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Research Article

Metric Dimension of Crystal Cubic Carbon Structure

Xiujun Zhang[1] **and Muhammad Naeem**[2]

[1]*School of Computer Science, Chengdu University, Chengdu, China*
[2]*Department of Mathematics and Statistics, Institute of Southern Punjab, Multan, Pakistan*

Correspondence should be addressed to Muhammad Naeem; naeempkn@gmail.com

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For any given graph G, we say $W \subseteq V(G)$ is a resolving set or resolves the graph G if every vertex of G is uniquely determined by its vector of distances to the vertices in W. The metric dimension of G is the minimum cardinality of all the resolving sets. The study of metric dimension of chemical structures is increasing in recent times and it has application about the topology of such structures. The carbon atoms can bond together in various ways, called allotropes of carbon, one of which is crystal cubic carbon structure $\mathrm{CCC}(n)$. The aim of this article is to find the metric dimension of $\mathrm{CCC}(n)$.

1. Introduction

Let G be a simple connected graph and let $W = \{w_1, w_2, \ldots, w_k\}$ be an ordered subset of the set of vertices $V(G)$ of G. The distance $d(u, v)$ of two vertices of G is the length of shortest path between u and v. The representation of a vertex u of G with respect to W is the k-vector $(d(u, w_1), d(u, w_2), \ldots, d(u, w_k))$ and it is denoted as $r(u|W)$. The set W is called the resolving set or to resolve G if the representation of distinct vertices is distinct. That is, if u and v are two distinct vertices, then $r(u|W) \neq (v|W)$. The metric dimension of a graph is the cardinality of the minimal resolving set and it is denoted as $\beta(G)$. As there may be many different resolving subsets in $V(G)$ of different sizes, the study of the minimal one is important and it has been studied over the years. Some authors also use the term basis for G which is a resolving set with minimum cardinal number (see [1]). This work is about a study of resolving sets in chemical structural graphs.

The metric dimension of a general metric space was introduced in 1953 in [2], but at that time, it attracted little attention. Then, about twenty years later, it was applied to the distances between vertices of a graph [3–5]. Since then, it has been frequently used in graph theory, chemistry, biology, robotics, and many other disciplines. For some literature studies, see [6–9].

From many parameters for the study of graphs, the metric dimension is one of those that has many applications, and these applications are diverse like in pharmaceutical chemistry [10, 11], robot navigation [12], and combinatorial optimization [13]. A chemical compound or material can be represented by many graph structures, but only one of them may express its topological properties. The chemists require mathematical forms for a set of chemical compounds to give distinct representations to distinct compound structures. The structure of chemical compounds or materials can be represented by a labeled graph whose vertex and edge labels specify the atom and bond types, respectively. Thus, a graph theoretic interpretation of this problem is to provide representations for the vertices of a graph in such a way that distinct vertices have distinct representations.

At very high pressures of above 1000 GPa (gigapascal), one of the forms of carbon, namely, diamond, is predicted to transform into the so-called C_8 structure, a body-centered cubic structure with 8 atoms in the unit cell. This cubic carbon phase might have importance in astrophysics. Its structure is known in one of the metastable phases of silicon and is similar to cubane. The structure of this phase was proposed in 2012 as carbon sodalite [14]. In 2017, Baig et al. [15] modified and extended this structure and named it crystal cubic carbon $\mathrm{CCC}(n)$. We are taking all the notations

as they were in [15]. The structure of crystal cubic carbon consists of cubes.

The molecular graph of crystal cubic carbon CCC(n) for the second level is depicted in Figure 1. Its structure starts from one unit cube and then by attaching cubes at each vertex of the unit cube by an edge. For the third level, the CCC(3) is constructed by attaching cube to each vertex of cubes of CCC(2) having degree 3 or you can say by attaching cubes by an edge to all the white vertices of CCC(2). So, at each level, a new set of cubes is attached by edges to the white vertices of cubes of the preceding level. The third level of CCC(n) is displayed in Figure 2 which is constructed and presented in a most suitable manner to explain the structure of CCC(n).

All the new attached cubes, at each level, will be called the outermost layer of cubes or outermost level of cubes, or you can say at each level, the cubes with white vertices will be called the outermost layer. As in CCC(2), the outermost layer of cubes consists of 8 cubes. Because there are 7×8 vertices of degree 3, so in CCC(3), the outermost layer of cubes will consist of 7×8 cubes. Similarly, this procedure is repeated to get the next level. The cardinality of vertices and edges in CCC(n) is given below, respectively.

$$\begin{aligned} |V(\mathrm{CCC}(n))| &= 2\left\{24\sum_{r=3}^{n}\left(2^3-1\right)^{r-3} + 31\left(2^3-1\right)^{n-2} + 2\sum_{r=0}^{n-2}\left(2^3-1\right)^{r} + 3\right\}, \\ |E(\mathrm{CCC}(n))| &= 4\left\{24\sum_{r=3}^{n}\left(2^3-1\right)^{r-3} + 24\left(2^3-1\right)^{n-2} + 2\sum_{r=0}^{n-2}\left(2^3-1\right)^{r} + 3\right\}. \end{aligned} \tag{1}$$

