-1}}}^{\delta(k)}(\tilde{x}_n) N_{\mathcal{Q}_{\text{CDTHP-P-2}}}^{\delta(k)}(\tilde{x}_n) + N_{\mathcal{Q}_{\text{CDTHP-S-1}}}^{\delta(k)}(\tilde{x}_n) N_{\mathcal{Q}_{\text{CDTHP-S-2}}}^{\delta(k)}(\tilde{x}_n) \right) \end{aligned} \right) \tag{15}$$

By using the Cauchy-Schwarz inequality,  $(x_1 y_1 + x_2 y_2 + \dots + x_n y_n)^2 \leq (x_1^2 + x_2^2 + \dots + x_n^2)(y_1^2 + y_2^2 + \dots + y_n^2)$ , we have







**Definition 9.** For any two CDT-2HFSs,  $\mathcal{Q}_{CDTH-1} = \left\{ \left( \begin{array}{l} (M_{\mathcal{Q}_{CDTHRP-p-1}}^{\delta(j)}(\tilde{x}_i) \cdot e^{i2\pi(M_{\mathcal{Q}_{CDTHRP-p-1}}^{\delta(j)}(\tilde{x}_i))), M_{\mathcal{Q}_{CDTHRP-s-1}}^{\delta(j)}(\tilde{x}_i) \cdot e^{i2\pi(M_{\mathcal{Q}_{CDTHRP-s-1}}^{\delta(j)}(\tilde{x}_i))), \\ (N_{\mathcal{Q}_{CDTHRP-p-1}}^{\delta(k)}(\tilde{x}_i) \cdot e^{i2\pi(N_{\mathcal{Q}_{CDTHRP-p-1}}^{\delta(k)}(\tilde{x}_i))), N_{\mathcal{Q}_{CDTHRP-s-1}}^{\delta(k)}(\tilde{x}_i) \cdot e^{i2\pi(N_{\mathcal{Q}_{CDTHRP-s-1}}^{\delta(k)}(\tilde{x}_i))} \end{array} \right), j, k = 1, 2, 3, \dots, n, m \right\}$  and  $\mathcal{Q}_{CDTH-2} = \left\{ \left( \begin{array}{l} (M_{\mathcal{Q}_{CDTHRP-p-2}}^{\delta(j)}(\tilde{x}_i) \cdot e^{i2\pi(M_{\mathcal{Q}_{CDTHRP-p-2}}^{\delta(j)}(\tilde{x}_i))), M_{\mathcal{Q}_{CDTHRP-s-2}}^{\delta(j)}(\tilde{x}_i) \cdot e^{i2\pi(M_{\mathcal{Q}_{CDTHRP-s-2}}^{\delta(j)}(\tilde{x}_i))), \\ (N_{\mathcal{Q}_{CDTHRP-p-2}}^{\delta(k)}(\tilde{x}_i) \cdot e^{i2\pi(N_{\mathcal{Q}_{CDTHRP-p-2}}^{\delta(k)}(\tilde{x}_i))), N_{\mathcal{Q}_{CDTHRP-s-2}}^{\delta(k)}(\tilde{x}_i) \cdot e^{i2\pi(N_{\mathcal{Q}_{CDTHRP-s-2}}^{\delta(k)}(\tilde{x}_i))} \end{array} \right), j, k = 1, 2, 3, \dots, n, m \right\}$

$\left. j, k = 1, 2, 3, \dots, \tilde{l}_{x_i}, \tilde{l}_{\tilde{x}_i} \right\}$ , the maximum-based correlation coefficient (MCC) is of the form

$$\zeta_{CDTHF-mcc}(\mathcal{Q}_{CDTH-1}, \mathcal{Q}_{CDTH-2}) = \frac{\zeta_{CDTHF-c}(\mathcal{Q}_{CDTH-1}, \mathcal{Q}_{CDTH-2})}{\max(\zeta_{CDTHF-c}(\mathcal{Q}_{CDTH-1}, \mathcal{Q}_{CDTH-1})^{(1/2)}, \zeta_{CDTHF-c}(\mathcal{Q}_{CDTH-1}, \mathcal{Q}_{CDTH-2})^{(1/2)})}$$

$$= \frac{\sum_{i=1}^n \left( \begin{array}{l} \left( \frac{1}{\tilde{l}_{x_i}} \sum_{j=1}^{\tilde{l}_{x_i}} (M_{\mathcal{Q}_{CDTHRP-p-1}}^{\delta(j)}(\tilde{x}_i) M_{\mathcal{Q}_{CDTHRP-p-2}}^{\delta(j)}(\tilde{x}_i) + M_{\mathcal{Q}_{CDTHRP-s-1}}^{\delta(j)}(\tilde{x}_i) M_{\mathcal{Q}_{CDTHRP-s-2}}^{\delta(j)}(\tilde{x}_i)) \right) + \\ \left( \frac{1}{\tilde{l}_{x_i}} \sum_{j=1}^{\tilde{l}_{x_i}} (M_{\mathcal{Q}_{CDTHRP-p-1}}^{\delta(j)}(\tilde{x}_i) M_{\mathcal{Q}_{CDTHRP-p-2}}^{\delta(j)}(\tilde{x}_i) + M_{\mathcal{Q}_{CDTHRP-s-1}}^{\delta(j)}(\tilde{x}_i) M_{\mathcal{Q}_{CDTHRP-s-2}}^{\delta(j)}(\tilde{x}_i)) \right) + \\ \left( \frac{1}{\tilde{l}_{x_i}} \sum_{k=1}^{\tilde{l}_{x_i}} (N_{\mathcal{Q}_{CDTHRP-p-1}}^{\delta(k)}(\tilde{x}_i) N_{\mathcal{Q}_{CDTHRP-p-2}}^{\delta(k)}(\tilde{x}_i) + N_{\mathcal{Q}_{CDTHRP-s-1}}^{\delta(k)}(\tilde{x}_i) N_{\mathcal{Q}_{CDTHRP-s-2}}^{\delta(k)}(\tilde{x}_i)) \right) + \\ \left( \frac{1}{\tilde{l}_{x_i}} \sum_{k=1}^{\tilde{l}_{x_i}} (N_{\mathcal{Q}_{CDTHRP-p-1}}^{\delta(k)}(\tilde{x}_i) N_{\mathcal{Q}_{CDTHRP-p-2}}^{\delta(k)}(\tilde{x}_i) + N_{\mathcal{Q}_{CDTHRP-s-1}}^{\delta(k)}(\tilde{x}_i) N_{\mathcal{Q}_{CDTHRP-s-2}}^{\delta(k)}(\tilde{x}_i)) \right) \end{array} \right)}{\max \left( \left( \begin{array}{l} \left( \frac{1}{\tilde{l}_{x_i}} \sum_{j=1}^{\tilde{l}_{x_i}} ((M_{\mathcal{Q}_{CDTHRP-p-1}}^{\delta(j)}(\tilde{x}_i))^2 + (M_{\mathcal{Q}_{CDTHRP-s-1}}^{\delta(j)}(\tilde{x}_i))^2) \right) + \\ \left( \frac{1}{\tilde{l}_{x_i}} \sum_{j=1}^{\tilde{l}_{x_i}} ((M_{\mathcal{Q}_{CDTHRP-p-1}}^{\delta(j)}(\tilde{x}_i))^2 + (M_{\mathcal{Q}_{CDTHRP-s-1}}^{\delta(j)}(\tilde{x}_i))^2) \right) + \\ \left( \frac{1}{\tilde{l}_{x_i}} \sum_{k=1}^{\tilde{l}_{x_i}} ((N_{\mathcal{Q}_{CDTHRP-p-1}}^{\delta(k)}(\tilde{x}_i))^2 + (N_{\mathcal{Q}_{CDTHRP-s-1}}^{\delta(k)}(\tilde{x}_i))^2) \right) + \\ \left( \frac{1}{\tilde{l}_{x_i}} \sum_{k=1}^{\tilde{l}_{x_i}} ((N_{\mathcal{Q}_{CDTHRP-p-1}}^{\delta(k)}(\tilde{x}_i))^2 + (N_{\mathcal{Q}_{CDTHRP-s-1}}^{\delta(k)}(\tilde{x}_i))^2) \right) \end{array} \right)^{(1/2)}, \left( \begin{array}{l} \left( \frac{1}{\tilde{l}_{x_i}} \sum_{j=1}^{\tilde{l}_{x_i}} ((M_{\mathcal{Q}_{CDTHRP-p-2}}^{\delta(j)}(\tilde{x}_i))^2 + (M_{\mathcal{Q}_{CDTHRP-s-2}}^{\delta(j)}(\tilde{x}_i))^2) \right) + \\ \left( \frac{1}{\tilde{l}_{x_i}} \sum_{j=1}^{\tilde{l}_{x_i}} ((M_{\mathcal{Q}_{CDTHRP-p-2}}^{\delta(j)}(\tilde{x}_i))^2 + (M_{\mathcal{Q}_{CDTHRP-s-2}}^{\delta(j)}(\tilde{x}_i))^2) \right) + \\ \left( \frac{1}{\tilde{l}_{x_i}} \sum_{k=1}^{\tilde{l}_{x_i}} ((N_{\mathcal{Q}_{CDTHRP-p-2}}^{\delta(k)}(\tilde{x}_i))^2 + (N_{\mathcal{Q}_{CDTHRP-s-2}}^{\delta(k)}(\tilde{x}_i))^2) \right) + \\ \left( \frac{1}{\tilde{l}_{x_i}} \sum_{k=1}^{\tilde{l}_{x_i}} ((N_{\mathcal{Q}_{CDTHRP-p-2}}^{\delta(k)}(\tilde{x}_i))^2 + (N_{\mathcal{Q}_{CDTHRP-s-2}}^{\delta(k)}(\tilde{x}_i))^2) \right) \end{array} \right)^{(1/2)} \right)} \quad (18)$$

**Proposition 2.** For any two CDT-2HFSs, QCDTH-1 and QCDTH-2, the MCC among CDT-2HFSs satisfies the following axioms:

- (1)  $0 \leq \zeta_{CDTH-mcc}(\mathcal{Q}_{CDTH-1}, \mathcal{Q}_{CDTH-2}) \leq 1$
- (2)  $\zeta_{CDTHF-mcc}(\mathcal{Q}_{CDTH-1}, \mathcal{Q}_{CDTH-2}) = 1 \Leftrightarrow \mathcal{Q}_{CDTH-1} = \mathcal{Q}_{CDTH-2}$

$$(3) \zeta_{CDTHF-mcc}(\mathcal{Q}_{CDTH-1}, \mathcal{Q}_{CDTH-2}) = \zeta_{CDTHF-mcc}(\mathcal{Q}_{CDTH-2}, \mathcal{Q}_{CDTH-1})$$

*Proof.* We prove the three above conditions by using equation (18). By using the inequality, it is clear that

$0 \leq \zeta_{\text{CDTH-mcc}}(\mathcal{Q}_{\text{CDTH-1}}, \mathcal{Q}_{\text{CDTH-2}})$ ; then, we only prove that  $\zeta_{\text{CDTH-mcc}}(\mathcal{Q}_{\text{CDTH-1}}, \mathcal{Q}_{\text{CDTH-2}}) \leq 1$ . For this, we choose that

$$\begin{aligned}
 \zeta_{\text{CDTH-c}}(\mathcal{Q}_{\text{CDTH-1}}, \mathcal{Q}_{\text{CDTH-2}}) = & \left( \begin{aligned} & \frac{1}{\tilde{L}_{x_i}} \sum_{j=1}^{\tilde{L}_{x_i}} \left( M_{\mathcal{Q}_{\text{CDTHP-P-1}}}^{\delta(j)}(\tilde{x}_i) M_{\mathcal{Q}_{\text{CDTHP-P-2}}}^{\delta(j)}(\tilde{x}_i) + M_{\mathcal{Q}_{\text{CDTHP-S-1}}}^{\delta(j)}(\tilde{x}_i) M_{\mathcal{Q}_{\text{CDTHP-S-2}}}^{\delta(j)}(\tilde{x}_i) \right) + \\ & \frac{1}{\tilde{L}_{x_i}} \sum_{j=1}^{\tilde{L}_{x_i}} \left( M_{\mathcal{Q}_{\text{CDTHP-P-1}}}^{\delta(j)}(\tilde{x}_i) M_{\mathcal{Q}_{\text{CDTHP-P-2}}}^{\delta(j)}(\tilde{x}_i) + M_{\mathcal{Q}_{\text{CDTHP-S-1}}}^{\delta(j)}(\tilde{x}_i) M_{\mathcal{Q}_{\text{CDTHP-S-2}}}^{\delta(j)}(\tilde{x}_i) \right) + \\ & \frac{1}{\tilde{L}_{x_i}} \sum_{k=1}^{\tilde{L}_{x_i}} \left( N_{\mathcal{Q}_{\text{CDTHP-P-1}}}^{\delta(k)}(\tilde{x}_i) N_{\mathcal{Q}_{\text{CDTHP-P-2}}}^{\delta(k)}(\tilde{x}_i) + N_{\mathcal{Q}_{\text{CDTHP-S-1}}}^{\delta(k)}(\tilde{x}_i) N_{\mathcal{Q}_{\text{CDTHP-S-2}}}^{\delta(k)}(\tilde{x}_i) \right) + \\ & \frac{1}{\tilde{L}_{x_i}} \sum_{k=1}^{\tilde{L}_{x_i}} \left( N_{\mathcal{Q}_{\text{CDTHP-P-1}}}^{\delta(k)}(\tilde{x}_i) N_{\mathcal{Q}_{\text{CDTHP-P-2}}}^{\delta(k)}(\tilde{x}_i) + N_{\mathcal{Q}_{\text{CDTHP-S-1}}}^{\delta(k)}(\tilde{x}_i) N_{\mathcal{Q}_{\text{CDTHP-S-2}}}^{\delta(k)}(\tilde{x}_i) \right) \end{aligned} \right) \\
 & \left( \begin{aligned} & \frac{1}{\tilde{L}_{x_1}} \sum_{j=1}^{\tilde{L}_{x_1}} \left( M_{\mathcal{Q}_{\text{CDTHP-P-1}}}^{\delta(j)}(\tilde{x}_1) M_{\mathcal{Q}_{\text{CDTHP-P-2}}}^{\delta(j)}(\tilde{x}_1) + M_{\mathcal{Q}_{\text{CDTHP-P-2-S-1}}}^{\delta(j)}(\tilde{x}_1) M_{\mathcal{Q}_{\text{CDTHP-P-2-S-2}}}^{\delta(j)}(\tilde{x}_1) \right) + \\ & \dots + \frac{1}{\tilde{L}_{x_n}} \sum_{j=1}^{\tilde{L}_{x_n}} \left( M_{\mathcal{Q}_{\text{CDTHP-P-1}}}^{\delta(j)}(\tilde{x}_n) M_{\mathcal{Q}_{\text{CDTHP-P-2-P-2}}}^{\delta(j)}(\tilde{x}_n) + M_{\mathcal{Q}_{\text{CDTHP-P-2-S-1}}}^{\delta(j)}(\tilde{x}_n) M_{\mathcal{Q}_{\text{CDTHP-P-2-S-2}}}^{\delta(j)}(\tilde{x}_n) \right) \\ & \frac{1}{\tilde{L}_{x_1}} \sum_{j=1}^{\tilde{L}_{x_1}} \left( M_{\mathcal{Q}_{\text{CDTHP-P-1}}}^{\delta(j)}(\tilde{x}_1) M_{\mathcal{Q}_{\text{CDTHP-P-2}}}^{\delta(j)}(\tilde{x}_1) + M_{\mathcal{Q}_{\text{CDTHP-S-1}}}^{\delta(j)}(\tilde{x}_1) M_{\mathcal{Q}_{\text{CDTHP-S-2}}}^{\delta(j)}(\tilde{x}_1) \right) + \\ & \dots + \frac{1}{\tilde{L}_{x_n}} \sum_{j=1}^{\tilde{L}_{x_n}} \left( M_{\mathcal{Q}_{\text{CDTHP-P-1}}}^{\delta(j)}(\tilde{x}_n) M_{\mathcal{Q}_{\text{CDTHP-P-2}}}^{\delta(j)}(\tilde{x}_n) + M_{\mathcal{Q}_{\text{CDTHP-S-1}}}^{\delta(j)}(\tilde{x}_n) M_{\mathcal{Q}_{\text{CDTHP-S-2}}}^{\delta(j)}(\tilde{x}_n) \right) \\ & \frac{1}{\tilde{L}_{x_1}} \sum_{k=1}^{\tilde{L}_{x_1}} \left( N_{\mathcal{Q}_{\text{CDTHP-P-1}}}^{\delta(k)}(\tilde{x}_1) N_{\mathcal{Q}_{\text{CDTHP-P-2}}}^{\delta(k)}(\tilde{x}_1) + N_{\mathcal{Q}_{\text{CDTHP-S-1}}}^{\delta(k)}(\tilde{x}_1) N_{\mathcal{Q}_{\text{CDTHP-S-2}}}^{\delta(k)}(\tilde{x}_1) \right) + \\ & \dots + \frac{1}{\tilde{L}_{x_n}} \sum_{k=1}^{\tilde{L}_{x_n}} \left( N_{\mathcal{Q}_{\text{CDTHP-P-1}}}^{\delta(k)}(\tilde{x}_n) N_{\mathcal{Q}_{\text{CDTHP-P-2}}}^{\delta(k)}(\tilde{x}_n) + N_{\mathcal{Q}_{\text{CDTHP-S-1}}}^{\delta(k)}(\tilde{x}_n) N_{\mathcal{Q}_{\text{CDTHP-S-2}}}^{\delta(k)}(\tilde{x}_n) \right) \\ & \frac{1}{\tilde{L}_{x_1}} \sum_{k=1}^{\tilde{L}_{x_1}} \left( N_{\mathcal{Q}_{\text{CDTHP-P-1}}}^{\delta(k)}(\tilde{x}_1) N_{\mathcal{Q}_{\text{CDTHP-P-2}}}^{\delta(k)}(\tilde{x}_1) + N_{\mathcal{Q}_{\text{CDTHP-S-1}}}^{\delta(k)}(\tilde{x}_1) N_{\mathcal{Q}_{\text{CDTHP-S-2}}}^{\delta(k)}(\tilde{x}_1) \right) + \\ & \dots + \frac{1}{\tilde{L}_{x_n}} \sum_{k=1}^{\tilde{L}_{x_n}} \left( N_{\mathcal{Q}_{\text{CDTHP-P-1}}}^{\delta(k)}(\tilde{x}_n) N_{\mathcal{Q}_{\text{CDTHP-P-2}}}^{\delta(k)}(\tilde{x}_n) + N_{\mathcal{Q}_{\text{CDTHP-S-1}}}^{\delta(k)}(\tilde{x}_n) N_{\mathcal{Q}_{\text{CDTHP-S-2}}}^{\delta(k)}(\tilde{x}_n) \right) \end{aligned} \right) \\
 = & \dots
 \end{aligned} \tag{19}$$

