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Research Article

Multiple-Sets Split Common Fixed-Point Problems for Demicontractive Mappings

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In this paper, we are concerned with the multiple-sets split common fixed-point problems whenever the involved mappings are demicontractive. We first study several properties of demicontractive mappings and particularly their connection with directed mappings. By making use of these properties, we propose some new iterative methods for solving multiple-sets split common fixed-point problems, as well as multiple-sets spit feasibility problems. Under mild conditions, we establish their weak convergence of the proposed methods.

1. Introduction

The split common fixed-point problem (SCFP) requires finding an element in a fixed-point set such that its image under a linear transformation belongs to another fixed-point set. Formally, it consists in finding $x \in H_1$ such that

$$x \in F(U), Ax \in F(T), \tag{1}$$

where $A: H_1 \longrightarrow H_2$ is a bounded linear mapping from a Hilbert space H_1 into another Hilbert space H_2, and $F(U)$ and $F(T)$ are respectively the fixed-point sets of nonlinear mappings $U: H_1 \longrightarrow H_1$ and $T: H_2 \longrightarrow H_2$. Specially, if U and T are both metric projections, then problem (1) is reduced to the well-known split feasibility problem (SFP) [1]. Actually, the SFP can be formulated as finding $x \in H_1$ such that

$$x \in C, Ax \in Q, \tag{2}$$

where $C \subseteq H_1$ and $Q \subseteq H_2$ are nonempty closed convex sets, and mapping A is as above. These two problems recently have been extensively investigated since they play an important role in various areas including signal processing and image reconstruction [2–6].

We assume throughout the paper that problem (1) is consistent, which means that its solution set is nonempty. Censor and Segal [7] studied problem (1) when U and T are directed mappings. In this situation, they proposed the following method:

$$x_{n+1} = U\left[x_n - \tau_n A^* (I - T)Ax_n\right], \tag{3}$$

where A^* is the conjugate of A, I stands for the identity mapping, and τ_n is a properly chosen stepsize. It is shown that if τ_n is chosen in $(0, 2/\|A\|^2)$, then (7) converges weakly to a solution of (1). Subsequently, this result was extended to more general cases (see, e.g., [8–17]). Since the choice of the stepsize is related to $\|A\|$, thus to implement (7), one has to compute (or at least estimate) the norm $\|A\|$, which is generally not easy in practice. A way avoiding this is to adopt variable stepsize which ultimately has no relation with $\|A\|$ [9, 10, 18]. In this connection, Wang and Cui [10] proposed the following stepsize:

$$\tau_n = \begin{cases} \dfrac{\left\|(I - T)Ax_n\right\|^2}{\left\|A^* (I - T)Ax_n\right\|^2}, & \left\|(I - T)Ax_n\right\| \neq 0; \\ 0, & \left\|(I - T)Ax_n\right\| = 0. \end{cases} \tag{4}$$

On the other hand, Wang [19] proposed a new method:

$$x_{n+1} = x_n - \tau_n\left[(I - U)x_n + A^* (I - T)Ax_n\right], \tag{5}$$

where $\{\tau_n\} \subset (0, \infty)$ is chosen such that

$$\tau_n = \frac{\|(I-U)x_n\|^2 + \|(I-T)Ax_n\|^2}{\|(I-U)x_n + A^*(I-T)Ax_n\|^2}. \quad (6)$$

It is clear that the selection of stepsizes (8) and (6) does not rely on the norm $\|A\|$, which in turn improves the performance of the original algorithm. Assume that U and T are both directed such that $I-T$ and $I-U$ are demiclosed at 0. It is shown that the sequence $\{x_n\}$ generated by (7) and (8) or (5) and (6) converges weakly to a solution of problem (1).

