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Research Article

Dynamic Response Analysis of a Forced Fractional Viscoelastic Beam*

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In this paper, dynamic response analysis of a forced fractional viscoelastic beam under moving external load is studied. The beauty of this study is that the effect of values of fractional order, the effect of internal damping, and the effect of intensity value of the moving force load on the dynamic response of the beam are analyzed. Constitutive equations for fractional order viscoelastic beam are constructed in the manner of Euler–Bernoulli beam theory. Solution of the fractional beam system is obtained by using Bernoulli collocation method. Obtained results are presented in the tables and graphical forms for two different beam systems, which are polybutadiene beam and butyl B252 beam.

1. Introduction

Theory and applications of beams are very important research area due to its wide usage areas in applied sciences. Especially after starting the space adventure of the mankind, the demand to more resistant structures has great importance. Beams are generally modeled based on Euler–Bernoulli beam theory, which is called classical beam theory. The background of beam theory goes on Newton's second law and some different aspects of beams, such as modeling, analysis of bending-buckling, and reinforcement and control, are hot topics of research papers since the beginning of the nineteenth century. The books can provide a general overview about the Euler–Bernoulli beam theory, please see [1–3]. Some important studies related to beams modeled in the sense of classical beam theory are also summarized as follows, but not limited to [4–15]. The beam systems in [1–15] have the integer order derivatives of the state function. In the beginning of 1930s, fractional derivative was introduced for describing the constitutive relation of some beam materials [16], and after 1980s, since fractional order equations have good memory and can be used to describe material properties more accurately with fewer parameters, they are considered to be good mathematical models for describing the dynamic mechanical behavior of materials [17]. In [18], the dynamic behavior of the thin plates resting on a fractionally damped viscoelastic foundation subjected to a moving point load is investigated and results show that the damping of the foundation system increases with increasing the order of the fractional derivative, which leads to a decrease in the dynamic response. In [19], the dynamic response spectra of fractionally damped viscoelastic beams subjected to concentrated moving load are presented and results reveal that with an increase in the order of the fractional derivative, the system damping of the system increases and the dynamic amplification factor (DAF) decreases, especially in the dynamic zone of the sweep parameter. In [20], the precise integration method (PIM) is extended to numerically integrate the equation of motion with fractional terms, which offers high accuracy and obtained numerical results indicate the viscoelastic dampers can enhance the seismic performance of structures significantly. In [21], the nonstationary free vibration and nonlinear dynamic behavior of the viscoelastic nanoplates are analyzed. Obtained results show that the viscoelastic model-based vibration is nonstationary unlike the elastic model.

Moreover, the damping mechanism of the viscoelasticity is amplitude dependent and the contribution of the viscoelastic damping terms at higher forcing conditions becomes noticeable. On the other hand, several numerical methods are developed and employed for better analyzing the fractional mechanical systems. Widely used methods for fractional systems are finite element method [22], Galerkin method [23], variational iteration method [24], and multiscale method [25, 26]. Especially, papers existing in the literature, which include a solution method for analyzing the dynamic response of a fractional order beam system, can be shortly listed as [19, 27–30]. In [19], the authors combined Galerkin method and Newton–Raphson method for analyzing the vibration of a fractional beam equation and they compared the results for only seeing the effects of fractional or integer derivatives. In [27], the author considered the dynamic response analyzing of a fractional order viscoelastic beam by means of green function method. In [27], the author only compared the results based on changes on the fractional derivative between $(0, 1)$. In [28], the authors employed the Adomian decomposition method for solving a fractional beam equation and they only observed the effect of the order of fractional derivative. In [29], the authors used the dynamic green function method for analyzing the dynamic response in a fractional beam equation and the beam equation does not include the damping term. Results are simulated for only indicating the effects of order of fractional derivative. In [30], the author employed the green function method for a fractional viscoelastic beam system subjected to a base excitation. After obtaining the solution, the author compared the results corresponding to different fractional order derivative. By comparing the present study with the studies existing in the literature, objectives of the present study are expressed as follows:

(i) In this paper, Bernoulli collocation method is firstly employed for analyzing the fractional viscoelastic beam equation. In the literature, especially for the fractional beam systems, green function method, Galerkin method, Newton–Raphson method, Adomian decomposition method, and Bernoulli collocation method in this paper were used, but by comparing these five methods, it is clear that Bernoulli collocation method is new and has less computational process and less work.

(ii) In the literature, the authors only considered and discussed the effects of order of fractional derivative on the dynamic response. But, we discussed both the effects of the order of fractional order derivative and the effects of damping coefficient term and the effect of density of moving force load. So, it is said that the present study has wider perspective than other studies.

