Research Article

An Approach for Generating Weights Using the Pairwise Comparison Matrix

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Data envelopment analysis (DEA) has been used for obtaining weights for the analytic hierarchy process (AHP), an approach known as DEAHP. This method sometimes identifies more than one decision criterion or alternative as DEAHP-efficient. To overcome this problem, this paper proposes a new approach that not only generates appropriate weights for the decision criteria or alternatives, but also differentiates between DEAHP-efficient decision criteria or alternatives. To this end, we propose a DEA model with an assurance region and a cross-weight model that prioritizes decision criteria or alternatives by considering their most unfavorable weights. Two numerical examples are also provided to illustrate the advantages and potential applications of the proposed model.

1. Introduction

An important issue in the analytic hierarchy process (AHP) is obtaining a priority vector from a pairwise comparison matrix, which has widely been investigated in AHP articles [1–3]. Therefore, the key issue is how to determine the weights of decision criteria or alternatives [4–6]. Since the advent of this technique, it has been theoretically developed multiple times and applied for various purposes (e.g., review studies of Kheybari et al. [7] and Emrouznejad and Marra [8]).

Several methods have been proposed in AHP-related articles for the determination of local weights based on the pairwise comparison matrices [9–18]. Each of these methods has specific advantages and disadvantages, and thus, none of them can be considered the best. Although the eigenvector method (EM) has been strongly recommended and preferred by Saaty [19], there is no general agreement on its superiority (e.g., Mikhailov [20] and Srdjevic [21]).

The data envelopment analysis (DEA) method, proposed by Charnes, Cooper [22], has been used for prioritization in AHP. In this approach, decision criteria or alternatives in a

pairwise comparison matrix are considered decision-making units (DMUs). Moreover, the elements on the rows of the pairwise comparison matrix are the outputs of DMUs, and the efficiency of DMUs is considered priorities of the pairwise comparison matrix. Accordingly, Ramanathan [23] proposed a DEAHP method to obtain both weights and cumulative weights in AHP. Although DEAHP can generate true weights for consistent pairwise comparison matrices, it generates illogical and meaningless weights when it comes to inconsistent pairwise comparison matrices. Wang, Chin [24] and Wang, Chin [25] detailed the disadvantages of DEAHP by providing numerical examples. To overcome these issues, they proposed a DEA model with an assurance region (AR) to generate weights in the AHP. The DEA/AR model can generate intuitive and even logical weights for both consistent and inconsistent pairwise comparison matrices. Wang, Parkan [26] proposed a linear programming method to generate the most desirable weights from pairwise comparison matrices. Wang and Chin [27] proposed a DEA model for prioritization in AHP. Instead of the efficiency of each DMU, the proposed model defines relative efficiency as its priority. As a result, it generates the most desirable

weights, which are close to the weights of the eigenvector of the pairwise comparison matrices.

The most desirable weights for each decision criterion or alternative are assessed from its view, which is called self-evaluation [28, 29]. When a decision criterion or alternative evaluates its best weight, it also evaluates other decision criteria or alternatives. Such weights, evaluated using other criteria or alternatives, are called cross-weights [30]. The evaluations done by other criteria or alternatives are called peer-evaluation. Obviously, cross-weights may not be desirable for decision criteria or alternatives. Therefore, the use of the most desirable weights is not suitable for a comprehensive decision-making.

To generate a comprehensive weight that is also logical for every decision criterion or alternative, Wang, Luo [28] proposed the cross-weight evaluation technique for weight differentiation. The cross-weight evaluation assessed the weights of a pairwise comparison matrix not only from its standpoint, but also from a peer standpoint. Therefore, the weights generated from the cross-weight evaluation technique are more logical and fair. The models proposed by Wang et al. [28] were among the optimistic models in DEA. Although those models are more optimal than available DEAHP models in differentiation, similar to other DEAHP models, they may sometimes evaluate some decision criteria or alternatives at the same level, making decision-making problematic.

The present paper proposed a new approach, called “analysis of the most undesirable weight,” to obtain the weights of decision criteria and alternatives. It enables us to rank a set of decision criteria and alternatives, even those assessed at the same level from an optimistic view. The principal condition for the analysis of the most undesirable weight is that, in addition to the most desirable weight, the most undesirable weight can also be assigned to every decision criterion and alternative. The most desirable and undesirable weights for a decision criterion or alternative define the weight interval of that criterion or alternative. Two numerical examples will be provided to show the advantages and potential applications of the proposed model.

The remaining parts of the paper are organized as follows: Section 2 briefly reviews the DEAHP models for the generation of the most desirable weights for pairwise comparison matrices. Section 3 deals with the cross-weight evaluation technique from a pessimistic standpoint. Section 4 presents numerical examples. Section 5 presents comparison with other prioritization methods. Section 6 is devoted to the conclusion.

