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Research Article

On the Study of Reverse Degree-Based Topological Properties for the Third Type of pth Chain Hex-Derived Network

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Vertices and edges are made from a network, with the degree of a vertex referring to the number of connected edges. The chance of every vertex possessing a given degree is represented by a network's degree appropriation, which reveals important global network characteristics. Many fields, including sociology, public health, business, medicine, engineering, computer science, and basic sciences, use network theory. Logistical networks, gene regulatory networks, metabolic networks, social networks, and driven networks are some of the most significant networks. In physical, theoretical, and environmental chemistry, a topological index is a numerical value assigned to a molecular structure/network that is used for correlation analysis. Hexagonal networks of dimension t are used to build hex-derived networks, which have a wide range of applications in computer science, medicine, and engineering. For the third type of hex-derived networks, topological indices of reverse degree based are discussed in this study.

1. Introduction

A topological descriptor is a numerical value that represents the complete structure of a graph. In the study of topological descriptors, graph theory has shown to be a fruitful field of study. The primary elements of topological indices link the many chemical and physical characteristics of fundamental chemical substances. Vertex-edge-based topological indices are employed in the research of QSAR/QSPR for the prediction of bio-activity of different chemical compounds. With the dimension p, hexagonal networks create hex-derived networks, which have a wide range of implementations in engineering, computer science, and also medicine. In [1], researchers created a new form of graph known as a "third type of hex-derived networks" [2, 3] and continued this work by calculating degree-based topological descriptors for these networks, in which they computed exact values of some vertex-edge named topological indices for this network.

Researchers have used graph theory to develop a range of helpful tools, including graph labeling, topological indices, and finding numbers. The subject of graph theory has several applications and implementations in various fields of study, including chemistry, medicine, and engineering. A polynomial, a series of integers, a numeric value, or a matrix can all be used to identify a graph. A chemical compound can be represented as a graph (or a diagram) or usually denoted as a molecular graph, nodes played a role of atoms, and the bonding between atoms is usually labeled as edges in the molecular graph theory. Recently, a new topic called cheminformatics was established, which is a mix of chemistry, information science, and mathematics, in which the QSPR/QSAR connection, bio-activity, and characterization of chemical compounds are investigated and reported in [4].

The topological descriptor is a numerical number associated with chemical compositions that maintain the relationship between chemical structures and a variety of physico-chemical characteristics, biological activity, and chemical reactivity. To describe the topology of a chemical network, it translated into a number, which is further used to create topological indices. Distance-based topological indices, degree-based topological indices, and counting-related topological indices are some of the most common forms of topological indices for graphs. Many academics have recently discovered topological indices for studying basic features of molecular graphs or networks. In [5–12], these networks have extremely compelling topological qualities that have been examined in distinct characteristics.

Let p and q represent the number of rows and number of triangles in each row of third-type pth chain hex-derived networks $G^3_{p,q}$, respectively, shown in Figure 1. Let G be a simple connected network, with a set of vertex and edges denoted by V and E, respectively. $|V|$ represents the order of G and $|E|$ represents the size of G. Let d_θ be the degree of a vertex $\theta \in V$ in G and $\mathfrak{R}_\theta$ be its reverse degree that was introduced by Kulli [13] and defined as $\mathfrak{R}_\theta = 1 + \Delta - d_\theta$, where Δ denoted the maximum degree of the given graph. Let $\mathbf{E}_{\mathfrak{R}_\theta,\mathfrak{R}_\vartheta}$ represent the edge partition of the given graph based on reverse degree of end vertices of an edge $\theta\vartheta \in E$ and $|\mathbf{E}_{\mathfrak{R}_\theta,\mathfrak{R}_\vartheta}|$ represent its cardinality.

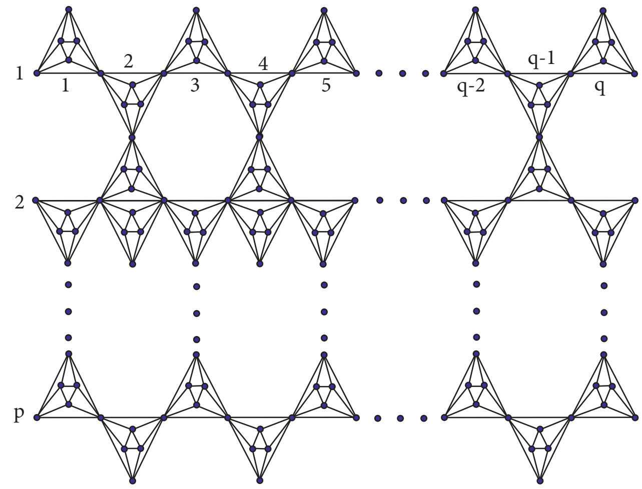

FIGURE 1: Third-type chain hex-derived network $G^3_{p,q}$.

We define general reverse degree topological invariant $\mathbf{T}(G)$ as follows:

$$\mathcal{T}(G) = \sum_{\theta\vartheta \in E(G)} \lambda(\mathfrak{R}_\theta, \mathfrak{R}_\vartheta), \tag{1}$$

$$\text{If } \lambda(\mathfrak{R}_\theta, \mathfrak{R}_\vartheta) = (\mathfrak{R}_\theta \times \mathfrak{R}_\vartheta)^\alpha, \text{ then } \mathcal{T}(G) \text{ represents the general reverse Randić index } (\mathfrak{R}\mathbb{R}_\alpha(G)) \text{ for } \alpha = \frac{1}{2}, -\frac{1}{2}, 1, -1, \\ \text{If } \alpha = 1, \text{ then it is known as the second reverse Zagreb index } (\mathfrak{R}\mathbb{M}_2(G)), \tag{2}$$

$$\text{If } \lambda(\mathfrak{R}_\theta, \mathfrak{R}_\vartheta) = \sqrt{\frac{\mathfrak{R}_\theta + \mathfrak{R}_\vartheta - 2}{\mathfrak{R}_\theta \times \mathfrak{R}_\vartheta}}, \text{ then } \mathcal{T}(G) \text{ represents the reverse atom} - \text{bond connectivity index } (\mathfrak{R}\mathbb{ABC}(G)), \tag{3}$$

$$\text{If } \lambda(\mathfrak{R}_\theta, \mathfrak{R}_\vartheta) = \frac{2\sqrt{\mathfrak{R}_\theta \times \mathfrak{R}_\vartheta}}{\mathfrak{R}_\theta + \mathfrak{R}_\vartheta}, \text{ then } \mathcal{T}(G) \text{ represents the reverse geometric} - \text{arithmetic index } (\mathfrak{R}\mathrm{GA}(G)), \tag{4}$$

$$\text{If } \lambda(\mathfrak{R}_\theta, \mathfrak{R}_\vartheta) = (\mathfrak{R}_\theta + \mathfrak{R}_\vartheta), \text{ then } \mathcal{T}(G) \text{ represents the first reverse Zagreb index } (\mathfrak{R}\mathbb{M}_1(G)), \tag{5}$$

$$\text{If } \lambda(\mathfrak{R}_\theta, \mathfrak{R}_\vartheta) = (\mathfrak{R}_\theta + \mathfrak{R}_\vartheta)^2, \text{ then } \mathcal{T}(G) \text{ represents the reverse hyper Zagreb index } (\mathfrak{R}\mathbb{H}\mathrm{M}(G)), \tag{6}$$

$$\text{If } \lambda(\mathfrak{R}_\theta, \mathfrak{R}_\vartheta) = \left((\mathfrak{R}_\theta)^2 + (\mathfrak{R}_\vartheta)^2\right), \text{ then } \mathcal{T}(G) \text{ represents the reverse forgotten index } (\mathfrak{R}\mathbb{F}(G)), \tag{7}$$

$$\begin{aligned} &\text{If } \lambda(\mathfrak{R}_\theta, \mathfrak{R}_\vartheta) = (\mathfrak{R}_\theta + \mathfrak{R}_\vartheta)^\alpha (\mathfrak{R}_\theta \times \mathfrak{R}_\vartheta)^\beta, \text{ then } \mathcal{T}(G) \text{ represents} \\ &\text{the first reverse redefined index} (\mathfrak{R}\mathbb{R}\mathbb{Z}_1(G)) \text{for } \alpha = 1, \beta = -1, \\ &\text{the second reverse redefined index} (\mathfrak{R}\mathbb{R}\mathbb{Z}_2(G)) \text{for } \alpha = -1, \beta = 1, \\ &\text{the third reverse redefined index } (\mathfrak{R}\mathbb{R}\mathbb{Z}_3(G)) \text{for } \alpha = 1, \beta = 1. \end{aligned} \tag{8}$$

For latest results on topological descriptors for different chemical and computer networks and for general graphs, we refer to see [14–25]. In this current research work, we determine the exact values of all the above reverse indices.