By using the Cauchy-Schwarz inequality,  $(x_1y_1 + x_2y_2 + \dots + x_ny_n)^2 \leq (x_1^2 + x_2^2 + \dots + x_n^2)(y_1^2 + y_2^2 + \dots + y_n^2)$ , we have

$$\begin{aligned}
 & \left( \zeta_{\text{CDTHP-c}}(\mathcal{Q}_{\text{CDTH-1}}, \mathcal{Q}_{\text{CDTH-2}}) \right)^2 \leq \left( \begin{aligned}
 & \frac{1}{\tilde{L}_{x_1}} \sum_{j=1}^{\tilde{x}_1} \left( M_{\mathcal{Q}_{\text{CDTHP-P-1}}}^{\delta(j)2}(\tilde{x}_1) + M_{\mathcal{Q}_{\text{CDTHP-S-1}}}^{\delta(j)2}(\tilde{x}_1) \right) + \\
 & \dots + \frac{1}{\tilde{L}_{x_n}} \sum_{j=1}^{\tilde{x}_n} \left( M_{\mathcal{Q}_{\text{CDTHP-P-1}}}^{\delta(j)2}(\tilde{x}_n) + M_{\mathcal{Q}_{\text{CDTHP-S-1}}}^{\delta(j)2}(\tilde{x}_n) \right) \\
 & \frac{1}{\tilde{L}_{x_1}} \sum_{j=1}^{\tilde{x}_1} \left( M_{\mathcal{Q}_{\text{CDTHP-P-1}}}^{\delta(j)2}(\tilde{x}_1) + M_{\mathcal{Q}_{\text{CDTHP-S-1}}}^{\delta(j)2}(\tilde{x}_1) \right) + \\
 & \dots + \frac{1}{\tilde{L}_{x_n}} \sum_{j=1}^{\tilde{x}_n} \left( M_{\mathcal{Q}_{\text{CDTHP-P-1}}}^{\delta(j)2}(\tilde{x}_n) + M_{\mathcal{Q}_{\text{CDTHP-S-1}}}^{\delta(j)2}(\tilde{x}_n) \right) \\
 & \frac{1}{\tilde{L}_{x_1}} \sum_{k=1}^{\tilde{x}_1} \left( N_{\mathcal{Q}_{\text{CDTHP-P-1}}}^{\delta(k)2}(\tilde{x}_1) + N_{\mathcal{Q}_{\text{CDTHP-P-1}}}^{\delta(k)2}(\tilde{x}_1) \right) + \\
 & \dots + \frac{1}{\tilde{L}_{x_n}} \sum_{k=1}^{\tilde{x}_n} \left( N_{\mathcal{Q}_{\text{CDTHP-P-1}}}^{\delta(k)2}(\tilde{x}_n) + N_{\mathcal{Q}_{\text{CDTHP-P-1}}}^{\delta(k)2}(\tilde{x}_n) \right) \\
 & \frac{1}{\tilde{L}_{x_1}} \sum_{k=1}^{\tilde{x}_1} \left( N_{\mathcal{Q}_{\text{CDTHP-P-1}}}^{\delta(k)2}(\tilde{x}_1) + N_{\mathcal{Q}_{\text{CDTHP-P-1}}}^{\delta(k)2}(\tilde{x}_1) \right) + \\
 & \dots + \frac{1}{\tilde{L}_{x_n}} \sum_{k=1}^{\tilde{x}_n} \left( N_{\mathcal{Q}_{\text{CDTHP-P-1}}}^{\delta(k)2}(\tilde{x}_n) + N_{\mathcal{Q}_{\text{CDTHP-P-1}}}^{\delta(k)2}(\tilde{x}_n) \right)
 \end{aligned} \right) \\
 & \times \left( \begin{aligned}
 & \frac{1}{\tilde{L}_{x_1}} \sum_{j=1}^{\tilde{x}_1} \left( M_{\mathcal{Q}_{\text{CDTHP-P-2}}}^{\delta(j)2}(\tilde{x}_1) + M_{\mathcal{Q}_{\text{CDTHP-S-2}}}^{\delta(j)2}(\tilde{x}_1) \right) + \\
 & \dots + \frac{1}{\tilde{L}_{x_n}} \sum_{j=1}^{\tilde{x}_n} \left( M_{\mathcal{Q}_{\text{CDTHP-P-2}}}^{\delta(j)2}(\tilde{x}_n) + M_{\mathcal{Q}_{\text{CDTHP-S-2}}}^{\delta(j)2}(\tilde{x}_n) \right) \\
 & \frac{1}{\tilde{L}_{x_1}} \sum_{j=1}^{\tilde{x}_1} \left( M_{\mathcal{Q}_{\text{CDTHP-P-2}}}^{\delta(j)2}(\tilde{x}_1) + M_{\mathcal{Q}_{\text{CDTHP-S-2}}}^{\delta(j)2}(\tilde{x}_1) \right) + \\
 & \dots + \frac{1}{\tilde{L}_{x_n}} \sum_{j=1}^{\tilde{x}_n} \left( M_{\mathcal{Q}_{\text{CDTHP-P-2}}}^{\delta(j)2}(\tilde{x}_n) + M_{\mathcal{Q}_{\text{CDTHP-S-2}}}^{\delta(j)2}(\tilde{x}_n) \right) \\
 & \frac{1}{\tilde{L}_{x_1}} \sum_{k=1}^{\tilde{x}_1} \left( N_{\mathcal{Q}_{\text{CDTHP-P-2}}}^{\delta(k)2}(\tilde{x}_1) + N_{\mathcal{Q}_{\text{CDTHP-P-2}}}^{\delta(k)2}(\tilde{x}_1) \right) + \\
 & \dots + \frac{1}{\tilde{L}_{x_n}} \sum_{k=1}^{\tilde{x}_n} \left( N_{\mathcal{Q}_{\text{CDTHP-P-2}}}^{\delta(k)2}(\tilde{x}_n) + N_{\mathcal{Q}_{\text{CDTHP-P-2}}}^{\delta(k)2}(\tilde{x}_n) \right) \\
 & \frac{1}{\tilde{L}_{x_1}} \sum_{k=1}^{\tilde{x}_1} \left( N_{\mathcal{Q}_{\text{CDTHP-P-2}}}^{\delta(k)2}(\tilde{x}_1) + N_{\mathcal{Q}_{\text{CDTHP-P-2}}}^{\delta(k)2}(\tilde{x}_1) \right) + \\
 & \dots + \frac{1}{\tilde{L}_{x_n}} \sum_{k=1}^{\tilde{x}_n} \left( N_{\mathcal{Q}_{\text{CDTHP-P-2}}}^{\delta(k)2}(\tilde{x}_n) + N_{\mathcal{Q}_{\text{CDTHP-P-2}}}^{\delta(k)2}(\tilde{x}_n) \right)
 \end{aligned} \right) \\
 & = \zeta_{\text{CDTHP-c}}(\mathcal{Q}_{\text{CDTH-1}}, \mathcal{Q}_{\text{CDTH-1}}) \cdot \zeta_{\text{CDTHP-c}}(\mathcal{Q}_{\text{CDTH-2}}, \mathcal{Q}_{\text{CDTH-2}}).
 \end{aligned}
 \tag{20}$$

$\zeta_{\text{CDTHF-c}}(Q_{\text{CDTH-1}}, Q_{\text{CDTH-2}}) \leq (\zeta_{\text{CDTHF-c}}(Q_{\text{CDTH-1}}, Q_{\text{CDTH-1}}))^{(1/2)} (\zeta_{\text{CDTHF-c}}(Q_{\text{CDTH-2}}, Q_{\text{CDTH-2}}))^{(1/2)}$ ; thus  $0 \leq \zeta_{\text{CDTHF-mcc}}(Q_{\text{CDTH-1}}, Q_{\text{CDTH-2}}) \leq 1$ . Furthermore, we prove the second part by using equation (18). By hypothesis, it is given that  $Q_{\text{CDTH-1}} = Q_{\text{CDTH-2}}$ ; then  $M_{\text{CDTHRP-p-1}}^{\delta(j)}(\tilde{x}_i) = M_{\text{CDTHRP-p-2}}^{\delta(j)}(\tilde{x}_i)$ ,  $M_{\text{CDTHIP-p-1}}^{\delta(j)}(\tilde{x}_i) = M_{\text{CDTHIP-p-2}}^{\delta(j)}(\tilde{x}_i)$ ,  $M_{\text{CDTHRP-s-1}}^{\delta(j)}(\tilde{x}_i) = M_{\text{CDTHRP-s-2}}^{\delta(j)}(\tilde{x}_i)$ ,  $M_{\text{CDTHIP-s-1}}^{\delta(j)}(\tilde{x}_i) =$

$N_{\text{CDTHRP-s-2}}^{\delta(k)}(\tilde{x}_i)$ ,  $N_{\text{CDTHRP-p-1}}^{\delta(k)} = N_{\text{CDTHRP-p-2}}^{\delta(k)}$ ,  $N_{\text{CDTHIP-p-2}}^{\delta(k)}(\tilde{x}_i) = N_{\text{CDTHIP-p-2}}^{\delta(k)}(\tilde{x}_i)$ , and  $N_{\text{CDTHRP-s-1}}^{\delta(k)} = N_{\text{CDTHRP-s-2}}^{\delta(k)}$ ,  $N_{\text{CDTHIP-s-1}}^{\delta(k)}(\tilde{x}_i) = N_{\text{CDTHIP-s-2}}^{\delta(k)}(\tilde{x}_i)$ ; then, by using equation (18), we get  $\zeta_{\text{CDTHF-mcc}}(\mathcal{Q}_{\text{CDTH-1}}, \mathcal{Q}_{\text{CDTH-2}}) = 1$ . Additionally, we prove the third condition such that

$$\zeta_{\text{CDTHF-mcc}}(\mathcal{Q}_{\text{CDTH-1}}, \mathcal{Q}_{\text{CDTH-2}}) = \frac{\sum_{i=1}^n \left( \begin{aligned} & \left( 1/\tilde{l}_{x_i} \right) \sum_{j=1}^{\tilde{l}_{x_i}} \left( M_{\text{CDTHRP-p-1}}^{\delta(j)}(\tilde{x}_i) M_{\text{CDTHRP-p-2}}^{\delta(j)}(\tilde{x}_i) + M_{\text{CDTHRP-s-1}}^{\delta(j)}(\tilde{x}_i) M_{\text{CDTHRP-s-2}}^{\delta(j)}(\tilde{x}_i) \right) \\ & \left( 1/\tilde{l}_{x_i} \right) \sum_{j=1}^{\tilde{l}_{x_i}} \left( M_{\text{CDTHIP-p-1}}^{\delta(j)}(\tilde{x}_i) M_{\text{CDTHIP-p-2}}^{\delta(j)}(\tilde{x}_i) + M_{\text{CDTHIP-s-1}}^{\delta(j)}(\tilde{x}_i) M_{\text{CDTHIP-s-2}}^{\delta(j)}(\tilde{x}_i) \right) \\ & \left( 1/\tilde{l}_{x_i} \right) \sum_{k=1}^{\tilde{l}_{x_i}} \left( N_{\text{CDTHRP-p-1}}^{\delta(k)}(\tilde{x}_i) N_{\text{CDTHRP-p-2}}^{\delta(k)}(\tilde{x}_i) + N_{\text{CDTHRP-p-1}}^{\delta(k)}(\tilde{x}_i) N_{\text{CDTHRP-p-2}}^{\delta(k)}(\tilde{x}_i) \right) \\ & \left( 1/\tilde{l}_{x_i} \right) \sum_{k=1}^{\tilde{l}_{x_i}} \left( N_{\text{CDTHIP-p-1}}^{\delta(k)}(\tilde{x}_i) N_{\text{CDTHIP-p-2}}^{\delta(k)}(\tilde{x}_i) + N_{\text{CDTHIP-p-1}}^{\delta(k)}(\tilde{x}_i) N_{\text{CDTHIP-p-2}}^{\delta(k)}(\tilde{x}_i) \right) \end{aligned} \right)}{\left( \left( \left( \left( 1/\tilde{l}_{x_i} \right) \sum_{j=1}^{\tilde{l}_{x_i}} \left( \left( M_{\text{CDTHRP-p-1}}^{\delta(j)}(\tilde{x}_i) \right)^2 + \left( M_{\text{CDTHRP-s-1}}^{\delta(j)}(\tilde{x}_i) \right)^2 \right) + \right. \right. \right. \right. \right. \\ \left. \left. \left( 1/\tilde{l}_{x_i} \right) \sum_{j=1}^{\tilde{l}_{x_i}} \left( \left( M_{\text{CDTHIP-p-1}}^{\delta(j)}(\tilde{x}_i) \right)^2 + \left( M_{\text{CDTHIP-s-1}}^{\delta(j)}(\tilde{x}_i) \right)^2 \right) + \right. \right. \\ \left. \left. \left( 1/\tilde{l}_{x_i} \right) \sum_{k=1}^{\tilde{l}_{x_i}} \left( \left( N_{\text{CDTHRP-p-1}}^{\delta(k)}(\tilde{x}_i) \right)^2 + \left( N_{\text{CDTHRP-s-1}}^{\delta(k)}(\tilde{x}_i) \right)^2 \right) + \right. \right. \\ \left. \left. \left( 1/\tilde{l}_{x_i} \right) \sum_{k=1}^{\tilde{l}_{x_i}} \left( \left( N_{\text{CDTHIP-p-1}}^{\delta(k)}(\tilde{x}_i) \right)^2 + \left( N_{\text{CDTHIP-s-1}}^{\delta(k)}(\tilde{x}_i) \right)^2 \right) \right) \right)^{(1/2)} \right) \max \left( \left( \left( \left( 1/\tilde{l}_{x_i} \right) \sum_{j=1}^{\tilde{l}_{x_i}} \left( \left( M_{\text{CDTHRP-p-2}}^{\delta(j)}(\tilde{x}_i) \right)^2 + \left( M_{\text{CDTHRP-s-2}}^{\delta(j)}(\tilde{x}_i) \right)^2 \right) + \right. \right. \right. \right. \right. \\ \left. \left. \left( 1/\tilde{l}_{x_i} \right) \sum_{j=1}^{\tilde{l}_{x_i}} \left( \left( M_{\text{CDTHIP-p-2}}^{\delta(j)}(\tilde{x}_i) \right)^2 + \left( M_{\text{CDTHIP-s-2}}^{\delta(j)}(\tilde{x}_i) \right)^2 \right) + \right. \right. \\ \left. \left. \left( 1/\tilde{l}_{x_i} \right) \sum_{k=1}^{\tilde{l}_{x_i}} \left( \left( N_{\text{CDTHRP-p-2}}^{\delta(k)}(\tilde{x}_i) \right)^2 + \left( N_{\text{CDTHRP-s-2}}^{\delta(k)}(\tilde{x}_i) \right)^2 \right) + \right. \right. \\ \left. \left. \left( 1/\tilde{l}_{x_i} \right) \sum_{k=1}^{\tilde{l}_{x_i}} \left( \left( N_{\text{CDTHIP-p-2}}^{\delta(k)}(\tilde{x}_i) \right)^2 + \left( N_{\text{CDTHIP-s-2}}^{\delta(k)}(\tilde{x}_i) \right)^2 \right) \right) \right)^{(1/2)} \right)$$

$$\begin{aligned}
 & \left( \sum_{i=1}^n \left( \frac{1}{\tilde{L}_{x_i}} \sum_{j=1}^{\tilde{L}_{x_i}} \left( M_{\mathcal{Q}_{\text{CDTHR-P-2}}}^{\delta(j)}(\tilde{x}_i) M_{\mathcal{Q}_{\text{CDTHR-P-1}}}^{\delta(j)}(\tilde{x}_i) + M_{\mathcal{Q}_{\text{CDTHR-S-2}}}^{\delta(j)}(\tilde{x}_i) M_{\mathcal{Q}_{\text{CDTHR-S-1}}}^{\delta(j)}(\tilde{x}_i) \right) \right) \right. \\
 & \left. \left( \frac{1}{\tilde{L}_{x_i}} \sum_{j=1}^{\tilde{L}_{x_i}} \left( M_{\mathcal{Q}_{\text{CDTHIP-P-2}}}^{\delta(j)}(\tilde{x}_i) M_{\mathcal{Q}_{\text{CDTHIP-P-1}}}^{\delta(j)}(\tilde{x}_i) + M_{\mathcal{Q}_{\text{CDTHIP-S-2}}}^{\delta(j)}(\tilde{x}_i) M_{\mathcal{Q}_{\text{CDTHIP-S-1}}}^{\delta(j)}(\tilde{x}_i) \right) \right) \right. \\
 & \left. \left( \frac{1}{\tilde{L}_{x_i}} \sum_{k=1}^{\tilde{L}_{x_i}} \left( N_{\mathcal{Q}_{\text{CDTHR-P-2}}}^{\delta(k)}(\tilde{x}_i) N_{\mathcal{Q}_{\text{CDTHR-P-1}}}^{\delta(k)}(\tilde{x}_i) + N_{\mathcal{Q}_{\text{CDTHR-S-2}}}^{\delta(k)}(\tilde{x}_i) N_{\mathcal{Q}_{\text{CDTHR-P-1}}}^{\delta(k)}(\tilde{x}_i) \right) \right) \right. \\
 & \left. \left( \frac{1}{\tilde{L}_{x_i}} \sum_{k=1}^{\tilde{L}_{x_i}} \left( N_{\mathcal{Q}_{\text{CDTHIP-P-2}}}^{\delta(k)}(\tilde{x}_i) N_{\mathcal{Q}_{\text{CDTHIP-P-1}}}^{\delta(k)}(\tilde{x}_i) + N_{\mathcal{Q}_{\text{CDTHIP-S-2}}}^{\delta(k)}(\tilde{x}_i) N_{\mathcal{Q}_{\text{CDTHIP-P-1}}}^{\delta(k)}(\tilde{x}_i) \right) \right) \right) \\
 & \left. \right)^{(1/2)} = \zeta_{\text{CDTHF-mcc}}(\mathcal{Q}_{\text{CDTH-1}}, \mathcal{Q}_{\text{CDTH-2}}). \\
 & \max \left( \left( \sum_{i=1}^n \left( \frac{1}{\tilde{L}_{x_i}} \sum_{j=1}^{\tilde{L}_{x_i}} \left( (M_{\mathcal{Q}_{\text{CDTHR-P-2}}}^{\delta(j)}(\tilde{x}_i))^2 + (M_{\mathcal{Q}_{\text{CDTHR-S-2}}}^{\delta(j)}(\tilde{x}_i))^2 \right) \right) \right) \right. \\
 & \left. \left( \frac{1}{\tilde{L}_{x_i}} \sum_{j=1}^{\tilde{L}_{x_i}} \left( (M_{\mathcal{Q}_{\text{CDTHIP-P-2}}}^{\delta(j)}(\tilde{x}_i))^2 + (M_{\mathcal{Q}_{\text{CDTHIP-S-2}}}^{\delta(j)}(\tilde{x}_i))^2 \right) \right) \right. \\
 & \left. \left( \frac{1}{\tilde{L}_{x_i}} \sum_{k=1}^{\tilde{L}_{x_i}} \left( (N_{\mathcal{Q}_{\text{CDTHR-P-2}}}^{\delta(k)}(\tilde{x}_i))^2 + (N_{\mathcal{Q}_{\text{CDTHR-S-2}}}^{\delta(k)}(\tilde{x}_i))^2 \right) \right) \right. \\
 & \left. \left( \frac{1}{\tilde{L}_{x_i}} \sum_{k=1}^{\tilde{L}_{x_i}} \left( (N_{\mathcal{Q}_{\text{CDTHIP-P-2}}}^{\delta(k)}(\tilde{x}_i))^2 + (N_{\mathcal{Q}_{\text{CDTHIP-S-2}}}^{\delta(k)}(\tilde{x}_i))^2 \right) \right) \right) \right)^{(1/2)} \\
 & \left( \sum_{i=1}^n \left( \frac{1}{\tilde{L}_{x_i}} \sum_{j=1}^{\tilde{L}_{x_i}} \left( (M_{\mathcal{Q}_{\text{CDTHIP-P-1}}}^{\delta(j)}(\tilde{x}_i))^2 + (M_{\mathcal{Q}_{\text{CDTHIP-S-1}}}^{\delta(j)}(\tilde{x}_i))^2 \right) \right) \right. \\
 & \left. \left( \frac{1}{\tilde{L}_{x_i}} \sum_{j=1}^{\tilde{L}_{x_i}} \left( (M_{\mathcal{Q}_{\text{CDTHIP-P-1}}}^{\delta(j)}(\tilde{x}_i))^2 + (M_{\mathcal{Q}_{\text{CDTHIP-S-1}}}^{\delta(j)}(\tilde{x}_i))^2 \right) \right) \right. \\
 & \left. \left( \frac{1}{\tilde{L}_{x_i}} \sum_{k=1}^{\tilde{L}_{x_i}} \left( (N_{\mathcal{Q}_{\text{CDTHIP-P-1}}}^{\delta(k)}(\tilde{x}_i))^2 + (N_{\mathcal{Q}_{\text{CDTHIP-S-1}}}^{\delta(k)}(\tilde{x}_i))^2 \right) \right) \right. \\
 & \left. \left( \frac{1}{\tilde{L}_{x_i}} \sum_{k=1}^{\tilde{L}_{x_i}} \left( (N_{\mathcal{Q}_{\text{CDTHIP-P-1}}}^{\delta(k)}(\tilde{x}_i))^2 + (N_{\mathcal{Q}_{\text{CDTHIP-S-1}}}^{\delta(k)}(\tilde{x}_i))^2 \right) \right) \right) \right)^{(1/2)} \tag{21}
 \end{aligned}$$

