Research Article

Sliding Mode Matrix-Projective Synchronization for Fractional-Order Neural Networks

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This work generalizes the projection scaling factor to a general constant matrix and proposes the matrix-projection synchronization (MPS) for fractional-order neural networks (FNNs) based on sliding mode control firstly. This kind of scaling factor is far more complex than the constant scaling factor, and it is highly variable and difficult to predict in the process of realizing the synchronization for the driving and response systems, which can ensure high security and strong confidentiality. Then, the fractional-order integral sliding surface and sliding mode controller for FNNs are designed. Furthermore, the criterion for realizing MPS is proved, and the reachability and stability of the synchronization error system are analyzed, so that the global MPS is realized for FNNs. Finally, a numerical application is given to demonstrate the feasibility of theory analysis. MPS is more general, so it is reduced to antisynchronization, complete synchronization, projective synchronization (PS), and modified PS when selecting different projective matrices. This work will enrich the synchronization theory of FNNs and provide a feasible method to study the MPS of other fractional-order dynamical models.

1. Introduction

Neural network is an important part of artificial intelligence, which is composed of a large number of highly connected neurons. It is a mathematical model based on the preliminary understanding of the physiological structure and activity mechanism of the brain. The neural network has the unique knowledge representation structure and can process information, learn, and adapt to the unknown system efficiently and quickly, which provide new research ideas for control problems and intelligent information processing. Integer order differential equations cannot describe the memory properties of neurons and the dependence on past history, but the fractional-order calculus [1–4], which has strong memory and hereditary characteristic, contains all of the information from the start point to the current moment and can describe the memory properties and dynamical behaviors of neurons more accurately. Therefore, FNNs can improve the computational ability of neurons, speed up the

information transmission of neurons, and solve the problem of parameter identification effectively. With the development of fractional-order calculus, FNNs are becoming more and more popular and their dynamical behaviors have been widely investigated, such as stability [5–8], bifurcations [9], chaos and hyperchaos [10], and synchronization [11–14].

Two systems are called PS when the drive and response systems are synchronized to a scaling factor. Recently, scholars have researched the PS of FNNs and achieved a lot of valuable results. In [15–17], by using different control methods, the authors have researched the PS for FNNs. In [18], Zhang et al. have used the adaptive control method to achieve the PS of FNNs in quaternion field. In [19–21], researchers have explored the PS for fractional-order memristive neural networks with different characters. In [22–24], the authors extend the FNNs to the complex domain and study their PS and quasiprojective synchronization by using different control strategies. In [25], Ding and Shen constructed the fractional-order integral sliding mode

surface, designed fractional-order sliding mode controller, and realized PS for two FNNs with different structures. In [26, 27], by means of the fractional Lyapunov-like method, the authors realized PS in finite-time and mixed $H\infty$ /passive projective synchronization for nonidentical FNNs via a new sliding mode controller. In [28], by using sliding mode control, Wu et al. realized the finite-time interlayer PS of fractional-order two-layer networks based on Caputo derivatives.

To the best of our knowledge, the scaling factor of PS in most research studies is a diagonal matrix or a fixed constant, but in fact this kind of scaling factor may not ensure high security of communication. Our work will generalize the proportion factor to a general constant matrix, which is far more complex than the constant scaling factor. Also, it is highly variable and difficult to predict in the process of realizing the synchronization for the driving and response systems, which can ensure the high security and strong confidentiality. Based on the characteristics of matrix scaling factor, our work presents a new kind of MPS for FNNs, whose complexity and unpredictability can effectively increase the difficulty for hackers to track the right path, improve the antiattack capability of the system, and enhance the confidentiality of secure communication.

As everyone knows, synchronization of FNNs can be reached via various control methods. Especially, sliding mode control strategy has many important and special advantages, which include low sensitivity to the parameter perturbation and external disturbance, implementation simplicity, and fast response. In our work, because of the complexity of the MPS and nonidentical FNNs, it greatly increases the difficulty of control in actual operation. Hence, it is extremely necessary to use fractional-order sliding mode control strategy to research the global MPS.