2. Structure of Third-Type Hex-Derived Networks

With the help of complete graphs of order 3 (K_3), Chen et al. [26] assembled a hexagonal mesh. In terms of chemistry, these K_3 graphs are also called oxide graphs. Figure 1 is obtained by joining these K_3 graphs. Two-dimensional mesh graph HX(2) (see Figure 2(a)) is obtained by joining six K_3 graphs and three-dimensional mesh graph HX(3) (see Figure 2(b)) is obtained by putting K_3 graphs around all sides of HX(2) [27]. Furthermore, repeating the same process by putting the tK_3 graph around each hexagon, we obtained the t^{th} hexagonal mesh. We have to note that the one-dimensional hexagonal mesh graph does not exist.

The novel network, labeled the third category of hex-derived networks, was developed in [1]. In [2, 3], they defined the graphically construction algorithm for the third type of hexagonal hex-derived network HHDN3(t). Huo *el at.* [28] explained the graphical construction algorithm for m^{th} chain hex-derived network of third type. In this paper, we denote it by $G^3_{p,q}$, and different priorities of p and q the chain hex-derived networks are shown in Figure 3. In [29–33], you may find related research that utilizes this idea and that may benefit from the new research's visions.

3. Main Results

In this section, we study the third-type pth chain hex-derived networks $G^3_{p,q}$ in the following three cases.

(i) Case 1: for $p = q$, $(p,q) \geq 1$.

(ii) Case 2: for $p < q$, p is odd and q is a natural number. For $p > q$, p is odd and q is a natural number. For $p < q$, p and q both are even. For $p > q$, p and q both are even.

(iii) Case 3: for $p < q$, p is even and q is odd. For $p > q$, p is even and q is odd.

3.1. Results for Case 1. We provide a formula that would be used to calculate any reverse degree topological descriptors of Case 1 for $G^3_{p,q}$.

Lemma 1. *Let $G^3_{p,q}$ be a third-type pth chain hex-derived networks. Then,*

$$\mathcal{T}_1\left(G^3_{p,q}\right) = 3pq(\lambda(1,1) + 2\lambda(1,5) + \lambda(5,5)) + 3(p+q)(\lambda(5,5) - \lambda(1,1)) + 2(\lambda(1,1) - 2\lambda(1,5) + \lambda(5,5)). \tag{9}$$

Proof. The graph $G^3_{p,q}$ contains $12pq$ edges and maximum degree in $G^3_{p,q}$ graph is 8. There are two types of reverse degree vertices in $G^3_{p,q}$ that are 1 and 5. Let us partition the edges of $G^3_{p,q}$ according to its reverse degrees according to Case 1 as

$$\begin{aligned}
\mathbf{E}_{1,1} &= \left\{\theta\vartheta \in E\left(G^3_{p,q}\right) : \mathfrak{R}_\theta = 1, \mathfrak{R}_\vartheta = 1\right\},\\
\mathbf{E}_{1,5} &= \left\{\theta\vartheta \in E\left(G^3_{p,q}\right) : \mathfrak{R}_\theta = 1, \mathfrak{R}_\vartheta = 5\right\},\\
\mathbf{E}_{5,5} &= \left\{\theta\vartheta \in E\left(G^3_{p,q}\right) : \mathfrak{R}_\theta = 5, \mathfrak{R}_\vartheta = 5\right\}.
\end{aligned} \tag{10}$$

Note that $E(G^3_{p,q}) = \mathbf{E}_{1,1} \cup \mathbf{E}_{1,5} \cup \mathbf{E}_{5,5}$ and $|\mathbf{E}_{1,1}| = 3pq - 3p - 3q + 2$, $|\mathbf{E}_{1,5}| = 6pq - 4$, and $|\mathbf{E}_{5,5}| = 3pq + 3p + 3q + 2$. Hence,

$$\begin{aligned}
\mathcal{T}_1\left(G^3_{p,q}\right) &= \sum_{\theta\vartheta \in E\left(G^3_{p,q}\right)} \lambda(\mathfrak{R}_\theta, \mathfrak{R}_\vartheta)\\
&= \sum_{\theta\vartheta\in\mathbf{E}_{1,1}} \lambda(1,1) + \sum_{\theta\vartheta\in\mathbf{E}_{1,5}} \lambda(1,5) + \sum_{\theta\vartheta\in\mathbf{E}_{5,5}} \lambda(5,5)\\
&= (3pq - 3p - 3q + 2)\lambda(1,1) + (6pq - 4)\lambda(1,5) + (3pq + 3p + 3q + 2)\lambda(5,5).
\end{aligned} \tag{11}$$

After simplification, we obtain

$$\mathcal{T}_1\left(G^3_{p,q}\right) = 3pq(\lambda(1,1) + 2\lambda(1,5) + \lambda(5,5)) + 3(p+q)(\lambda(5,5) - \lambda(1,1)) + 2(\lambda(1,1) - 2\lambda(1,5) + \lambda(5,5)). \tag{12}$$

□

Theorem 1. *The general reverse Randić index of $G^3_{p,q}$ is equal to*

$$\mathfrak{R}\mathbb{R}_\alpha\left(G^3_{p,q}\right) = \begin{cases}
108pq + 72(p+q) + 32, & \text{for } \alpha = 1,\\
(18 + 6\sqrt{5})pq + 12(p+q) + 12 - 4\sqrt{5}, & \text{for } \alpha = \dfrac{1}{2},\\
\left(\dfrac{18}{5} + \dfrac{6\sqrt{5}}{5}\right)pq - \dfrac{12}{5}(p+q) + \dfrac{12}{5} - \dfrac{4\sqrt{5}}{5}, & \text{for } \alpha = \dfrac{-1}{2},\\
\dfrac{108}{25}pq - \dfrac{72}{25}(p+q) + \dfrac{32}{25}, & \text{for } \alpha = -1.
\end{cases} \tag{13}$$

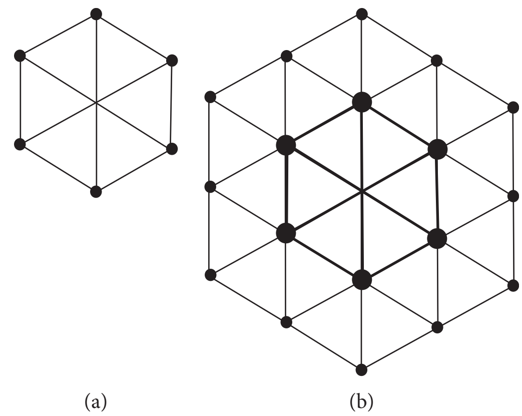

Figure 2: Hexagonal meshes: (a) HX (2) and (b) HX (3).