Now, let us consider the multiple-sets split common fixed-point problem (MSCFP) that is more general than the SCFP. Formally, it consists in finding $x \in H_1$ such that

$$x \in \bigcap_{i=1}^{t} F(U_i), Ax \in \bigcap_{j=1}^{s} F(T_j), \quad (7)$$

where t and s are two positive integers, $A: H_1 \longrightarrow H_2$ is a bounded linear mapping from a Hilbert space H_1 into another Hilbert space H_2, and $F(U_i)$ and $F(T_j)$ are respectively the fixed-point sets of nonlinear mappings $U_i: H_1 \longrightarrow H_1$, $i = 1, 2, \ldots, t$ and $T_j: H_2 \longrightarrow H_2$, $j = 1, 2, \ldots, s$. Specially, if these nonlinear mappings are all metric projections, problem (7) is reduced to the well-known MSFP [20]. Actually, it can be formulated as the problem of finding $x \in H_1$ such that

$$x \in \bigcap_{i=1}^{t} C_i, Ax \in \bigcap_{j=1}^{s} Q_j, \quad (8)$$

where t and s are two positive integers, $A: H_1 \longrightarrow H_2$ is as above, and $\{C_i\}_{i=1}^{t} \subset H_1$ and $\{Q_j\}_{j=1}^{s} \subset H_2$ are two classes of nonempty convex closed subsets.

Inspired by the works mentioned above, we are aimed to introduce and analyze iterative methods for solving the MSCFP in Hilbert spaces. We first study several properties of demicontractive mappings and especially find its connection with the directed mapping. By making use of these properties, we propose a new iterative algorithm for solving the MSCFP, as well as MSFP. Under mild conditions, we obtain the weak convergence of the proposed algorithm. Our results extend the related works from the case of two-sets to the case of multiple-sets.

2. Preliminary

Throughout the paper, assume that H, H_1, H_2 are real Hilbert spaces, and $F(T)$ denotes its fixed-point set of a mapping T. The following formula plays an important role in the subsequent analysis.

Lemma 1 (see [21]). *Let $s, t \in \mathbb{R}$ and $x, y \in H$. It then follows that*

$$\|tx + sy\|^2 = t(t+s)\|x\|^2 + s(t+s)\|y\|^2 - ts\|x-y\|^2. \quad (9)$$

We next recall the definition of several important classes of nonlinear mappings.

Definition 1 (see [21]). Let T be a mapping from H into H.

(i) T is nonexpansive if

$$\|Tx - Ty\| \le \|x - y\|, \quad \forall x, y \in H. \quad (10)$$

(ii) T is firmly nonexpansive if

$$\|Tx - Ty\|^2 \le \|x-y\|^2 - \|(I-T)x - (I-T)y\|^2, \quad \forall x, y \in H. \quad (11)$$

(iii) T is k-strictly pseudocontractive $(k < 1)$ if

$$\|Tx - Ty\|^2 \le \|x-y\|^2 + k\|(I-T)x - (I-T)y\|^2, \quad \forall x, y \in H. \quad (12)$$

Definition 2 (see [21]). Let $T: H \longrightarrow H$ be a mapping with $F(T) \ne \varnothing$.

(i) T is quasinonexpansive if

$$\|Tx - y\| \le \|x - y\|, \quad \forall (x, y) \in H \times F(T). \quad (13)$$

(ii) T is directed if

$$\|Tx - y\|^2 \le \|x - y\|^2 - \|(I-T)x\|^2, \quad \forall (x, y) \in H \times F(T). \quad (14)$$

(iii) T is k-demicontractive $(k < 1)$ if

$$\|Tx - y\|^2 \le \|x - y\|^2 + k\|((I-T))x\|^2, \quad \forall (x, y) \in H \times F(T). \quad (15)$$

It is clear that a directed mapping is -1-demicontractive, while a quasinonexpansive mapping is 0-demicontractive. It is also clear that a firmly nonexpansive mapping is -1-strictly pseudocontractive, while a nonexpansive mapping is 0-strictly pseudocontractive.

It is well known that a mapping T is firmly nonexpansive if and only if $2T - I$ is nonexpansive (cf. [21]). Analogously, we can easily get the following lemma, which presents a characteristic of directed mappings by using quasinonexpansive mappings.

Lemma 2 *A mapping T is directed if and only if $2T - I$ is quasinonexpansive.*

We now study properties of demicontractive mappings.

Lemma 3 (see [22]). *Let $T: H \longrightarrow H$ be k-demicontractive $(k < 1)$ with $F(T) \ne \varnothing$. Then, the following hold.*

(i) $\langle Tx - z, (I - T)x\rangle \geq 0, \quad \forall z \in F(T), x \in H$;

(ii) $\langle x - z, (I - T)x\rangle \geq \|(I - T)x\|^2, \quad \forall z \in F(T), x \in H$.