(iii) Also, in the literature, results are obtained for one beam system. In this paper, effects of order of fractional derivative, effects of damping coefficient term, and the effect of density of moving force load are observed and compared for two different beam systems which are polybutadiene beam and butyl B252 beam.

For theoretical and experimental review about the fractional Euler–Bernoulli beams, please see [31]. Specifically, in the present paper, displacement analysis of a forced fractional viscoelastic beam is studied. External moving force load perfectly moves on the beam with the velocity $v(t)$ from the left edge to the right edge of the beam. The solution of the fractional beam system is obtained by means of Bernoulli collocation method. The main advantage of the Bernoulli collocation method is that employing the Bernoulli polynomials is easier than Chebyshev, Bessel polynomials, and Haar wavelets [32–34]. These advantages of Bernoulli polynomials provide us for obtaining the solution by making less computational process in shorter time. In the step of employing the Bernoulli collocation method, some external moving force loads having different load intensities are considered and also the effects of internal damping and fractional order of the derivative are searched for a fractional beam system. In the simulations, two different beam systems, which are polybutadiene beam and butyl B252 beam, are taken into account for being compared each other in the aspects of internal damping effects and resistance to effect of external moving force. Comparison results of the beam systems are presented in tables and graphics. The rest of the paper is organized as follows: in the next section, definition of the displacement analysis problem for a fractional viscoelastic beam is presented and scheme of the beam is overviewed. In the third section, short definition of the fractional derivative in the Caputo sense is introduced. In the fourth section, Bernoulli collocation method is explained and adopted to the present problem. In the fifth section, obtained results are given and discussions are made in the light of employing the Bernoulli collocation method to fractional viscoelastic beam system.

2. Definition of the Problem

The motion equation of the fractional viscoelastic homogeneous beam is obtained by considering the Euler–Bernoulli beam theory by ignoring shear deformation factor and rotary inertia of the beam. The beam is considered as a uniform viscoelastic beam and mechanical energy dissipation inside the beam is modeled by fractional order differential equations. By taking into account the [35], stress-strain constitutive relation of a fractional viscoelastic beam is given as follows:

$$\sigma = E\varepsilon(t) + E'_{\gamma} D_t^{\gamma}[\varepsilon(t)] = E\left(\varepsilon + \mu_{\gamma}\frac{d^{\gamma}\varepsilon(t)}{\mathrm{d}t^{\gamma}}\right), \quad (1)$$

in which E is the Young's modulus of the viscoelastic beam, μ_{γ} is the damping coefficient, and D_t^{γ} is the fractional derivative operator with the order γ with respect to t. The simply supported viscoelastic beam initially is at rest and nondeformed. The beam is subjected to a horizontally moving constant force load with the velocity $v(t)$ from the left edge to right edge of the beam, respect to x axis. In the light of [27], let us introduce the formulation of a fractional viscoelastic beam structure illustrated in Figure 1.

$$A\rho \frac{\partial^2 w(t,x)}{\partial t^2} + \mathrm{EI}\mu_\gamma \left[\frac{d^\gamma}{dt^\gamma} \frac{\partial^4 w(t,x)}{\partial x^4}\right] + \mathrm{EI}\frac{\partial^4 w(t,x)}{\partial x^4} = P\delta(x - v(t)), \tag{2}$$

in which w is the deflection of the viscoelastic beam in $\mathscr{C} = \{(t,x)\colon t \in (0,\mathrm{tf}), x \in (0,\ell)\}$, t is the time variable, t_f is the final time observed duration, x is the space variable, ℓ is the length of the viscoelastic beam, A is the cross-section area of the structure, ρ is the material mass density of the viscoelastic beam, I is the axial moment of inertia of the beam, P is a constant showing intensity of the external moving force load, δ is the Dirac-delta function, and $v(t)$ is the velocity of the moving force load with the condition $0 \le v(t) \le \ell$. Equation (2) is subjected to the following boundary conditions:

$$w(t,x) = 0,\ w_{xx}(t,x) = 0 \text{ at } x = 0, \ell, \tag{3}$$

and the following initial conditions:

$$w(t,x) = w_0(x),\ w_t(t,x) = w_1(x) \text{ at } t = 0, \tag{4}$$

in which $w_0(x) \in H^1(0,\ell) = \{w_0(x) \in L^2(0,\ell)\colon \partial w_0(x)/\partial x \in L^2(0,\ell)\}$, $w_1(x) \in L^2(0,\ell)$. $L^2(\mathscr{C})$ means to square-integrable functions space in the manner of Hilbert in the domain $\mathscr{C}$ in the Lebesgue sense with the following norm and inner product:

$$\|\eta\|^2 = <\eta,\eta>, \quad <\eta,\rho>_{\mathscr{C}} = \int_{\mathscr{C}} \rho\eta \, d\mathscr{C}. \tag{5}$$