According to the above discussion and main research content, our work is divided into the following chapters. Section 2 introduces some lemmas and establishes the FNNs. In Section 3, by using the fractional-order sliding mode control method, MPS is defined and sufficient criterion is proved. As applications, a numerical application is given to demonstrate theory analysis in Section 4. Section 5 concludes the whole work.

2. Preliminaries

Definition 1 (see [1]). Fractional integral for function $f: [t_0, +\infty) \rightarrow R$ is

$${}_t I_t^\alpha f(t) = \frac{1}{\Gamma(\alpha)} \int_{t_0}^t (t-\xi)^{\alpha-1} f(\xi) d\xi, \quad (\alpha > 0, t \geq t_0), \quad (1)$$

where $\Gamma(\alpha) = \int_0^{+\infty} e^{-t} t^{\alpha-1} dt$ is gamma function.

Definition 2 (see [1]). Caputo derivative for a function $f(t) \in C^n([t_0, +\infty), R)$ is defined as

$${}_t^C D_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \int_{t_0}^t \frac{f^{(n)}(\xi)}{(t-\xi)^{\alpha-n+1}} d\xi, \quad (\alpha > 0, t \geq t_0), \quad (2)$$

where n is the positive integer satisfying $n-1 < \alpha < n$, $f^{(n)}$ is the n -th-order derivative of $f(x)$, and $C^n([t_0, +\infty), R)$ is the space which is composed of n order continuous differentiable functions from $[t_0, +\infty)$ to R . In particular, when $0 < \alpha < 1$, ${}_t^C D_t^\alpha f(t) = (1/\Gamma(1-\alpha)) \int_{t_0}^t (f'(\xi)/(t-\xi)^\alpha) d\xi$; when $\alpha = 1$, the Caputo derivative operation ${}_t^C D_t^\alpha f(t)$ is identified with integer order ones ($df(t)/dt$).

Lemma 1. If Caputo derivative ${}_t^C D_t^\alpha f(t)$ is integrable, then

$${}_t^C I_t^\alpha ({}_t^C D_t^\alpha f(t)) = f(t) - \sum_{k=0}^{n-1} \frac{f^{(k)}(t_0)}{k!} (t-t_0)^k. \quad (3)$$

Especially, for $0 < \alpha < 1$, one can obtain ${}_t^C I_t^\alpha ({}_t^C D_t^\alpha f(t)) = f(t) - f(t_0)$.

Lemma 2. For Caputo derivative ${}_t^C D_t^\alpha f(t)$, the following equality holds:

$${}_t^C D_t^{1-\alpha} {}_t^C D_t^\alpha f(t) = {}_t^C D_t^1 f(t) = \frac{df(t)}{dt}. \quad (4)$$

Lemma 3. If the constant $C \neq 0$, then ${}_t^C D_t^\alpha C = 0$.

Lemma 4. If function $f(t) \in L^\infty(t_0, t)$, for $\alpha > 0$ and $\beta > 0$, then

$${}_t^C D_t^{\alpha+C} {}_t^C I_t^\beta f(t) = {}_t^C D_t^{\alpha-\beta} f(t), \quad (5)$$

when $\alpha = \beta$,

$${}_t^C D_t^\alpha ({}_t^C I_t^\alpha f(t)) = f(t). \quad (6)$$

Lemma 5 (see [2]). For any time instant $t \geq 0$, if $x(t) \in R$ is continuously differentiable, then the inequality $(1/2) {}_a^C D_t^\alpha [x^2(t)] \leq x(t) {}_a^C D_t^\alpha x(t)$ ($0 < \alpha < 1$) holds; if $x(t) \in R^n$, the inequality $(1/2) {}_a^C D_t^\alpha x^T(t)x(t) \leq x^T(t) {}_a^C D_t^\alpha x(t)$ also holds.

Lemma 6 (see [6]). If $f(t) \in C^1([0, +\infty), R)$ is a continuously differentiable function, the inequality

$${}_0^C D_t^\alpha |f(t^+)|_{a.e.} \leq \operatorname{sgn}(f(t)) {}_0^C D_t^\alpha f(t), \quad (7)$$

holds almost everywhere, where $0 < \alpha < 1$ and $f(t^+) \triangleq \lim_{\tau \rightarrow t^+} f(\tau)$.