**Definition 10.** For any two CDT-2HFSs,  $\mathcal{Q}_{\text{CDTH-1}} = \left\{ \left( M_{\mathcal{Q}_{\text{CDTHR-P-1}}}^{\delta(j)}(\tilde{x}_i) \cdot e^{i2\pi(M_{\mathcal{Q}_{\text{CDTHIP-P-1}}}^{\delta(j)}(\tilde{x}_i))} \right), M_{\mathcal{Q}_{\text{CDTHR-S-1}}}^{\delta(j)}(\tilde{x}_i) \cdot e^{i2\pi(M_{\mathcal{Q}_{\text{CDTHIP-S-1}}}^{\delta(j)}(\tilde{x}_i))} \right), \left( N_{\mathcal{Q}_{\text{CDTHR-P-1}}}^{\delta(k)}(\tilde{x}_i) \cdot e^{i2\pi(N_{\mathcal{Q}_{\text{CDTHIP-P-1}}}^{\delta(k)}(\tilde{x}_i))} \right), N_{\mathcal{Q}_{\text{CDTHR-S-1}}}^{\delta(k)}(\tilde{x}_i) \cdot e^{i2\pi(N_{\mathcal{Q}_{\text{CDTHIP-S-1}}}^{\delta(k)}(\tilde{x}_i))} \right) \right\}$ ,  $j, k = 1, 2, 3, \dots, n, m$  and  $\mathcal{Q}_{\text{CDTH-2}} =$

$\left\{ \left( M_{\mathcal{Q}_{\text{CDTHR-P-2}}}^{\delta(j)}(\tilde{x}_i) \cdot e^{i2\pi(M_{\mathcal{Q}_{\text{CDTHIP-P-2}}}^{\delta(j)}(\tilde{x}_i))} \right), M_{\mathcal{Q}_{\text{CDTHR-S-2}}}^{\delta(j)}(\tilde{x}_i) \cdot e^{i2\pi(M_{\mathcal{Q}_{\text{CDTHIP-S-2}}}^{\delta(j)}(\tilde{x}_i))} \right), \left( N_{\mathcal{Q}_{\text{CDTHR-P-2}}}^{\delta(k)}(\tilde{x}_i) \cdot e^{i2\pi(N_{\mathcal{Q}_{\text{CDTHIP-P-2}}}^{\delta(k)}(\tilde{x}_i))} \right), N_{\mathcal{Q}_{\text{CDTHR-S-2}}}^{\delta(k)}(\tilde{x}_i) \cdot e^{i2\pi(N_{\mathcal{Q}_{\text{CDTHIP-S-2}}}^{\delta(k)}(\tilde{x}_i))} \right) \right\}$ , the weighted correlation coefficient is of the form

$$\zeta_{\text{CDTHF-wcc}}(\mathcal{Q}_{\text{CDTH-1}}, \mathcal{Q}_{\text{CDTH-2}}) = \frac{\zeta_{\text{CDTHF-wc}}(\mathcal{Q}_{\text{CDTH-1}} - \mathcal{Q}_{\text{CDTH-2}})}{\zeta_{\text{CDTHF-wc}}(\mathcal{Q}_{\text{CDTH-1}}, \mathcal{Q}_{\text{CDTH-1}})^{(1/2)} \times \zeta_{\text{CDTHF-wc}}(\mathcal{Q}_{\text{CDTH-1}}, \mathcal{Q}_{\text{CDTH-2}})^{(1/2)}}$$

$$\begin{aligned}
 & \left( \begin{aligned}
 & \left( 1/\tilde{\tau}_{x_i} \right) \sum_{j=1}^{\tilde{\tau}_{x_i}} M_{\mathcal{Q}_{CDTHP-P-1}}^{\delta(j)}(\tilde{x}_i) M_{\mathcal{Q}_{CDTHP-P-2}}^{\delta(j)}(\tilde{x}_i) + M_{\mathcal{Q}_{CDTHP-S-1}}^{\delta(j)}(\tilde{x}_i) M_{\mathcal{Q}_{CDTHP-S-2}}^{\delta(j)}(\tilde{x}_i) + \\
 & \left( 1/\tilde{\tau}_{x_i} \right) \sum_{j=1}^{\tilde{\tau}_{x_i}} M_{\mathcal{Q}_{CDTHP-P-1}}^{\delta(j)}(\tilde{x}_i) M_{\mathcal{Q}_{CDTHP-P-2}}^{\delta(j)}(\tilde{x}_i) + M_{\mathcal{Q}_{CDTHP-S-1}}^{\delta(j)}(\tilde{x}_i) M_{\mathcal{Q}_{CDTHP-S-2}}^{\delta(j)}(\tilde{x}_i) + \\
 & \left( 1/\tilde{\tau}_{x_i} \right) \sum_{j=1}^{\tilde{\tau}_{x_i}} N_{\mathcal{Q}_{CDTHP-P-1}}^{\delta(k)}(\tilde{x}_i) N_{\mathcal{Q}_{CDTHP-P-2}}^{\delta(k)}(\tilde{x}_i) + N_{\mathcal{Q}_{CDTHP-S-1}}^{\delta(k)}(\tilde{x}_i) N_{\mathcal{Q}_{CDTHP-S-2}}^{\delta(k)}(\tilde{x}_i) + \\
 & \left( 1/\tilde{\tau}_{x_i} \right) \sum_{j=1}^{\tilde{\tau}_{x_i}} N_{\mathcal{Q}_{CDTHP-P-1}}^{\delta(k)}(\tilde{x}_i) N_{\mathcal{Q}_{CDTHP-P-2}}^{\delta(k)}(\tilde{x}_i) + N_{\mathcal{Q}_{CDTHP-S-1}}^{\delta(k)}(\tilde{x}_i) N_{\mathcal{Q}_{CDTHP-S-2}}^{\delta(k)}(\tilde{x}_i)
 \end{aligned} \right) \\
 = & \frac{\left( \begin{aligned}
 & \left( \begin{aligned}
 & \left( \Omega_i/\tilde{\tau}_{x_i} \right) \sum_{j=1}^{\tilde{\tau}_{x_i}} \left( M_{\mathcal{Q}_{CDTHP-P-1}}^{\delta(j)}(\tilde{x}_i) \right)^2 + \left( M_{\mathcal{Q}_{CDTHP-S-1}}^{\delta(j)}(\tilde{x}_i) \right)^2 + \\
 & \left( \Omega_i/\tilde{\tau}_{x_i} \right) \sum_{j=1}^{\tilde{\tau}_{x_i}} \left( M_{\mathcal{Q}_{CDTHP-P-1}}^{\delta(j)}(\tilde{x}_i) \right)^2 + \left( M_{\mathcal{Q}_{CDTHP-S-1}}^{\delta(j)}(\tilde{x}_i) \right)^2 + \\
 & \left( \Omega_i/\tilde{\tau}_{x_i} \right) \sum_{k=1}^{\tilde{\tau}_{x_i}} \left( N_{\mathcal{Q}_{CDTHP-P-1}}^{\delta(k)}(\tilde{x}_i) \right)^2 + \left( N_{\mathcal{Q}_{CDTHP-S-1}}^{\delta(k)}(\tilde{x}_i) \right)^2 + \\
 & \left( \Omega_i/\tilde{\tau}_{x_i} \right) \sum_{k=1}^{\tilde{\tau}_{x_i}} \left( N_{\mathcal{Q}_{CDTHP-P-1}}^{\delta(k)}(\tilde{x}_i) \right)^2 + \left( N_{\mathcal{Q}_{CDTHP-S-1}}^{\delta(k)}(\tilde{x}_i) \right)^2 +
 \end{aligned} \right)^{(1/2)} \times \\
 & \left( \begin{aligned}
 & \left( \Omega_i/\tilde{\tau}_{x_i} \right) \sum_{j=1}^{\tilde{\tau}_{x_i}} \left( M_{\mathcal{Q}_{CDTHP-P-2}}^{\delta(j)}(\tilde{x}_i) \right)^2 + \left( M_{\mathcal{Q}_{CDTHP-S-2}}^{\delta(j)}(\tilde{x}_i) \right)^2 + \\
 & \left( \Omega_i/\tilde{\tau}_{x_i} \right) \sum_{j=1}^{\tilde{\tau}_{x_i}} \left( M_{\mathcal{Q}_{CDTHP-P-2}}^{\delta(j)}(\tilde{x}_i) \right)^2 + \left( M_{\mathcal{Q}_{CDTHP-S-2}}^{\delta(j)}(\tilde{x}_i) \right)^2 + \\
 & \left( \Omega_i/\tilde{\tau}_{x_i} \right) \sum_{k=1}^{\tilde{\tau}_{x_i}} \left( N_{\mathcal{Q}_{CDTHP-P-2}}^{\delta(k)}(\tilde{x}_i) \right)^2 + \left( N_{\mathcal{Q}_{CDTHP-S-1}}^{\delta(k)}(\tilde{x}_i) \right)^2 + \\
 & \left( \Omega_i/\tilde{\tau}_{x_i} \right) \sum_{k=1}^{\tilde{\tau}_{x_i}} \left( N_{\mathcal{Q}_{CDTHP-P-2}}^{\delta(k)}(\tilde{x}_i) \right)^2 + \left( N_{\mathcal{Q}_{CDTHP-S-1}}^{\delta(k)}(\tilde{x}_i) \right)^2 +
 \end{aligned} \right)^{(1/2)}
 \end{aligned} \right) \tag{22}
 \end{aligned}$$

where  $\Omega = (\Omega_1, \Omega_2, \dots, \Omega_n)^T$  represents weight vector with the condition that  $\sum_{i=1}^n \Omega_i = 1, \Omega_i \in [0, 1]$ .

**Proposition 3.** For any two CDT-2HFSs, QCDTH-1 and QCDTH-2, the CC among CDT-2HFSs satisfies the following axioms:

- (1)  $0 \leq \zeta_{CDTH-wcc}(\mathcal{Q}_{CDTH-1}, \mathcal{Q}_{CDTH-2}) \leq 1$
- (2)  $\zeta_{CDTH-wcc}(\mathcal{Q}_{CDTH-1}, \mathcal{Q}_{CDTH-2}) = 1 \Leftrightarrow \mathcal{Q}_{CDTH-1} = \mathcal{Q}_{CDTH-2}$
- (3)  $\zeta_{CDTH-wcc}(\mathcal{Q}_{CDTH-1}, \mathcal{Q}_{CDTH-2}) = \zeta_{CDTH-wcc}(\mathcal{Q}_{CDTH-2}, \mathcal{Q}_{CDTH-1})$

*Proof.* The proof is straightforward.

**Definition 11.** For any two CDT-2HFSs,  $\mathcal{Q}_{CDTH-1} = \left\{ \left( \begin{aligned} & \left( M_{\mathcal{Q}_{CDTHP-P-1}}^{\delta(j)}(\tilde{x}_i) \cdot e^{i2\pi(M_{\mathcal{Q}_{CDTHP-P-1}}^{\delta(j)}(\tilde{x}_i))} \right), M_{\mathcal{Q}_{CDTHP-S-1}}^{\delta(j)}(\tilde{x}_i) \cdot e^{i2\pi(M_{\mathcal{Q}_{CDTHP-S-1}}^{\delta(j)}(\tilde{x}_i))} \right), \right. \\ & \left. \left( N_{\mathcal{Q}_{CDTHP-P-1}}^{\delta(k)}(\tilde{x}_i) \cdot e^{i2\pi(N_{\mathcal{Q}_{CDTHP-P-1}}^{\delta(k)}(\tilde{x}_i))} \right), N_{\mathcal{Q}_{CDTHP-S-1}}^{\delta(k)}(\tilde{x}_i) \cdot e^{i2\pi(N_{\mathcal{Q}_{CDTHP-S-1}}^{\delta(k)}(\tilde{x}_i))} \right) \right\}$ ,  $j, k = 1, 2, 3, \dots, n, m$  and  $\mathcal{Q}_{CDTH-2} = \left\{ \left( \begin{aligned} & \left( M_{\mathcal{Q}_{CDTHP-P-2}}^{\delta(j)}(\tilde{x}_i) \cdot e^{i2\pi(M_{\mathcal{Q}_{CDTHP-P-2}}^{\delta(j)}(\tilde{x}_i))} \right), M_{\mathcal{Q}_{CDTHP-S-2}}^{\delta(j)}(\tilde{x}_i) \cdot e^{i2\pi(M_{\mathcal{Q}_{CDTHP-S-2}}^{\delta(j)}(\tilde{x}_i))} \right), \right. \\ & \left. \left( N_{\mathcal{Q}_{CDTHP-P-2}}^{\delta(k)}(\tilde{x}_i) \cdot e^{i2\pi(N_{\mathcal{Q}_{CDTHP-P-2}}^{\delta(k)}(\tilde{x}_i))} \right), N_{\mathcal{Q}_{CDTHP-S-2}}^{\delta(k)}(\tilde{x}_i) \cdot e^{i2\pi(N_{\mathcal{Q}_{CDTHP-S-2}}^{\delta(k)}(\tilde{x}_i))} \right) \right\}$ ,  $j, k = 1, 2, 3, \dots, \tilde{\tau}_{x_i}, \tilde{\tau}_{x_i}$ , the maximum-based weighted correlation coefficient (MCC) is of the form

$$\zeta_{CDTH-mwcc}(\mathcal{Q}_{CDTH-1}, \mathcal{Q}_{CDTH-2}) = \frac{\zeta_{CDTH-wc}(\mathcal{Q}_{CDTH-1} - \mathcal{Q}_{CDTH-2})}{\max(\zeta_{CDTH-wc}(\mathcal{Q}_{CDTH-1}, \mathcal{Q}_{CDTH-1})^{(1/2)} \times \zeta_{CDTH-wc}(\mathcal{Q}_{CDTH-1}, \mathcal{Q}_{CDTH-2})^{(1/2)})}$$



$$\begin{aligned}
 & \sum_{i=1}^n \Omega_i \left( \begin{aligned} & \left( 1/\tilde{r}_{x_i} \sum_{j=1}^{\tilde{x}_i} M_{\mathcal{Q}_{CDTHRP-p-1}}^{\delta(j)}(\tilde{x}_i) M_{\mathcal{Q}_{CDTHRP-p-2}}^{\delta(j)}(\tilde{x}_i) + M_{\mathcal{Q}_{CDTHRP-s-1}}^{\delta(j)}(\tilde{x}_i) M_{\mathcal{Q}_{CDTHRP-s-2}}^{\delta(j)}(\tilde{x}_i) \right) + \\ & \left( 1/\tilde{r}_{x_i} \sum_{j=1}^{\tilde{x}_i} M_{\mathcal{Q}_{CDTHIP-p-1}}^{\delta(j)}(\tilde{x}_i) M_{\mathcal{Q}_{CDTHIP-p-2}}^{\delta(j)}(\tilde{x}_i) + M_{\mathcal{Q}_{CDTHIP-s-1}}^{\delta(j)}(\tilde{x}_i) M_{\mathcal{Q}_{CDTHIP-s-2}}^{\delta(j)}(\tilde{x}_i) \right) + \\ & \left( 1/\tilde{r}_{x_i} \sum_{j=1}^{\tilde{x}_i} N_{\mathcal{Q}_{CDTHRP-p-1}}^{\delta(k)}(\tilde{x}_i) N_{\mathcal{Q}_{CDTHRP-p-2}}^{\delta(k)}(\tilde{x}_i) + N_{\mathcal{Q}_{CDTHRP-s-1}}^{\delta(k)}(\tilde{x}_i) N_{\mathcal{Q}_{CDTHRP-s-2}}^{\delta(k)}(\tilde{x}_i) \right) + \\ & \left( 1/\tilde{r}_{x_i} \sum_{j=1}^{\tilde{x}_i} N_{\mathcal{Q}_{CDTHIP-p-1}}^{\delta(k)}(\tilde{x}_i) N_{\mathcal{Q}_{CDTHIP-p-2}}^{\delta(k)}(\tilde{x}_i) + N_{\mathcal{Q}_{CDTHIP-s-1}}^{\delta(k)}(\tilde{x}_i) N_{\mathcal{Q}_{CDTHIP-s-2}}^{\delta(k)}(\tilde{x}_i) \right) \end{aligned} \right) \\
 = & \frac{\left( \begin{aligned} & \left( \left( \Omega_i/\tilde{r}_{x_i} \sum_{j=1}^{\tilde{x}_i} \left( \left( M_{\mathcal{Q}_{CDTHRP-p-1}}^{\delta(j)}(\tilde{x}_i) \right)^2 + \left( M_{\mathcal{Q}_{CDTHRP-s-1}}^{\delta(j)}(\tilde{x}_i) \right)^2 \right) + \right) \right)^{(1/2)} \\ & \left( \left( \Omega_i/\tilde{r}_{x_i} \sum_{j=1}^{\tilde{x}_i} \left( \left( M_{\mathcal{Q}_{CDTHIP-p-1}}^{\delta(j)}(\tilde{x}_i) \right)^2 + \left( M_{\mathcal{Q}_{CDTHIP-s-1}}^{\delta(j)}(\tilde{x}_i) \right)^2 \right) + \right) \right)^{(1/2)} \\ & \left( \left( \Omega_i/\tilde{r}_{x_i} \sum_{k=1}^{\tilde{x}_i} \left( \left( N_{\mathcal{Q}_{CDTHRP-p-1}}^{\delta(k)}(\tilde{x}_i) \right)^2 + \left( N_{\mathcal{Q}_{CDTHRP-s-1}}^{\delta(k)}(\tilde{x}_i) \right)^2 \right) + \right) \right)^{(1/2)} \\ & \left( \left( \Omega_i/\tilde{r}_{x_i} \sum_{k=1}^{\tilde{x}_i} \left( \left( N_{\mathcal{Q}_{CDTHIP-p-1}}^{\delta(k)}(\tilde{x}_i) \right)^2 + \left( N_{\mathcal{Q}_{CDTHIP-s-1}}^{\delta(k)}(\tilde{x}_i) \right)^2 \right) + \right) \right)^{(1/2)} \end{aligned} \right) \times \\
 \max & \left( \begin{aligned} & \left( \left( \Omega_i/\tilde{r}_{x_i} \sum_{j=1}^{\tilde{x}_i} \left( \left( M_{\mathcal{Q}_{CDTHRP-p-2}}^{\delta(j)}(\tilde{x}_i) \right)^2 + \left( M_{\mathcal{Q}_{CDTHRP-s-2}}^{\delta(j)}(\tilde{x}_i) \right)^2 \right) + \right) \right)^{(1/2)} \\ & \left( \left( \Omega_i/\tilde{r}_{x_i} \sum_{j=1}^{\tilde{x}_i} \left( \left( M_{\mathcal{Q}_{CDTHIP-p-2}}^{\delta(j)}(\tilde{x}_i) \right)^2 + \left( M_{\mathcal{Q}_{CDTHIP-s-2}}^{\delta(j)}(\tilde{x}_i) \right)^2 \right) + \right) \right)^{(1/2)} \\ & \left( \left( \Omega_i/\tilde{r}_{x_i} \sum_{k=1}^{\tilde{x}_i} \left( \left( N_{\mathcal{Q}_{CDTHRP-p-2}}^{\delta(k)}(\tilde{x}_i) \right)^2 + \left( N_{\mathcal{Q}_{CDTHRP-s-1}}^{\delta(k)}(\tilde{x}_i) \right)^2 \right) + \right) \right)^{(1/2)} \\ & \left( \left( \Omega_i/\tilde{r}_{x_i} \sum_{k=1}^{\tilde{x}_i} \left( \left( N_{\mathcal{Q}_{CDTHIP-p-2}}^{\delta(k)}(\tilde{x}_i) \right)^2 + \left( N_{\mathcal{Q}_{CDTHIP-s-1}}^{\delta(k)}(\tilde{x}_i) \right)^2 \right) + \right) \right)^{(1/2)} \end{aligned} \right) \end{aligned} \tag{23}$$

where  $\Omega = (\Omega_1, \Omega_2, \dots, \Omega_n)^T$  represents weight vector with the condition that  $\sum_{i=1}^n \Omega_i = 1, \Omega_i \in [0, 1]$ .

**Proposition 4.** For any two CDT-2HFSs, QCDTH-1 and QCDTH-2, the MCC among CDT-2HFSs satisfies the following axioms:

- (1)  $0 \leq \zeta_{CDTH-mwcc}(\mathcal{Q}_{CDTH-1}, \mathcal{Q}_{CDTH-2}) \leq 1$
- (2)  $\zeta_{CDTH-mwcc}(\mathcal{Q}_{CDTH-1}, \mathcal{Q}_{CDTH-2}) = 1 \Leftrightarrow \mathcal{Q}_{CDTH-1} = \mathcal{Q}_{CDTH-2}$
- (3)  $\zeta_{CDTH-mwcc}(\mathcal{Q}_{CDTH-1}, \mathcal{Q}_{CDTH-2}) = \zeta_{CDTH-mwcc}(\mathcal{Q}_{CDTH-2}, \mathcal{Q}_{CDTH-1})$

*Proof.* The proof is straightforward.

The explored notions, which are stated in the form of equations (13)–(23), are more proficient and more modified than the existing drawbacks; for instance, if we choose the

imaginary part of equations (13)–(23) to be zero, then equations (13)–(23) convert it for DT-2HFS.