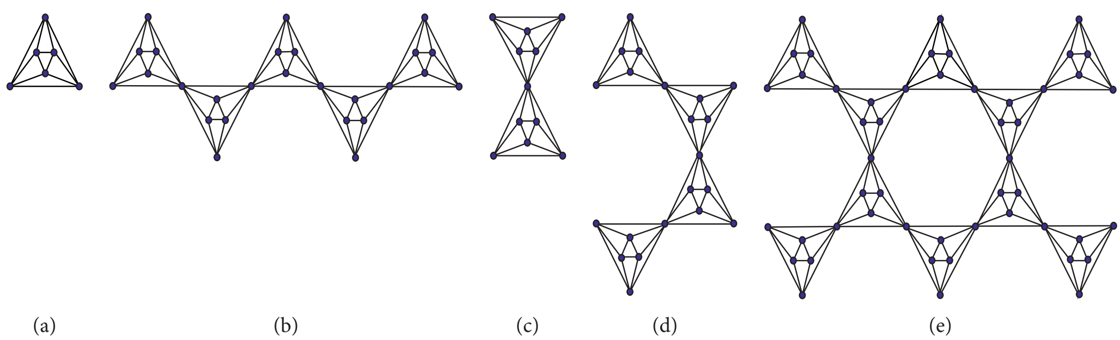

Figure 3: Chain hex-derived networks $G^3_{p,q}$ for different priorities of p and q.

Proof. For $\mathfrak{R}\mathbb{R}_\alpha(G^3_{p,q})$ which is the general reverse Randić index of $G^3_{p,q}$, from equation (2), we have $\lambda(\mathfrak{R}_\theta, \mathfrak{R}_\vartheta) = (\mathfrak{R}_\theta \times \mathfrak{R}_\vartheta)^\alpha$; therefore, $\lambda(1,1) = 1$, $\lambda(1,5) = (5)^\alpha$, and $\lambda(5,5) = (25)^\alpha$. Thus, by Lemma 1,

$$\mathfrak{R}\mathbb{R}_\alpha\left(G^3_{p,q}\right)\big) = 3pq\left(1 + 2(5)^\alpha + (25)^\alpha\right) + 3(p+q)\left((25)^\alpha - 1\right) + 2\left(1 - 2(5)^\alpha + (25)^\alpha\right). \tag{14}$$

Put $\alpha = 1$, and we have

$$\mathfrak{R}\mathbb{R}_1\left(G^3_{p,q}\right)\big) = 108pq + 72(p+q) + 32. \tag{15}$$

Put $\alpha = (1/2)$, and we have

$$\mathfrak{R}\mathbb{R}_{(1/2)}\left(G^3_{p,q}\right)\big) = (18 + 6\sqrt{5})pq + 12(p+q) + 12 - 4\sqrt{5}. \tag{16}$$

Put $\alpha = (-1/2)$, and we have

$$\mathfrak{R}\mathbb{R}_{(-1/2)}\left(G^3_{p,q}\right)\big) = \left(\frac{18}{5} + \frac{6\sqrt{5}}{5}\right)pq - \frac{12}{5}(p+q) + \frac{12}{5} - \frac{4\sqrt{5}}{5}. \tag{17}$$

Put $\alpha = -1$, and we have

$$\mathfrak{R}\mathbb{R}_{-1}\left(G^3_{p,q}\right)\big) = \frac{108}{25}pq - \frac{72}{25}(p+q) + \frac{32}{25}. \tag{18}$$

□

Theorem 2. *Let $G^3_{p,q}$ be a third-type p th chain hex-derived networks. Then, the reverse atom-bond connectivity index is*

$$\mathfrak{R}\mathbb{ABC}\left(G^3_{p,q}\right) = \left(\frac{12\sqrt{5}}{5} + \frac{6\sqrt{2}}{5}\right)pq - \frac{8\sqrt{5}}{5} + \frac{2(3p + 3q + 2)\sqrt{2}}{5}. \tag{19}$$

The reverse geometric-arithmetic index is

$$\mathfrak{R}\mathrm{GA}\left(G^3_{p,q}\right) = (6 + 2\sqrt{5})pq + 4 - \frac{4\sqrt{5}}{3}. \tag{20}$$

The first reverse Zagreb index is

$$\mathfrak{R}\mathbb{M}_1\left(G^3_{p,q}\right)\big) = 72pq + 24p + 24q. \tag{21}$$

The reverse hyper-Zagreb index is

$$\mathfrak{R}\mathbb{HM}\left(G^3_{p,q}\right) = 528pq + 288p + 288q + 64. \tag{22}$$

The reverse forgotten index is

$$\mathfrak{R}\mathbb{F}\left(G^3_{p,q}\right) = 312pq + 144p + 144q. \tag{23}$$

Proof. For $\mathfrak{R}\mathbb{ABC}(G^3_{p,q})$ which is the reverse atom-bond connectivity index of $G^3_{p,q}$, from equation (3), we have $\lambda(\mathfrak{R}_\theta, \mathfrak{R}_\vartheta) = \sqrt{(\mathfrak{R}_\theta + \mathfrak{R}_\vartheta - 2)/\mathfrak{R}_\theta \times \mathfrak{R}_\vartheta}$; therefore, $\lambda(1,1)$

$= 0$, $\lambda(1,5) = \sqrt{4/5}$, and $\lambda(5,5) = (\sqrt{8}/5)$. Thus, by Lemma 1 and after simplification,

$$\mathfrak{R}\mathbb{ABC}\left(G_{p,q}^3\right) = \left(\frac{12\sqrt{5}}{5} + \frac{6\sqrt{2}}{5}\right)pq - \frac{8\sqrt{5}}{5} + \frac{2(3p+3q+2)\sqrt{2}}{5}. \tag{24}$$

For $\mathfrak{R}\mathrm{GA}(G_{p,q}^3)$ which is the reverse geometric-arithmetic index of $G_{p,q}^3$, from equation (4), we have $\lambda(\mathfrak{R}_\theta, \mathfrak{R}_\vartheta) = (2\sqrt{\mathfrak{R}_\theta \times \mathfrak{R}_\vartheta}/(\mathfrak{R}_\theta + \mathfrak{R}_\vartheta))$; therefore, $\lambda(1,1) = 1$, $\lambda(1,5) = (\sqrt{5}/3)$, and $\lambda(5,5) = 1$. Thus, by Lemma 1 and after simplification,

$$\mathfrak{R}\mathrm{GA}\left(G_{p,q}^3\right) = (6 + 2\sqrt{5})pq + 4 - \frac{4\sqrt{5}}{3}. \tag{25}$$

For $\mathfrak{R}\mathbb{M}_1(G_{p,q}^3)$ which is the first reverse Zagreb index of $G_{p,q}^3$, from equation (5), we have $\lambda(\mathfrak{R}_\theta, \mathfrak{R}_\vartheta) = (\mathfrak{R}_\theta + \mathfrak{R}_\vartheta)$; therefore, $\lambda(1,1) = 2$, $\lambda(1,5) = 6$, and $\lambda(5,5) = 10$. Thus, by Lemma 1 and after simplification,

$$\mathfrak{R}\mathbb{M}_1\left(G_{p,q}^3\right)) = 72pq + 24p + 24q. \tag{26}$$

For $\mathfrak{R}\mathbb{HM}(G_{p,q}^3)$ which is the first reverse hyper-Zagreb index of $G_{p,q}^3$, from equation (6), we have $\lambda(\mathfrak{R}_\theta, \mathfrak{R}_\vartheta) = (\mathfrak{R}_\theta + \mathfrak{R}_\vartheta)^2$; therefore, $\lambda(1,1) = 4$, $\lambda(1,5) = 36$, and $\lambda(5,5) = 100$. Thus, by Lemma 1 and after simplification,

$$\mathfrak{R}\mathbb{HM}\left(G_{p,q}^3\right) = 528pq + 288p + 288q + 64. \tag{27}$$

For $\mathfrak{R}\mathbb{F}(G_{p,q}^3)$ which is the reverse forgotten index of $G_{p,q}^3$, from equation (7), we have $\lambda(\mathfrak{R}_\theta, \mathfrak{R}_\vartheta) = ((\mathfrak{R}_\theta)^2 + (\mathfrak{R}_\vartheta)^2)$; therefore, $\lambda(1,1) = 2$, $\lambda(1,5) = 26$, and $\lambda(5,5) = 50$. Thus, by Lemma 1 and after simplification,

$$\mathfrak{R}\mathbb{F}\left(G_{p,q}^3\right) = 312pq + 144p + 144q. \tag{28}$$

□

Theorem 3. *Let $G_{p,q}^3$ be a third-type p th chain hex-derived networks. Then, the first reverse redefined index is*

$$\mathfrak{R}\mathbb{RZ}_1\left(G_{p,q}^3\right) = \frac{72pq}{5} - \frac{24p}{5} - \frac{24q}{5}. \tag{29}$$