Let us assume that

$$w(t,x) = \sum_{n=1}^{N} z_n(t)\sqrt{2}\sin\left(\frac{n\pi x}{\ell}\right). \tag{6}$$

After substituting the equations (6) into (2) and multiplying both sides of equation (2) with $\sqrt{2}\sin(n\pi x/\ell)$, integrating on $(0,\ell)$, we obtain the following ordinary differential equation as follows:

$$A\rho z_n''(t) + \mathrm{EI}(n\pi)^4\mu_\gamma\left[\frac{d^\gamma}{\mathrm{d}t^\gamma} z_n(t)\right] + \mathrm{EI}(n\pi)^4 z_n(t) = P\sqrt{2}\sin\left(\frac{n\pi v(t)}{\ell}\right), \quad n = 1,\ldots,N. \tag{7}$$

Equation (7) is subjected to the following initial conditions:

$$z_n(0) = \sqrt{2}\int_0^\ell w_0(x)\sin\left(\frac{n\pi x}{\ell}\right)\mathrm{d}x, \quad z_n'(0) = \sqrt{2}\int_0^\ell w_1(x)\sin\left(\frac{n\pi x}{\ell}\right)\mathrm{d}x. \tag{8}$$

3. The Fractional Derivative in the Caputo Sense

Definition. The Caputo definition of the fractional-order derivative is

$$D^\gamma f(x) = \frac{1}{\Gamma(n-\gamma)}\int_0^x \frac{f^{(n)}(t)}{(x-t)^{\gamma+1-n}}\mathrm{d}t, \quad n-1 < \gamma \le n,\ n \in \mathbb{N}, \tag{9}$$

where $\gamma > 0$ is the order of the derivative and n is the smallest integer greater than γ. For the Caputo derivative, we have

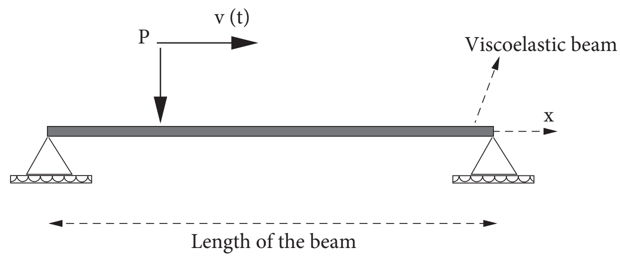


FIGURE 1: Schematic of the viscoelastic beam under moving force load P with the velocity $v(t)$.

$$
\begin{aligned}
&D^{\gamma}C = 0, \quad C \text{ is constant},\\
&D^{\gamma}x^{q} = \begin{cases} 0, & \text{for } q \in \mathbb{N}_0 \text{ and } q < \lceil \gamma \rceil, \\ \dfrac{\Gamma(q+1)}{\Gamma(q+1-\gamma)} x^{q-\gamma}, & \text{for } q \in \mathbb{N}_0 \text{ and } q \geq \lceil \gamma \rceil \text{ or } q \notin \mathbb{N} \text{ and } q > \lfloor \gamma \rfloor. \end{cases}
\end{aligned} \tag{10}
$$

4. Bernoulli Collocation Method

The recurrence relation of the Bernoulli polynomials is defined by

$$B_n(x) = 2xB_{n-1}(x) + B_{n-2}(x). \tag{11}$$

For $n \geq 3.$, $B_1(x) = 1$, $B_2(x) = 2x$. The first few Bernoulli polynomials are

$$B_1(x) = 1, \tag{12}$$

$$
\begin{aligned}
B_2(x) &= x - \frac{1}{2},\\
B_3(x) &= x^2 - x - \frac{1}{6},\\
B_4(x) &= x^3 - \frac{3}{2}x^2 + \frac{x}{2}.\\
&\vdots
\end{aligned} \tag{13}
$$