### 5. Entropy Measures for Complex Dual Type-2 Hesitant Fuzzy Sets

The aim of this study is to present the novel of two types of entropy measures (EMs). The special cases of the explored measures are also explored.

*Definition 12.* For any two CDT-2HFSs  $\mathcal{Q}_{CDTH-1} = \left\{ \left( \left( M_{\mathcal{Q}_{CDTHRP-p-1}}^{\delta(j)}(\tilde{x}_i) \cdot e^{i2\pi(M_{\mathcal{Q}_{CDTHIP-p-1}}^{\delta(j)}(\tilde{x}_i))} \right), M_{\mathcal{Q}_{CDTHRP-s-1}}^{\delta(j)}(\tilde{x}_i) \cdot e^{i2\pi(M_{\mathcal{Q}_{CDTHIP-s-1}}^{\delta(j)}(\tilde{x}_i))} \right), \left( N_{\mathcal{Q}_{CDTHRP-p-1}}^{\delta(k)}(\tilde{x}_i) \cdot e^{i2\pi(N_{\mathcal{Q}_{CDTHIP-p-1}}^{\delta(k)}(\tilde{x}_i))} \right), N_{\mathcal{Q}_{CDTHRP-s-1}}^{\delta(k)}(\tilde{x}_i) \cdot e^{i2\pi(N_{\mathcal{Q}_{CDTHIP-s-1}}^{\delta(k)}(\tilde{x}_i))} \right) \right\}$ , the two EMs are defined by

$$\mathcal{F}_1(\mathcal{Q}_{\text{CDTH-1}}) = \frac{1}{n} \sum_{i=1}^n \left\{ \begin{array}{l} \left( \begin{array}{l} 2 + \left(1/\tilde{r}_{\tilde{x}_i}\right) \sum_{j=1}^{\tilde{r}_{\tilde{x}_i}} \left( M_{\mathcal{Q}_{\text{CDTHRP-P-1}}}^{\delta(j)}(\tilde{x}_i) + M_{\mathcal{Q}_{\text{CDTHRP-S-1}}}^{\delta(j)}(\tilde{x}_i) \right) - \\ \left(1/\tilde{r}_{\tilde{x}_i}\right) \sum_{k=1}^{\tilde{r}_{\tilde{x}_i}} \left( N_{\mathcal{Q}_{\text{CDTHRP-P-1}}}^{\delta(k)}(\tilde{x}_i) + N_{\mathcal{Q}_{\text{CDTHRP-S-1}}}^{\delta(k)}(\tilde{x}_i) \right) + \\ \left(1/\tilde{r}_{\tilde{x}_i}\right) \sum_{j=1}^{\tilde{r}_{\tilde{x}_i}} \left( M_{\mathcal{Q}_{\text{CDTHIP-P-1}}}^{\delta(j)}(\tilde{x}_i) + M_{\mathcal{Q}_{\text{CDTHIP-S-1}}}^{\delta(j)}(\tilde{x}_i) \right) - \\ \left(1/\tilde{r}_{\tilde{x}_i}\right) \sum_{k=1}^{\tilde{r}_{\tilde{x}_i}} \left( N_{\mathcal{Q}_{\text{CDTHIP-P-1}}}^{\delta(k)}(\tilde{x}_i) + N_{\mathcal{Q}_{\text{CDTHIP-S-1}}}^{\delta(k)}(\tilde{x}_i) \right) \end{array} \right) \\ \sin \frac{\phantom{}}{8} \end{array} \right\} \quad (24)$$
  

$$\left\{ \begin{array}{l} \left( \begin{array}{l} 2 + \left(1/\tilde{r}_{\tilde{x}_i}\right) \sum_{j=1}^{\tilde{r}_{\tilde{x}_i}} \left( M_{\mathcal{Q}_{\text{CDTHRP-P-1}}}^{\delta(j)}(\tilde{x}_i) + M_{\mathcal{Q}_{\text{CDTHRP-S-1}}}^{\delta(j)}(\tilde{x}_i) \right) + \\ \left(1/\tilde{r}_{\tilde{x}_i}\right) \sum_{k=1}^{\tilde{r}_{\tilde{x}_i}} \left( N_{\mathcal{Q}_{\text{CDTHRP-P-1}}}^{\delta(k)}(\tilde{x}_i) + N_{\mathcal{Q}_{\text{CDTHRP-S-1}}}^{\delta(k)}(\tilde{x}_i) \right) - \\ \left(1/\tilde{r}_{\tilde{x}_i}\right) \sum_{j=1}^{\tilde{r}_{\tilde{x}_i}} \left( M_{\mathcal{Q}_{\text{CDTHIP-P-1}}}^{\delta(j)}(\tilde{x}_i) + M_{\mathcal{Q}_{\text{CDTHIP-S-1}}}^{\delta(j)}(\tilde{x}_i) \right) + \\ \left(1/\tilde{r}_{\tilde{x}_i}\right) \sum_{k=1}^{\tilde{r}_{\tilde{x}_i}} \left( N_{\mathcal{Q}_{\text{CDTHIP-P-1}}}^{\delta(k)}(\tilde{x}_i) + N_{\mathcal{Q}_{\text{CDTHIP-S-1}}}^{\delta(k)}(\tilde{x}_i) \right) \end{array} \right) \\ \pi \times \sin \frac{\phantom{}}{8} - 1 \end{array} \right\}$$

$$\mathcal{F}_1 (@_{CDTH-1}) = \frac{1}{n} \sum_{i=1}^n \left\{ \begin{array}{l} \left( \begin{array}{l} \pi \times \\ \cos \end{array} \frac{\left( \begin{array}{l} 2 + \left(1/\tilde{r}_{\tilde{x}_i}\right) \sum_{j=1}^{\tilde{x}_i} \left( M_{@_{CDTHRP-P-1}}^{\delta(j)} \left( \tilde{x}_i \right) + M_{@_{CDTHRP-S-1}}^{\delta(j)} \left( \tilde{x}_i \right) \right) + \\ \left(1/\tilde{r}_{\tilde{x}_i}\right) \sum_{k=1}^{\tilde{x}_i} \left( N_{@_{CDTHRP-P-1}}^{\delta(k)} \left( \tilde{x}_i \right) + N_{@_{CDTHRP-S-1}}^{\delta(k)} \left( \tilde{x}_i \right) \right) - \\ \left(1/\tilde{r}_{\tilde{x}_i}\right) \sum_{j=1}^{\tilde{x}_i} \left( M_{@_{CDTHIP-P-1}}^{\delta(j)} \left( \tilde{x}_i \right) + M_{@_{CDTHIP-S-1}}^{\delta(j)} \left( \tilde{x}_i \right) \right) + \\ \left(1/\tilde{r}_{\tilde{x}_i}\right) \sum_{k=1}^{\tilde{x}_i} \left( N_{@_{CDTHIP-P-1}}^{\delta(k)} \left( \tilde{x}_i \right) + N_{@_{CDTHIP-S-1}}^{\delta(k)} \left( \tilde{x}_i \right) \right) \end{array} \right)}{8} \right\} \quad (25) \\
 \left( \begin{array}{l} \pi \times \\ \cos \end{array} \frac{\left( \begin{array}{l} 2 + \left(1/\tilde{r}_{\tilde{x}_i}\right) \sum_{j=1}^{\tilde{x}_i} \left( M_{@_{CDTHRP-P-1}}^{\delta(j)} \left( \tilde{x}_i \right) + M_{@_{CDTHRP-S-1}}^{\delta(j)} \left( \tilde{x}_i \right) \right) + \\ \left(1/\tilde{r}_{\tilde{x}_i}\right) \sum_{k=1}^{\tilde{x}_i} \left( N_{@_{CDTHRP-P-1}}^{\delta(k)} \left( \tilde{x}_i \right) + N_{@_{CDTHRP-S-1}}^{\delta(k)} \left( \tilde{x}_i \right) \right) - \\ \left(1/\tilde{r}_{\tilde{x}_i}\right) \sum_{j=1}^{\tilde{x}_i} \left( M_{@_{CDTHIP-P-1}}^{\delta(j)} \left( \tilde{x}_i \right) + M_{@_{CDTHIP-S-1}}^{\delta(j)} \left( \tilde{x}_i \right) \right) + \\ \left(1/\tilde{r}_{\tilde{x}_i}\right) \sum_{k=1}^{\tilde{x}_i} \left( N_{@_{CDTHIP-P-1}}^{\delta(jk)} \left( \tilde{x}_i \right) + N_{@_{CDTHIP-S-1}}^{\delta(k)} \left( \tilde{x}_i \right) \right) \end{array} \right)}{8} - 1 \right)
 \end{array} \right.$$

The two EMs based on CDT-2HFSs has the following properties:

- (1) If  $M_{\mathcal{Q}_{CDTHRP-p-1}}^{\delta(j)} = M_{\mathcal{Q}_{CDTHRP-S-1}}^{\delta(j)} = 1, N_{\mathcal{Q}_{CDTHRP-S-1}}^{\delta(k)} = 0, M_{\mathcal{Q}_{CDTHIP-p-1}}^{\delta(j)} = M_{\mathcal{Q}_{CDTHIP-S-1}}^{\delta(j)} = 1, N_{\mathcal{Q}_{CDTHIP-p-1}}^{\delta(k)} = N_{\mathcal{Q}_{CDTHIP-S-1}}^{\delta(k)} = 0$  or  $M_{\mathcal{Q}_{CDTHRP-p-2}}^{\delta(j)} = 1, N_{\mathcal{Q}_{CDTHRP-S-2}}^{\delta(k)} = N_{\mathcal{Q}_{CDTHRP-p-2}}^{\delta(k)} = 0, M_{\mathcal{Q}_{CDTHIP-p-2}}^{\delta(j)} = M_{\mathcal{Q}_{CDTHIP-S-2}}^{\delta(j)} = 1, N_{\mathcal{Q}_{CDTHIP-p-2}}^{\delta(k)} = 0$
- (2) If  $M_{\mathcal{Q}_{CDTHRP-p-1}}^{\delta(j)}, M_{\mathcal{Q}_{CDTHRP-S-1}}^{\delta(j)} \leq M_{\mathcal{Q}_{CDTHRP-p-2}}^{\delta(j)}, M_{\mathcal{Q}_{CDTHRP-S-2}}^{\delta(j)}, N_{\mathcal{Q}_{CDTHRP-p-1}}^{\delta(k)}, N_{\mathcal{Q}_{CDTHRP-S-1}}^{\delta(k)} \geq N_{\mathcal{Q}_{CDTHRP-p-2}}^{\delta(k)}, N_{\mathcal{Q}_{CDTHRP-S-2}}^{\delta(k)}, M_{\mathcal{Q}_{CDTHIP-p-1}}^{\delta(j)}, M_{\mathcal{Q}_{CDTHIP-S-1}}^{\delta(j)} \leq M_{\mathcal{Q}_{CDTHIP-p-1}}^{\delta(j)}, M_{\mathcal{Q}_{CDTHIP-S-1}}^{\delta(j)}, N_{\mathcal{Q}_{CDTHIP-p-1}}^{\delta(k)}, N_{\mathcal{Q}_{CDTHIP-S-1}}^{\delta(k)} \geq N_{\mathcal{Q}_{CDTHIP-p-2}}^{\delta(k)}, N_{\mathcal{Q}_{CDTHIP-S-2}}^{\delta(k)}$  then,  $\mathcal{F}_1(\mathcal{Q}_{CDTH-1}) \leq \mathcal{F}_1(\mathcal{Q}_{CDTH-2})$ , and if we change  $\leq$  into  $\geq$ , then  $\mathcal{F}_1(\mathcal{Q}_{CDTH-1}) \geq \mathcal{F}_1(\mathcal{Q}_{CDTH-2})$
- (3)  $\mathcal{F}_1(\mathcal{Q}_{CDTH-1}) = \mathcal{F}_1(\mathcal{Q}_{CDTH-1}^c)$  and  $\mathcal{F}_2(\mathcal{Q}_{CDTH-1}) = \mathcal{F}_2(\mathcal{Q}_{CDTH-1}^c)$ .

The explored notions, which are stated in the form of (23) and (24)fd25, are more proficient and more modified than the existing drawbacks; for instance, if we choose the imaginary part of (23) and (24)fd25 to be zero, then (23) and (24)fd25 convert it for DT-2HFS.

### 6. TOPSIS Method Based on CDT-2HFSs

Basically, a novel TOPSIS method using CC and EM is provided to handle the MADM problems based on Cq-ROFS. Previously, TOPSIS method was proposed based on sample SMs, but in our proposed work we considered the CC and EM. The DM cannot accurately examine the proximity of each alternative to ideal solution in some particular cases. So, we replace the TOPSIS method with the

CC instead of DM to check the efficacy and effectiveness of the proposed work.

**6.1. Problem Description.** Consider an MADM problem, whose  $m$  alternatives and  $n$  attributes are denoted by  $C = \{c_1, c_2, c_3, \dots, c_m\}$  and  $U = (u_1, u_2, u_3, \dots, u_n)$ , respectively, with respect to weight vectors represented by  $\Omega = (\Omega_1, \Omega_2, \dots, \Omega_n)^T$  with the conditions that  $\Omega \in \mathbb{O}, 1$  and  $\sum_{i=1}^n \Omega_i = 1$ . Each attribute of each alternative is simplified using CDT-2HFSs  $\mathcal{Q}_{CDTH-z} = \left\{ \left( M_{\mathcal{Q}_{CDTHRP-p-z}}^{\delta(k)}(\tilde{x}_i) \cdot e^{i2\pi(M_{\mathcal{Q}_{CDTHIP-p-z}}^{\delta(k)}(\tilde{x}_i))}, M_{\mathcal{Q}_{CDTHRP-S-z}}^{\delta(k)}(\tilde{x}_i) \cdot e^{i2\pi(M_{\mathcal{Q}_{CDTHIP-S-z}}^{\delta(k)}(\tilde{x}_i))}, \right. \right\}$   
 $\left. \left( N_{\mathcal{Q}_{CDTHRP-p-z}}^{\delta(k)}(\tilde{x}_i) \cdot e^{i2\pi(N_{\mathcal{Q}_{CDTHIP-p-z}}^{\delta(k)}(\tilde{x}_i))}, N_{\mathcal{Q}_{CDTHRP-S-z}}^{\delta(k)}(\tilde{x}_i) \cdot e^{i2\pi(N_{\mathcal{Q}_{CDTHIP-S-z}}^{\delta(k)}(\tilde{x}_i))} \right) \right\}$ ,  $j, k = 1, 2, 3, \dots, n, m1, 2, 3, \dots, n, m$ ,  $z = 1, 2, 3, \dots, n$ , satisfying the following conditions:  $0 \leq \max(M_{\mathcal{Q}_{CDTHRP-p}}^{\delta(j)}) + \max(N_{\mathcal{Q}_{CDTHRP-p}}^{\delta(k)}) \leq 1, 0 \leq \max(M_{\mathcal{Q}_{CDTHIP-p}}^{\delta(j)}) + \max(N_{\mathcal{Q}_{CDTHIP-p}}^{\delta(k)}) \leq 1, 0 \leq \max(M_{\mathcal{Q}_{CDTHRP-S}}^{\delta(j)}) + \max(N_{\mathcal{Q}_{CDTHIP-S}}^{\delta(k)}) \leq 1$  and  $0 \leq \max(M_{\mathcal{Q}_{CDTHIP-S}}^{\delta(j)}) + \max(N_{\mathcal{Q}_{CDTHIP-p}}^{\delta(k)}) \leq 1$ . All the attributes values of the alternatives are in the form of CDT-2HF decision matrix (CDT-2HFD $M$ ); that is,  $[\mathcal{Q}_{CDTH-yz}]_{m \times n} = \left\{ \left( M_{\mathcal{Q}_{CDTHRP-p-yz}}^{\delta(k)}(\tilde{x}_i) \cdot e^{i2\pi(M_{\mathcal{Q}_{CDTHIP-p-yz}}^{\delta(k)}(\tilde{x}_i))}, M_{\mathcal{Q}_{CDTHRP-S-yz}}^{\delta(k)}(\tilde{x}_i) \cdot e^{i2\pi(M_{\mathcal{Q}_{CDTHIP-S-yz}}^{\delta(k)}(\tilde{x}_i))}, \right. \right\}$   
 $\left. \left( N_{\mathcal{Q}_{CDTHRP-p-yz}}^{\delta(k)}(\tilde{x}_i) \cdot e^{i2\pi(N_{\mathcal{Q}_{CDTHIP-p-yz}}^{\delta(k)}(\tilde{x}_i))}, N_{\mathcal{Q}_{CDTHRP-S-yz}}^{\delta(k)}(\tilde{x}_i) \cdot e^{i2\pi(N_{\mathcal{Q}_{CDTHIP-S-yz}}^{\delta(k)}(\tilde{x}_i))} \right) \right\}$ ,  $j, k = 1, 2, 3, \dots, n, m1, 2, 3, \dots, n, m$ ,  $y, z = 1, 2, 3, \dots, m, n$ .

Due to tension, time limitations of the decision-makers (DMs), and complication of problems, it is awkward to give the weight information of attribute in advance. To handle such type of issue, we compute the weights of attributes and consider the proposed EM examining the weight of each attribute as

$$\Omega_j = \frac{1 - \mathcal{H}_j}{n - \sum_{i=1}^n \mathcal{H}_i} \tag{26}$$

where  $\mathcal{H}_j \in [0, 1], j = 1, 2, 3, \dots, n$ , is defined as

$$\mathcal{H}_j = \frac{1}{n} \sum_{i=1}^n \left\{ \begin{aligned} & \left( \begin{aligned} & \left( \begin{aligned} & 2 + \left(1/\tilde{r}_{x_i}\right) \sum_{j=1}^{\tilde{r}_{x_i}} \left( M_{\mathcal{Q}_{CDTHRP-P-i}}^{\delta(j)}(\tilde{x}_i) + M_{\mathcal{Q}_{CDTHRP-S-i}}^{\delta(j)}(\tilde{x}_i) \right) - \\ & \left(1/\tilde{r}_{x_i}\right) \sum_{k=1}^{\tilde{r}_{x_i}} \left( N_{\mathcal{Q}_{CDTHRP-P-i}}^{\delta(k)}(\tilde{x}_i) + N_{\mathcal{Q}_{CDTHRP-S-i}}^{\delta(k)}(\tilde{x}_i) \right) + \\ & \left(1/\tilde{r}_{x_i}\right) \sum_{j=1}^{\tilde{r}_{x_i}} \left( M_{\mathcal{Q}_{CDTHIP-P-i}}^{\delta(j)}(\tilde{x}_i) + M_{\mathcal{Q}_{CDTHIP-S-i}}^{\delta(j)}(\tilde{x}_i) \right) - \\ & \left(1/\tilde{r}_{x_i}\right) \sum_{k=1}^{\tilde{r}_{x_i}} \left( N_{\mathcal{Q}_{CDTHIP-P-i}}^{\delta(k)}(\tilde{x}_i) + N_{\mathcal{Q}_{CDTHIP-S-i}}^{\delta(k)}(\tilde{x}_i) \right) \end{aligned} \right) \\ & \sin \frac{\pi \times}{8} \end{aligned} \right) + \\ & \left( \begin{aligned} & \left( \begin{aligned} & 2 - \left(1/\tilde{r}_{x_i}\right) \sum_{j=1}^{\tilde{r}_{x_i}} \left( M_{\mathcal{Q}_{CDTHRP-P-i}}^{\delta(j)}(\tilde{x}_i) + M_{\mathcal{Q}_{CDTHRP-S-i}}^{\delta(j)}(\tilde{x}_i) \right) + \\ & \left(1/\tilde{r}_{x_i}\right) \sum_{k=1}^{\tilde{r}_{x_i}} \left( N_{\mathcal{Q}_{CDTHRP-P-i}}^{\delta(k)}(\tilde{x}_i) + N_{\mathcal{Q}_{CDTHRP-S-i}}^{\delta(k)}(\tilde{x}_i) \right) - \\ & \left(1/\tilde{r}_{x_i}\right) \sum_{j=1}^{\tilde{r}_{x_i}} \left( M_{\mathcal{Q}_{CDTHIP-P-i}}^{\delta(j)}(\tilde{x}_i) + M_{\mathcal{Q}_{CDTHIP-S-i}}^{\delta(j)}(\tilde{x}_i) \right) + \\ & \left(1/\tilde{r}_{x_i}\right) \sum_{k=1}^{\tilde{r}_{x_i}} \left( N_{\mathcal{Q}_{CDTHIP-P-i}}^{\delta(k)}(\tilde{x}_i) + N_{\mathcal{Q}_{CDTHIP-S-i}}^{\delta(k)}(\tilde{x}_i) \right) \end{aligned} \right) \\ & \sin \frac{\pi \times}{8} \end{aligned} \right) - 1 \end{aligned} \right\}, \tag{27}$$