There are some articles that describe the different topological properties of CCC(n) structure, the famous of those topological indices are Randic, ABC, and Zagreb indices and other degree-based indices of CCC(n) which are computed in [15–18]. In the articles [19, 20], theauthors calculated eccentricity and Szeged-type topological indices of CCC(n). The aim of this article is to compute the metric dimension of CCC(n). Note that if $W = \{w_1, w_2, \ldots, w_k\}$ is the ordered set of vertices of a graph G, then ξ^{th} component of $r(c|W)$ is $0 \Longleftrightarrow c = w_\xi$. Thus, in order to show that W is a resolving set, it suffices to verify that $r(a|W) \neq r(b|W)$ for each pair of distinct vertices $a, b \in V(G)\backslash W$.

2. Main Result

In this section, we will present the main result about the $\beta(\mathrm{CCC}(n))$. But before going further, let us discuss the very simple case of CCC(1) which is just a cube. We claim that $\beta(\mathrm{CCC}(1)) = 3$ indeed is true, let us see how.

Assume that $\beta(\mathrm{CCC}(1)) = 1$, and because of symmetry, we can take any vertex of cube to be the resolving set as in Figure 3(a), say $W = \{a\}$, then $r(b|W) = r(c|W)$, which is a contradiction. So, $\beta(\mathrm{CCC}(1)) > 1$. Assume that $\beta(\mathrm{CCC}(1)) = 2$. Then, there are two possibilities for the elements of the resolving set W of CCC(1) because of its symmetric shape. The possible cases are as follows:

(I) The two elements of W are the vertices on the main diagonal of CCC(1).

(II) The two elements of W are on the same face of the cube. In this case, the both elements are either on the main diagonal of a face or on the same edge of a face.

Without loss of generality, we can assume that $W = \{a, f\}$ for case (I). For case (II) without loss of generality, we can assume $W = \{a, c\}$ and $W = \{b, c\}$, respectively. Then, Figures 3(b)–3(d) show that $\beta(\mathrm{CCC}(1)) \neq 2$; the ordered pairs in these Figures denote the representations of the vertices. Thus, from Figure 3(e), it is proved that $\beta(\mathrm{CCC}(1)) = 3$.

Now, we will prove the main result of this article.

Theorem 1. *The metric dimension of crystal cubic carbon structure CCC(n) is $7^{n-2} \times 16$, for all $n \geq 2$, that is, $\beta(CCC(n)) = 7^{n-2} \times 16, \forall n \geq 2$.*

Proof. Let $G = \mathrm{CCC}(n)$ be the crystal cubic carbon structure and $n \geq 2$. To show that the $\beta(\mathrm{CCC}(n)) = 7^{n-2} \times 16$ firstly, we will show that $\beta(\mathrm{CCC}(n)) \geq 7^{n-2} \times 16$. Let Q_n be a cube on the outermost layer of CCC(n), as depicted in Figure 4 (note that there are no cubes attached to the vertices $b_1, b_2, b_3, c_1, c_2, c_3$, and u). In other words, all these vertices are of degree 3 and they belong to only one cube which is Q_n. Observe that the red vertex of cube Q_n is attached with red edge to a cube Q_{n-1} of the preceding level at its blue vertex. Also, note that $d(b_1, a) = 1 = d(b_2, a) = d(b_3, a)$ and $d(c_1, a) = 2 = d(c_2, a) = d(c_3, a)$ and $d(u, a) = 3$.

Let $W = \{w_1, w_2, \ldots, w_k\}$ be a resolving set of CCC(n). We claim that at least two vertices of Q_n belong to W. Suppose on contrary that no vertex of Q_n belongs to W and let $r(a|W)$ be a representation of vertex $a \in V(Q_n)$. Note that all the shortest paths from any vertex of Q_n to any vertex of W contain the vertex a of Q_n. So, we can say that all such paths pass through vertex a (path may end at it). Then,

$$\begin{aligned} r(b_1|W) &= (d(b_1, w_1), d(b_1, w_2), \ldots, d(b_1, w_k)) \\ &= (d(a, w_1) + 1, d(a, w_2) + 1, \ldots, d(a, w_k) + 1) \\ &= (d(b_2, w_1), d(b_2, w_2), \ldots, d(b_2, w_k)) \\ &= r(b_2|W); \end{aligned} \tag{2}$$

this is a contradiction. Now, assume that exactly one vertex from the set $V(Q_n)$ belongs to W. Without loss of generality, we can assume that this common vertex is w_1.