Our goal is to get the approximate solution as the truncated Bernoulli series defined by

$$y(x) = \sum_{n=1}^{N+1} c_n B_n(x), \tag{14}$$

where $B_n(x)$ denotes the Bernoulli polynomials; $c_n\,(1 \leq n \leq N+1)$ are the unknown coefficients for Bernoulli polynomial, and N is any positive integer which possess $N \geq m$. Let us assume that linear combination of Bernoulli polynomials equation (14) is an approximate solution of equation (7). Our purpose is to determine the matrix forms of equation (7) by using (14). Firstly, we can write Bernoulli polynomials (12) in the matrix form

$$\mathbf{B}(x) = \mathbf{T}(x)\mathbf{M}, \tag{15}$$

where $B(x) = [B_1(x)\, B_2(x) \cdots B_{N+1}(x)]$, $\mathbf{T}(x) = (1\, x\, x^2\, x^3 \ldots x^N)$, $\mathbf{C} = (c_1\, c_2 \cdots c_{N+1})^T$, and

$$
\mathbf{M} = \begin{pmatrix}
1 & -\frac{1}{2} & \frac{1}{6} & 0 & -\frac{1}{30} & 0 & \frac{1}{42} & 0 & -\frac{1}{30}\\
0 & 1 & -1 & \frac{1}{2} & 0 & -\frac{1}{6} & 0 & \frac{1}{6} & 0\\
0 & 0 & 1 & -\frac{3}{2} & 1 & 0 & -\frac{1}{2} & 0 & \frac{2}{3}\\
0 & 0 & 0 & 1 & -2 & \frac{5}{3} & 0 & -\frac{7}{6} & 0\\
0 & 0 & 0 & 0 & 1 & -\frac{5}{2} & \frac{5}{2} & 0 & -\frac{7}{3}\\
0 & 0 & 0 & 0 & 0 & 1 & -3 & \frac{7}{2} & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -\frac{7}{2} & \frac{14}{3}\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -4\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}. \tag{16}
$$

The matrix form of equation (14) by a truncated Bernoulli series is given by

$$y(x) = \mathbf{B}(x)\mathbf{C}. \tag{17}$$

By using equations (15) and (17), the matrix relation is expressed as

$$
\begin{aligned}
y(x) &\cong y_N(x) = \mathbf{T}(x)\mathbf{MC},\\
y^{(\gamma)}(x) &\cong y_N^{(\gamma)}(x) = \mathbf{T}(x)\mathbf{X}_{(\gamma)}(x)\mathbf{D}_{(\gamma)}\mathrm{MC},\\
y''(x) &\cong y_N''(x) = \mathbf{T}(x)\mathbf{D}^2\mathbf{MC},
\end{aligned} \tag{18}
$$

where

$$\mathbf{X}_{(\gamma)}(x) = \left[0, x^{1-\gamma}, x^{2-\gamma}, \ldots, x^{N-\gamma}\right], \tag{19}$$

$$\mathbf{D} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 0 & 5 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \ddots & N \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, \mathbf{D}^0 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 1 \end{bmatrix},$$

$$\mathbf{T} = \begin{bmatrix} \mathbf{T}(x_0) \\ \mathbf{T}(x_1) \\ \vdots \\ \mathbf{T}(x_N) \end{bmatrix} = \begin{bmatrix} 1 & x_0 & \ldots & x_0^N \\ 1 & x_1 & \ldots & x_1^N \\ 1 & \vdots & \ldots & \vdots \\ 1 & x_N & \ldots & x_N^N \end{bmatrix}, \mathbf{D}_{(\gamma)} = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & \dfrac{\Gamma(2)}{\Gamma(2-\gamma)} & 0 & \cdots & 0 \\ 0 & 0 & \dfrac{\Gamma(3)}{\Gamma(3-\gamma)} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \dfrac{\Gamma(N)}{\Gamma(N-\gamma)} \end{bmatrix}. \tag{20}$$

By using equation (18), we obtain the following relation:

$$\mathbf{Y}^{(k)}(x) = \mathbf{T}(x)\mathbf{D}^k\mathbf{MC}. \tag{21}$$

By substituting the Bernoulli collocation points given by

$$x_i = a + \frac{(b-a)i}{N}, \quad i = 0, 1, \ldots, N, \tag{22}$$

into equation (21), we obtain

$$\mathbf{Y}^{(k)}(x_i) = \mathbf{T}(x_i)\mathbf{D}^k\mathbf{MC}, \quad k = 0, \gamma, 2. \tag{23}$$

and the compact form of the relation (23) becomes

$$\mathbf{Y}^{(k)} = \mathbf{TD}^k\mathbf{MC}, \quad k = 0, \gamma, 2. \tag{24}$$

In this way, the unknown Bernoulli coefficients c_n, $n = 1, 2, \ldots, N+1$ are obtained by solving the system. Then, these coefficients are substituted into (14), and the approximate solution is obtained. For more details, see [36].