2. DEAHP Models for Generating Most Desirable Weights

Assume that

$$A = (a_{ij})_{n \times n} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}, \quad (1)$$

is a pairwise comparison matrix with $a_{ii} = 1$ and $a_{ji} = 1/a_{ij}$ for $j \neq i$ and $W = (w_1, \dots, w_n)^T$ is its priority vector. In DEAHP, each row of A is considered a DMU, and each column is considered an output. Accordingly, Wang, Chin [25] proposed DEA model (2) to generate weights from pairwise comparison matrices A :

$$\begin{aligned} \max \quad & w_o, \\ \text{s.t.} \quad & \begin{cases} w_i = \sum_{j=1}^n a_{ij} v_j \leq 1, & i = 1, \dots, n, \\ \frac{w_j}{\beta} \leq v_j \leq \frac{w_j}{n}, & j = 1, \dots, n, \end{cases} \end{aligned} \quad (2)$$

where o refers to the criterion or alternative under the evaluation. By solving model (2) for each w_o ($o = 1, \dots, n$), the most desirable weights for n decision criteria or alternatives can be obtained. v_j ($j = 1, \dots, n$) are decision variables, and $w_j/\beta \leq v_j \leq w_j/n$ ($j = 1, \dots, n$) are the AR imposed on the DEA model. β is the upper boundary of the maximum eigenvector of the pairwise comparison matrix $A = (a_{ij})_{n \times n}$, which is determined from the following equation:

$$\beta = \min \left\{ \max_i \left(\frac{1}{r_i} \sum_{j=1}^n a_{ij} r_j \right), \max_i \left(\frac{1}{c_i} \sum_{j=1}^n a_{ji} c_j \right) \right\}, \quad (3)$$

where $r_1, \dots, r_n$ and $c_1, \dots, c_n$ are, respectively, the sums of rows and columns of $A = (a_{ij})_{n \times n}$. If there is a set of positive weights v_j^* , ($j = 1, \dots, n$) to fulfill $w_o^* = 1$, the decision criterion or alternative is called DEAHP efficient; otherwise, it is called DEAHP nonefficient. Typically, the best decision criterion or alternative can be selected from DEAHP-efficient decision criteria or alternatives.

Another technique for generating weight in the DEAHP is the use of cross-weight evaluation to obtain priorities. Model (2) only considers the most desirable weights obtained from self-evaluation and may generate several optimal weight vectors. To solve the problem of nonuniqueness, Wang et al. [28] proposed the cross-weight model (4) that generates unique weights:

$$\begin{aligned} & \max \sum_{j=1, j \neq o}^n w_j, \\ & \text{s.t.} \begin{cases} \sum_{j=1}^n a_{oj} v_j = w_o^*, \\ w_i = \sum_{j=1}^n a_{ij} v_j \leq 1, \quad i = 1, \dots, n, \\ \frac{w_j}{\beta} \leq v_j \leq \frac{w_j}{n}, \quad j = 1, \dots, n, \end{cases} \end{aligned} \quad (4)$$

where w_o^* is the most desirable weight of criterion or alternative under investigation based on model (2).

3. DEAHP Models for Generating Most Undesirable Weights

Model (2) may recognize more than one decision criterion or alternative as efficient. These efficient decision criteria or alternatives cannot be differentiated based on their most desirable weights. However, we can consider the most undesirable weights. The most undesirable weight is in contrast to the most desirable weight and represents the weight of each decision criterion or alternative in the most undesirable condition. Theoretically, the most desirable and undesirable weights should be obtained in the same range, which should present an interval for each decision criterion or alternative. For example, they can be measured in the interval of $[\alpha, 1]$, where $\alpha > 0$ is a predefined parameter [31–33]. The corresponding model can be created as follows:

$$\begin{aligned} & \min \hat{w}_o, \\ & \text{s.t.} \begin{cases} \hat{w}_i = \sum_{j=1}^n a_{ij} v_j \geq \alpha, \quad i = 1, \dots, n, \\ \frac{\hat{w}_j}{\beta} \leq v_j \leq \frac{\hat{w}_j}{n}, \quad j = 1, \dots, n, \end{cases} \end{aligned} \quad (5)$$

Nevertheless, if the most undesirable weight is measured in this way, the major problem is the need to predefine an appropriate value for α . To avoid the problem of determining the value of α , the most undesirable weight of each decision criterion or alternative can be measured with the following model:

$$\begin{aligned} & \min \hat{w}_o \\ & \text{s.t.} \begin{cases} \hat{w}_i = \sum_{j=1}^n a_{ij} v_j \geq 1, \quad i = 1, \dots, n \\ \hat{w}_j / \beta \leq v_j \leq \hat{w}_j / n, \quad j = 1, \dots, n. \end{cases} \end{aligned} \quad (6)$$

This model can be solved for all considered decision criteria and alternatives. The decision criterion or alternative with the greatest weight can be selected. The advantage of model (6) relative to model (2) is that it allows for differentiating between decision criteria and alternatives of DEAHP efficient. If there was a set of positive weights for which the value of an optimal objective function of model (6) is 1 or $\hat{w}_o^* = 1$, it would be said that the decision criterion or alternative is DEAHP inefficient; otherwise, it is called DEAHP non-inefficient.

Theorem 1. If $A = (a_{ij})_{n \times n}$ was a perfectly consistent pairwise comparison matrix, then model (6) would generate the following weights:

$$\hat{w}_i^* = \frac{\hat{w}_i}{\min_{j \in \{1, \dots, n\}} \{\hat{w}_j\}}, \quad i = 1, \dots, n, \quad (7)$$

which are the normalization of true weights $\hat{w}_i$, ($i = 1, \dots, n$) of the pairwise comparison matrix $A = (a_{ij})_{n \times n}$.