Consider two nonidentical FNNs as the drive and response system:

$$\begin{cases} {}^C_0 D_t^\alpha x(t) = -Cx(t) + Af(x(t)), \\ z(t) = Px(t), \end{cases} \quad (8)$$

$$\begin{cases} {}^C_0 D_t^\alpha y(t) = -Dy(t) + Bg(y(t)) + u(t), \\ \hat{z}(t) = Py(t), \quad (0 < \alpha < 1), \end{cases} \quad (9)$$

where $x(t) = (x_1(t), x_2(t), \dots, x_n(t))^T \in R^n$ and $y(t) = (y_1(t), y_2(t), \dots, y_n(t))^T \in R^n$ denote the state variables and $z(t)$ and $\hat{z}(t)$ are the outputs. $f(x(t)) = (f_1(x_1(t)), f_2(x_2(t)), \dots, f_n(x_n(t)))^T \in R^n$ and $g(y(t)) = (g_1(y_1(t)), g_2(y_2(t)), \dots, g_n(y_n(t)))^T \in R^n$ are neuron activation functions at time t . n is the number of units. $C = \text{diag}(C_1, C_2, \dots, C_n)$ and $D = \text{diag}(D_1, D_2, \dots, D_n)$ are self-regulating parameters of neurons. $A = (A_{ij})_{n \times n}$ and $B = (B_{ij})_{n \times n}$ denote synaptic connection weight matrices. $u(t)$ is the controller to be designed later.

Assumption 1. The continuous neuron activation functions f and g meet Lipschitz condition

$$\begin{aligned} |f(\phi) - f(\varphi)| &< F|\phi - \varphi|, \\ |g(\phi) - g(\varphi)| &< G|\phi - \varphi|, \quad (\phi, \varphi \in R), \end{aligned} \quad (10)$$

where $F, G > 0$ are Lipschitz constants.

3. Main Results

In this part, based on the fractional sliding mode control method, the sliding mode controllers will be designed to research the MPS between systems (8) and (9).

The error function of MPS is

$$e(t) = y(t) - \Lambda x(t), \quad (11)$$

where $e = (e_1, e_2, \dots, e_n)^T$ and $\Lambda = (\Lambda_{ij})_{n \times n}$ is a general constant matrix.

Definition 3. If any two solutions $x(t)$ and $y(t)$ with initial values $x(0)$ and $y(0)$ meet

$$\lim_{t \rightarrow +\infty} \|e(t)\|_1 = \lim_{t \rightarrow +\infty} \|y(t) - \Lambda x(t)\| = 0, \quad (12)$$

systems (8) and (9) are said to be MPS, where $\| \cdot \|_1 = \sum_{i=1}^n$ denotes the Euclidean norm.

Taking Caputo derivative of both sides of error function $e(t) = y(t) - \Lambda x(t)$ and substituting into (8) and (9), the error system can be obtained as

$$\begin{aligned} {}^C_0 D_t^\alpha e(t) &= {}^C_0 D_t^\alpha y(t) - \Lambda {}^C_0 D_t^\alpha x(t) \\ &= -Dy(t) + Bg(y(t)) + u(t) - \Lambda(-Cx(t) + Af(x(t))) \\ &= -D(y(t) - \Lambda x(t)) - D\Lambda x(t) + B(g(y(t)) - g(\Lambda x(t))) + Bg(\Lambda x(t)) + u(t) \\ &\quad - \Lambda(-Cx(t) + Af(x(t))) \end{aligned} \quad (13)$$

$$\begin{aligned} &= -De(t) + B(g(y(t)) - g(\Lambda x(t))) - D\Lambda x(t) + Bg(\Lambda x(t)) \\ &\quad - \Lambda(-Cx(t) + Af(x(t))) + u(t), \end{aligned}$$

$$\bar{z}(t) = Pe(t). \quad (14)$$

3.1. Sliding Mode Controller Design. The design principle of sliding mode control is usually to design a suitable sliding surface as required and then construct a controller to force the system to move on the sliding surface and stay on it forever. First, the fractional integral sliding surface is designed as