$$\mathcal{H}_j = \frac{1}{n} \sum_{i=1}^n \left\{ \begin{aligned} & \left( \begin{aligned} & \left( 2 + \left( 1/\tilde{r}_{x_i} \right) \sum_{j=1}^{\tilde{r}_{x_i}} \left( M_{\mathcal{Q}_{CDTHRP-p-i}}^{\delta(j)} \left( \tilde{x}_i \right) + M_{\mathcal{Q}_{CDTHRP-s-i}}^{\delta(j)} \left( \tilde{x}_i \right) \right) - \right. \\ & \left( 1/\tilde{r}_{x_i} \right) \sum_{k=1}^{\tilde{r}_{x_i}} \left( N_{\mathcal{Q}_{CDTHRP-p-i}}^{\delta(k)} \left( \tilde{x}_i \right) + N_{\mathcal{Q}_{CDTHRP-s-i}}^{\delta(k)} \left( \tilde{x}_i \right) \right) + \\ & \left( 1/\tilde{r}_{x_i} \right) \sum_{j=1}^{\tilde{r}_{x_i}} \left( M_{\mathcal{Q}_{CDTHIP-p-i}}^{\delta(j)} \left( \tilde{x}_i \right) + M_{\mathcal{Q}_{CDTHIP-s-i}}^{\delta(j)} \left( \tilde{x}_i \right) \right) - \\ & \left. \left( 1/\tilde{r}_{x_i} \right) \sum_{k=1}^{\tilde{r}_{x_i}} \left( N_{\mathcal{Q}_{CDTHIP-p-i}}^{\delta(k)} \left( \tilde{x}_i \right) + N_{\mathcal{Q}_{CDTHIP-s-i}}^{\delta(k)} \left( \tilde{x}_i \right) \right) \right) \right) \\ & \text{COS} \frac{\pi \times}{8} + \end{aligned} \right. \\ & \left. \left( \begin{aligned} & \left( 2 - \left( 1/\tilde{r}_{x_i} \right) \sum_{j=1}^{\tilde{r}_{x_i}} \left( M_{\mathcal{Q}_{CDTHRP-p-i}}^{\delta(j)} \left( \tilde{x}_i \right) + M_{\mathcal{Q}_{CDTHRP-s-i}}^{\delta(j)} \left( \tilde{x}_i \right) \right) + \right) \right. \\ & \left( 1/\tilde{r}_{x_i} \right) \sum_{k=1}^{\tilde{r}_{x_i}} \left( N_{\mathcal{Q}_{CDTHRP-p-i}}^{\delta(k)} \left( \tilde{x}_i \right) + N_{\mathcal{Q}_{CDTHRP-s-i}}^{\delta(k)} \left( \tilde{x}_i \right) \right) - \\ & \left( 1/\tilde{r}_{x_i} \right) \sum_{j=1}^{\tilde{r}_{x_i}} \left( M_{\mathcal{Q}_{CDTHIP-p-i}}^{\delta(j)} \left( \tilde{x}_i \right) + M_{\mathcal{Q}_{CDTHIP-s-i}}^{\delta(j)} \left( \tilde{x}_i \right) \right) + \\ & \left. \left( 1/\tilde{r}_{x_i} \right) \sum_{k=1}^{\tilde{r}_{x_i}} \left( N_{\mathcal{Q}_{CDTHIP-p-i}}^{\delta(k)} \left( \tilde{x}_i \right) + N_{\mathcal{Q}_{CDTHIP-s-i}}^{\delta(k)} \left( \tilde{x}_i \right) \right) \right) \right) \\ & \text{COS} \frac{\pi \times}{8} - 1 \end{aligned} \right\} \tag{28}
 \end{aligned}$$

When we consider that the imaginary part of (26) and (27) will be zero, (26) and (27) will be converted for DT-2HFS.

Procedure for MADM problem based on the above analysis by considering the proposed CDT-2HF TOPSIS method using CC and EM is explained below.

6.2. Application. The steps of the CDT-2HF TOPSIS method using WCC are as follows:

normalized the decision matrix by considering the following formula. We have

(i) Step 1: some decision-making problems also contain benefits and cost types of informations, so for this, we

$$r_{yz} = \left( \begin{array}{l} \left( \left( M_{\mathcal{Q}_{CDTHRP-p-yz}}^{\delta(j)}(\tilde{x}_i) \cdot e^{i2\pi \left( M_{\mathcal{Q}_{CDTHIP-p-yz}}^{\delta(j)}(\tilde{x}_i) \right)}, M_{\mathcal{Q}_{CDTHRP-S-yz}}^{\delta(j)}(\tilde{x}_i) \cdot e^{i2\pi \left( M_{\mathcal{Q}_{CDTHIP-S-yz}}^{\delta(j)}(\tilde{x}_i) \right)} \right), \right. \\ \left. \left( N_{\mathcal{Q}_{CDTHRP-p-yz}}^{\delta(k)}(\tilde{x}_i) \cdot e^{i2\pi \left( N_{\mathcal{Q}_{CDTHIP-p-yz}}^{\delta(k)}(\tilde{x}_i) \right)}, N_{\mathcal{Q}_{CDTHRP-S-yz}}^{\delta(k)}(\tilde{x}_i) \cdot e^{i2\pi \left( N_{\mathcal{Q}_{CDTHIP-S-yz}}^{\delta(k)}(\tilde{x}_i) \right)} \right) \right) \\ \left. \left. \left. \left. \left( \left( M_{\mathcal{Q}_{CDTHRP-p-yz}}^{\delta(j)}(\tilde{x}_i) \cdot e^{i2\pi \left( M_{\mathcal{Q}_{CDTHIP-p-yz}}^{\delta(j)}(\tilde{x}_i) \right)}, M_{\mathcal{Q}_{CDTHRP-S-yz}}^{\delta(j)}(\tilde{x}_i) \cdot e^{i2\pi \left( M_{\mathcal{Q}_{CDTHIP-S-yz}}^{\delta(j)}(\tilde{x}_i) \right)} \right), \right. \right. \right. \right. \right. \right. \\ \left. \left. \left. \left. \left( N_{\mathcal{Q}_{CDTHRP-p-yz}}^{\delta(k)}(\tilde{x}_i) \cdot e^{i2\pi \left( N_{\mathcal{Q}_{CDTHIP-p-yz}}^{\delta(k)}(\tilde{x}_i) \right)}, N_{\mathcal{Q}_{CDTHRP-S-yz}}^{\delta(k)}(\tilde{x}_i) \cdot e^{i2\pi \left( N_{\mathcal{Q}_{CDTHIP-S-yz}}^{\delta(k)}(\tilde{x}_i) \right)} \right) \right) \right) \right) \right) \right) \right. \\ \left. \left. \left. \left. \left( \left( N_{\mathcal{Q}_{CDTHRP-p-yz}}^{\delta(k)}(\tilde{x}_i) \cdot e^{i2\pi \left( N_{\mathcal{Q}_{CDTHIP-p-yz}}^{\delta(k)}(\tilde{x}_i) \right)}, N_{\mathcal{Q}_{CDTHRP-S-yz}}^{\delta(k)}(\tilde{x}_i) \cdot e^{i2\pi \left( N_{\mathcal{Q}_{CDTHIP-S-yz}}^{\delta(k)}(\tilde{x}_i) \right)} \right), \right. \right. \right. \right. \right. \right. \\ \left. \left. \left. \left. \left( M_{\mathcal{Q}_{CDTHRP-p-yz}}^{\delta(j)}(\tilde{x}_i) \cdot e^{i2\pi \left( M_{\mathcal{Q}_{CDTHIP-p-yz}}^{\delta(j)}(\tilde{x}_i) \right)}, M_{\mathcal{Q}_{CDTHRP-S-yz}}^{\delta(j)}(\tilde{x}_i) \cdot e^{i2\pi \left( M_{\mathcal{Q}_{CDTHIP-S-yz}}^{\delta(j)}(\tilde{x}_i) \right)} \right) \right) \right) \right) \right) \right) \right. \end{array} \right) = \left. \begin{array}{l} \text{for benefit types of attributes,} \\ \text{for cost types of attributes.} \end{array} \right) \tag{29}$$

Step 2: by using (25), we examine the weight vector of the attributes.

Step 3: by using (30) and (31), we examine the PIS and NIS among the alternatives.

$$R^+ = (r_{i1}^+, r_{i2}^+, r_{i3}^+, \dots, r_{in}^+),$$

$$\cdot r_{ij}^+ \left( \left( \left( \max M_{\mathcal{Q}_{CDTHRP-p-yz}}^{\delta(j)}(\tilde{x}) \cdot e^{i2\pi \left( M_{\mathcal{Q}_{CDTHIP-p-yz}}^{\delta(j)}(\tilde{x}) \right)}, \max M_{\mathcal{Q}_{CDTHRP-S-yz}}^{\delta(j)}(\tilde{x}) \cdot e^{i2\pi \left( M_{\mathcal{Q}_{CDTHIP-S-yz}}^{\delta(j)}(\tilde{x}) \right)} \right), \right. \right. \\ \left. \left. \left( \min N_{\mathcal{Q}_{CDTHRP-p-yz}}^{\delta(k)}(\tilde{x}) \cdot e^{i2\pi \left( N_{\mathcal{Q}_{CDTHIP-p-yz}}^{\delta(k)}(\tilde{x}) \right)}, \min N_{\mathcal{Q}_{CDTHRP-S-yz}}^{\delta(k)}(\tilde{x}) \cdot e^{i2\pi \left( N_{\mathcal{Q}_{CDTHIP-S-yz}}^{\delta(k)}(\tilde{x}) \right)} \right) \right) \right), \tag{30}$$

$$R^- = (r_{i1}^-, r_{i2}^-, r_{i3}^-, \dots, r_{in}^-),$$

$$\cdot r_{ij}^- \left( \left( \left( \min M_{\mathcal{Q}_{CDTHRP-p-yz}}^{\delta(j)}(\tilde{x}) \cdot e^{i2\pi \left( M_{\mathcal{Q}_{CDTHIP-p-yz}}^{\delta(j)}(\tilde{x}) \right)}, \min M_{\mathcal{Q}_{CDTHRP-S-yz}}^{\delta(j)}(\tilde{x}) \cdot e^{i2\pi \left( M_{\mathcal{Q}_{CDTHIP-S-yz}}^{\delta(j)}(\tilde{x}) \right)} \right), \right. \right. \\ \left. \left. \left( \max N_{\mathcal{Q}_{CDTHRP-p-yz}}^{\delta(k)}(\tilde{x}) \cdot e^{i2\pi \left( N_{\mathcal{Q}_{CDTHIP-p-yz}}^{\delta(k)}(\tilde{x}) \right)}, \max N_{\mathcal{Q}_{CDTHRP-S-yz}}^{\delta(k)}(\tilde{x}) \cdot e^{i2\pi \left( N_{\mathcal{Q}_{CDTHIP-S-yz}}^{\delta(k)}(\tilde{x}) \right)} \right) \right) \right). \tag{31}$$

Step 4: by using (21), we examine the CDT-2HF PIS, and we have

$$\zeta_{\text{CDTHF-wcc-y}}(r_{yz}, R^+). \tag{32}$$

We also examine complex CDT-2HF NIS by using (21), and we have

$$\zeta_{\text{CDTHF-wcc-y}}(r_{yz}, R^-). \tag{33}$$

Step 5: by using (34), we examine the closeness of each of the alternatives, and we have

$$\mathcal{P}_i = \frac{\zeta_{\text{CDTHF-wcc-y}}(r_{yz}, R^+)}{\zeta_{\text{CDTHF-wcc-y}}(r_{yz}, R^+) + \zeta_{\text{CDTHF-wcc-y}}(r_{yz}, R^-)} \rightarrow . \tag{34}$$

Step 6: we rank all alternatives and examine the best optimal one.

Step 7: the end.

*Example 2.* The company of intranet is usually attacked by malicious intrusions. To enhance the security of the intranet, the company plans to purchase the firewall production and put it between the intranet and extranet for blocking illegal access. Basically, there are four types of firewall productions given to be considered, which are detailed as follows:  $C = \{c_1, c_2, c_3, c_4\}$ . When choosing the firewall production, the company pays attention to the factors detailed as follows:

$u_1 \rightarrow$  the promotion,  $u_2 \rightarrow$  configuration simplicity,  $u_3 \rightarrow$  Security level, and  $u_4 \rightarrow$  maintenance sever level, the weight vector of which is denoted and defined by  $\Omega_j \in [0, 1]$ ,  $\sum_{j=1}^n \Omega_j = 1$ , and  $\Omega = (\Omega_1, \Omega_2, \Omega_3, \Omega_4)^T$ . To examine the firewall production with respect to their factors, we consider the following matrix, and the decision matrix is given in the form of Table 1.

The steps of the proposed complex dual type-2 hesitant fuzzy TOPSIS method are as follows:

- (i) Step 1: some decision-making problems also contain benefits and cost types of informations, so for this, we normalized the decision matrix by considering (29), but the considered information cannot be normalized. So, we have used the information available in Table 1 and go to step 2.
- (ii) Step 2: by using (26), we examine the weight vector of the attributes.

$$\Omega = \{0.342, 0.155, 0.067, 0.2, 0.236\}^T. \tag{35}$$

- (iii) Step 3: by using (30) and (31), we examine the PIS and NIS among the alternatives.

$$R^+ = \left[ \left[ \left\{ \left( \begin{matrix} 0.62e^{i2\pi(0.62)} \\ 0.12e^{i2\pi(0.12)} \end{matrix} \right), \left( \begin{matrix} 0.22e^{i2\pi(0.22)} \\ 0.24e^{i2\pi(0.24)} \end{matrix} \right), \left( \begin{matrix} 0.21e^{i2\pi(0.21)} \\ 0.25e^{i2\pi(0.25)} \end{matrix} \right) \right\}, \left[ \left\{ \left( \begin{matrix} 0.5e^{i2\pi(0.5)} \\ 0.04e^{i2\pi(0.04)} \end{matrix} \right), \left( \begin{matrix} 0.31e^{i2\pi(0.31)} \\ 0.33e^{i2\pi(0.33)} \end{matrix} \right), \left( \begin{matrix} 0.04e^{i2\pi(0.04)} \\ 0.43e^{i2\pi(0.43)} \end{matrix} \right) \right\}, \right. \right. \\ \left. \left. \left\{ \left( \begin{matrix} 0.23e^{i2\pi(0.23)} \\ 0.23e^{i2\pi(0.23)} \end{matrix} \right), (0.1e^{i2\pi(0.1)}, 0.23e^{i2\pi(0.23)}) \right\}, \left\{ \left( \begin{matrix} 0.12e^{i2\pi(0.12)} \\ 0.1e^{i2\pi(0.1)} \end{matrix} \right), (0.01e^{i2\pi(0.01)}, 0.01e^{i2\pi(0.01)}) \right\} \right. \right. \\ \left. \left. \left[ \left\{ \left( \begin{matrix} 0.32e^{i2\pi(0.32)} \\ 0.33e^{i2\pi(0.33)} \end{matrix} \right), \left( \begin{matrix} 0.33e^{i2\pi(0.33)} \\ 0.43e^{i2\pi(0.34)} \end{matrix} \right), \left( \begin{matrix} 0.4e^{i2\pi(0.43)} \\ 0.5e^{i2\pi(0.5)} \end{matrix} \right) \right\}, \left[ \left\{ \left( \begin{matrix} 0.31e^{i2\pi(0.31)} \\ 0.32e^{i2\pi(0.32)} \end{matrix} \right), \left( \begin{matrix} 0.32e^{i2\pi(0.32)} \\ 0.33e^{i2\pi(0.33)} \end{matrix} \right), \left( \begin{matrix} 0.33e^{i2\pi(0.33)} \\ 0.4e^{i2\pi(0.4)} \end{matrix} \right) \right\}, \right. \right. \\ \left. \left. \left\{ \left( \begin{matrix} 0.1e^{i2\pi(0.1)} \\ 0.12e^{i2\pi(0.12)} \end{matrix} \right), (0.2e^{i2\pi(0.2)}, 0.3e^{i2\pi(0.3)}) \right\}, \left\{ \left( \begin{matrix} 0.24e^{i2\pi(0.6)} \\ 0.23e^{i2\pi(0.23)} \end{matrix} \right), (0.2e^{i2\pi(0.2)}, 0.1e^{i2\pi(0.1)}) \right\} \right. \right. \\ \left. \left. \left[ \left\{ \left( \begin{matrix} 0.36e^{i2\pi(0.36)} \\ 0.12e^{i2\pi(0.12)} \end{matrix} \right), \left( \begin{matrix} 0.12e^{i2\pi(0.12)} \\ 0.14e^{i2\pi(0.24)} \end{matrix} \right), \left( \begin{matrix} 0.1e^{i2\pi(0.1)} \\ 0.15e^{i2\pi(0.15)} \end{matrix} \right) \right\}, \left[ \left\{ \left( \begin{matrix} 0.15e^{i2\pi(0.15)} \\ 0.01e^{i2\pi(0.01)} \end{matrix} \right), \left( \begin{matrix} 0.31e^{i2\pi(0.31)} \\ 0.13e^{i2\pi(0.13)} \end{matrix} \right), \left( \begin{matrix} 0.04e^{i2\pi(0.04)} \\ 0.14e^{i2\pi(0.14)} \end{matrix} \right) \right\}, \right. \right. \\ \left. \left. \left\{ \left( \begin{matrix} 0.33e^{i2\pi(0.33)} \\ 0.33e^{i2\pi(0.33)} \end{matrix} \right), (0.34e^{i2\pi(0.34)}, 0.31e^{i2\pi(0.31)}) \right\}, \left\{ \left( \begin{matrix} 0.32e^{i2\pi(0.32)} \\ 0.31e^{i2\pi(0.31)} \end{matrix} \right), (0.04e^{i2\pi(0.04)}, 0.4e^{i2\pi(0.4)}) \right\} \right. \right. \\ \left. \left. \left[ \left\{ \left( \begin{matrix} 0.12e^{i2\pi(0.12)} \\ 0.31e^{i2\pi(0.31)} \end{matrix} \right), \left( \begin{matrix} 0.13e^{i2\pi(0.13)} \\ 0.14e^{i2\pi(0.14)} \end{matrix} \right), \left( \begin{matrix} 0.14e^{i2\pi(0.14)} \\ 0.25e^{i2\pi(0.25)} \end{matrix} \right) \right\}, \left[ \left\{ \left( \begin{matrix} 0.1e^{i2\pi(0.1)} \\ 0.2e^{i2\pi(0.2)} \end{matrix} \right), \left( \begin{matrix} 0.2e^{i2\pi(0.2)} \\ 0.3e^{i2\pi(0.3)} \end{matrix} \right), \left( \begin{matrix} 0.3e^{i2\pi(0.3)} \\ 0.4e^{i2\pi(0.4)} \end{matrix} \right) \right\}, \right. \right. \\ \left. \left. \left\{ \left( \begin{matrix} 0.31e^{i2\pi(0.31)} \\ 0.32e^{i2\pi(0.32)} \end{matrix} \right), (0.32e^{i2\pi(0.2)}, 0.33e^{i2\pi(0.33)}) \right\}, \left\{ \left( \begin{matrix} 0.41e^{i2\pi(0.41)} \\ 0.33e^{i2\pi(0.33)} \end{matrix} \right), (0.32e^{i2\pi(0.32)}, 0.31e^{i2\pi(0.31)}) \right\} \right. \right. \end{matrix} \right] \right] \tag{36}$$



TABLE 1: Original decision matrix, all items of which are in the form of complex numbers.