Proof. Since $A = (a_{ij})_{n \times n}$ is a perfectly consistent pairwise comparison matrix, it can be recognized by the weights of eigenvector $\hat{w}_j = 1 / \sum_{i=1}^n a_{ij}$ ($j = 1, \dots, n$) in the form of $a_{ij} = \hat{w}_i / \hat{w}_j$ ($i, j = 1, \dots, n$). Accordingly, $\sum_{j=1}^n (v_j / \hat{w}_j) \geq 1 / \hat{w}_i$, ($i = 1, \dots, n$) can be obtained from $\sum_{j=1}^n a_{ij} v_j = \sum_{j=1}^n (\hat{w}_i / \hat{w}_j) v_j = \hat{w}_i \sum_{j=1}^n (v_j / \hat{w}_j) \geq 1$, ($i = 1, \dots, n$). Therefore, we have $\sum_{j=1}^n (v_j / \hat{w}_j) \geq \max_i (1 / \hat{w}_i) = 1 / \min_{j \in \{1, \dots, n\}} \{\hat{w}_j\}$. The minimum value of the objective function of model (6) can be obtained in the form of $\hat{w}_o^* = \hat{w}_o \sum_{j=1}^n (v_j^* / \hat{w}_j) = \hat{w}_o / \min_{j \in \{1, \dots, n\}} \{\hat{w}_j\}$, where $\hat{w}_o \in \{\hat{w}_1, \dots, \hat{w}_n\}$.

Model (6) only considers the most undesirable weights obtained from self-evaluation and may generate several optimal weight vectors. To solve the problem of non-uniqueness, cross-weight model (8), which generates unique weights, is recommended:

$$\begin{aligned} & \min \sum_{j=1, j \neq o}^n \hat{w}_j, \\ & \text{s.t.} \begin{cases} \sum_{j=1}^n a_{oj} v_j = \hat{w}_o^*, \\ \hat{w}_i = \sum_{j=1}^n a_{ij} v_j \geq 1, \quad i = 1, \dots, n, \\ \hat{w}_j / \beta \leq v_j \leq \hat{w}_j / n, \quad j = 1, \dots, n, \end{cases} \end{aligned} \quad (8)$$

where $\hat{w}_o^*$ is the most undesirable weight of criterion or alternative under investigation, based on model (6). $\square$

4. Numerical Examples

In this part, two numerical examples were investigated using the proposed approach to show its application and capability in the differentiation of efficient decision criteria or alternatives.

Example 1. Consider the following pairwise comparison matrix of Wang and Chin [27]:

$$A = \begin{bmatrix} 1 & 1/3 & 1/3 & 1 & 1/2 \\ 3 & 1 & 1/2 & 1/2 & 1 \\ 3 & 2 & 1 & 1/2 & 2 \\ 1 & 2 & 2 & 1 & 1 \\ 2 & 1 & 1/2 & 1 & 1 \end{bmatrix}. \quad (9)$$

This pairwise comparison matrix has a good consistency $CR = 0.0999$. $\beta = 6.3810$ was obtained for this pairwise comparison matrix. Table 1 shows the priorities obtained from EM and DEA/AR models (2) and (6). According to Table 1, it can be seen that EM and the DEA/AR model (6) evaluate C_3 as the most important criterion or alternative. DEA/AR models (2) and (6) differ in the evaluation of C_3 and C_4 . The DEA/AR model (2) assesses C_3 and C_4 at the same level (defines C_3 and C_4 as efficient) and cannot differentiate between them, while the DEA/AR model (6) can easily differentiate C_3 from C_4 , and evaluates C_3 as more important than C_4 . In other words, DEA/AR model (6) provides a general rank of decision criteria or alternatives.

Tables 2 and 3 show the results from the evaluation of cross-weights obtained from DEA/AR models (2), (4), (6), and (8). Obviously, the mean cross-weights generated using the DEA/AR models (2) and (4) (Table 2) indicate that C_3 and C_4 are at the same level of importance, whereas the mean cross-weights generated using the DEA/AR models (6) and (8) (Table 3) indicate that C_3 is more important than C_4 . Therefore, the mean cross-weights obtained from DEA/AR models (6) and (8) have greater differentiation performance than the mean cross-weights obtained from DEA/AR models (2) and (3).

Example 2. Consider the supplier selection problem of BEKO (a major TV manufacturer in Turkey), derived from Sevki et al. [34]. The hierarchical structure of the supplier selection problem for purchasing CRTs is presented in Figure 1. It consists of six evaluation criteria, 25 subcriteria, and three suppliers. The six evaluation criteria are as follows:

- C_1 : performance assessment
- C_2 : human resources
- C_3 : quality system assessment
- C_4 : manufacturing
- C_5 : business criteria
- C_6 : using information technology

Each criterion is divided into 3–6 subcriteria, forming a total of 25 subcriteria. These 25 subcriteria are as follows:

- C_{11} : shipment Quality
- C_{12} : delivery
- C_{13} : cost analysis
- C_{21} : number of Employees
- C_{22} : organizational structure
- C_{23} : training

- C_{24} : number of technical staff
- C_{31} : management commitment
- C_{32} : inspection
- C_{33} : quality planning
- C_{34} : quality assurance
- C_{41} : production capacity
- C_{42} : predictive and preventive maintenance
- C_{43} : lead-time
- C_{44} : up-to-date techniques and equipment
- C_{45} : transportation–storage and packaging
- C_{46} : new product development
- C_{51} : reputation
- C_{52} : geographic location
- C_{53} : price
- C_{54} : patent
- C_{55} : technical capacity
- C_{61} : radio-frequency identification (RFID)
- C_{62} : electronic data interchange (EDI)
- C_{63} : internet

For anonymity, the three supplier companies are named by numbers 1, 2, and 3. In Tables 4–16, the pairwise comparison matrix has been provided for six evaluation criteria, 25 subcriteria, and three suppliers. Moreover, their local weights are reported in the last two columns of Tables 4–16, using the EM and model (8).

