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Research Article

Insider Trading with Memory under Random Deadline

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In this paper, we study a model of continuous-time insider trading in which noise traders have some memories and the trading stops at a random deadline. By a filtering theory on fractional Brownian motion and the stochastic maximum principle, we obtain a necessary condition of the insider's optimal strategy, an equation satisfied. It shows that when the volatility of noise traders is constant and the noise traders' memories become weaker and weaker, the optimal trading intensity and the corresponding residual information tend to those, respectively, when noise traders have no any memory. And, numerical simulation illustrates that if both the trading intensity of the insider and the volatility of noise trades are independent of trading time, the insider's expected profit is always lower than that when the asset value is disclosed at a finite fixed time; this is because the trading time ahead is a random deadline which yields the loss of the insider's information.

1. Introduction

In 1985, Kyle [1] proposed an equilibrium model of continuous insider trading, in which there are three types of traders in a market with a risky asset, whose values is normally distributed. Subsequently, Back [2] formulated and studied a revised-version of Kyle's model [1] in continuous time and, by dynamic programming principle, obtained the existence and uniqueness of equilibrium pricing rule within a certain class of insider trading. This model has been known as a classical Kyle–Back model. Later, by the same method, Caldentey and Stacchetti [3] investigated the equilibrium for a class of insider trading with a dynamic asset driven by a Brownian motion and with trading time stopped at a random deadline. Moreover, Collins-Dufresne and Fos [4] studied the impact of a stochastic volatility of noise trades on the equilibrium for some kind of insider trading with a static risky asset. Alternatively, by a filtering theory on Brownian motion and maximum principle, Aase et al. [5] solved a model of insider trading problem with volatility of noise trades' time varying. Then, Zhou [6] also obtained a close form of equilibrium when market makers have some partial observations on a risky asset. Furthermore, Ma et al. [7] studied a more general model of insider trading with an asset value driven by a conditional mean-field Ornstein–Uhlenbeck-type dynamics and obtained a closed form of optimal intensity of trading strategy as well as dynamic pricing rule. In fact, there are many research studies about insider trading, see [8–12] and so on.

Note that, in those insider trading models mentioned above, it is always assumed that noise trades are driven by Brownian motion; that is, noise traders have no memory on the history of their trades. In fact, in real financial markets, noise traders often have some memories on the history; in other words, mathematically, noise trades follow a dynamics driven by fractional Brownian motion with Hurst parameter $H > (1/2)$, which will have some impacts on the insider's benefit. Biagini etal. [13] studied insider trading in this setting, obtained an equation for the optimal trading intensity, proved that when $H \longrightarrow (1/2)$, the solution converges to the solution in the classical case driven by Brownian motion ($H = (1/2)$), and pointed out that both the optimal insider trading intensity and the expect profit of an insider decreases with increasing $H \in [1/2, 1)$; the larger the Hurst parameter H is, the stronger the memory ability of noise traders is. Then, based on the model in [4], Yang et al.

[14] proposed an insider trading model with long memory, in which the volume of noise trades is driven by a fractional Brownian motion with the Hurst parameter $H > (1/2)$, with its volatility following a general stochastic process, and found that the optimal trading strategy of the insider turns out to possess the property of long memory, and the price impact is also affected by the fractional noise.

From the abovementioned studies, in this paper, based on the two models in [3, 13], we will continue to study a model of continuous-time insider trading in which noise traders have some memories and the trading stops at a random deadline. By a filtering theory on fractional Brownian motion and the maximum principle, we will look for some necessary conditions of the insider's optimal strategy and investigate the impact of noise traders' memory and random deadline on equilibrium, especially on the insider's profit.

The rest of this paper is organized into six sections. Section 2 presents some necessary preliminaries. In Section 3, our model with equilibrium concept is proposed. In Section 4, our main theorem about a necessary condition of optimal trading intensity is stated. In Section 5, we discuss the impact of noise traders' memory and random deadline on equilibrium. Conclusions are drawn in Section 6.

2. Preliminaries

Assume that all randomness comes from a common filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P)$ satisfying the usual conditions [15]. Let us introduce the concept of fractional Brownian motion B^H with Hurst parameter $H \in (0,1)$ [13, 16].

Definition 1. A fractional Brownian motion (fBm) B^H with Hurst parameter (or scaling factor) $H \in (0,1)$ is a process satisfying these following properties:

(1) All paths are almost surely continuous.