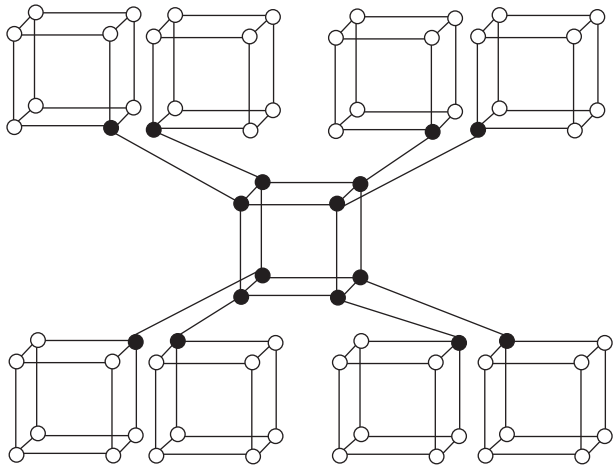

Figure 1: Crystal cubic carbon structure CCC(2).

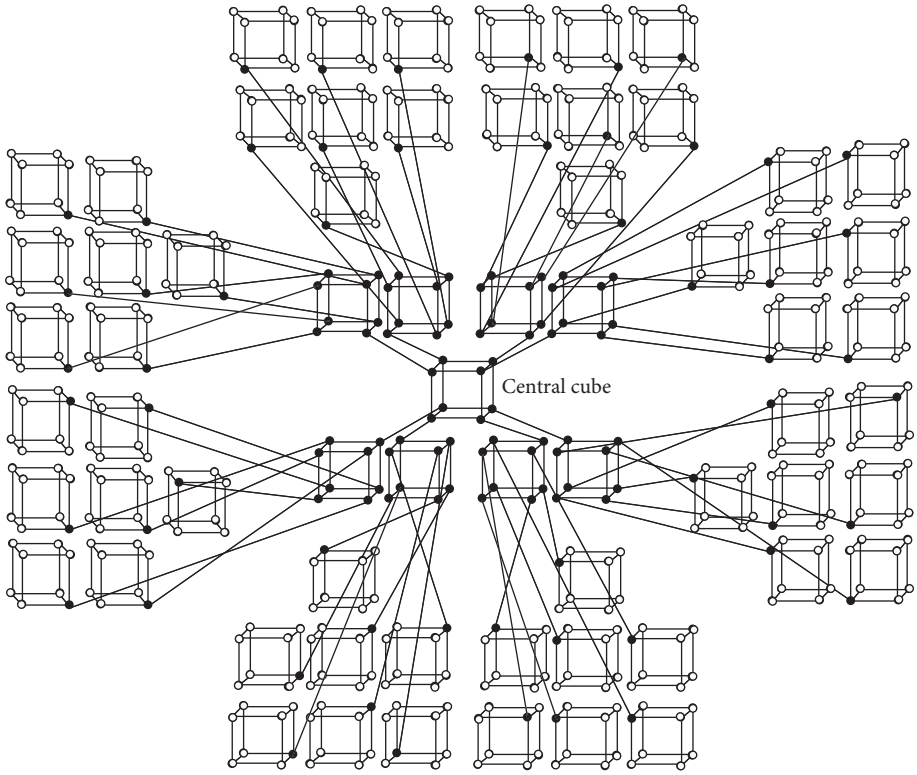

Figure 2: Crystal cubic carbon structure CCC(3), with CCC(1) as the central cube.

Case 1. If $w_1 = a$, then

$$\begin{aligned} r(b_1|W) &= (1, d(b_1, w_2), \ldots, d(b_1, w_k)) \\ &= (1, d(b_2, w_2), \ldots, d(b_2, w_k)) \\ &= r(b_2|W), \text{ a contradiction.} \end{aligned} \tag{3}$$

Case 2. If $w_1 = b_1$, then $d(c_1, w_1) = 1 = d(c_2, w_1)$

$$\begin{aligned} r(c_1|W) &= (1, d(c_1, w_2), \ldots, d(c_1, w_k)) \\ &= (1, d(c_2, w_2), \ldots, d(c_2, w_k)) \\ &= r(c_2|W), \text{ again a contradiction.} \end{aligned} \tag{4}$$