$$S(t) = e(t) + {}^C_0 I_t^\alpha [(C + KP)e(t) - B(g(y(t)) - g(\Lambda x(t)))], \quad (15)$$

where $K \in R^{n \times n}$ is the gain matrix. Integrate system (13) from 0 to t , and using Lemma 1, we have

$$\begin{aligned} {}^C_0 I_t^\alpha ({}^C_0 D_t^\alpha e(t)) &= e(t) - e(0) \\ &= {}^C_0 I_t^\alpha [-De(t) + B(g(y(t)) - g(\Lambda x(t))) - D\Lambda x(t) + Bg(\Lambda x(t)) \\ &\quad - \Lambda(-Cx(t) + Af(x(t))) + u(t)], \end{aligned} \quad (16)$$

then

$$e(t) = e(0) + {}^C_0I_t^\alpha [-De(t) + B(g(y(t)) - g(\Lambda x(t))) - D\Lambda x(t) + Bg(\Lambda x(t)) - \Lambda(-Cx(t) + Af(x(t))) + u(t)]. \quad (17)$$

Next, from equations (15) and (17), we have

$$S(t) = e(0) + {}^C_0I_t^\alpha [(C - D + KP)e(t) - D\Lambda x(t) + Bg(\Lambda x(t)) - \Lambda(-Cx(t) + Af(x(t))) + u(t)], \quad (18)$$

and sliding surface (18) and (15) are equivalent. Based on the sliding mode control method, when error system (13) moves on sliding surface, the formulas $S(t) = 0$ and $\dot{S}(t) = 0$ have to be satisfied. Then, one can obtain $\dot{S}(t) = {}^C_0D_t^{1-\alpha} {}^C_0D_t^\alpha S(t) = 0$ by using Lemma 2, which means ${}^C_0D_t^\alpha S(t) = 0$. Using (18) and Lemmas 3 and 4, we obtain

$$\begin{aligned} {}^C_0D_t^\alpha S(t) &= {}^C_0D_t^\alpha e(0) + {}^C_0D_t^\alpha {}^C_0I_t^\alpha [(C - D + KP)e(t) - D\Lambda x(t) + Bg(\Lambda x(t)) \\ &\quad - \Lambda(-Cx(t) + Af(x(t))) + u(t)] \\ &= (C - D + KP)e(t) - D\Lambda x(t) + Bg(\Lambda x(t)) - \Lambda(-Cx(t) + Af(x(t))) + u(t) = 0. \end{aligned} \quad (19)$$

From equation (19), the equivalent sliding mode controller is designed as

$$u_{eq}(t) = -(C - D + KP)e(t) + D\Lambda x(t) - Bg(\Lambda x(t)) + \Lambda(-Cx(t) + Af(x(t))). \quad (20)$$

Substituting controller (20) into (13), the sliding mode error system is described as

$${}^C_0D_t^\alpha e(t) = -(C + KP)e(t) + B(g(y(t)) - g(\Lambda x(t))). \quad (21)$$

Obviously, $e(t) = 0$ is an equilibrium point of (21). In order to make the trajectories of the system reach the sliding surface more efficiently and faster, the reaching law is selected as

$$u_r(t) = -k^* \operatorname{sgn}(S(t)), \quad (22)$$

where the switching gain is $k^* > 0$ and $\operatorname{sgn}(S(t)) = \begin{cases} -1 & S(t) < 0 \\ 0 & S(t) = 0 \\ 1 & S(t) > 0 \end{cases}$. Then, the sliding mode control $u(t)$ is denoted by

$$\begin{aligned} u(t) &= u_{eq}(t) + u_r(t) \\ &= -(C - D + KP)e(t) + D\Lambda x(t) - Bg(\Lambda x(t)) + \Lambda(-Cx(t) + Af(x(t))) - k^* \operatorname{sgn}(S(t)). \end{aligned} \quad (23)$$

Theorem 1. According to sliding switching surface (15) and sliding mode controller (23), synchronization error trajectories (13) asymptotically reach sliding surface $S(t) = 0$.