5. Simulation Results and Discussion

Bernoulli collocation method for obtaining the solution of fractional viscoelastic beam equation is employed. Hence, displacement analysis of a forced fractional viscoelastic beam is investigated by taking into account the different moving force loads, different values of internal damping coefficient, and different values of fractional order of derivative. Obtained results are simulated and presented in the tables and graphical forms. The velocity, from left to right, of the external moving force on beam $v(t)$ is considered as $\sin(\pi t)$. In order to observe the dynamic response of the viscoelastic beams under the different intensity of external moving force, the intensity constant of the external moving force load on the beam is involved to computation as $P = 1, 25, 50$. Also, the values in Tables 1–6 are computed on $x = 0.5$, which is the middle point of the fractional viscoelastic beams. Observed duration of time is $t_f = 1$. In the first case, forced displacement analysis of a polybutadiene beam is observed for different values of moving force load and results are presented in Table 1. The length and material density of the fractional viscoelastic beam are taken into account as $\ell = 1\,m$ and $\rho = 160\,\mathrm{kg/m^3}$, respectively. The cross-sectional area A is $0.72\,\mathrm{m^2}$, moment of inertia J is $(0.1)^4/12$, and Young's modulus E is 8.15×10^5 for a fractional viscoelastic polybutadiene beam. Also, the order of fractional derivative γ is evaluated as 0.528 for the results in Figure 2 and Tables 1 and 3. By observing Figure 2, it is concluded that while the intensity of the external moving load force increases, namely, P is 1 to 25 and 50, the displacement of the fractional viscoelastic polybutadiene beam also increases. Also, parallel observation results to Figure 2 are obtained by taking into account Table 1. For example, on the moment $t = 0.5$, the amount of the displacement of the polybutadiene beam is measured as 0.001 1 for $P = 1$, 0.028 for $P = 25$, and 0.056 for $P = 50$. This observation is valid the entire time interval $t = 0, \ldots, 1$ for polybutadiene beam. Also, the effect of internal damping on the displacement is presented in Table 3 for polybutadiene beam. The internal damping

Table 1: Some values of $w(t, x)$ for $P = 1, 25, 50$ (for a polybutadiene beam).

t	$w_{P=1}$	$w_{P=25}$	$w_{P=50}$
0.1	0.000 025 2	0.000 632 0	0.001 264 0
0.2	0.000 176 1	0.004 402 5	0.008 805 0
0.3	0.000 463 7	0.011 593 4	0.023 186 8
0.4	0.000 805 8	0.020 145 2	0.040 290 4
0.5	0.001 115 0	0.027 877 2	0.055 754 5
0.6	0.001 350 9	0.033 774 5	0.067 549 0
0.7	0.001 529 4	0.038 236 1	0.076 472 1
0.8	0.001 709 4	0.042 735 6	0.085 471 2
0.9	0.001 933 0	0.048 326 2	0.096 652 5
1.0	0.002 026 3	0.050 657 4	0.101 315 0

Table 2: Some values of $w(t, x)$ for $P = 1, 25, 50$ (for a butyl B252 beam).

t	$w_{P=1}$	$w_{P=25}$	$w_{P=50}$
0.1	0.000 020 2	0.000 505 3	0.001 010 7
0.2	0.000 140 7	0.003 517 7	0.007 035 4
0.3	0.000 370 1	0.009 253 1	0.018 506 4
0.4	0.000 642 1	0.016 053 0	0.032 106 1
0.5	0.000 886 5	0.022 164 5	0.044 328 9
0.6	0.001 070 9	0.026 772 4	0.053 544 8
0.7	0.001 207 9	0.030 198 1	0.060 396 2
0.8	0.001 345 0	0.033 624 9	0.067 249 8
0.9	0.001 515 9	0.037 899 2	0.075 798 4
1.0	0.001 581 7	0.039 544 7	0.079 089 3

Table 3: Some values of $w(t, x)$ for different values of μ for $P = 1$ (for a polybutadiene beam).

$t\mu$	0.2	0.4	0.6	0.8	1.0
0.1	0.000 024 90	0.000 024 24	0.000 023 62	0.000 023 03	0.000 022 48
0.2	0.000 170 50	0.000 161 04	0.000 152 42	0.000 144 54	0.000 137 32
0.3	0.000 440 42	0.000 402 52	0.000 369 54	0.000 340 70	0.000 315 37
0.4	0.000 748 72	0.000 659 75	0.000 586 20	0.000 524 91	0.000 473 41
0.5	0.001 011 32	0.000 856 73	0.000 735 83	0.000 640 02	0.000 563 11
0.6	0.001 194 99	0.000 973 52	0.000 810 05	0.000 687 00	0.000 592 56
0.7	0.001 323 26	0.001 044 56	0.000 850 35	0.000 711 06	0.000 608 30
0.8	0.001 459 32	0.001 136 46	0.000 922 48	0.000 774 52	0.000 667 97
0.9	0.001 645 41	0.001 287 75	0.001 058 63	0.000 902 75	0.000 790 71
1.0	0.001 713 62	0.001 340 71	0.001 112 91	0.000 964 62	0.000 862 85

Table 4: Some values of $w(t, x)$ for different values of μ for $P = 1$ (for a butyl B252 beam).