(2) B_t^H is centered Gaussian with covariance function

$$E\left[B_t^H B_s^H\right] = \frac{1}{2}\left(t^{2H} + s^{2H} - |t-s|^{2H}\right). \tag{1}$$

(3) B_t^H has stationary increments.

For $H = 1/2$, B_t^H is a standard Brownian motion (B_m) denoted by B; the increments of the process are independent. Contrary to $H = 1/2$, the increments are not independent. More exactly, if $t - s = nh$, then the covariance between $B_{t+h}^H - B_t^H$ and $B_{s+h}^H - B_s^H$ is expressed as

$$\rho_H(n) = \frac{1}{2}h^{2H}\left[(n+1)^{2H} + (n-1)^{2H} - 2n^{2H}\right]. \tag{2}$$

In particular, for $H > (1/2)$, the two increments of form $B_{t+h}^H - B_t^H$ and $B_{t+2h}^H - B_{t+h}^H$ are positively correlated. Furthermore,

$$\lim_{n\to\infty} \frac{\rho_H(n)}{H(2H-1)n^{2H-2}} = 1, \tag{3}$$

which means that the dependence between the increments $B_{t+h}^H - B_t^H$ and $B_{s+h}^H - B_s^H$ decays slowly as n tends to infinity, or the two increments of B^H have the long-range dependence property [16] and

$$\sum_{n=1}^{\infty} \rho_H(n) = \infty. \tag{4}$$

And, paths of the process are more regular than those of B presenting an aggregation behavior, which is used to describe systems with memory and persistence.

While, for $H < (1/2)$, the two increments of form $B_{t+h}^H - B_t^H$ and $B_{t+2h}^H - B_{t+h}^H$ are negatively correlated with

$$\sum_{n=1}^{\infty} \left|\rho_H(n)\right| < \infty, \tag{5}$$

the paths of B^H are less regular than those of B, such that systems have phenomena of turbulence and antipersistence.

In this paper, we restrict ourselves to the case $H \geq (1/2)$ and understand the integral of deterministic functions with respect to fBm in the sense of that described in [17], and others are usual Lebesgue or Lebesgue–Stieltjes integral; if we denote

$$B_t^* = \int_0^t k_H(t,s)\mathrm{d}B_s^H, \tag{6}$$

where $k_H = 2H\Gamma((3/2) - H)\Gamma((1/2) + H)$, then B_t^* is a Gaussian martingale with $\langle B^*\rangle_t = \lambda_H^{-1}t^{2-2H}$ and $\lambda_H = (2H\Gamma(3-2H)\Gamma(H + (1/2))/\Gamma((3/2) - H))$. For convenience, denote L_H^2, the space of equivalence of measurable functions f, on $[0,\infty)$ such that

$$\langle\langle\langle f, g\rangle\rangle_H = H(2H-1)\int_0^\infty \int_0^\infty f(s)g(t)|s-t|^{2H-2}\mathrm{d}s\mathrm{d}t < \infty. \tag{7}$$

3. The Model with Equilibrium

There is a risky asset traded in continuous time with value v normally distributed $N(0, \sigma_v^2)$, which is made public at a random time τ. The random time τ has a geometric distribution with probability of failure $e^{-\mu t}$ for some $\mu > 0$ and is independent of the history of transactions and prices [3]. And, there are three types of agents in the market:

(i) *An insider*, who has private information of the liquidation value v at the trade beginning and submits her/his order x_t at time t.

(ii) *Noise traders*, who have no information of the liquidation value v and submit a total order z_t in the form:

$$z_t = \int_0^t \sigma_{zs}\mathrm{d}B_s^H, \tag{8}$$

where σ_z is a deterministic continuously differentiable function satisfying $\sigma_{zt} > 0$ and belongs to L_H^2.

(iii) *Market makers*, who receive the total traded volume (cannot distinguish between the insiders and noise traders),

$$y_t = x_t + z_t, \tag{9}$$

and set the market price p_t for the asset value v in a semistrong way as in [1]:

$$p_t = E\left[v|\mathscr{F}_t^M\right], \tag{10}$$

where $\mathscr{F}_t^M = \sigma\{y_s, 0 \le s \le t\}$.