Proof. Construct a Lyapunov function as

$$V(t) = \frac{1}{2} S^T(t) S(t), \quad (24)$$

taking Caputo derivative of (24) with respect to time t and based on Lemma 5, we obtain

$$\begin{aligned} {}^C_0D_t^\alpha V(t) &\leq S^T(t) {}^C_0D_t^\alpha S(t) \\ &= S^T(t) [(C - D + KP)e(t) - D\Lambda x(t) + Bg(\Lambda x(t)) - \Lambda(-Cx(t) + Af(x(t))) + u(t)] \\ &= S^T(t) [-k^* \operatorname{sgn}(S(t))] \\ &= -k^* \|S(t)\|. \end{aligned} \quad (25)$$

As $k^* > 0$, system trajectories asymptotically converge to $S(t) = 0$, which means synchronization error trajectories (21) reach the predetermined sliding surface globally and stay on it forever. $\square$

3.2. Stability Analysis

Theorem 2. *If Assumption 1 and the inequality*

$$C_i > \sum_{j=1}^n \left(\sum_{l=1}^n |k_{jl} p_{li}| \right) + \sum_{j=1}^n (|b_{ji}| m_i), \quad (26)$$

hold, then the drive system (8) and response system (9) can realize MPS based on controller (23).

Proof. First, error system (21) is converted as

$$\begin{aligned} {}^C_0 D_t^\alpha e_i(t) = & -C_i e_i(t) - \sum_{j=1}^n \left(\sum_{l=1}^n k_{il} p_{lj} \right) e_j(t) \\ & + \sum_{j=1}^n b_{ij} \left(g_j(y_j(t)) - t g_j \left(\sum_{i=1}^n \Lambda_{ji} x_i(t) \right) \right). \end{aligned} \quad (27)$$

Next, design Lyapunov function $V(t) = \|e(t)\|_1 = \sum_{i=1}^n |e_i(t)|$, according to Assumption 1 and Lemma 6, taking Caputo derivative of Lyapunov function $V(t)$ along error trajectories (21), then

$$\begin{aligned} {}^C_0 D_t^\alpha V(t) &= {}^C_0 D_t^\alpha \left(\sum_{i=1}^n |e_i(t)| \right) = \sum_{i=1}^n {}^C_0 D_t^\alpha |e_i(t)| \leq \sum_{i=1}^n \operatorname{sgn}(e_i(t)) {}^C_0 D_t^\alpha e_i(t) \\ &= \sum_{i=1}^n \operatorname{sgn}(e_i(t)) \left(-C_i e_i(t) - \sum_{j=1}^n \left(\sum_{l=1}^n k_{il} p_{lj} \right) e_j(t) + \sum_{j=1}^n b_{ij} \left(g_j(y_j(t)) - g_j \left(\sum_{i=1}^n \Lambda_{ji} x_i(t) \right) \right) \right) \\ &= \sum_{i=1}^n \operatorname{sgn}(e_i(t)) (-C_i e_i(t)) - \sum_{i=1}^n \operatorname{sgn}(e_i(t)) \left(\sum_{j=1}^n \left(\sum_{l=1}^n k_{il} p_{lj} \right) e_j(t) \right) \\ &\quad + \sum_{i=1}^n \operatorname{sgn}(e_i(t)) \left(\sum_{j=1}^n b_{ij} \left(g_j(y_j(t)) - g_j \left(\sum_{i=1}^n \Lambda_{ji} x_i(t) \right) \right) \right) \\ &\leq \sum_{i=1}^n \left[-C_i |e_i(t)| + \sum_{j=1}^n \left(\sum_{l=1}^n |k_{il} p_{lj}| \right) |e_j(t)| + \sum_{j=1}^n |b_{ij}| |G_j| |e_j(t)| \right] \\ &= \sum_{i=1}^n \left[-C_i |e_i(t)| + \sum_{j=1}^n \left(\sum_{l=1}^n |k_{jl} p_{li}| \right) |e_i(t)| + \sum_{j=1}^n |b_{ji}| |G_i| |e_i(t)| \right] \\ &= - \sum_{i=1}^n \left[C_i - \sum_{j=1}^n \left(\sum_{l=1}^n |k_{jl} p_{li}| \right) - \sum_{j=1}^n |b_{ji}| |G_i| \right] |e_i(t)|. \end{aligned} \quad (28)$$