$t\mu$	0.2	0.4	0.6	0.8	1.0
0.1	0.000 019 90	0.000 019 35	0.000 018 85	0.000 018 37	0.000 017 91
0.2	0.000 136 09	0.000 128 33	0.000 121 26	0.000 114 82	0.000 108 94
0.3	0.000 350 96	0.000 319 92	0.000 293 02	0.000 269 58	0.000 249 07
0.4	0.000 595 32	0.000 522 71	0.000 463 02	0.000 413 52	0.000 372 12
0.5	0.000 801 77	0.000 676 17	0.000 578 62	0.000 501 79	0.000 440 45
0.6	0.000 943 91	0.000 764 93	0.000 633 94	0.000 536 04	0.000 461 36
0.7	0.001 040 91	0.000 817 11	0.000 662 68	0.000 552 79	0.000 472 22
0.8	0.001 459 32	0.000 886 19	0.000 717 38	0.000 601 55	0.000 667 97
0.9	0.001 285 91	0.001 002 83	0.000 823 34	0.000 701 97	0.000 518 57
1.0	0.001 333 74	0.001 041 33	0.000 864 75	0.000 750 70	0.000 615 00

coefficient is evaluated from 0.2 to 1 and by examining Table 3; it reveals that when internal damping coefficient increases, the displacement of the polybutadiene beam decreases under the same conditions. The effect of the fractional order to system is observed from Table 5 and it can be concluded that while increasing the values of the

TABLE 5: Some values of $w(t,x)$ for different values of γ for $P = 1$ (for a polybutadiene beam).

$t\gamma$	0.2	0.4	0.6	0.8	1.0
0.1	0.000 025 28	0.000 025 28	0.000 025 28	0.000 025 27	0.000 025 27
0.2	0.000 176 11	0.000 176 10	0.000 176 09	0.000 176 07	0.000 176 04
0.3	0.000 463 79	0.000 463 76	0.000 463 71	0.000 463 63	0.000 463 49
0.4	0.000 805 96	0.000 805 88	0.000 805 75	0.000 805 56	0.000 805 26
0.5	0.001 115 39	0.001 115 23	0.001 114 99	0.001 114 65	0.001 114 16
0.6	0.001 351 46	0.001 351 20	0.001 350 84	0.001 350 34	0.001 349 69
0.7	0.001 530 10	0.001 529 73	0.001 529 25	0.001 528 64	0.001 527 88
0.8	0.001 710 24	0.001 709 78	0.001 709 20	0.001 708 52	0.001 707 72
0.9	0.001 933 99	0.001 933 45	0.001 932 81	0.001 932 09	0.000 790 71
1.0	0.002 027 31	0.002 026 71	0.002 026 06	0.002 025 38	0.001 931 32

TABLE 6: Some values of $w(t,x)$ for different values of γ for $P = 1$ (for a butyl B252 beam).

$t\gamma$	0.2	0.4	0.6	0.8	1.0
0.1	0.000 020 21	0.000 020 21	0.000 020 21	0.000 020 21	0.000 020 21
0.2	0.000 140 71	0.000 140 71	0.000 140 70	0.000 140 69	0.000 140 66
0.3	0.000 370 16	0.000 370 14	0.000 370 11	0.000 370 06	0.000 369 97
0.4	0.000 642 21	0.000 642 16	0.000 642 08	0.000 641 96	0.000 641 77
0.5	0.000 886 76	0.000 886 66	0.000 886 51	0.000 886 29	0.000 885 98
0.6	0.001 071 19	0.001 071 02	0.001 070 79	0.001 070 48	0.001 070 07
0.7	0.001 208 33	0.001 208 09	0.001 207 79	0.001 207 41	0.001 206 93
0.8	0.001 345 49	0.001 345 20	0.001 344 84	0.001 344 41	0.001 343 92
0.9	0.001 516 53	0.001 516 20	0.001 515 80	0.001 515 36	0.001 514 89
1.0	0.001 582 39	0.001 582 02	0.001 581 62	0.001 581 21	0.001 580 82