As in [1, 2, 5], assume that the dynamics of the insider's strategy x_t is as

$$dx_t = \beta_t (v - p_t)dt, \quad x_0 = 0, \tag{11}$$

where β is a deterministic, positive, and smooth function on $[0, \infty)$, called the insider trading intensity in [1]. Then, informally, the total expected payoff of the insider is as in [3]:

$$E\int_0^\tau (v - p_s)dx_s = \int_0^{+\infty} \beta_s \exp(-\mu s)\Sigma_s ds, \tag{12}$$

where $\Sigma_t = E(v - p_t)^2$, called residual information in [1].

Note that (β, p) should satisfy $E[\int_0^\infty \exp(-\mu t)\beta_t (v - p_t)^2 dt] < \infty$. The collection of all these profiles (β, p) is denoted by $\mathbb{S}$.

Similar to those in [2] or [1], we give the following definition.

Definition 2. An equilibrium is a profile (β, p) in $\mathbb{S}$ such that

(i) Maximization of profit: it is given that p and β maximize

$$\int_0^\infty \beta_s \exp(-\mu s)\Sigma_s ds. \tag{13}$$

(ii) Market efficiency: it is given β and p satisfy the semistrong pricing condition:

$$p_t = E\left[v|\mathscr{F}_t^M\right]. \tag{14}$$

4. The Main Theorem

Before establishing the central theorem, we give an important lemma, whose proof can be found in [4] or [13].

Lemma 1. *Let $\Sigma_0 = 1$; then the residual information*

$$\Sigma_t = \frac{1}{1 + \int_0^t \rho_s^2 dl_s} \tag{15}$$

and

$$\frac{d}{dy}\Big|_{y=0}\Sigma(\beta + y\xi) = 2\Sigma_t^2 \int_0^t \left(\int_r^t \rho_s' K_H(s,r)ds - \rho_t K_H(t,r)\right)\frac{\xi_r}{\sigma_{zr}} dr, \tag{16}$$

where ξ is an arbitrary deterministic and smooth function on $[0, \infty)$, and the other functions or processes above are defined as follows:

$$\rho_t = \frac{d}{dl_t}\int_0^t k_H(t,s)\frac{\beta_s}{\sigma_{zs}} ds, \tag{17}$$

$$\begin{aligned} \Gamma(t) &= \int_0^{+\infty} x^{t-1} e^{-x} dx, \\ dl_t &= \frac{(2-2H)t^{1-2H}}{2H\Gamma((3/2)-H)\Gamma((1/2)+H)} dt, \end{aligned} \tag{18}$$

and

$$k_H(t,s) = \frac{s^{(1/2)-H}(t-s)^{(1/2)-H}}{2H\Gamma((3/2)-H)\Gamma((1/2)+H)}, \quad 0 < s < t. \tag{19}$$

Now, we will establish our central theorem below.

Theorem 1. *Suppose that the insider takes the optimal control β with its optimal expected profit:*

$$J^H(\beta) = \int_0^\infty \exp(-\mu t)\beta\Sigma(\beta)dt. \tag{20}$$

Then, the residual information Σ_t satisfies the following equation:

$$\begin{aligned} \Sigma_t = -2\sigma_{zt}^{-1} \int_t^\infty \beta_r \Big(\exp(\mu t - \mu r)\Sigma_r^2 \\ \left(\left(\int_t^r \rho_s' K_H(s,t)ds - \rho_r K_H(r,t)\right)\right)\Big)dr. \end{aligned} \tag{21}$$

Proof. Assume that β is an optimal strategy of the insider. Then, if we define a real function g by

$$g(y) = J^H(\beta + y\xi), \quad y \in R, \tag{22}$$

where $y \in R$ and ξ is an arbitrary function on $[0, \infty)$, then g is maximal at $y = 0$, or

$$0 = \frac{d}{dy}\Big|_{y=0} J^H(\beta + y\xi) = \frac{d}{dy}\Big|_{y=0}\int_0^\infty \exp(-\mu t)(\beta + y\xi)\Sigma_t(\beta + y\xi)dt. \tag{23}$$

From Lemma 1, we obtain

$$\begin{aligned} 0 = \int_0^\infty \exp(-\mu t)\xi\Sigma_t dt + 2\int_0^\infty \beta_t \exp(-\mu t)\Sigma_t^2 \\ \left\{\int_0^t \left(\int_r^t \rho_s' K_H((s,r)ds - \rho_t K_H(t,r)\right)\frac{\xi_r}{\sigma_{zr}} dr\right\} dt. \end{aligned} \tag{24}$$