The local weights of three suppliers given each sub-criterion of each criterion are presented in Tables 17–22 to obtain a compound weight vector of each criterion. In Table 23, the compound weight vectors are accumulated in the form of a comprehensive weight vector relative to the supplier accumulation. In Tables 17–23, the weights of EM and model (8) are both normalized until their maximum value equates to 1. According to Table 23, EM and model (8) generate similar final decisions. They assessed Supplier 2 as the best supplier.

5. Comparison with Other Prioritization Methods

This section compares the proposed prioritization method with Eigenvector Method (EM), Least Squares Method (LSM) [35], Weighted Least Square Method (WLSM) [10], Logarithmic Least Squares Method (LLSM) [11], and Correlation Coefficient Maximization Approach (CCMA) [36] to indicate their differences, advantages, and disadvantages.

According to Saaty [2], EM calculates eigenvector weights by solving the following eigenvector equations:

$$\sum_{j=1}^n a_{ij} w_j = \lambda_{\max} w_i, \quad i = 1, \dots, n, \quad (10)$$

where $\lambda_{\max}$ is the maximum eigenvalue of pairwise comparison matrix A , w_i ($i = 1, \dots, n$) applies in $w_i > 0$, and we have $\sum_{i=1}^n w_i = 1$. A weakness of EM is its non-linearity.

TABLE 1: Priority of pairwise comparison matrix A and their ranks.

Model	Decision criteria or alternatives				
	C_1	C_2	C_3	C_4	C_5
EM	0.1129 (5)	0.1763 (4)	0.2661 (1)	0.2648 (2)	0.1799 (3)
DEA/AR model (2)	0.4454 (5)	0.6836 (4)	1.0000 (1)	1.0000 (1)	0.6945 (3)
DEA/AR model (6)	1.0000 (5)	1.4716 (4)	2.2291 (1)	2.1827 (2)	1.5448 (3)

TABLE 2: Results from the cross-weight evaluation based on DEA/AR models (2) and (4).

Criteria or alternatives	Target criterion or alternative					Average cross-weights	Rank
	C_1	C_2	C_3	C_4	C_5		
C_1	0.4454	0.4454	0.4454	0.4454	0.4454	0.4454	5
C_2	0.6836	0.6836	0.6836	0.6836	0.6836	0.6836	4
C_3	1	1	1	1	1	1	1
C_4	1	1	1	1	1	1	1
C_5	0.6945	0.6945	0.6945	0.6945	0.6945	0.6945	3

TABLE 3: Results from the cross-weight evaluation based on DEA/AR models (6) and (8).

Criteria or alternatives	Target criterion or alternative					Average cross-weights	Rank
	C_1	C_2	C_3	C_4	C_5		
C_1	1	1	1	1	1	1	5
C_2	1.4774	1.4716	1.5240	1.5683	1.4716	1.5026	4
C_3	2.2421	2.2431	2.2291	2.3184	2.2431	2.2552	2
C_4	2.2878	2.2993	2.2628	2.1827	2.2993	2.2663	1
C_5	1.5449	1.5448	1.5528	1.5866	1.5448	1.5548	3

Should the matrix have greater dimensions, it will be time-consuming to calculate eigenvalues and eigenvectors.

Saaty and Vargas [35] formulated LSM as follows [37]:

$$\begin{aligned}
 \min \quad & z = \sum_{i=1}^n \sum_{j=1}^n \left(\frac{a_{ij} - w_i}{w_j} \right)^2 \\
 \text{s.t.} \quad & \sum_{i=1}^n w_i = 1, \\
 & w_i \geq 0, \\
 & i = 1, \dots, n.
 \end{aligned} \tag{11}$$

Model (11) is clearly non-linear.

Chu et al. [10] formulated WLSM as follows:

$$\begin{aligned}
 \min \quad & z = \sum_{i=1}^n \sum_{j=1}^n (a_{ij} w_j - w_i)^2 \\
 \text{s.t.} \quad & \sum_{i=1}^n w_i = 1, \\
 & w_i \geq 0, \\
 & i = 1, \dots, n.
 \end{aligned} \tag{12}$$

Model (12) is also non-linear.

Crawford [11] formulated LLSM as follows:

$$\begin{aligned}
 \min \quad & z = \sum_{i=1}^n \sum_{j=1}^n (\ln a_{ij} - \ln(w_i/w_j))^2 \\
 \text{s.t.} \quad & \sum_{i=1}^n w_i = 1, \\
 & w_i \geq 0, \\
 & i = 1, \dots, n.
 \end{aligned} \tag{13}$$

By solving Model (13), which is non-linear too, w_i ($i = 1, \dots, n$) will be computed.

By an overview, it can be stated that Ordinary Least Squares (OLS) and LLSM minimize arithmetic mean of errors and geometric mean of errors respectively.

Wang et al. [36] formulated CCMA as follows:

$$\begin{aligned}
 \text{Max } J = & \sum_{i=1}^n \sum_{j=1}^n \left(a_{ij} - \frac{1/n + \beta \hat{w}_i^*}{1/n + \beta \hat{w}_j^*} \right)^2 \\
 \text{s.t.} \quad & \beta \geq 0,
 \end{aligned} \tag{14}$$

where $\hat{w}_i^*$ is calculated by the following equations:

$$\hat{w}_i^* = \frac{\sum_{j=1}^n b_{ij}}{\sqrt{\sum_{i=1}^n (\sum_{j=1}^n b_{ij})^2}}, \quad i = 1, \dots, n, \tag{15}$$

$$b_{ij} = \frac{a_{ij} - \bar{a}_j}{\sqrt{\sum_{k=1}^n (a_{kj} - \bar{a}_j)^2}}, \quad i, j = 1, \dots, n,$$

where $\bar{a}_j = 1/n \sum_{i=1}^n a_{ij}$.