The second reverse redefined index is

$$\mathfrak{R}\mathbb{RZ}_2\left(G_{p,q}^3\right) = 14pq + 6p + 6q + \frac{8}{3}. \tag{30}$$

The third reverse redefined index is

$$\mathfrak{R}\mathbb{RZ}_3\left(G_{p,q}^3\right) = 936pq + 744p + 744q + 384. \tag{31}$$

Proof. For $\mathfrak{R}\mathbb{RZ}_1(G_{p,q}^3)$ which is the first reverse redefined index of $G_{p,q}^3$, from equation (8), we have $\lambda(\mathfrak{R}_\theta, \mathfrak{R}_\vartheta) = (\mathfrak{R}_\theta + \mathfrak{R}_\vartheta)^1(\mathfrak{R}_\theta \times \mathfrak{R}_\vartheta)^{-1}$; therefore, $\lambda(1,1) = 2$, $\lambda(1,5) = (6/5)$, and $\lambda(5,5) = (2/5)$. Thus, by Lemma 1 and after simplification,

$$\mathfrak{R}\mathbb{RZ}_1\left(G_{p,q}^3\right) = \frac{72pq}{5} - \frac{24p}{5} - \frac{24q}{5}. \tag{32}$$

For $\mathfrak{R}\mathbb{RZ}_2(G_{p,q}^3)$ which is the second reverse redefined index of $G_{p,q}^3$, from equation (8), we have $\lambda(\mathfrak{R}_\theta, \mathfrak{R}_\vartheta) = (\mathfrak{R}_\theta + \mathfrak{R}_\vartheta)^{-1}(\mathfrak{R}_\theta \times \mathfrak{R}_\vartheta)^1$; therefore, $\lambda(1,1) = (1/2)$, $\lambda(1,5) = (5/6)$, and $\lambda(5,5) = (5/2)$. Thus, by Lemma 1 and after simplification,

$$\mathfrak{R}\mathbb{RZ}_2\left(G_{p,q}^3\right) = 14pq + 6p + 6q + \frac{8}{3}. \tag{33}$$

For $\mathfrak{R}\mathbb{RZ}_3(G_{p,q}^3)$ which is the third reverse redefined index of $G_{p,q}^3$, from equation (8), we have $\lambda(\mathfrak{R}_\theta, \mathfrak{R}_\vartheta) = (\mathfrak{R}_\theta + \mathfrak{R}_\vartheta)(\mathfrak{R}_\theta \times \mathfrak{R}_\vartheta)$; therefore, $\lambda(1,1) = 2$, $\lambda(1,5) = 30$, and $\lambda(5,5) = 250$. Thus, by Lemma 1 and after simplification,

$$\mathfrak{R}\mathbb{RZ}_3\left(G_{p,q}^3\right) = 936pq + 744p + 744q + 384. \tag{34}$$

□

3.2. Results for Case 2. We provide a formula that would be used to calculate any reverse degree topological descriptors of Case 2 for $G_{p,q}^3$.

Lemma 2. *Let $G_{p,q}^3$ be a third-type p th chain hex-derived networks. Then,*

$$\begin{aligned}\mathcal{T}_2\left(G_{p,q}^3\right) = {} & 3pq(\lambda(1,1) + 2\lambda(1,5) + \lambda(5,5)) \\ & + 2(2p+q)(\lambda(5,5) - \lambda(1,1)) + 2(\lambda(1,1) \\ & - 2\lambda(1,5) + \lambda(5,5)).\end{aligned} \tag{35}$$

Proof. The graph $G_{p,q}^3$ contains $12pq$ edges and maximum degree in $G_{p,q}^3$ graph is 8. There are two types of reverse degree vertices in $G_{p,q}^3$ that are 1 and 5. Let us partition the edges of $G_{p,q}^3$ according to its reverse degrees according to Case 2 as

$$\begin{aligned}\mathbf{E}_{1,1} &= \left\{\theta\vartheta \in E\left(G_{p,q}^3\right) : \mathfrak{R}_\theta = 1, \mathfrak{R}_\vartheta = 1\right\}, \\ \mathbf{E}_{1,5} &= \left\{\theta\vartheta \in E\left(G_{p,q}^3\right) : \mathfrak{R}_\theta = 1, \mathfrak{R}_\vartheta = 5\right\}, \\ \mathbf{E}_{5,5} &= \left\{\theta\vartheta \in E\left(G_{p,q}^3\right) : \mathfrak{R}_\theta = 5, \mathfrak{R}_\vartheta = 5\right\}.\end{aligned} \tag{36}$$

Note that $E(G_{p,q}^3) = \mathbf{E}_{1,1} \cup \mathbf{E}_{1,5} \cup \mathbf{E}_{5,5}$ and $|\mathbf{E}_{1,1}| = 3pq - 4p - 2q + 2$, $|\mathbf{E}_{1,5}| = 6pq - 4$, and $|\mathbf{E}_{5,5}| = 3pq + 4p + 2q + 2$. Hence,

$$\begin{aligned}\mathcal{T}_2\left(G_{p,q}^3\right) &= \sum_{\theta\vartheta \in E\left(G_{p,q}^3\right)} \lambda(\mathfrak{R}_\theta, \mathfrak{R}_\vartheta) \\ &= \sum_{\theta\vartheta \in \mathbf{E}_{1,1}} \lambda(1,1) + \sum_{\theta\vartheta \in \mathbf{E}_{1,5}} \lambda(1,5) + \sum_{\theta\vartheta \in \mathbf{E}_{5,5}} \lambda(5,5) \\ &= (3pq - 4p - 2q + 2)\lambda(1,1) + (6pq - 4)\lambda(1,5) \\ &\quad + (3pq + 4p + 2q + 2)\lambda(5,5).\end{aligned} \tag{37}$$

After simplification, we obtain

$$\mathcal{T}_2(G^3_{p,q}) = 3pq(\lambda(1,1) + 2\lambda(1,5) + \lambda(5,5)) + 2(2p+q)(\lambda(5,5) - \lambda(1,1)) + 2(\lambda(1,1) - 2\lambda(1,5) + \lambda(5,5)). \quad (38)$$

□

Theorem 4. *The general reverse Randić index of* $G^3_{p,q}$ *is equal to*

$$\mathfrak{R}\mathbb{R}_\alpha(G^3_{p,q}) = \begin{cases} 108pq + 96p + 48q + 32, & \text{for } \alpha = 1, \\ (18 + 6\sqrt{5})pq + 16p + 8q + 12 - 4\sqrt{5}, & \text{for } \alpha = \frac{1}{2}, \\ \left(\frac{18}{5} + \frac{6\sqrt{5}}{5}\right)pq - \frac{16p}{5} - \frac{8}{5}q + \frac{12}{5} - \frac{4\sqrt{5}}{5}, & \text{for } \alpha = \frac{-1}{2}, \\ \frac{108}{25}pq - \frac{96}{25}p - \frac{48}{25}q + \frac{32}{25}, & \text{for } \alpha = -1. \end{cases} \quad (39)$$

Proof. For $\mathfrak{R}\mathbb{R}_\alpha(G^3_{p,q})$ which is the general reverse Randić index of $G^3_{p,q}$, from equation (2), we have $\lambda(\mathfrak{R}_\theta, \mathfrak{R}_\vartheta) = (\mathfrak{R}_\theta \times \mathfrak{R}_\vartheta)^\alpha$; therefore, $\lambda(1,1) = 1$, $\lambda(1,5) = (5)^\alpha$, and $\lambda(5,5) = (25)^\alpha$. Thus, by Lemma 2,

$$\mathfrak{R}\mathbb{R}_\alpha(G^3_{p,q})) = 3pq(1 + 2(5)^\alpha + (25)^\alpha) + 2(2p+q)((25)^\alpha - 1) + 2(1 - 2(5)^\alpha + (25)^\alpha). \quad (40)$$