Let

$$I_1 = \int_0^\infty \exp(-\mu t)\xi\Sigma_t \mathrm{d}t, \tag{25}$$

$$I_2 = 2\int_0^\infty \beta_t \exp(-\mu t)\Sigma_t^2 \left\{\int_0^t\left(\int_r^t \rho_s' K_H(s,r)\mathrm{d}s - \rho_t K_H(t,r)\right)\frac{\xi_r}{\sigma_{zr}}\mathrm{d}r\right\}\mathrm{d}t. \tag{26}$$

Changing the order of integration in (26), we have

$$I_2 = 2\int_0^\infty \frac{\xi_t}{\sigma_{zt}}\left(\int_t^\infty \left(\beta_r \exp(-\mu r)\Sigma_r^2 \left(\left(\int_t^r \rho_s' K_H(s,t)\mathrm{d}s - \rho_r K_H(r,t)\right)\right)\right)\mathrm{d}r\right)\mathrm{d}t. \tag{27}$$

Then, taking both I_2 and (25) into (23), we obtain

$$\begin{aligned} 0 &= \int_0^\infty \exp(-\mu t)\xi_t\Sigma_t \mathrm{d}t + 2\int_0^\infty \frac{\xi_t}{\sigma_{zt}}\left(\int_t^\infty \left(\beta_r \exp(-\mu r)\Sigma_r^2 \left(\left(\int_t^r \rho_s' K_H(s,t)\mathrm{d}s - \rho_r K_H(r,t)\right)\right)\right)\mathrm{d}r\right)\mathrm{d}t \\ &= \int_0^\infty \exp(-\mu t)\xi_t\left[\Sigma_t + \frac{2}{\sigma_{zt}}\left(\int_t^\infty \left(\beta_r \exp(\mu t - \mu r)\Sigma_r^2 \left(\left(\int_t^r \rho_s' K_H(s,t)\mathrm{d}s - \rho_r K_H(r,t)\right)\right)\right)\mathrm{d}r\right)\right]\mathrm{d}t. \end{aligned} \tag{28}$$

Since ξ is arbitrary, we have

$$\Sigma_t = -2\sigma_{zt}^{-1}\int_t^\infty \beta_r\left(\exp(\mu t - \mu r)\Sigma_r^2 \left(\left(\int_t^r \rho_s' K_H(s,t)\mathrm{d}s - \rho_r K_H(r,t)\right)\right)\right)\mathrm{d}r. \tag{29}$$

5. The Impact of Memory and Random Deadline on Equilibrium

The motive for is to find out the impact of memory and random deadline on equilibrium with Hurst parameter H varying.

Note that both the assumed optimal trading intensity β_t in Section 2 and the corresponding residual information Σ_t depend on Hurst parameter H, which can be written, respectively, as

$$\begin{aligned} \beta_t &= \beta_t(H), \\ \Sigma_t &= \Sigma_t(H). \end{aligned} \tag{30}$$

Equations (15) and (29) can be turned, respectively, as

$$\Sigma_t(H) = \frac{1}{1 + \int_0^t \rho_s^2(H)\mathrm{d}l_s}, \tag{31}$$

$$\Sigma_t(H) = -2\sigma_{zt}^{-1}\int_t^\infty \beta_r(H)\left(\exp(\mu t - \mu r)\Sigma_r^2(H) \left(\left(\int_t^r \rho_s' K_H(s,t)\mathrm{d}s - \rho_r K_H(r,t)\right)\right)\right)\mathrm{d}r. \tag{32}$$

Unfortunately, we are not able to solve our general equation to obtain $\beta(H)$ and $\Sigma(H)$ explicitly. However, we have the following limit theorem.

Theorem 2. *Assume that*

$$\begin{aligned} \lim_{H\longrightarrow(1/2)} \beta_t(H) &= \beta_t, \\ \lim_{H\longrightarrow(1/2)} \Sigma_t(H) &= \Sigma_t, \\ \sigma_{zt} &\equiv 1. \end{aligned} \tag{33}$$

Then,

$$\begin{aligned} \Sigma_t &= \exp(-2\mu t), \\ \beta_t &= \sqrt{2\mu}\exp(\mu t). \end{aligned} \tag{34}$$