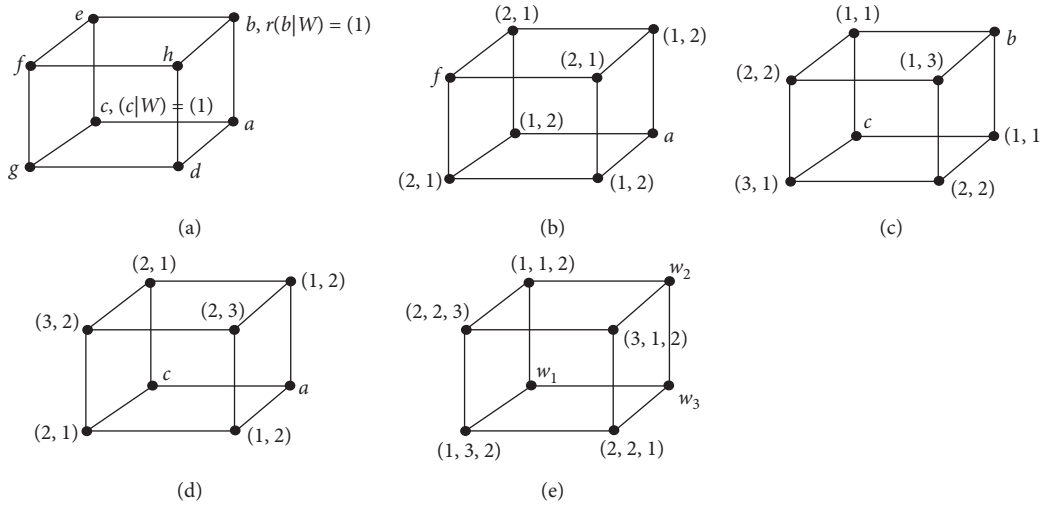

Figure 3: The graph of CCC(1) with all options of possible resolving sets.

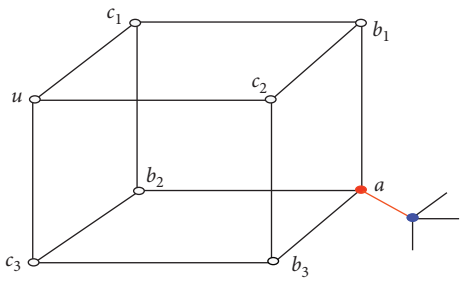

Figure 4: An arbitrary cube Q_n in the outermost layer of cubes of CCC(n).

Similar contradictions appear for $w_1 = b_2$ and $w_1 = b_3$, let us look at it.

Case 3. If $w_1 = b_2$, then $d(c_1, w_1) = 1 = d(c_3, w_1)$

$$\begin{aligned} r(c_1|W) &= (1, d(c_1, w_2), \ldots, d(c_1, w_k)) \\ &= (1, d(c_3, w_2), \ldots, d(c_3, w_k)) \\ &= r(c_3|W), \text{ a contradiction.} \end{aligned} \tag{5}$$

Case 4. If $w_1 = b_3$, then $d(c_2, w_1) = 1 = d(c_3, w_1)$

$$\begin{aligned} r(c_2|W) &= (1, d(c_2, w_2), \ldots, d(c_2, w_k)) \\ &= (1, d(c_3, w_2), \ldots, d(c_3, w_k)) \\ &= r(c_3|W), \text{ a contradiction.} \end{aligned} \tag{6}$$

Case 5. If $w_1 = c_1$, then $d(b_1, w_1) = 1 = d(b_2, w_1)$

$$\begin{aligned} r(b_1|W) &= (1, d(b_1, w_2), \ldots, d(b_1, w_k)) \\ &= (1, d(b_2, w_2), \ldots, d(b_2, w_k)) \\ &= r(b_2|W), \text{ a contradiction.} \end{aligned} \tag{7}$$

Case 6. If $w_1 = c_2$, then $d(b_1, w_1) = 1 = d(b_3, w_1)$

$$\begin{aligned} r(b_1|W) &= (1, d(b_1, w_2), \ldots, d(b_1, w_k)) \\ &= (1, d(b_3, w_2), \ldots, d(b_3, w_k)) \\ &= r(b_3|W), \text{ a contradiction.} \end{aligned} \tag{8}$$

Case 7. If $w_1 = c_3$, then $d(b_2, w_1) = 1 = d(b_3, w_1)$

$$\begin{aligned} r(b_2|W) &= (1, d(b_2, w_2), \ldots, d(b_2, w_k)) \\ &= (1, d(b_3, w_2), \ldots, d(b_3, w_k)) \\ &= r(b_3|W), \text{ a contradiction.} \end{aligned} \tag{9}$$

Case 8. If $w_1 = u$, then $d(b_1, w_1) = 1 = d(b_3, w_1)$

$$\begin{aligned} r(b_1|W) &= (1, d(b_1, w_2), \ldots, d(b_1, w_k)) \\ &= (1, d(b_3, w_2), \ldots, d(b_3, w_k)) \\ &= r(b_3|W), \text{ a contradiction.} \end{aligned} \tag{10}$$

The contradiction in all the cases proved our claim. So, at least two vertices from the vertex set of Q_n are in the resolving set W of CCC(n). Since Q_n was taken arbitrary, so W contains at least two vertices from each of the cube in the outermost layer of cubes of CCC(n). By the construction of CCC(n), we can see that at each step or at each level, the cubes in CCC(n) are increased by a number equal to 7 multiplied by the number of cubes in the outermost layer of the previous level. For example, in CCC(2), we have 8 cubes in the outer layer, and in CCC(3), we have 7×8 cubes in the outermost layer. Thus, there are exactly $7^{n-2} \times 8$ cubes in the outermost layer of CCC(n). Since from each such cube there are at least two vertices in W, so $\beta(\text{CCC}(n)) \geq 7^{n-2} \times 16$. □