Put $\alpha = 1$, and we have

$$\mathfrak{R}\mathbb{R}_1(G^3_{p,q})) = 108pq + 96p + 48q + 32. \quad (41)$$

Put $\alpha = (1/2)$, and we have

$$\mathfrak{R}\mathbb{R}_{(1/2)}(G^3_{p,q})) = (18 + 6\sqrt{5})pq + 16p + 8q + 12 - 4\sqrt{5}. \quad (42)$$

Put $\alpha = (-1/2)$, and we have

$$\mathfrak{R}\mathbb{R}_{(-1/2)}(G^3_{p,q})) = \left(\frac{18}{5} + \frac{6\sqrt{5}}{5}\right)pq - \frac{16p}{5} - \frac{8}{5}q + \frac{12}{5} - \frac{4\sqrt{5}}{5}. \quad (43)$$

Put $\alpha = -1$, and we have

$$\mathfrak{R}\mathbb{R}_{-1}(G^3_{p,q})) = \frac{108}{25}pq - \frac{96}{25}p - \frac{48}{25}q + \frac{32}{25}. \quad (44)$$

□

Theorem 5. *Let* $G^3_{p,q}$ *be a third-type* p *th chain hex-derived networks. Then, the reverse atom-bond connectivity index is*

$$\mathfrak{R}\mathbb{ABC}(G^3_{p,q}) = \left(\frac{12\sqrt{5}}{5} + \frac{6\sqrt{2}}{5}\right)pq - \frac{8\sqrt{5}}{5} + \frac{2(4p+2q+2)\sqrt{2}}{5}. \quad (45)$$

The reverse geometric-arithmetic index is

$$\mathfrak{R}\mathrm{GA}(G^3_{p,q}) = (6 + 2\sqrt{5})pq + 4 - \frac{4\sqrt{5}}{3}. \quad (46)$$

The first reverse Zagreb index is

$$\mathfrak{R}\mathbb{M}_1(G^3_{p,q})) = 72pq + 32p + 16q. \quad (47)$$

The reverse hyper-Zagreb index is

$$\mathfrak{R}\mathbb{HM}(G^3_{p,q}) = 528pq + 384p + 192q + 64. \quad (48)$$

The reverse forgotten index is

$$\mathfrak{R}\mathbb{F}(G^3_{p,q}) = 312pq + 192p + 96q. \quad (49)$$

Proof. For $\mathfrak{R}\mathbb{ABC}(G^3_{p,q})$ which is the reverse atom-bond connectivity index of $G^3_{p,q}$, from equation (3), we have $\lambda(\mathfrak{R}_\theta, \mathfrak{R}_\vartheta) = \sqrt{(\mathfrak{R}_\theta + \mathfrak{R}_\vartheta - 2)/\mathfrak{R}_\theta \times \mathfrak{R}_\vartheta}$; therefore, $\lambda(1,1) = 0$, $\lambda(1,5) = \sqrt{4/5}$, and $\lambda(5,5) = (\sqrt{8}/5)$. Thus, by Lemma 2 and after simplification,

$$\mathfrak{R}\mathbb{ABC}(G^3_{p,q}) = \left(\frac{12\sqrt{5}}{5} + \frac{6\sqrt{2}}{5}\right)pq - \frac{8\sqrt{5}}{5} + \frac{2(4p+2q+2)\sqrt{2}}{5}. \quad (50)$$

For $\mathfrak{R}\mathrm{GA}(G^3_{p,q})$ which is the reverse geometric-arithmetic index of $G^3_{p,q}$, from equation (4), we have $\lambda(\mathfrak{R}_\theta, \mathfrak{R}_\vartheta) = (2\sqrt{\mathfrak{R}_\theta \times \mathfrak{R}_\vartheta}/(\mathfrak{R}_\theta + \mathfrak{R}_\vartheta))$; therefore, $\lambda(1,1) = 1$, $\lambda(1,5) = (\sqrt{5}/3)$, and $\lambda(5,5) = 1$. Thus, by Lemma 2 and after simplification,

$$\mathfrak{R}\mathrm{GA}(G^3_{p,q}) = (6 + 2\sqrt{5})pq + 4 - \frac{4\sqrt{5}}{3}. \quad (51)$$

For $\mathfrak{R}\mathbb{M}_1(G^3_{p,q})$ which is the first reverse Zagreb index of $G^3_{p,q}$, from equation (5), we have $\lambda(\mathfrak{R}_\theta, \mathfrak{R}_\vartheta) = (\mathfrak{R}_\theta + \mathfrak{R}_\vartheta)$; therefore, $\lambda(1,1) = 2$, $\lambda(1,5) = 6$, and $\lambda(5,5) = 10$. Thus, by Lemma 2 and after simplification,

$$\mathfrak{R}\mathbb{M}_1\left(G^3_{p,q}\right)) = 72pq + 32p + 16q. \quad (52)$$

For $\mathfrak{R}\mathbb{HM}(G^3_{p,q})$ which is the first reverse hyper-Zagreb index of $G^3_{p,q}$, from equation (6), we have $\lambda(\mathfrak{R}_\theta, \mathfrak{R}_\vartheta) = (\mathfrak{R}_\theta + \mathfrak{R}_\vartheta)^2$; therefore, $\lambda(1,1) = 4$, $\lambda(1,5) = 36$, and $\lambda(5,5) = 100$. Thus, by Lemma 2 and after simplification,

$$\mathfrak{R}\mathbb{HM}\left(G^3_{p,q}\right) = 528pq + 384p + 192q + 64. \quad (53)$$

For $\mathfrak{R}\mathbb{F}(G^3_{p,q})$ which is the reverse forgotten index of $G^3_{p,q}$, from equation (7), we have $\lambda(\mathfrak{R}_\theta, \mathfrak{R}_\vartheta) = ((\mathfrak{R}_\theta)^2 + (\mathfrak{R}_\vartheta)^2)$; therefore, $\lambda(1,1) = 2$, $\lambda(1,5) = 26$, and $\lambda(5,5) = 50$. Thus, by Lemma 2 and after simplification,

$$\mathfrak{R}\mathbb{F}\left(G^3_{p,q}\right) = 312pq + 192p + 96q. \quad (54)$$

□

Theorem 6. *Let $G^3_{p,q}$ be a third-type pth chain hex-derived networks. Then, the first reverse redefined index is*

$$\mathfrak{R}\mathbb{RZ}_1\left(G^3_{p,q}\right) = \frac{72pq}{5} - \frac{32p}{5} - \frac{16q}{5}. \quad (55)$$

The second reverse redefined index is

$$\mathfrak{R}\mathbb{RZ}_2\left(G^3_{p,q}\right) = 14pq + 8p + 4q + \frac{8}{3}. \quad (56)$$

The third reverse redefined index is

$$\mathfrak{R}\mathbb{RZ}_3\left(G^3_{p,q}\right) = 936pq + 992p + 496q + 384. \quad (57)$$

Proof. For $\mathfrak{R}\mathbb{RZ}_1(G^3_{p,q})$ which is the first reverse redefined index of $G^3_{p,q}$, from equation (8), we have $\lambda(\mathfrak{R}_\theta, \mathfrak{R}_\vartheta) = (\mathfrak{R}_\theta + \mathfrak{R}_\vartheta)^1(\mathfrak{R}_\theta \times \mathfrak{R}_\vartheta)^{-1}$; therefore, $\lambda(1,1) = 2$, $\lambda(1,5) = (6/5)$, and $\lambda(5,5) = (2/5)$. Thus, by Lemma 2 and after simplification,

$$\mathfrak{R}\mathbb{RZ}_1\left(G^3_{p,q}\right) = \frac{72pq}{5} - \frac{32p}{5} - \frac{16q}{5}. \quad (58)$$