Proof. Since $\lim_{H\to(1/2)} K_H(t,s) = 1$, it is easy to see that, by equations (31) and (32),

$$\Sigma_t = \left(1 + \int_0^t \rho_s^2 \mathrm{d}l(s)\right)^{-1}, \tag{35}$$

$$\Sigma_t = 2\beta_t\int_t^\infty \beta_r\Sigma_r^2\exp(\mu t - \mu r)\mathrm{d}r. \tag{36}$$

Then, by equation (35), it is easy to check that

$$\begin{aligned} \frac{\mathrm{d}\Sigma_t}{\mathrm{d}t} &= -\beta_t^2\Sigma_t^2, \\ \Sigma_0 &= 1. \end{aligned} \tag{37}$$

And, by the structure of equation (36), we can assume that Σ_t is the form of $\exp(-k\mu t)$, where $k > 0$. Now, taking it into (37), we have

$$\beta_t = \sqrt{k\mu}\exp\left(\frac{k\mu t}{2}\right). \tag{38}$$

Then, taking it back into (36), we get $k = 2$. Therefore, the conclusion holds.

Remark 1. In [3], Caldentey and Stacchetti proposed a model of insider trading with a random deadline and found that the residual information Σ_t in the equilibrium is an

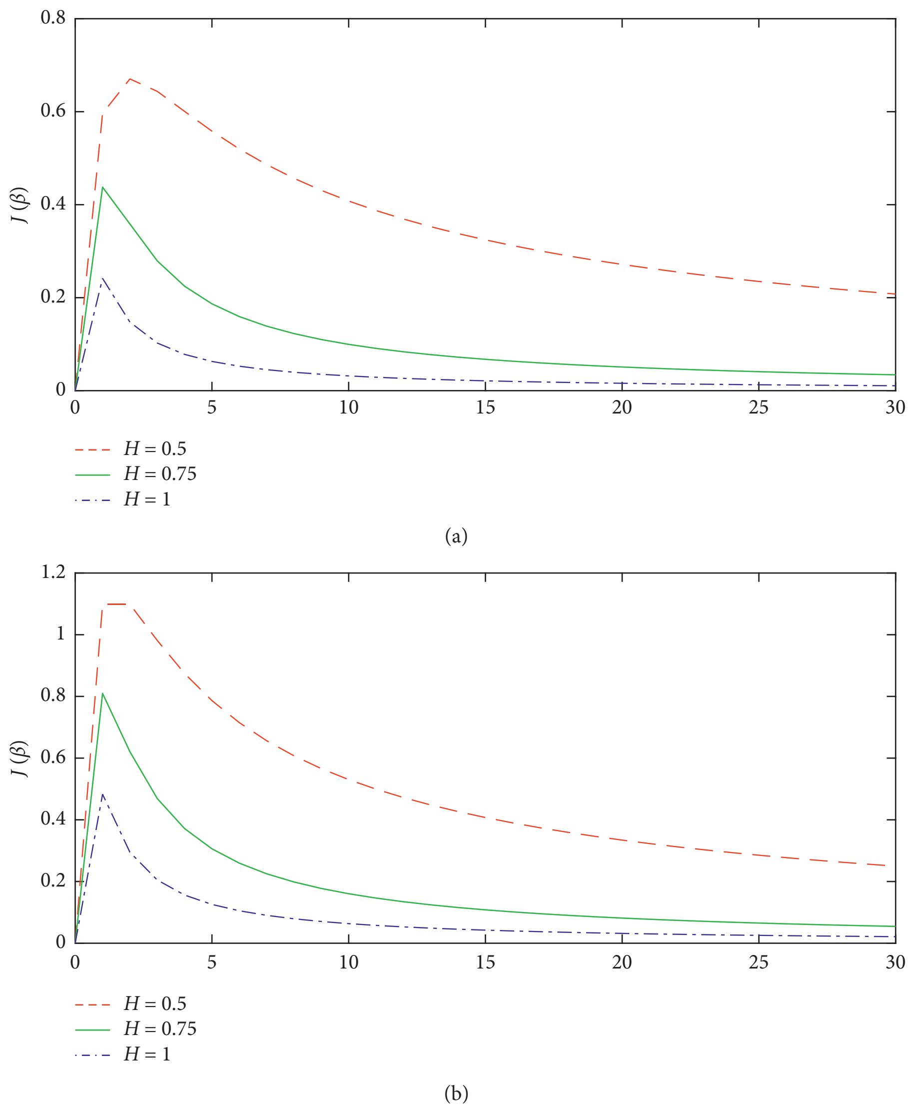

FIGURE 1: Hurst parameter $H = 0.5$ (top curve), Hurst parameter $H = 0.75$ (second from top), and Hurst parameter $H = 1$ (third from top). (a) Plot of the function $J(\beta)$ for 3 different values of H x-axis is β. (b) Plot of the function $J(\beta)$ for 3 different values of H x-axis is β.