Let $\lambda_i = C_i - \sum_{j=1}^n (\sum_{l=1}^n |k_{jl} p_{li}|) - \sum_{j=1}^n |b_{ji}| |G_i|$, and we can obtain

$$\lambda = \min_{1 \leq i \leq n} \lambda_i = \min_{1 \leq i \leq n} \left(C_i - \sum_{j=1}^n \left(\sum_{l=1}^n |k_{jl} p_{li}| \right) - \sum_{j=1}^n |b_{ji}| |G_i| \right) > 0, \quad (29)$$

then

$${}^C_0 D_t^\alpha V(t) \leq -\lambda V(t) = -\lambda \|e(t)\|_1 \leq 0, \quad (t \geq 0), \quad (30)$$

and $V(t)$ is monotonously nonincreasing. Hence, $V(t) \leq V(0) (t \geq 0)$ and $e_i(t)$ is bounded on $t \geq 0$ from the definition of $V(t)$. There exists a positive constant satisfying $|{}^C_0 D_t^\alpha \|e(t)\|_1| \leq M$; then, we claim that $\lim_{t \rightarrow \infty} \|e(t)\|_1 = 0$ whose proof is similar to Theorem 1 in [16]. Therefore, error system (21) is globally asymptotically stable and MPS between systems (8) and (9) is realized based on controller (23). $\square$

Remark 1. When activating controller (23), response system (9) converts to

$$\begin{aligned} {}^C_0 D_t^\alpha y(t) = & -(C + KP)y(t) + (C\Lambda - \Lambda C)x(t) + kp\Lambda x(t) \\ & + Bg(y(t)) - Bg(\Lambda x(t)) + \Lambda Af(x(t)), \end{aligned} \quad (31)$$

and then based on drive system (8), controlled response system (31), and error system (27), the MPS behaviors between systems (8) and (9) can be analyzed furthermore.

Remark 2. Theorems 1 and 2 are still true and new for $\alpha = 1$.

In particular, the MPS is a kind of more general synchronization. Selecting different projection matrices and controllers, it degenerates to some special synchronization types as Remark 3.

FIGURE 1: Chaotic behaviors of systems (34) and (35) at $\alpha = 0.98$. (a) $x_1 - x_2 - x_3$ phase portrait of system (34). (b) $x_1 - x_2$ phase portrait of system (34). (c) $y_2 - y_1 - y_3$ phase portrait of system (35). (d) $y_2 - y_3$ phase portrait of system (35).

Remark 3

- (1) If $\Lambda = I$, the sliding mode controller becomes

$$\begin{aligned} u(t) &= u_{eq}(t) + u_r(t) \\ &= -(C - D + KP)y(t) + KPx(t) \\ &\quad - Bg(x(t)) + Af(x(t)) - k^* \text{sgn}(S(t)), \end{aligned} \quad (32)$$

where sliding surface $S(t)$ is given by (15), and then systems (8) and (9) can achieve complete synchronization.

- (2) If $\Lambda = -I$, based on sliding surface (15), the sliding mode controller becomes

$$\begin{aligned} u(t) &= u_{eq}(t) + u_r(t) \\ &= -(C - D + KP)y(t) - Bg(-x(t)) \\ &\quad - KPx(t) - Af(x(t)) - k^* \text{sgn}(S(t)), \end{aligned} \quad (33)$$

and then systems (8) and (9) can realize antisynchronization.

- (3) If $\Lambda = cI$ ($c = \text{const}$ and $c \neq \pm 1$), the sliding mode controller is similar to (23) and sliding surface $S(t)$ is given by (15), and then systems (8) and (9) can achieve the PS.
- (4) If $\Lambda = \text{diag}(c_1, c_2, \dots, c_n)$ ($c_i = \text{const}$, $i = 1, 2, \dots, n$) and at least two c_i of them are unequal, the sliding mode controller is similar to (23) and sliding surface $S(t)$ is given by (15), and then systems (8) and (9) can achieve the modified PS.