Lemma 4. *For each $i = 1, 2, \ldots, t$, assume that $T_i\colon H \longrightarrow H$ is k_i-demicontractive with $k_i < 1$. Let $T = 1/2\sum_{i=1}^{t}\omega_i((1 + k_i)I + (1 - k_i)T_i)$, where $0 < \omega_i < 1, \sum_{i=1}^{t}\omega_i = 1$. If $\cap_{i=1}^{t}F(T_i)$ is nonempty, then*

$$F(T) = \bigcap_{i=1}^{t} F(T_i). \tag{16}$$

Proof. We first show $\cap_{i=1}^{t}F(T_i) \subseteq F(T)$. Pick $x \in \cap_{i=1}^{t}F(T_i)$. It then follows that

$$\begin{aligned} Tx &= \frac{1}{2}\sum_{i=1}^{t}\omega_i((1 + k_i)x + (1 - k_i)T_i x) \\ &= \frac{1}{2}\sum_{i=1}^{t}\omega_i((1 + k_i)x + (1 - k_i)x) \\ &= \frac{1}{2}\sum_{i=1}^{t}\omega_i 2x = x. \end{aligned} \tag{17}$$

Since x is chosen arbitrarily, we have $\cap_{i=1}^{t}F(T_i) \subseteq F(T)$.

It suffices to show that $F(T) \subseteq \cap_{i=1}^{t}F(T_i)$. Fix $z \in \cap_{i=1}^{t}F(T_i)$ and choose any $x \in F(T)$. Since $Tx = x$ and T_i is k_i-demicontractive, we have

$$\begin{aligned} 0 &= 4\langle Tx - x, x - z\rangle \\ &= 2\sum_{i=1}^{t}\omega_i(1 - k_i)\langle T_i x - x, x - z\rangle \\ &\geq \sum_{i=1}^{t}\omega_i(1 - k_i)^2\|T_i x - x\|^2. \end{aligned} \tag{18}$$

Thus, $\sum_{i=1}^{t}\omega_i(1 - k_i)^2\|x - T_i x\|^2 = 0$. Since $\omega_i(1 - k_i) > 0$, we have $\|x - T_i x\| = 0$ for all $i = 1, 2 \ldots t$. Moreover, since x is chosen arbitrarily, we get $F(T) \subseteq \cap_{i=1}^{t}F(T_i)$. Hence, the proof is complete. □

Lemma 5. *For each $i = 1, 2 \ldots t$, assume that $T_i\colon H \longrightarrow H$ is k_i-demicontractive with $k_i < 1$. Let $T = 1/2\sum_{i=1}^{t}\omega_i((1 + k_i)I + (1 - k_i)T_i)$, where $0 < \omega_i < 1, \sum_{i=1}^{t}\omega_i = 1$. If $\cap_{i=1}^{t}F(T_i)$ is nonempty, then T is directed. Moreover, if for each $i = 1, 2 \ldots t$, $I - T_i$ is demiclosed at 0, then $I - T$ is also demiclosed at 0.*

Proof. By Lemma 4, we have $F(T) = \cap_{i=1}^{t}F(T_i) \neq \emptyset$. By Lemma 2, it suffices to show that $2T - I = \sum_{i=1}^{t}\omega_i(k_i I + (1 - k_i)T_i)$ is quasinonexpansive. To this end, fix any $(x, z) \in H \times F(T)$. By Lemma 1 and the property of demicontractions that

$$\begin{aligned} \|(k_i x + (1 - k_i)T_i x) - z\|^2 &= \|k_i(x - z) + (1 - k_i)(T_i x - z)\|^2 \\ &= k_i\|x - z\|^2 + (1 - k_i)\|T_i x - z\|^2 - k_i(1 - k_i)\|(I - T_i)x\|^2 \\ &\leq k_i\|x - z\|^2 + (1 - k_i)\left(\|x - z_i\|^2 + k_i\|(I - T_i)x\|^2\right) - k_i(1 - k_i)\|(I - T_i)x\|^2 \\ &= \|x - z\|^2, \end{aligned} \tag{19}$$