2.1. Second Part of Proof. In this part, we will show that $\beta(\mathrm{CCC}(n)) \leq 7^{n-2} \times 16$. Let $W = \{w_1, w_2, \ldots, w_k\}$ be the collection of all the vertices of type b_1 and b_2 just like we have discussed in part one of the proof and depicted in Figure 4. Then, $k = 7^{n-2} \times 16$. We claim that W is a resolving set of $\mathrm{CCC}(n)$. The representations of the two arbitrary vertices of $\mathrm{CCC}(n)$ can be compared in five different cases and they are discussed as follows:

(1) The two arbitrary selected vertices are on the same cube in the outermost level of $\mathrm{CCC}(n)$ (see Figure 4).

(2) The two arbitrary selected vertices are on the same cube, but this cube is not the outer most cube and neither the central cube (i.e., $\mathrm{CCC}(1)$), as depicted in Figure 5.

(3) The two arbitrary selected vertices are on the central cube, as displayed in Figure 6.

(4) The two arbitrary selected vertices are on a chain of cubes with one end being the cube of the outermost level (see Figure 7).

(5) The two arbitrary selected vertices are on distinct chains of cubes and those chains are connecting at a cube which we can call a branching cube. As explained in Figure 8, in which B cube is the branching cube, S-cube and T-cube are on different chains each containing one of the selected vertices.

Case (1). This can be proved by a direct computation for the representation of all the vertices in this cube (Figure 4). Without loss of generality, we can assume that $w_1 = b_1, w_2 = b_2 \in W$, then $r(a|W) = (1, 1, d(a, w_3), \ldots, d(a, w_k))$ and

$$\begin{aligned} r(b_3|W) &= (2, 2, d(a, w_3) + 1, \ldots, d(a, w_k) + 1), \\ r(c_1|W) &= (1, 1, d(a, w_3) + 2, \ldots, d(a, w_k) + 2), \\ r(c_2|W) &= (1, 3, d(a, w_3) + 2, \ldots, d(a, w_k) + 2), \\ r(c_3|W) &= (3, 1, d(a, w_3) + 2, \ldots, d(a, w_k) + 2), \\ r(u|W) &= (2, 2, d(a, w_3) + 3, \ldots, d(a, w_k) + 3). \end{aligned} \tag{11}$$

We can see from the above that these representations are all distinct in this case.

Case (2). Let the two arbitrary selected vertices be on the same cube and this cube is not on the outermost cube and neither is it the central cube. A visualization of such cube is given in Figure 5. We can label the vertices of this cube Q_A, as shown in Figure 5. Without loss of generality, we can assume that w_1, w_2 are on the cube in the outermost layer of cubes and that cube is connected to cube Q_A at vertex u_1 by a chain of cubes. Similarly, we can assume that w_{2i-1}, w_{2i} are on the cube in the outermost layer of cubes and those cubes are connected to cube Q_A at vertices u_i, $i = 2, \ldots, 7$, by a chain of cubes, respectively.

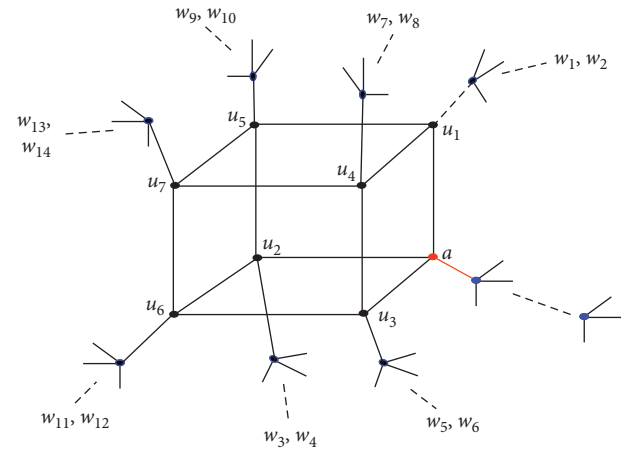

FIGURE 5: Arbitrary cube Q_A not in the outermost layer of cubes of $\mathrm{CCC}(n)$ and nor the central cube. This cube is connected to the central cube by a chain of cubes at vertex a.