4. Numerical Application

Two nonidentical drive and response systems are considered as

FIGURE 2: MPS behaviors between systems (34) and (35). Time history of (a) x_1, y_1 ; (b) x_2, y_2 ; (c) x_3, y_3 ; (d) $x_1 + x_2 - 4x_3, y_1$; (e) $-0.5x_1 - 2x_2 + x_3, y_2$; and (f) $-3x_1 - 0.5x_2 - x_3, y_3$.

FIGURE 3: Trajectories of synchronization errors e_1, e_2 , and e_3 .

$$\begin{cases} {}^C_0 D_t^\alpha x(t) = -Cx(t) + Af(x(t)), \\ z(t) = Px(t), \end{cases} \quad (34)$$

$$\begin{cases} {}^C_0 D_t^\alpha y(t) = -Dy(t) + Bg(y(t)) + u(t), \\ \hat{z}(t) = Py(t), \end{cases} \quad (35)$$

where $C = D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, $A = \begin{pmatrix} 2 & -1.2 & 0 \\ 1.8 & 1.71 & 1.15 \\ -4.75 & 0 & 1.1 \end{pmatrix}$, $B = \begin{pmatrix} 2.5 & -1.5 & 0 \\ 2 & 1.7 & 1.2 \\ -4.7 & 0 & 1 \end{pmatrix}$, $P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, $f(x(t)) = \tanh(x(t))$, $g(y(t)) = \tanh(y(t))$, and $u(t)$ is the controller. In order to analyze chaotic synchronization for systems (34) and (35), the value of derivative order is chosen as $\alpha = 0.98$, which causes the two systems to exhibit chaotic behaviors described in Figures 1(a)–1(d) with initial conditions $[x_1(0), x_2(0), x_3(0)] = [y_1(0), y_2(0), y_3(0)] = [0.5, -0.6, 0.4]$.

Projection matrix Λ is chosen as

$$\Lambda = \begin{pmatrix} 1 & 1 & -4 \\ -0.5 & -2 & 1 \\ -3 & -0.5 & -1 \end{pmatrix}, \quad (36)$$

and then error functions are calculated as

$$\begin{aligned} e_1 &= y_1 - \sum_{j=1}^3 \Lambda_{1j} x_j = y_1 - (x_1 + x_2 - 4x_3), \\ e_2 &= y_2 - \sum_{j=1}^3 \Lambda_{2j} x_j = y_2 - (-0.5x_1 - 2x_2 + x_3), \\ e_3 &= y_3 - \sum_{j=1}^3 \Lambda_{3j} x_j = y_3 - (-3x_1 - 0.5x_2 - x_3). \end{aligned} \quad (37)$$

Next, choosing $k_1 = 5, k_2 = 5, k_3 = 5$, and $k^* = 20$ and according to Theorems 1 and 2, the MPS between systems (34) and (35) is realized.

Figures 2(a)–2(c) depict the time trajectories of variables $x_1 \sim y_1$, $x_2 \sim y_2$, and $x_3 \sim y_3$. Simultaneously, it is clearly seen that variables $x_1 + x_2 - 4x_3 \sim y_1$, $-0.5x_1 - 2x_2 + x_3 \sim y_2$, and $-3x_1 - 0.5x_2 - x_3 \sim y_3$ realize the complete synchronization as depicted in Figures 2(d)–2(f). Figure 3 describes that the synchronization error trajectories converge to zero asymptotically for drive system (34) and response system (35). These numerical simulations and figures clearly indicate the applicability and effectiveness of the sliding controller for MPS.

5. Conclusions

For general FNNs, the paper comes up with the global MPS, constructs the fractional-order sliding surface, designs a sliding mode controller, establishes and proves the sufficient condition, and then realizes the global MPS. Our theory analysis provides important theoretical basis and technical support for enhancing signal security by using MPS of FNNs and contributes to the development of artificial intelligence. In future works, we will extend the MPS to the fractional-order memristive neural networks and consider applying them into secret communication.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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