hence $\|(k_i x + (1 - k_i)T_i x) - z\| \leq \|x - z\|$ for all $i = 1, 2 \ldots t$. It then follows that

$$\begin{aligned} \|(2T - I)x - z\| &= \left\|\sum_{i=1}^{t}\omega_i(k_i x + (1 - k_i)T_i x) - z\right\| \\ &\leq \sum_{i=1}^{t}\omega_i\|(k_i x + (1 - k_i)T_i x) - z\| \\ &\leq \sum_{i=1}^{t}\omega_i\|x - z\| \\ &= \|x - z\|. \end{aligned} \tag{20}$$

Thus, $2T - I$ is quasinonexpansive, which implies T is directed.

Let us now prove the second assertion. By Lemma 4, we have $F(T) = \cap_{i=1}^{t}F(T_i) \neq \emptyset$. Let $\{x_n\} \subset H$ be such that $x_n \rightharpoonup x$ and $\|x_n - Tx_n\| \longrightarrow 0$ as $n \longrightarrow \infty$. Fix $z \in F(T)$. Since T_i is k_i-demicontractive, we have

$$\begin{aligned} 4\langle Tx_n - x_n, x_n - z\rangle &= 2\sum_{i=1}^{t}\omega_i(1 - k_i)\langle T_i x_n - x_n, x_n - z\rangle \\ &\geq \sum_{i=1}^{t}\omega_i(1 - k_i)^2\|T_i x_n - x_n\|^2. \end{aligned} \tag{21}$$

Since $\omega_i(1 - k_i) > 0$, we have $\lim_n\|x_n - T_i x_n\| = 0$, which, by our hypothesis, implies $\lim_n\|x - T_i x\| = 0$ for all $i = 1, 2 \ldots t$, that is, $x \in \cap_{i=1}^{t}F(T_i)$. By Lemma 4, the proof is complete. □

Finally, we end this section by recalling two weak convergence theorems of iterative methods for approximating a solution of the two-sets SCFP (1).

Theorem 1 (see [10], Theorem 3.1). *(Assume that U and T are both directed such that $I - U$ and $I - T$ are both demiclosed at 0. Then, the sequence $\{x_n\}$, generated by (7) and (8), converges weakly to a solution of problem (1).*

Theorem 2 (see [19], Theorem 3.4). *Assume that U and T are both directed such that $I-U$ and $I-T$ are both demiclosed at 0. Then, the sequence $\{x_n\}$, generated by (5) and (6), converges weakly to a solution of problem (1).*

3. The Case for Demicontractive Mappings

In this section, we are concerned with the multiple-sets split common feasibility problem and we assume that (7) is consistent, which means that its solution set is nonempty. First, motivated by (7) and (8), we propose the first algorithm for solving problem (7).

Algorithm 1. Let x_0 be arbitrary and choose $\{\alpha_i\}_{i=1}^t \subset (0,1)$ with $\sum_{i=1}^t \alpha_i = 1$, $\{\beta_j\}_{j=1}^s \subset (0,1)$ with $\sum_{j=1}^s \beta_j = 1$. Given x_n, update the next iteration via

$$\begin{cases} y_n = x_n - \tau_n \sum_{j=1}^s \beta_j \left(1-l_j\right) A^* \left(I-T_j\right) A x_n \\ x_{n+1} = \dfrac{1}{2} \sum_{i=1}^t \alpha_i \left((1+k_i) y_n + (1-k_i) U_i y_n\right), \end{cases} \tag{22}$$

where $\tau_n = 0$ if $\|\sum_{j=1}^s \beta_j (1-l_j)(I-T_j)Ax_n\| = 0$; otherwise,

$$\tau_n = \frac{\left\|\sum_{j=1}^s \beta_j \left(1-l_j\right)\left(I-T_j\right) A x_n\right\|^2}{\left\|\sum_{j=1}^s \beta_j \left(1-l_j\right) A^* \left(I-T_j\right) A x_n\right\|^2}. \tag{23}$$