$$\begin{aligned} &d(u_1, w_1) \neq d(u_i, w_1), \quad i = 1, \ldots, 7 \text{ and } i \neq 1, \\ &d(u_2, w_3) \neq d(u_i, w_3), \quad i = 1, \ldots, 7 \text{ and } i \neq 2, \\ &d(u_3, w_5) \neq d(u_i, w_5)) \quad i = 1, \ldots, 7 \text{ and } i \neq 3, \\ &d(u_4, w_7) \neq d(u_i, w_7), \quad i = 1, \ldots, 7 \text{ and } i \neq 4, \\ &d(u_5, w_9) \neq d(u_i, w_9), \quad i = 1, \ldots, 7 \text{ and } i \neq 5, \\ &d(u_6, w_{11}) \neq d(u_i, w_{11}), \quad i = 1, \ldots, 7 \text{ and } i \neq 6, \\ &d(u_7, w_1 3) \neq d(u_i, w_1 3), \quad i = 1, \ldots, 7 \text{ and } i \neq 7. \end{aligned} \tag{12}$$

Also,

$$\begin{aligned} d(a, w_1) &= d(u_1, w_1) + 1, \\ d(a, w_3) &= d(u_2, w_3) + 1, \\ d(a, w_5) &= d(u_3, w_5) + 1, \\ d(a, w_7) &= d(u_4, w_7) + 2, \\ d(a, w_9) &= d(u_5, w_9) + 2, \\ d(a, w_{11}) &= d(u_6, w_{11}) + 2, \end{aligned} \tag{13}$$

and $d(a, w_{13}) = d(u_7, w_{13}) + 3$. All these computations show that $r(u_i|W) \neq r(u_j|W)$ for $i \neq j$ and $r(a|W) \neq r(u_i|W)$ for $i = 1, \ldots, 7$. This completes the proof in this case.

Case (3). Assume that the two arbitrary selected vertices are on the central cube, as displayed in Figure 6, where just like in the previous case (2), we have labeled all 8 vertices with $u_1, u_2, \ldots, u_8$. Again, without loss of generality, we assume that $w_{2i-1}, w_{2i}, i = 1, \ldots, 8$, are on the cube in the outermost layer of cubes and those outermost cubes containing w_{2i-1}, w_{2i} are connected to the central cube $\mathrm{CCC}(1)$ at vertices $u_i, i = 1, 2, \ldots, 8$, by a chain of cubes, respectively. These assumptions imply that

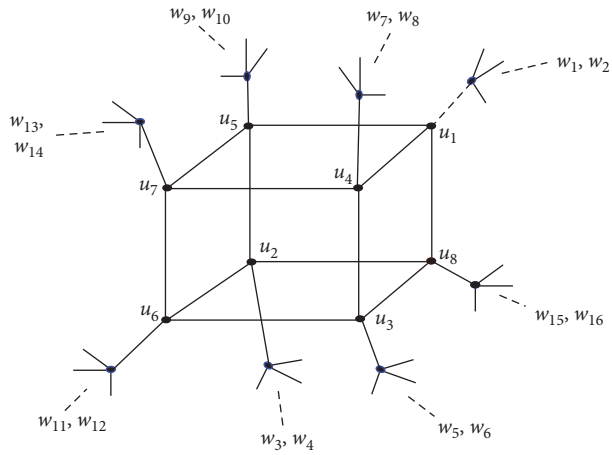

Figure 6: Central cube of CCC(n), that is, the cube CCC(1).

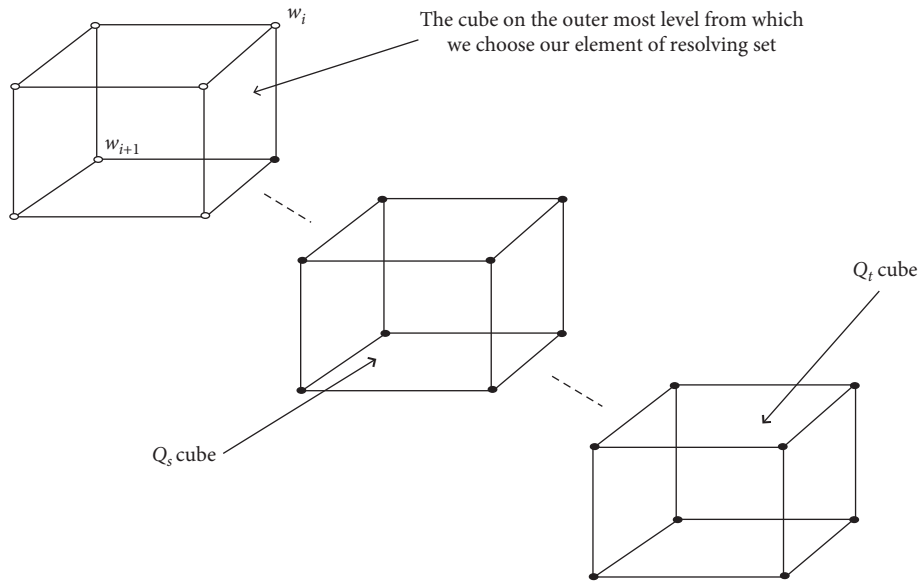

Figure 7: A chain of cubes with one end being the cube of the outermost level; Q_s and Q_t are arbitrary cubes on the chain but not the outermost cubes.