exponential structure, so our exponential form of Σ_t or β_t in Theorem 2 is natural and reasonable. In fact, as in [18], when noise traders have no memories, that is, $H = (1/2)$, the optimal trading intensity and the corresponding residual information are the same as those in Theorem 2. So, Theorem 2 means that, as memories of noise traders become weaker and weaker, the optimal trading intensity and the corresponding residual information tend to those, respectively, in insider trading without any memory in [18].

Next, we will analyse the impact of memory on insider expected profit when $H \in ((1/2), 1]$. To make some conclusions about this influence in some special case, we also restrict ourselves to both insider trading intensity and the volatility of noise trades are constant numbers as in [13]; if we denote $\beta_t(H) \equiv \beta > 0$ and $\sigma_{zt} \equiv 1$, then ρ_t in (17) can be written as follows:

$$\rho_t = \frac{\mathrm{d}}{\mathrm{d}l_t}\int_0^t k_H(t,s)\frac{\beta_s}{\sigma_{zs}}\mathrm{d}s = \frac{\Gamma((3/2)-H)^2\beta}{\Gamma(3-2H)}. \tag{39}$$

Hence, together with (18), we work out

$$\Sigma_t = \left(1 + \frac{\beta^2\Gamma((3/2)-H)^3}{2H\Gamma(3-2H)^2\Gamma((1/2)+H)}t^{2-2H}\right)^{-1}. \tag{40}$$

Then, by equation (12), we have

$$J^H(\beta) = \beta\int_0^\infty \exp(-\mu s)\left(1 + \frac{\beta^2\Gamma((3/2)-H)^3}{2H\Gamma(3-2H)^2\Gamma((1/2)+H)}s^{2-2H}\right)^{-1}\mathrm{d}s. \tag{41}$$

It is easy to see that

$$J^H(0) = 0,$$
$$\lim_{\beta \longrightarrow \infty} J^H(\beta) = 0, \quad (42)$$

and $J^H(\beta)$ is a continuous function of β and can attain its maximum value. Now, we choose different parameters H to show the relation between β and $J^H(\beta)$ numerically as plots. Because the trading deadline is random time τ with parameter μ, we choose $\mu = 1$ and $T = 10000$ and plot the function $J(\beta)$ for the Hurst parameters $H = 0.5, 0.75, 1$ as in Figure 1(a); to compare with the setting in [13], we also plot the function $J(\beta)$ for the Hurst parameters $H = 0.5, 0.75, 1$, as in Figure 1(b).

According to Figure 1, the expected profit in our model is always lower than that in [13]; this is because our trading deadline is random, leading to the loss of the insider's information.

6. Conclusions

In this paper, we study a model of continuous-time insider trading in which noise traders have some memories and the trading stops at a random deadline. By the filtering theorem on fractional Brownian motion in [19] and the stochastic maximum principle as in [5], the necessary condition of the insider's optimal strategy is obtained, as the equation in Theorem 1 should be satisfied by the insider's optimal strategy.

As in Theorem 2, it shows that when noise traders' memories become weaker and weaker, the optimal trading intensity and the corresponding residual information tend to those, respectively, when noise traders have any memory in [18]. And, as in Figure 1, numerical simulation shows that if both insider trading intensity and the volatility of noise trades assume two constant numbers, the expected profit with a random deadline is always lower than that when the asset value is disclosed at a finite fixed time in [13]; this is because trading time in our model is a random deadline which yields the loss of the insider's information.

Beyond that, as shown in Figure 1(a), as $H \in [1/2, 1)$ increases, both the utility value of an insider and the optimal insider trading intensity $\beta^*(H)$ decrease.

As in [13], we also think that there are two ways to reduce the complexity of the noise with increasing H: the one is that the noise process becomes long-range dependence, and the other is that the paths of the noise process become more regular, which makes market makers more easier to excavate insider's private information. And, together with trading at a random deadline, the insider's information advantage will lose more, which makes the profit more less.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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