FIGURE 1: Hierarchical structure of supplier selection problem.

TABLE 4: Pairwise comparison matrix for six criteria and their local weight.

Criteria	C_1	C_2	C_3	C_4	C_5	C_6	EM weights	Model (8)
C_1	1	4	3	2	1/2	7	0.245	7.586
C_2	1/4	1	1/2	1/3	1/8	2	0.054	1.690
C_3	1/3	2	1	1/2	1/5	4	0.094	2.928
C_4	1/2	3	2	1	1/3	5	0.153	4.732
C_5	2	8	5	3	1	9	0.422	13.108
C_6	1/7	1/2	4	1/5	1/9	1	0.032	1.000

Consistency ratio 0.0128.

TABLE 5: Pairwise comparison matrix for three subcriteria of C_1 and their local weights.

Criteria	C_{11}	C_{12}	C_{13}	EM weights	Model (8)
C_{11}	1	5	6	0.726	7.101
C_{12}	1/5	1	2	0.172	1.684
C_{13}	1/6	1/2	1	0.102	1.000

Consistency ratio 0.0251.

TABLE 6: Pairwise comparison matrix for four subcriteria of C_2 and their local weights.

Criteria	C_{21}	C_{22}	C_{23}	C_{24}	EM weights	Model (8)
C_{21}	1	1/5	1/6	1/3	0.061	1.000
C_{22}	5	1	1/2	3	0.310	5.011
C_{23}	6	2	1	4	0.495	8.022
C_{24}	3	1/3	1/4	1	0.134	2.165

Consistency ratio 0.0292.

TABLE 7: Pairwise comparison matrix for four subcriteria of C_3 and their local weights.

Criteria	C_{31}	C_{32}	C_{33}	C_{34}	EM weights	Model (8)
C_{31}	1	1/4	1/4	1/6	0.060	1.000
C_{32}	4	1	1/3	1/3	0.156	2.564
C_{33}	4	3	1	1/3	0.273	4.383
C_{34}	6	3	3	1	0.512	8.460

Consistency ratio 0.0717.

TABLE 8: Pairwise comparison matrix for six subcriteria of C_4 and their local weights.

Criteria	C_{41}	C_{42}	C_{43}	C_{44}	C_{45}	C_{46}	EM weights	Model (8)
C_{41}	1	2	3	1/2	4	1/3	0.152	3.956
C_{42}	1/2	1	1/2	1/4	2	1/7	0.061	1.597
C_{43}	1/3	2	1	1/3	2	1/6	0.079	2.044
C_{44}	2	4	3	1	6	1/2	0.241	6.266
C_{45}	1/4	1/2	1/2	1/6	1	1/9	0.038	1.000
C_{46}	3	7	6	2	9	1	0.428	11.143

Consistency ratio 0.0168.

TABLE 9: Pairwise comparison matrix for five subcriteria of C_5 and their local weights.

Criteria	C_{51}	C_{52}	C_{53}	C_{54}	C_{55}	EM weights	Model (8)
C_{51}	1	2	1/8	1/3	1/7	0.052	1.499
C_{52}	1/2	1	1/9	1/5	1/8	0.035	1.000
C_{53}	8	9	1	5	2	0.479	13.745
C_{54}	3	5	1/5	1	1/3	0.128	3.654
C_{55}	7	8	1/2	3	1	0.306	8.754

Consistency ratio 0.0294.

TABLE 10: Pairwise comparison matrix for three subcriteria of C_6 and their local weights.

Criteria	C_{61}	C_{62}	C_{63}	EM weights	Model (8)
C_{61}	1	1/6	1/7	0.070	1.000
C_{62}	6	1	1/2	0.348	5.005
C_{63}	7	2	1	0.582	8.361

Consistency ratio 0.0279.

TABLE 11: Pairwise comparison matrices for three suppliers relative to subcriteria of C_1 and their local weights.

Supplier	Supplier1	Supplier 2	Supplier 3	EM weights	Model (8)
<i>A. Comparison of suppliers with respect to shipment quality (C_{11}) (consistency ratio 0.0332)</i>					
Supplier 1	1	3	5	0.637	6.066
Supplier 2	1/3	1	3	0.258	2.461
Supplier 3	1/5	1/3	1	0.105	1.000
<i>B. Comparison of suppliers with respect to delivery (C_{12}) (consistency ratio 0.0251)</i>					
Supplier 1	1	1/5	1/6	0.081	1.000
Supplier 2	5	1	1/2	0.342	4.211
Supplier 3	6	2	1	0.577	7.101
<i>C. Comparison of suppliers with respect to cost (C_{13}) (consistency ratio 0)</i>					
Supplier 1	1	1/3	1/6	0.100	1.000
Supplier 2	3	1	1/2	0.300	3.000
Supplier 3	6	2	1	0.600	6.000

TABLE 12: Pairwise comparison matrices for three suppliers relative to subcriteria of C_2 and their local weights.