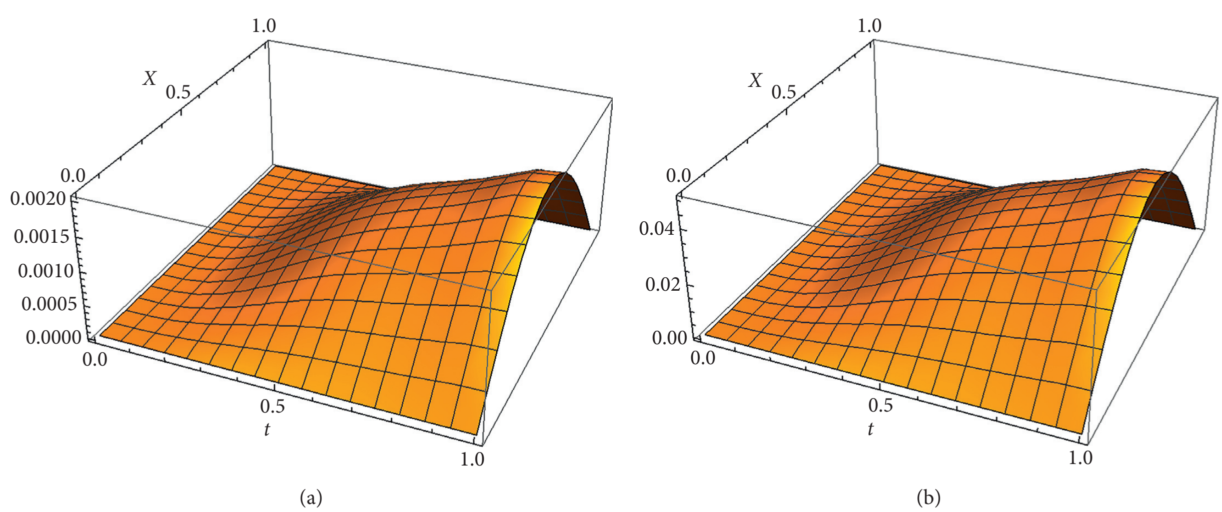
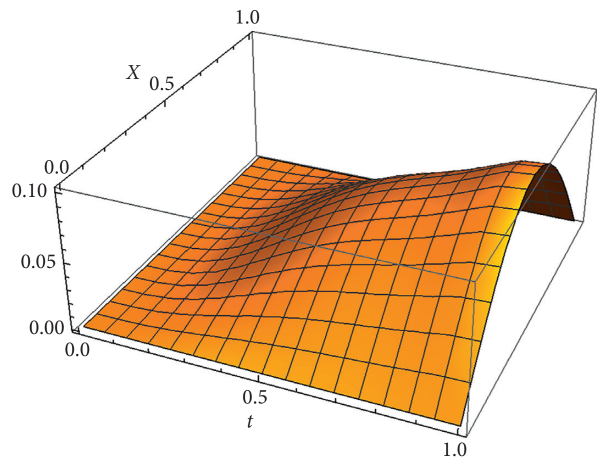
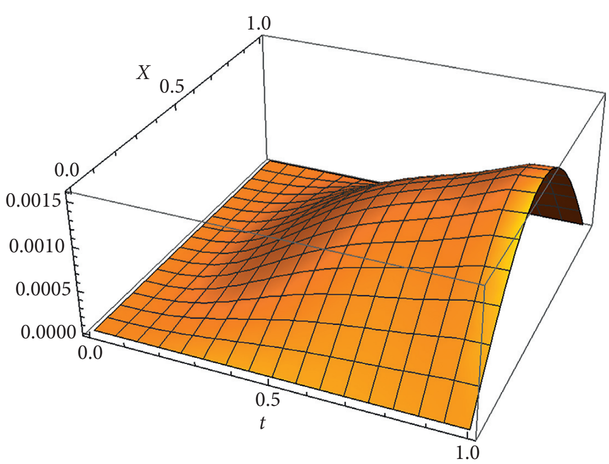


FIGURE 2: Continued.