Symbols	$u_1$	$u_2$
$c_1$	$\left\{ \left\{ \left( 0.6e^{i2\pi(0.6)}, 0.1e^{i2\pi(0.1)} \right), \left( 0.2e^{i2\pi(0.2)}, 0.4e^{i2\pi(0.4)} \right), \left( 0.1e^{i2\pi(0.1)}, 0.5e^{i2\pi(0.5)} \right) \right\}, \left\{ \left( 0.3e^{i2\pi(0.3)}, 0.3e^{i2\pi(0.3)} \right), (0.1e^{i2\pi(0.1)}, 0.3e^{i2\pi(0.3)}) \right\} \right\}$	$\left\{ \left\{ \left( 0.5e^{i2\pi(0.5)}, 0.01e^{i2\pi(0.01)} \right), \left( 0.1e^{i2\pi(0.1)}, 0.3e^{i2\pi(0.3)} \right), \left( 0.01e^{i2\pi(0.01)}, 0.4e^{i2\pi(0.4)} \right) \right\}, \left\{ \left( 0.2e^{i2\pi(0.2)}, 0.1e^{i2\pi(0.1)} \right), (0.01e^{i2\pi(0.01)}, 0.03e^{i2\pi(0.03)}) \right\} \right\}$
$c_2$	$\left\{ \left\{ \left( 0.61e^{i2\pi(0.61)}, 0.11e^{i2\pi(0.11)} \right), \left( 0.12e^{i2\pi(0.12)}, 0.14e^{i2\pi(0.14)} \right), \left( 0.11e^{i2\pi(0.11)}, 0.15e^{i2\pi(0.15)} \right) \right\}, \left\{ \left( 0.13e^{i2\pi(0.13)}, 0.31e^{i2\pi(0.13)} \right), (0.11e^{i2\pi(0.11)}, 0.13e^{i2\pi(0.13)}) \right\} \right\}$	$\left\{ \left\{ \left( 0.15e^{i2\pi(0.15)}, 0.02e^{i2\pi(0.02)} \right), \left( 0.11e^{i2\pi(0.11)}, 0.13e^{i2\pi(0.13)} \right), \left( 0.02e^{i2\pi(0.02)}, 0.14e^{i2\pi(0.14)} \right) \right\}, \left\{ \left( 0.12e^{i2\pi(0.12)}, 0.11e^{i2\pi(0.11)} \right), (0.02e^{i2\pi(0.02)}, 0.01e^{i2\pi(0.01)}) \right\} \right\}$
$c_3$	$\left\{ \left\{ \left( 0.62e^{i2\pi(0.62)}, 0.12e^{i2\pi(0.12)} \right), \left( 0.22e^{i2\pi(0.22)}, 0.24e^{i2\pi(0.24)} \right), \left( 0.21e^{i2\pi(0.21)}, 0.25e^{i2\pi(0.25)} \right) \right\}, \left\{ \left( 0.23e^{i2\pi(0.23)}, 0.23e^{i2\pi(0.23)} \right), (0.21e^{i2\pi(0.21)}, 0.23e^{i2\pi(0.23)}) \right\} \right\}$	$\left\{ \left\{ \left( 0.25e^{i2\pi(0.25)}, 0.03e^{i2\pi(0.03)} \right), \left( 0.21e^{i2\pi(0.21)}, 0.23e^{i2\pi(0.23)} \right), \left( 0.03e^{i2\pi(0.03)}, 0.24e^{i2\pi(0.24)} \right) \right\}, \left\{ \left( 0.22e^{i2\pi(0.22)}, 0.21e^{i2\pi(0.21)} \right), (0.02e^{i2\pi(0.02)}, 0.02e^{i2\pi(0.02)}) \right\} \right\}$
$c_4$	$\left\{ \left\{ \left( 0.36e^{i2\pi(0.36)}, 0.31e^{i2\pi(0.31)} \right), \left( 0.32e^{i2\pi(0.32)}, 0.34e^{i2\pi(0.34)} \right), \left( 0.31e^{i2\pi(0.31)}, 0.35e^{i2\pi(0.35)} \right) \right\}, \left\{ \left( 0.33e^{i2\pi(0.33)}, 0.33e^{i2\pi(0.33)} \right), (0.31e^{i2\pi(0.31)}, 0.33e^{i2\pi(0.33)}) \right\} \right\}$	$\left\{ \left\{ \left( 0.35e^{i2\pi(0.35)}, 0.04e^{i2\pi(0.04)} \right), \left( 0.31e^{i2\pi(0.31)}, 0.33e^{i2\pi(0.33)} \right), \left( 0.04e^{i2\pi(0.04)}, 0.43e^{i2\pi(0.43)} \right) \right\}, \left\{ \left( 0.32e^{i2\pi(0.32)}, 0.31e^{i2\pi(0.31)} \right), (0.04e^{i2\pi(0.04)}, 0.04e^{i2\pi(0.04)}) \right\} \right\}$
$c_1$	$\left\{ \left\{ \left( 0.2e^{i2\pi(0.2)}, 0.3e^{i2\pi(0.3)} \right), \left( 0.3e^{i2\pi(0.3)}, 0.4e^{i2\pi(0.4)} \right), \left( 0.4e^{i2\pi(0.4)}, 0.5e^{i2\pi(0.5)} \right) \right\}, \left\{ \left( 0.01e^{i2\pi(0.01)}, 0.2e^{i2\pi(0.2)} \right), (0.2e^{i2\pi(0.2)}, 0.3e^{i2\pi(0.3)}) \right\} \right\}$	$\left\{ \left\{ \left( 0.1e^{i2\pi(0.1)}, 0.2e^{i2\pi(0.6)} \right), \left( 0.2e^{i2\pi(0.6)}, 0.3e^{i2\pi(0.3)} \right), \left( 0.3e^{i2\pi(0.3)}, 0.4e^{i2\pi(0.4)} \right) \right\}, \left\{ \left( 0.4e^{i2\pi(0.4)}, 0.3e^{i2\pi(0.3)} \right), (0.2e^{i2\pi(0.2)}, 0.1e^{i2\pi(0.1)}) \right\} \right\}$
$c_2$	$\left\{ \left\{ \left( 0.12e^{i2\pi(0.12)}, 0.31e^{i2\pi(0.31)} \right), \left( 0.13e^{i2\pi(0.13)}, 0.14e^{i2\pi(0.14)} \right), \left( 0.14e^{i2\pi(0.14)}, 0.51e^{i2\pi(0.51)} \right) \right\}, \left\{ \left( 0.11e^{i2\pi(0.11)}, 0.12e^{i2\pi(0.12)} \right), (0.21e^{i2\pi(0.12)}, 0.31e^{i2\pi(0.31)}) \right\} \right\}$	$\left\{ \left\{ \left( 0.11e^{i2\pi(0.11)}, 0.21e^{i2\pi(0.21)} \right), \left( 0.21e^{i2\pi(0.21)}, 0.31e^{i2\pi(0.31)} \right), \left( 0.31e^{i2\pi(0.31)}, 0.41e^{i2\pi(0.41)} \right) \right\}, \left\{ \left( 0.41e^{i2\pi(0.41)}, 0.31e^{i2\pi(0.31)} \right), (0.21e^{i2\pi(0.21)}, 0.11e^{i2\pi(0.11)}) \right\} \right\}$
$c_3$	$\left\{ \left\{ \left( 0.22e^{i2\pi(0.22)}, 0.32e^{i2\pi(0.23)} \right), \left( 0.23e^{i2\pi(0.23)}, 0.24e^{i2\pi(0.24)} \right), \left( 0.24e^{i2\pi(0.24)}, 0.25e^{i2\pi(0.25)} \right) \right\}, \left\{ \left( 0.21e^{i2\pi(0.21)}, 0.22e^{i2\pi(0.22)} \right), (0.22e^{i2\pi(0.22)}, 0.23e^{i2\pi(0.23)}) \right\} \right\}$	$\left\{ \left\{ \left( 0.21e^{i2\pi(0.21)}, 0.22e^{i2\pi(0.22)} \right), \left( 0.22e^{i2\pi(0.22)}, 0.23e^{i2\pi(0.23)} \right), \left( 0.23e^{i2\pi(0.23)}, 0.24e^{i2\pi(0.24)} \right) \right\}, \left\{ \left( 0.24e^{i2\pi(0.24)}, 0.23e^{i2\pi(0.23)} \right), (0.22e^{i2\pi(0.22)}, 0.12e^{i2\pi(0.12)}) \right\} \right\}$
$c_4$	$\left\{ \left\{ \left( 0.32e^{i2\pi(0.32)}, 0.33e^{i2\pi(0.33)} \right), \left( 0.33e^{i2\pi(0.33)}, 0.43e^{i2\pi(0.43)} \right), \left( 0.34e^{i2\pi(0.43)}, 0.35e^{i2\pi(0.35)} \right) \right\}, \left\{ \left( 0.31e^{i2\pi(0.31)}, 0.32e^{i2\pi(0.32)} \right), (0.32e^{i2\pi(0.32)}, 0.33e^{i2\pi(0.33)}) \right\} \right\}$	$\left\{ \left\{ \left( 0.31e^{i2\pi(0.31)}, 0.32e^{i2\pi(0.32)} \right), \left( 0.32e^{i2\pi(0.32)}, 0.33e^{i2\pi(0.33)} \right), \left( 0.33e^{i2\pi(0.33)}, 0.34e^{i2\pi(0.34)} \right) \right\}, \left\{ \left( 0.34e^{i2\pi(0.34)}, 0.33e^{i2\pi(0.33)} \right), (0.32e^{i2\pi(0.32)}, 0.31e^{i2\pi(0.31)}) \right\} \right\}$

(iv) Step 4: by using (22), we examine the complex dual type-2 hesitant fuzzy PIS, and we have

$$\begin{aligned}
 K_1(r_{1j}, R^+) &= 0.7782, \\
 K_2(r_{2j}, R^+) &= 0.6645, \\
 K_3(r_{3j}, R^+) &= 0.6612, \\
 K_4(r_{4j}, R^+) &= 0.5534.
 \end{aligned}
 \tag{37}$$

We also examine complex dual type-2 hesitant NIS by using equation (34), and we have

$$\begin{aligned}
 K_1(r_{1j}, R^-) &= 0.7146, \\
 K_2(r_{2j}, R^-) &= 0.5627, \\
 K_3(r_{3j}, R^-) &= 0.5537, \\
 K_4(r_{4j}, R^-) &= 0.5718.
 \end{aligned}
 \tag{38}$$

(v) Step 5: by using (32), we examine the closeness of each of the alternatives, and we have

$$\begin{aligned}
 \mathcal{P}_1 &= 0.5213, \\
 \mathcal{P}_2 &= 0.5415, \\
 \mathcal{P}_3 &= 0.5442, \\
 \mathcal{P}_4 &= 0.4918.
 \end{aligned}
 \tag{39}$$

(v) Step 6: we rank all alternatives and examine the best optimal one.

$$\mathcal{P}_3 \geq \mathcal{P}_2 \geq \mathcal{P}_1 \geq \mathcal{P}_4.
 \tag{40}$$

(vii) Step 7: the end. The comparative analysis of the explored measure with existing measures [30] is discussed in the form of Table 2.

6.3. *Medical Diagnosis.* In this study, we explored the idea of the algorithm which is taken from [31] which is very effective and meaningful for explored works. The new algorithm utilizes the complex dual type-2 hesitant fuzzy similarity and entropy measures which obtain excellent results in application.

Problem statement: suppose that five patients, namely, Lil, Jones, Deby, Ramot, and Inas, visit a given laboratory for medical diagnosis. They are observed to have the following symptoms: heart pain, temperature, cough, liver pain, and kidney pain. That is, the set of patients QCDTH is as follows:

$$\mathcal{Q}_{CDTH} = \{\text{viz, Lil, Jones, Deby, Ramot, Inas}\},
 \tag{41}$$

and the set of symptoms X is as follows:

$$X = \{x_1 \text{ (heart pain), } x_2 \text{ (temperature), } x_3 \text{ (cough), } x_4 \text{ (liver pain), } x_5 \text{ (kidney pain)}\}.
 \tag{42}$$

Then we will find which patient has which kind of disease. The information related to this problem is given in Example 3, which is discussed below.

*Example 3.* Suppose a set of diagnoses  $\mathcal{Q}_{CDTH} = \{ \mathcal{Q}_{CDTH-1} \text{ (Heart problem), } \mathcal{Q}_{CDTH-2} \text{ (Fever), } \mathcal{Q}_{CDTH-3} \text{ (Flu), } \mathcal{Q}_{CDTH-4} \text{ (Liver problem), } \mathcal{Q}_{CDTH-5} \text{ (Kidney problem)} \}$

and a set of symptoms  $X = \{ x_1 \text{ (heart problem), } x_2 \text{ (temperature), } x_3 \text{ (cough), } x_4 \text{ (liver pain), } x_5 \text{ (kidney pain)} \}$ . Suppose a sick person according to all symptoms is represented by the CDT-2HFSs given as follows:

$$\mathcal{Q}_{CDTH} = \left\{ \begin{aligned} & \left( 0.91e^{i2\pi(0.91)}, 0.92e^{i2\pi(0.91)} \right), \left( 0.93e^{i2\pi(0.93)}, 0.94e^{i2\pi(0.94)} \right), \left( 0.95e^{i2\pi(0.95)}, 0.86e^{i2\pi(0.85)} \right), \\ & \left( 0.002e^{i2\pi(0.003)}, 0.004e^{i2\pi(0.003)} \right), \left( 0.003e^{i2\pi(0.003)}, 0.001e^{i2\pi(0.001)} \right) \end{aligned} \right\}.
 \tag{43}$$

All diagnoses  $\mathcal{Q}_{CDTH-i} (i = 1, 2, 3, 4, 5)$  that can be represented as CDT-2HFSs according to all symptoms are given below:

$$\begin{aligned}
 \mathcal{Q}_{CDTH-1} \text{ (heart problem)} &= \left\{ \left\{ \left( 0.81e^{i2\pi(0.71)}, 0.76e^{i2\pi(0.57)} \right), \left( 0.82e^{i2\pi(0.72)}, 0.77e^{i2\pi(0.57)} \right), \left( 0.85e^{i2\pi(0.75)}, 0.8e^{i2\pi(0.61)} \right) \right\}, \right. \\ & \quad \left. \left\{ \left( 0.07e^{i2\pi(0.06)}, 0.06e^{i2\pi(0.07)} \right), \left( 0.02e^{i2\pi(0.02)}, 0.07e^{i2\pi(0.08)} \right) \right\} \right\}, \\
 \mathcal{Q}_{CDTH-1} \text{ (fever)} &= \left\{ \left\{ \left( 0.82e^{i2\pi(0.72)}, 0.77e^{i2\pi(0.58)} \right), \left( 0.83e^{i2\pi(0.73)}, 0.78e^{i2\pi(0.59)} \right), \left( 0.86e^{i2\pi(0.76)}, 0.81e^{i2\pi(0.62)} \right) \right\}, \right. \\ & \quad \left. \left\{ \left( 0.08e^{i2\pi(0.07)}, 0.07e^{i2\pi(0.08)} \right), \left( 0.03e^{i2\pi(0.03)}, 0.08e^{i2\pi(0.05)} \right) \right\} \right\}, \\
 \mathcal{Q}_{CDTH-4} \text{ (flu)} &= \left\{ \left\{ \left( 0.83e^{i2\pi(0.73)}, 0.78e^{i2\pi(0.59)} \right), \left( 0.837e^{i2\pi(0.74)}, 0.79e^{i2\pi(0.6)} \right), \left( 0.87e^{i2\pi(0.77)}, 0.82e^{i2\pi(0.63)} \right) \right\}, \right. \\ & \quad \left. \left\{ \left( 0.01e^{i2\pi(0.01)}, 0.03e^{i2\pi(0.06)} \right), \left( 0.04e^{i2\pi(0.01)}, 0.00e^{i2\pi(0.00)} \right) \right\} \right\}, \\
 \mathcal{Q}_{CDTH-4} \text{ (liver problem)} &= \left\{ \left\{ \left( 0.83e^{i2\pi(0.73)}, 0.78e^{i2\pi(0.59)} \right), \left( 0.84e^{i2\pi(0.74)}, 0.79e^{i2\pi(0.6)} \right), \left( 0.87e^{i2\pi(0.77)}, 0.82e^{i2\pi(0.63)} \right) \right\}, \right. \\ & \quad \left. \left\{ \left( 0.03e^{i2\pi(0.04)}, 0.05e^{i2\pi(0.06)} \right), \left( 0.06e^{i2\pi(0.03)}, 0.02e^{i2\pi(0.08)} \right) \right\} \right\}, \\
 \mathcal{Q}_{CDTH-4} \text{ (kidney problem)} &= \left\{ \left\{ \left( 0.84e^{i2\pi(0.74)}, 0.77e^{i2\pi(0.6)} \right), \left( 0.84e^{i2\pi(0.75)}, 0.8e^{i2\pi(0.61)} \right), \left( 0.88e^{i2\pi(0.78)}, 0.83e^{i2\pi(0.64)} \right) \right\}, \right. \\ & \quad \left. \left\{ \left( 0.04e^{i2\pi(0.03)}, 0.05e^{i2\pi(0.05)} \right), \left( 0.06e^{i2\pi(0.02)}, 0.03e^{i2\pi(0.07)} \right) \right\} \right\}.
 \end{aligned}
 \tag{44}$$

TABLE 2: Comparison of the explored work with existing work.

Method	Score values	Ranking
Karaaslan et al. [30]	$\mathcal{P}_1 = 0.5249, \mathcal{P}_2 = 0.5417, \mathcal{P}_3 = 0.5439, \mathcal{P}_4 = 0.4923$	$\mathcal{P}_1 \geq \mathcal{P}_2 \geq \mathcal{P}_1 \geq \mathcal{P}_4$
Explored approach	$\mathcal{P}_1 = 0.5213, \mathcal{P}_2 = 0.5415, \mathcal{P}_3 = 0.5442, \mathcal{P}_4 = 0.4918$	$\mathcal{P}_3 \geq \mathcal{P}_2 \geq \mathcal{P}_1 \geq \mathcal{P}_4$

The geometrical representation of the explored measures, which are discussed in Table 2, is described with the help of Figure 1.

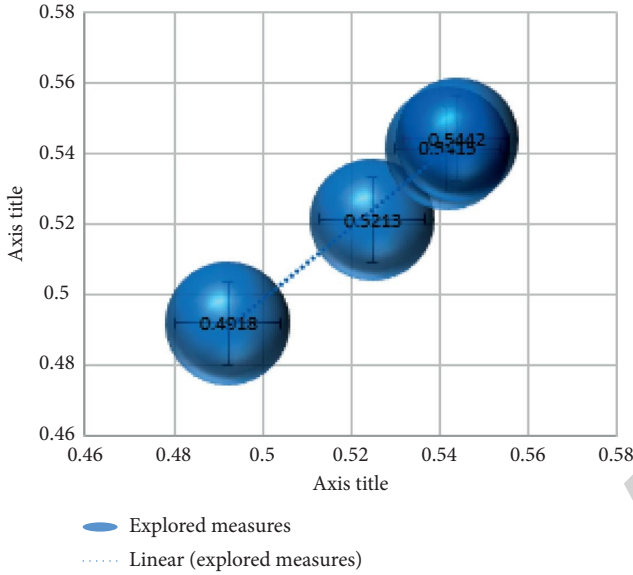


FIGURE 1: Graphical representation for the information of Table 2.

The aim of this work is to examine the best alternative from the family of alternatives by using the measures. The information of the resultant values of the explored measure and some existing measures is stated in Table 3. The,  $\zeta_{CDTH-mcc-i}(Q_{CDTH-1}, Q_{CDTH-2}) = P_i, i = 1, 2, 3, 4, 5,$  are follow as.

6.4. *Pattern Recognition.* The instruments of likeness measures have applications in design classification. In such a marvel, the class of an obscure example or item is discovered utilizing some likeness estimating devices and a few inclinations of leaders. In this segment, the likeness estimates that grew so far in Section 3 are applied to an example acknowledgment (building design acknowledgment) issue, where the class of an obscure structure material should have been assessed. The outcomes got utilizing the similitude proportions of CHFSSs are then examined for portrayal of the benefits of proposed work and the constraints of existing work. To clarify the marvel, an illustrative model adjusted from [31] is discussed.

To evaluate the proficiency of the explored measures, we adopt the pattern recognition model form [31]. The purpose of this application is to find the reliability and skill of the presented measures; we solve a numerical example that contains the CDT-2HFNs and utilized it in the environment of pattern recognition.