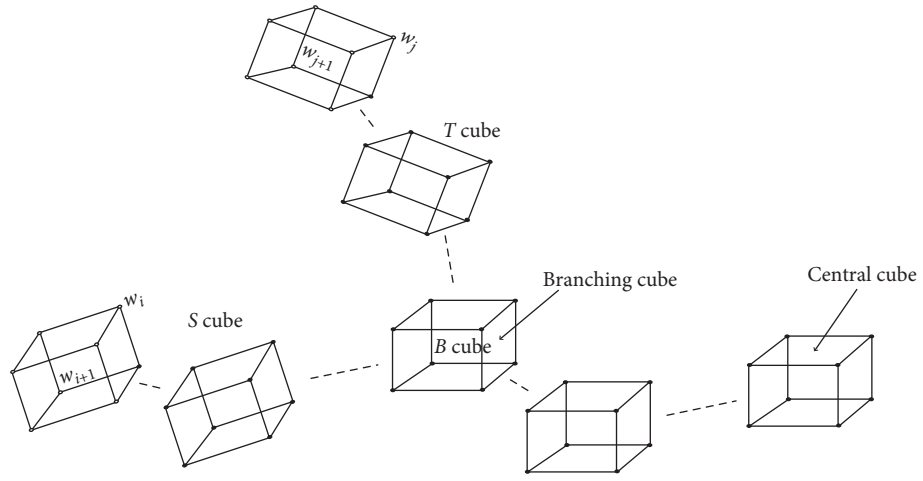

Figure 8: Branching cube and chain of cubes in CCC(n).

$$\begin{aligned}
d(u_1, w_1) &\neq d(u_i, w_1), \quad 1 \leq i \leq 8 \text{ and } i \neq 1,\\
d(u_2, w_3) &\neq d(u_i, w_3), \quad 1 \leq i \leq 8 \text{ and } i \neq 2,\\
d(u_3, w_5) &\neq d(u_i, w_5), \quad 1 \leq i \leq 8 \text{ and } i \neq 3,\\
d(u_4, w_7) &\neq d(u_i, w_7), \quad 1 \leq i \leq 8 \text{ and } i \neq 4,\\
d(u_5, w_9) &\neq d(u_i, w_9), \quad 1 \leq i \leq 8 \text{ and } i \neq 5,\\
d(u_6, w_{11}) &\neq d(u_i, w_{11}), \quad 1 \leq i \leq 8 \text{ and } i \neq 6,\\
d(u_7, w_{13}) &\neq d(u_i, w_{13}), \quad 1 \leq i \leq 8 \text{ and } i \neq 7,\\
d(u_8, w_{15}) &\neq d(u_i, w_{15}), \quad 1 \leq i \leq 8 \text{ and } i \neq 8.
\end{aligned} \tag{14}$$

So, we get the conclusion that, in this case, again $r(u_i|W) \neq r(u_i|W)$ for $i \neq j$ and $1 \leq i \leq 8$, $1 \leq j \leq 8$.

Case (4). Now, we are going to discuss case (4). Assume that the two arbitrary selected vertices s, t are on two distinct cubes and those cubes are on a chain of cubes, see Figure 7. Assume that one end of this chain is the outermost cube containing two arbitrary resolving elements, say w_1, w_2 (without loss of generality, we can assume that those vertices are w_1, w_2), and the other end is the central cube.

As depicted in Figure 7, let t be a vertex of cube Q_t and s be a vertex of cube Q_s, then $d(s, w_1) < d(t, w_1)$, and therefore, $r(s|W) \neq r(t|W)$. This completes the proof in this case.

Case (5). Finally, suppose that the two arbitrary selected vertices s, t are on distinct chains of cubes and those chains are connecting at a cube which we can call a branching cube; this branching cube can also be the central cube. As explained in Figure 8, in which B cube is the branching cube, S cube and T cube are on different chains each containing one of the selected vertices, that is, $s \in V(S)$ and $t \in V(T)$. Both of the two cubes S and T or any one of these cube can also be the cubes in the outermost level of cubes.

Note: in the idea of case (4), we can say that someone can select two vertices on different cubes such that there is chain of cube connecting them and both ends of this chain are the cubes on the outermost level of cubes. But then, there must be a cube (which we call branching cube) in this chain that connects to the central cube by the chain of cubes.) Without loss of generality, we can assume that $w_i = w_1, w_{i+1} = w_2$ and $w_j = w_3, w_{j+1} = w_4$. We can see that the length of the shortest path from vertex w_1 to vertex t of cube T is greater than the length of the shortest path from vertex w_1 to vertex s of cube S. Thus, $d(s, w_1) \neq d(t, w_1)$, so this implies that $r(s|W) \neq r(t|W)$.

All these five cases prove that $W = \{w_1, w_2, \ldots, w_k\}$ is a resolving set. Since there are $7^{n-2} \times 16$ number of elements in W, therefore the proof of theorem concludes.