Theorem 3. *Assume that U_i and T_j are respectively k_i and l_j-demicontractive such that $I-U_i$ and $I-T_j$ are demiclosed at 0 for $i=1,2,\ldots,t$ and $j=1,2,\ldots,s$. Then, the sequence $\{x_n\}$, generated by Algorithm 1, converges weakly to a solution of (7).*

Proof. Let $U = 1/2\sum_{i=1}^t \alpha_i ((1+k_i)I + (1-k_i)U_i)$ and $T = 1/2\sum_{j=1}^s \beta_j ((1+l_j)I + (1-l_j)T_j)$. Thus, we can rewrite Algorithm 1 as

$$x_{n+1} = U\left(x_n - \tau_n A^* (I-T) A x_n\right), \tag{24}$$

where $\tau_n = 0$ if $\|(I-T)Ax_n\| = 0$; otherwise,

$$\tau_n = \frac{\left\|(I-T)Ax_n\right\|^2}{\left\|A^*(I-T)Ax_n\right\|^2}. \tag{25}$$

By Lemma 5, U and T are both directed such as $I-T$ and $I-U$ are demiclosed at 0. It then follows from Theorem 1 that $\{x_n\}$ weakly converges to a point x that satisfies $x \in F(U)$ and $Ax \in F(T)$. Moreover, by Lemma 4, we conclude that $x \in \cap_i F(U_i)$ and $Ax \in \cap_j F(T_j)$, that is, x is a solution of problem (7). □

Motivated by (5) and (6), we propose the second algorithm for solving problem (7).

Algorithm 2. Let x_0 be arbitrary and choose $\{\alpha_i\}_{i=1}^t \subset (0,1)$ with $\sum_{i=1}^t \alpha_i = 1$, $\{\beta_j\}_{j=1}^s \subset (0,1)$ with $\sum_{j=1}^s \beta_j = 1$. Given x_n, if

$$\left\|\sum_{i=1}^t \alpha_i (1-k_i)(I-U_i)x_n + \sum_{j=1}^s \beta_j \left(1-l_j\right) A^* \left(I-T_j\right) A x_n\right\| = 0, \tag{26}$$

then stop; otherwise, update the next iteration via

$$x_{n+1} = x_n - \tau_n \left[\sum_{i=1}^t (1-k_i)(I-U_i)x_n + \sum_{j=1}^s \beta_j \left(1-l_j\right) A^* \left(I-T_j\right) A x_n\right], \tag{27}$$

where

$$\tau_n = \frac{\left\|\sum_{i=1}^t \alpha_i (I-U_i)(1-k_i)x_n\right\|^2 + \left\|\sum_{j=1}^s \beta_j \left(1-l_j\right) A^* \left(I-T_j\right) A x_n\right\|^2}{2\left\|\sum_{i=1}^t (1-k_i)(I-U_i)x_n + \sum_{j=1}^s \beta_j \left(1-l_j\right) A x_n\right\|^2}. \tag{28}$$

Theorem 4. *Assume that U_i and T_j are respectively k_i and l_j-demicontractive such that $I-U_i$ and $I-T_j$ are demiclosed at 0 for $i=1,2,\ldots,t$ and $j=1,2,\ldots,s$. Then, the sequence $\{x_n\}$, generated by Algorithm 2, converges weakly to a solution of (7).*

Proof. Let $U = 1/2\sum_{i=1}^t \alpha_i ((1+k_i)I + (1-k_i)U_i)$ and $T = 1/2\sum_{j=1}^s \beta_j ((1+l_j)I + (1-l_j)T_j)$. Thus, we can rewrite Algorithm 2 as $x_{n+1} = x_n - \tau_n [(I-U)x_n + A^*(I-T)Ax_n]$, where

$$\tau_n = \frac{\left\|(I-U)x_n\right\|^2 + \left\|(I-T)Ax_n\right\|^2}{\left\|(I-U)x_n + A^*(I-T)Ax_n\right\|^2}. \tag{29}$$

By Lemma 5, U and T are both directed such as $I-T$ and $I-U$ are demiclosed at 0. It then follows from Theorem 2 that $\{x_n\}$ weakly converges to a point x that satisfies $x \in F(U)$ and $Ax \in F(T)$. Moreover, by Lemma 4, we conclude that $x \in \cap_i F(U_i)$ and $Ax \in \cap_j F(T_j)$, that is, x is a solution of problem (7). □

4. Multiple-Sets Split Feasibility Problem

In this section, we apply the previous result to approximate a solution of the multiple-sets split feasibility problem (MSFP). Also, we assume that problem (8) is consistent, which means that its solution set is nonempty. By applying Algorithm 1, we obtain the first algorithm for solving (8).