*Example 4.* We consider five knowns with their class labels being represented as follows:  $P_{CQ-1}, P_{CQ-2}, P_{CQ-3}, P_{CQ-4},$  and  $P_{CQ-5}$  and  $Q_{CDTH-1}, Q_{CDTH-2}, Q_{CDTH-3}, Q_{CDTH-4},$  and  $Q_{CDTH-5}$ . The information of the above patterns is in the form of CDT-2HFNs for universal set  $X_{UNI} = \{x_1, x_2, x_3, x_4, x_5\}$ , which is stated as follows:

$$\begin{aligned}
 Q_{CDTH-1} &= \left\{ \left\{ (0.8e^{i2\pi(0.7)}, 0.75e^{i2\pi(0.56)}), (0.81e^{i2\pi(0.71)}, 0.76e^{i2\pi(0.57)}), (0.84e^{i2\pi(0.74)}, 0.79e^{i2\pi(0.60)}) \right\}, \right. \\
 &\quad \left. \left\{ (0.08e^{i2\pi(0.07)}, 0.07e^{i2\pi(0.08)}), (0.03e^{i2\pi(0.03)}, 0.08e^{i2\pi(0.09)}) \right\} \right\}, \\
 Q_{CDTH-2} &= \left\{ \left\{ (0.81e^{i2\pi(0.71)}, 0.76e^{i2\pi(0.57)}), (0.82e^{i2\pi(0.72)}, 0.77e^{i2\pi(0.58)}), (0.85e^{i2\pi(0.75)}, 0.8e^{i2\pi(0.61)}) \right\}, \right. \\
 &\quad \left. \left\{ (0.09e^{i2\pi(0.08)}, 0.08e^{i2\pi(0.09)}), (0.04e^{i2\pi(0.04)}, 0.09e^{i2\pi(0.06)}) \right\} \right\}, \\
 Q_{CDTH-3} &= \left\{ \left\{ (0.82e^{i2\pi(0.72)}, 0.77e^{i2\pi(0.58)}), (0.83e^{i2\pi(0.73)}, 0.78e^{i2\pi(0.59)}), (0.86e^{i2\pi(0.76)}, 0.81e^{i2\pi(0.62)}) \right\}, \right. \\
 &\quad \left. \left\{ (0.02e^{i2\pi(0.03)}, 0.04e^{i2\pi(0.05)}), (0.05e^{i2\pi(0.02)}, 0.01e^{i2\pi(0.07)}) \right\} \right\}, \\
 Q_{CDTH-4} &= \left\{ \left\{ (0.83e^{i2\pi(0.73)}, 0.78e^{i2\pi(0.59)}), (0.84e^{i2\pi(0.74)}, 0.79e^{i2\pi(0.6)}), (0.87e^{i2\pi(0.77)}, 0.82e^{i2\pi(0.63)}) \right\}, \right. \\
 &\quad \left. \left\{ (0.03e^{i2\pi(0.03)}, 0.05e^{i2\pi(0.06)}), (0.06e^{i2\pi(0.03)}, 0.02e^{i2\pi(0.08)}) \right\} \right\}, \\
 Q_{CDTH-5} &= \left\{ \left\{ (0.84e^{i2\pi(0.74)}, 0.77e^{i2\pi(0.6)}), (0.84e^{i2\pi(0.75)}, 0.8e^{i2\pi(0.61)}), (0.88e^{i2\pi(0.78)}, 0.83e^{i2\pi(0.64)}) \right\}, \right. \\
 &\quad \left. \left\{ (0.04e^{i2\pi(0.03)}, 0.05e^{i2\pi(0.05)}), (0.06e^{i2\pi(0.02)}, 0.03e^{i2\pi(0.07)}) \right\} \right\}.
 \end{aligned} \tag{45}$$

TABLE 3: Comparison of the explored work with existing work.

Method	Measures	Ranking
Karaaslan et al. [30]	$\mathcal{P}_1 = 0.803, \mathcal{P}_2 = 0.853, \mathcal{P}_3 = 0.889, \mathcal{P}_4 = 0.902, \mathcal{P}_5 = 0.943$	$\mathcal{P}_5 \geq \mathcal{P}_4 \geq \mathcal{P}_3 \geq \mathcal{P}_2 \geq \mathcal{P}_1$
Explored approach, equation (18)	$\mathcal{P}_1 = 0.834, \mathcal{P}_2 = 0.851, \mathcal{P}_3 = 0.888, \mathcal{P}_4 = 0.916, \mathcal{P}_5 = 0.951$	$\mathcal{P}_5 \geq \mathcal{P}_4 \geq \mathcal{P}_3 \geq \mathcal{P}_2 \geq \mathcal{P}_1$

The geometrical representation of the explored measures, which is discussed in Table 3, is described with the help of Figure 2.

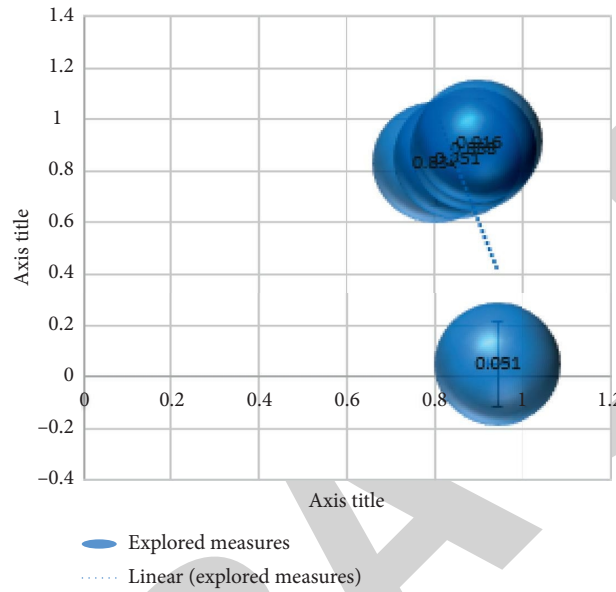


FIGURE 2: Graphical representation for the information of Table 3.

Their unknown pattern is stated as follows:

$$Q_{CDTH} = \left\{ \begin{array}{l} (0.9e^{i2\pi(0.9)}, 0.91e^{i2\pi(0.91)}), (0.92e^{i2\pi(0.92)}, 0.93e^{i2\pi(0.93)}), (0.94e^{i2\pi(0.94)}, 0.85e^{i2\pi(0.85)}) \\ (0.02e^{i2\pi(0.03)}, 0.04e^{i2\pi(0.03)}), (0.03e^{i2\pi(0.03)}, 0.01e^{i2\pi(0.01)}) \end{array} \right\}. \quad (46)$$

The aim of this work is to examine the best alternative from the family of alternatives by using the measures. The information of the resultant values of the explored measure and some existing measures is stated in Table 4. The,  $\zeta_{CDTHF-mcc-i}(Q_{CCDTH-1}, Q_{CDTH-2}) = \mathcal{P}_i, i = 1, 2, 3, 4, 5,$  are follow as.

The geometrical representation of the explored measures, which is discussed in Table 4, is described with the help of Figure 3.

6.5. Clustering Algorithm Based on CDT-2HFSs. The aim of this study is to present the clustering algorithm based on the novel approach of CDT-2HFSs to examine the reliability and proficiency of the explored approach. For this, we choose the set of alternatives and their attributes with weight vectors, whose expressions are in the form of  $A_{AL} = \{A_{AL-1}, A_{AL-2}, \dots, A_{AL-m}\}, C_{AT} = \{C_{AT-1}, C_{AT-2}, \dots, C_{AT-n}\},$  and  $\Omega = (\Omega_1, \Omega_2, \dots, \Omega_n)^T$  and  $\Omega_j \in [0, 1], \sum_{j=1}^n \Omega_j = 1.$  The

technique of the clustering algorithm is summarized as follows:

Step 1: construct the decision matrix, all entities of which are in the form of CDT-2HFSs.

Step 2: construct the correlation matrix by using (12). The correlation matrix is expressed by  $\zeta_{CDTHF-cc} = \zeta_{CDTHF-cc-yz}(Q_{CDTH-y}, Q_{CDTH-z})$  where  $(\zeta_{CDTHF-cc})_{m \times m} = C.$

Step 3: we checked whether the correlation matrix  $C$  satisfies  $C^2 \subseteq C,$  where  $C = C \circ C = (\zeta_{CDTHF-cc-yz})_{m \times m}, \zeta_{CDTHF-cc-yz} = \max_z \{ \min \{ \zeta_{CDTHF-cc-yx}, \zeta_{CDTHF-cc-xz} \} \}.$  If  $C$  does not satisfy condition  $C^2 \subseteq C,$  then the equivalent correlation matrix  $C^{2^k}$  will be formed:  $C \rightarrow C^2 \rightarrow C^4 \rightarrow \dots \rightarrow C^{2^k} \rightarrow \dots$  until  $C^{2^{k+1}}.$

Step 4: furthermore, we examine the  $\lambda$ -cutting matrix by using

TABLE 4: Comparison of the explored work with existing work.

Method	Measures	Ranking
Karaaslan and Ozlu [30]	$\mathcal{P}_1 = 0.814, \mathcal{P}_2 = 0.862, \mathcal{P}_3 = 0.892, \mathcal{P}_4 = 0.911, \mathcal{P}_5 = 0.951$	$\mathcal{P}_5 \geq \mathcal{P}_4 \geq \mathcal{P}_3 \geq \mathcal{P}_2 \geq \mathcal{P}_1$
Explored approach equation (18)	$\mathcal{P}_1 = 0.845, \mathcal{P}_2 = 0.866, \mathcal{P}_3 = 0.897, \mathcal{P}_4 = 0.913, \mathcal{P}_5 = 0.947$	$\mathcal{P}_5 \geq \mathcal{P}_4 \geq \mathcal{P}_3 \geq \mathcal{P}_2 \geq \mathcal{P}_1$

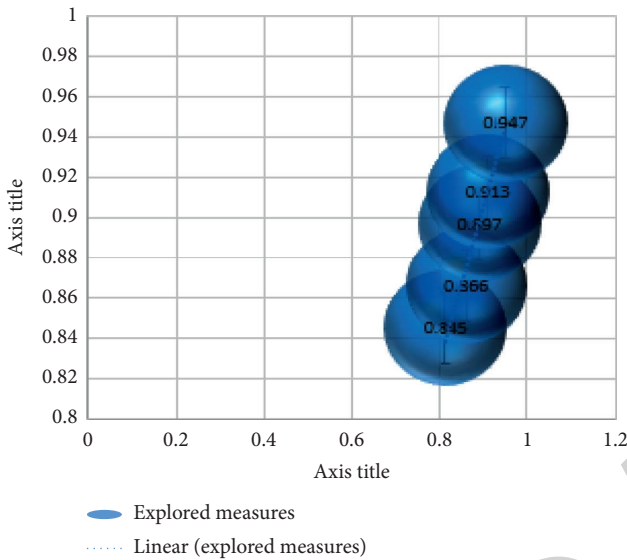


FIGURE 3: Graphical representation for the information of Table 4.

$$C_\lambda = (\lambda \zeta_{\text{CDTHF-cc-}yz})_{m \times m} = \begin{cases} 0, & \zeta_{\text{CDTHF-cc-}yz} < \lambda, \\ 1, & \zeta_{\text{CDTHF-cc-}yz} \geq \lambda, \end{cases} \quad (47)$$

where  $\lambda \in [0, 1]$  denotes the confidence level.

Step 5: we build up all possible classifications based on  $\lambda$ -cutting matrix. If all elements of the  $y$ th line (column) in  $C_\lambda$  are the same as the corresponding elements of the  $z$ th line (column) in  $C_\lambda$ , then the CDT-2HFS  $D_y$  and  $D_z$  are of the same type. For simplicity, we draw the

$$\zeta_{\text{CDTHF-cc}}^2 = \begin{bmatrix} 1 & 0.79 & 0.85 & 0.81 & 0.8 & 0.72 & 0.75 & 0.66 & 0.67 & 0.7 \\ 0.79 & 1 & 0.83 & 0.81 & 0.75 & 0.72 & 0.65 & 0.65 & 0.64 & 0.69 \\ 0.85 & 0.83 & 1 & 0.89 & 0.89 & 0.88 & 0.82 & 0.75 & 0.74 & 0.78 \\ 0.81 & 0.81 & 0.89 & 1 & 0.87 & 0.8 & 0.76 & 0.72 & 0.69 & 0.79 \\ 0.8 & 0.75 & 0.89 & 0.87 & 1 & 0.85 & 0.83 & 0.73 & 0.75 & 0.8 \\ 0.72 & 0.72 & 0.88 & 0.8 & 0.85 & 1 & 0.87 & 0.79 & 0.83 & 0.82 \\ 0.75 & 0.65 & 0.82 & 0.76 & 0.83 & 0.87 & 1 & 0.84 & 0.83 & 0.85 \\ 0.66 & 0.65 & 0.75 & 0.75 & 0.73 & 0.79 & 0.84 & 1 & 0.92 & 0.89 \\ 0.67 & 0.64 & 0.74 & 0.69 & 0.75 & 0.83 & 0.83 & 0.92 & 1 & 0.86 \\ 0.7 & 0.69 & 0.78 & 0.79 & 0.8 & 0.82 & 0.85 & 0.89 & 0.86 & 1 \end{bmatrix}. \quad (48)$$

graphical shape of the explored clustering algorithm, which is stated with the help of Table 5.

*Example 5.* (see [30]). Consider a speculation organization that needs to put a total cash in the most ideal choice and along these lines organization officials decide five choices by considering different standards to recognize the best choice to put away the cash: (a)  $\mathcal{A}_{\text{AL-5}}$  is a structure organization; (b)  $\mathcal{A}_{\text{AL-2}}$  is an airplane organization; (c)  $\mathcal{A}_{\text{AL-3}}$  is a food organization; (d)  $\mathcal{A}_{\text{AL-4}}$  is an electronic things organization; (e)  $\mathcal{A}_{\text{AL-5}}$  is a cowhide organization; (f)  $\mathcal{A}_{\text{AL-6}}$  is a vehicle organization; (g)  $\mathcal{A}_{\text{AL-7}}$  is a correspondence organization; (h)  $\mathcal{A}_{\text{AL-8}}$  is a product organization; (I)  $\mathcal{A}_{\text{AL-9}}$  is a paper creation organization; (j)  $\mathcal{A}_{\text{AL-10}}$  is a plastic creation organization. The speculation organization must make a choice as indicated by the five rules: (a)  $x_1$  is the transportation; (b)  $x_2$  is the work; (c)  $x_3$  is an ecological effect; (d)  $x_4$  is the vicinity to crude material; (e)  $x_5$  is the experience. The loads of standards  $x_1, x_2, x_3, x_4$  and  $x_5$  are given by  $\Omega_1 = 0.2, \Omega_2 = 0.25, \Omega_3 = 0.1, \Omega_4 = 0.3, \Omega_5 = 0.15$ , individually. The 10 choices are assessed under the standards by etymological evaluations yielded in Table 6 and given by decision-makers. The technique of the clustering algorithm is summarized as follows:

Step 1: we construct the decision matrix, all entities of which are in the form of CDT-2HFSs; see Table 5.

Step 2: we construct the correlation matrix by using equation (22). The correlation matrix is expressed by  $\zeta_{\text{CDTHF-cc}} = \zeta_{\text{CDTHF-cc-}yz}(\mathcal{Q}_{\text{CDTH-}y}, \mathcal{Q}_{\text{CDTH-}z})$ , where  $(\zeta_{\text{CDTHF-cc}})_{m \times m} = C$ , such that

Step 3: we checked whether the correlation matrix  $C$  satisfies  $C^2 \subseteq C$ , where  $C^2 = C \circ C = (\zeta_{CDTHF-cc-yz})_{m \times m}$   
 $\zeta_{CDTHF-cc-yz} = \max_z \{ \min \{ \zeta_{CDTHF-cc-xy}, \zeta_{CDTHF-cc-xz} \} \}$ .

$$\zeta_{CDTHF-cc}^2 = \begin{bmatrix} 1 & 0.83 & 0.85 & 0.85 & 0.85 & 0.85 & 0.85 & 0.82 & 0.75 & 0.75 & 0.8 \\ 0.83 & 1 & 0.83 & 0.83 & 0.83 & 0.83 & 0.83 & 0.82 & 0.75 & 0.75 & 0.79 \\ 0.85 & 0.83 & 1 & 0.89 & 0.89 & 0.88 & 0.87 & 0.82 & 0.83 & 0.82 & \\ 0.85 & 0.83 & 0.89 & 1 & 0.89 & 0.88 & 0.83 & 0.79 & 0.8 & 0.8 & \\ 0.85 & 0.83 & 0.89 & 0.89 & 1 & 0.88 & 0.85 & 0.83 & 0.83 & 0.83 & \\ 0.85 & 0.83 & 0.88 & 0.88 & 0.88 & 1 & 0.87 & 0.84 & 0.83 & 0.85 & \\ 0.82 & 0.82 & 0.87 & 0.83 & 0.85 & 0.87 & 1 & 0.85 & 0.85 & 0.85 & \\ 0.75 & 0.75 & 0.82 & 0.79 & 0.83 & 0.84 & 0.85 & 1 & 0.92 & 0.89 & \\ 0.75 & 0.75 & 0.83 & 0.8 & 0.83 & 0.83 & 0.85 & 0.92 & 1 & 0.89 & \\ 0.8 & 0.79 & 0.82 & 0.8 & 0.83 & 0.85 & 0.85 & 0.89 & 0.89 & 1 & \end{bmatrix}, \tag{49}$$

where  $C$  does not satisfy the condition  $C^2 \subseteq C$ ; then the equivalent correlation matrix  $C^{2^k}$  will be formed:  
 $C \rightarrow C^2 \rightarrow C^4 \rightarrow \dots \rightarrow C^{2^k} \rightarrow \dots$  until  $C^{2^k} = C^{2^{k+1}}$ . Then

$$\zeta_{CDTHF-cc}^4 = \begin{bmatrix} 1 & 0.83 & 0.85 & 0.85 & 0.85 & 0.85 & 0.85 & 0.85 & 0.84 & 0.83 & 0.85 \\ 0.83 & 1 & 0.83 & 0.83 & 0.83 & 0.83 & 0.83 & 0.83 & 0.83 & 0.83 & 0.83 \\ 0.85 & 0.83 & 1 & 0.89 & 0.89 & 0.88 & 0.87 & 0.85 & 0.85 & 0.85 & \\ 0.85 & 0.83 & 0.89 & 1 & 0.89 & 0.88 & 0.87 & 0.84 & 0.83 & 0.85 & \\ 0.85 & 0.83 & 0.89 & 0.89 & 1 & 0.88 & 0.87 & 0.85 & 0.85 & 0.85 & \\ 0.85 & 0.83 & 0.88 & 0.88 & 0.88 & 1 & 0.87 & 0.85 & 0.85 & 0.85 & \\ 0.85 & 0.83 & 0.87 & 0.87 & 0.87 & 0.87 & 1 & 0.85 & 0.85 & 0.85 & \\ 0.84 & 0.83 & 0.85 & 0.84 & 0.85 & 0.85 & 0.85 & 1 & 0.92 & 0.89 & \\ 0.83 & 0.83 & 0.85 & 0.83 & 0.85 & 0.85 & 0.85 & 0.92 & 1 & 0.89 & \\ 0.8 & 0.79 & 0.82 & 0.8 & 0.83 & 0.85 & 0.85 & 0.89 & 0.89 & 1 & \end{bmatrix},$$

$$\zeta_{CDTHF-cc}^8 = \begin{bmatrix} 1 & 0.83 & 0.85 & 0.85 & 0.85 & 0.85 & 0.85 & 0.85 & 0.85 & 0.85 & \\ 0.83 & 1 & 0.83 & 0.83 & 0.83 & 0.83 & 0.83 & 0.83 & 0.83 & 0.83 & \\ 0.85 & 0.83 & 1 & 0.89 & 0.89 & 0.88 & 0.87 & 0.85 & 0.85 & 0.85 & \\ 0.85 & 0.83 & 0.89 & 1 & 0.89 & 0.88 & 0.87 & 0.84 & 0.83 & 0.85 & \\ 0.85 & 0.83 & 0.89 & 0.89 & 1 & 0.88 & 0.87 & 0.85 & 0.85 & 0.85 & \\ 0.85 & 0.83 & 0.88 & 0.88 & 0.88 & 1 & 0.87 & 0.85 & 0.85 & 0.85 & \\ 0.85 & 0.83 & 0.87 & 0.87 & 0.87 & 0.87 & 1 & 0.85 & 0.85 & 0.85 & \\ 0.85 & 0.83 & 0.85 & 0.85 & 0.85 & 0.85 & 0.85 & 1 & 0.92 & 0.89 & \\ 0.85 & 0.83 & 0.85 & 0.85 & 0.85 & 0.85 & 0.85 & 0.92 & 1 & 0.89 & \\ 0.85 & 0.83 & 0.85 & 0.85 & 0.85 & 0.85 & 0.85 & 0.89 & 0.89 & 1 & \end{bmatrix}$$

$$\zeta_{\zeta_{CDTHF-cc}}^{16} = \begin{bmatrix} 1 & 0.83 & 0.85 & 0.85 & 0.85 & 0.85 & 0.85 & 0.85 & 0.85 & 0.85 \\ 0.83 & 1 & 0.83 & 0.83 & 0.83 & 0.83 & 0.83 & 0.83 & 0.83 & 0.83 \\ 0.85 & 0.83 & 1 & 0.89 & 0.89 & 0.88 & 0.87 & 0.85 & 0.85 & 0.85 \\ 0.85 & 0.83 & 0.89 & 1 & 0.89 & 0.88 & 0.87 & 0.85 & 0.85 & 0.85 \\ 0.85 & 0.83 & 0.89 & 0.89 & 1 & 0.88 & 0.87 & 0.85 & 0.85 & 0.85 \\ 0.85 & 0.83 & 0.88 & 0.88 & 0.88 & 1 & 0.87 & 0.85 & 0.85 & 0.85 \\ 0.85 & 0.83 & 0.87 & 0.87 & 0.87 & 0.87 & 1 & 0.85 & 0.85 & 0.85 \\ 0.85 & 0.83 & 0.85 & 0.85 & 0.85 & 0.85 & 0.85 & 1 & 0.92 & 0.89 \\ 0.85 & 0.83 & 0.85 & 0.85 & 0.85 & 0.85 & 0.85 & 0.92 & 1 & 0.89 \\ 0.85 & 0.83 & 0.85 & 0.85 & 0.85 & 0.85 & 0.85 & 0.89 & 0.89 & 1 \end{bmatrix} = \zeta_{\zeta_{CDTHF-cc}}^8. \tag{50}$$

Therefore, we get  $\zeta_{\zeta_{CDTHF-cc}}^{16} = \zeta_{\zeta_{CDTHF-cc}}^8$ .

where  $\lambda \in 0, 1$  denotes the confidence level, such that

Step 4: furthermore, we examine the  $\lambda$ -cutting matrix by using

$$C_\lambda = (\lambda \zeta_{\zeta_{CDTHF-cc-yz}})_{m \times m} = \begin{cases} 0, & \zeta_{\zeta_{CDTHF-cc-yz}} < \lambda, \\ 1, & \zeta_{\zeta_{CDTHF-cc-yz}} \geq \lambda, \end{cases} \tag{51}$$

$$\begin{aligned}
 C_{0 < \lambda \leq 0.83} &= \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}, & C_{0.83 < \lambda \leq 0.85} &= \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}, \\
 C_{0.85 < \lambda \leq 0.87} &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}, & C_{0.87 < \lambda \leq 0.88} &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix},
 \end{aligned}$$

TABLE 5: All possible classifications based on  $\lambda$ -cutting matrices.