For $\mathfrak{R}\mathbb{RZ}_2(G^3_{p,q})$ which is the second reverse redefined index of $G^3_{p,q}$, from equation (8), we have $\lambda(\mathfrak{R}_\theta, \mathfrak{R}_\vartheta) = (\mathfrak{R}_\theta + \mathfrak{R}_\vartheta)^{-1}(\mathfrak{R}_\theta \times \mathfrak{R}_\vartheta)^1$; therefore, $\lambda(1,1) = (1/2)$, $\lambda(1,5) = (5/6)$, and $\lambda(5,5) = (5/2)$. Thus, by Lemma 2 and after simplification,

$$\mathfrak{R}\mathbb{RZ}_2\left(G^3_{p,q}\right) = 14pq + 8p + 4q + \frac{8}{3}. \quad (59)$$

For $\mathfrak{R}\mathbb{RZ}_3(G^3_{p,q})$ which is the third reverse redefined index of $G^3_{p,q}$, from equation (8), we have $\lambda(\mathfrak{R}_\theta, \mathfrak{R}_\vartheta) = (\mathfrak{R}_\theta + \mathfrak{R}_\vartheta)(\mathfrak{R}_\theta \times \mathfrak{R}_\vartheta)$; therefore, $\lambda(1,1) = 2$, $\lambda(1,5) = 30$, and $\lambda(5,5) = 250$. Thus, by Lemma 2 and after simplification,

$$\mathfrak{R}\mathbb{RZ}_3\left(G^3_{p,q}\right) = 936pq + 992p + 496q + 384. \quad (60)$$

□

3.3. Results for Case 3. We provide a formula that would be used to calculate any reverse degree topological descriptors of Case 3 for $G^3_{p,q}$.

Lemma 3. *Let $G^3_{p,q}$ be a third-type pth chain hex-derived networks. Then,*

$$\begin{aligned}\mathcal{T}_3\left(G^3_{p,q}\right) &= 3pq(\lambda(1,1) + 2\lambda(1,5) + \lambda(5,5)) \\ &\quad + 2(\lambda(5,5) - \lambda(1,1))(2p+q) \\ &\quad + 2(\lambda(1,1) - 4\lambda(1,5) + 3\lambda(5,5)).\end{aligned} \quad (61)$$

Proof. The graph $G^3_{p,q}$ contains $12pq$ edges, and maximum degree in $G^3_{p,q}$ graph is 8. There are two types of reverse degree vertices in $G^3_{p,q}$ that are 1 and 5. Let us partition the edges of $G^3_{p,q}$ according to its reverse degrees according to Case 3 as

$$\begin{aligned}\mathbf{E}_{1,1} &= \left\{\theta\vartheta \in E\left(G^3_{p,q}\right): \mathfrak{R}_\theta = 1, \mathfrak{R}_\vartheta = 1\right\}, \\ \mathbf{E}_{1,5} &= \left\{\theta\vartheta \in E\left(G^3_{p,q}\right): \mathfrak{R}_\theta = 1, \mathfrak{R}_\vartheta = 5\right\}, \\ \mathbf{E}_{5,5} &= \left\{\theta\vartheta \in E\left(G^3_{p,q}\right): \mathfrak{R}_\theta = 5, \mathfrak{R}_\vartheta = 5\right\}.\end{aligned} \quad (62)$$

Note that $E(G^3_{p,q}) = \mathbf{E}_{1,1} \cup \mathbf{E}_{1,5} \cup \mathbf{E}_{5,5}$ and $|\mathbf{E}_{1,1}| = 3pq - 4p - 2q + 2$, $|\mathbf{E}_{1,5}| = 6pq - 8$, and $|\mathbf{E}_{5,5}| = 3pq + 4p + 2q + 6$. Hence,

$$\begin{aligned}\mathcal{T}_1\left(G^3_{p,q}\right) &= \sum_{\theta\vartheta \in E\left(G^3_{p,q}\right)} \lambda(\mathfrak{R}_\theta, \mathfrak{R}_\vartheta) \\ &= \sum_{\theta\vartheta \in \mathbf{E}_{1,1}} \lambda(1,1) + \sum_{\theta\vartheta \in \mathbf{E}_{1,5}} \lambda(1,5) + \sum_{\theta\vartheta \in \mathbf{E}_{5,5}} \lambda(5,5) \\ &= (3pq - 4p - 2q + 2)\lambda(1,1) + (6pq - 8)\lambda(1,5) \\ &\quad + (3pq + 4p + 2q + 6)\lambda(5,5).\end{aligned} \quad (63)$$

After simplification, we obtain

$$\begin{aligned}\mathcal{T}_3\left(G^3_{p,q}\right) &= 3pq(\lambda(1,1) + 2\lambda(1,5) + \lambda(5,5)) \\ &\quad + 2(\lambda(5,5) - \lambda(1,1))(2p+q) + 2(\lambda(1,1) \\ &\quad - 4\lambda(1,5) + 3\lambda(5,5)).\end{aligned} \quad (64)$$

□

Theorem 7. *The general reverse Randić index of $G^3_{p,q}$ is equal to*

$$\Re\mathbb{R}_{\alpha}\left(G_{p,q}^{3}\right)=\begin{cases}108pq+96p+48q+112, & \text{for } \alpha=1,\\ (18+6\sqrt{5})pq+16p+8q+32-8\sqrt{5}, & \text{for } \alpha=\dfrac{1}{2},\\ \left(\dfrac{18}{5}+\dfrac{6\sqrt{5}}{5}\right)pq-\dfrac{16p}{5}-\dfrac{8}{5}q+\dfrac{16}{5}-\dfrac{8\sqrt{5}}{5}, & \text{for } \alpha=\dfrac{-1}{2},\\ \dfrac{108pq}{25}-\dfrac{96p}{25}-\dfrac{48q}{25}+\dfrac{16}{25}, & \text{for } \alpha=-1.\end{cases} \tag{65}$$

Proof. For $\Re\mathbb{R}_{\alpha}(G_{p,q}^{3})$ which is the general reverse Randić index of $G_{p,q}^{3}$, from equation (2), we have $\lambda(\Re_{\theta},\Re_{\vartheta})=(\Re_{\theta}\times\Re_{\vartheta})^{\alpha}$; therefore, $\lambda(1,1)=1$, $\lambda(1,5)=(5)^{\alpha}$, and $\lambda(5,5)=(25)^{\alpha}$. Thus, by Lemma 3,

$$\Re\mathbb{R}_{\alpha}\left(G_{p,q}^{3}\right))=3pq\left(1+2(5)^{\alpha}+(25)^{\alpha}\right)+2\left((25)^{\alpha}-1\right)(2p+q)+2\left(1-4(5)^{\alpha}+3(25)^{\alpha}\right). \tag{66}$$

Put $\alpha=1$, and we have

$$\Re\mathbb{R}_{1}\left(G_{p,q}^{3}\right))=108pq+96p+48q+112. \tag{67}$$

Put $\alpha=(1/2)$, and we have

$$\Re\mathbb{R}_{(1/2)}\left(G_{p,q}^{3}\right))=(18+6\sqrt{5})pq+16p+8q+32-8\sqrt{5}. \tag{68}$$

Put $\alpha=(-1/2)$, and we have

$$\Re\mathbb{R}_{(-1/2)}\left(G_{p,q}^{3}\right))=\left(\frac{18}{5}+\frac{6\sqrt{5}}{5}\right)pq-\frac{16p}{5}-\frac{8}{5}q+\frac{16}{5}-\frac{8\sqrt{5}}{5}. \tag{69}$$

Put $\alpha=-1$, and we have

$$\Re\mathbb{R}_{-1}\left(G_{p,q}^{3}\right))=\frac{108pq}{25}-\frac{96p}{25}-\frac{48q}{25}+\frac{16}{25}. \tag{70}$$