3. Conclusion

In this article, we have studied the metric dimension of the crystal cubic carbon structure and we gave a formula for its metric dimension. We have found that the metric dimension of CCC(n) is not constant and find its closed form.

Data Availability

All the proofs and exemplary data of this study are included in the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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References

[1] M. Baca, E. T. Baskoro, A. N. M. Salman, S. W. Saputro, and D. Suprijanto, "On metric dimension of regular bipartite graphs," *Bulletin mathématiques de la Société des sciences mathématiques de Roumanie*, vol. 54, pp. 15–28, 2011.

[2] L. M. Blumenthal, *Theory and Applications of Distance Geometry*, Oxford University Press, Oxford, UK, 1953.

[3] F. Harary and R. A. Melter, "On the metric dimension of a graph," *Ars Combinatoria*, vol. 2, pp. 191–195, 1976.

[4] P. J. Slater, "Leaves of trees," *Congressus Numerantium*, vol. 14, pp. 549–559, 1975.

[5] P. J. Slater, "Dominating and reference sets in a graph," *Journal of Mathematical and Physical Sciences*, vol. 22, no. 4, pp. 445–455, 1988.

[6] J.-B. Liu, M. F. Nadeem, H. M. A. Siddiqui, and W. Nazir, "Computing metric dimension of certain families of toeplitz graphs," *IEEE Access*, vol. 7, pp. 126734–126741, 2019.

[7] M. A. Mohammed, A. J. Munshid, H. M. A. Siddiqui, and M. R. Farahani, "Computing metric and partition dimension of tessellation of plane by boron nanosheets," *Eurasian Chemical Communications*, vol. 2, no. 10, pp. 1064–1071, 2020.

[8] H. M. A. Siddiqui, S. Hayat, A. Khan, M. Imran, A. Razzaq, and J.-B. Liu, "Resolvability and fault-tolerant resolvability structures of convex polytopes," *Theoretical Computer Science*, vol. 796, pp. 114–128, 2019.

[9] B. Yang, M. Rafiullah, H. M. A. Siddiqui, and S. Ahmad, "On resolvability parameters of some wheel-related graphs," *Journal of Chemistry*, vol. 2019, Article ID 9259032, 9 pages, 2019.

[10] P. J. Cameron and J. H. VanLint, *Designs, Graphs, Codes and their Links in London Mathematical Society Student Texts*, Cambridge University Press, Cambridge, UK, 1991.

[11] G. Chartrand, L. Eroh, M. A. Johnson, and O. R. Oellermann, "Resolvability in graphs and metric dimension of a graph," *Discrete Applied Mathematics*, vol. 105, pp. 99–113, 2000.

[12] S. Khuller, B. Raghavachari, and A. Rosenfeld, "Localization in graphs," Technical Report CS-TR-3326, University of Maryland at College Park, College Park, MD, USA, 1994.

[13] A. Sebo and E. Tannier, "On metric generators of graphs," *Mathematics of Operations Research*, vol. 29, pp. 383–393, 2004.

[14] A. Pokropivny and S. Volz, ""C8" phase: supercubane, tetrahedral, BC-8 or carbon sodalite," *Physica Status Solidi B*, vol. 249, no. 9, pp. 1704–2170, 2012.

[15] A. Q. Baig, M. Imran, W. Khalid, and M. Naeem, "Molecular description of carbon graphite and crystal cubic carbon structures," *Canadian Journal of Chemistry*, vol. 95, no. 6, pp. 674–686, 2017.

[16] W. Gao, M. Siddiqui, M. Naeem, and N. Rehman, "Topological characterization of carbon graphite and crystal cubic carbon structures," *Molecules*, vol. 22, no. 9, p. 1496, 2017.
[17] H. Yang, M. Naeem, and M. K. Siddiqui, "Molecular properties of carbon crystal cubic structures," *Open Chemistry*, vol. 18, no. 1, pp. 339–346, 2020.
[18] M. A. Zahid, M. Naeem, A. Q. Baig, and W. Gao, "General fifth M-zagreb indices and fifth M-zagreb polynomials of crystal cubic carbon," *Utilitas Mathematica*, vol. 109, pp. 263–270, 2018.
[19] M. Imran, M. Naeem, A. Q. Baig, M. K. Siddiqui, M. A. Zahid, and W. Gao, "Modified eccentric descriptors of crystal cubic carbon," *Journal of Discrete Mathematical Sciences & Cryptography*, vol. 22, no. 7, pp. 1215–1228, 2019.
[20] H. Yang, M. Naeem, A. Q. Baig, H. Shaker, and M. K. Siddiqui, "Vertex Szeged index of crystal cubic carbon structure," *Journal of Discrete Mathematical Sciences & Cryptography*, vol. 22, no. 7, pp. 1177–1187, 2019.