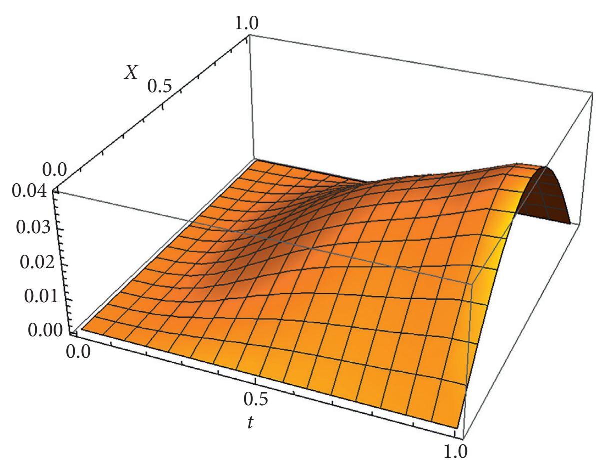


(c)

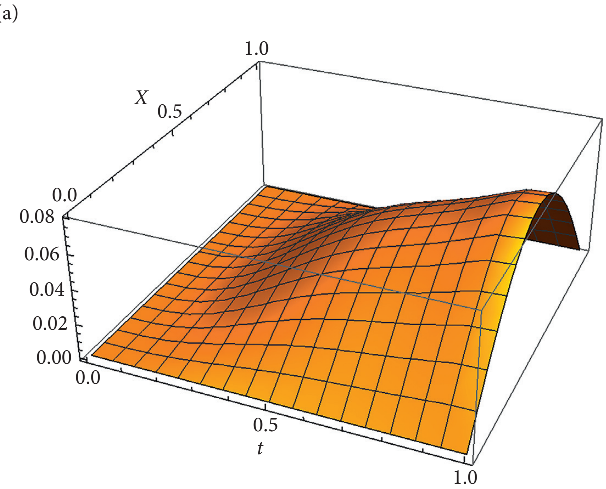
Figure 2: Displacements of a polybutadiene beam for $P = 1, 25, 50$.



(a)



(b)



(c)

Figure 3: Displacements of a butyl B252 beam for $P = 1, 25, 50$.

fractional derivative, the value of the displacement is decreasing. In the second case, a butyl B252 beam is taken into account by the coefficients; the cross-sectional area A is $0.72\,\mathrm{m}^2$, moment of inertia J is $(0.1)^4/12$, and Young's modulus E is 1.05×10^6. The order of fractional derivative γ is considered as 0.519 for Figure 3 and Tables 2 and 4. By checking Figure 3, it is easy to see that displacements corresponding to much bigger intensity of moving force load are much bigger. For example, on the moment $t=0.5$, while $P=1$ to $P=25, 50$, corresponding displacements are calculated as 0.00089, 0.022, and 0.044, respectively. This is effective along the observation duration. In Table 4, some results related to the effect of internal damping are presented and internal damping coefficient is included in the computation as 0.2 to 1. After looking at Table 4, it is concluded that while internal damping coefficient decreases, the displacement of the butyl B252 beam increases and relation between the effects of internal damping and displacements is inversely proportional. The relation between the displacement and fractional order in the system is vice versa. As understood from Table 6, while decreasing the values of the fractional derivative, the value of the displacement is increasing. These observation results of the present study are also compatible with the results existing in the literature. By taking into account Tables 1–6 and Figures 2 and 3 and comparing these two kinds of fractional viscoelastic beams, it is seen that the polybutadiene beam has more greater displacements than butyl B252 beam under same conditions. Also, the effect of internal damping coefficient is more visible on the butyl B252 beam according to polybutadiene beam. These observations make clear that butyl B252 beam is stronger and preferable than the polybutadiene beam.

6. Conclusion

In this study, the Bernoulli collocation method as a new solution method for obtaining the approximate solution of a fractional viscoelastic beam model subjected to moving force load is employed. Dynamic response analysis of the fractional viscoelastic beam model is investigated for two different specific beams: polybutadiene beam and butyl B252 beam. Displacement analysis of a point on the fractional viscoelastic beams is studied for different moving force loads and also effect of the internal damping to displacement is observed for different internal damping coefficients. Moreover, dynamic response of the fractional viscoelastic beam is examined for different values of the fractional order. Obtained results are presented in tables and graphics and results reveal that Bernoulli collocation method is very effective and powerful solution method for obtaining the solution of fractional order viscoelastic beam models. After observing Figures 2 and 3, it is easy to conclude that as the moving force load increases, the displacement of a point on the beams also increases. Also, numerical results, presented in Tables 1–4, show that under the same moving force load with the same internal damping effect, the displacement of a point on the polybutadiene beam is greater than that corresponding to butyl B252 beam. Moreover, under the same moving force load, changes in the displacements of a point on the beams are examined in the aspect of different internal damping effects and observations made clear that butyl B252 beam better reflects the effect of internal damping to displacement of a point on the beam. By comparing polybutadiene beam and butyl B252 beam, it is concluded that polybutadiene beam is more open to destructive effects of vibrations under the same conditions with the butyl B252 beam.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

The authors completed this study and wrote and approved the final version of the manuscript.

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