□

Theorem 8. *Let $G_{p,q}^{3}$ be a third-type p th chain hex-derived networks. Then, the reverse atom-bond connectivity index is*

$$\Re\mathbb{ABC}\left(G_{p,q}^{3}\right)=\left(\frac{12\sqrt{5}}{5}+\frac{6\sqrt{2}}{5}\right)pq-\frac{16\sqrt{5}}{5}+\frac{2(4p+2q+6)\sqrt{2}}{5}. \tag{71}$$

The reverse geometric-arithmetic index is

$$\Re\mathrm{GA}\left(G_{p,q}^{3}\right)=(6+2\sqrt{5})pq+8-\frac{8\sqrt{5}}{3}. \tag{72}$$

The first reverse Zagreb index is

$$\Re\mathbb{M}_{1}\left(G_{p,q}^{3}\right))=72pq+32p+16q+16. \tag{73}$$

The reverse hyper-Zagreb index is

$$\Re\mathbb{HM}\left(G_{p,q}^{3}\right)=528pq+384p+192q+320. \tag{74}$$

The reverse forgotten index is

$$\Re\mathbb{F}\left(G_{p,q}^{3}\right)=312pq+192p+96q+96. \tag{75}$$

Proof. For $\Re\mathbb{ABC}(G_{p,q}^{3})$ which is the reverse atom-bond connectivity index of $G_{p,q}^{3}$, from equation (3), we have $\lambda(\Re_{\theta},\Re_{\vartheta})=\sqrt{\Re_{\theta}+\Re_{\vartheta}-2/\Re_{\theta}\times\Re_{\vartheta}}$; therefore, $\lambda(1,1)=0$, $\lambda(1,5)=\sqrt{4/5}$, and $\lambda(5,5)=(\sqrt{8}/5)$. Thus, by Lemma 3 and after simplification,

$$\Re\mathbb{ABC}\left(G_{p,q}^{3}\right)=\left(\frac{12\sqrt{5}}{5}+\frac{6\sqrt{2}}{5}\right)pq-\frac{16\sqrt{5}}{5}+\frac{2(4p+2q+6)\sqrt{2}}{5}. \tag{76}$$

For $\Re\mathrm{GA}(G_{p,q}^{3})$ which is the reverse geometric-arithmetic index of $G_{p,q}^{3}$, from equation (4), we have $\lambda(\Re_{\theta},\Re_{\vartheta})=(2\sqrt{\Re_{\theta}\times\Re_{\vartheta}}/\Re_{\theta}+\Re_{\vartheta})$; therefore, $\lambda(1,1)=1$, $\lambda(1,5)=(\sqrt{5}/3)$, and $\lambda(5,5)=1$. Thus, by Lemma 3 and after simplification,

$$\Re\mathrm{GA}\left(G_{p,q}^{3}\right)=(6+2\sqrt{5})pq+8-\frac{8\sqrt{5}}{3}. \tag{77}$$

Table 1: Numerical comparison of $\mathfrak{R}\mathbb{ABC}$, $\mathfrak{R}$GA, $\mathfrak{R}\mathbb{M}_1$, $\mathfrak{R}\mathbb{H}$M, and $\mathfrak{R}\mathbb{F}$ for Case 1.

[p, q]	$\mathfrak{R}\mathbb{ABC}$	$\mathfrak{R}$GA	$\mathfrak{R}\mathbb{M}_1$	$\mathfrak{R}\mathbb{H}$M	$\mathfrak{R}\mathbb{F}$
[1, 1]	8.0114	11.491	120	1168	600
[2, 2]	32.597	42.907	384	3328	1824
[3, 3]	71.309	95.269	792	6544	3672
[4, 4]	124.15	168.57	1344	10816	6144
[5, 5]	191.12	262.82	2040	16144	9240
[6, 6]	272.21	378.02	2880	22528	12960
[7, 7]	367.43	514.16	3864	29968	17304
[8, 8]	476.78	671.25	4992	38464	22272
[9, 9]	600.26	849.27	6264	48016	27864
[10, 10]	737.87	1048.2	7680	58624	34080

Table 2: Numerical comparison of $\mathfrak{R}\mathbb{ABC}$, $\mathfrak{R}$GA, $\mathfrak{R}\mathbb{M}_1$, $\mathfrak{R}\mathbb{H}$M, and $\mathfrak{R}\mathbb{F}$ for Case 2.

[p, q]	$\mathfrak{R}\mathbb{ABC}$	$\mathfrak{R}$GA	$\mathfrak{R}\mathbb{M}_1$	$\mathfrak{R}\mathbb{H}$M	$\mathfrak{R}\mathbb{F}$
[5, 6]	227.56	315.19	2416	18976	10896
[5, 7]	264.01	367.55	2792	21808	12552
[5, 8]	300.47	419.91	3168	24640	14208
[4, 4]	124.15	168.57	1344	10816	6144
[6, 6]	272.21	378.02	2880	22528	12960
[8, 8]	476.78	671.25	4992	38464	22272
[9, 8]	535.55	755.02	5600	43072	24960
[9, 7]	470.85	660.77	4936	38128	22056
[10, 8]	594.32	838.80	6208	47680	27648
[12, 10]	883.66	1257.7	9184	69952	40704

Table 3: Numerical comparison of $\mathfrak{R}\mathbb{ABC}$, $\mathfrak{R}$GA, $\mathfrak{R}\mathbb{M}_1$, $\mathfrak{R}\mathbb{H}$M, and $\mathfrak{R}\mathbb{F}$ for Case 3.

[p, q]	$\mathfrak{R}\mathbb{ABC}$	$\mathfrak{R}$GA	$\mathfrak{R}\mathbb{M}_1$	$\mathfrak{R}\mathbb{H}$M	$\mathfrak{R}\mathbb{F}$
[4, 5]	152.22	211.48	1664	13376	7584
[6, 7]	314.41	441.87	3344	26144	15024
[8, 9]	533.10	756.03	5600	43136	24960
[10, 11]	808.31	1154.0	8432	64352	37392
[12, 13]	1140.0	1635.7	11840	89792	52320
[4, 3]	93.448	127.70	1056	8768	4896
[6, 5]	227.38	316.20	2448	19424	11088
[8, 7]	417.83	588.47	4416	34304	19776
[10, 9]	664.78	944.53	6960	53408	30960
[12, 11]	968.24	1384.4	10080	76736	44640

For $\mathfrak{R}\mathbb{M}_1(G^3_{p,q})$ which is the first reverse Zagreb index of $G^3_{p,q}$, from equation (5), we have $\lambda(\mathfrak{R}_\theta, \mathfrak{R}_\vartheta) = (\mathfrak{R}_\theta + \mathfrak{R}_\vartheta)$; therefore, $\lambda(1,1) = 2$, $\lambda(1,5) = 6$, and $\lambda(5,5) = 10$. Thus, by Lemma 3 and after simplification,

$$\mathfrak{R}\mathbb{M}_1\left(G^3_{p,q}\right)) = 72pq + 32p + 16q + 16. \tag{78}$$

For $\mathfrak{R}\mathbb{H}\mathrm{M}(G^3_{p,q})$ which is the first reverse hyper-Zagreb index of $G^3_{p,q}$, from equation (6), we have $\lambda(\mathfrak{R}_\theta, \mathfrak{R}_\vartheta) = (\mathfrak{R}_\theta + \mathfrak{R}_\vartheta)^2$; therefore, $\lambda(1,1) = 4$, $\lambda(1,5) = 36$, and $\lambda(5,5) = 100$. Thus, by Lemma 3 and after simplification,

$$\mathfrak{R}\mathbb{H}\mathrm{M}\left(G^3_{p,q}\right) = 528pq + 384p + 192q + 320. \tag{79}$$