Supplier	Supplier 1	Supplier 2	Supplier 3	EM weights	Model (8)
<i>A. Comparison of suppliers with respect to the number of employees (C_{21}) (consistency ratio 0.0032)</i>					
Supplier 1	1	1/3	1/5	0.109	1.000
Supplier 2	3	1	1/2	0.309	2.823
Supplier 3	5	2	1	0.582	5.313
<i>B. Comparison of suppliers with respect to organization structure (C_{22}) (consistency ratio 0.0462)</i>					
Supplier 1	1	4	6	0.691	7.526
Supplier 2	1/4	1	3	0.218	2.373
Supplier 3	1/6	1/3	1	0.091	1.000
<i>C. Comparison of suppliers with respect to the number training (C_{23}) (consistency ratio 0.0462)</i>					
Supplier 1	1	4	6	0.691	7.526
Supplier 2	1/4	1	3	0.218	2.373
Supplier 3	1/6	1/3	1	0.091	1.000
<i>D. Comparison of suppliers with respect to the number of technical staff (C_{24}) (consistency ratio 0.0032)</i>					
Supplier 1	1	1/3	2	0.230	1.882
Supplier 2	3	1	5	0.648	5.313
Supplier 3	1/2	1/5	1	0.122	1.000

TABLE 13: Pairwise comparison matrices for three suppliers relative to subcriteria of C_3 and their local weights.

Supplier	Supplier 1	Supplier 2	Supplier 3	EM weights	Model (8)
<i>A. Comparison of suppliers with respect to management commitment (C_{31}) (consistency ratio 0.0739)</i>					
Supplier 1	1	4	5	0.674	6.635
Supplier 2	1/4	1	3	0.226	2.226
Supplier 3	1/5	1/3	1	0.100	1.000
<i>B. Comparison of suppliers with respect to inspection and control (C_{32}) (consistency ratio 0.0559)</i>					
Supplier 1	1	5	7	0.731	8.972
Supplier 2	1/5	1	3	0.188	2.316
Supplier 3	1/7	1/3	1	0.081	1.000
<i>C. Comparison of suppliers with respect to quality planning (C_{33}) (consistency ratio 0.0187)</i>					
Supplier 1	1	7	9	0.793	10.416
Supplier 2	1/7	1	2	0.131	1.724
Supplier 3	1/9	1/2	1	0.076	1.000
<i>D. Comparison of suppliers with respect to quality assurance (C_{34}) (consistency ratio 0.0559)</i>					
Supplier 1	1	5	7	0.731	8.972
Supplier 2	1/5	1	3	0.188	2.316
Supplier 3	1/7	1/3	1	0.081	1.000

TABLE 14: Pairwise comparison matrices for three suppliers relative to subcriteria of C_4 and their local weights.

Supplier	Supplier 1	Supplier 2	Supplier 3	EM weights	Model (8)
<i>A. Comparison of suppliers with respect to production capacity (C_{41}) (consistency ratio 0.0739)</i>					
Supplier 1	1	1/5	1/4	0.094	1.000
Supplier 2	5	1	3	0.627	6.635
Supplier 3	4	1/3	1	0.280	2.968
<i>B. Comparison of suppliers with respect to predictive and preventive maintenance (C_{42}) (consistency ratio 0.0332)</i>					
Supplier 1	1	3	5	0.637	6.066
Supplier 2	1/3	1	3	0.258	2.461
Supplier 3	1/5	1/3	1	0.105	1.000
<i>C. Comparison of suppliers with respect to lead-time (C_{43}) (consistency ratio 0.0212)</i>					
Supplier 1	1	2	1/4	0.200	1.708
Supplier 2	1/2	1	1/5	0.117	1.000
Supplier 3	4	5	1	0.683	5.840
<i>D. Comparison of suppliers with respect to up-to-date techniques and equipment (C_{44}) (consistency ratio 0)</i>					
Supplier 1	1	1	4	0.444	4.000
Supplier 2	1	1	4	0.444	4.000

TABLE 14: Continued.

Supplier	Supplier 1	Supplier 2	Supplier 3	EM weights	Model (8)
Supplier 3	1/4	1/4	1	0.111	1.000
<i>E. Comparison of suppliers with respect to transportation-storage and packaging (C_{45}) (consistency ratio 0.0634)</i>					
Supplier 1	1	3	1/3	0.268	2.277
Supplier 2	1/3	1	1/4	0.117	1.000
Supplier 3	3	4	1	0.614	5.205
<i>F. Comparison of suppliers with respect to new product development (C_{46}) (consistency ratio 0.0634)</i>					
Supplier 1	1	1/3	6	0.285	4.554
Supplier 2	3	1	8	0.653	10.409
Supplier 3	1/6	1/8	1	0.062	1.000

TABLE 15: Pairwise comparison matrices for three suppliers relative to subcriteria of C_5 and their local weights.

Supplier	Supplier 1	Supplier 2	Supplier 3	EM weights	Model (8)
<i>A. Comparison of suppliers with respect to reputation (C_{51}) (consistency ratio 0.0634)</i>					
Supplier 1	1	3	8	0.653	10.409
Supplier 2	1/3	1	6	0.285	4.554
Supplier 3	1/8	1/6	1	0.062	1.000
<i>B. Comparison of suppliers with respect to geographical location (C_{52}) (consistency ratio 0.0739)</i>					
Supplier 1	1	4	5	0.674	6.635
Supplier 2	1/4	1	3	0.226	2.226
Supplier 3	1/5	1/3	1	0.101	1.000
<i>C. Comparison of suppliers with respect to price (C_{53}) (consistency ratio 0.0634)</i>					
Supplier 1	1	1/3	1/4	0.117	1.000
Supplier 2	3	1	1/3	0.268	2.277
Supplier 3	4	3	1	0.614	5.205
<i>D. Comparison of suppliers with respect to patents (C_{54}) (consistency ratio 0)</i>					
Supplier 1	1	1	1	0.333	1.000
Supplier 2	1	1	1	0.333	1.000
Supplier 3	1	1	1	0.333	1.000
<i>E. Comparison of suppliers with respect to technical capability (C_{55}) (consistency ratio 0.0614)</i>					
Supplier 1	1	1/4	5	0.231	3.312
Supplier 2	4	1	9	0.709	11.666
Supplier 3	1/5	1/9	1	0.060	1.000

TABLE 16: Pairwise comparison matrices for three suppliers relative to subcriteria of C_6 and their local weights.