Classifications	Representations	Limitations
$\mathcal{A}_{AL-i}$ of the same characteristic	$\left\{ \mathcal{A}_{AL-1}, \mathcal{A}_{AL-2}, \mathcal{A}_{AL-3}, \mathcal{A}_{AL-4}, \mathcal{A}_{AL-5}, \mathcal{A}_{AL-6}, \mathcal{A}_{AL-7}, \mathcal{A}_{AL-8}, \mathcal{A}_{AL-9}, \mathcal{A}_{AL-10} \right\}$	$0 < \lambda \leq 0.83$
$\mathcal{A}_{AL-i}$ of the two characteristics	$\left\{ \mathcal{A}_{AL-1}, \mathcal{A}_{AL-2}, \mathcal{A}_{AL-3}, \mathcal{A}_{AL-4}, \mathcal{A}_{AL-5}, \mathcal{A}_{AL-6}, \mathcal{A}_{AL-7}, \mathcal{A}_{AL-8}, \mathcal{A}_{AL-9}, \mathcal{A}_{AL-10} \right\}, \left\{ \mathcal{A}_{AL-2} \right\}$	$0.83 < \lambda \leq 0.85$
$\mathcal{A}_{AL-i}$ of the three characteristics	$\left\{ \mathcal{A}_{AL-1} \right\}, \left\{ \mathcal{A}_{AL-3}, \mathcal{A}_{AL-4}, \mathcal{A}_{AL-5}, \mathcal{A}_{AL-6}, \mathcal{A}_{AL-7} \right\}, \left\{ \mathcal{A}_{AL-2} \right\}, \left\{ \mathcal{A}_{AL-8}, \mathcal{A}_{AL-9}, \mathcal{A}_{AL-10} \right\}$	$0.85 < \lambda \leq 0.87$
$\mathcal{A}_{AL-i}$ of the five characteristics	$\left\{ \mathcal{A}_{AL-1} \right\}, \left\{ \mathcal{A}_{AL-3}, \mathcal{A}_{AL-4}, \mathcal{A}_{AL-5}, \mathcal{A}_{AL-6} \right\}, \left\{ \mathcal{A}_{AL-2} \right\}, \left\{ \mathcal{A}_{AL-7} \right\}, \left\{ \mathcal{A}_{AL-8}, \mathcal{A}_{AL-9}, \mathcal{A}_{AL-10} \right\}$	$0.87 < \lambda \leq 0.88$
$\mathcal{A}_{AL-i}$ of the six characteristics	$\left\{ \mathcal{A}_{AL-1} \right\}, \left\{ \mathcal{A}_{AL-3}, \mathcal{A}_{AL-4}, \mathcal{A}_{AL-5} \right\}, \left\{ \mathcal{A}_{AL-2} \right\}, \left\{ \mathcal{A}_{AL-6} \right\}, \left\{ \mathcal{A}_{AL-7} \right\}, \left\{ \mathcal{A}_{AL-8}, \mathcal{A}_{AL-9}, \mathcal{A}_{AL-10} \right\}$	$0.88 < \lambda \leq 0.89$
$\mathcal{A}_{AL-i}$ of the nine characteristics	$\left\{ \mathcal{A}_{AL-1} \right\}, \left\{ \mathcal{A}_{AL-3} \right\}, \left\{ \mathcal{A}_{AL-4} \right\}, \left\{ \mathcal{A}_{AL-5} \right\}, \left\{ \mathcal{A}_{AL-2} \right\}, \left\{ \mathcal{A}_{AL-6} \right\}, \left\{ \mathcal{A}_{AL-7} \right\}, \left\{ \mathcal{A}_{AL-8}, \mathcal{A}_{AL-9}, \mathcal{A}_{AL-10} \right\}$	$0.89 < \lambda \leq 0.92$
$\mathcal{A}_{AL-i}$ of the ten characteristics	$\left\{ \mathcal{A}_{AL-1} \right\}, \left\{ \mathcal{A}_{AL-2} \right\}, \left\{ \mathcal{A}_{AL-3} \right\}, \left\{ \mathcal{A}_{AL-4} \right\}, \left\{ \mathcal{A}_{AL-5} \right\}, \left\{ \mathcal{A}_{AL-6} \right\}, \left\{ \mathcal{A}_{AL-7} \right\}, \left\{ \mathcal{A}_{AL-8} \right\}, \left\{ \mathcal{A}_{AL-9} \right\}, \left\{ \mathcal{A}_{AL-10} \right\}$	$0.92 < \lambda \leq 1$

$$\begin{aligned}
 C_{0.88 < \lambda \leq 0.89} &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}, \quad C_{0.89 < \lambda \leq 0.92} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}, \\
 C_{0.92 < \lambda \leq 1} &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \tag{52}
 \end{aligned}$$

Step 5: we build up all possible classifications based on  $\lambda$ -cutting matrix. If all elements of the  $y$ th line (column) in  $C_\lambda$  are the same as the corresponding elements of the  $z$ th line (column) in  $C_\lambda$ , then the CDT-2HFS  $D_y$  and  $D_z$  are of the same type. For simplicity, we draw the graphical shape of the explored clustering algorithm, which is stated with the help of Table 5.

Therefore, from the above analysis, we get the result that the explored notions and their measures are more powerful

and more proficient than exiting measures. CDT-2HFS is a proficient technique to cope with unpredictable and awkward information in realistic decision problems. CDT-2HFS is composed of the grade of truth and the grade of falsity, and the grade of truth (also for falsity grade) contains the grade of primary and secondary parts in the form of polar coordinates with the condition that the sum of the maximum of the real part (also for imaginary part) of the primary grade (also for secondary grade) cannot exceed the unit interval.



TABLE 6: Common evaluations of alternatives performed by decision-makers.

	$x_1$	$x_2$	$x_3$
$\mathcal{A}_{AL-1}$	$\left\{ \begin{array}{l} \{(0.6, 0.1), (0.2, 0.8), (0.1, 0.5)\}, \\ \{(0.3, 0.6), (0.1, 0.3)\} \end{array} \right\}$	$\left\{ \begin{array}{l} \{(0.59, 0.1), (0.4, 0.72), (0.3, 0.2)\}, \\ \{(0.4, 0.5), (0.3, 0.1)\} \end{array} \right\}$	$\left\{ \begin{array}{l} \{(0.7, 0.2), (0.6, 0.1), (0.2, 0.4)\}, \\ \{(0.3, 0.3), (0.2, 0.0)\} \end{array} \right\}$
$\mathcal{A}_{AL-2}$	$\left\{ \begin{array}{l} \{(0.35, 0.4), (0.2, 0.9), (0.1, 0.1)\}, \\ \{(0.3, 0.5), (0.2, 0.8)\} \end{array} \right\}$	$\left\{ \begin{array}{l} \{(0.4, 0.5), (0.1, 0.9), (0.1, 0.62)\}, \\ \{(0.5, 0.3), (0.4, 0.3)\} \end{array} \right\}$	$\left\{ \begin{array}{l} \{(0.7, 0.2), (0.6, 0.1), (0.2, 0.4)\}, \\ \{(0.3, 0.3), (0.2, 0.0)\} \end{array} \right\}$
$\mathcal{A}_{AL-3}$	$\left\{ \begin{array}{l} \{(0.65, 0.12), (0.21, 0.7), (0.2, 0.3)\}, \\ \{(0.35, 0.1), (0.34, 0.065)\} \end{array} \right\}$	$\left\{ \begin{array}{l} \{(0.7, 0.), (0.35, 0.71), (0.3, 0.2)\}, \\ \{(0.3, 0.1), (0.2, 0.5)\} \end{array} \right\}$	$\left\{ \begin{array}{l} \{(0.7, 0.6), (0.6, 0.1), (0.2, 0.4)\}, \\ \{(0.3, 0.3), (0.15, 0.0)\} \end{array} \right\}$
$\mathcal{A}_{AL-4}$	$\left\{ \begin{array}{l} \{(0.85, 0.12), (0.21, 0.7), (0.2, 0.3)\}, \\ \{(0.15, 0.1), (0.14, 0.65)\} \end{array} \right\}$	$\left\{ \begin{array}{l} \{(0.3, 0.2), (0.15, 0.71), (0.15, 0.1)\}, \\ \{(0.3, 0.1), (0.2, 0.5)\} \end{array} \right\}$	$\left\{ \begin{array}{l} \{(0.5, 0.5), (0.2, 0.4), (0.2, 0.1)\}, \\ \{(0.3, 0.3), (0.15, 0.0)\} \end{array} \right\}$
$\mathcal{A}_{AL-5}$	$\left\{ \begin{array}{l} \{(0.25, 0.12), (0.21, 0.7), (0.1, 0.3)\}, \\ \{(0.75, 0.1), (0.14, 0.65)\} \end{array} \right\}$	$\left\{ \begin{array}{l} \{(0.6, 0.2), (0.45, 0.71), (0.35, 0.1)\}, \\ \{(0.3, 0.1), (0.2, 0.5)\} \end{array} \right\}$	$\left\{ \begin{array}{l} \{(0.5, 0.8), (0.3, 0.1), (0.14, 0.4)\}, \\ \{(0.45, 0.0), (0.3, 0.3)\} \end{array} \right\}$
$\mathcal{A}_{AL-6}$	$\left\{ \begin{array}{l} \{(0.35, 0.12), (0.11, 0.9), (0.1, 0.8)\}, \\ \{(0.55, 0.1), (0.14, 0.65)\} \end{array} \right\}$	$\left\{ \begin{array}{l} \{(0.6, 0.3), (0.35, 0.71), (0.25, 0.1)\}, \\ \{(0.4, 0.1), (0.1, 0.9)\} \end{array} \right\}$	$\left\{ \begin{array}{l} \{(0.5, 0.8), (0.3, 0.1), (0.14, 0.4)\}, \\ \{(0.35, 0.1), (0.3, 0.3)\} \end{array} \right\}$
$\mathcal{A}_{AL-7}$	$\left\{ \begin{array}{l} \{(0.45, 0.12), (0.11, 0.75), (0.1, 0.15)\}, \\ \{(0.45, 0.1), (0.14, 0.65)\} \end{array} \right\}$	$\left\{ \begin{array}{l} \{(0.9, 0.3), (0.25, 0.71), (0.25, 0.1)\}, \\ \{(0.1, 0.1), (0.0, 0.9)\} \end{array} \right\}$	$\left\{ \begin{array}{l} \{(0.5, 0.8), (0.3, 0.9), (0.3, 0.3)\}, \\ \{(0.35, 0.1), (0.2, 0.9)\} \end{array} \right\}$
$\mathcal{A}_{AL-8}$	$\left\{ \begin{array}{l} \{(0.95, 0.82), (0.11, 0.75), (0.1, 0.55)\}, \\ \{(0.05, 0.10), (0.14, 0.65)\} \end{array} \right\}$	$\left\{ \begin{array}{l} \{(0.9, 0.3), (0.65, 0.1), (0.25, 0.71)\}, \\ \{(0.1, 0.7), (0.0, 0.9)\} \end{array} \right\}$	$\left\{ \begin{array}{l} \{(0.4, 0.8), (0.3, 0.6), (0.2, 0.3)\}, \\ \{(0.35, 0.1), (0.2, 0.9)\} \end{array} \right\}$
$\mathcal{A}_{AL-9}$	$\left\{ \begin{array}{l} \{(0.55, 0.22), (0.21, 0.75), (0.0, 0.95)\}, \\ \{(0.45, 0.1), (0.24, 0.15)\} \end{array} \right\}$	$\left\{ \begin{array}{l} \{(0.67, 0.1), (0.5, 0.3), (0.25, 0.71)\}, \\ \{(0.3, 0.9), (0.23, 0.70)\} \end{array} \right\}$	$\left\{ \begin{array}{l} \{(0.4, 0.8), (0.3, 0.6), (0.2, 0.3)\}, \\ \{(0.35, 0.1), (0.2, 0.9)\} \end{array} \right\}$
$\mathcal{A}_{AL-10}$	$\left\{ \begin{array}{l} \{(0.83, 0.82), (0.21, 0.75), (0.1, 0.65)\}, \\ \{(0.17, 0.1), (0.04, 0.55)\} \end{array} \right\}$	$\left\{ \begin{array}{l} \{(0.9, 0.2), (0.25, 0.71), (0.25, 0.1)\}, \\ \{(0.1, 0.7), (0.0, 0.9)\} \end{array} \right\}$	$\left\{ \begin{array}{l} \{(0.7, 0.8), (0.35, 0.65), (0.22, 0.23)\}, \\ \{(0.25, 0.1), (0.2, 0.9)\} \end{array} \right\}$
	$x_4$	$x_5$	
$\mathcal{A}_{AL-1}$	$\left\{ \begin{array}{l} \{(0.3, 0.1), (0.25, 0.65), (0.2, 0.2)\}, \\ \{(0.7, 0.3), (0.66, 0.60)\} \end{array} \right\}$	$\left\{ \begin{array}{l} \{(0.8, 0.1), (0.27, 0.89), (0.14, 0.5)\}, \\ \{(0.2, 0.6), (0.1, 0.34)\} \end{array} \right\}$	
$\mathcal{A}_{AL-2}$	$\left\{ \begin{array}{l} \{(0.35, 0.45), (0.3, 0.1), (0.1, 0.3)\}, \\ \{(0.55, 0.1), (0.3, 0.2)\} \end{array} \right\}$	$\left\{ \begin{array}{l} \{(0.45, 0.45), (0.2, 0.1), (0.1, 0.4)\}, \\ \{(0.55, 0.1), (0.5, 0.2)\} \end{array} \right\}$	
$\mathcal{A}_{AL-3}$	$\left\{ \begin{array}{l} \{(0.4, 0.1), (0.25, 0.75), (0.2, 0.2)\}, \\ \{(0.6, 0.3), (0.5, 0.6)\} \end{array} \right\}$	$\left\{ \begin{array}{l} \{(0.35, 0.1), (0.35, 0.1), (0.25, 0.15)\}, \\ \{(0.6, 0.3), (0.3, 0.6)\} \end{array} \right\}$	
$\mathcal{A}_{AL-4}$	$\left\{ \begin{array}{l} \{(0.4, 0.1), (0.25, 0.75), (0.2, 0.2)\}, \\ \{(0.5, 0.6), (0.45, 0.3)\} \end{array} \right\}$	$\left\{ \begin{array}{l} \{(0.5, 0.4), (0.48, 0.1), (0.15, 0.75)\}, \\ \{(0.5, 0.6), (0.35, 0.3)\} \end{array} \right\}$	
$\mathcal{A}_{AL-5}$	$\left\{ \begin{array}{l} \{(0.57, 0.1), (0.25, 0.75), (0.23, 0.2)\}, \\ \{(0.45, 0.6), (0.35, 0.3)\} \end{array} \right\}$	$\left\{ \begin{array}{l} \{(0.37, 0.1), (0.25, 0.15), (0.23, 0.25)\}, \\ \{(0.35, 0.9), (0.33, 0.6)\} \end{array} \right\}$	
$\mathcal{A}_{AL-6}$	$\left\{ \begin{array}{l} \{(0.57, 0.1), (0.25, 0.75), (0.23, 0.2)\}, \\ \{(0.35, 0.3), (0.13, 0.9)\} \end{array} \right\}$	$\left\{ \begin{array}{l} \{(0.15, 0.65), (0.15, 0.2), (0.15, 0.1)\}, \\ \{(0.85, 0.3), (0.13, 0.9)\} \end{array} \right\}$	
$\mathcal{A}_{AL-7}$	$\left\{ \begin{array}{l} \{(0.85, 0.1), (0.35, 0.75), (0.25, 0.1)\}, \\ \{(0.15, 0.3), (0.13, 0.9)\} \end{array} \right\}$	$\left\{ \begin{array}{l} \{(0.75, 0.1), (0.25, 0.75), (0.25, 0.1)\}, \\ \{(0.25, 0.3), (0.23, 0.9)\} \end{array} \right\}$	
$\mathcal{A}_{AL-8}$	$\left\{ \begin{array}{l} \{(0.55, 0.1), (0.45, 0.2), (0.05, 0.75)\}, \\ \{(0.15, 0.3), (0.13, 0.9)\} \end{array} \right\}$	$\left\{ \begin{array}{l} \{(0.65, 0.2), (0.55, 0.1), (0.4, 0.45)\}, \\ \{(0.25, 0.3), (0.03, 0.9)\} \end{array} \right\}$	
$\mathcal{A}_{AL-9}$	$\left\{ \begin{array}{l} \{(0.55, 0.1), (0.45, 0.2), (0.05, 0.75)\}, \\ \{(0.15, 0.3), (0.13, 0.9)\} \end{array} \right\}$	$\left\{ \begin{array}{l} \{(0.65, 0.55), (0.35, 0.2), (0.35, 0.1)\}, \\ \{(0.25, 0.3), (0.23, 0.9)\} \end{array} \right\}$	
$\mathcal{A}_{AL-10}$	$\left\{ \begin{array}{l} \{(0.65, 0.25), (0.45, 0.2), (0.45, 0.1)\}, \\ \{(0.15, 0.35), (0.13, 0.5)\} \end{array} \right\}$	$\left\{ \begin{array}{l} \{(0.45, 0.25), (0.35, 0.1), (0.25, 0.2)\}, \\ \{(0.43, 0.7), (0.11, 0.35)\} \end{array} \right\}$	

### 7. Conclusion

The theory of CDT-2HFS is a mixture of two different modifications of FS, called CFS and DT-2HFS. CDT-2HFS is a proficient technique to cope with unpredictable and awkward information in realistic decision problems. The intention of this manuscript is to determine the novel methodology of CDT-2HFSs and to discuss their operational laws. These operational laws are also defensible with the

assistance of examples. Furthermore, based on novel CDT-2HFS, we reconnoitered the CC and EMs, and their special cases are also discussed. TOPSIS method based on CDT-2HFS is also explored. Then, we applied our explored measures based on CDT-2HFSs in the environment of TOPSIS method, medical diagnosis, pattern recognition, and clustering algorithm to cope with awkward and complicated information in realistic decision issues. Finally, some numerical examples are given and discussed to

examine the proficiency and validity of the explored measures. Comparative analysis, advantages, and graphical interpretation of the explored measures with some other existing measures are also discussed.

In the future, the concept of complex dual type-2 hesitant fuzzy sets can be applied to group MADM problems. Moreover, the problems discussed in this manuscript can be discussed in the environment of complex q-rung orthopair fuzzy sets [32–39], T-spherical fuzzy sets [40, 41], and some others [42–45].

## Data Availability

The data used in this article are artificial and hypothetical, and anyone can use these data without prior permission by just citing this article.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

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