For $\mathfrak{R}\mathbb{F}(G^3_{p,q})$ which is the reverse forgotten index of $G^3_{p,q}$, from equation (7), we have $\lambda(\mathfrak{R}_\theta, \mathfrak{R}_\vartheta) = ((\mathfrak{R}_\theta)^2 + (\mathfrak{R}_\vartheta)^2)$; therefore, $\lambda(1,1) = 2$, $\lambda(1,5) = 26$ and $\lambda(5,5) = 50$. Thus, by Lemma 3 and after simplification,

$$\mathfrak{R}\mathbb{F}\left(G^3_{p,q}\right) = 312pq + 192p + 96q + 96. \tag{80}$$

□

Theorem 9. *Let $G^3_{p,q}$ be a third-type pth chain hex-derived networks. Then, the first reverse redefined index is*

$$\mathfrak{R}\mathbb{R}\mathbb{Z}_1\left(G^3_{p,q}\right) = \frac{72pq}{5} - \frac{32p}{5} - \frac{16q}{5} - \frac{16}{5}. \tag{81}$$

The second reverse redefined index is

$$\mathfrak{R}\mathbb{R}\mathbb{Z}_2\left(G^3_{p,q}\right) = 14pq + 8p + 4q + \frac{28}{3}. \tag{82}$$

The third reverse redefined index is

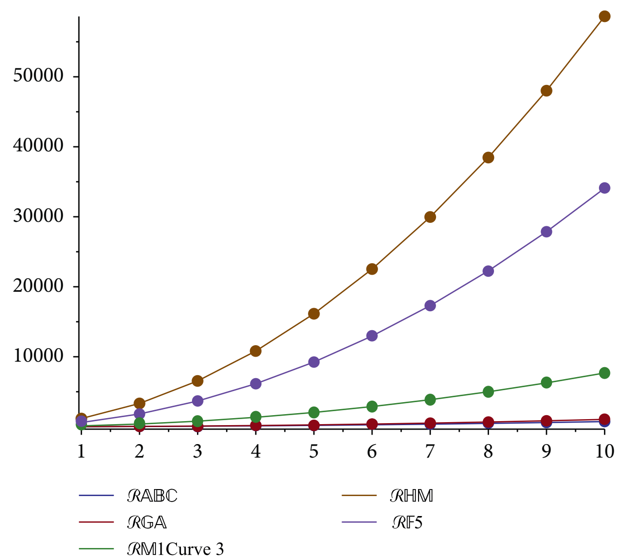

Figure 4: Graphical representation of Table 1.

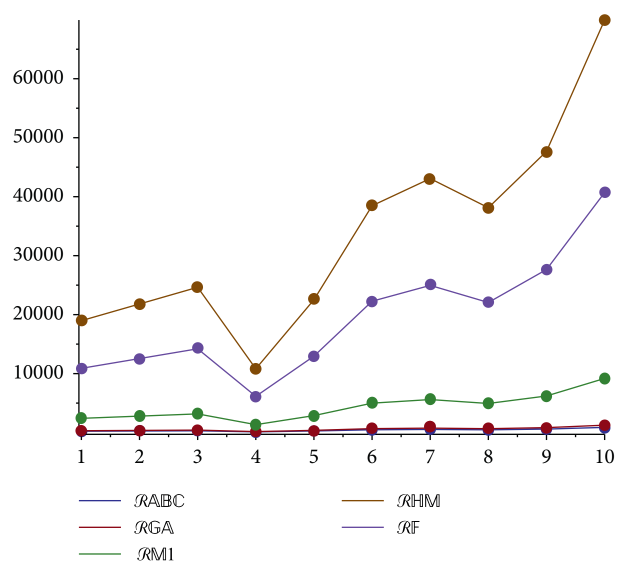

Figure 5: Graphical representation of Table 2.

$$\mathfrak{R}\mathbb{R}\mathbb{Z}_3\left(G^3_{p,q}\right) = 936pq + 992p + 496q + 1264. \qquad (83)$$

Proof. For $\mathfrak{R}\mathbb{R}\mathbb{Z}_1(G^3_{p,q})$ which is the first reverse redefined index of $G^3_{p,q}$, from equation (8), we have $\lambda(\mathfrak{R}_\theta, \mathfrak{R}_\vartheta) = (\mathfrak{R}_\theta + \mathfrak{R}_\vartheta)^1 (\mathfrak{R}_\theta \times \mathfrak{R}_\vartheta)^{-1}$; therefore, $\lambda(1,1) = 2$, $\lambda(1,5) = (6/5)$, and $\lambda(5,5) = (2/5)$. Thus, by Lemma 3 and after simplification,

$$\mathfrak{R}\mathbb{R}\mathbb{Z}_1\left(G^3_{p,q}\right) = \frac{72pq}{5} - \frac{32p}{5} - \frac{16q}{5} - \frac{16}{5}. \qquad (84)$$

For $\mathfrak{R}\mathbb{R}\mathbb{Z}_2(G^3_{p,q})$ which is the second reverse redefined index of $G^3_{p,q}$, from equation (8), we have $\lambda(\mathfrak{R}_\theta, \mathfrak{R}_\vartheta) = (\mathfrak{R}_\theta + \mathfrak{R}_\vartheta)^{-1} (\mathfrak{R}_\theta \times \mathfrak{R}_\vartheta)^1$; therefore, $\lambda(1,1) = (1/2)$, $\lambda(1,5) = (5/6)$, and $\lambda(5,5) = (5/2)$. Thus, by Lemma 3 and after simplification,

$$\mathfrak{R}\mathbb{R}\mathbb{Z}_2\left(G^3_{p,q}\right) = 14pq + 8p + 4q + \frac{28}{3}. \qquad (85)$$

For $\mathfrak{R}\mathbb{R}\mathbb{Z}_3(G^3_{p,q})$ which is the third reverse redefined index of $G^3_{p,q}$, from equation (8), we have $\lambda(\mathfrak{R}_\theta, \mathfrak{R}_\vartheta) = (\mathfrak{R}_\theta + \mathfrak{R}_\vartheta)(\mathfrak{R}_\theta \times \mathfrak{R}_\vartheta)$; therefore, $\lambda(1,1) = 2$, $\lambda(1,5) = 30$, and $\lambda(5,5) = 250$. Thus, by Lemma 3 and after simplification,

$$\mathfrak{R}\mathbb{R}\mathbb{Z}_3\left(G^3_{p,q}\right) = 936pq + 992p + 496q + 1264. \qquad (86)$$

□

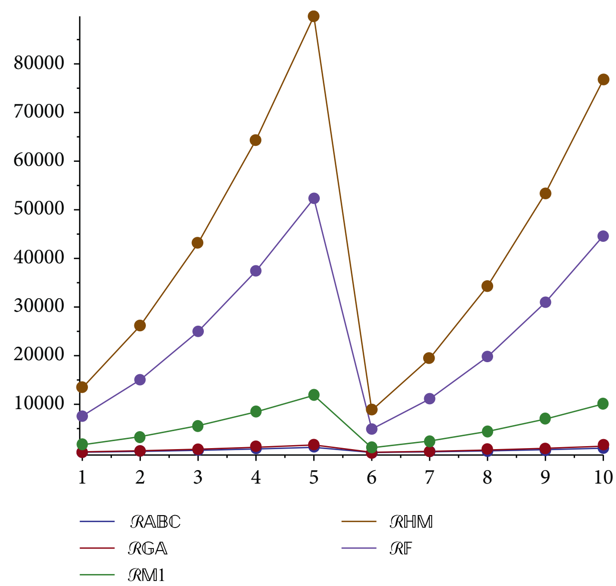

Figure 6: Graphical representation of Table 3.

4. Numerical and Graphical Representation

In this section, we determine the numerical values of $\mathfrak{R}\mathbb{ABC}$, $\mathfrak{R}$GA, $\mathfrak{R}\mathbb{M}_1$, $\mathfrak{R}\mathbb{H}$M, and $\mathfrak{R}\mathbb{F}$ in Tables 1–3, for Case 1, Case 2, and Case 3, respectively. We represent these results graphically in Figures 4–6.

Data Availability

No data were used to support the findings of the study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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