Supplier	Supplier 1	Supplier 2	Supplier 3	EM weights	Model (8)
<i>A. Comparison of suppliers with respect to RFID (C_{61}) (consistency ratio 0)</i>					
Supplier 1	1	1	3	0.429	3.000
Supplier 2	1	1	3	0.429	3.000
Supplier 3	1/3	1/3	1	0.143	1.000
<i>B. Comparison of suppliers with respect to EDI (C_{62}) (consistency ratio 0)</i>					
Supplier 1	1	1	3	0.429	3.000
Supplier 2	1	1	3	0.429	3.000
Supplier 3	1/3	1/3	1	0.143	1.000
<i>C. Comparison of suppliers with respect to internet (C_{63}) (consistency ratio 0)</i>					
Supplier 1	1	1	3	0.429	3.000
Supplier 2	1	1	3	0.429	3.000
Supplier 3	1/3	1/3	1	0.143	1.000

TABLE 17: Compound weights of three suppliers from the view of C_1 .

	C_{11}	C_{12}	C_{13}	Compound weight	Normalisation
<i>A. EM weights are normalised to make their maximum one</i>					
Subcriteria weight	1.000	0.237	0.141		
Supplier 1	1.000	0.141	0.167	1.057	1.000
Supplier 2	0.405	0.593	0.500	0.616	0.583
Supplier 3	0.164	1.000	1.000	0.542	0.513
<i>B. Model (8) weights are normalised to make their maximum one</i>					
Subcriteria weight	1.000	0.237	0.141		
Supplier 1	1.000	0.141	0.167	1.057	1.000
Supplier 2	0.406	0.593	0.500	0.617	0.584
Supplier 3	0.165	1.000	1.000	0.543	0.514

TABLE 18: Compound weights of three suppliers from the view of C_2 .

	C_{21}	C_{22}	C_{23}	C_{24}	Compound weight	Normalisation
<i>A. EM weights are normalised to make their maximum one</i>						
Subcriteria weight	0.124	0.627	1.000	0.270		
Supplier 1	0.188	1.000	1.000	0.354	1.746	1.000
Supplier 2	0.531	0.315	0.323	1.000	0.856	0.490
Supplier 3	1.000	0.132	0.136	0.188	0.394	0.225
<i>B. Model (8) weights are normalised to make their maximum one</i>						
Subcriteria weight	0.125	0.625	1.000	0.270		
Supplier 1	0.188	1.000	1.000	0.354	1.746	1.000
Supplier 2	0.531	0.315	0.315	1.000	0.856	0.490
Supplier 3	1.000	0.133	0.133	0.188	0.394	0.225

TABLE 19: Compound weights of three suppliers from the view of C_3 .

	C_{31}	C_{32}	C_{33}	C_{34}	Compound weight	Normalisation
<i>A. EM weights are normalised to make their maximum one</i>						
Subcriteria weight	0.117	0.304	0.533	1.000		
Supplier 1	1.000	1.000	1.000	1.000	1.955	1.000
Supplier 2	0.335	0.258	0.166	0.258	0.464	0.237
Supplier 3	0.149	0.111	0.096	0.111	0.213	0.109
<i>B. Model (8) weights are normalised to make their maximum one</i>						
Subcriteria weight	0.118	0.303	0.518	1.000		
Supplier 1	1.000	1.000	1.000	1.000	1.939	1.000
Supplier 2	0.335	0.258	0.166	0.258	0.462	0.238
Supplier 3	0.151	0.111	0.096	0.111	0.212	0.109

TABLE 20: Compound weights of three suppliers from the view of C_4 .

	C_{41}	C_{42}	C_{43}	C_{44}	C_{45}	C_{46}	Compound weight	Normalisation
<i>A. EM weights are normalised to make their maximum one</i>								
Subcriteria weight	0.355	0.143	0.184	0.563	0.090	1.000		
Supplier 1	0.149	1.000	0.292	1.000	0.437	0.437	1.289	0.637
Supplier 2	1.000	0.405	0.171	1.000	0.191	1.000	2.025	1.000
Supplier 3	0.446	0.164	1.000	0.250	1.000	0.095	0.692	0.342
<i>B. Model (8) weights are normalised to make their maximum one</i>								
Subcriteria weight	0.355	0.143	0.183	0.562	0.090	1.000		
Supplier 1	0.151	1.000	0.292	1.000	0.437	0.438	1.289	0.637
Supplier 2	1.000	0.406	0.171	1.000	0.192	1.000	2.023	1.000
Supplier 3	0.447	0.165	1.000	0.250	1.000	0.096	0.692	0.342

TABLE 21: Compound weights of three suppliers from the view of C_5 .

	C_{51}	C_{52}	C_{53}	C_{54}	C_{55}	Compound weight	Normalisation
<i>A. EM weights are normalised to make their maximum one</i>							
Subcriteria weight	0.109	0.072	1.000	0.266	0.639		
Supplier 1	1.000	1.000	0.191	1.000	0.326	0.847	0.599
Supplier 2	0.437	0.335	0.437	1.000	1.000	1.414	1.000
Supplier 3	0.095	0.149	1.000	1.000	0.085	1.342	0.949
<i>B. Model (8) weights are normalised to make their maximum one</i>							
Subcriteria weight	0.109	0.073	1.000	0.266	0.637		
Supplier 1	1.000	1.000	0.192	1.000	0.284	0.821	0.581
Supplier 2	0.438	0.335	0.437	1.000	1.000	1.412	1.000
Supplier 3	0.096	0.151	1.000	1.000	0.086	1.342	0.950

TABLE 22: Compound weights of three suppliers from the view of C_6 .