Algorithm 3. Let x_0 be arbitrary and choose $\{\alpha_i\}_{i=1}^t \subset (0,1)$ with $\sum_{i=1}^t \alpha_i = 1$, $\{\beta_j\}_{j=1}^s \subset (0,1)$ with $\sum_{j=1}^s \beta_j = 1$. Given x_n, update the next iteration via

$$x_{n+1} = \sum_{i=1}^{t} \alpha_i P_{C_i}\left[x_n - \tau_n A^* \sum_{j=1}^{s} \beta_j \left(I - P_{Q_j}\right) A x_n\right], \tag{30}$$

where $\tau_n = 0$ if $\|\sum_{j=1}^s \beta_j (1 - l_j)(I - T_j) A x_n\| = 0$; otherwise,

$$\tau_n = \frac{\left\|\sum_{j=1}^s \beta_j \left(I - P_{Q_j}\right) A x_n\right\|^2}{\left\|\sum_{j=1}^s \beta_j A^* \left(I - P_{Q_j}\right) A x_n\right\|^2}. \tag{31}$$

Theorem 5. *The sequence $\{x_n\}$, generated by Algorithm 3, converges weakly to a solution of (2).*

Proof. It suffices to notice that both P_{C_i} and P_{Q_j} are -1-demicontractive, which implies $k_i = l_j = -1$ for all $i = 1, \ldots, t$, $j = 1, \ldots, s$. Applying Theorem 3 yields the desired assertion. □

Next, we propose the second algorithm for solving (8) by applying Algorithm 2.

Algorithm 4. Let x_0 be arbitrary and choose $\{\alpha_i\}_{i=1}^t \subset (0,1)$ with $\sum_{i=1}^t \alpha_i = 1$, $\{\beta_j\}_{j=1}^s \subset (0,1)$ with $\sum_{j=1}^s \beta_j = 1$. Given x_n, if

$$\left\|\sum_{i=1}^{t} \alpha_i \left(I - P_{C_i}\right) x_n + \sum_{j=1}^{s} \beta_j A^* \left(I - P_{Q_j}\right) A x_n\right\| = 0, \tag{32}$$

then stop; otherwise, update the next iteration via

$$x_{n+1} = x_n - \tau_n \left[\sum_{i=1}^{t} \alpha_i \left(I - P_{C_i}\right) x_n + \sum_{j=1}^{s} \beta_j A^* \left(I - P_{Q_j}\right) A x_n\right], \tag{33}$$

where

$$\tau_n = \frac{\left\|\sum_{i=1}^t \alpha_i \left(I - P_{C_i}\right) x_n\right\|^2 + \left\|\sum_{j=1}^s \beta_j \left(I - P_{Q_j}\right) A x_n\right\|^2}{\left\|\sum_{i=1}^t \alpha_i \left(I - P_{C_i}\right) x_n + \sum_{j=1}^s \beta_j A^* \left(I - P_{Q_j}\right) A x_n\right\|^2}. \tag{34}$$

Theorem 6. *The sequence $\{x_n\}$, generated by Algorithm 4, converges weakly to a solution of (8).*

Proof. It suffices to notice that both P_{C_i} and P_{Q_j} are -1-demicontractive, which implies $k_i = l_j = -1$ for all $i = 1, \ldots, t$, $j = 1, \ldots, s$. Applying Theorem 4 yields the desired assertion. □

5. Conclusion

In this paper, we consider the MSCFP whenever the involved mappings are demicontractive. We obtained several properties of demicontractive mappings and particularly their connection with directed mappings. These properties enable us to propose some new iterative methods for solving MSCFP, as well as MSFP. Under mild conditions, we establish their weak convergence of the proposed methods. Our results extend the existing works from the case of two-sets to the case of multiple-sets.

Data Availability

No data were used to support this study.

Conflicts of Interest

The author declares no conflicts of interest.

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