	C_{61}	C_{62}	C_{63}	Compound weight	Normalisation
<i>A. EM weights are normalised to make their maximum one</i>					
Subcriteria weight	0.119	0.598	1.000		
Supplier 1	1.000	1.000	1.000	1.718	1.000
Supplier 2	1.000	1.000	1.000	1.718	1.000
Supplier 3	0.333	0.333	0.333	0.573	0.333
<i>B. Model (8) weights are normalised to make their maximum one</i>					
Subcriteria weight	0.119	0.598	1.000		
Supplier 1	1.000	1.000	1.000	1.718	1.000
Supplier 2	1.000	1.000	1.000	1.718	1.000
Supplier 3	0.333	0.333	0.333	0.573	0.333

TABLE 23: Global weights of the three suppliers with respect to the total Goa.

	C_1	C_2	C_3	C_4	C_5	C_6	Global weight	Normalisation
<i>A. EM weights are normalised to make their maximum one</i>								
Criteria weight	0.580	0.129	0.224	0.362	1.000	0.076		
Supplier 1	1.000	1.000	1.000	0.637	0.599	1.000	1.838	0.971
Supplier 2	0.583	0.490	0.237	1.000	1.000	1.000	1.892	1.000
Supplier 3	0.513	0.225	0.109	0.342	0.949	0.333	1.449	0.766
<i>B. Model (8) weights are normalised to make their maximum one</i>								
Criteria weight	0.579	0.129	0.223	0.361	1.000	0.076		
Supplier 1	1.000	1.000	1.000	0.637	0.581	1.000	1.818	0.961
Supplier 2	0.584	0.490	0.238	1.000	1.000	1.000	1.891	1.000
Supplier 3	0.514	0.225	0.109	0.342	0.950	0.333	1.450	0.767

TABLE 24: Priority vectors and ranking order obtained from different prioritization methods.

	DEA/AR Model (8)	EM	WLSM	LSM	LLSM	CCMA
w_1	9.0062 (3)	0.1730 (3)	0.1427 (2)	0.2202 (2)	0.1748 (2)	0.1815 (3)
w_2	2.8338 (5)	0.0540 (5)	0.0543 (5)	0.0468 (5)	0.0626 (5)	0.0825 (5)
w_3	10.4515 (2)	0.1881 (2)	0.1213 (3)	0.1494 (4)	0.1487 (4)	0.1508 (4)
w_4	1.0000 (8)	0.0175 (8)	0.0301 (8)	0.0290 (8)	0.0193 (8)	0.0214 (8)
w_5	1.7471 (7)	0.0310 (7)	0.0465 (6)	0.0413 (7)	0.0356 (7)	0.0426 (7)
w_6	2.0066 (6)	0.0363 (6)	0.0465 (6)	0.0416 (6)	0.0423 (6)	0.0565 (6)
w_7	8.8996 (4)	0.1668 (4)	0.0891 (4)	0.2029 (3)	0.1670 (3)	0.1818 (2)
w_8	18.1511 (1)	0.3332 (1)	0.4695 (1)	0.2687 (1)	0.3496 (1)	0.2829 (1)

Example 3. Consider the following inconsistent comparison matrix studied by Lipovetsky and Conklin [38]:

$$A = \begin{bmatrix} 1 & 5 & 3 & 7 & 6 & 6 & 1/3 & 1/4 \\ 1/5 & 1 & 1/3 & 5 & 3 & 3 & 1/5 & 1/7 \\ 1/3 & 3 & 1 & 6 & 3 & 4 & 6 & 1/5 \\ 1/7 & 1/5 & 1/6 & 1 & 1/3 & 1/4 & 1/7 & 1/8 \\ 1/6 & 1/3 & 1/3 & 3 & 1 & 1/2 & 1/5 & 1/6 \\ 1/6 & 1/3 & 1/4 & 4 & 2 & 1 & 1/5 & 1/6 \\ 3 & 5 & 1/6 & 7 & 5 & 5 & 1 & 1/2 \\ 4 & 7 & 5 & 8 & 6 & 6 & 2 & 1 \end{bmatrix}. \quad (16)$$

In this pairwise comparison matrix, we have $\beta = 13.2372$

Table 24 shows priority vectors obtained from different prioritization methods. As can be seen, various prioritization methods generate different rankings. This is mainly due to the fact that the initialization vector A was highly inconsistent and should be therefore adjusted. Lipovetsky and Conklin [37] showed that the order of the adjusted ranking is $asw_8 > w_7 > w_1 > w_3 > w_2 > w_6 > w_5 > w_4$. Ranking in DEA/AR Model (8) is exactly of the same order as ranking in EM.

6. Conclusion

Although obtaining weights in the AHP has already been investigated by different studies, this problem needs more clarification. This paper proposed a new approach for generating weights from a pairwise comparison matrix. The proposed approach considers the most undesirable weight for each decision criterion or alternative and selects a decision criterion or alternative with the greatest weight. Two numerical examples, including BEKO in Turkey on CRT supplier selection, conducted by Sevklı, Lenny Koh [34], were investigated using the proposed approach. Moreover, the proposed method proved its ability to effectively differentiate decision criteria and alternative of DEAHP efficient.

Data Availability

All of our data are clear, and we have explained their source.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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